The cusp anomalous dimension in QCD and its supersymmetric extensions

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arXiv:1510.07803, 1409.0023

Aspects of Amplitudes, June 21, 2016

Outline

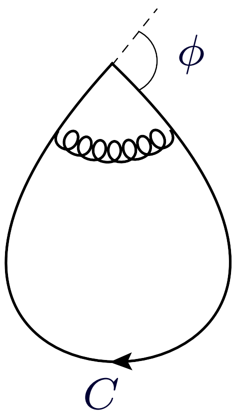
- ✔ Cusp anomalous dimension
- ✔ General properties, interesting limits
- ✔ Three-loop calculation
- ✔ Surprising result leading to few conjectures

What is cusp anomalous dimension

Wilson loops in gauge theories

$$W_C = \frac{1}{N_R} \langle 0 | \, \mathrm{tr}_R \, P \exp \left(ig \oint_C dx^\mu A_\mu^a(x) T^a \right) |0\rangle$$

- ✔ Nonlocal gauge invariant functional of the integration contour C
- ✔ Equations of motions in Yang-Mills theories $=$ Loop (Makeenko-Migdal) equations for W_C
- ✔ Cusped Wilson loops develop UV divergences making loop equations inconsistent with quantum corrections [Polyakov'80]

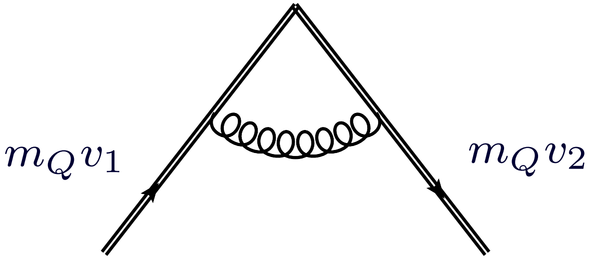
$W_C =$ 

$$\sim g^2 T^a T^a \int ds dt \frac{(\dot{x}(s)\dot{x}(t))}{[(x(s)-x(t))^2]^{1-\epsilon}} \sim \frac{1}{\epsilon} g^2 C_R (\phi \cot \phi - 1)$$

- ✔ Resurrection: cusp singularities can be used to control IR divergences of scattering amplitudes [GK,Radyushkin'86]

Applications

Scattering of a heavy quark off an external potential ($m_Q \to \infty$ and $(v_1 v_2) = \cos\phi$)



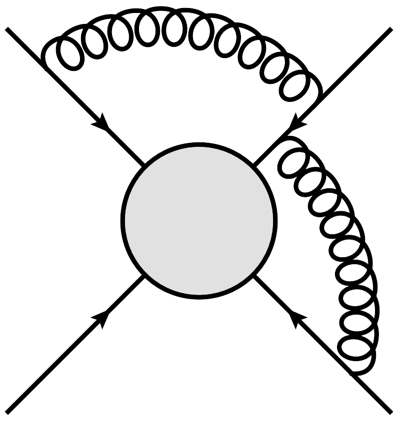
$$\sim g^2 C_R \int d^4k \frac{(v_1 v_2)}{k^2 (kv_1)(kv_2)} = \underbrace{-\frac{g^2 C_R}{4\pi^2}(\phi\cot\phi - 1)}_{\text{cusp anom.dim.}} \ln\frac{\mu_{\rm UV}}{\mu_{\rm IR}}$$

All-loop result

$$W \sim \exp\left(-\Gamma_{\rm cusp}(\phi, g^2) \ln\frac{\mu_{\rm UV}}{\mu_{\rm IR}}\right)$$

IR and UV divergences are in the one-to-one correspondence [GK,Radyushkin'86]

Infrared structure of (planar) scattering amplitudes



$$\sim \exp\left(\ln\mu_{\rm IR} \sum_i \Gamma_{\rm cusp}(\phi_i, g^2)\right)$$

Interesting limits

- ✔ Small cusp angle $\phi \to 0$

$$\Gamma_{\text{cusp}}(\phi) = -\phi^2 B(\alpha_s) + O(\phi^4)$$

$B(\alpha_s)$ bremsstrahlung function

- ✔ Large Minkowskian cusp angle $x = \mathrm{e}^{i\phi} \to 0$

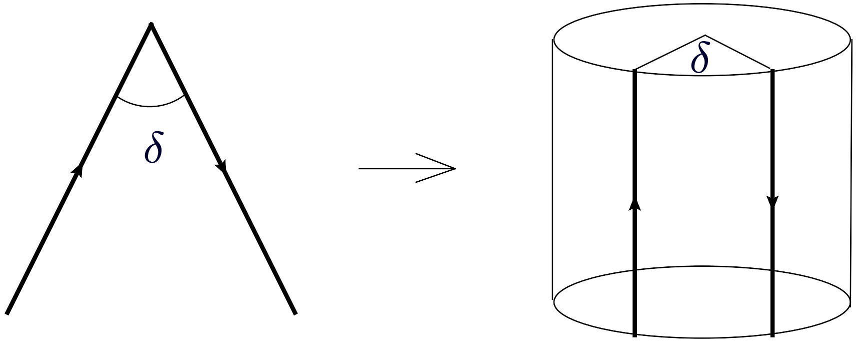
$$\Gamma_{\text{cusp}}(\phi) = K(\alpha_s) \ln(1/x) + O(x^0)$$

