Carving out EFT Space via Soft Limits

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1607.xxxxx w/ C. Cheung, K. Kampf, J. Novotny, J. Trnka

Outline

- Motivation—EFTs from Amplitudes
- Classification Scheme and Tools
- Carving out EFT Space
- Exceptional EFTs
- Conclusion & Outlook

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Hidden Simplicity in Amplitudes

- Simplicity in amplitudes v.s. Lagrangian

  $\mathcal{N}$=4 sYM and $\mathcal{N}$=8 SUGRA as simplest QFTs.

  - Many exciting developments

- Any analog in EFTs?

  Where is the simplicity?

$$\mathcal{L} = \sum_{m,n=0}^{\infty} \lambda_{m,n}\, \partial^m \phi^n$$

Scalar EFTs from Soft Limits

- Focus on soft limits: [See also Marotta's talk]

$$A(p \to 0) \sim p^{\sigma}$$

- Amplitudes have non-trivial structure if the soft limit is enhanced

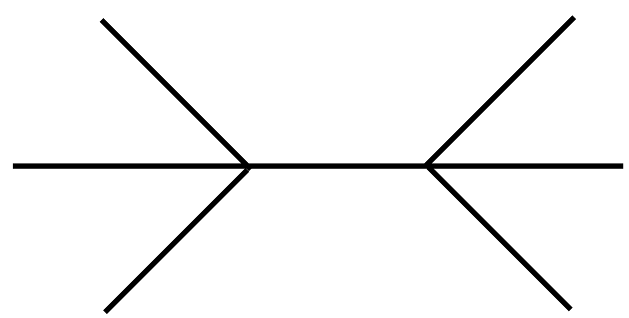
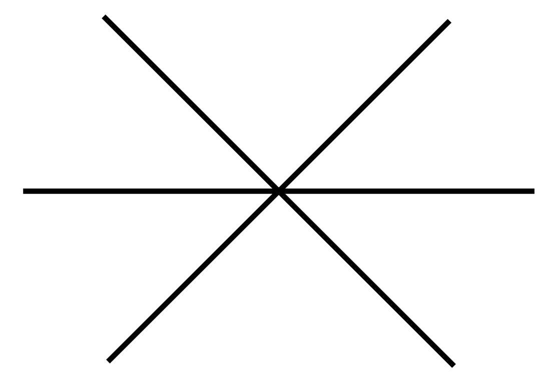
$$\sigma > \frac{m}{n} \quad \text{for} \quad \mathcal{L} \supset \lambda_{m,n}\, \partial^m \phi^n$$

Scalar EFTs from Soft Limits

- Example: a two-derivative theory

$$\mathcal{L} \supset \lambda_{2,4}\partial^2\phi^4 + \lambda_{2,6}\partial^2\phi^6$$

- Vanishing of six point amplitude requires non-trivial cancellation

$\lambda_{2,4}$  $+ \lambda_{2,6}$ 

- Relate higher point vertices to lower point

Scalar EFTs from Soft Limits

[Cheung, Kampf, Novotny, Trnka 1412.4095]

- Bottom-up search found well-known EFTs as non-trivial ones

σ	EFT
1	NLSM
2	DBI/Galieon
3	special Galieon (sGal)

- Natural Classification of EFTs from soft limits

- Anymore nontrivial theory? What's the space of nontrivial EFTs?

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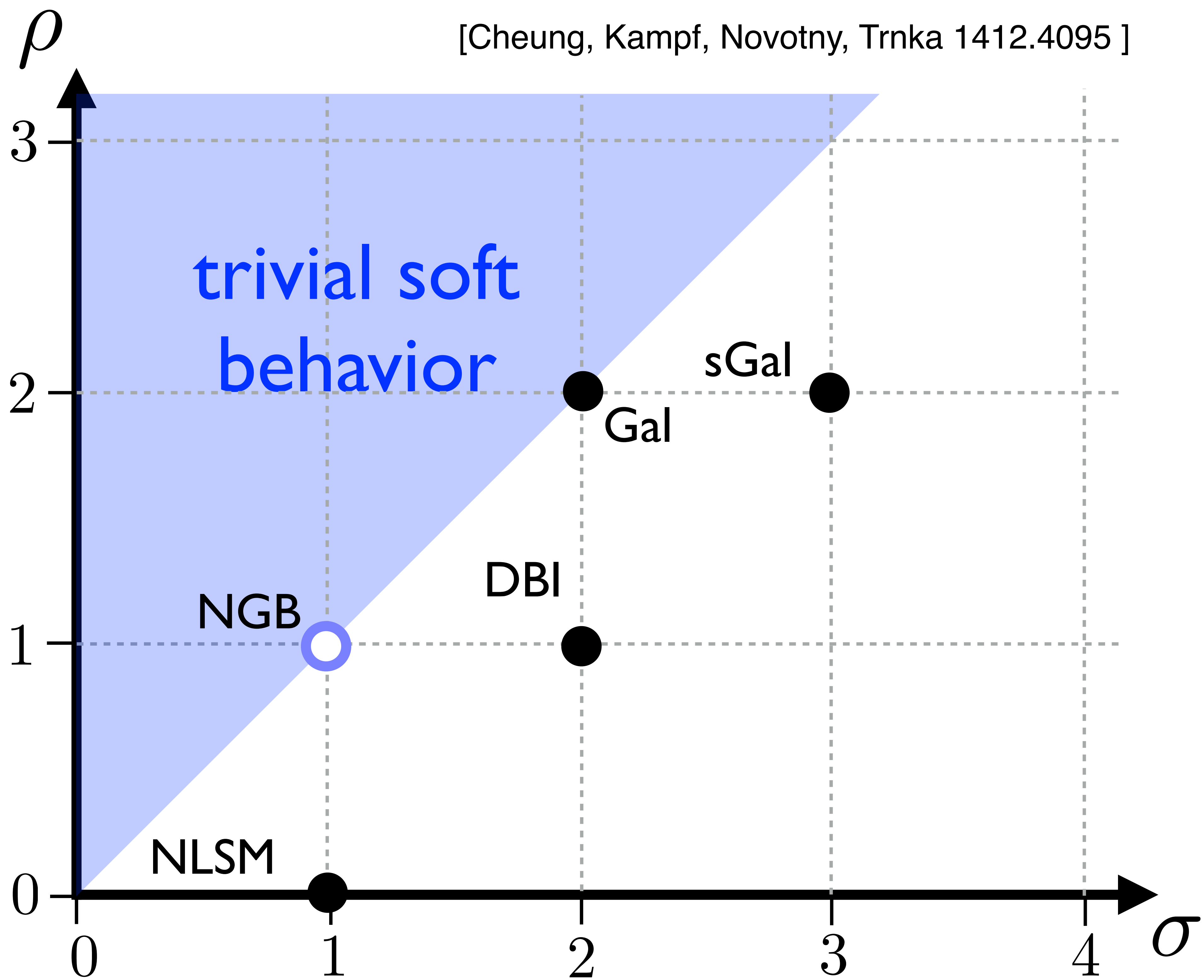
Theory Space

