Numerical Unitarity Method for Two-Loop Amplitudes in QCD

based on: [arXiv 1510.05626, HI]

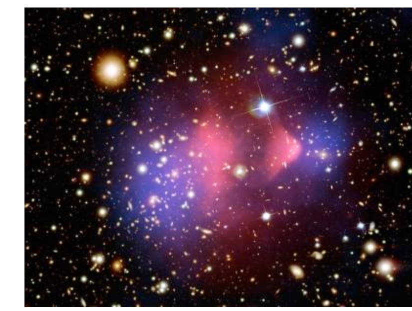
Harald Ita

University of Freiburg, Germany

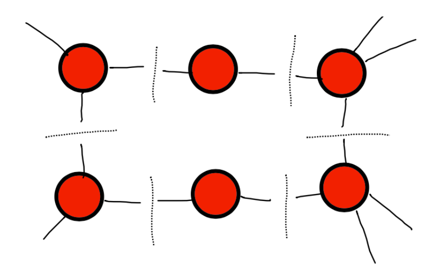
Aspects of Amplitudes program, Nordita
Stockholm, June 27th 2016

Content

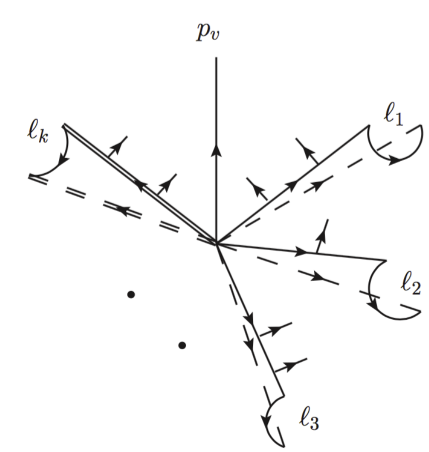
Physics Motivation



Loop Computations



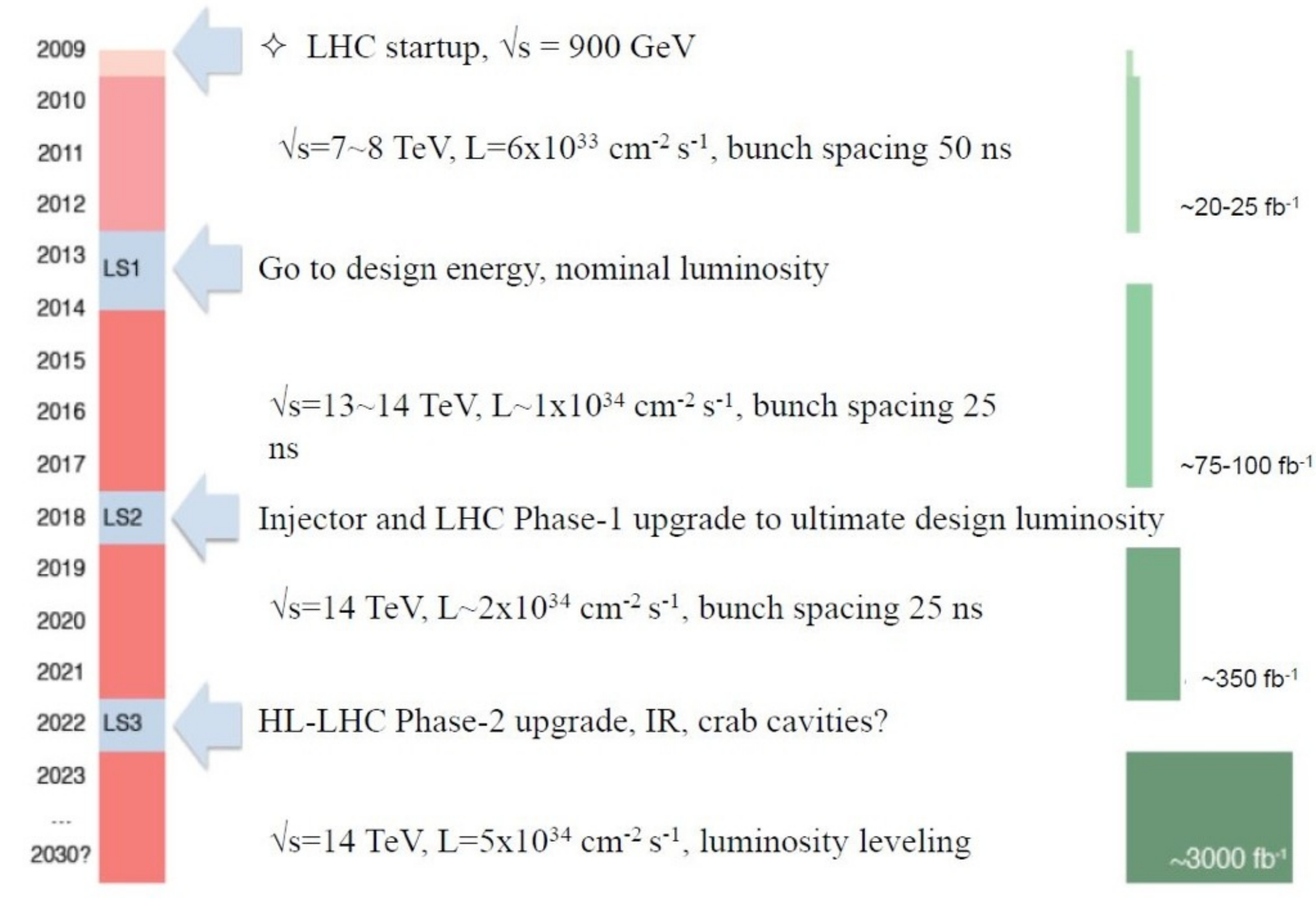
Surface-Terms



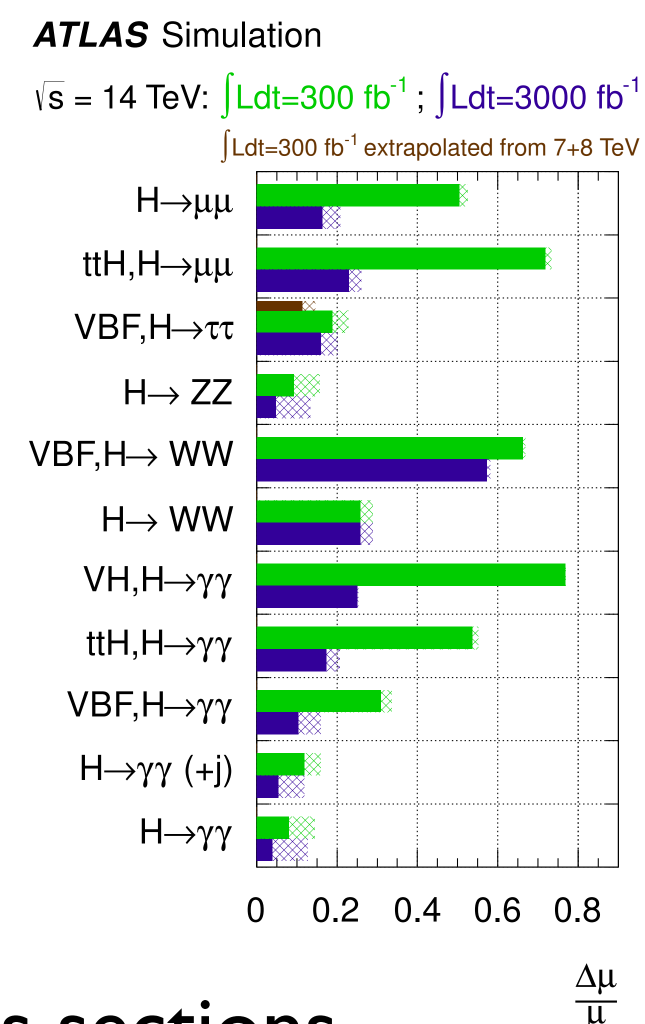
Formal Structures

LHC Timeline

Taken from Rolf Heuer in CERN General Meeting January 2013.