$K(\alpha_s)$ light-like cusp anomalous dimension

- ✔ Backtracking limit $\phi = \pi - \delta$ with $\delta \to 0$

$$\Gamma_{\text{cusp}}(\phi) = -\frac{V(\alpha_s)}{\delta} + O(\delta^0)$$

If conformal symmetry is not broken, $V(\alpha_s)/\delta$ gives a quark-antiquark potential



From QCD to $\mathcal{N} = 4$ SYM

Two classes of Yang-Mills theories:

(i) QCD – gauge field coupled to n_f fermions in the fundamental representation of the $SU(N)$

(ii) Supersymmetric extensions – gauge field coupled to interacting n_s scalars and n_f fermions all in the adjoint representation of the $SU(N)$:

$$\mathcal{N} = 1: \quad \big(n_f = 1, n_s = 0\big)\,,$$
$$\mathcal{N} = 2: \quad \big(n_f = 2, n_s = 2\big)\,,$$
$$\mathcal{N} = 4: \quad \big(n_f = 4, n_s = 6\big)$$

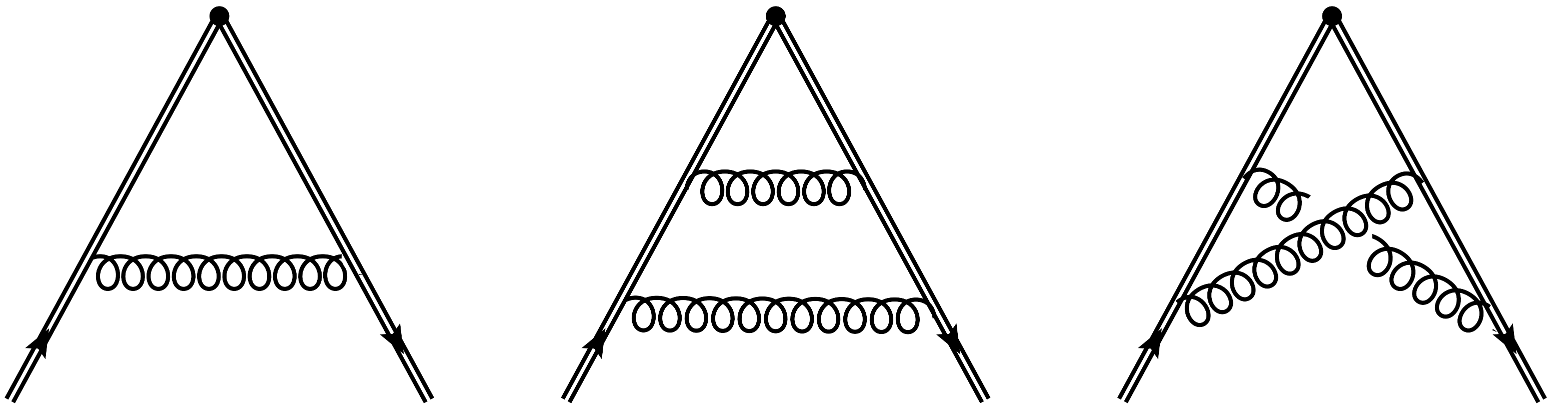
Main questions for this talk:

- ✔ Are there any common properties of the cusp anomalous dimension in QCD and SYM theories?
- ✔ How does the cusp anomaly depends on the choice of the $R-$representation (Casimir scaling?)

Compute the cusp anomalous dimension at three loops for arbitrary number of fermions/scalars to see more structure

General properties

Nonabelian exponentiation:



Relation between the Feynman integrals

$$\frac{1}{2}\,[I_{\mathsf{a}}]^2 = I_{\mathsf{b}} + I_{\mathsf{c}}$$

- ✔ Need only planar integrals!
- ✔ Only maximally nonabelian color factors contribute

$$\Gamma_{\mathrm{cusp}}(\phi,\alpha_s) \sim \sum_{\ell} \alpha_s^{\ell} \big[C_R\, C_A^{\ell-1}\gamma_\ell + 0 \times\, C_R^2 C_A^{\ell-2} + \cdots + 0 \times\, C_R^{\ell}\big] + O(n_f, n_s)$$

