From maximal to minimal supersymmetry in string loop amplitudes

Marcus Berg
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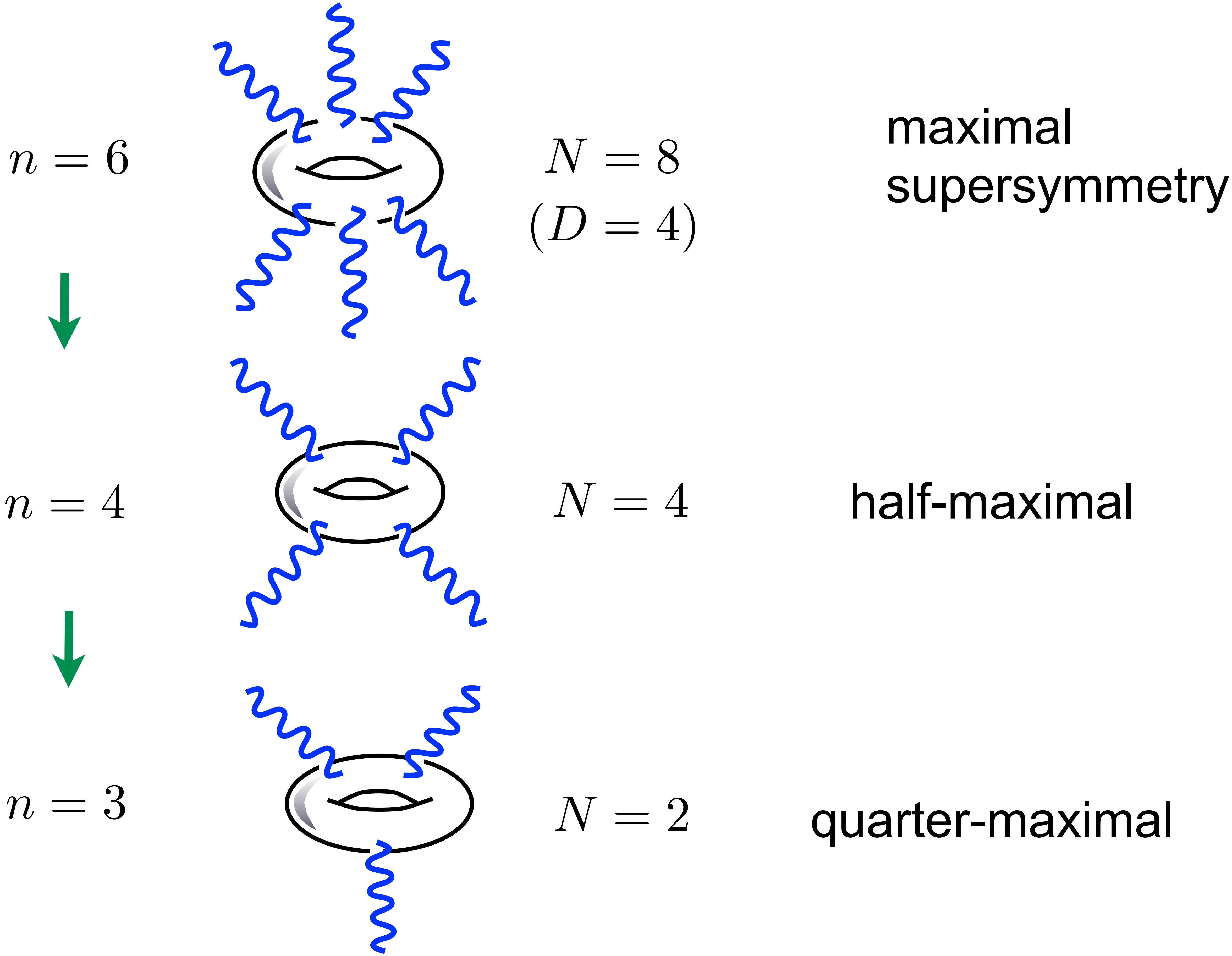
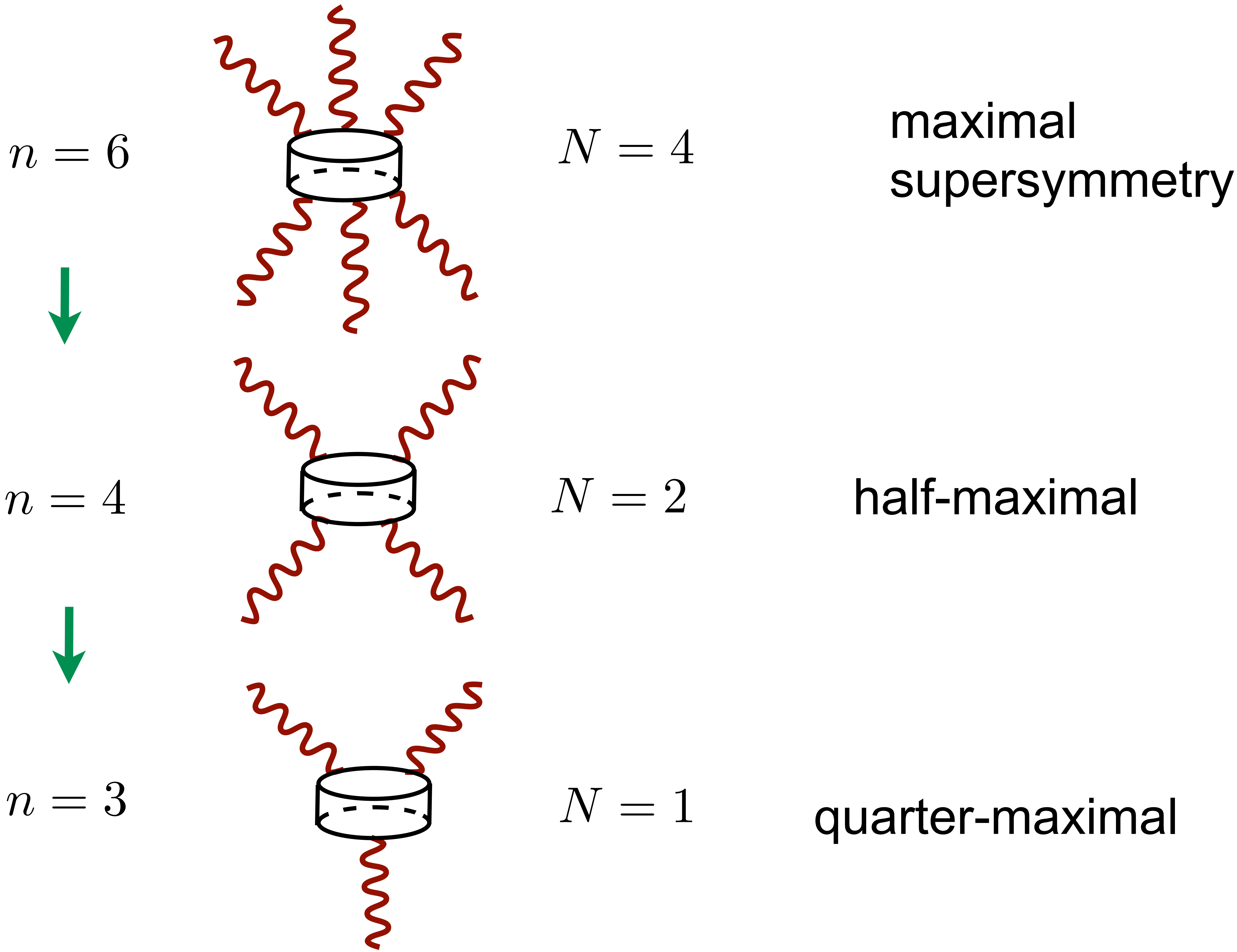
M.B., Buchberger, Schlotterer 1603.05262
M.B., Buchberger, Schlotterer 16xx.xxxxx

Nordita
2016

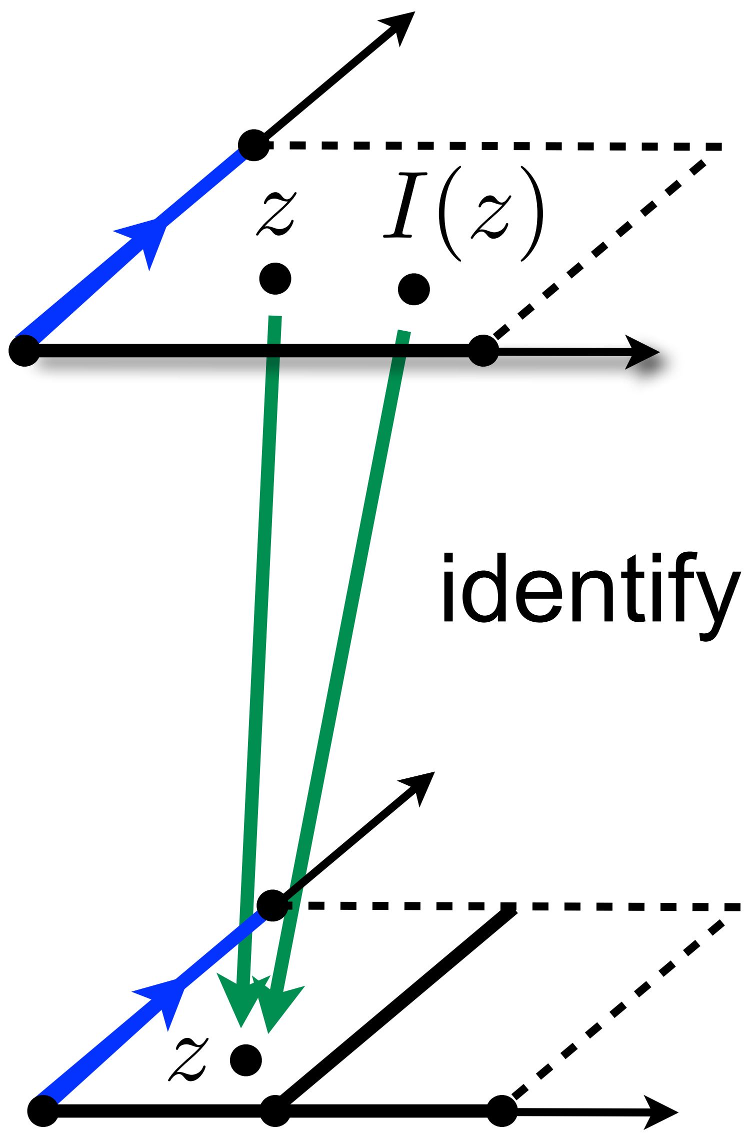


$n = 6$
$N = 8$
$(D = 4)$
maximal
supersymmetry
$n = 4$
$N = 4$
half-maximal
$n = 3$
$N = 2$
quarter-maximal

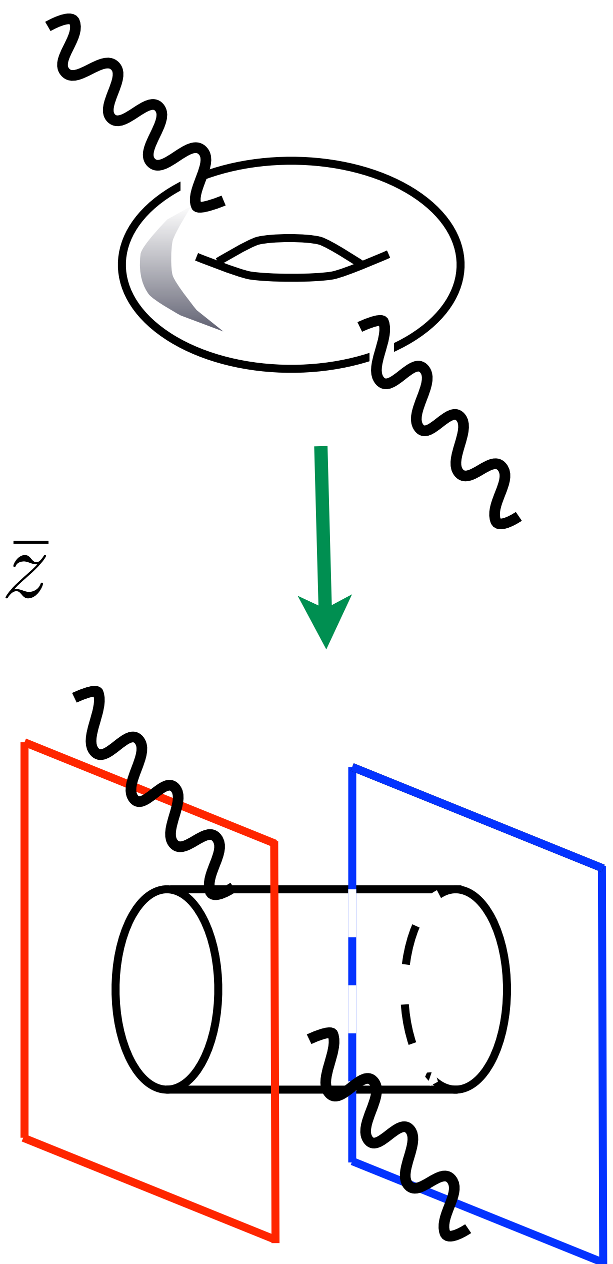

$n = 6$
$N = 4$
maximal supersymmetry
$n = 4$
$N = 2$
half-maximal
$n = 3$
$N = 1$
quarter-maximal