(LS = Long Shutdown)



Potential: new physics & percent-level cross sections.

Theory Input

- Precision theory predictions at the few percent level (input parameters, backgrounds, predictions)
- Broad range of predictions for indirect detection of New Physics
- Go deep by inclusion of multiple final states (new channels, as probes of hard scattering process)
- Complete including of higher-order effects (electroweak, final-state description)

Complex dynamic of proton collisions:

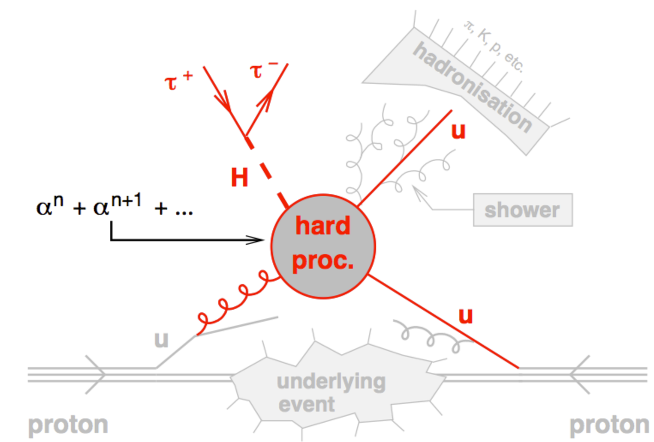


diagram from G. Salam

Precise predictions are a multi-layered problem:

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{ij,NNLO} = & \int_{\mathrm{d}\Phi_{n+2}} [\mathrm{d}\hat{\sigma}^{RR}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{S}_{ij,NNLO}] \\
& + \int_{\mathrm{d}\Phi_{n+1}} [\mathrm{d}\hat{\sigma}^{RV}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{T}_{ij,NNLO}] \\
& + \int_{\mathrm{d}\Phi_{n}} [\mathrm{d}\hat{\sigma}^{VV}_{ij,NNLO} - \mathrm{d}\hat{\sigma}^{U}_{ij,NNLO}]
\end{aligned}
$$

[Antenna subtraction; Gehrmann-De Ridder, Gehrmann, Glover; Kosower]

Current frontier: NNLO computations in QCD

NLO Progress

Driving ideas & methods:

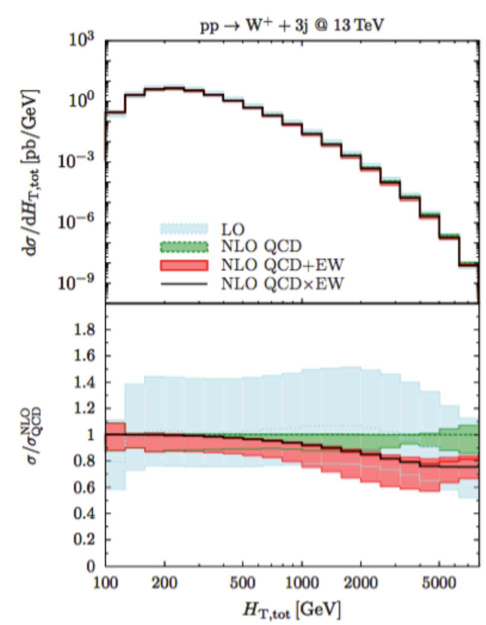
- Unitarity and numerical unitarity methods [Bern, Dixon, Kosowers; Britto, Cachazo Feng; Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov]
- Integral reduction algorithms [e.g. COLLIER; Denner, Dittmaier; GOLEM; Binoth, Guillet, Pilon, Heinrich, Schubert]
- Refined techniques; single cuts, spinor helicity, color tricks, recursions [e.g. OpenLoops, many contributions from amplitudes field]

Current mature tools:

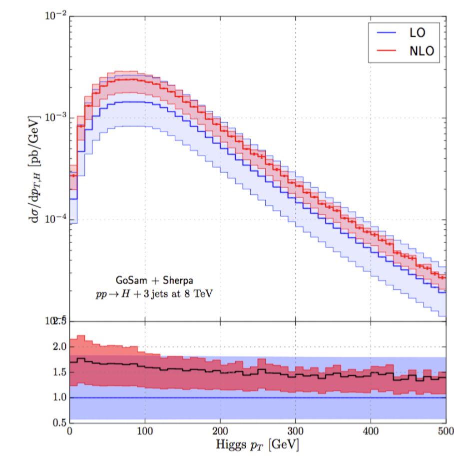
- Sherpa, Munich, Madgraph,…
- BlackHat, Hawk, GoSam, MCFM, Madgraph, NJet, OpenLoops, Prophecy4f, Prospino, Recola, Rocket,…

Some state-of-the-art predictions:

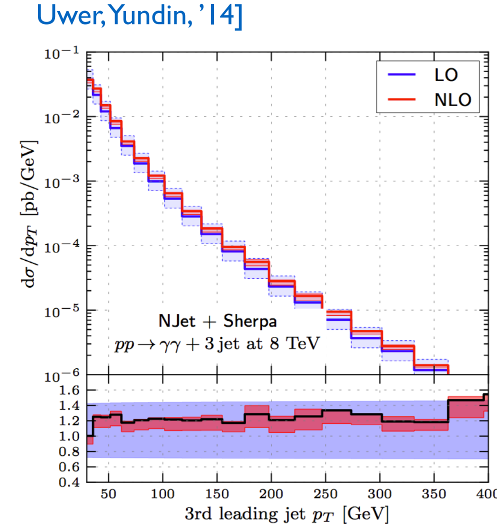
[OpenLoops; Kallweit, Lindert, Maierhofer, Pozzorini, Schonherr, '14]



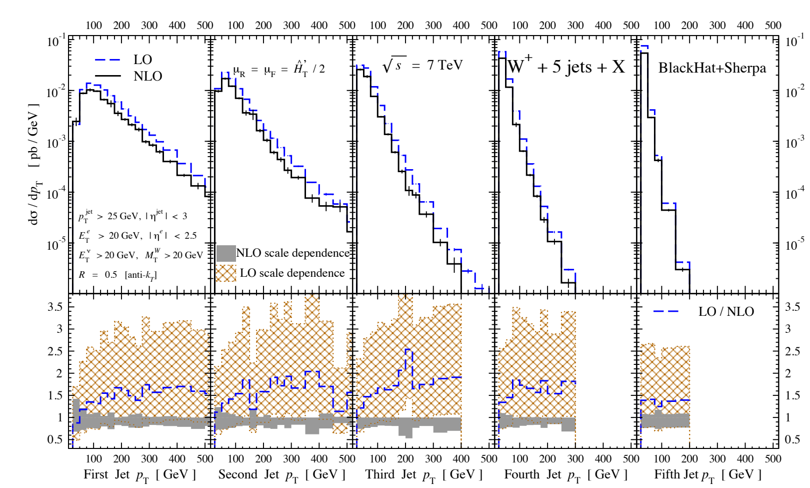
[GoSam; Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano, '13]



[NJet; Badger, Biedermann, Guffanti, Uwer, Yundin, '14]



[BlackHat; Bern, Dixon, Febres Cordero, HI, Kosower, Maitre, '13]



NNLO Progress (see D. Kosower's talk)

Recent highlights for proton collisions:

- di-photon: [Catani, Cieri, di Florian, Ferrera, Grazzini 11; Campbell, Ellis, Ye Li, Williams]
- W+1-jet: [Boughezal, Focke, Liu, Petriello 15]
- Z+1-jet: [Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan 15; Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello 15]
- H+1-jet: [Chen, Gehrmann, Glove, Jaquier 14; Caola, Melnikov, Schulze 15; Boughezal, Focke, Giele, Liu, Periello 15]
- tt: [Czakon, Fiedler, Mitov15]
- WW: [Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Mannteuffel, Pozzerini, Rathlev, Tancredi 14]
- ZZ: [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, v.Mannteuffel, Pozzerini, Rathlev, Tancredi, Weihs 14]
- Zphoton: [Grazzini, Kallweit, Rathlev, Torre 14]

A frontier:

- 3jets : amplitudes [Badger, Frellesvig, Zhang 15; Gehrmann, Henn, Lo Presti 15]

Three/more final states? Masses? Jets as probes of interactions?
Complexity bottleneck?