$C_A = N$ the Casimir in the adjoint of the $SU(N)$

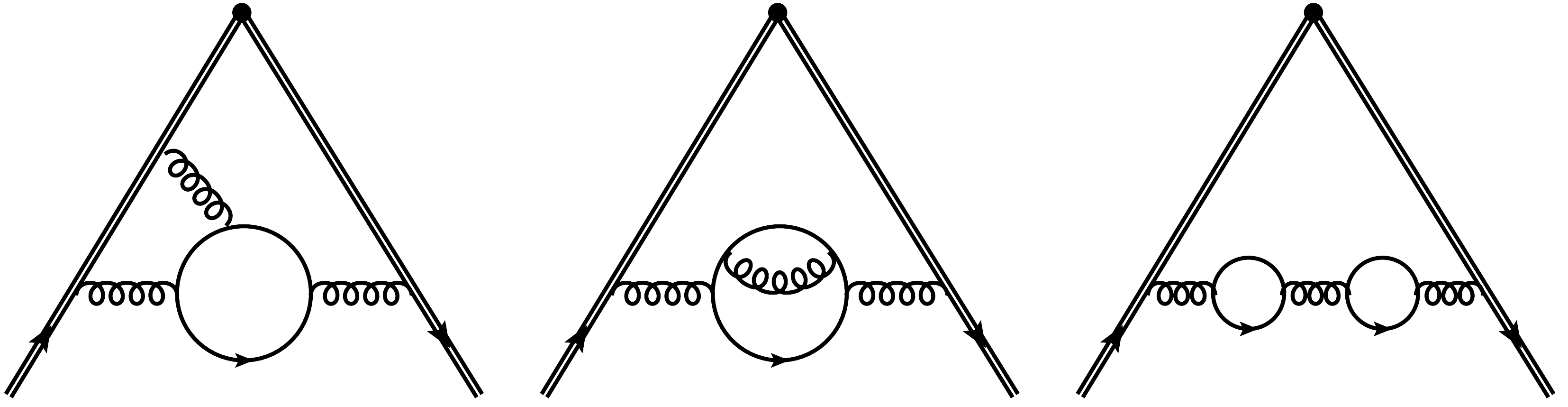
General properties II

General form of the cusp anomalous dimension in QCD

$$\Gamma_{\mathrm{cusp,QCD}}(\phi,\alpha_s) = C_R\Big[\frac{\alpha_s}{\pi}\gamma + \Big(\frac{\alpha_s}{\pi}\Big)^2 \left(C_A\gamma_A + T_F n_f \gamma_f\right)$$

$$+ \Big(\frac{\alpha_s}{\pi}\Big)^3 \Big(C_A^2\gamma_{AA} + C_A T_F n_f \gamma_{Af} + C_F T_F n_f \gamma_{Ff} + \left(T_F n_f\right)^2 \gamma_{ff}\Big)\Big] + \mathcal{O}(\alpha_s^4)$$

Sample diagrams contributing to $C_A T_F n_f$, $C_F T_F n_f$ and $(T_F n_f)^2$ terms, respectively



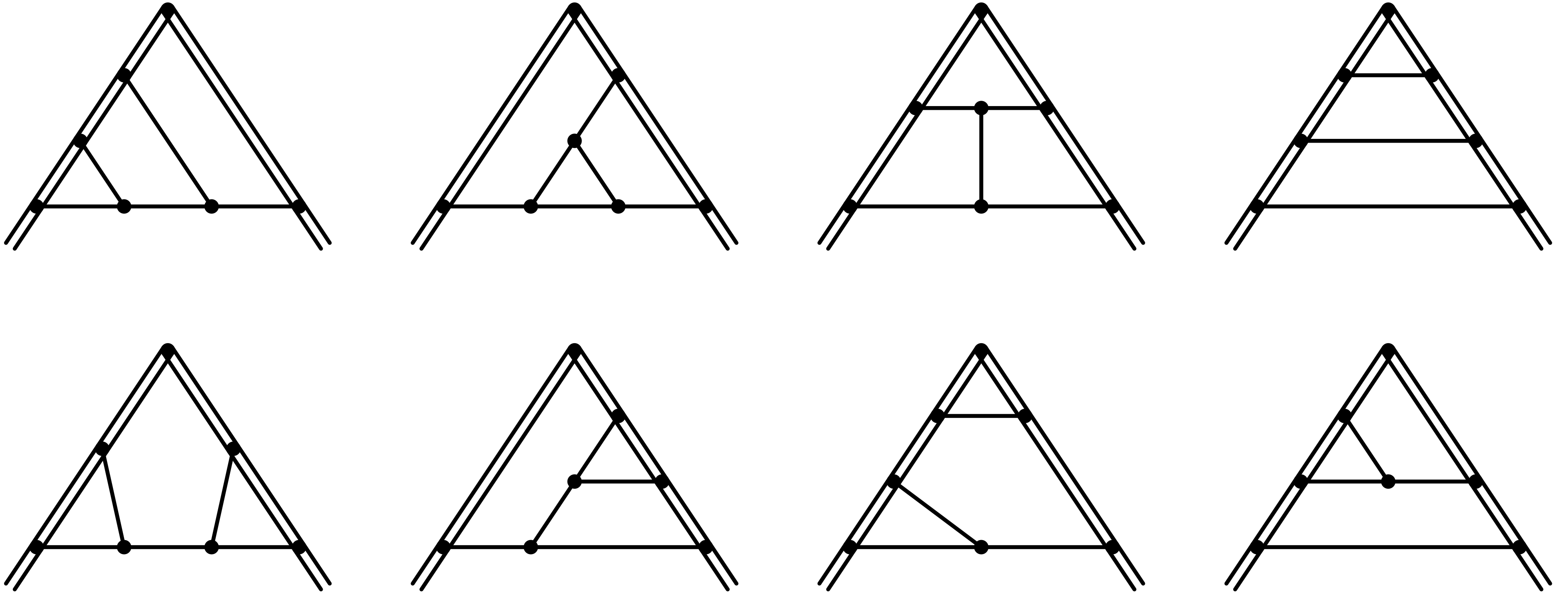
Time scale:

one loop	1980	[Polyakov]
two loops	1987	[GK,Radyushkin]
three loops	2015	[Grozin,Henn,GK,Marquard]