Open and unoriented worldsheets by identification

M.B., Haack, Kang, Sjörs ’14



identify $I(z) = 1 - \bar{z}$

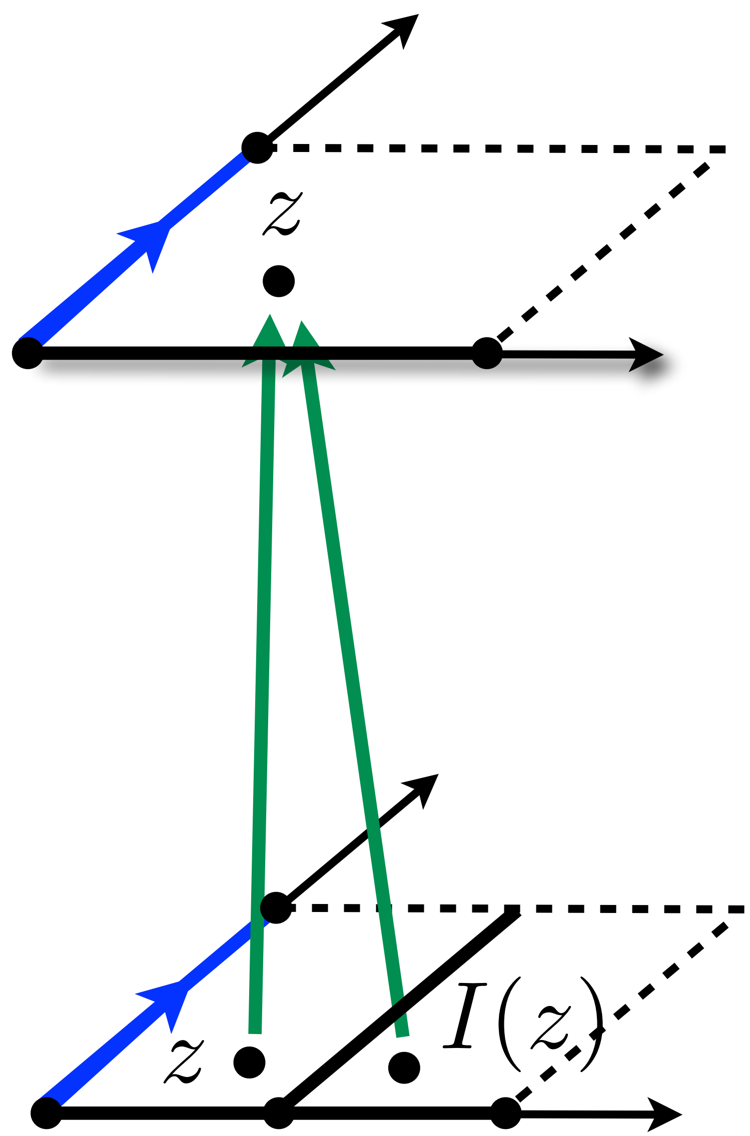


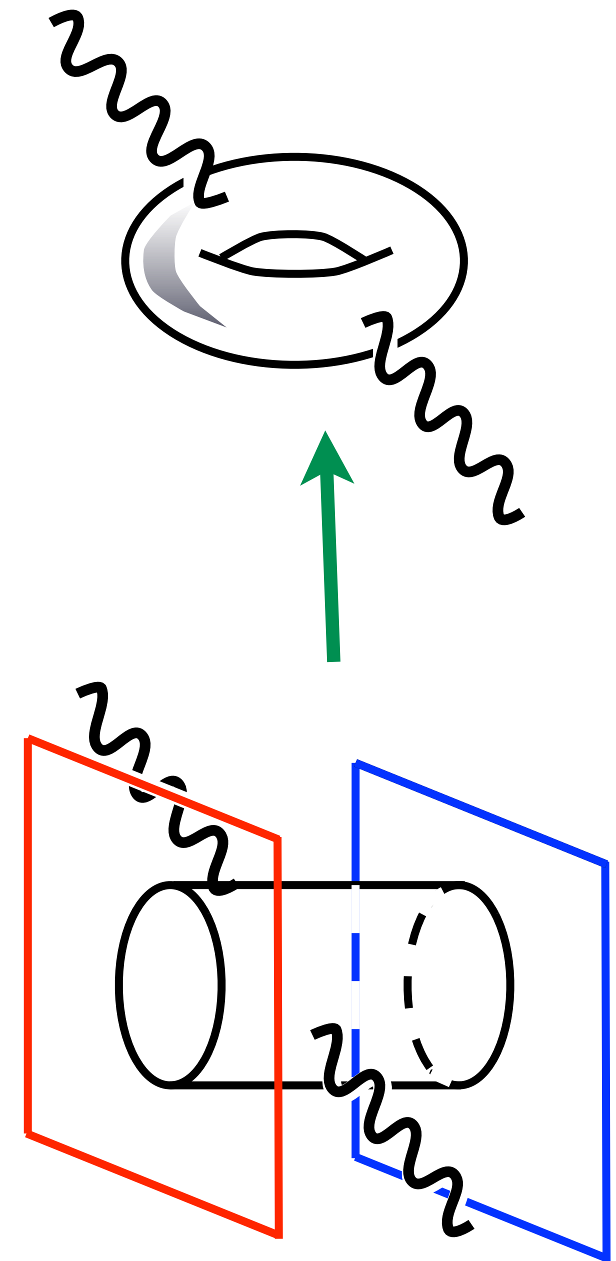
fixed line = boundary

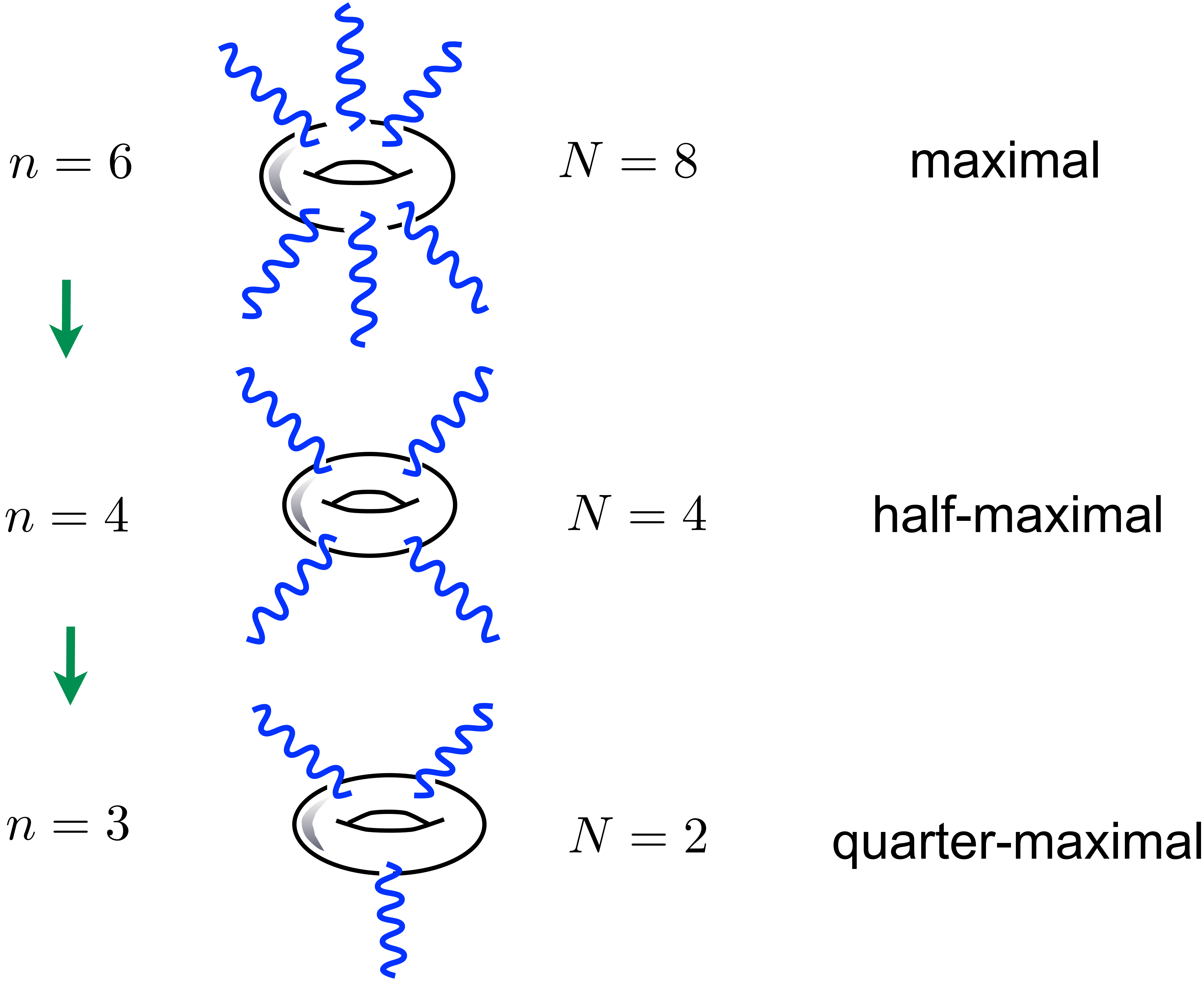
Lift to covering torus

M.B., Haack, Kang, Sjörs '14

$$\int_{\Sigma} d^2z \; (f(z) + f(I(z))) = \int_{\mathcal{T}} d^2z \; f(z)$$







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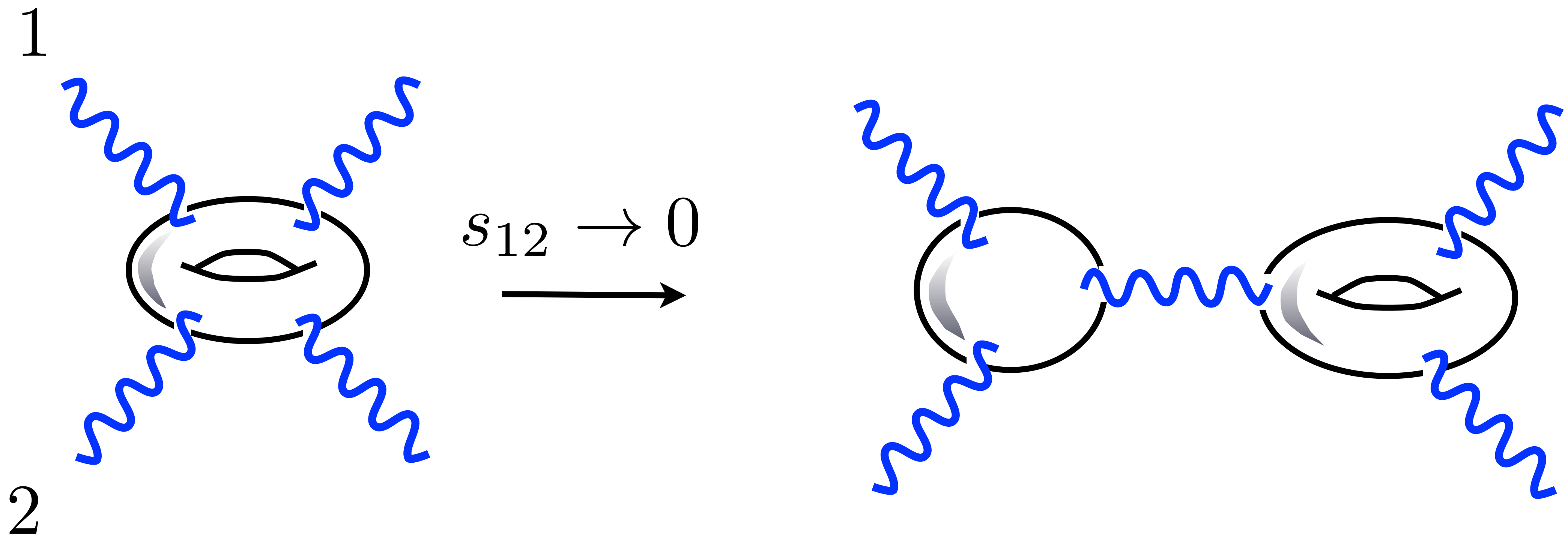


infrared regularization

Minahan '87

Motivations

- moduli stabilization needs quantum effective action

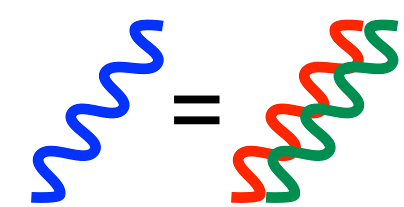
Balasubramanian, Berglund, Conlon, Quevedo ’05

(example: Large volume scenario)

... dynamics of light moduli lead to phenomenology, cosmology

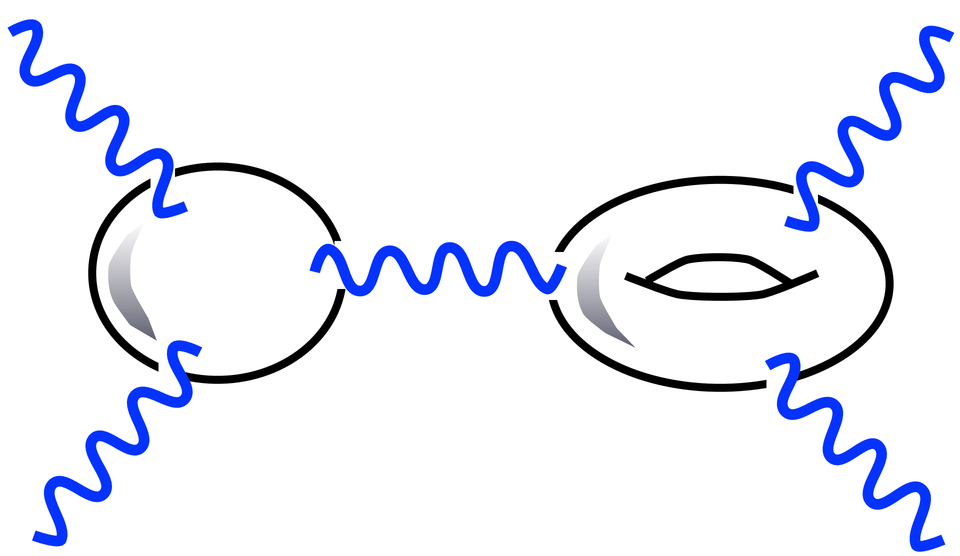
- interplay string/field theory amplitudes (KLT, BCJ, ...)