- Parameters: (ρ, σ, d, v)
  - $\rho = \dfrac{m-2}{n-2}$ for $\mathcal{L} \supset \partial^m \phi^n$
    ~average number of derivatives
  - σ : soft degree
  - $d \geq 4$: spacetime dimension
  - v : valency of leading vertex
- Nontrivial regime: $\rho \leq \sigma$

Theory Space

[Cheung, Kampf, Novotny, Trnka 1412.4095]



Soft Momentum Shifts

- Probing soft limits via momentum deformation

$$p_i \to p_i(1 - z a_i)$$

(BCFW $p \to p - zq$ doesn't see soft limit)

- All-line Soft Shift
  - Momentum conservation $\sum_{i=1}^{n} a_i p_i = 0$ and distinct $\{a_i\}$ requires $n > d + 1$

[Cheung, Kampf, Novotny, CHS, Trnka 1509.03309]

Soft Momentum Shifts

- All-but-one-line Soft Shift

$$p_i \to p_i(1 - z a_i), \quad 1 \le i \le n-1$$

$$p_n \to p_n + z q_n, \quad q_n = \sum a_i p_i$$

  - Applicability: $n > 4$

- All-but-two-line Soft Shift

$$p_i \to p_i(1 - z a_i), \quad 1 \le i \le n-2$$

$$p_{n-1} \to p_{n-1} + z q_{n-1}$$

$$p_n \to p_n + z q_n, \quad q_n + q_{n-1} = \sum a_i p_i$$

  - Applicability: $n \ge 4$

Soft Recursion Relations

[Cheung, Kampf, Novotny, CHS, Trnka 1509.03309; see also Feng's talk]

- Amplitudes are fixed by

Factorization + Soft limits ("gauge invariance")

$$\oint \frac{dz}{z} \frac{A_n(z)}{F_n(z)} = 0$$

- Denominator $F_n(z) = \prod_{i=1}^{n_s} (1 - a_i z)^\sigma$ tames large z behavior
- No new poles: $A(z \to \frac{1}{a_i}) \sim (1 - a_i z)^\sigma$

Poles from factorization $\longrightarrow$ recursion relations!

Soft Recursion Relations

- Recursion Relations

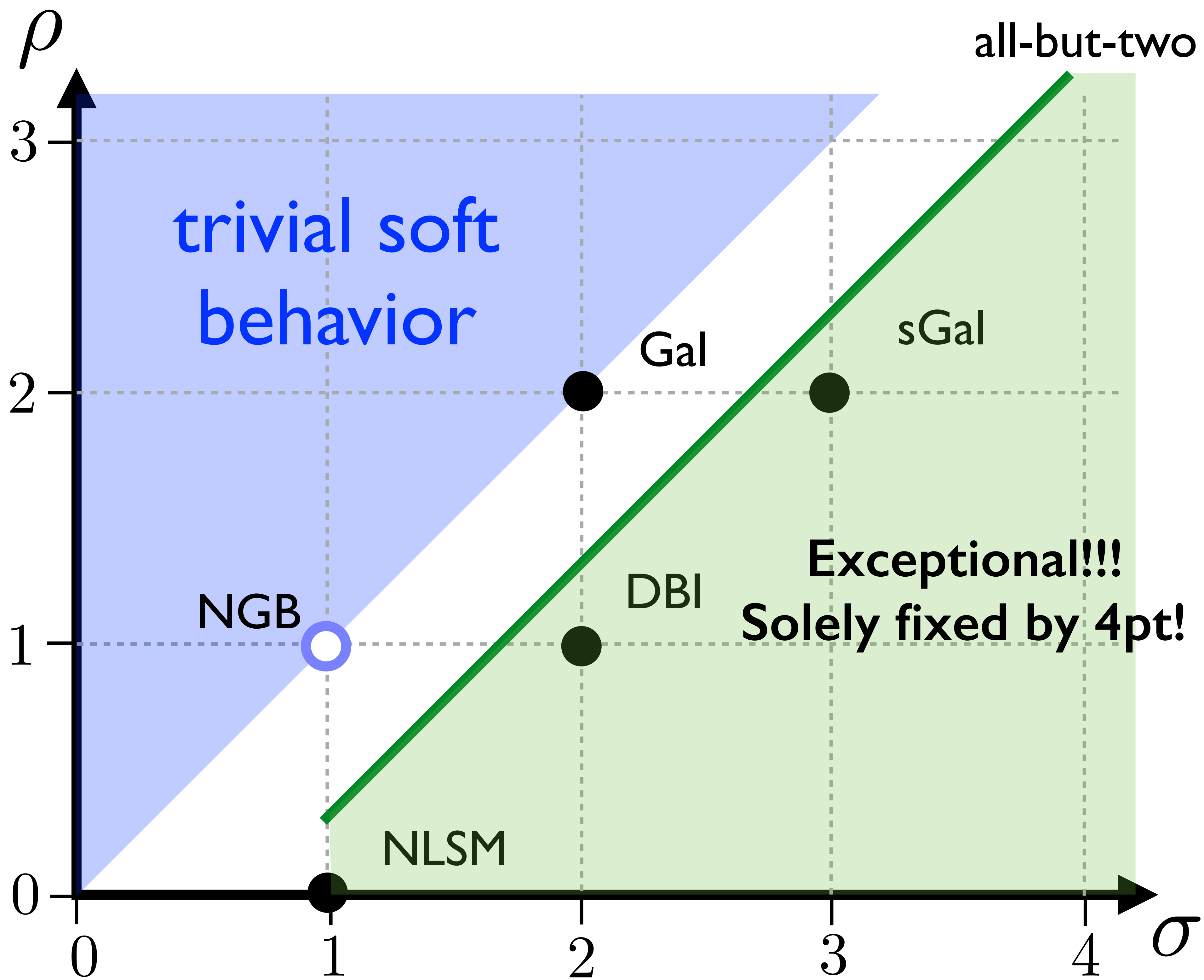
$$A_n(0) = \sum_I \frac{1}{P_I^2} \frac{A_{L,R}(z_{I-})}{(1 - z_{I-}/z_{I+})F(z_{I-})} + (z_{I+} \leftrightarrow z_{I-})$$

