Scattering via Riemann Spheres

Song He

Institute of Theoretical Physics, Chinese Academy of Sciences

Based on works with Freddy Cachazo & Ellis Yuan (2013-15)
+ to appear with Yong Zhang

Aspects of Amplitudes, Nordita

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Parke-Taylor formula

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$$M_n(1^+,\ldots,i^-,\ldots,j^-,\ldots,n^+) = \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\cdots\langle n{-}1\; n\rangle\langle n\, 1\rangle}\,.$$

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$$j_A(z)\, j_B(z') = \frac{f_{AB}^C\, j_C}{z-z'} + \ldots\,, \to \quad PT_n := \frac{1}{(z_1-z_2)\,(z_2-z_3)\,\cdots\,(z_n-z_1)}\,.$$

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3. other twistor-string formulas e.g. for $\mathcal{N}=8$ supergravity: replace PT by determinants [Cachazo, Geyer; Cachazo, Skinner '12...]

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String origin: ambi-twistor strings [Mason, Skinner '13], "chiral" field-theory limit [Berkovits '13; Siegel '15,...]... [c.f. talks last week]

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in 4d equivalent to RSV equations without spinors [CHY '13]
also saddle points in high-energy limit [Gross, Mende '80]. ???

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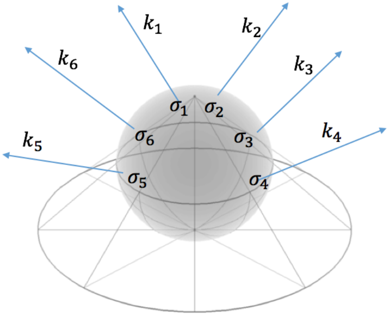

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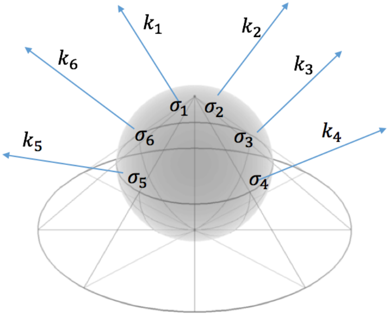

Determine locations of n punctures in terms of n-pt kinematics

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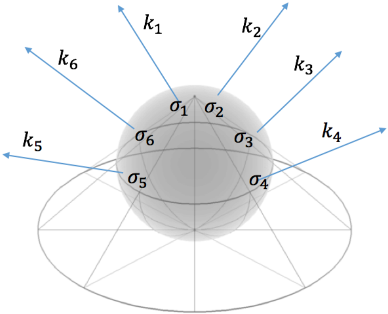

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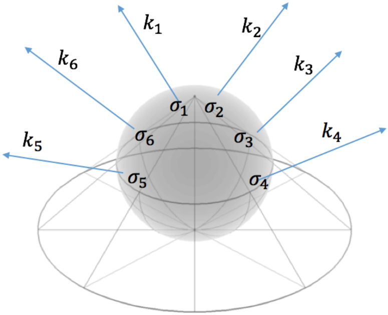

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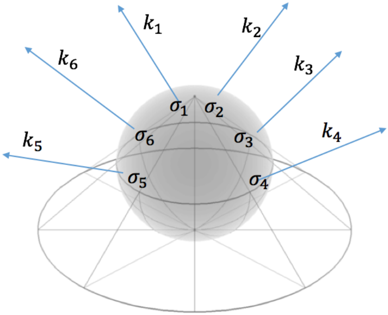

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$n-3$ eqs for $n-3$ variables; non-trivially $(n-3)!$ solutions

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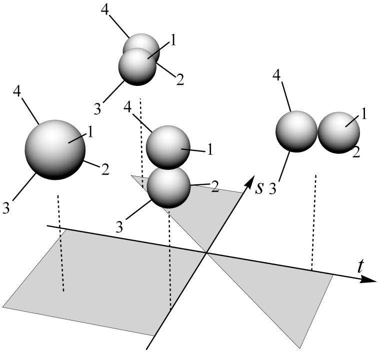

