Yang-Mills origin of gravitational symmetries

1309.0546 1312.6523 1402.4649 1408.4434 1502.02578 1502.05359

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Outline

Motivation

The main motivation for studying the classical double copy comes from:

- Amplitudes [Kawai-Lewellen-Tye ’85, Bern-Carrasco-Johansson ’08, Cachazo-He-Yuan ’14]
- Classical solutions [Monteiro-O’Connell-White ’14]

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Study this via symmetries:

- Spacetime
- Global internal
- Local internal

Framework

$$\Big(\text{linear super-Yang-Mills}\Big)^2 = \Big(\text{linear supergravity}\Big)$$

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Spacetime	Global sPoincaré	Global sPoincaré	(1)
Global internal	$R \times E$	G/H	(2)
Local internal	YM gauge	graviton, gravitino and p-form gauge	(3)

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What about non-linear?

Dictionary

How should the sYM fields combine? [Siegel '88, '95]

$$\Phi_\mu^{iA}(x) \otimes \tilde{\Phi}_\alpha^{i'A'}(x) \tag{4}$$

Dictionary

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Propose:

- Decompose the $SO(1, D-1)$ tensor product into irreps.
- Keep the $R \times E \times \tilde{R} \times \tilde{E}$ indices.
- Kill the gauge indices using a biadjoint scalar. [Hodges '11, '12]
- Use a convolution to combine the field dependence.

$$\begin{aligned}\left(\Phi_\mu^{i} \circ \tilde{\Phi}_\alpha^{i'}\right)(x) &\equiv \left(\Phi_\mu^{iA} \star \Phi_{AA'} \star \tilde{\Phi}_\alpha^{i'A'}\right)(x) \\ &= \int \mathrm{d}^D y \, \mathrm{d}^D z \; \Phi_\mu^{iA}(y)\Phi_{AA'}(z-y)\tilde{\Phi}_\alpha^{i'A'}(x-z) \end{aligned} \tag{5}$$

Example: $\mathcal{N}_L \times \mathcal{N}_R = 4 \times 4$

Focus on global symmetries and thus work with on-shell fields and drop gauge indices.

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Each theory has $U(1)st \times SU(4)$ under which the content is the vector $\mathbf{1}^{-2} + \mathbf{1}^{2}$, four complex spinors $\mathbf{4}^{-1} + \bar{\mathbf{4}}^{1}$ and six real scalars $\mathbf{6}^{0}$.

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The resulting theory will have $U(1)_{st} \times SU(4)_L \times SU(4)_R \times U(1)_d$ where:

$$U(1)_{st} = U(1)^L_{st} + U(1)^R_{st} \tag{6}$$

$$U(1)_d = U(1)^L_{st} - U(1)^R_{st} \tag{7}$$

The squaring table

$\mathbf{V}_4 \backslash \mathbf{V}_4$	$\mathbf{1}^{-2}+\mathbf{1}^{2}$	$\mathbf{4}^{-1}+\bar{\mathbf{4}}^{1}$	$\mathbf{6}^{0}$
$\mathbf{1}^{-2}+\mathbf{1}^{2}$	$(\mathbf{1},\mathbf{1})_0^4+(\mathbf{1},\mathbf{1})_0^{-4}$ $+(\mathbf{1},\mathbf{1})_4^0+(\mathbf{1},\mathbf{1})_{-4}^0$	$(\mathbf{1},\mathbf{4})_3^1+(\mathbf{1},\bar{\mathbf{4}})_{-3}^{-1}$ $+(\mathbf{1},\mathbf{4})_{-1}^{-3}+(\mathbf{1},\bar{\mathbf{4}})_1^3$	$(\mathbf{1},\mathbf{6})_2^2+(\mathbf{1},\mathbf{6})_{-2}^{-2}$
$\mathbf{4}^{-1}+\bar{\mathbf{4}}^{1}$	$(\mathbf{4},\mathbf{1})_{-3}^1+(\bar{\mathbf{4}},\mathbf{1})_3^{-1}$ $+(\mathbf{4},\mathbf{1})_1^{-3}+(\bar{\mathbf{4}},\mathbf{1})_{-1}^3$	$(\mathbf{4},\mathbf{4})_0^{-2}+(\bar{\mathbf{4}},\bar{\mathbf{4}})_0^2$ $+(\mathbf{4},\bar{\mathbf{4}})_{-2}^0+(\bar{\mathbf{4}},\mathbf{4})_2^0$	$(\mathbf{4},\mathbf{6})_{-1}^{-1}+(\bar{\mathbf{4}},\mathbf{6})_1^1$
$\mathbf{6}^{0}$	$(\mathbf{6},\mathbf{1})_{-2}^2+(\mathbf{6},\mathbf{1})_2^{-2}$	$(\mathbf{6},\mathbf{4})_1^{-1}+(\mathbf{6},\bar{\mathbf{4}})_{-1}^1$	$(\mathbf{6},\mathbf{6})_0^0$