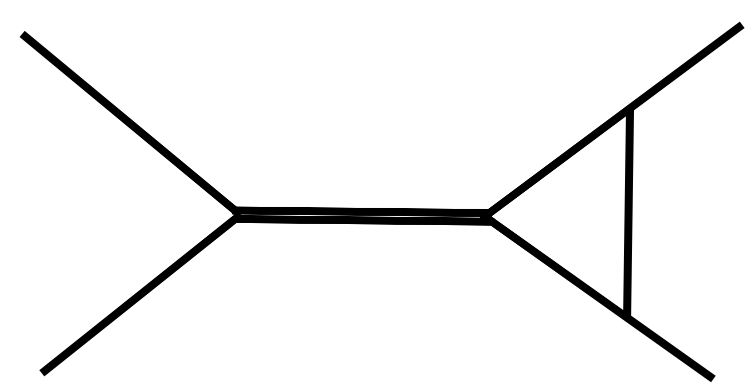
$$e_{\mu\nu} = e_\mu \tilde{e}_\nu$$



...

Stieberger, Taylor ’15





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$N=4$ YANG–MILLS AND $N=8$ SUPERGRAVITY AS LIMITS OF STRING THEORIES*

Michael B. GREEN[1] and John H. SCHWARZ

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Received 29 December 1981

The formulation of supersymmetric string theories in ten dimensions is generalized to incorporate compactified dimensions. Expressions for the one-loop four-particle S-matrix elements of $N=4$ Yang–Mills and $N=8$ supergravity in four dimensions are obtained by studying the string-theory loop amplitudes in the limit that the radii of the compactified dimensions and the Regge slope parameter simultaneously approach zero. If certain patterns that emerge should persist in the higher orders of perturbation theory, then $N=4$ Yang–Mills in four dimensions would be ultraviolet finite to all orders, whereas $N=8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops.

1. Introduction

A light-cone-gauge action for supersymmetric strings in ten-dimensional space-time was recently formulated [1]. Depending on the choice of boundary conditions,

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"In order to decide whether theory I or theory II is the more promising candidate for phenomenology, some further theoretical developments may be necessary. For example, it is clearly important to incorporate symmetry breaking..."

persist in the higher orders of perturbation theory, then $N = 4$ Yang–Mills in four dimensions would be ultraviolet finite to all orders, whereas $N = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops.

1. Introduction

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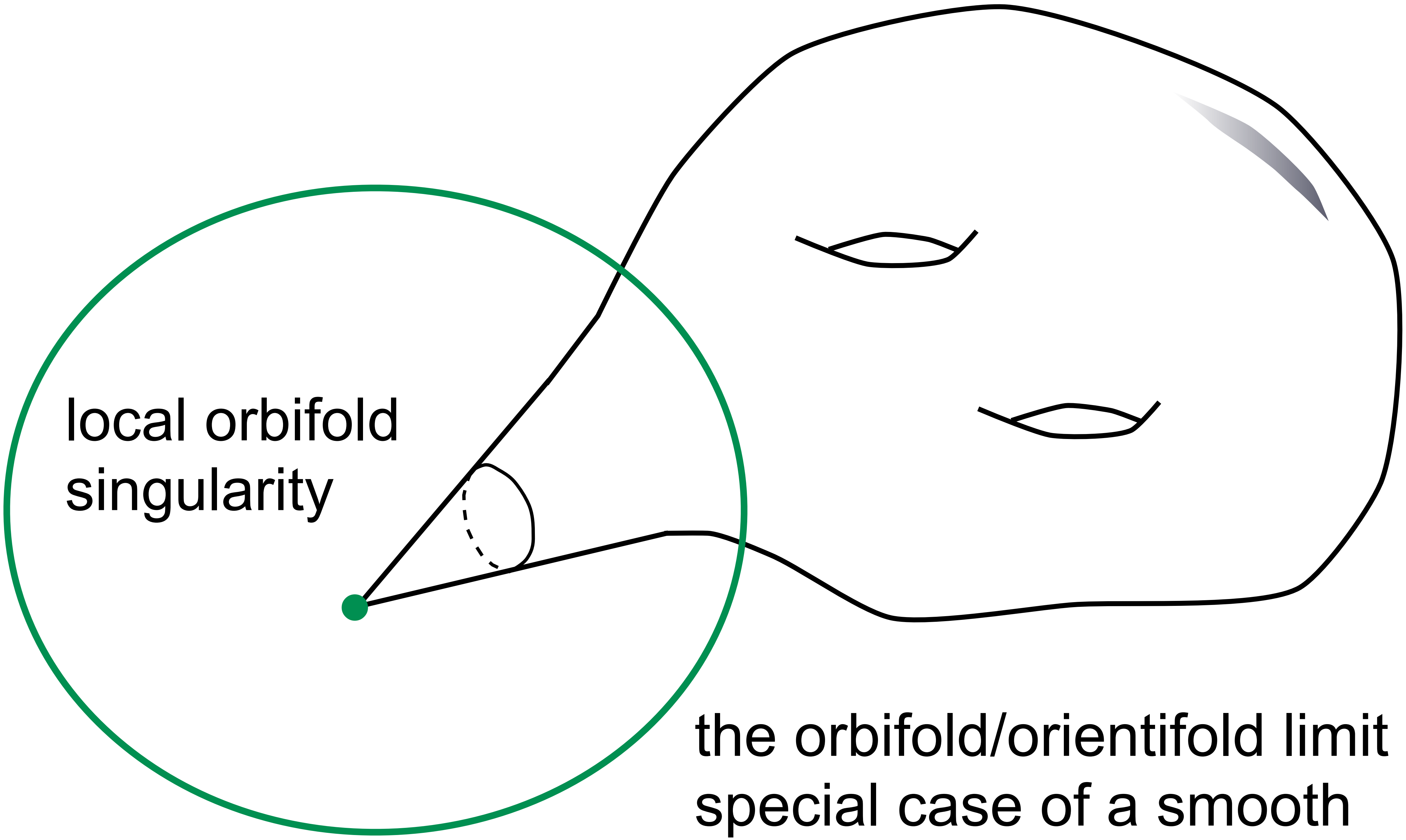
Simple models for extra dimensions

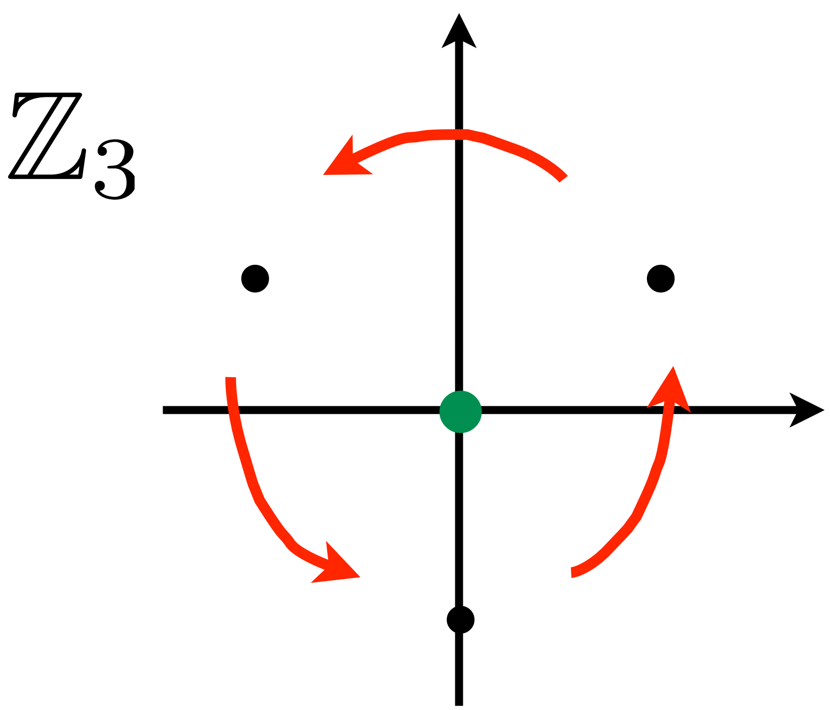
moduli: S, T, U, ϕ



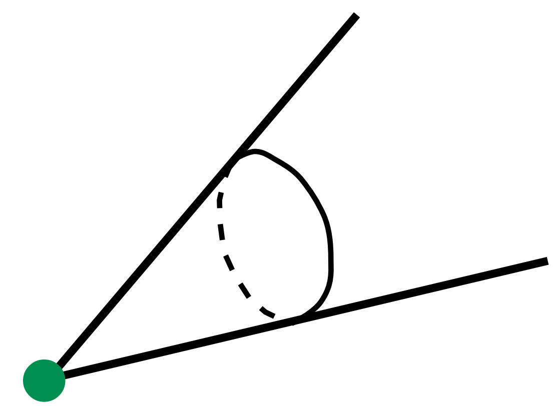
the orbifold/orientifold limit is a special case of a smooth Calabi-Yau manifold.

Simple models for extra dimensions

Orbifold: Identify under discrete spatial rotation



makes cone



$$\Theta Z^1 = e^{2\pi i v_1} Z^1 \text{ , here } v_1 = \frac{1}{3}$$

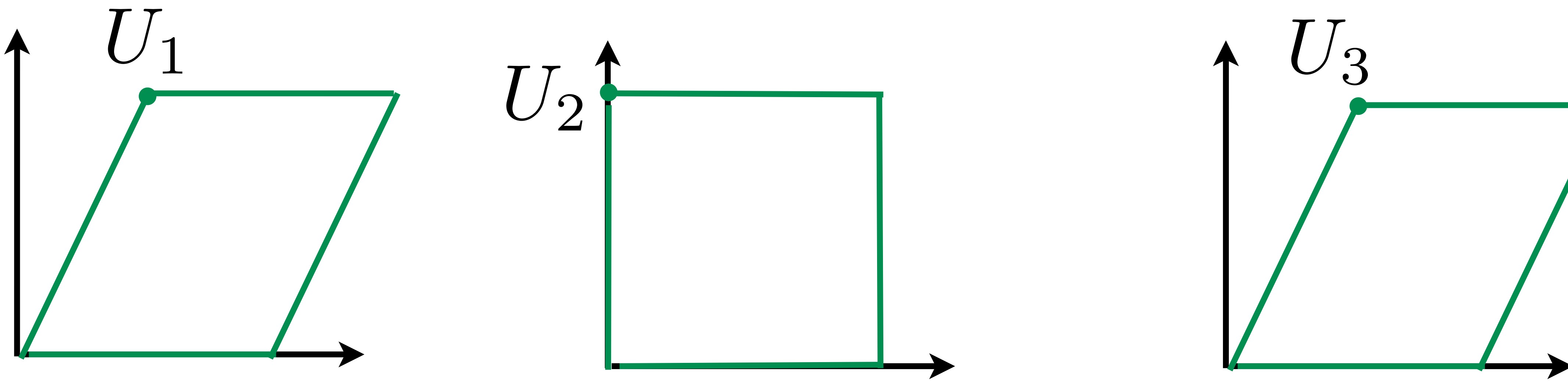
$$(Z^1 = X^4 + \bar{U} X^5)$$

Simple models for extra dimensions

sample $\mathcal{N}=1$ orbifold: $\mathbb{T}^6/\mathbb{Z}_6'$

$(Z^1 = X^4 + \bar{U}X^5)$

$\Theta Z^1 = e^{2\pi i v_1} Z^1 \quad \Theta Z^2 = e^{2\pi i v_2} Z^2 \quad \Theta Z^3 = e^{2\pi i v_3} Z^3$



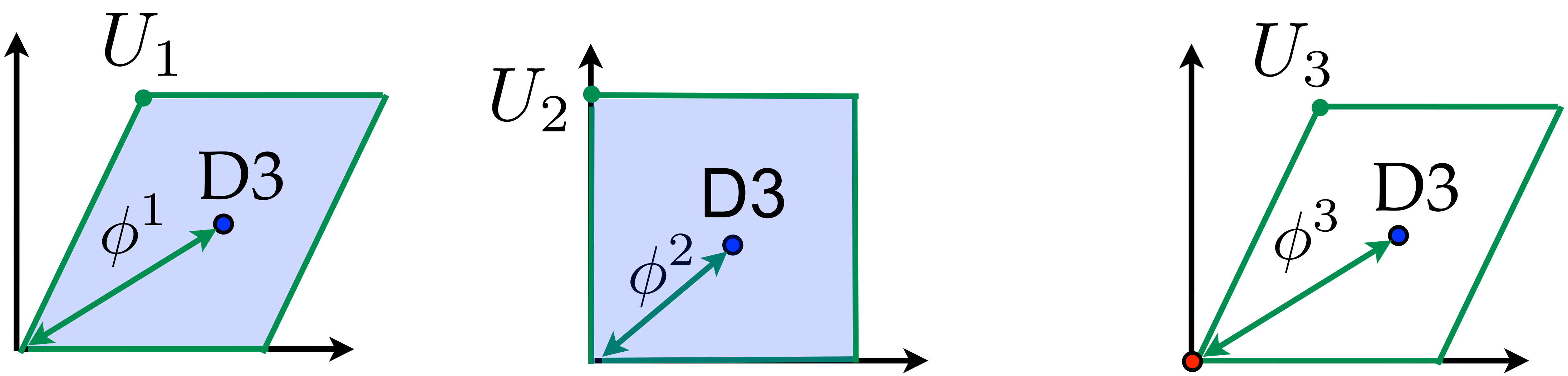
$$\mathbb{Z}_6' : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$$