Unitarity Approach @ Two-loops

Status:

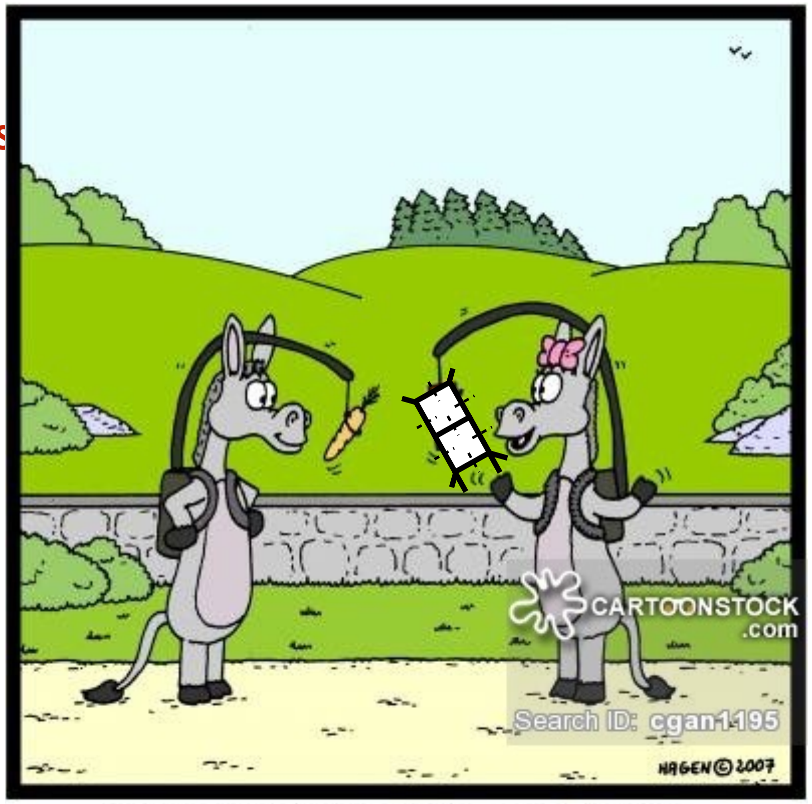
- Selected analytic computations of QCD amplitudes (see S. Badger's talk)
- Many formal computations: super YM and gravity theories

Discussed here:

- Extend one-loop ideas for multi-loop amplitudes
- Numerical and stable algorithm for QFT

Spin-off:

- Methods for integral-reduction (see also K. Larsen's talk)
- A master integral count
- Better understanding of perturbative QFT



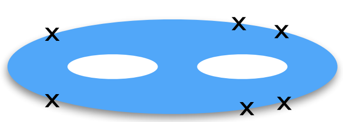
Carrots just didn't get me going anymore, so I switched to NNLO instead...

Loop Computations

Loop Computations

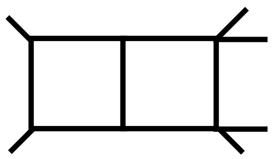
Feynman amplitude:

$$A(p_i) = \int [d^{(nD)}\ell] \; \tilde{A}(\ell, p_i)$$



Universal ansatz:

$$A(p_i) = \sum_{\text{integral basis}} c_j(p_i)\, \mathcal{I}_j(p_i)$$



$$\int [d^{(nD)}\ell] \; \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

Challenges: number of diagrams, integral reduction, integrals, algebra, numerical stability

Is there a viable process-independent approach to obtain coefficients?

Canonical Approach

Isolate tensor integrals:

$$\int [d^{(nD)}\ell] \; \tilde{A}(\ell, p_i) \longrightarrow \sum_{\text{tensor topologies}} T^{k}_{\mu_1\cdots\mu_m} \mathcal{I}_k^{\mu_1\cdots\mu_m}$$

Integral reduction:

$$\int [d^{(nD)}\ell] \; \frac{\ell^{\mu_1}\cdots\tilde{\ell}^{\mu_m}}{\rho^1\cdots\rho^N}$$

$$\sum_{\substack{\text{k in tensor integrals}\\ \text{j in integral basis}}} T_{k,\mu_1\cdots\mu_m} C_{k,j}^{\mu_1\cdots\mu_m} \, \mathcal{I}_j$$

2-loop methods: Tensor reduction [Tarasov 96; Anastasiou, Glover, Oleari 99], Integration-by-parts identities [Tkachov, Chetyrkin 81], Lorentz invariance identities [Gehrmann, Remiddi 99], Laporta algorithm [Laporta 01]

Programs in use: Reduze [Mannteuffel, Studerus], AIR [Anastasiou, Lazopoulos], FIRE [Smirnov, Smirnov], LiteRed [Lee], SecDec [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]; COLLIER [Denner, Dittmaier], GOLEM [Binoth, Guillet, Pilon, Heinrich, Schubert]

Unitarity Approach

Exploit analytic structure and use cutting rules:

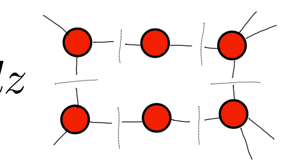
$$\int_{\mathcal{C}} [\mathrm{dLIPS}]\ \tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i)$$

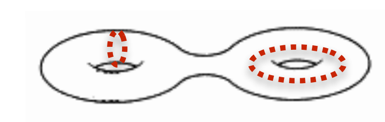
$$= \sum_{\text{integrals with cuts}} c_j(p_i) \int_{\mathcal{C}} [\mathrm{dLIPS}] \frac{m_j(\ell, p_i)}{(\text{uncut propagator terms})}$$

Properties:

- Fine-grained set of equations
- Organise equations: from maximal to minimal number of cut propagators
- Remaining integration
- Analytic approach for very compact results

Multi-loop pioneers [Bern, Dixon, Kosower, Dunbar 94; Bern, Dixon, Dunbar, Perelstein, Rozowsky 98; Bern, Dixon, Kosower 00]

$\oint_{\mathcal{C}} dz$ 

$\mathcal{C}$: 

Recently: duality between master integrals and contours [Kosower, Larsen 11; Caron-Huot, Larsen 12; Georgoudis, Zhang 15; Sogaard, Zhang 14; HI 15]

Unitarity Approach - Integrands

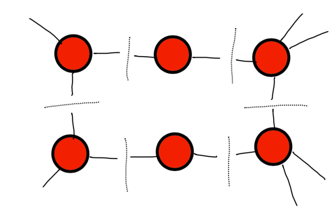
Use integrand basis and remove integration:

$$\tilde{A}(\ell, p_i) = \sum_{\text{j in integrand basis}} c_j(p_i) \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

Classification of integrands [Badger, Frellesvig, Zhang. 13; Mastrolia, Mirabella, Ossola, Peraro 12]

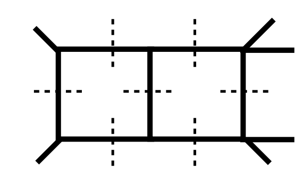
Factorisation in loop momenta [Ellis, Giele, Kunszt]:

$$\lim_{\{\rho^i\}\to 0} \tilde{A}(\ell, p_i) \to \tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i) \frac{1}{(\text{large propagator terms})}$$