Setup of three-loop calculation

Generate all three-loop Feynman diagrams :

- ✔ 315 diagrams involving gluons and fermions plus 100 additional diagrams involving scalars
- ✔ In the planar limit there are only 120 diagrams, plus 32 diagrams with scalars
- ✔ Their contribution is expressed in terms of Feynman integrals of 8 different topologies



Computation of master integrals

- ✔ Step 1: Integration-by-parts reduction yields a set of 71 master scalar integrals [FIRE, Crusher]

$$\vec{f}(x;\epsilon) = \{f_1, \ldots, f_{71}\}\,, \qquad x = e^{i\phi}\,, \qquad D = 4 - 2\epsilon$$

- ✔ Step 2: Choose basis of uniform transcendental weight functions

$$\vec{f}(x;\epsilon) = \sum_{k\geq 0} \epsilon^k \vec{f}^{(k)}(x)$$

$\vec{f}^{(k)}(x) = \mathbb{Q}-$linear combination of harmonic polylogarithms of weight k

- ✔ Step 3: Differential equations in this basis [Henn]

$$\partial_x \vec{f}(x;\epsilon) = \epsilon \left[\frac{a}{x} + \frac{b}{x-1} + \frac{c}{x+1}\right] \vec{f}(x;\epsilon)$$

a, b, c constant 71×71 matrices

- ✔ Step 4: Boundary conditions trivially from $x = 1$ (straight line limit $\phi \to 0$)

- ✔ Step 5: Solution in terms of harmonic polylogarithms [Remiddi,Vermaseren]

$$\vec{f}(x;\epsilon) = \vec{f}(1,\epsilon) + \epsilon \int_1^x dy \left[\frac{a}{y} + \frac{b}{y-1} + \frac{c}{y+1}\right] \vec{f}(y;\epsilon)$$

Calculation at three loops

- Compute UV divergent part of the cusped Wilson loop in dimensional regularization

$$\log W = -\frac{1}{2\epsilon}\Big(\frac{\alpha_s}{\pi}\Big)\Gamma^{(1)} + \Big(\frac{\alpha_s}{\pi}\Big)^2\left[\frac{\beta_0\Gamma^{(1)}}{16\epsilon^2} - \frac{\Gamma^{(2)}}{4\epsilon}\right] + \Big(\frac{\alpha_s}{\pi}\Big)^3\left[-\frac{\beta_0^2\Gamma^{(1)}}{96\epsilon^3} + \frac{\beta_1\Gamma^{(1)} + 4\beta_0\Gamma^{(2)}}{96\epsilon^2} - \frac{\Gamma^{(3)}}{6\epsilon}\right]$$

Renormalized coupling constant $\alpha_s = g_{\rm YM}^2/(4\pi)$ in $\overline{\rm MS}$ scheme

$$\frac{d\log\alpha_s}{d\log\mu} = -2\epsilon - 2\beta(\alpha_s) = -2\epsilon - 2\left[\beta_0\frac{\alpha_s}{4\pi} + \beta_1\Big(\frac{\alpha_s}{4\pi}\Big)^2 + \dots\right]$$

- Extract the cusp anomalous dimension in $\overline{\rm MS}$ scheme

$$\Gamma_{\rm cusp}(\phi,\alpha_s) = \frac{d\log W}{d\log\mu} = \frac{\alpha_s}{\pi}\Gamma^{(1)} + \Big(\frac{\alpha_s}{\pi}\Big)^2\Gamma^{(2)} + \Big(\frac{\alpha_s}{\pi}\Big)^3\Gamma^{(3)} + \dots$$

- To restore the supersymmetry convert into $\overline{\rm DR}$ scheme

$$\Gamma_{\rm cusp}^{\overline{\rm DR}}(\phi,\alpha_s^{\overline{\rm DR}}) = \Gamma_{\rm cusp}^{\overline{\rm MS}}(\phi,\alpha_s^{\overline{\rm MS}}) \qquad \alpha_s^{\overline{\rm DR}} = \alpha_s^{\overline{\rm MS}}\left[1 + c_1\frac{\alpha_s^{\overline{\rm MS}}}{\pi} + c_2\left(\frac{\alpha_s^{\overline{\rm MS}}}{\pi}\right)^2 + \dots\right]$$

- Checks:
  - Reproduce known results for $\beta-$function coefficients
  - Dependence on gauge parameter disappears from $\Gamma_{\rm cusp}$

Two-loop result

$$\Gamma^{\overline{\mathrm{MS}}}_{\mathrm{QCD}} = \frac{\alpha_s}{\pi} C_R \tilde{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 C_R \left[\frac{1}{2} C_A \left(\tilde{A}_2 + \tilde{A}_3\right) + \left(\frac{67}{36} C_A - \frac{5}{9} T_F n_f\right) \tilde{A}_1\right]$$