Simple models for extra dimensions

sample $\mathcal{N}=1$ orbifold: $\mathbb{T}^6/\mathbb{Z}_6'$

$(Z^1 = X^4 + \bar{U} X^5)$

$\Theta Z^1 = e^{2\pi i v_1} Z^1$ $\Theta Z^2 = e^{2\pi i v_2} Z^2$ $\Theta Z^3 = e^{2\pi i v_3} Z^3$



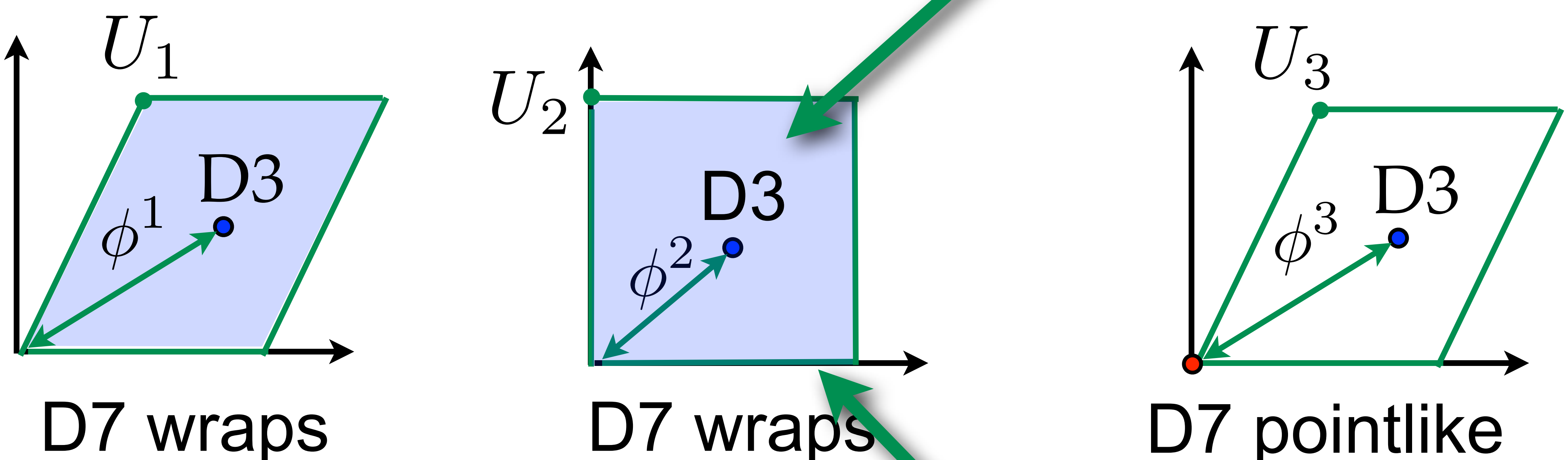
D7 wraps D7 wraps D7 pointlike

$$\mathbb{Z}_6' : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$$

Θ	Θ^2
"N=1 sector"	"N=2 sector"
"completely twisted"	"partially twisted"

$$(Z^1 = X^4 + \bar{U} X^5)$$

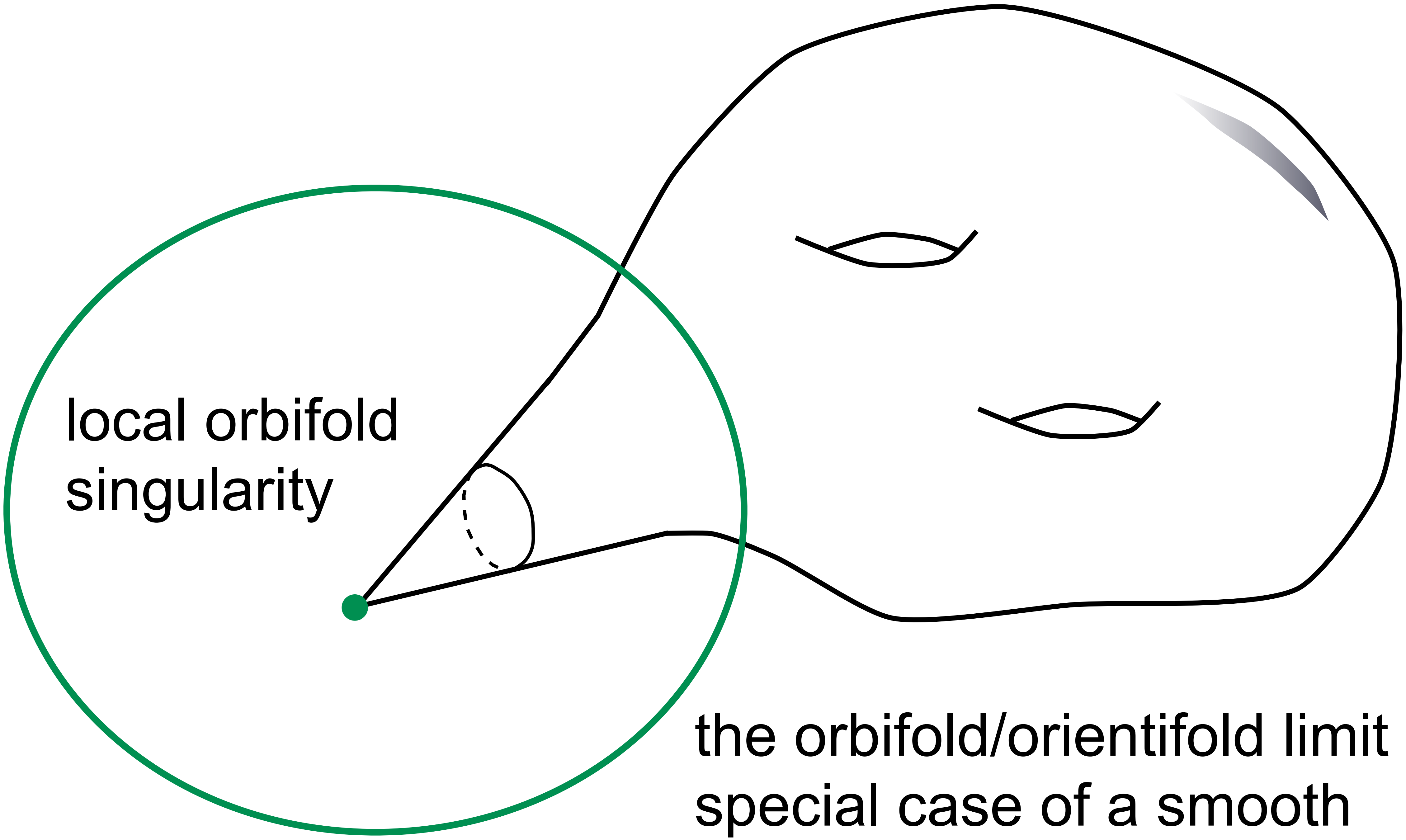
$$\Theta Z^1 = e^{2\pi i v_1} Z^1 \qquad \Theta Z^2 = e^{2\pi i v_2} Z^2 \qquad \Theta Z^3 = e^{2\pi i v_3} Z^3$$



$$\mathbb{Z}'_6 : (v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$$

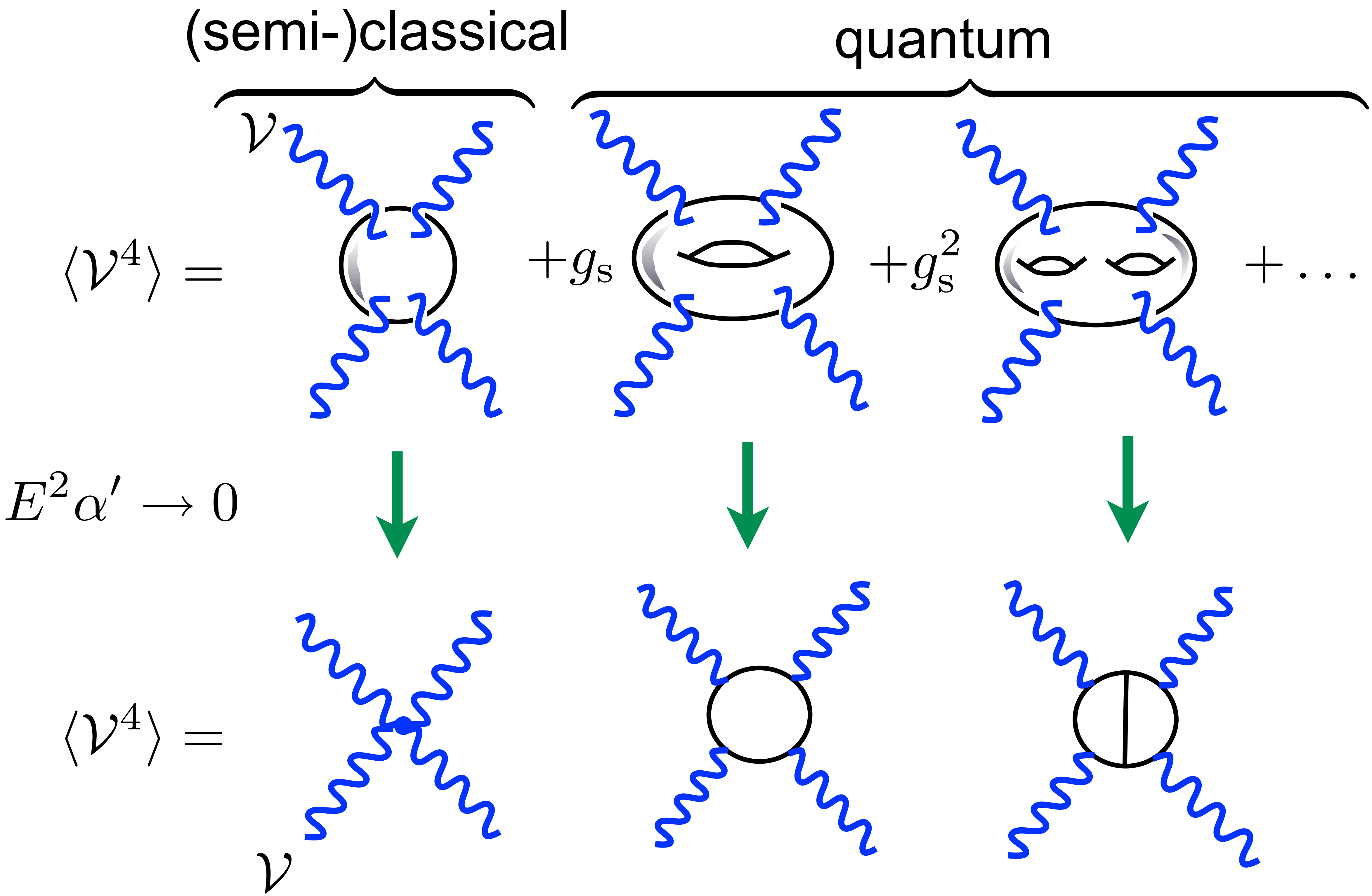
Simple models for extra dimensions

moduli: S, T, U, ϕ



the orbifold/orientifold limit is a special case of a smooth Calabi-Yau manifold.

String perturbation theory: two expansions



Curvature 1-loop corrections

review in
M.B., Buchberger, Schlotterer '16

	D=10			D=6			D=4		
1-loop term	IIA	IIB	Het	IIA/K3	IIB/K3	Het/K3	IIA/CY	IIB/CY	Het/CY
R	×	×	×						
R^2	×	×	×	✓	×	✓	✓	✓	✓
R^3	×	×	×	×	×	×	×	×	×
R^4	✓	✓	✓	✓	✓	✓	✓	✓	✓

"input"

Curvature 1-loop corrections

review in
M.B., Buchberger, Schlotterer '16

	D=10			D=6			D=4		
1-loop term	IIA	IIB	Het	IIA/K3	IIB/K3	Het/K3	IIA/CY	IIB/CY	Het/CY
R	×	×	×	×	×	×	✓	✓	×
R^2	×	×	×	✓	×	✓	✓	✓	✓
R^3	×	×	×	×	×	×	×	×	×
R^4	✓	✓	✓	✓	✓	✓	✓	✓	✓

"input"

"output"
(in principle known,
but more work needed)

Matching to gravity

$$R_{mnpq} = \left(\frac{1}{2}h_{mq,np} + \frac{1}{8}\left(h_{mq,r}h_{np}{}^{,r} + \left(h_{rm,q} + h_{rq,m} - 2h_{mq,r}\right)\left(h^{r}{}_{n,p} + h^{r}{}_{p,n}\right)\right)\right.$$
$$\left. - (m \leftrightarrow n) - (p \leftrightarrow q) + ((m,n) \leftrightarrow (p,q))\right)$$

cross term in $R_{mnpq}R^{mnpq}$

Known facts about R^2 loop corrections

Harvey, Moore '96
Gregori, Kiritsis, ... '96
...

consider half-maximal ($N = 4$ in $D = 4$)

- even-even $(k_1 e_2 k_3)(k_2 e_1 e_3 k_2) + 5 \text{ perm.}$
- odd-odd $e_1 e_2 e_3\, k_1 k_2 \epsilon\; k_1 k_2 \epsilon\; \eta$ contracts to same invariant as even-even, times constant *c*

$$(c \pm c) R_{mnpq} R^{mnpq}$$ cancel in IIB

consistent with duality:
there should be R^2 in IIA from heterotic *tree-level* R^2

Known facts about R^3 loop corrections

Grisaru '77

...

Bergshoeff, de Roo '89

...

- naively vanish at any order "by supersymmetry" (no cubic superinvariant)

$$\cancel{R^3}$$

- can be nonzero in *non-flat* background

...

Maldacena, Pimentel '11

$$\alpha' H^2 R^3$$

...

Liu, Minasian '15

...