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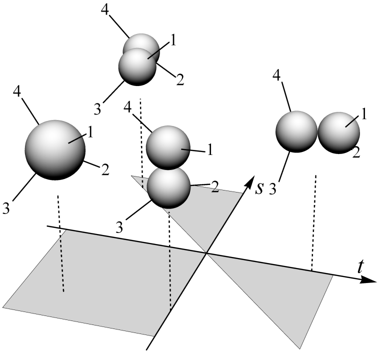

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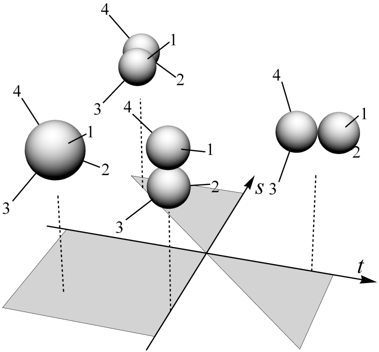

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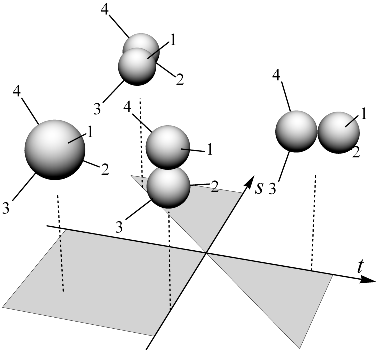

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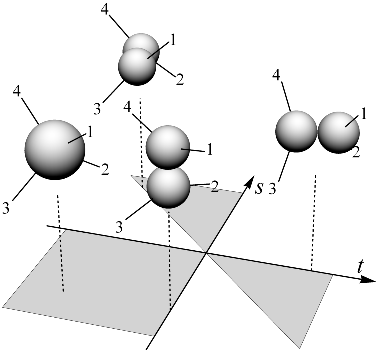

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well known in string theory: Riemann knows a lot of physics!

CHY formulation

Tree amps = contour integral in $\mathcal{M}_{n,0}$ = sum over solutions

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Goal: find "dynamic part" (CHY integrand) for a given theory

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What does the formula compute?

Trivalent diagrams from CHY

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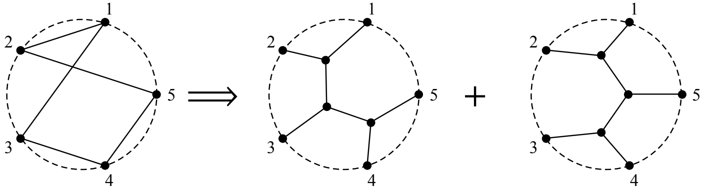

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ϕ^3 theory with flavors, e.g. in bi-adjoint of $\mathrm{U}(N)\times\mathrm{U}(N')$:
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CHY formula for bi-adjoint ϕ^3 amplitudes: gives sum of all $m[\pi|\rho]$'s with flavor factors (note permutation invariance)

$$M_n^{\phi^3} = \int d\mu_n \, \mathcal{C} \, \mathcal{C}' = \sum_{\pi,\rho} \mathrm{Tr}(T^{I_{\pi(1)}} \cdots T^{I_{\pi(n)}}) \, \mathrm{Tr}(T^{I_{\rho(1)}} \cdots T^{I_{\rho(n)}}) \; m[\pi|\rho]$$

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$$A_{a,b} := \begin{cases} \frac{k_a \cdot k_b}{\sigma_{a,b}} & a \neq b \\ 0 & a = b \end{cases}, \quad B_{a,b} := \begin{cases} \frac{\epsilon_a \cdot \epsilon_b}{\sigma_{a,b}} & a \neq b \\ 0 & a = b \end{cases},$$

