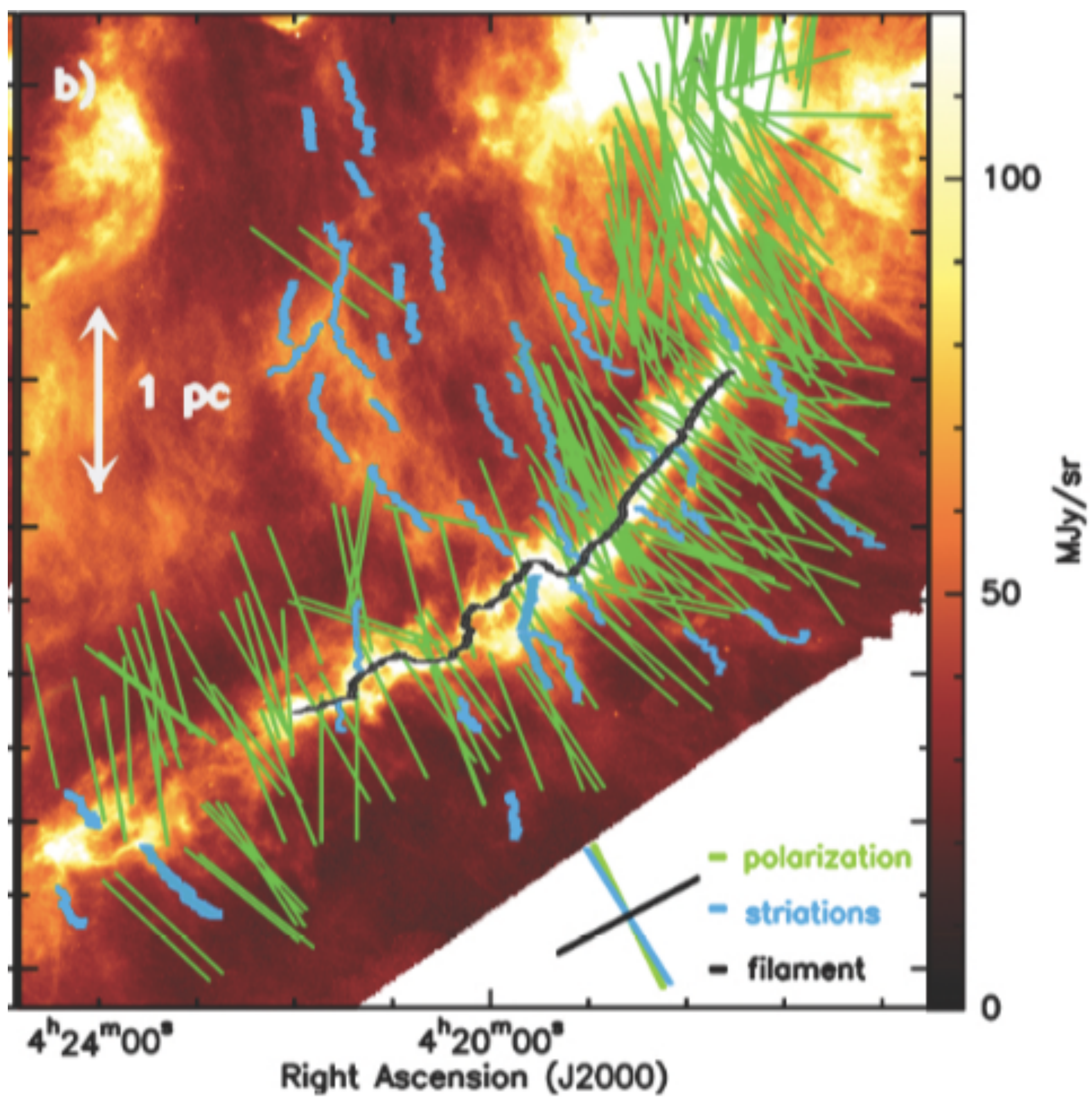
Fragmentation of Filamentary Molecular Clouds Threaded by Perpendicular Magnetic Field