Generating function of z dependence

$$\Omega(z,\alpha) \equiv \exp\left(2\pi i\alpha \, \frac{\mathrm{Im}\, z}{\mathrm{Im}\, \tau}\right) \frac{\vartheta_1'(0)\vartheta_1(z+\alpha)}{\vartheta_1(z)\vartheta_1(\alpha)} \equiv \sum_{n=0}^{\infty} \alpha^{n-1} f^{(n)}(z) \ ,$$

$$f^{(0)}(z) \equiv 1 \ , \qquad f^{(1)}(z) = \partial \ln \vartheta_1(z) + 2\pi i \, \frac{\mathrm{Im}\, z}{\mathrm{Im}\, \tau}$$

$$f^{(2)}(z) \equiv \frac{1}{2}\left\{ \left(\partial \ln \vartheta_1(z) + 2\pi i \, \frac{\mathrm{Im}\, z}{\mathrm{Im}\, \tau}\right)^2 + \partial^2 \ln \vartheta_1(z) - \frac{\vartheta_1'''(0)}{3\vartheta_1'(0)} \right\}$$

...

Fay identity (like partial fraction decomposition):

$$f^{(1)}_{12} f^{(1)}_{13} + f^{(1)}_{21} f^{(1)}_{23} + f^{(1)}_{31} f^{(1)}_{32} = f^{(2)}_{12} + f^{(2)}_{13} + f^{(2)}_{23} \ ,$$

Basic definitions (open strings)

$$\mathcal{A}_{1/2}(1,2,\ldots,n) = \int \mathrm{d}\mu_{12\ldots n}^{D=6} \left\{ \Gamma_{\mathcal{C}}^{(4)}\, \mathcal{I}_{n,\max} + \sum_{k=1}^{N-1} \hat{\chi}_k\, \mathcal{I}_{n,1/2}(\vec{v}_k) \right\}$$

measure:

$$\int \mathrm{d}\mu_{12\ldots n}^{D} \equiv \frac{V_D}{8N} \int_0^\infty \frac{\mathrm{d}\tau_2}{(8\pi^2\alpha'\tau_2)^{D/2}} \int_{0\leq \mathrm{Im}\,(z_1)\leq \mathrm{Im}\,(z_2)\leq \ldots \leq \mathrm{Im}\,(z_n)\leq \tau_2} \mathrm{d}z_1\, \mathrm{d}z_2\, \ldots\, \mathrm{d}z_n\, \delta(z_1)\Pi_n$$

even+odd:

$$\mathcal{I}_{n,1/2}(\vec{v}_k) \equiv \mathcal{I}^e_{n,1/2}(\vec{v}_k) + \mathcal{I}^o_{n,D=6}$$

even:

$$\mathcal{I}^e_{n,1/2}(\vec{v}_k) \equiv \frac{1}{\Pi_n} \sum_{\nu=2}^{4} (-1)^\nu \left[\frac{\vartheta_\nu(0,\tau)}{\vartheta_1'(0,\tau)}\right]^2 \left[\frac{\vartheta_\nu(kv,\tau)}{\vartheta_1(kv,\tau)}\right]^2 \langle V_1^{(0)}(z_1) V_2^{(0)}(z_2)\, \ldots\, V_n^{(0)}(z_n)\rangle_\nu$$

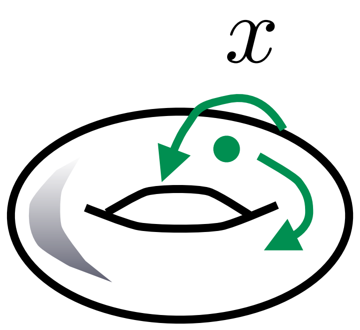
Known spin sum

Maximal supersymmetry:

$$\mathcal{G}_N(x_1, x_2, \ldots, x_N) \equiv \sum_{\nu=2,3,4} (-1)^{\nu-1} \left(\frac{\vartheta_\nu(0)}{\vartheta_1'(0)}\right)^4 S_\nu(x_1) S_\nu(x_2) \ldots S_\nu(x_N)$$

Fermion Green's function (Szegö kernel)

$$S_\nu(x) \equiv \frac{\vartheta_1'(0)\vartheta_\nu(x)}{\vartheta_\nu(0)\vartheta_1(x)} \, .$$



Known systematics for maximal

Tsuchiya '88
...
Stieberger, Taylor '02
Dolan, Goddard '07
...
Broedel, Mafra, Matthes Schlotterer '14
...

$$\mathcal{G}_N(x_1, x_2, \ldots, x_N) = 0\,, \qquad N \leq 3$$

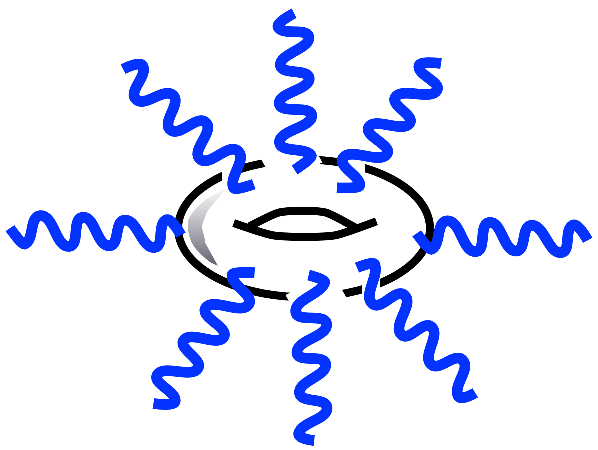
$$\mathcal{G}_4(x_1, x_2, x_3, x_4) = 1$$

$$\mathcal{G}_5(x_1, x_2, \ldots, x_5) = \sum_{j=1}^{5} f_j^{(1)}$$

$$\mathcal{G}_6(x_1, x_2, \ldots, x_6) = \sum_{j=1}^{6} f_j^{(2)} + \sum_{1\leq j<k}^{6} f_j^{(1)} f_k^{(1)}$$

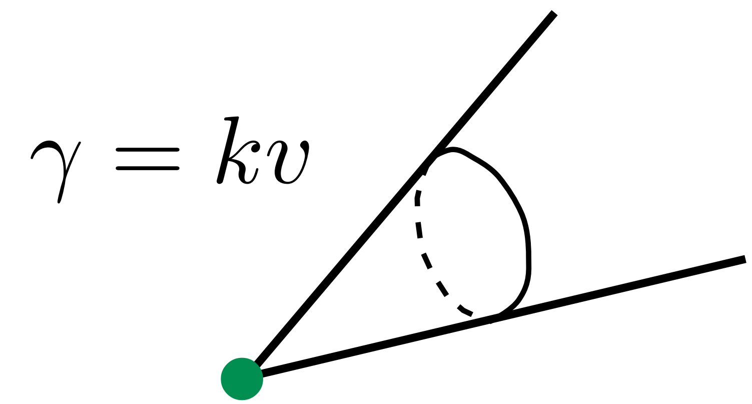
$$\mathcal{G}_7(x_1, x_2, \ldots, x_7) = \sum_{j=1}^{7} f_j^{(3)} + \sum_{1\leq j<k}^{7} (f_j^{(2)} f_k^{(1)} + f_j^{(1)} f_k^{(2)}) + \sum_{1\leq j<k<l}^{7} f_j^{(1)} f_k^{(1)} f_l^{(1)}$$

$$\mathcal{G}_8(x_1, x_2, \ldots, x_8) = \sum_{j=1}^{8} f_j^{(4)} + \sum_{1\leq j<k}^{8} (f_j^{(3)} f_k^{(1)} + f_j^{(2)} f_k^{(2)} + f_j^{(1)} f_k^{(3)}) + \sum_{1\leq j<k<l<m}^{8} f_j^{(1)} f_k^{(1)} f_l^{(1)} f_m^{(1)}$$

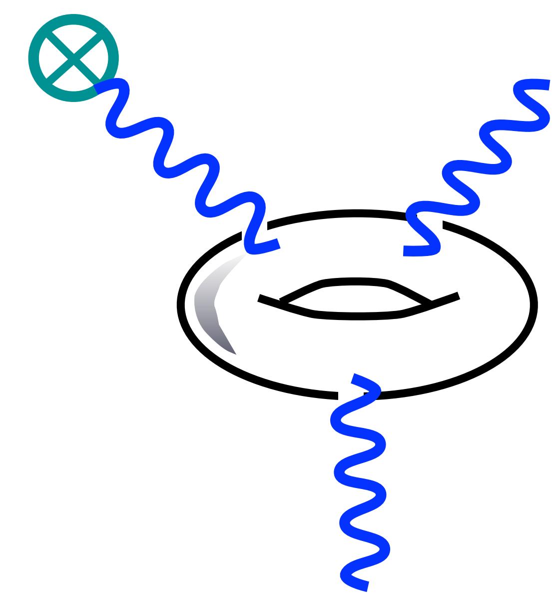


$$+ \sum_{1\leq j<k<l}^{8} (f_j^{(2)} f_k^{(1)} f_l^{(1)} + f_j^{(1)} f_k^{(2)} f_l^{(1)} + f_j^{(1)} f_k^{(1)} f_l^{(2)}) + 3G_4\,,$$

Reducing supersymmetry (orbifold)



Trick: orbifold partition function is like
Green's function evaluated "at twist"



$$F^{(0)}_{1/2}(\gamma) \equiv 1 \ , \qquad F^{(2)}_{1/2}(\gamma) \equiv 2f^{(2)}(\gamma) - f^{(1)}(\gamma)^2$$

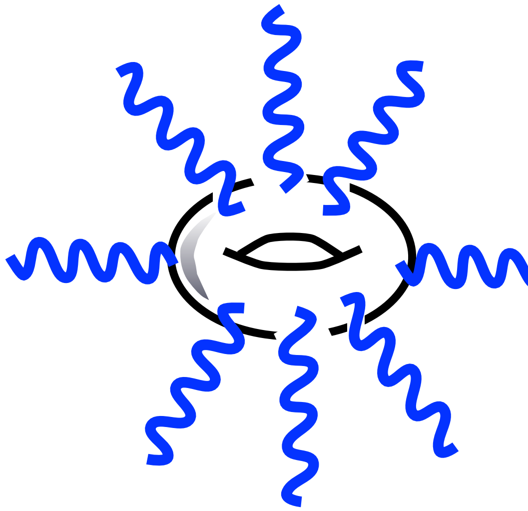
$$F^{(4)}_{1/2}(\gamma) \equiv 2f^{(4)}(\gamma) - 2f^{(3)}(\gamma)f^{(1)}(\gamma) + f^{(2)}(\gamma)^2$$

$$\mathcal{I}^e_{n,1/2}(\vec{v}_k) = \frac{1}{\Pi_n}\sum_{\nu=2}^{4}(-1)^{\nu-1}\left[\frac{\vartheta_\nu(0)}{\vartheta_1'(0)}\right]^4 S_\nu(kv)S_\nu(-kv)\,\langle V_1^{(0)}(z_1)V_2^{(0)}(z_2)\ \ldots\ V_n^{(0)}(z_n)\rangle_\nu$$

$$\sum_{\nu=2}^{4}(-1)^{\nu-1}\left[\frac{\vartheta_\nu(0)}{\vartheta_1'(0)}\right]^4 S_\nu(\gamma)S_\nu(-\gamma)\,S_\nu(x_1)\ldots S_\nu(x_n) = \mathcal{G}_{n+2}(x_1,x_2,\ldots,x_n,\gamma,-\gamma)$$

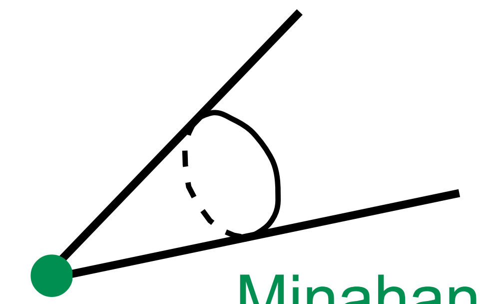
Reducing supersymmetry (orbifold)

$$V_m(x_1, x_2, \ldots, x_n) \equiv \left(\alpha^n \Omega(x_1, \alpha)\Omega(x_2, \alpha) \ldots \Omega(x_n, \alpha)\right)\Big|_{\alpha^m}$$



$$\begin{aligned}
\mathcal{G}_{2+2}(\gamma, -\gamma, x_1, x_2) &= 1 \\
\mathcal{G}_{2+3}(\gamma, -\gamma, x_1, x_2, x_3) &= V_1(x_1, x_2, x_3) = f^{(1)}(x_1) + f^{(1)}(x_2) + f^{(1)}(x_3) \\
\mathcal{G}_{2+4}(\gamma, -\gamma, x_1, \ldots, x_4) &= F^{(2)}_{1/2}(\gamma) + V_2(x_1, \ldots, x_4) \\
\mathcal{G}_{2+5}(\gamma, -\gamma, x_1, \ldots, x_5) &= F^{(2)}_{1/2}(\gamma)V_1(x_1, \ldots, x_5) + V_3(x_1, \ldots, x_5) \\
\mathcal{G}_{2+6}(\gamma, -\gamma, x_1, \ldots, x_6) &= F^{(4)}_{1/2}(\gamma) + 3G_4 + F^{(2)}_{1/2}(\gamma)V_2(x_1, \ldots, x_6) + V_4(x_1, \ldots, x_6) \\
\mathcal{G}_{2+7}(\gamma, -\gamma, x_1, \ldots, x_7) &= (F^{(4)}_{1/2}(\gamma) + 3G_4)V_1(x_1, \ldots, x_7) \\
&\quad + F^{(2)}_{1/2}(\gamma)V_3(x_1, \ldots, x_7) + V_5(x_1, \ldots, x_7) \\
\mathcal{G}_{2+8}(\gamma, -\gamma, x_1, \ldots, x_8) &= F^{(6)}_{1/2}(\gamma) + 10G_6 + F^{(4)}_{1/2}(\gamma)V_2(x_1, \ldots, x_8) + F^{(2)}_{1/2}(\gamma)V_4(x_1, \ldots, x_8) \\
&\quad + 3G_4(F^{(2)}_{1/2}(\gamma) + V_2(x_1, \ldots, x_8)) + V_6(x_1, \ldots, x_8)\ ,
\end{aligned}$$

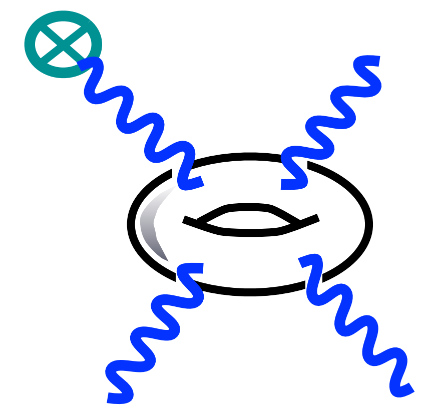
IR regularization of (2-pt), 3-pt, ...