- Validity depends on derivative counting v.s. soft degree

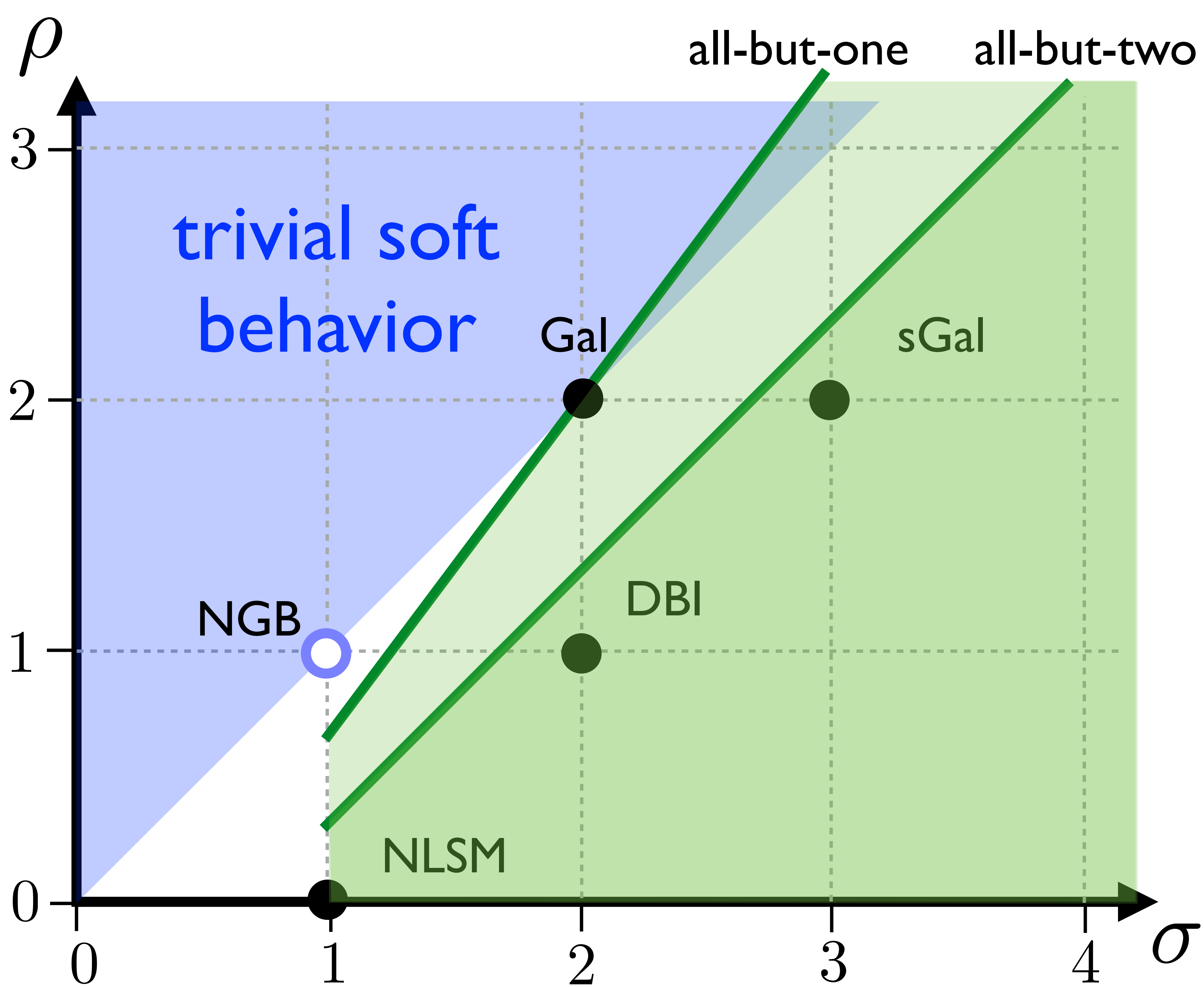
$$\rho < \frac{n_s \sigma - 2}{n - 2} \to \sigma$$