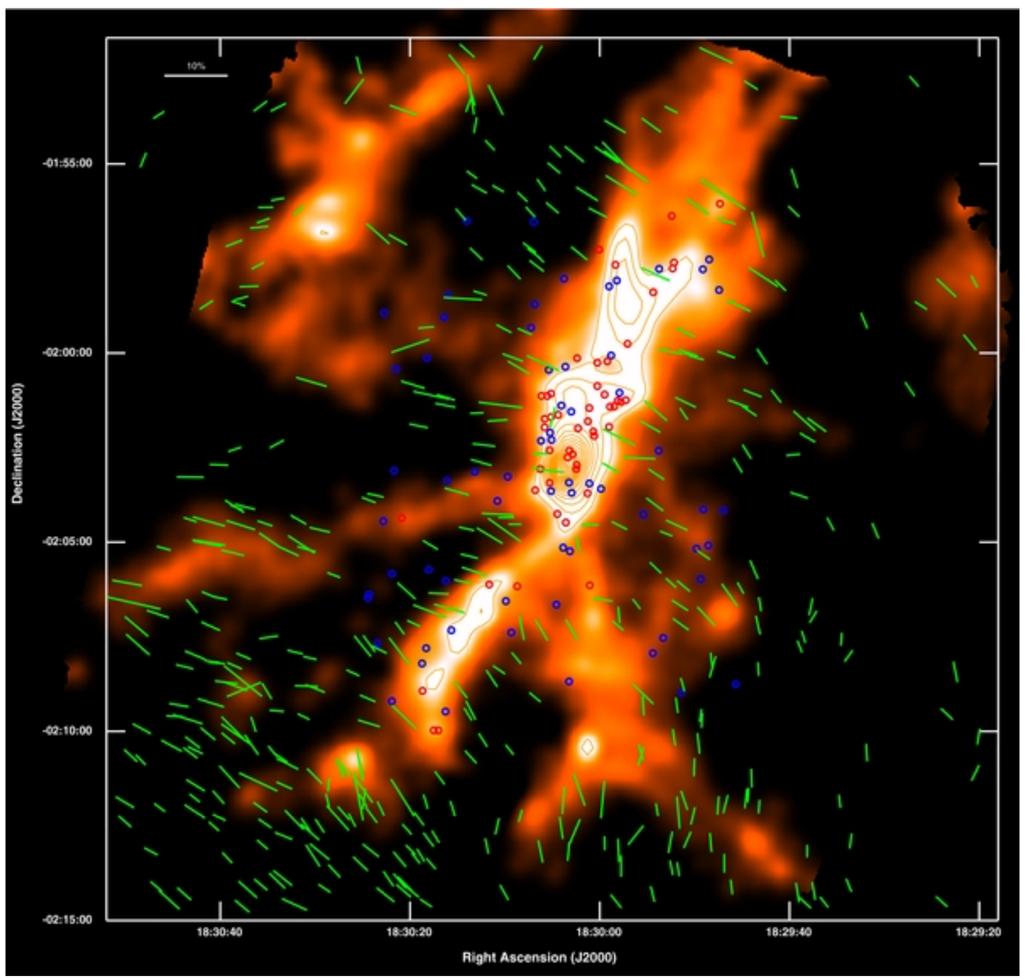
Taurus Palmeirim+ 13
250 cont.＋Magnetic Field (Polarization)

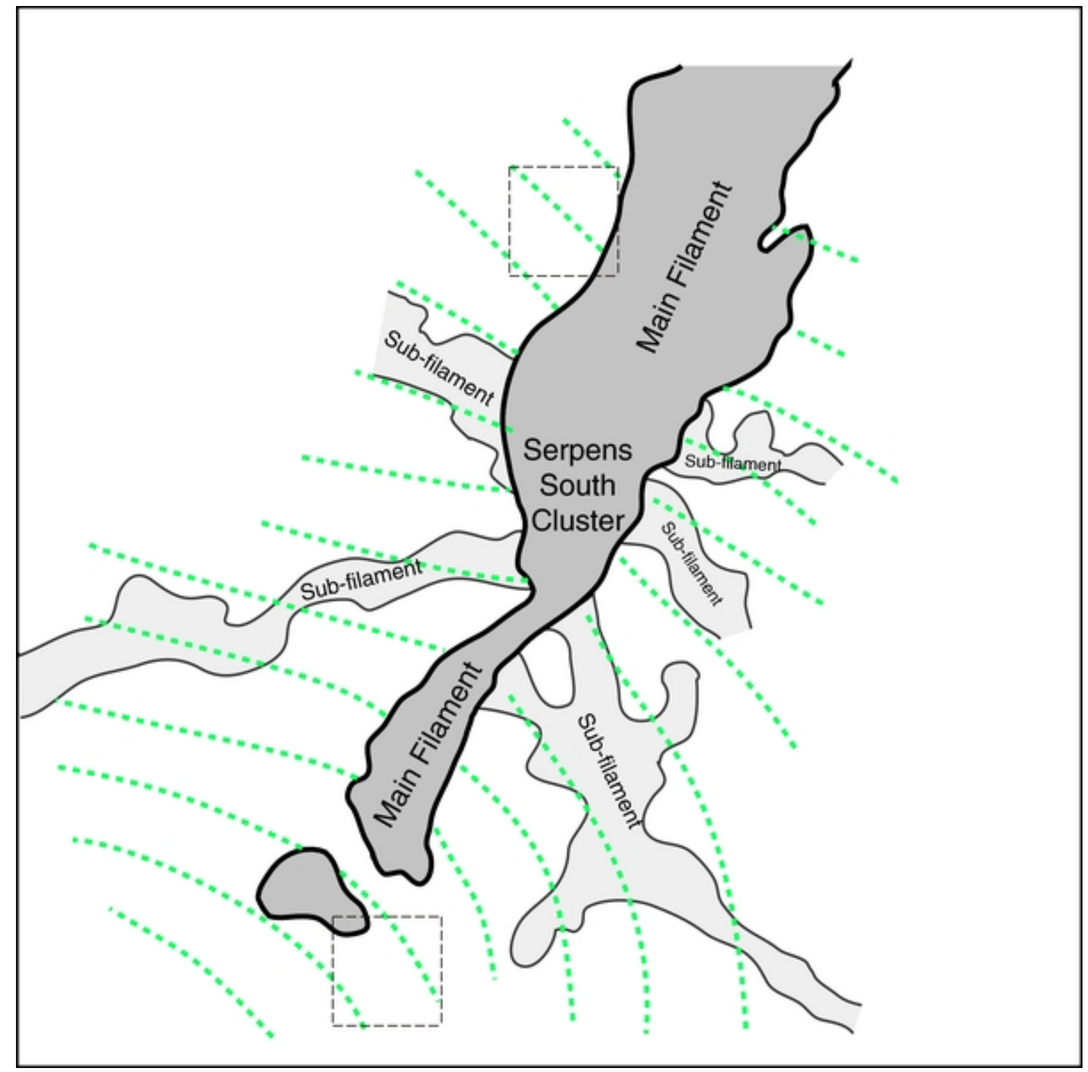
B direction matters.

submitted to ApJ

Tomoyuki Hanawa (U. Chiba)
Takahiro Kudo (U. Nagasaki)
Kohji Tomisaka (NAOJ)

Magnetic Field is Perpendicular to the Main Filaments. Striations (sub filaments) are parallel to B.