Coefficient functions

$$\tilde{A}_i(x) = A_i(x) - A_i(1)$$

$$A_1(x) = \xi \, \frac{1}{2} H_1(y) \,, \qquad A_2(x) = \left[\frac{\pi^2}{3} + \frac{1}{2} H_{1,1}(y)\right] + \xi \left[-H_{0,1}(y) - \frac{1}{2} H_{1,1}(y)\right] \,,$$

$$A_3(x) = \xi \left[-\frac{\pi^2}{6} H_1(y) - \frac{1}{4} H_{1,1,1}(y)\right] + \xi^2 \left[\frac{1}{2} H_{1,0,1}(y) + \frac{1}{4} H_{1,1,1}(y)\right]$$

Scaling variables

$$x = \mathrm{e}^{i\phi} \,, \qquad \xi = \frac{1+x^2}{1-x^2} = i \cot\phi \,, \qquad y = 1 - x^2$$

The harmonic polylogarithms

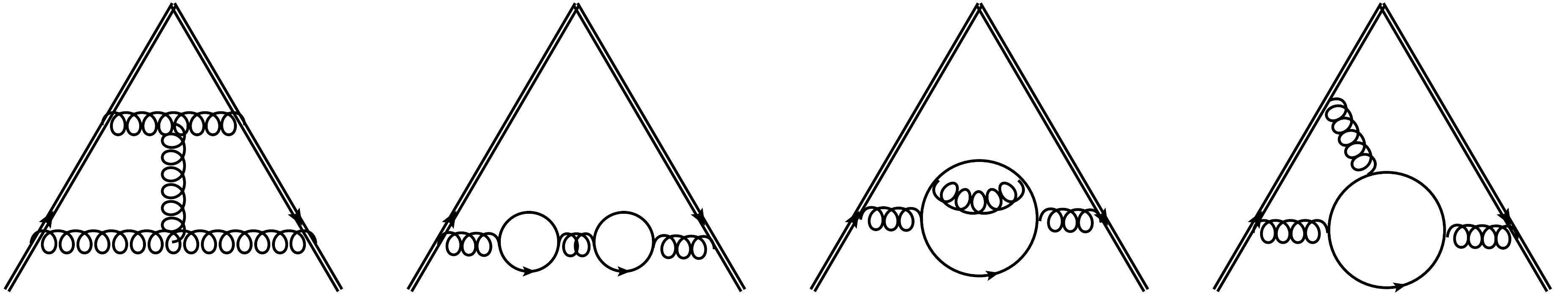
$$H_1(x) = -\log(1-x) \,, \qquad H_0(x) = \log(x) \,, \qquad H_{-1}(x) = \log(1+x)$$

$$H_{a_1,a_2,\ldots,a_n}(y) = \int_0^y \frac{dt}{t+a_1} H_{a_2,\ldots,a_n}(t) dt \,,$$

Three-loop result

$$\Gamma_{\rm QCD}^{\overline{\rm MS}} = \cdots + \left(\frac{\alpha_s}{\pi}\right)^3 C_R \left[C_A^2\, \gamma_{AA} + (T_F n_f)^2 \gamma_{ff} + C_F T_F n_f \gamma_{Ff} + C_A T_F n_f \gamma_{Af} \right]$$

Sample diagrams



Coefficient functions

$$\gamma_{AA} = \frac{1}{4}\left(\tilde{A}_5 + \tilde{A}_4 + \tilde{B}_5 + \tilde{B}_3\right) + \frac{67}{36}\tilde{A}_3 + \frac{29}{18}\tilde{A}_2 + \left(\frac{245}{96} + \frac{11}{24}\zeta_3\right)\tilde{A}_1\,,$$

$$\gamma_{ff} = -\frac{1}{27}\tilde{A}_1\,, \qquad \gamma_{Ff} = \left(\zeta_3 - \frac{55}{48}\right)\tilde{A}_1\,,$$

$$\gamma_{Af} = -\frac{5}{9}\left(\tilde{A}_2 + \tilde{A}_3\right) - \frac{1}{6}\left(7\zeta_3 + \frac{209}{36}\right)\tilde{A}_1$$

γ_{ff} agrees with large n_f calculation [Gracey'94], [Braun,Beneke'95]

Three-loop result (2)

Coefficient functions of uniform transcendental weight

$$\tilde{A}_i(x) = A_i(x) - A_i(1)\,, \qquad \tilde{B}_i(x) = B_i(x) - B_i(1)$$

$$A_4(x) = \left[-\frac{\pi^2}{6}H_{1,1}(y) - \frac{1}{4}H_{1,1,1,1}(y)\right] + \xi\left[\frac{\pi^2}{3}H_{0,1}(y) + \frac{\pi^2}{6}H_{1,1}(y) + 2H_{1,1,0,1}(y) + \frac{3}{2}H_{0,1,1,1}(y) + \frac{7}{4}H_{1,1,1,1}(y)\right.$$
$$\left. + 3\zeta_3 H_1(y)\right] + \xi^2\left[-2H_{1,0,0,1}(y) - 2H_{0,1,0,1}(y) - 2H_{1,1,0,1}(y) - H_{1,0,1,1}(y) - H_{0,1,1,1}(y) - \frac{3}{2}H_{1,1,1,1}(y)\right],$$