- Nontrivial theories are constructible

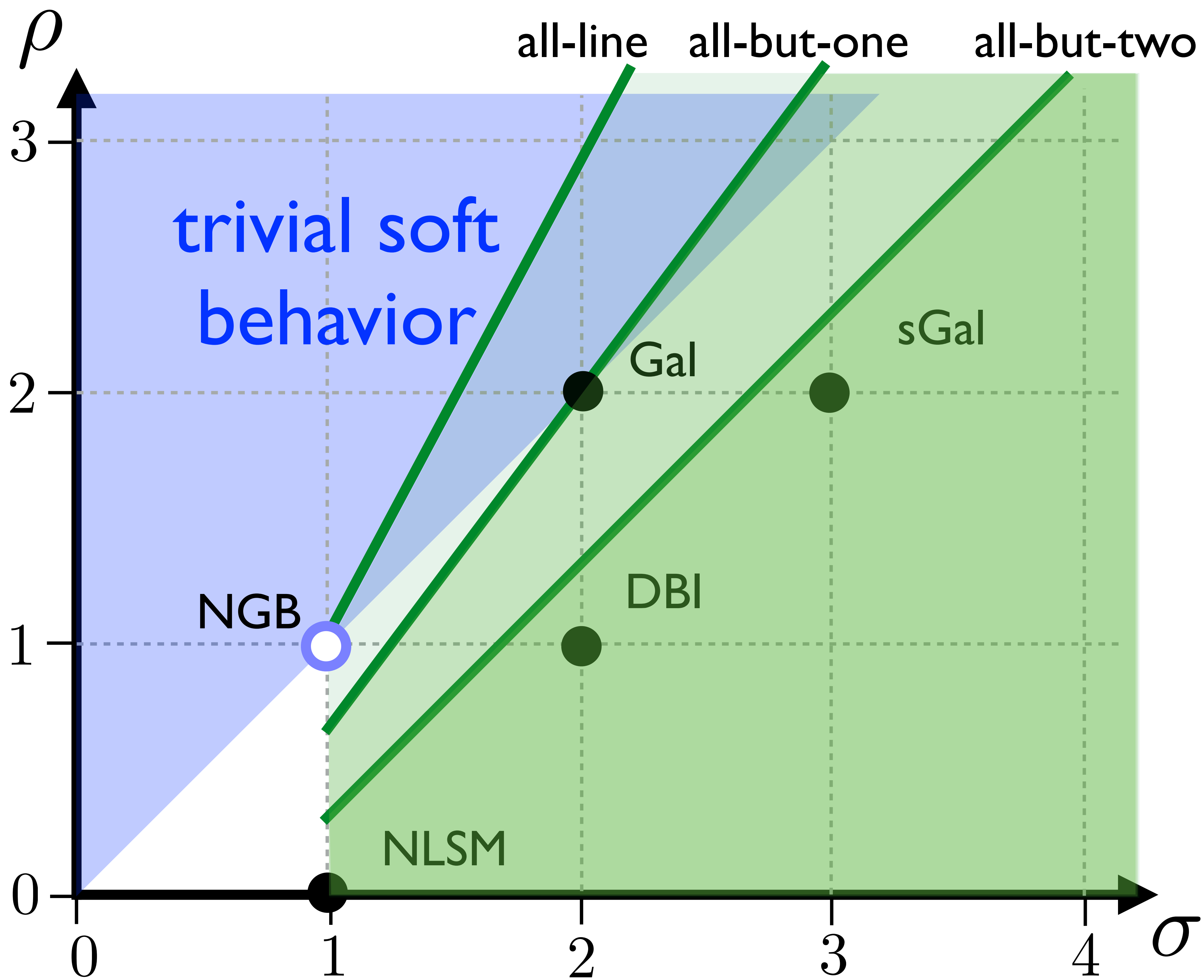
Constructibility



Constructibility



Constructibility



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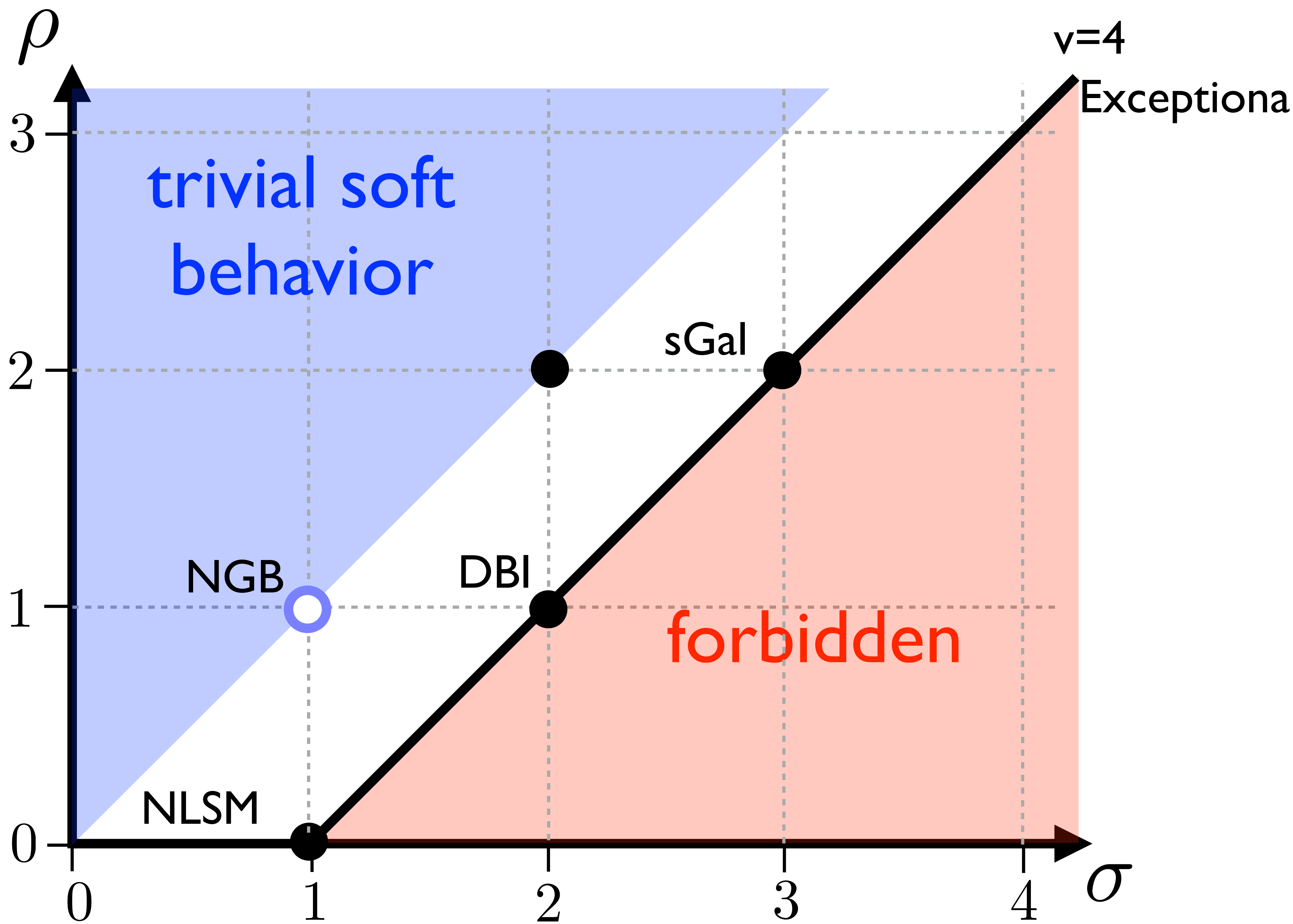
Soft Limit from Contact Amplitudes