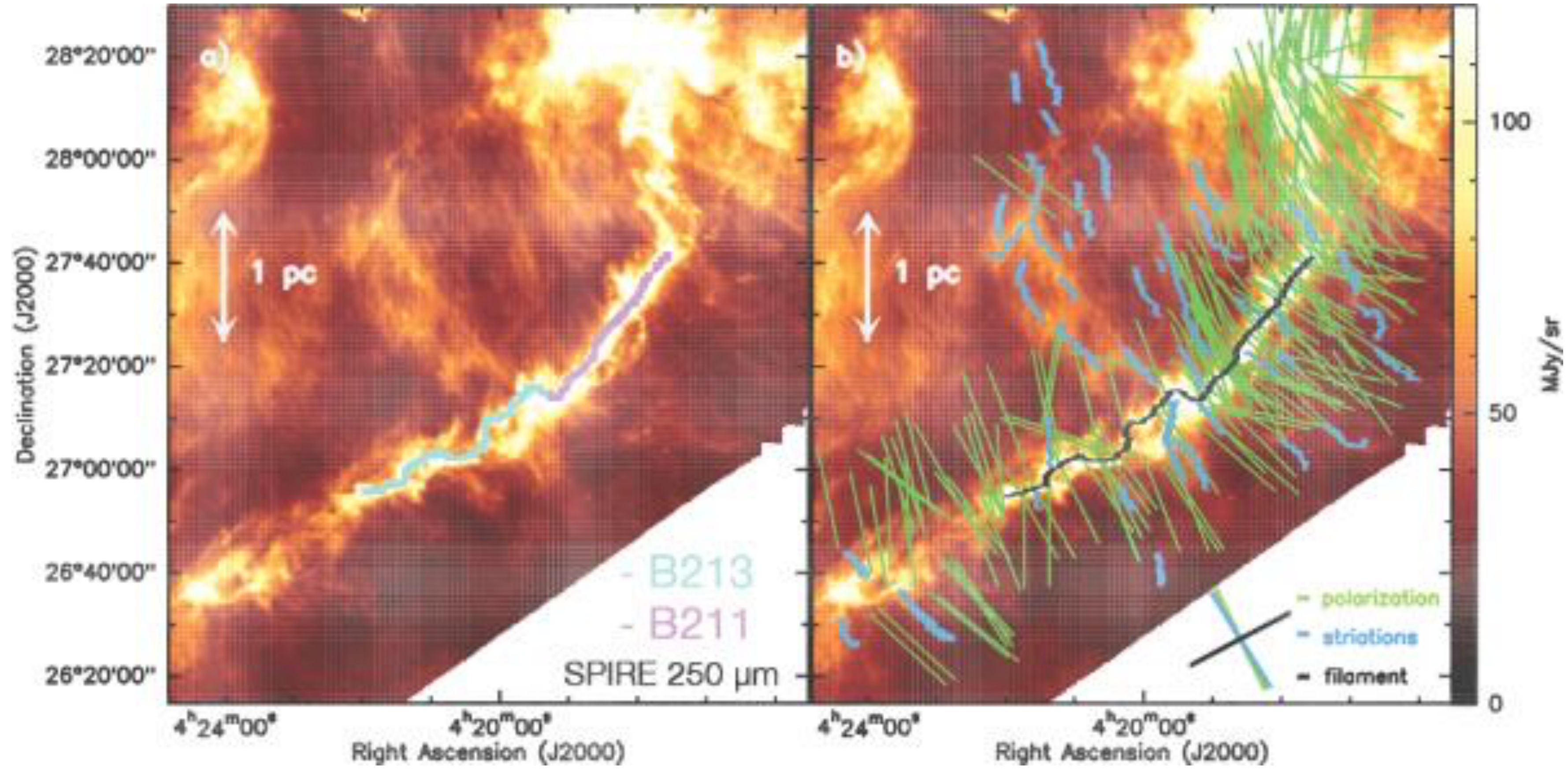




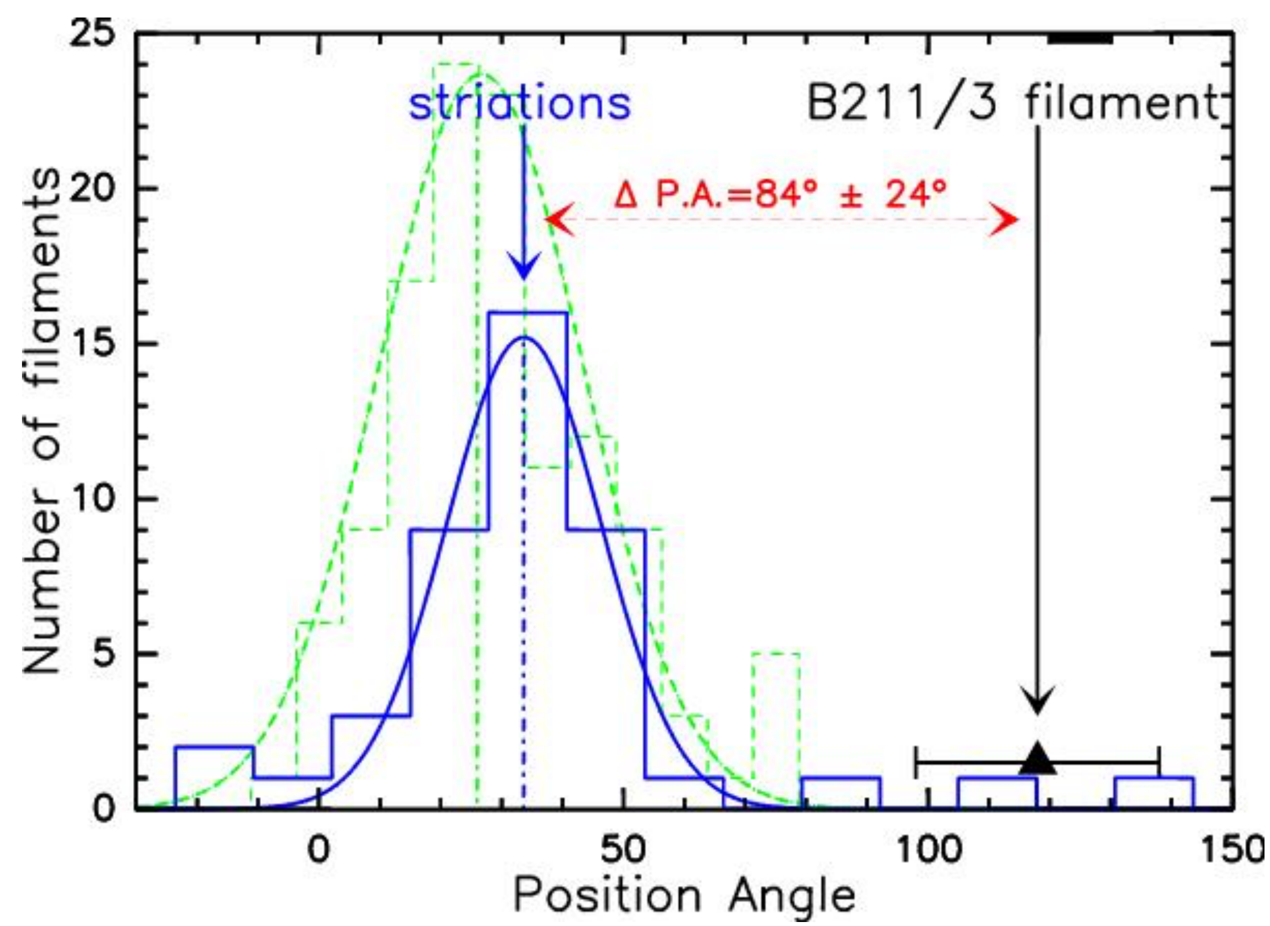
Serpens South (K-band) Sugitani+11