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The origin of $\mathrm{Pf}'\Psi$: by scattering equations, it is exactly given by open-string correlators in the field-theory limit

$$\mathrm{Pf}'\Psi \sim \langle V^{(0)}(\sigma_1) \ldots V^{(-1)}(\sigma_i) \ldots V^{(-1)}(\sigma_j) \ldots V^{(0)}(\sigma_n) \rangle$$

Gauge invariance

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$$
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Substituting $\epsilon_1 \to k_1$ $\mathrm{Pf}'\Psi = 0$ for each solution of scattering equations $\Longrightarrow$ gauge invariance manifest from CHY formula

CHY formula for gravity

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In general $\epsilon^{\mu}\epsilon'^{\nu}$ gives $h^{\mu\nu} + B^{\mu\nu} + \phi$; CHY formula for gravity

$$\boxed{M_n^{h+B+\phi} = \int d\mu_n \, \mathrm{Pf}'\Psi(\epsilon) \, \mathrm{Pf}'\Psi(\epsilon') \longrightarrow M_n^{\mathrm{GR}} = \int d\mu_n \, \det{}' \, \Psi(\epsilon)}$$

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GR $\sim$ YM$\otimes$ YM" or precisely "GR$=$YM$^2/\phi^3$" [KLT '86, BCJ' 08].

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$\mathrm{Pf}'\Psi(\epsilon) \times \mathrm{Pf}'\Psi(\epsilon')$ correspond to closed-string correlator by using scattering equations: closed-string = open-string [2]

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$$M_n = \int d\mu_n \, L \, R \; = \sum_{I=1}^{(n-3)!} \frac{L_I \; R_I}{J_I} = \mathbf{L} \cdot \mathbf{J}^{-1} \cdot \mathbf{R} \, ,$$

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General double-copy relations from splitting CHY formula into two $\rightarrow$ BCJ for partial amps: $M_n[\pi] = \sum_{\alpha,\beta}\; m[\pi|\alpha]\; m^{-1}[\alpha|\beta]\; M_n[\beta]$.

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Why these EFT's special? Goldstone bosons with enhanced Adler's zero! For NLSM, scalar DBI, sGal, $M_n \sim \tau^1, \tau^2, \tau^3 \to 0$ with soft emission $p^\mu \sim \tau \to 0$ [Cheung et al '14; CHY '14].

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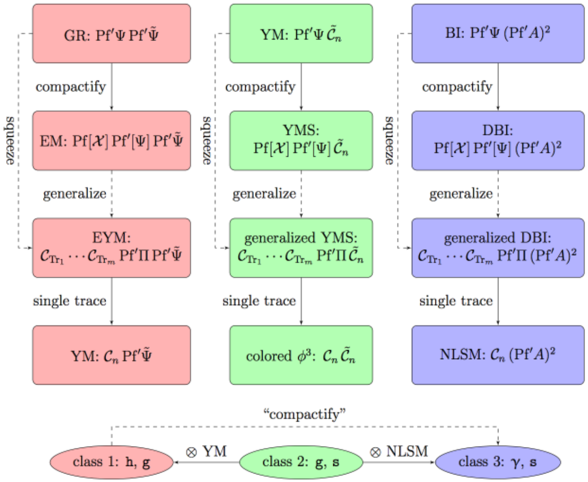
Compact formula for all gluon-graviton amps in GR $\oplus$ YM (& YM $\oplus$ ϕ^3). New ambitwistor-string models [Geyer et al' 15].

A landscape of massless theories



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Imposing $\delta(\mathcal{E})$'s gives formula for one-loop amplitudes

$$M_n^{(1)} = \int d^D \ell \frac{1}{\ell^2} \int d\mu_n^{(1)}\, \mathcal{I}_n(\{\sigma, k, \epsilon\}; \ell)\,,$$

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$$\boxed{\mathcal{E}_a = \sum_{b \neq a} \frac{k_a \cdot k_b}{\sigma_a - \sigma_b} + \frac{k_a \cdot \ell}{\sigma_a}\,, \qquad \text{for } a = 1, \ldots, n\,.}$$