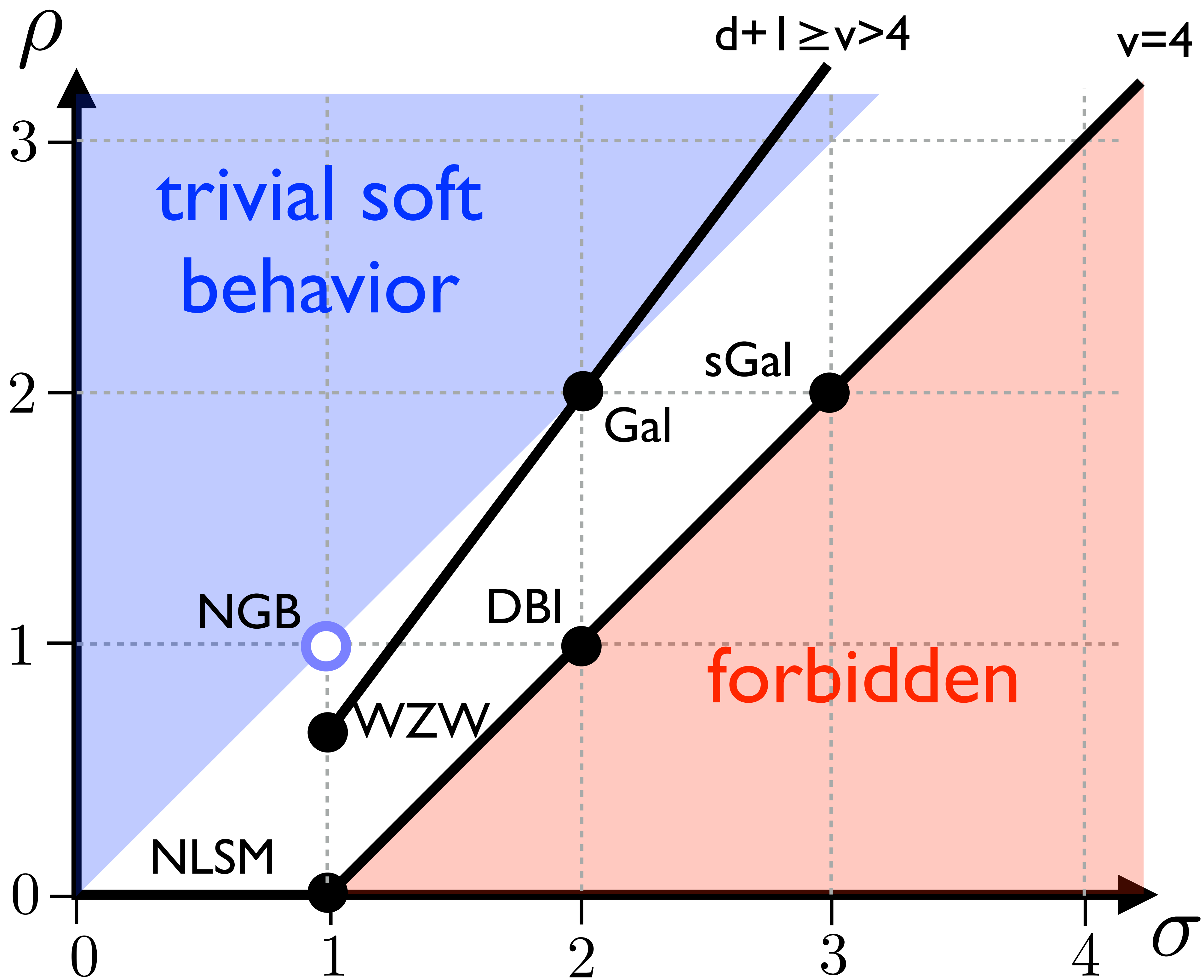
- What's the best soft limit from a contact amplitude?
  - Contact: no pole from factorization
  - Constructible: no pole at infinity
- Not compatible! Soft degree cannot be too strong