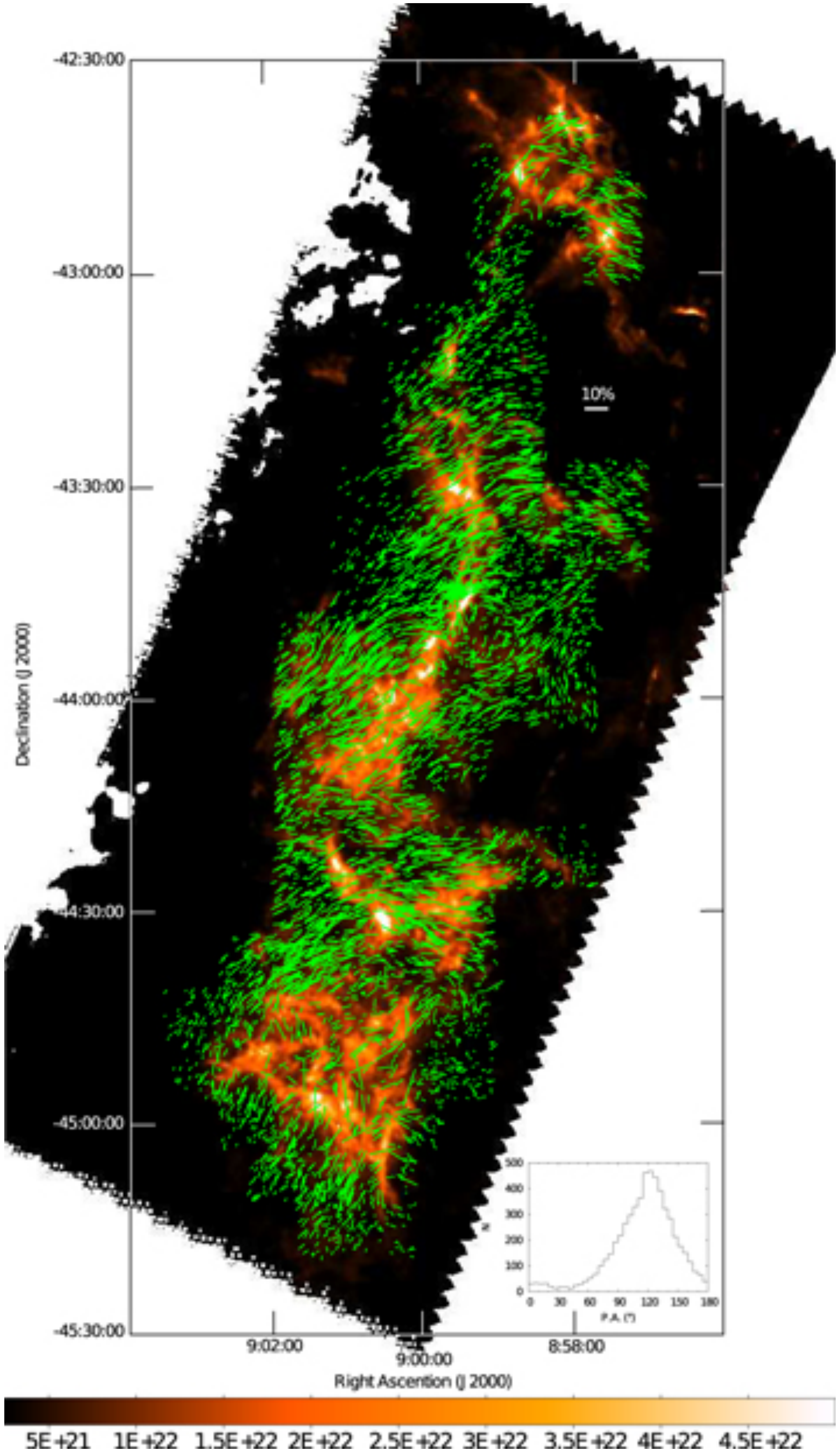
Serpens South (K-band) Sugitani+11

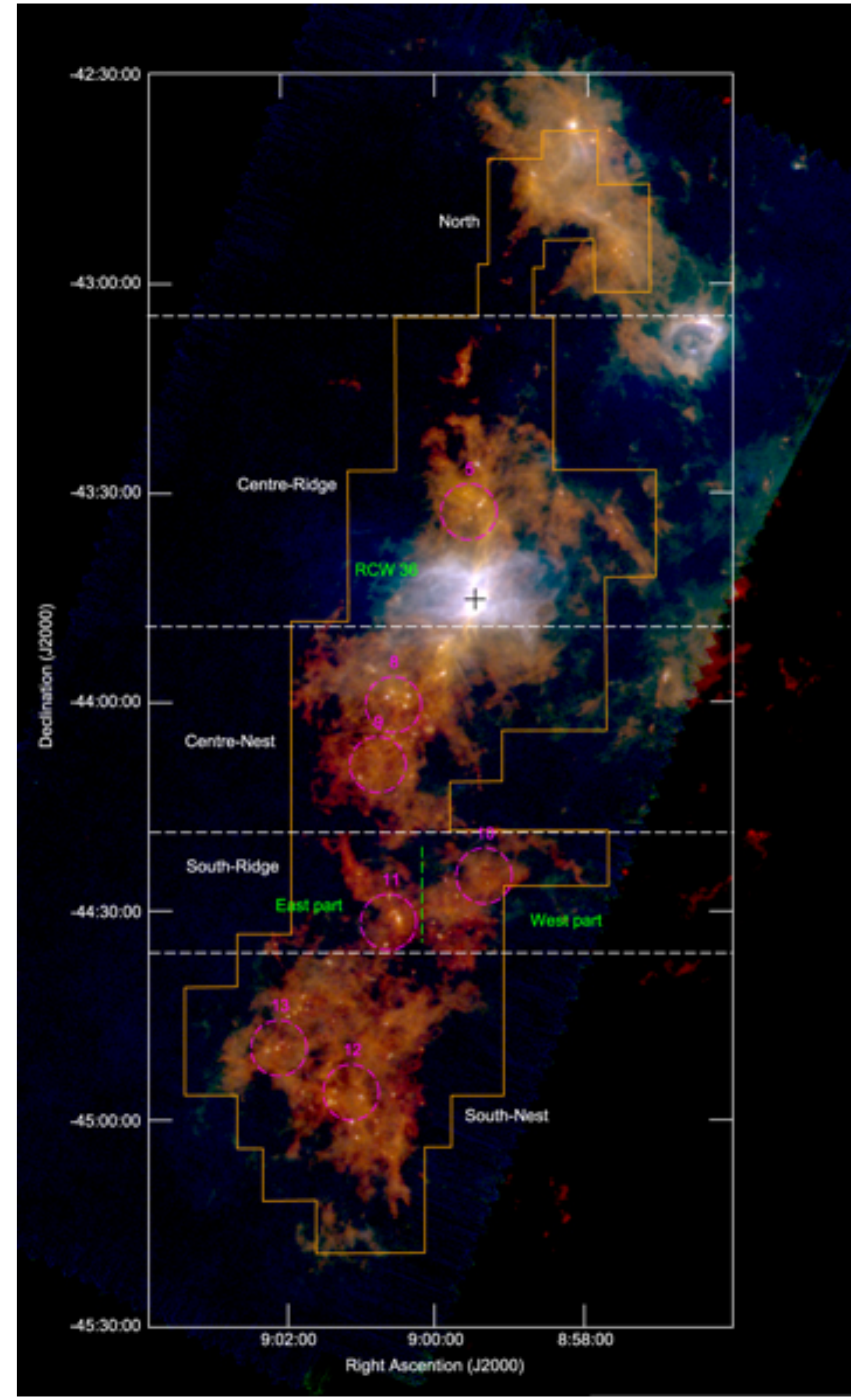
B211/3
Palmeirim+13



Cloud cores are formed through fragmentation of the main filament.