Imposing $\delta(\mathcal{E})$'s gives formula for one-loop amplitudes

$$M_n^{(1)} = \int d^D\ell \frac{1}{\ell^2} \int d\mu_n^{(1)}\, \mathcal{I}_n(\{\sigma, k, \epsilon\}; \ell)\,,$$

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New rep of loop integrands: a rational function with no ambiguities (treat all propagators equally) [c.f. Baadsgaard et al '15]

Loops from trees

Loops from trees

One-loop amp as forward limit of tree amp in higher dim:

$$M_n^{1-\mathrm{loop}} \sim \int \frac{d^D\ell}{\ell^2} \sum_{I_+=I_-,\epsilon_+=(\epsilon_-)^*} M_{n+2}^{\mathrm{tree}}(\ \{(k_i;0)\},\ \ \pm(\ell,|\ell|)\ ),$$

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One-loop "Pfaffians" from forward-limit of tree ones, e.g.

$$\text{Pf}_{\mathbf{s}}^{(1)} = \frac{1}{\sigma_{+,-}^2}\text{Pf}\Psi_n(\ell), \quad \text{Pf}_{\mathbf{g}}^{(1)} = \sum_{\epsilon_+=(\epsilon_-)^*} \text{Pf}'\Psi_{n+2}(\ell), \quad \text{Pf}_{\mathbf{f}}^{(1)} = \ldots$$

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Formulas for ϕ^3, Yang-Mills and gravity at one loop

$$\mathcal{I}_n^{\phi^3} = (\mathrm{PT}_n^{(1)})^2, \quad \mathcal{I}_n^{\mathrm{YM}} = \mathrm{PT}_n^{(1)}\,\mathrm{Pf}_{\mathbf{g}}^{(1)}, \quad \mathcal{I}_n^{\mathrm{GR}} = (\mathrm{Pf}_{\mathbf{g}}^{(1)})^2 - c_d(\mathrm{Pf}_{\mathbf{f}}^{(1)})^2\,,$$

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Gauge invariance, soft theorems, unitarity cuts, SUSY ...
natural one-loop KLT and BCJ relations at integrand level: e.g.

$$\mathrm{SUGRA} = \sum_{\alpha,\beta=1}^{(n-1)!-2(n-2)!} \mathrm{SYM}[\alpha]\,(\phi_3)^{-1}[\alpha|\beta]\,\mathrm{SYM}[\alpha] .$$

Back to four dimensions [CHY, 13′]

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What is special in 4d? scattering eqs fall into $n-3$ sectors, $k = 2, 3, \ldots, n-2$; solutions $(n-3)! = \sum_{k=2}^{n-2} \langle\langle n-3, k-2 \rangle\rangle$.

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$$P^2(z) = 0 \quad \Leftrightarrow \quad \exists\, \lambda(z), \tilde\lambda(z), \text{ s.t. } P^{\alpha\,\dot\alpha}(z) = \lambda^\alpha(z)\tilde\lambda^{\dot\alpha}(z)\,.$$

$$\text{ansatz :} \quad \lambda(z) := \sum_{I=1}^{k} \frac{t_I \lambda_I}{z - \sigma_I}\,, \qquad \tilde\lambda(z) := \sum_{i=k+1}^{n} \frac{t_i \tilde\lambda_i}{z - \sigma_i}\,,$$

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$(a\,b) := \frac{(\sigma_a-\sigma_b)}{t_a t_b}$; GL$(2,\mathbb{C})$: 4 for momentum-conservation.
Equivalent to RSV-Witten equations (GL(k)-fixed) [He et al ’16].

Back to four dimensions

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Remarkable factorization identity: on any solution of sector k,

$$\det{}'\left|\frac{\partial\{E\}}{\partial\{\sigma\}}\right| = J_{n,k}\,\det{}'H_k\,\det{}'\tilde{H}_{n-k}\,; \quad H_{I\neq J} = \frac{\langle I\,J\rangle}{(I\,J)}\,,\ \tilde{H}_{i\neq j} = \frac{[i\,j]}{(i\,j)}\,,$$

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4d measure $d\mu^{\text{4d}}_{n,k}$ ($2n{-}4$ integrals & delta functions); 4d integrand $\mathcal{I}^{\text{4d}}_{n,k} := \mathcal{I}_n/(\det' H_k \det' \tilde{H}_{n-k})$, from a sector-dependent reduction.