Algebraic equations using tree-level data:

$$\tilde{A}_1(\ell, p_i) \times \cdots \times \tilde{A}_m(\ell, p_i) = \sum_{\text{j in large integrands}} c_j(p_i)\, m_j(\ell, p_i) + \text{previously computed topologies}$$



Properties:

- Universal and numerical
- But: additional integral reduction required

Numerical Unitarity Approach

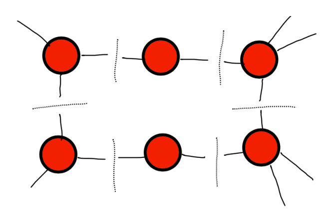
Integrand basis as direct sum of vanishing integrals and master integrals:

$$\tilde{A}(\ell, p_i) = \sum_{\text{j in master integrands}} c_j(p_i)\, \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N} + \sum_{\text{j in surface terms}} \hat{c}_j(p_i)\, \frac{\hat{m}_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$

@ one-loop [Ossola Papadopoulos, Pittau 07; Ellis, Giele Kunszt 07; Giele Kunszt, Melnikov 08]

@ two-loop [HI15]

Properties:



- Unitarity cuts give fine set of equations for coefficients
- Integral reduction implicit: just drop surface terms
- Process & multiplicity independent numerical approach

How to construct vanishing integrals?

- Use particular integration-by-parts identities [HI 15]!

IBPs

Vanishing Integrals

Surface Terms

Integration-by-parts identities: [Tkachov, Chetyrkin 81]

$$0 = \int [d^{((n=2)D)}\ell] \left[\partial_\mu \left(\frac{u^\mu}{\rho^1 \cdots \rho^N} \right) + \tilde{\partial}_\nu \left(\frac{\tilde{u}^\nu}{\rho^1 \cdots \rho^N} \right) \right]$$

Doubled propagators entangle unitarity equations:

$$\partial_\mu \left(\frac{u^\mu}{\rho^i} \right) = \frac{1}{\rho^i} \partial_\mu u^\mu - \frac{1}{(\rho^i)^2} u^\mu \partial_\mu \rho^i$$

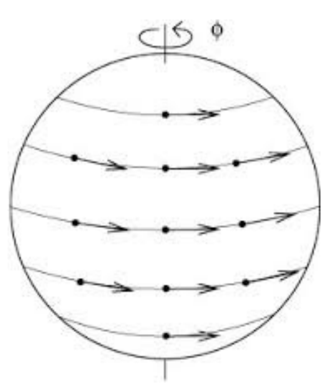
IBP-generating vectors: [Gluza, Kajda, Kosower 10]

$$(u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu)\rho^i = f^i(\ell, \tilde{\ell})\rho^i \qquad \text{for all } \rho^i$$

Geometric interpretation [HI 15, see similar Zhang 14]:

- Unitarity-cut surface: $\{\rho^i = 0\}$
- $(u^\mu, \tilde{u}^\nu)$ tangent vector to unitarity cut surface
- Intrinsic structure of unitarity cut

Unitarity-cut surface and IBP-generating vector field:



Natural Coordinates (1)

Need to solve

$$(u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu)\rho^i = f^i(\ell, \tilde{\ell})\rho^i \qquad \text{for all } \rho^i$$

solutions with computer algebra [Gluza, Kajda, Kosower 10; Schabinger 11]; explicit solutions [HI 15]; see also [Larsen, Zhang 15]

General coordinate transformation [HI 15]

inverse propagators

$$(\ell, \tilde{\ell}) \quad \to \quad (\rho^i, \alpha^a)$$

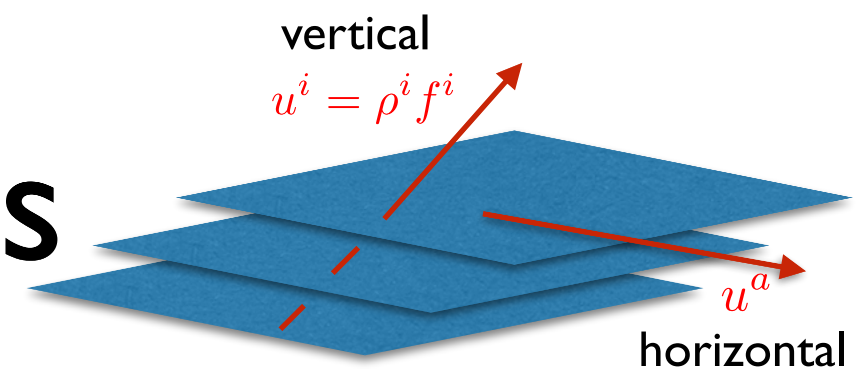
auxiliary coordinates

$$\left(u^j \frac{\partial}{\partial \rho^j} + u^a \frac{\partial}{\partial \alpha^a}\right)\rho^i = u^j \delta^i_j = f^i \rho^i \quad \longrightarrow \quad (u^i, u^a) = (f^i \rho^i, u^a)$$

Properties:

- Simple solution
- But: want polynomial vector fields for polynomial surface terms; careful treatment of algebraic structure — no problem

Organising IBPs



foliation of momentum space
in ρ^i = const. slices

Classification of IBP-generating vectors:

- Horizontal:

$$(u^i, u^a) = (0, u^a)$$ → relations within integral topology

- Vertical:

$$(u^i, u^a) = (f^i \rho^i, 0)$$ → relations between distinct integral topologies

- Mixed:

$$(u^i, u^a) = (f^i \rho^i, u^a)$$ → massless integrals, D-dependent relations

Role of intrinsic vector component u^a:

- Local translations
- Conformal transformations (massless integrals)

IBP-generating vectors (1)

van Neerven Vermaseren basis:

- External momenta

$$\{p_1, p_2, \ldots p_{N-1}\}$$

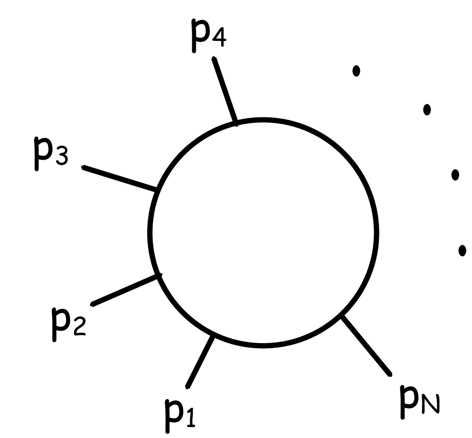
- Dual basis

$$v^{i\mu} = (G^{-1})^{ij} p_j^\mu, \quad \text{with} \quad G_{ij} = p_i^\mu p_{j\mu}$$
$$\rightarrow \quad v^{i\mu} p_{j\mu} = (G^{-1}G)^i_j = \delta^i_j$$
$$\rightarrow \quad v^{i\mu} \quad \text{in span of} \quad p_i^\mu$$

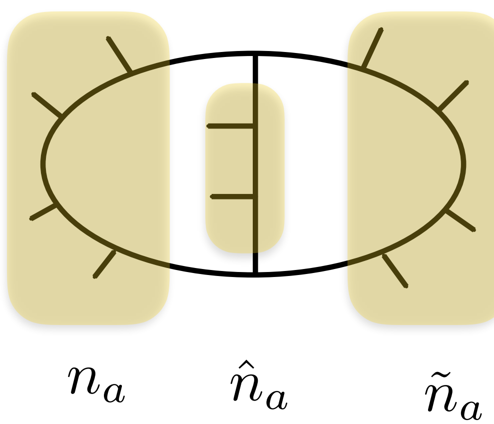
- Transverse space

$$n_a, \quad \text{with} \quad n_a^\mu p_{i\mu} = 0$$

one NV basis: n_a



three NV bases:



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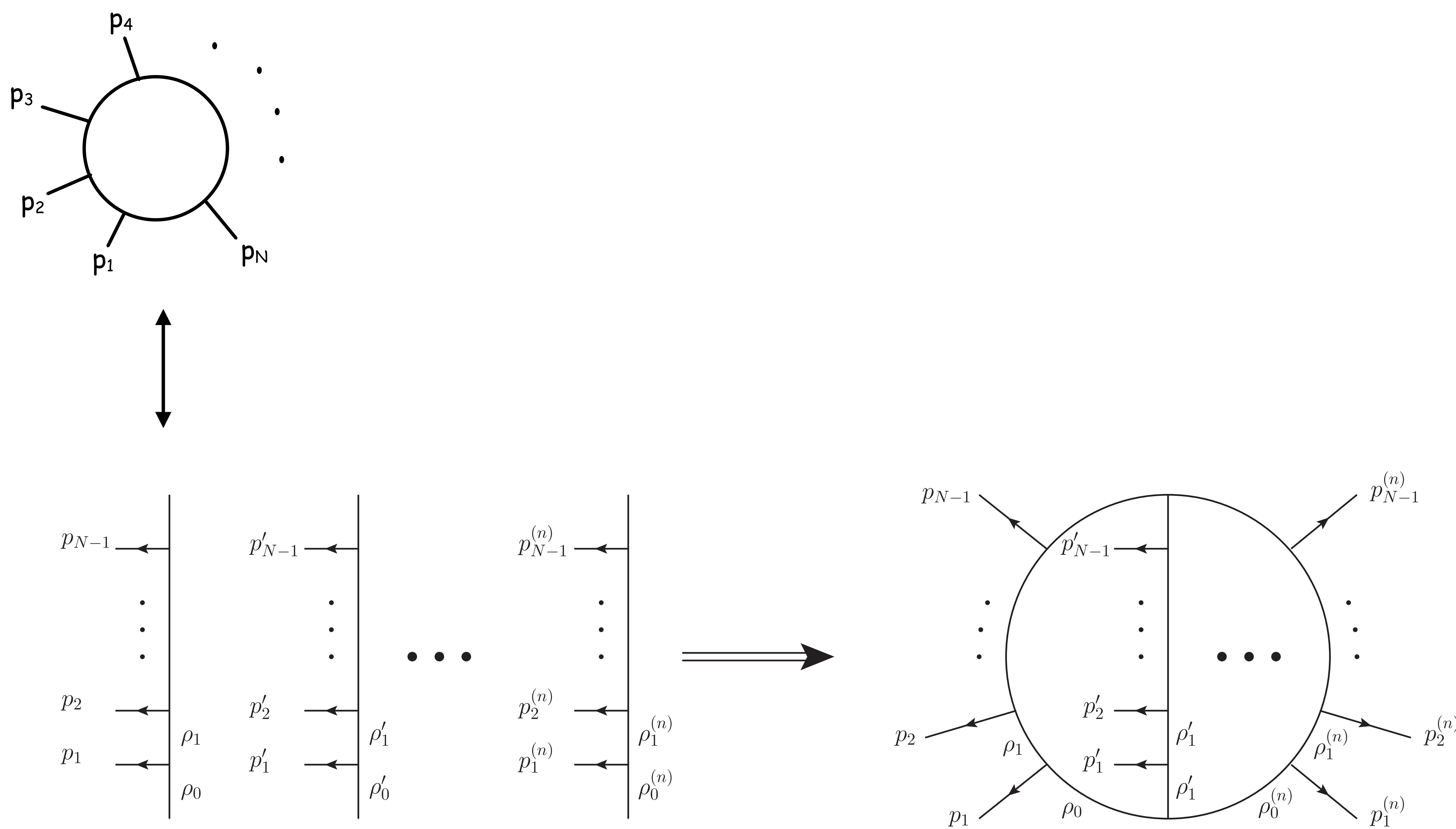


graphic from M. Jaquier

IBP-generating vectors (2)

One-loop integrals:

- Inverse propagators

$$\ell^2, (\ell - p_1)^2 - m_1^2, (\ell - p_1 - p_2)^2 - m_2^2, \ldots$$

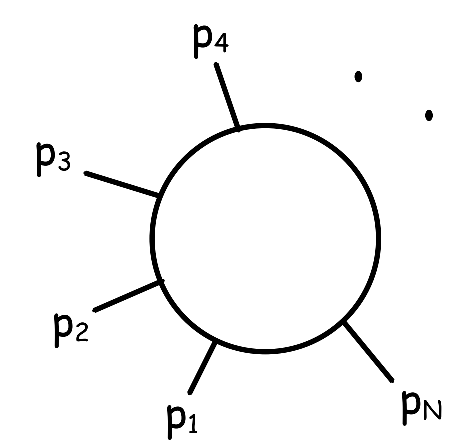
- IBP-generating vectors are rotation generators

$$u^{\nu}_{[ab]}\partial_{\nu} = n^{\mu}_{[a|}\ell_{\mu} n^{\nu}_{|b]}\partial_{\nu}$$

- Check

$$u^{\nu}_{[ab]}\partial_{\nu}(\ell^2) = 2\, n^{\mu}_{[a|}\ell_{\mu} n^{\nu}_{|b]}\ell_{\nu} = 0 \quad \text{(anti-symmetry)}$$
$$u^{\nu}_{[ab]}\partial_{\nu}(\ell^{\sigma} p_i^{\sigma}) = 2\, n^{\mu}_{[a|}\ell_{\mu} n^{\nu}_{|b]} p_{i\nu} = 0 \quad \text{(transversality)}$$

one NV basis: n_a



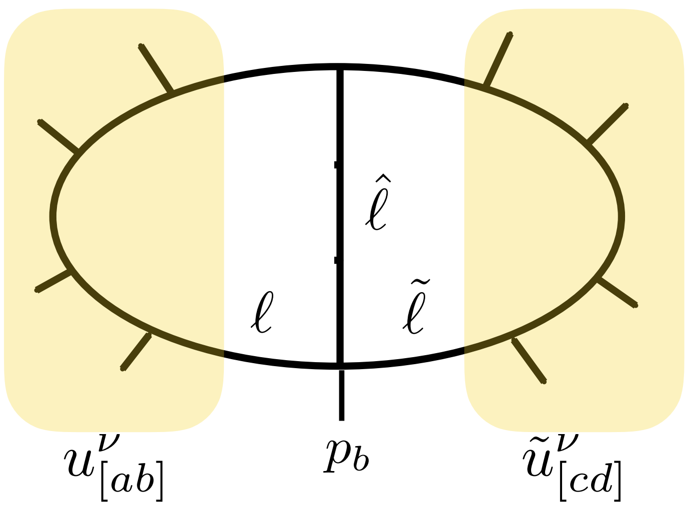
Remark: complete for generic massive integrals

IBP-generating vectors (3)

Two-loop integrals: planar, massive

Combine vectors of sub loops:

- Type (a) $u^{\nu}_{[abc]}\partial_\nu = n^{\kappa}_{[a|}\hat{\ell}_\kappa n^{\mu}_{|b|}\ell_\mu n^{\nu}_{|c]}\partial_\nu$
- Type (b) $n^{\mu}_{[a|}\ell_\mu n^{\nu}_{|b]}\partial_\nu + n^{\mu}_{[a|}\tilde{\ell}_\mu n^{\nu}_{|b]}\tilde{\partial}_\nu \quad \text{for} \quad n_a = \tilde{n}_a$
- Type (c) $\tilde{u}^{\nu}_{[ab]}\tilde{\partial}_\nu(\hat{\ell}^2)u^{\mu}_{[cd]}\partial_\mu - u^{\mu}_{[cd]}\partial_\mu(\hat{\ell}^2)\tilde{u}^{\nu}_{[ab]}\tilde{\partial}_\nu$



Properties:

- Integral topology determines types and number of vectors
- Multiplication with loop momentum polynomial give complete set of IBP-generating vectors
- Relations from type (b) known as `spurious terms' [Badger, Frellesvig, Zhang; Mastrolia, Ossola]

Surface terms

Properties of IBP-generating vectors:

- Horizontal: $[u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu]\rho^i = 0$
- Divergence free (observation): $\partial_\mu u^\mu + \tilde{\partial}_\nu \tilde{u}^\nu = 0$

Surface terms: $$\left[\partial_\mu \left(\frac{g\, u^\mu}{\rho^1 \cdots \rho^N}\right) + \tilde{\partial}_\nu \left(\frac{g\, \tilde{u}^\nu}{\rho^1 \cdots \rho^N}\right)\right] = \frac{[u^\mu \partial_\mu + \tilde{u}^\nu \tilde{\partial}_\nu] g}{\rho^1 \cdots \rho^N}$$

Construction algorithm:

- Input: numerator polynomials
- Directional derivatives
- Remove linear dependent terms

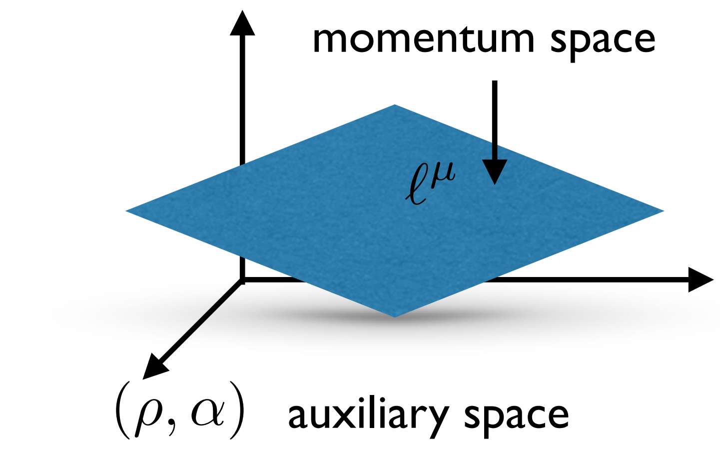
Reproduces known results @ one-loop level.

Construction of IBP-vectors

Natural Coordinates (2)

Loop momentum parametrization:

$$\ell = v^i r_i + \alpha^a n^a \quad \text{and} \quad \tilde{\ell} = \tilde{v}^i \tilde{r}_i + \tilde{\alpha}^a \tilde{n}^a$$



- Linear equations: $(p_i, \ell) = -\frac{1}{2}\left((\rho^i + m_i^2 - q_i^2) - (\rho^{i-1} + m_{i-1}^2 - q_{i-1}^2)\right) = r_i$
- Quadratic equations (implicit):

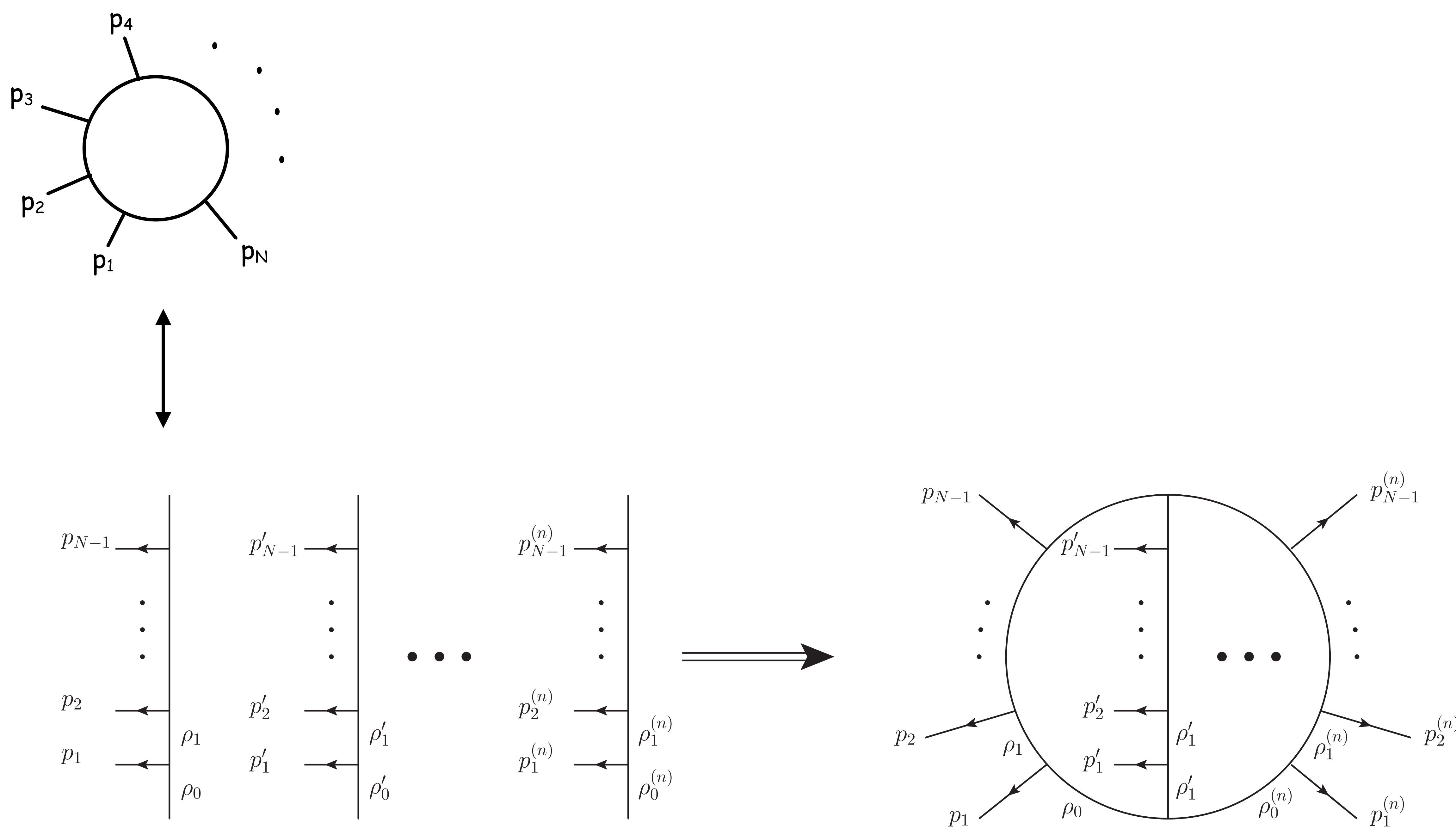
$$c(\rho, \alpha) := (\ell^2 - m_0^2) - \rho^0 = \sum_{a=1}^{D_t} (\alpha^a)^2 + (G^{-1})^{ij} r_i r_j - \rho^0 - m_0^2 = 0$$

- Each strand gives a quadratic expression:

@ 1-loops: $c(\rho, \alpha)$

@ 2-loops: $c(\rho, \alpha)$, $\tilde{c}(\tilde{\rho}, \tilde{\alpha})$ and $\hat{c}(\rho, \alpha, \tilde{\rho}, \tilde{\alpha}) = \left(\ell + \tilde{\ell} + p_b\right)^2 - \hat{\rho}^0 - \widehat{m}_0^2$