$$\rho \geq \frac{n_s \sigma - 2}{n - 2}$$

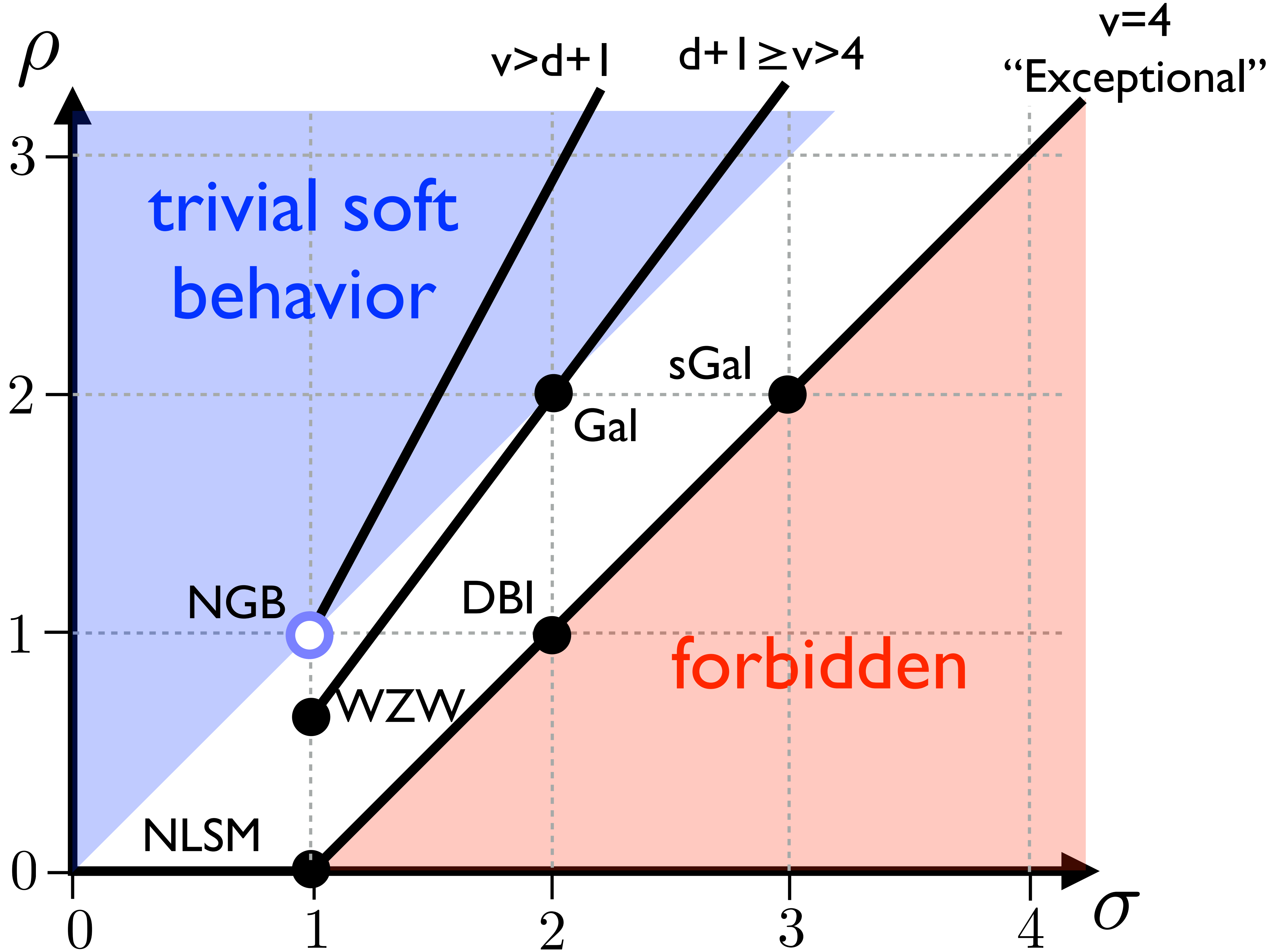
Soft Limit from Contact Amplitudes



Soft Limit from Contact Amplitudes



Soft Limit from Contact Amplitudes



Locality in Higher Point Amplitudes

- Still, infinite number of theories are allowed
- Constraint from higher point amplitudes?
- Non-local expression is very common