Back to four dimensions

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Pfaffian as a filter: solution-sector k and helicity-sector k'

$$\mathrm{Pf}'\Psi(1^-,\ldots,k'^-,(k'+1)^+,\ldots,n^+)|_{\mathrm{soln}.k} = \delta_k^{k'}\,\det{}'H_k\,\det{}'\tilde{H}_{n-k}\,.$$

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Natural for SUSY (fermionic delta functions in $d\mu_{n,k}$); equivalent to RSV-Witten & Cachazo-Skinner forms [Geyer et al '14; He et al '16]

More theories in four dimensions

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Similarly for $\mathrm{Pf}'A$: only non-vanishing for middle sector $k = \frac{n}{2}$:

$$\mathrm{Pf}'A|_{\mathrm{soln.}\,\frac{n}{2}} = \det{}' H_{\frac{n}{2}} \det{}' \tilde{H}_{\frac{n}{2}} \frac{\prod_{I<J}(I\,J) \prod_{i<j}(i\,j)}{\prod_{I,i}(I\,i)} \, .$$

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More realistic theories, e.g. incorporating quarks, Higgs boson etc.?

Massless QCD

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Including gluinos with a Jacobian from integrating out η's:

$$M_n = \int d\mu_{n,k}\, \mathrm{PT}_n\, \det J_{|\psi|\times|\psi|}\,, \qquad J_{I\in\psi^-,i\in\bar{\psi}^+} = \frac{\delta^{A_I\,A_i}}{(I\,i)}\,.$$

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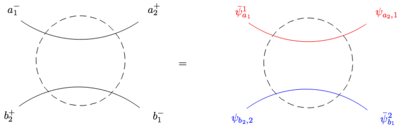

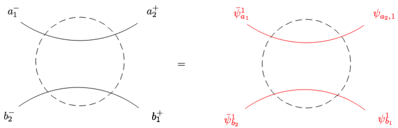

Massless QCD

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3 pairs (a,b & c): alternating and splitting cases as before, also

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Remarkable simplicities in 4d; CHY vs. interaction vertices???

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$$M_{\mathrm{MHV}}(\phi; n_g) = \frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}\,, \quad M_{\mathrm{all-minus}}(\phi; n_g) = \frac{m_H^4}{[1\,2][2\,3]\cdots[n\,1]}\,,\ \text{etc.}$$

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Canonical rep [Arkani-Hamed et al 08, 10]: connected formula for $\mathcal{N} = 4$/gluons → BCFW/CSW form; now we expect a whole zoo of new reps, also for QCD, Higgs, form factors etc.

Outlook

New picture: massless particles scattering via punctures on a sphere. Suggest a weak-weak duality of QFT & strings for S-matrix?

Web of theories connected by e.g. $\oplus$ (interaction) & $\otimes$ (double-copy)

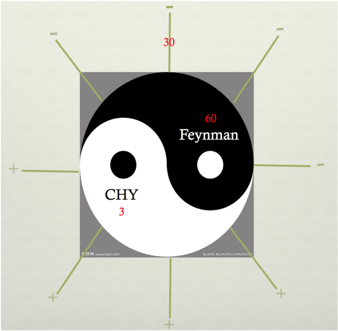
Huge simplifications in 4d $\rightarrow$ old and new connected formulas

QCD, Higgs, form factor? Scope of QFTs natural in CHY?

Loops: integrands $\rightarrow$ CHY for integrated amplitudes?

Stringy origin: twistor vs. "chiral" strings, Gross-Mende limit, ...?

Thank you!



taken from C.S. Lam's talk