… as before

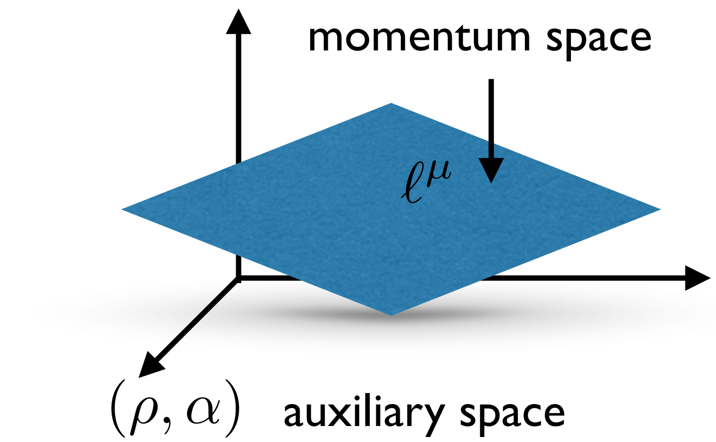


graphic from M. Jaquier

Polynomial data

Numerators give polynomials

$$t^{\mu_1 \cdots \mu_k} \ell_{\mu_1} \cdots \ell_{\mu_k} \quad \sim \quad (\rho, \alpha) - \text{polynomials}$$



- Auxiliary space as coordinate space
- Momentum space as algebraic variety from $\{c, \tilde{c}, \hat{c}\} = 0$

Vector fields:

- Map polynomials to polynomials — polynomial components
- Tangent to momentum space $(u^a \partial_a + u^k \partial_k) \{c, \tilde{c}, \hat{c}\} = 0$
- From above: $(u^i, u^a) = (f^i \rho^i, u^a)$

for

$\{c, \tilde{c}, \hat{c}\} = 0$

syzygy equations

equations give rotation generators as solutions

One-loop Example

One-loop relation:

$$c(\alpha, \rho, \mu) := (\ell^2 - m_0^2) - \rho^0 = C(\rho) + \sum_{a=1}^{D_t} (\alpha^a)^2 \ + \sum (\mu^b)^2 = 0$$

Rotation IBP vector:

$$u_{[ab]} \ = \ \alpha^a \frac{\partial}{\partial \alpha^b} - \alpha^b \frac{\partial}{\partial \alpha^a}$$

Scaling IBP vector:

$$u_s = f_s \, \alpha^a \frac{\partial}{\partial \alpha^a} + \sum_i f_s^i \rho^i \frac{\partial}{\partial \rho^i} \qquad \text{for} \qquad C(0) = 0$$

f's fixed by syzygy equations

D-dim rotation:

$$u_a \ = \ \alpha^a \mu^b \partial_{\mu^b} - (\mu^b \mu^b) \partial_{\alpha^a}$$

Numerator Basis

Numerators and unitarity cuts:

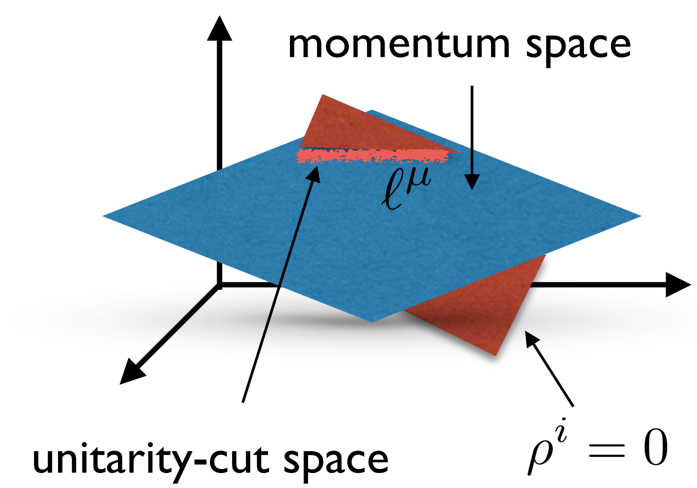
- Numerators modulo lower topologies = modulo inverse propagators: $\rho^i \sim 0$
- Consider functions on on-shell varieties

$$\rho^i \sim 0 \quad \text{and} \quad \{c, \tilde{c}, \hat{c}\} \sim 0$$

- Remove redundant surface terms on-shell

independent integrals

$$\int [d^{(nD)}\ell] \, \frac{m_j(\ell, p_i)}{\rho^1 \cdots \rho^N}$$



Finally use the linear independent surface terms in off-shell version.

Formal Structures

Adapted Coordinates

Coordinate change exposes fiber structure:

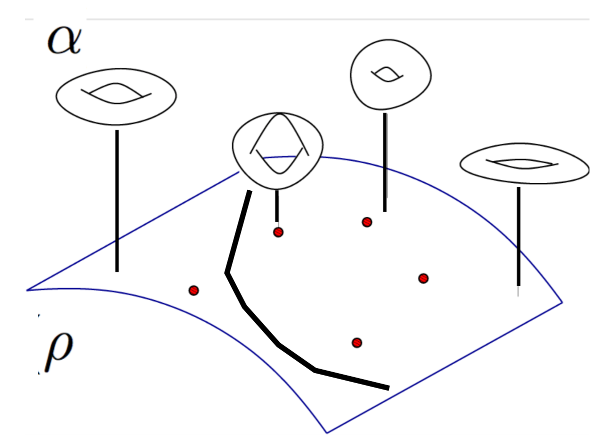
$$\mathcal{I}[t] = \int \frac{[d\rho]}{\rho^0 \ldots \tilde{\rho}^{(\tilde{N}-1)}} \times t(\rho, \alpha)\, \mu(\rho, \alpha)[d\alpha]$$

Functions on internal spaces important for full integral.

@ one loop: internal spaces complexified spheres; all non-constant harmonic functions integrate to zero = IBP relations, IBP-generating vectors are rotation generators

In general: need to understand topology of internal space to understand non-trivial integrals.

Integrate over fibers first



Counting IBP-relations

holomorphic form

Cutting loop integral:

$$\int \frac{t}{\rho^1 \cdots \rho^N} J[d\alpha d\rho] \xrightarrow{\text{cut}} \int t\, J[d\alpha]$$

exact form

IBP-relations give exact forms on shell:

$$\int \left[\left(\sum_i \frac{\rho^i \partial_i (f^i\, J)}{\rho^1 \cdots \rho^N} \right) + \left(\frac{\partial_a (u^a\, J)}{\rho^1 \cdots \rho^N} \right) \right] [d\alpha d\rho] \xrightarrow{\text{cut}} \int \partial_a (u^a\, J)[d\alpha]$$

Master integrands:

- (holomorphic forms) modulo (exact forms) = (cohomology of unitarity cut variety)

Topological count gives master integrands

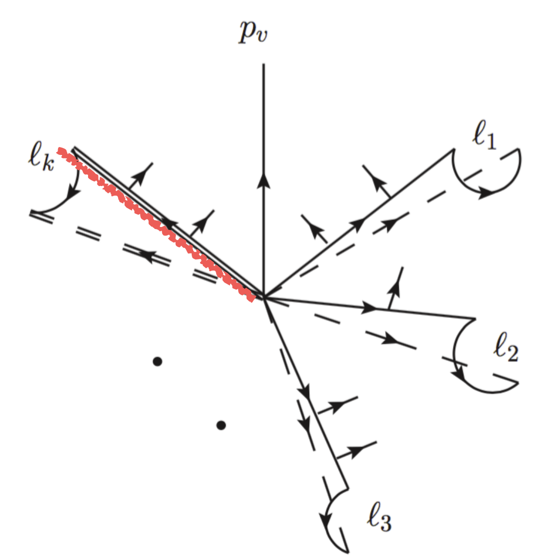
Completeness of generators:

- #(exact forms) = #(on-shell IBP-relations) — OK!

Formal structures

IBP-generating vectors:

- Rotation/scaling generators for each rung, consistent with momentum conservation at vertices.