Vela C Kusune+16

Fragmentation of a Filamentary Cloud is a ***Classic*** Problem.

ACTA ASTRONOMICA
Vol. **13**, (1963) No 1

1963

a half century ago!

On the Gravitational Instability of Some Magneto-hydrodynamical Systems of Astrophysical Interest

Part III

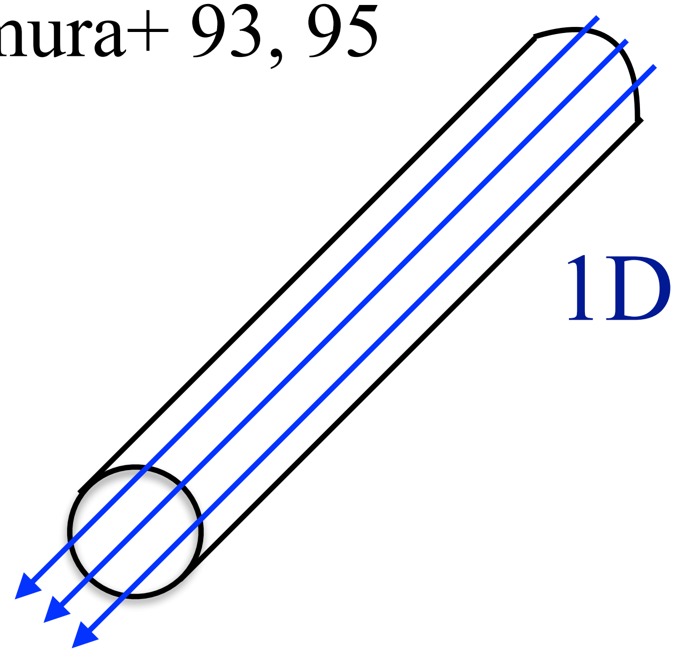
by

J. S. Stodółkiewicz

Stodolkiewcz 63, Ostriker 64, Nagasawa 87, Nakamura+93, 95, Hanawa+93, Matsumoto+94, Fiege & Pudritz 00

Equilibrium model:
Longitudinal Magnetic Field

cf. Nagasawa 87
Nakamura+ 93, 95



symmetric around the axis

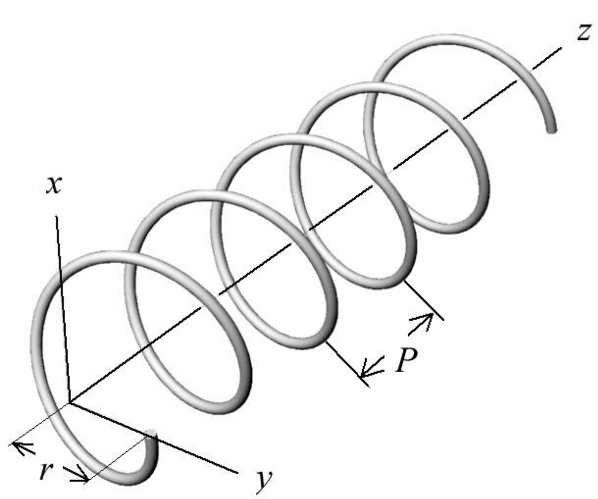
$$\rho(r) = \rho_0 \left(1 + \frac{r^2}{8H^2}\right)^{-2}$$

$$B_z(r) = B_0 \left(1 + \frac{r^2}{8H^2}\right)^{-1}$$

$$4\pi G \rho_0 H^2 = c_s^2 + \frac{B_0^2}{8\pi\rho_0}$$

supported in part by magnetic fields.
Stodolkiewicz 63

B_φ: hoop stress
cf. Fiege & Pudritz 00

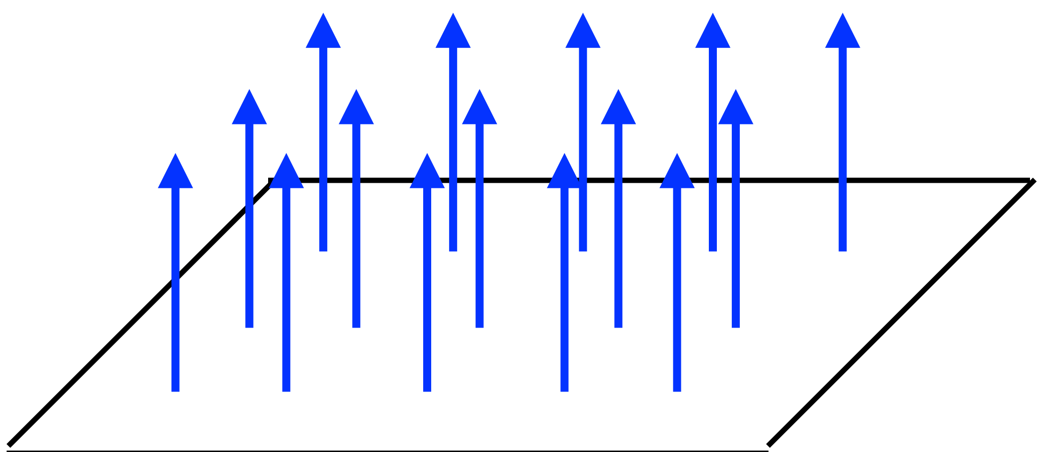


Unstable against fragmentation
Jeans wavelength
several times of the filament diameter