Minahan '87

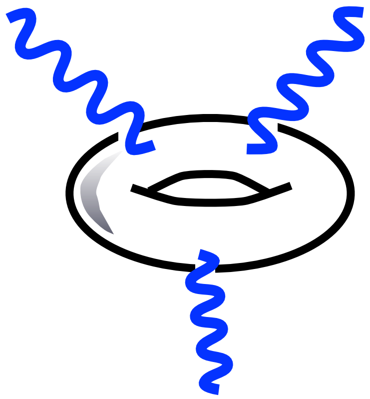
• relax momentum conservation

or



e.g. Gregori et al
K3 and K3 x T^2

• complex momenta,
as used in spinor helicity formalism



3-pt function

$$Q_i^m \equiv \sum_{j \neq i} k_j^m f_{ij}^{(1)}$$

left-movers:

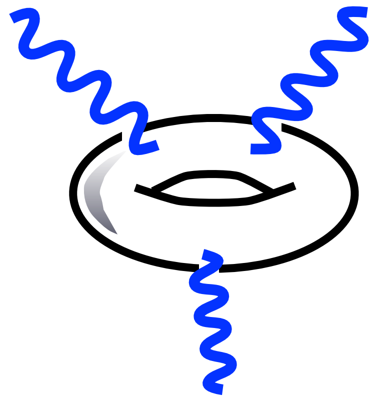
$$\mathcal{I}_3 = \mathcal{G}_2(\gamma_1, \gamma_2)\big[\big(\partial f_{12}^{(1)}(e_1 \cdot e_2)(e_3 \cdot Q_3) + (3 \leftrightarrow 2, 1)\big) - (e_1 \cdot Q_1)(e_2 \cdot Q_2)(e_3 \cdot Q_3)\big]$$
$$+ \big[\mathcal{G}_4(\gamma_1, \gamma_2, z_{12}, z_{21}) t(1,2)(e_3 \cdot Q_3) + (3 \leftrightarrow 2, 1)\big] + \mathcal{G}_5(\gamma_1, \gamma_2, z_{12}, z_{23}, z_{31}) t(1,2,3) \;.$$

$$\mathcal{I}_3 = f_{12}^{(1)} K_{12|3} + (12 \leftrightarrow 13, 23)$$

$$K_{12|3} = t(1,2,3) + (e_1 \cdot k_2) t(2,3) - (e_2 \cdot k_1) t(1,3) = s_{12}(e_1 \cdot e_2)(e_3 \cdot k_1) \;.$$

Lorentz traces of linearized field strength:

$$t(1,2) \equiv (e_1 \cdot k_2)(e_2 \cdot k_1) - (e_1 \cdot e_2)(k_1 \cdot k_2)$$
$$\begin{aligned} t(1,2,3) \equiv\; & (e_1 \cdot k_2)(e_2 \cdot k_3)(e_3 \cdot k_1) - (e_1 \cdot k_2)(e_2 \cdot e_3)(k_3 \cdot k_1) \\ & - (e_1 \cdot e_2)(k_2 \cdot k_3)(e_3 \cdot k_1) + (e_1 \cdot e_2)(k_2 \cdot e_3)(k_3 \cdot k_1) \\ & - (k_1 \cdot k_2)(e_2 \cdot k_3)(e_3 \cdot e_1) + (k_1 \cdot k_2)(e_2 \cdot e_3)(k_3 \cdot e_1) \\ & + (k_1 \cdot e_2)(k_2 \cdot k_3)(e_3 \cdot e_1) - (k_1 \cdot e_2)(k_2 \cdot e_3)(k_3 \cdot e_1) \end{aligned}$$
$$t(1,2,\ldots,n) \equiv (e_1 \cdot k_2)(e_2 \cdot k_3)(e_3 \cdot k_4) \ldots (e_{n-1} \cdot k_n)(e_n \cdot k_1)$$



3-pt function

$$Q_i^m \equiv \sum_{j\neq i} k_j^m f_{ij}^{(1)}$$

left-movers:

$$\mathcal{I}_3 = \mathcal{G}_2(\gamma_1,\gamma_2)\Big[\big(\partial f_{12}^{(1)}(e_1\cdot e_2)(e_3\cdot Q_3) + (3\leftrightarrow 2,1)\big) - (e_1\cdot Q_1)(e_2\cdot Q_2)(e_3\cdot Q_3)\Big]$$
$$+\Big[\mathcal{G}_4(\gamma_1,\gamma_2,z_{12},z_{21})t(1,2)(e_3\cdot Q_3) + (3\leftrightarrow 2,1)\Big] + \mathcal{G}_5(\gamma_1,\gamma_2,z_{12},z_{23},z_{31})t(1,2,3)\ .$$

$$\mathcal{I}_3 = f_{12}^{(1)} K_{12|3} + (12 \leftrightarrow 13, 23)$$

$$K_{12|3} = t(1,2,3) + (e_1\cdot k_2)t(2,3) - (e_2\cdot k_1)t(1,3) = s_{12}(e_1\cdot e_2)(e_3\cdot k_1)\ .$$

full graviton:

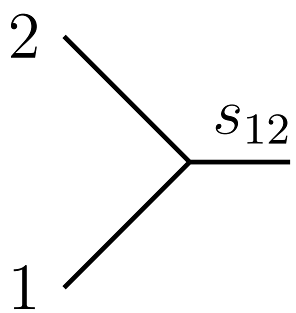
~~R^3~~

$$\mathcal{J}_{3,1/2} \equiv \mathcal{I}_{3,1/2}\tilde{\mathcal{I}}_{3,1/2} + \frac{\pi}{\operatorname{Im}\tau}\mathcal{I}^m_{3,1/2}\tilde{\mathcal{I}}^m_{3,1/2}$$

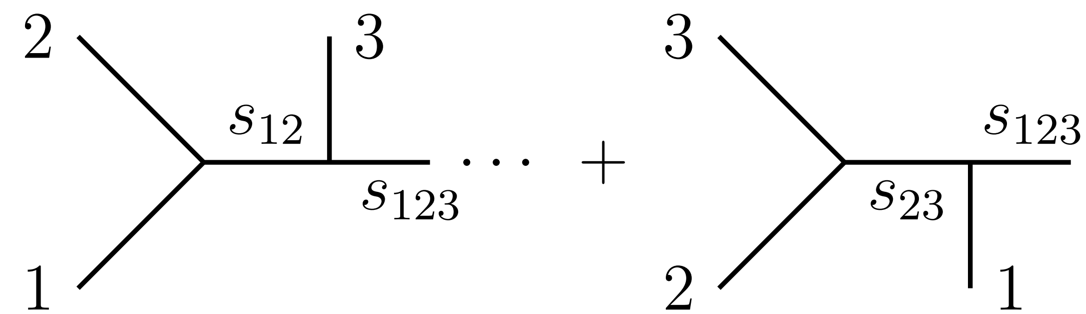
$$\mathcal{I}^m_{3,1/2} = M^m_{1|2,3}$$

Berends-Giele building blocks

$\mathfrak{e}_{12}^m, \mathfrak{f}_{12}^{mn} \leftrightarrow$



$\cdots$, $\mathfrak{e}_{123}^m, \mathfrak{f}_{123}^{mn} \leftrightarrow$



Berends, Giele '87

...

in supersymmetry & string theory: Mafra, Schlotterer, Stieberger '11

Mafra, Schlotterer '14-'15

$$\mathfrak{e}_1^m \equiv e_1^m$$

$$\mathfrak{e}_{12}^m \equiv \frac{1}{2s_{12}}\big[\mathfrak{e}_2^m(k_2\cdot\mathfrak{e}_1) + (\mathfrak{e}_2)_n\mathfrak{f}_1^{mn} - (1\leftrightarrow 2)\big]$$

$$\mathfrak{e}_{123}^m \equiv \frac{1}{2s_{123}}\Big\{\big[\mathfrak{e}_3^m(k_3\cdot\mathfrak{e}_{12}) + (\mathfrak{e}_3)_n\mathfrak{f}_{12}^{mn} - (12\leftrightarrow 3)\big] + \big[\mathfrak{e}_{23}^m(k_{23}\cdot\mathfrak{e}_1) + (\mathfrak{e}_{23})_n\mathfrak{f}_1^{mn} - (1\leftrightarrow 23)\big]\Big\}$$

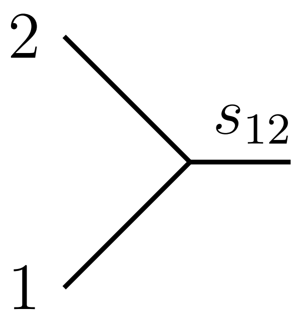
$$\mathfrak{f}_1^{mn} \equiv 2k_1^{[m}\mathfrak{e}_1^{n]}$$

$$\mathfrak{f}_{12}^{mn} \equiv 2k_{12}^{[m}\mathfrak{e}_{12}^{n]} - 2\mathfrak{e}_1^{[m}\mathfrak{e}_2^{n]}$$

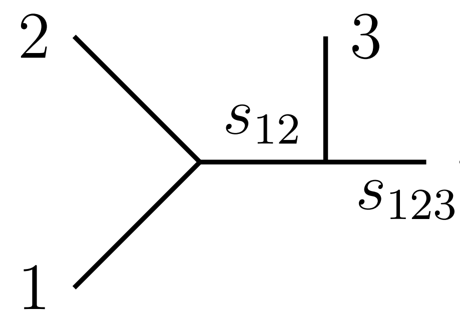
$$\mathfrak{f}_{123}^{mn} \equiv 2k_{123}^{[m}\mathfrak{e}_{123}^{n]} - 2(\mathfrak{e}_{12}^{[m}\mathfrak{e}_3^{n]} + \mathfrak{e}_1^{[m}\mathfrak{e}_{23}^{n]})$$

Berends-Giele building blocks

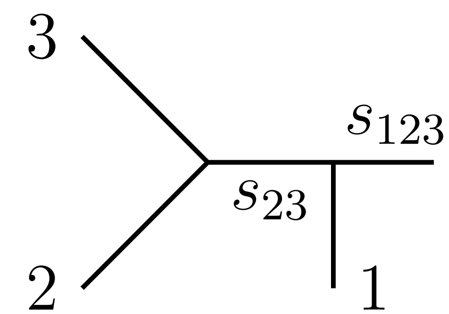
$\mathfrak{e}^m_{12}, \mathfrak{f}^{mn}_{12} \leftrightarrow$



$\cdots \; , \qquad \mathfrak{e}^m_{123}, \mathfrak{f}^{mn}_{123} \leftrightarrow$



$\cdots \; +$



$\cdots$

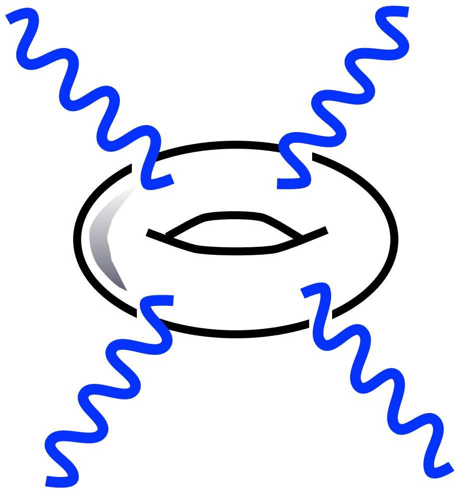
$$M_{A,B} \equiv -\frac{1}{2}\mathfrak{f}^{mn}_A \mathfrak{f}^{mn}_B = M_{B,A}$$

$$\mathcal{E}^m_{A|B,C} \equiv \frac{i}{4}\epsilon^m{}_{npqrs}\mathfrak{e}^n_A \mathfrak{f}^{pq}_B \mathfrak{f}^{rs}_C = \mathcal{E}^m_{A|C,B}$$

building blocks

$$M^m_{A|B,C} \equiv \mathfrak{e}^m_A M_{B,C} + \mathfrak{e}^m_B M_{A,C} + \mathfrak{e}^m_C M_{A,B} + \mathcal{E}^m_{A|B,C} = M^m_{A|C,B}$$