$$\oint \frac{dz}{z} \frac{A_n(z)}{F_n(z)} = 0$$

$$F_n(z) = \prod_{i=1}^{n_s} (1 - a_i z)^{\sigma}$$

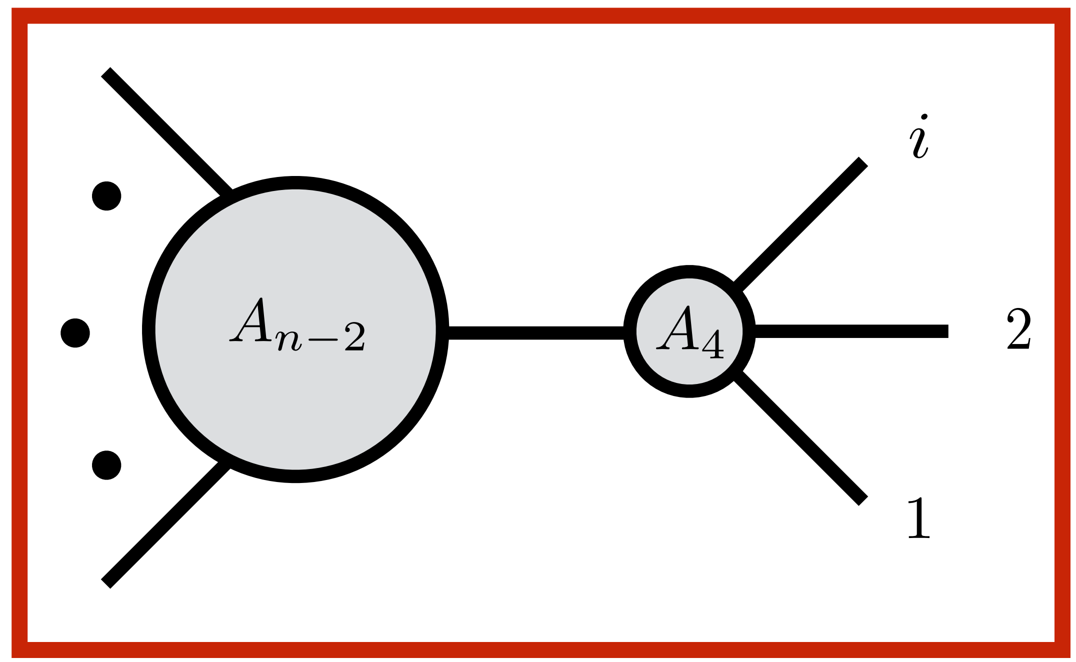
- The physical amplitudes should be free from $\{a_i\}$
  - Cancellation of non-local (spurious) poles

Locality in Higher Point Amplitudes

- When does spurious pole $a_1 - a_2$ appears?

$$A_n(0) = \sum_I \frac{1}{P_I^2} \frac{A_{L,R}(z_{I-})}{(1 - z_{I-}/z_{I+})F(z_{I-})} + (z_{I+} \leftrightarrow z_{I-})$$

- Singularity $(1 - a_i z_\pm) = 0$
  - Soft limit: $z \sim 1/a_1 \sim 1/a_2$
  - Factorization: $P^2(z = z_\pm) = 0$
- Spurious pole should cancel upon summing over channels



Locality in Higher Point Amplitudes

- The spurious pole is roughly the double soft limit!
  - “Hard” part is universal
  - “Soft” part only depends on $A_4 = \sum_{r=0}^{\rho+1} c_r\, u^r\, s^{\rho+1-r}$
- Leading Spurious pole:

$$\frac{c_r \sum_{i=3}^{n} (2p_{12}\bar{p}_i)^{r-1}}{(a_1 - a_2)^r}$$

  - r=1: need cancellation of different couplings
  - r=2: saved by momentum conservation $\sum_{i=3}^{n} \bar{p}_i = 0$
  - r>2: no cancellation!