$$A_5(x) = \xi\left[\frac{\pi^4}{12}H_1(y) + \frac{\pi^2}{4}H_{1,1,1}(y) + \frac{5}{8}H_{1,1,1,1,1}(y)\right] + \xi^2\left[-\frac{\pi^2}{6}H_{1,0,1}(y) - \frac{\pi^2}{3}H_{0,1,1}(y) - \frac{\pi^2}{4}H_{1,1,1}(y)\right.$$
$$\left. - H_{1,1,1,0,1}(y) - \frac{3}{4}H_{1,0,1,1,1}(y) - H_{0,1,1,1,1}(y) - \frac{11}{8}H_{1,1,1,1,1}(y) - \frac{3}{2}\zeta_3 H_{1,1}(y)\right]$$
$$+ \xi^3\left[H_{1,1,0,0,1}(y) + H_{1,0,1,0,1}(y) + H_{1,1,1,0,1}(y) + \frac{1}{2}H_{1,1,0,1,1}(y) + \frac{1}{2}H_{1,0,1,1,1}(y) + \frac{3}{4}H_{1,1,1,1,1}(y)\right],$$

$$B_3(x) = \left[-H_{1,0,1}(y) + \frac{1}{2}H_{0,1,1}(y) - \frac{1}{4}H_{1,1,1}(y)\right] + \xi\left[2H_{0,0,1}(y) + H_{1,0,1}(y) + H_{0,1,1}(y) + \frac{1}{4}H_{1,1,1}(y)\right],$$

$$B_5(x) = \frac{x}{1-x^2}\left[-\frac{\pi^4}{60}H_{-1}(x) - \frac{\pi^4}{60}H_1(x) - 4H_{-1,0,-1,0,0}(x) + 4H_{-1,0,1,0,0}(x) - 4H_{1,0,-1,0,0}(x)\right.$$
$$\left. + 4H_{1,0,1,0,0}(x) + 4H_{-1,0,0,0,0}(x) + 4H_{1,0,0,0,0}(x) + 2\zeta_3 H_{-1,0}(x) + 2\zeta_3 H_{1,0}(x)\right]$$

Variables $\xi = (1+x^2)/(1-x^2)$ and $y = 1 - x^2$

Checks of result

- ✔ Casimir scaling at 3 loops

$$\Gamma_{\text{cusp}}(\phi, \alpha_s) \sim C_R$$

Broken at four loops due to appearance of higher Casimirs

- ✔ Light-like limit $\phi \to i\infty$

$$\Gamma_{\text{cusp}}(\phi, \alpha_s) = K(\alpha_s)\phi + \mathcal{O}(\phi^0)$$

$K(\alpha_s)$ the light-like cusp anomalous dimension

$$K^{\overline{\text{MS}}}_{\text{QCD}}(\alpha_s) = C_R \Big\{ \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[C_A \left(\frac{67}{36} - \frac{\pi^2}{12} \right) - \frac{5}{9} T_F n_f \right]$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3 \Big[C_A^2 \left(\frac{245}{96} - \frac{67\pi^2}{216} + \frac{11\pi^4}{720} + \frac{11}{24}\zeta_3 \right) - \frac{1}{27}(T_F n_f)^2$$
$$+ C_A T_F n_f \left(-\frac{209}{216} + \frac{5\pi^2}{54} - \frac{7}{6}\zeta_3 \right) + C_F T_F n_f \left(\zeta_3 - \frac{55}{48} \right) \Big] \Big\}.$$

Agrees with the known result [Moch,Vermaseren,Vogt'2004]

$$K^{\overline{\text{DR}}}_{\mathcal{N}=4}(\alpha_s) = C_R \left[\frac{\alpha_s}{\pi} - \frac{\pi^2}{12}\left(\frac{\alpha_s}{\pi}\right)^2 C_A + \frac{11}{720}\pi^4 \left(\frac{\alpha_s}{\pi}\right)^3 C_A^2 \right] + \mathcal{O}(\alpha_s^4)\,,$$

Agrees with integrability prediction [Kotikov et al'2004]

Checks of result (2)

Backtracking limit $\phi = \pi - \delta$ and $\delta \to 0$

$$\Gamma_{\rm cusp}(\pi-\delta,\alpha_s) \sim -C_R \frac{\alpha_s}{\delta} V_{\rm cusp}(\alpha_s) + O(\alpha_s^4 \log\delta/\delta)$$