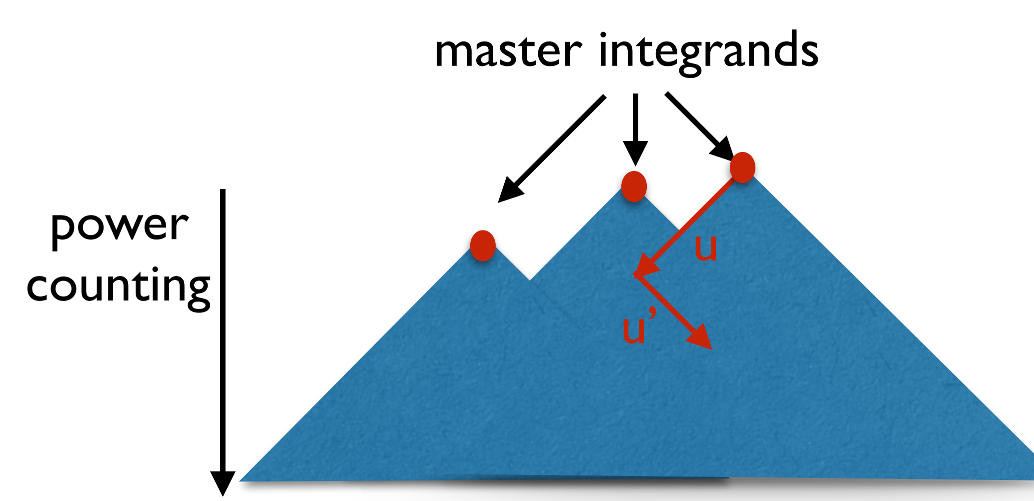
Geometric structures:

- Lie-algebra & representation theory:

$$[u_a, u_b] = f_{ab}^c(\ell, p_i) u_c$$

- Role of purely vertical IBP-generators
- Counting cycles…

Numerator polynomials:



Conclusions

Methods for numerical unitarity approach presented

- Numerical, universal approach

Next steps:

- Special cases (massless topologies, D-dim)
- Explicit proof-of-principle computation

Structures:

- Unitarity-cut phase spaces
- Coordinates adapted to integrals
- Classification of IBP-generating vectors

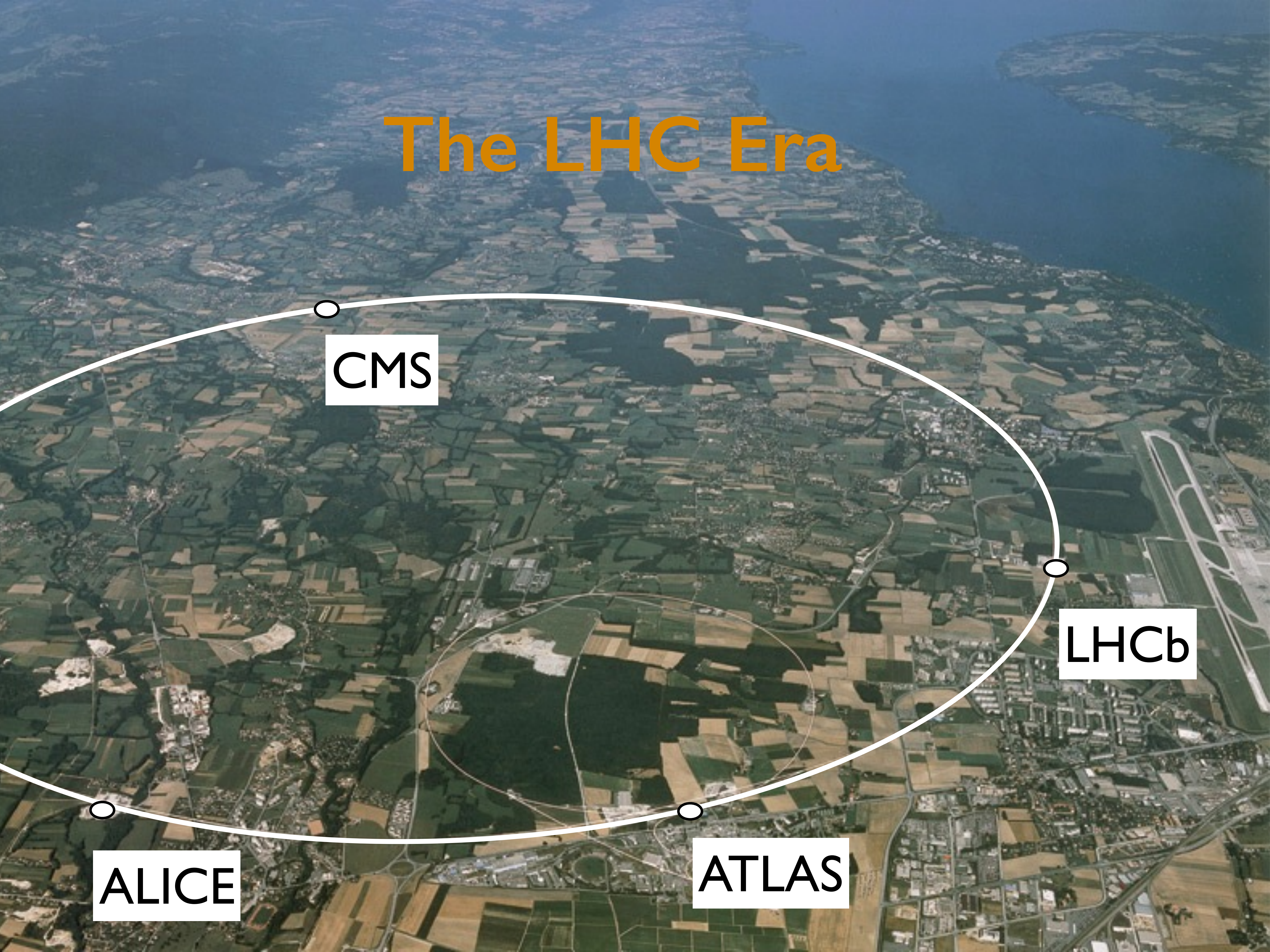
… continued

Open questions:

- Direkt understanding of current algebra?
- Further lessons from natural coordinates (integrals, diff. equations)?
- Cycles of unitarity-cut phase-spaces?
- Efficiency of numerical approach?
- Doubled propagators (cut-less approach).

Thanks!

The LHC Era

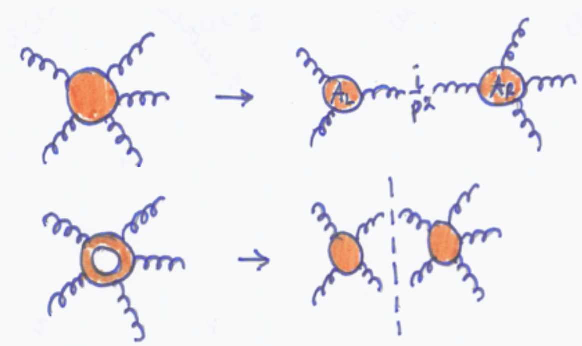


History of Unitarity Method

Origins in bootstrap program for strong interactions in 60s:

[Chew, Mandelstam; Eden, Landshoff, Olive, Polkinghorne; Veneziano; Virasoro, Shapiro; …]

- Reconstruct amplitudes directly from analytic properties:
  - Poles explained by factorization.
  - Branch cuts from optical theorem.



Replaced in 70s by rise of field theory (QCD) and Feynman rules.
Return of analyticity within perturbative field theory:

[Bern, Dixon, Dunbar, Kosover]

- Cutting rules, on-shell and unitarity methods.
- Adding to field theory methods.

Numerical unitarity methods are algorithm to construct complicated loop amplitudes from simpler tree amplitudes:

Many contributors: [Badger, Berger, Bern, Bjerrum-Bohr, Brandhumber, Britto, Cachazo, Dixon, Dunbar, Ellis, Febres-Cordero, Feng, Forde, Giele, Harmeren, Kosower, Kunszt, Mastrolia, Melnikov, Ossola, Papadopoulos, Pittau, Schwinn, Spence, Travaglini, Weinzierl, Witten]

- Built from analytic inspiration, N=4 SYM, etc