4-pt function



$$X_{23} = s_{23} f_{23}^{(1)}$$

regular under integration

one side (which will be double-copied):

$$\mathcal{I}_{3,1/2} = X_{23} C_{1|23}$$

$$\mathcal{I}_{4,1/2} = -2F_{1/2}^{(2)}(\gamma) t_8(1,2,3,4) + X_{23,4} C_{1|234} + X_{24,3} C_{1|243} + \left[s_{12} f_{12}^{(2)} P_{1|2|3,4} + (2 \leftrightarrow 3,4) \right] + \left[s_{23} f_{23}^{(2)} P_{1|(23)|4} + (23 \leftrightarrow 24, 34) \right]$$

built from *M*’s

C's built from Berends-Giele M's

gauge invariance from
"particle 1 + gauge invariant completion"

$$C_{1|23} \equiv M_{1,23} + M_{12,3} - M_{13,2}$$

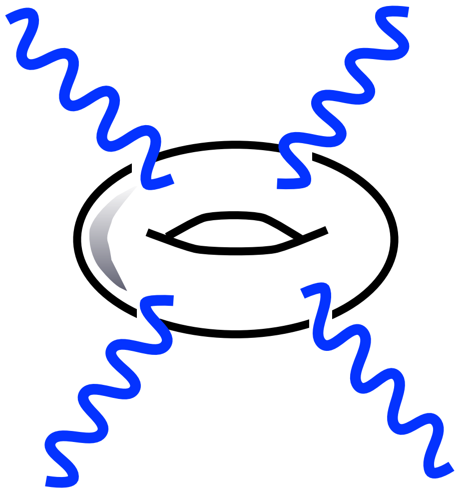
$$C_{1|234} \equiv M_{1,234} + M_{123,4} + M_{412,3} + M_{341,2} + M_{12,34} + M_{41,23}$$

$$C^m_{1|2,3} \equiv M^m_{1|2,3} + k^m_2 M_{12,3} + k^m_3 M_{13,2}$$

$$C^m_{1|23,4} \equiv M^m_{1|23,4} + M^m_{12|3,4} - M^m_{13|2,4} - k^m_2 M_{132,4} + k^m_3 M_{123,4} - k^m_4 (M_{41,23} + M_{412,3} - M_{413,2})$$

$$C^{mn}_{1|2,3,4} \equiv M^{mn}_{1|2,3,4} + 2\big[k^{(m}_2 M^{n)}_{12|3,4} + (2 \leftrightarrow 3,4)\big] - 2\big[k^{(m}_2 k^{n)}_3 M_{213,4} + (23 \leftrightarrow 24, 34)\big]$$

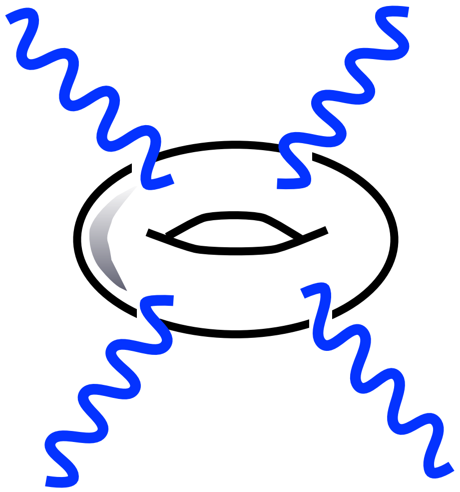
Finally: closed string 4-pt function



$$\mathcal{J}_{4,1/2} \equiv \Big| X_{23,4} C_{1|234} + X_{24,3} C_{1|243} + \big[s_{12} f^{(2)}_{12} P_{1|2|3,4} + (2 \leftrightarrow 3,4) \big] + \big[s_{23} f^{(2)}_{23} P_{1|(23)|4} + (23 \leftrightarrow 24, 34) \big] - 2 F^{(2)}_{1/2}(\gamma) t_8(1,2,3,4) \Big|^2 + \frac{\pi}{\operatorname{Im}\tau} \big(X_{23} C^m_{1|23,4} + X_{24} C^m_{1|24,3} + X_{34} C^m_{1|34,2} \big) \big(\bar{X}_{23} \tilde{C}^m_{1|23,4} + \bar{X}_{24} \tilde{C}^m_{1|24,3} + \bar{X}_{34} \tilde{C}^m_{1|34,2} \big) + \left(\frac{\pi}{\operatorname{Im}\tau} \right)^2 \big(\tfrac{1}{2} C^{mn}_{1|2,3,4} \tilde{C}^{mn}_{1|2,3,4} - P_{1|2|3,4} \tilde{P}_{1|2|3,4} - P_{1|3|2,4} \tilde{P}_{1|3|2,4} - P_{1|4|2,3} \tilde{P}_{1|4|2,3} \big) \ .$$

[see 1603.04790 for closely related 6-pt max. SUSY]

Finally: closed string 4-pt function



$$\mathcal{J}_{4,1/2} \equiv \Big| X_{23,4} C_{1|234} + X_{24,3} C_{1|243} + \big[s_{12} f_{12}^{(2)} P_{1|2|3,4} + (2 \leftrightarrow 3,4) \big]$$
$$+ \big[s_{23} f_{23}^{(2)} P_{1|(23)|4} + (23 \leftrightarrow 24,34) \big] - 2 F_{1/2}^{(2)}(\gamma) t_8(1,2,3,4) \Big|^2$$
$$+ \frac{\pi}{\operatorname{Im}\tau} (X_{23} C^m_{1|23,4} + X_{24} C^m_{1|24,3} + X_{34} C^m_{1|34,2})(\bar{X}_{23} \tilde{C}^m_{1|23,4} + \bar{X}_{24} \tilde{C}^m_{1|24,3} + \bar{X}_{34} \tilde{C}^m_{1|34,2})$$
$$+ \Big(\frac{\pi}{\operatorname{Im}\tau} \Big)^2 \boxed{\big(\tfrac{1}{2} C^{mn}_{1|2,3,4} \tilde{C}^{mn}_{1|2,3,4} - P_{1|2|3,4} \tilde{P}_{1|2|3,4} - P_{1|3|2,4} \tilde{P}_{1|3|2,4} - P_{1|4|2,3} \tilde{P}_{1|4|2,3} \big)} \,.$$

apparent obstruction to double copy
(cf. Mafra-Schlotterer 1410.0668: obstruction in 6-point, related to chiral anomaly in hexagon, but: Green-Schwarz!)

Recreating low-energy results

R^2

$$\mathcal{M}^{R^2}(1,2,3)\,\Big|_{\text{even}} = M^m_{1|2,3}\tilde{M}^m_{1|2,3}\,\Big|_{\text{even}} = \begin{cases} -2\epsilon^m(e_1,k_2,e_2,k_3,e_3)\epsilon_m(\tilde{e}_1,k_2,\tilde{e}_2,k_3,\tilde{e}_3) & : \text{ IIA} \\ 0 & : \text{ IIB} \end{cases}$$

$B \wedge R \wedge R$

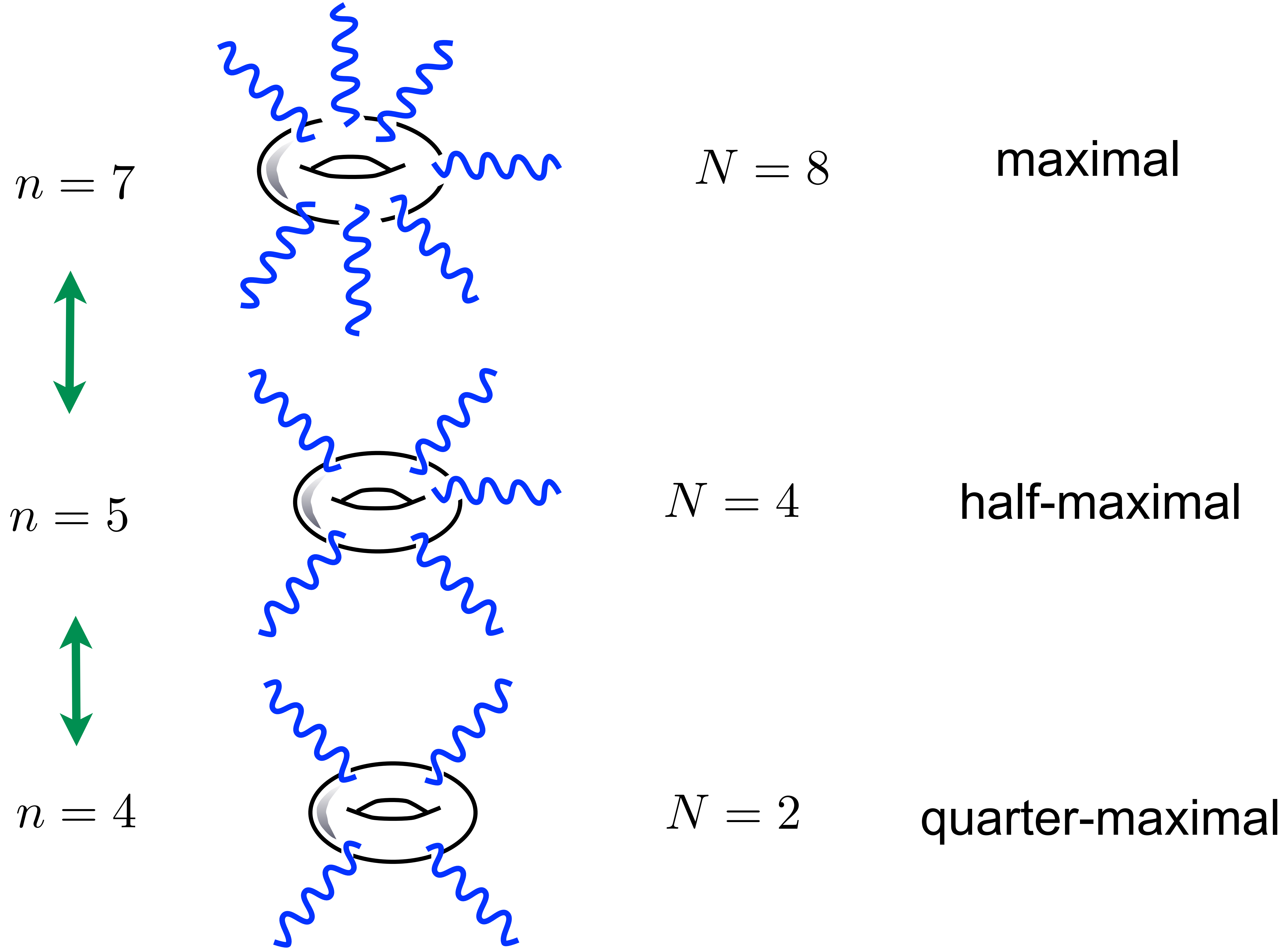
$$\mathcal{M}^{R^2}(1,2,3)\,\Big|_{\text{odd}} = M^m_{1|2,3}\tilde{M}^m_{1|2,3}\,\Big|_{\text{odd}}$$

$$= \begin{cases} i\big[e_1^m(e_2\cdot k_3)(e_3\cdot k_2) + \text{cyc}(1,2,3)\big]\epsilon_m(\tilde{e}_1,k_2,\tilde{e}_2,k_3,\tilde{e}_3) - (e_i \leftrightarrow \tilde{e}_i) & : \text{ IIA, odd \# of B-field} \\ i\big[e_1^m(e_2\cdot k_3)(e_3\cdot k_2) + \text{cyc}(1,2,3)\big]\epsilon_m(\tilde{e}_1,k_2,\tilde{e}_2,k_3,\tilde{e}_3) + (e_i \leftrightarrow \tilde{e}_i) & : \text{ IIB, two B-fields} \\ 0 & : \text{ otherwise} \end{cases}$$