Three-loop results:

$$V^{\overline{\rm MS}}_{\rm cusp,\,QCD} = 1 + \frac{\alpha_s}{\pi}\left(\frac{31}{36}C_A - \frac{5}{9}n_f T_F\right) + \left(\frac{\alpha_s}{\pi}\right)^2 \left[C_A^2\left(\frac{23}{288} + \frac{\pi^2}{4} - \frac{\pi^4}{64} + \frac{11}{24}\zeta_3\right)\right.$$
$$\left. - \frac{1}{27}(n_f T_F)^2 + C_F n_f T_F\left(\zeta_3 - \frac{55}{48}\right) + C_A n_f T_F\left(-\frac{7}{6}\zeta_3 + \frac{31}{216}\right)\right]$$

$$V^{\overline{\rm DR}}_{\rm cusp,\,\mathcal{N}=4} = 1 - \frac{\alpha_s}{\pi}C_A + \left(\frac{\alpha_s}{\pi}\right)^2 C_A^2\left(\frac{5}{4} + \frac{\pi^2}{4} - \frac{\pi^4}{64}\right)$$

Relation to quark-antiquark potential in $\mathcal{N}=4$ SYM [Prausa,Steinhauser'2013]

$$V_{\rm cusp,\,\mathcal{N}=4}(\alpha_s) - V_{Q\bar{Q},\mathcal{N}=4}(\alpha_s) = 0$$

and in QCD (up to conformal symmetry breaking corrections) [Peter'1997][Schröder'1998]

$$V_{\rm cusp,\,QCD}(\alpha_s) - V_{Q\bar{Q},{\rm QCD}}(\alpha_s) = \beta(\alpha_s)C(\alpha_s)$$

Universal scaling function

- ✔ Introduce a new effective coupling constant

$$a = \frac{\pi}{C_R} K(\alpha_s) = \alpha_s \left[1 + \frac{\alpha_s}{\pi} K^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^2 K^{(3)} + O(\alpha_s^3)\right]$$

$K^{(2)}$ and $K^{(3)}$ depend on n_f and n_s

- ✔ Define a new function

$$\Omega(\phi, a) := \Gamma_{\text{cusp}}(\phi, \alpha_s)$$

General form

$$\Omega(\phi, a) = \sum_{k \geq 0} \left(\frac{a}{\pi}\right)^k \Omega^{(k)}(\phi)\,, \qquad \Gamma_{\text{cusp}}(\phi, \alpha_s) = \sum_{k \geq 0} \left(\frac{\alpha_s}{\pi}\right)^k \Gamma^{(k)}(\phi)$$

Iterative solution

$$\Omega^{(1)} = \Gamma^{(1)}$$

$$\Omega^{(2)} = \Gamma^{(2)} - K^{(2)}\Gamma^{(1)}$$

$$\Omega^{(3)} = \Gamma^{(3)} - K^{(3)}\Gamma^{(1)} - 2K^{(2)}\Gamma^{(2)} + 2(K^{(2)})^2\Gamma^{(1)}$$

$$\Omega^{(4)} = \Gamma^{(4)} - K^{(4)}\Gamma^{(1)} + \text{[lower loops terms]}$$

$K^{(\ell)}$ and $\Gamma^{(\ell)}$ (with $\ell \geq 2$) depend on n_f and n_s

Universal scaling function (2)

- ✔ Large angle limit is one-loop exact, $\phi = i\ln(1/x)$ with $x \to 0$

$$\Omega(\phi, a) = \frac{a}{\pi} C_R \ln(1/x) + O(x^0)$$

- ✔ $\Omega_{\rm QCD}$ is independent on the number of fermions

- ✔ $\Omega_{\rm SUSY}$ is independent on the number of fermions and scalars

- ✔ The two functions coincide at three loops !

$$\Omega(\phi, a) = C_R \bigg[\frac{a}{\pi} \tilde{A}_1 + \left(\frac{a}{\pi}\right)^2 \frac{N}{2} \left(\frac{\pi^2}{6} \tilde{A}_1 + \tilde{A}_2 + \tilde{A}_3 \right)$$

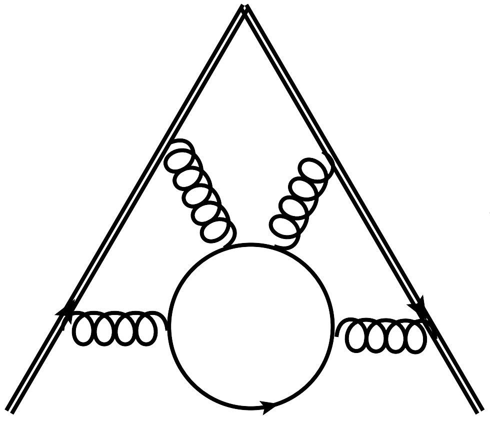
$$+ \left(\frac{a}{\pi}\right)^3 \frac{N^2}{4} \left(-\tilde{A}_2 + \tilde{A}_4 + \tilde{A}_5 + \tilde{B}_3 + \tilde{B}_5 - \frac{\pi^4}{180} \tilde{A}_1 + \frac{\pi^2}{3} (\tilde{A}_2 + \tilde{A}_3) \right) \bigg]$$

- ✔ *The function $\Omega(\phi, a)$ is the same in any gauge theory, to three loops at least !!!*

$$\Omega_{\mathcal{N}=4}(\phi, a) = \Omega_{\textsf{QCD}}(\phi, a) = \Omega_{\textsf{YM}}(\phi, a)$$