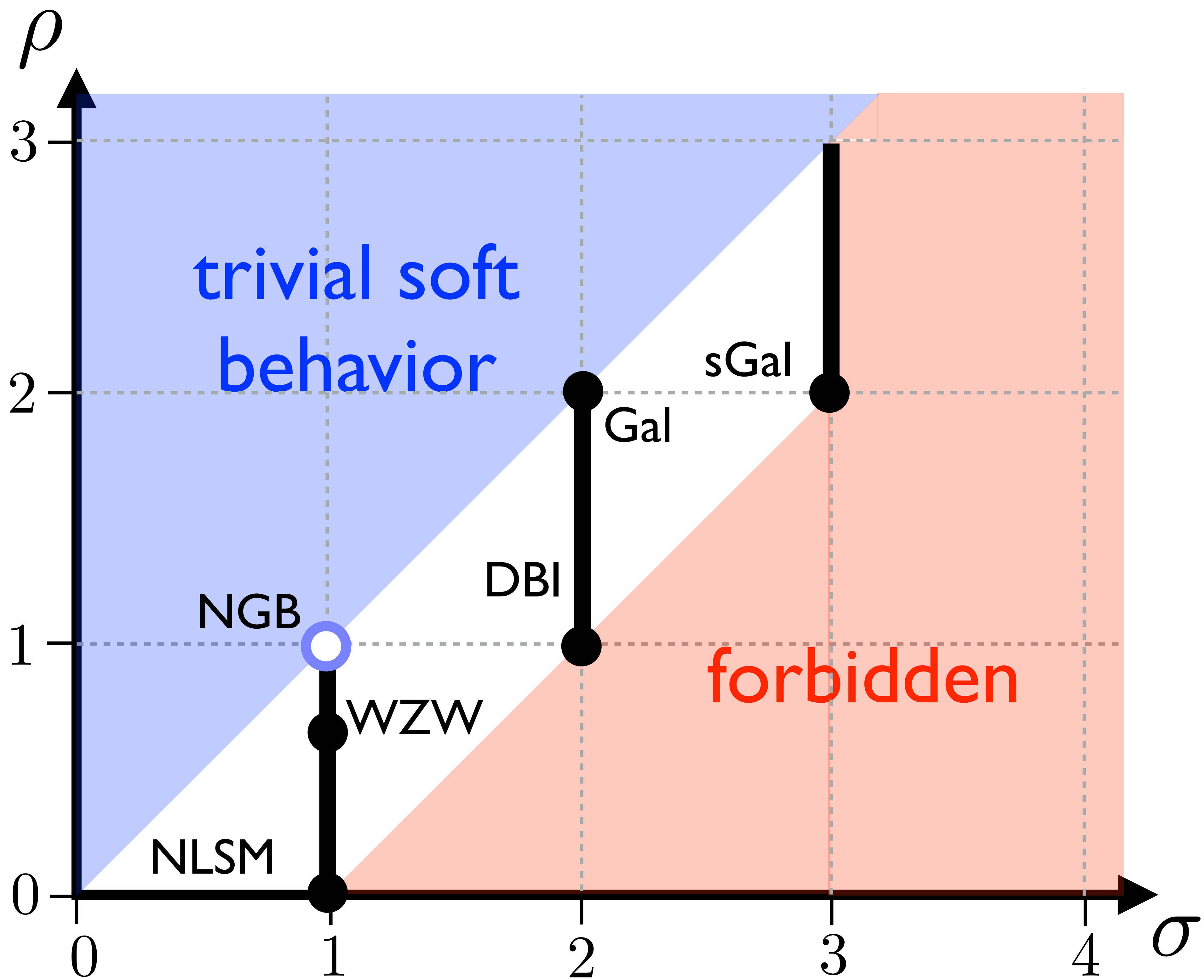
Locality in Higher Point Amplitudes

- The power of s,t,u cannot be greater than two
- Rule out non-trivial theory with $\sigma > 3 \Leftrightarrow$ **no higher spin**
- Rule out color-ordered exceptional theory beyond NLSM
- Sanity Check
  - NLSM: r=1 $A_4(612I) = -s_{16} - s_{12}, \quad A_4(123I) = +s_{13}$ ✓
  - DBI: r=2, global momentum conservation ✓
  - sGal: Naively r=3, but r=2 on-shell

$$A_4 = s^3 + t^3 + u^3 = -3s^2u - 3su^2$$ ✓

- Very similar to gauge invariance of soft factor in YM/GR!

Periodic Table of EFTs



Exceptional EFTs

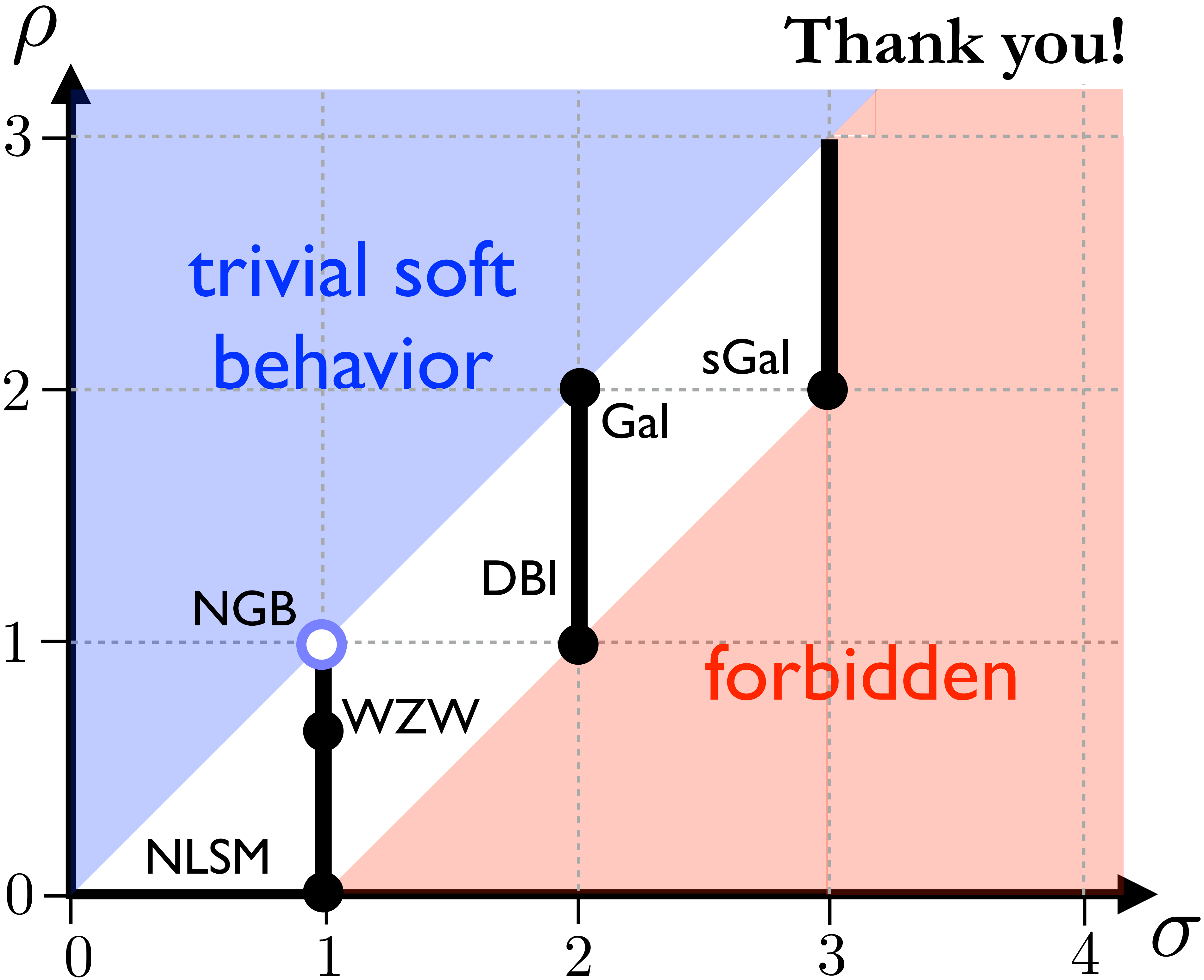
- NLSM, DBI, sGal—simplest EFTs?
  - All tree amplitudes are fixed by 4pt
  - Natural EFTs appear in CHY formalism

    [Cachazo, He, Yuan, 1412.3479; see He's talk]

Conclusion & Outlook

- Classification of scalar EFTs via soft limits
- No $\sigma > 3$ non-trivial theory;
  No $\sigma > 1$ color-ordered exceptional theory
  - Contact amplitudes+locality of higher points
- NLSM, DBI, and sGal as Exceptional EFTs
- Classification beyond vanishing soft limits?
  [See also Marotta's talk]
- Classification beyond scalar EFTs?
  [1509.07840,1512.03316,1512.06801,1605.08697,…]

Thank you!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3
2
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0
trivial soft
behavior
sGal
Gal
DBI
NGB
WZW
NLSM
forbidden
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σ