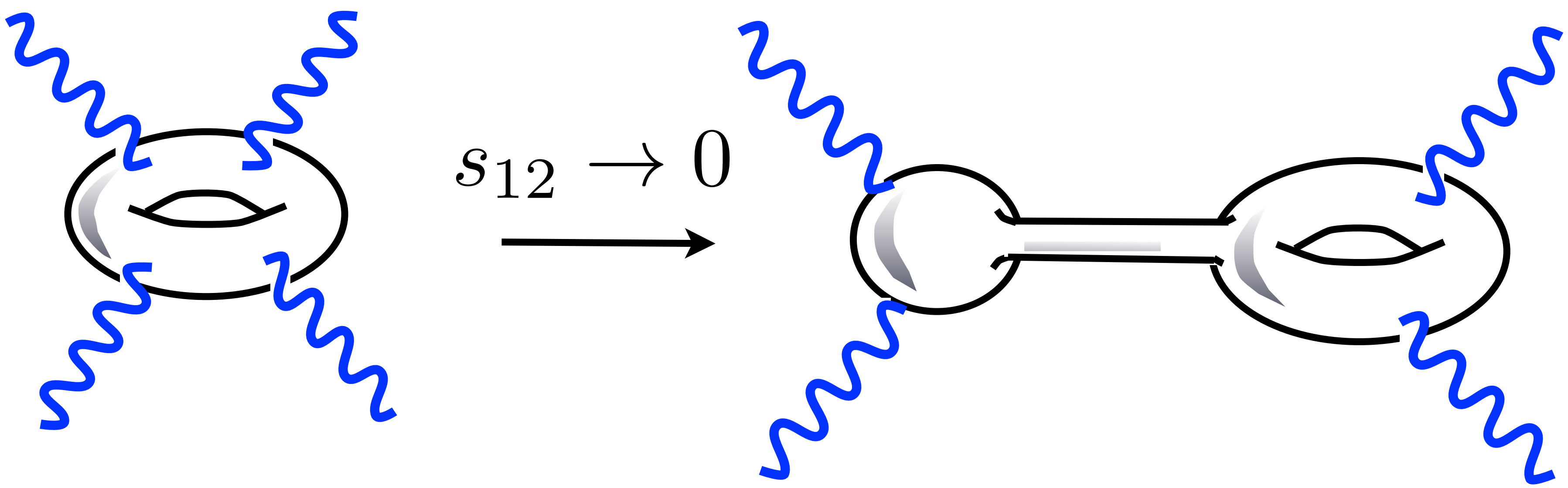
$H \wedge H \wedge R$

Half to quarter supersymmetry

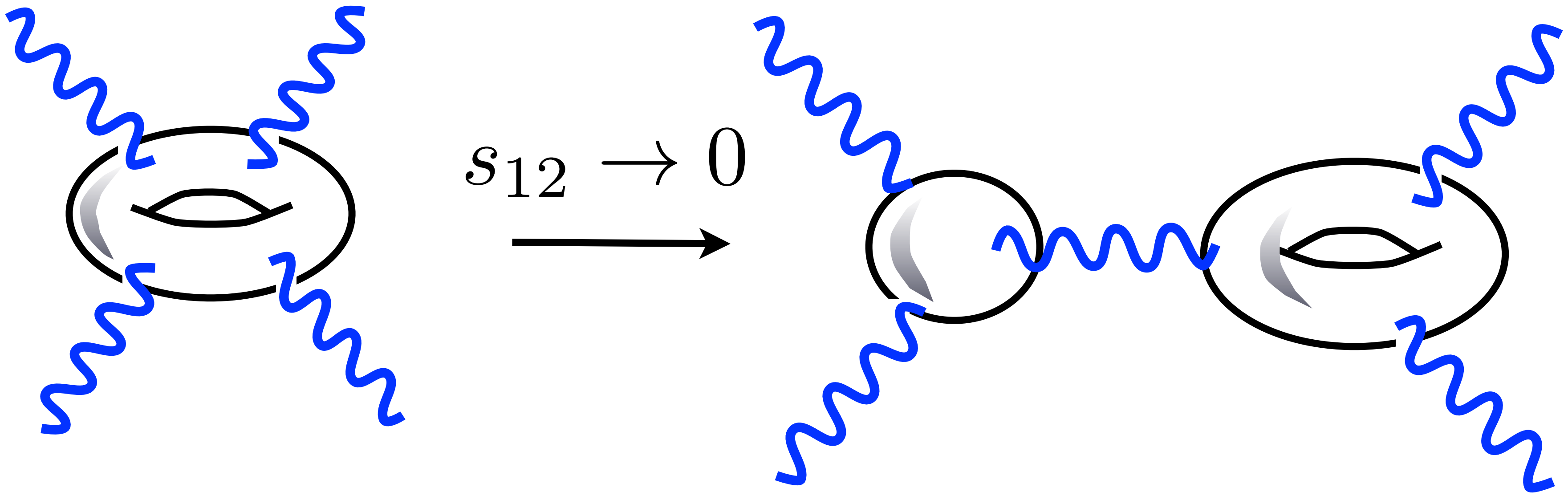
$$\mathcal{J}^{e,e}_{n,1/4} = \mathcal{J}^{e,e}_{n,1/2}\Big|_{F^{(k)}_{1/2}(\gamma_{k,k'}) \to F^{(k+1)}_{1/4}(\gamma^{j}_{k,k'}),\ \bar{F}^{(k)}_{1/2}(\bar{\gamma}_{k,k'}) \to \bar{F}^{(k+1)}_{1/4}(\bar{\gamma}^{j}_{k,k'})}$$


$n = 7$
$N = 8$
maximal
$n = 5$
$N = 4$
half-maximal
$n = 4$
$N = 2$
quarter-maximal

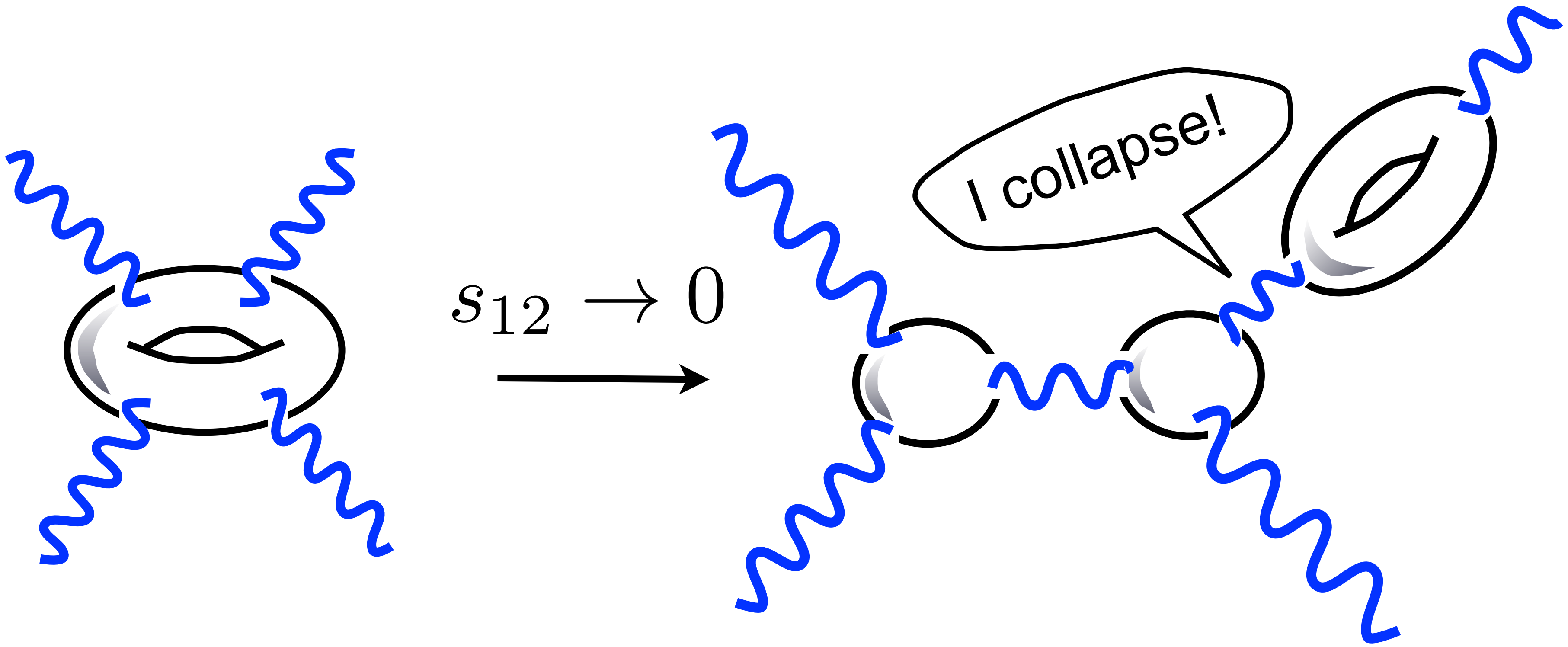
Factorization

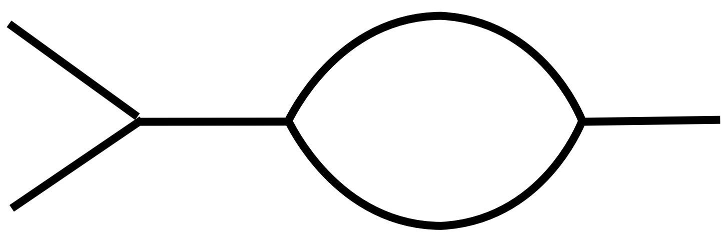


Factorization

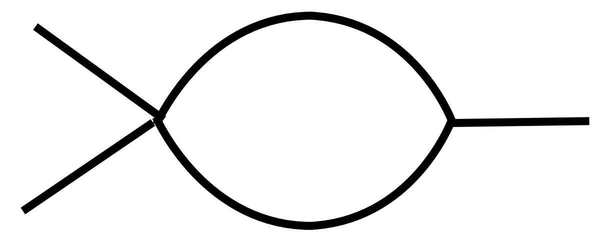


Factorization



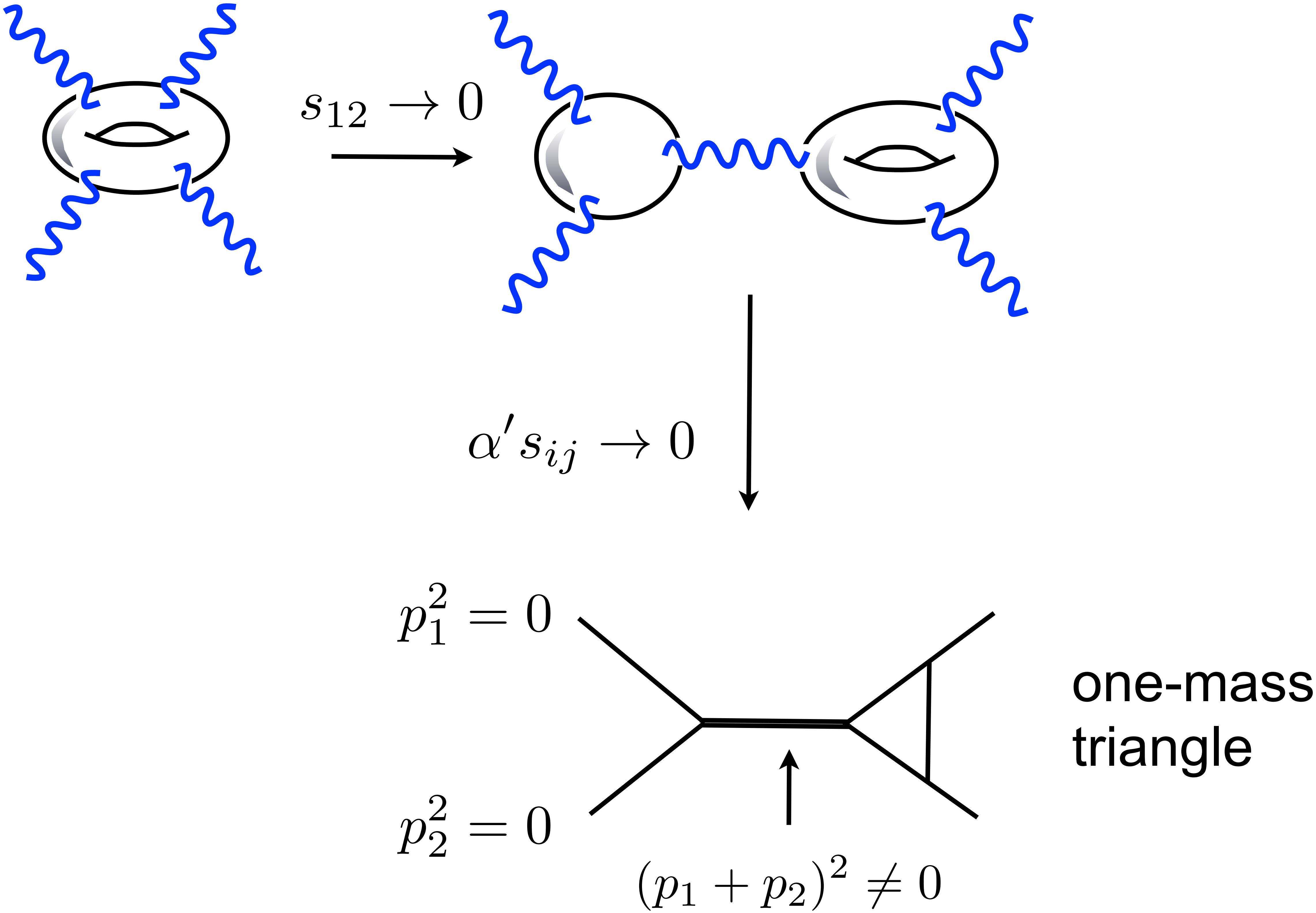


snail



shy snail

Factorization



N=1,2 amplitude

$$A^{\text{1-loop}}_{\mathcal{N}=1,2}(1,2,3,4) \quad = \quad c_2^{K3}(\gamma_i) A^{\text{1-loop}}_{\mathcal{N}=4}(1,2,3,4) + \int \frac{\mathrm{d}^D \ell}{(2\pi)^D}\, 2 \Bigg\{ \overbrace{\frac{C_{1|234}}{2\ell^2 \ell^2_{234}}}^{\text{bubble}} + \overbrace{\frac{\ell_m C^m_{1|23,4}}{\ell^2 \ell^2_{234} \ell^2_4} + \frac{\ell_m C^m_{1|2,34}}{\ell^2 \ell^2_{234} \ell^2_{34}}}^{\text{triangles}}$$

$$- \underbrace{\frac{P_{1|4|2,3}}{\ell^2_{234}\ell^2_{34}\ell^2_4}}_{\text{triangle/box}} + \underbrace{\frac{\ell_m \ell_n C^{mn}_{1,2,3,4} + \ell_m (s_{23} C^m_{1|23,4} + s_{24} C^m_{1|24,3} + s_{34} C^m_{1|2,34}) - \frac{1}{6} s_{23} s_{34} C_{1|234}}{\ell^2 \ell^2_{234} \ell^2_{34} \ell^2_4}}_{\text{box}} \Bigg\}$$

interplay locality / gauge invariance

[see 1410.0668 for closely related 6-pt max. SUSY]

MHV calculation of 4-pt vector

Bianchi, Consoli '15

- Can combine different kinematic factors, leads to very simple expression

$$\mathcal{A}_4^{\text{1-loop}}[1^-,2^-,3^+,4^+] = \frac{\alpha'^4 g_s^4}{8} F^4_{--++} \int_0^\infty \frac{dT}{T} \int d\mu^{(4)} \Big[4\mathcal{F}_\mathcal{N} + \mathcal{E}_\mathcal{N} \left(\mathcal{Y}_{12} + \mathcal{Y}_{34} - \mathcal{Y}_{13} - \mathcal{Y}_{24} - \mathcal{Y}_{14} - \mathcal{Y}_{23} \right) \Big] \Pi$$

no poles!

no factorization! believe 0/0 problem, but more work needed.

surprising in 4-pt function!

[see 1603.04790 for closely related 6-pt max. SUSY]

Outlook

- Scattering equations: relation?

  Cachazo, He, Yuan,
  ...

- Field theory amplitudes:
  more on BCJ from string amplitudes

  e.g. Mafra, Schlotterer, Stieberger, Tsimpis '10
  Ochirov, Tourkine '14
  ...

- Hybrid formalism in orbifold,
  AdS/plane wave loop corrections

- Approach from integrability: relation?

  Hoare, Tseytlin, ...,

- Ambitwistor orbifolds?