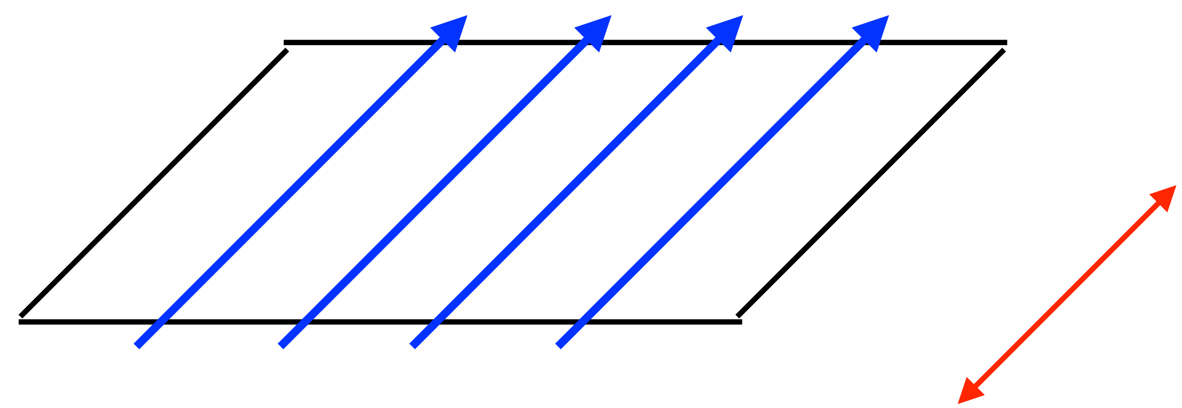
Stability of Magnetized Sheet Cloud (Nakano & Nakamura 78, Nakamura+ 91, Nagai+98)

$B > 2\pi\sqrt{G}\Sigma$ stable

always unstable (supercritical)