The supergravity

Collect fields of the same helicity under $SU(4)_L \times SU(4)_R \times U(1)_d$:

$$
\begin{aligned}
g^* : &\quad (\mathbf{1},\mathbf{1})_0 \\
\Psi^* : &\quad (\mathbf{4},\mathbf{1})_1 + (\mathbf{1},\mathbf{4})_{-1} \\
A^* : &\quad (\mathbf{6},\mathbf{1})_2 + (\mathbf{1},\mathbf{6})_{-2} + (\mathbf{4},\mathbf{4})_0 \\
\psi^* : &\quad (\bar{\mathbf{4}},\mathbf{1})_3 + (\mathbf{1},\bar{\mathbf{4}})_{-3} + (\mathbf{6},\mathbf{4})_1 + (\mathbf{4},\mathbf{6})_{-1} \\
\varphi : &\quad (\mathbf{1},\mathbf{1})_4 + (\mathbf{1},\mathbf{1})_{-4} + (\bar{\mathbf{4}},\mathbf{4})_2 + (\mathbf{4},\bar{\mathbf{4}})_{-2} + (\mathbf{6},\mathbf{6})_0
\end{aligned}
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\begin{array}{lll}
g^*: & (\mathbf{1},\mathbf{1})_0 & \rightarrow \mathbf{1} \\
\Psi^*: & (\mathbf{4},\mathbf{1})_1 + (\mathbf{1},\mathbf{4})_{-1} & \rightarrow \mathbf{8} \\
A^*: & (\mathbf{6},\mathbf{1})_2 + (\mathbf{1},\mathbf{6})_{-2} + (\mathbf{4},\mathbf{4})_0 & \rightarrow \mathbf{28} \\
\psi^*: & (\bar{\mathbf{4}},\mathbf{1})_3 + (\mathbf{1},\bar{\mathbf{4}})_{-3} + (\mathbf{6},\mathbf{4})_1 + (\mathbf{4},\mathbf{6})_{-1} & \rightarrow \mathbf{56} \\
\varphi: & (\mathbf{1},\mathbf{1})_4 + (\mathbf{1},\mathbf{1})_{-4} + (\bar{\mathbf{4}},\mathbf{4})_2 + (\mathbf{4},\bar{\mathbf{4}})_{-2} + (\mathbf{6},\mathbf{6})_0 & \rightarrow \mathbf{70}
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Therefore $H = SU(8)$ and $G = E_{7(7)}$

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Can we find a formula which takes as input $(D, \mathcal{Q}_L, \mathcal{Q}_R)$ and gives out the respective H and G?

The FRT magic square

[Freudenthal '54, Rosenfeld '56, Tits '66]

$\mathbb{A}(L) \setminus \mathbb{A}(R)$	$\mathbb{O}$	$\mathbb{H}$	$\mathbb{C}$	$\mathbb{R}$
$\mathbb{O}$	$\mathfrak{e}_{8(8)}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_{6(-14)}$	$\mathfrak{f}_{4(-20)}$
$\mathbb{H}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{so}(8,4)$	$\mathfrak{su}(4,2)$	$\mathfrak{sp}(2,1)$
$\mathbb{C}$	$\mathfrak{e}_{6(-14)}$	$\mathfrak{su}(4,2)$	$2\mathfrak{su}(2,1)$	$\mathfrak{su}(2,1)$
$\mathbb{R}$	$\mathfrak{f}_{4(-20)}$	$\mathfrak{sp}(2,1)$	$\mathfrak{su}(2,1)$	$\mathfrak{so}(2,1)$

$$\mathfrak{L}_{1,2} = \mathfrak{tri}\Big(\mathbb{A}(L)\Big) \oplus \mathfrak{tri}\Big(\mathbb{A}(R)\Big) + 3\Big(\mathbb{A}(L) \otimes \mathbb{A}(R)\Big) \tag{8}$$

The $D = 3$ magic square

[Borsten-Duff-Hughes-Nagy '13]

$\mathcal{N}_L \setminus \mathcal{N}_R$	8	4	2	1
8	$\mathfrak{e}_{8(8)}$	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_{6(-14)}$	$\mathfrak{f}_{4(-20)}$
4	$\mathfrak{e}_{7(-5)}$	$\mathfrak{so}(8,4)$	$\mathfrak{su}(4,2)$	$\mathfrak{sp}(2,1)$
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$$\mathfrak{g} = \left(\mathfrak{int}(\mathcal{Q}_L) \oplus \mathfrak{int}(\mathcal{Q}_R) + \mathbf{s}^{\mathcal{N}_L} \otimes \mathbf{s}^{\mathcal{N}_R}\right) + \left(\mathbf{s}^{\mathcal{N}_L} \otimes \mathbf{s}^{\mathcal{N}_R} + \mathbf{S}(\mathcal{Q}_L) \otimes \mathbf{S}(\mathcal{Q}_R)\right) \quad (9)$$

Extension to $3 \leq D \leq 10$

Start with $\mathfrak{p}$ because it is the space parametrised by the supergravity scalars:

$$\mathfrak{p}(3, \mathcal{Q}_L, \mathcal{Q}_R) = \mathbf{s}(3)^{\mathcal{N}_L} \otimes \mathbf{s}(3)^{\mathcal{N}_R} + \mathbf{S}(3, \mathcal{Q}_L) \otimes \mathbf{S}(3, \mathcal{Q}_R) \qquad (10)$$