Conjecture for nonplanar corrections

Violation of the Casimir scaling at four loops



$$\sim \mathrm{tr}_F(t^a t^b t^c t^d)\,\mathrm{tr}_F(t^a t^b t^c t^d) = \frac{d_F^{abcd} d_F^{abcd}}{2N_F} + \dots = C_F \frac{18 - 6N^2 + N^4}{96N^2} + \dots$$

Produces nonplanar corrections

$$\Gamma_{\mathrm{cusp}}(\alpha_s, x) = \frac{n_f}{64}\left(\frac{\alpha_s}{\pi}\right)^4 g(x) C_F C_4 + \dots\,, \qquad C_4 = \frac{N^2}{96}\left(1 - \frac{1}{16} N^{-2} + \frac{3}{16} N^{-4}\right)$$

Assuming that $\Omega(a, x)$ remains n_f independent at four loops yields

$$g(x) = \kappa_F \tilde{A}_1(x)$$

Fix constant κ_F from the relation to quark-antiquark potential [Smirnov,Smirnov,Steinhauser'2008]

$$\kappa_F = -56.83(1)$$

Prediction for nonplanar correction to the light-like cusp anomalous dimension $K(\alpha_s)$

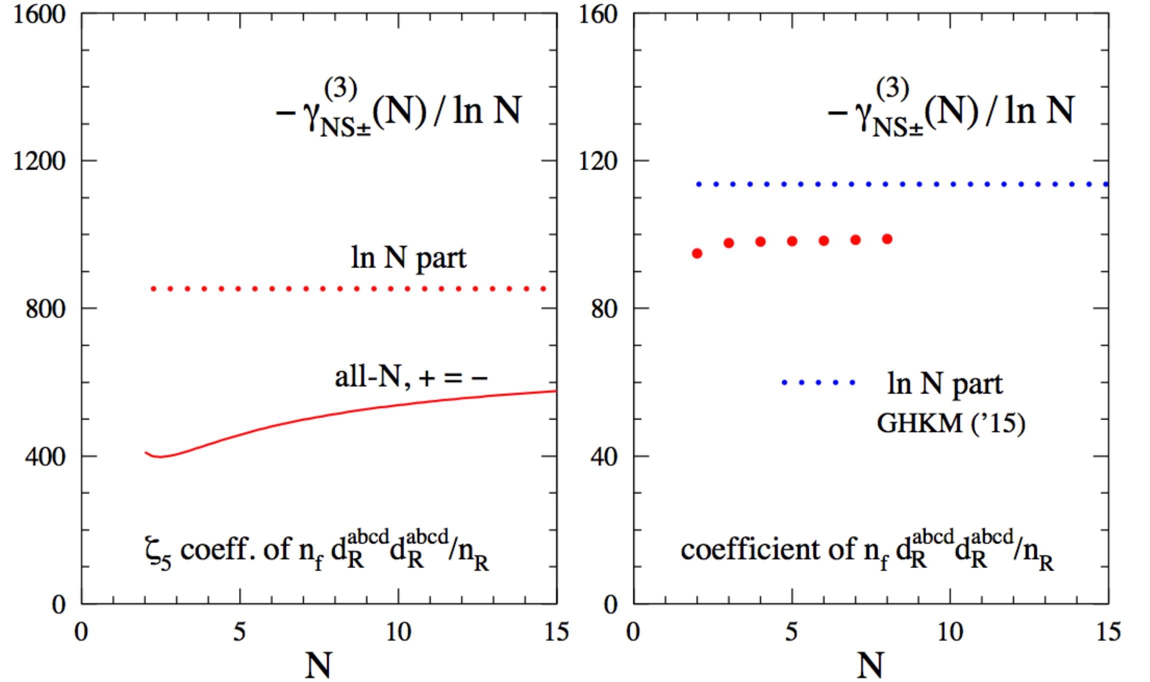
Recent development

B. Ruijl, T. Ueda, J. A. M. Vermaseren, J. Davies and A. Vogt,

"First Forcer results on deep-inelastic scattering and related quantities," arXiv:1605.08408 [hep-ph]

From A. Vogt's talk at *"Loops and Legs in Quantum Field Theory"*

n_f quartic-Casimir contribution to $\gamma_{\mathrm{ns}}^{(3)}(N)$



So far all-N form only for ζ_5 part – wrong in Velizhanin ('14). $N \leq 8$ values consistent with $\gamma_{\mathrm{cusp}}|_{(d_R^{abcd})^2}$ of Grozin, Henn, Korchemski, Marquardt ('15)

$$\gamma_{\mathrm{NS}}(N) = 2\Gamma_{\mathrm{cusp}} \ln N + O(N^0)$$

Summary

- ✔ Full analytic three-loop result in QCD and SYM theories
- ✔ Predicts infrared divergences of all planar 3-loop amplitudes
- ✔ n_f and n_s dependence is surprisingly simple!
- ✔ Leads to predictions/conjectures
  - ✗ Universality of $\Omega-$function
  - ✗ Violation of the Casimir scaling
  - ✗ Nonplanar corrections at four loops
- ✔ What is the reason for universality of $\Omega-$function?
- ✔ Can $\mathcal{N} = 4$ SYM result be computed from integrability?