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(12)

(13)

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$$\begin{aligned} \mathfrak{p}(D, \mathcal{Q}_L, \mathcal{Q}_R) = & (1 - \delta_{3D} + i\delta_{4D})\mathbb{R} \otimes \mathbb{R} + \mathbb{D}[\mathcal{N}_L, \mathcal{N}_R] \\ & + \mathbf{S}(D, \mathcal{Q}_L) \otimes \mathbf{S}(D, \mathcal{Q}_R) \end{aligned} \qquad (11)$$

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Use this to build the compact subgroup $\mathfrak{h}$:

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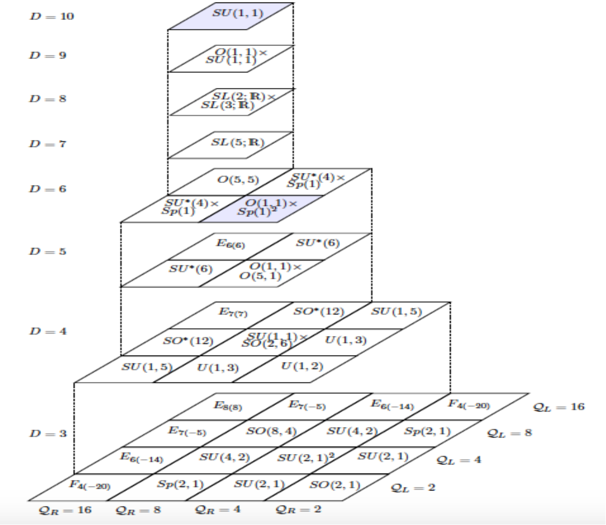
A magic pyramid of supergravities



Yang-Mills origin of H

Can we find a Yang-Mills interpretation of H?

Can we find a Yang-Mills origin of all the $\dim(H)$ parameters?

How do we transform the Yang-Mills pieces such that the net effect is a full H tranformation on the supergravity fields?

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$$\underset{H}{\delta}\left(\Phi^i \circ \Phi^{i'}\right) = \underset{int_L}{\delta} \Phi^i \circ \Phi^{i'} + \Phi^i \circ \underset{int_R}{\delta} \Phi^{i'} + \text{Scalar}\left(\underset{\epsilon_L}{\delta}\Phi^i \circ \underset{\epsilon_R}{\delta}\Phi^{i'}\right) \tag{15}$$

The transformations

Focus on local symmetries and thus work with off-shell fields and drop global internal indices. vector $\otimes$ vector.

$$\delta A^A_\mu = \partial_\mu \vartheta^A - f_{BC}{}^A \theta^B A^C_\mu \tag{16}$$

$$\delta \tilde{A}^{A'}_\nu = \partial_\nu \tilde{\vartheta}^{A'} - f_{B'C'}{}^{A'} \tilde{\theta}^{B'} \tilde{A}^{C'}_\nu \tag{17}$$

$$\delta \Phi_{AA'} = f_{BA}{}^C \theta^B \Phi_{CA'} + f_{B'A'}{}^{C'} \theta^{B'} \Phi_{AC'} \tag{18}$$

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Using:

$$Z_{\mu\nu}(x) \equiv \left(A_\mu \circ \tilde{A}_\nu\right)(x) = \left(A^A_\mu \star \Phi_{AA'} \star \tilde{A}^{A'}_\nu\right)(x) \tag{19}$$

we want to reproduce:

$$\delta Z_{\mu\nu} = \partial_\mu \alpha_\nu + \partial_\nu \beta_\mu \tag{20}$$

[AA-Borsten-Duff-Hughes-Nagy '14]

The relation

The transformation calculation gives:

$$\delta Z_{\mu\nu}(x) = \left(\delta A^{A}_{\mu} \star \Phi_{AA'} \star \tilde{A}^{A'}_{\nu} + A^{A}_{\mu} \star \delta\Phi_{AA'} \star \tilde{A}^{A'}_{\nu} + A^{A}_{\mu} \star \Phi_{AA'} \star \delta\tilde{A}^{A'}_{\nu} \right)(x) \tag{21}$$

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&= \partial_\mu \left(\vartheta^A \star \Phi_{AA'} \star \tilde{A}^{A'}_\nu \right)(x) - \left(f_{BC}{}^A \theta^B A^C_\mu \star \Phi_{AA'} \star \tilde{A}^{A'}_\nu \right)(x) \\
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&= \partial_\mu \left(\vartheta^A \star \Phi_{AA'} \star \tilde{A}^{A'}_\nu \right)(x) + \partial_\nu \left(A^A_\mu \star \Phi_{AA'} \star \tilde{\vartheta}^{A'} \right)(x) \qquad (21)
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\end{aligned}$$

A similar calculation reproduces the gauge transformations for the gravitini and p-form gauge potentials.

Outlook

The two most obvious improvements would be to:

- Find the Yang-Mills origin of the non-compact generators in G.
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Thank you!