B suppresses fragmentation

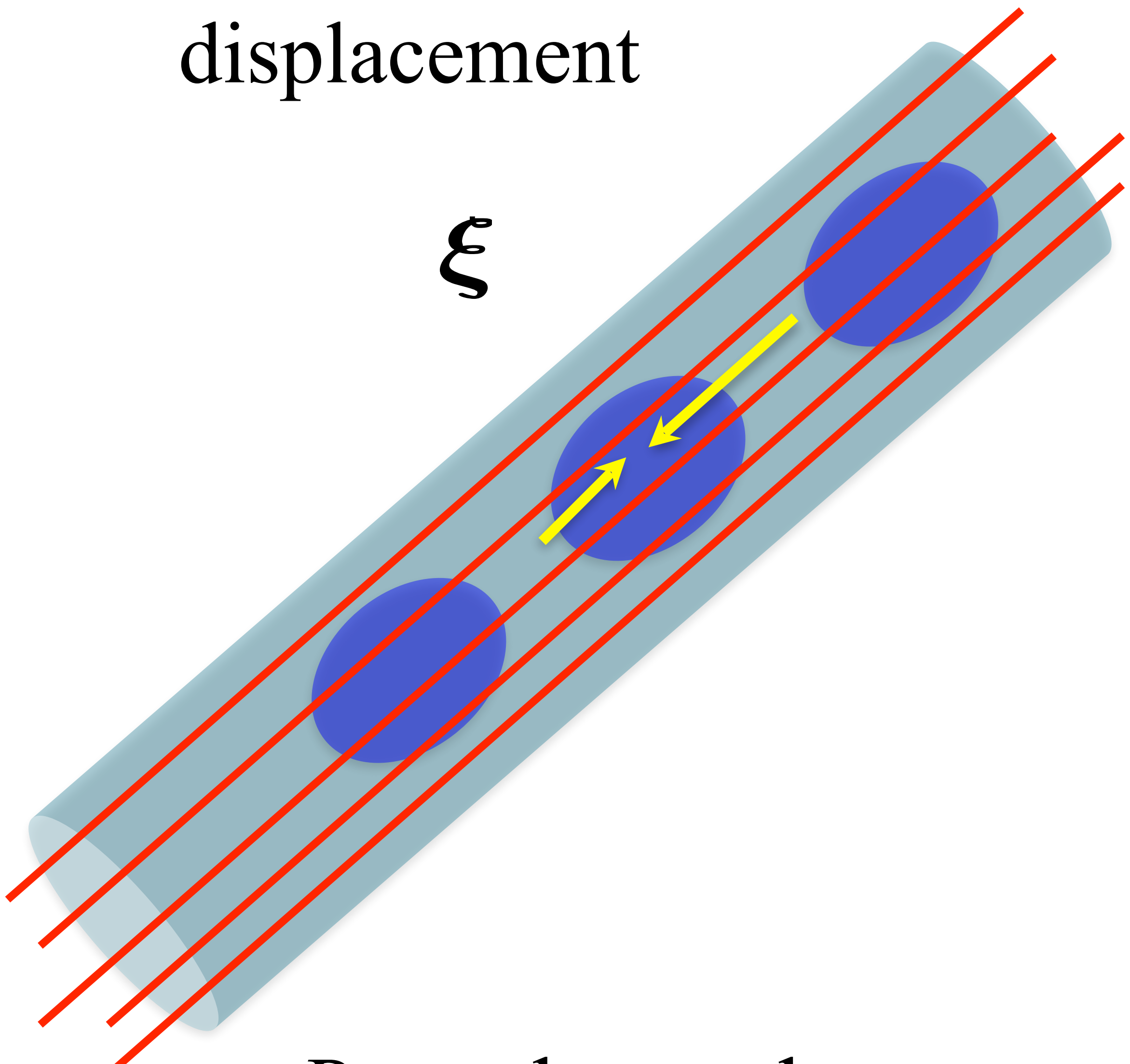


fragmentation

no magnetic force parallel to B

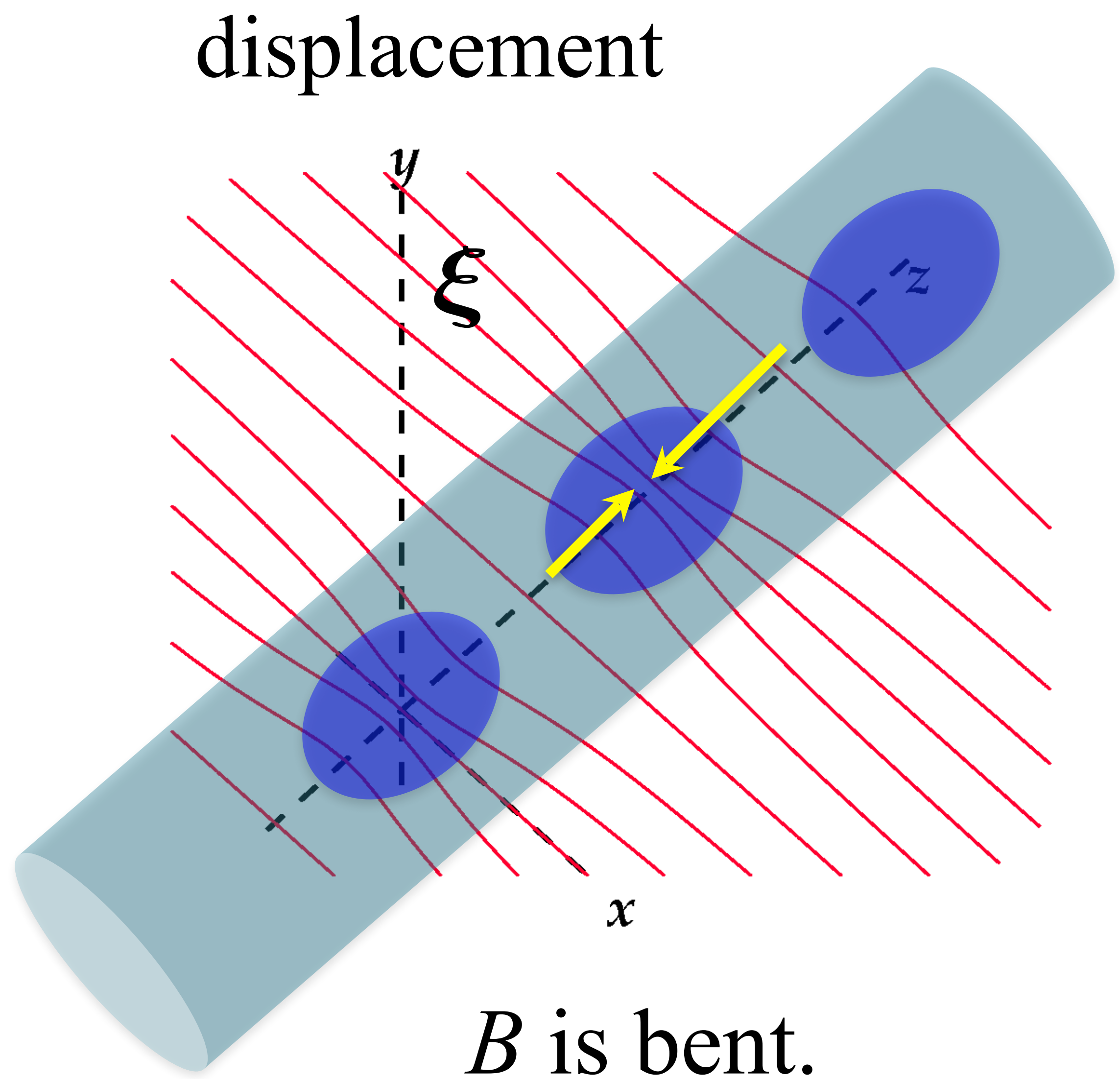
B direction matters.

1. Magnetic Force is Perpendicular to $\boldsymbol{B}$.
2. Critical Mass, $B_{\rm cr} = 2\pi\sqrt{G}\,\Sigma$



$B_{\parallel}$ unchanged

always supercritical



B is bent.

may be subcritical

Idealized Equilibrium Model

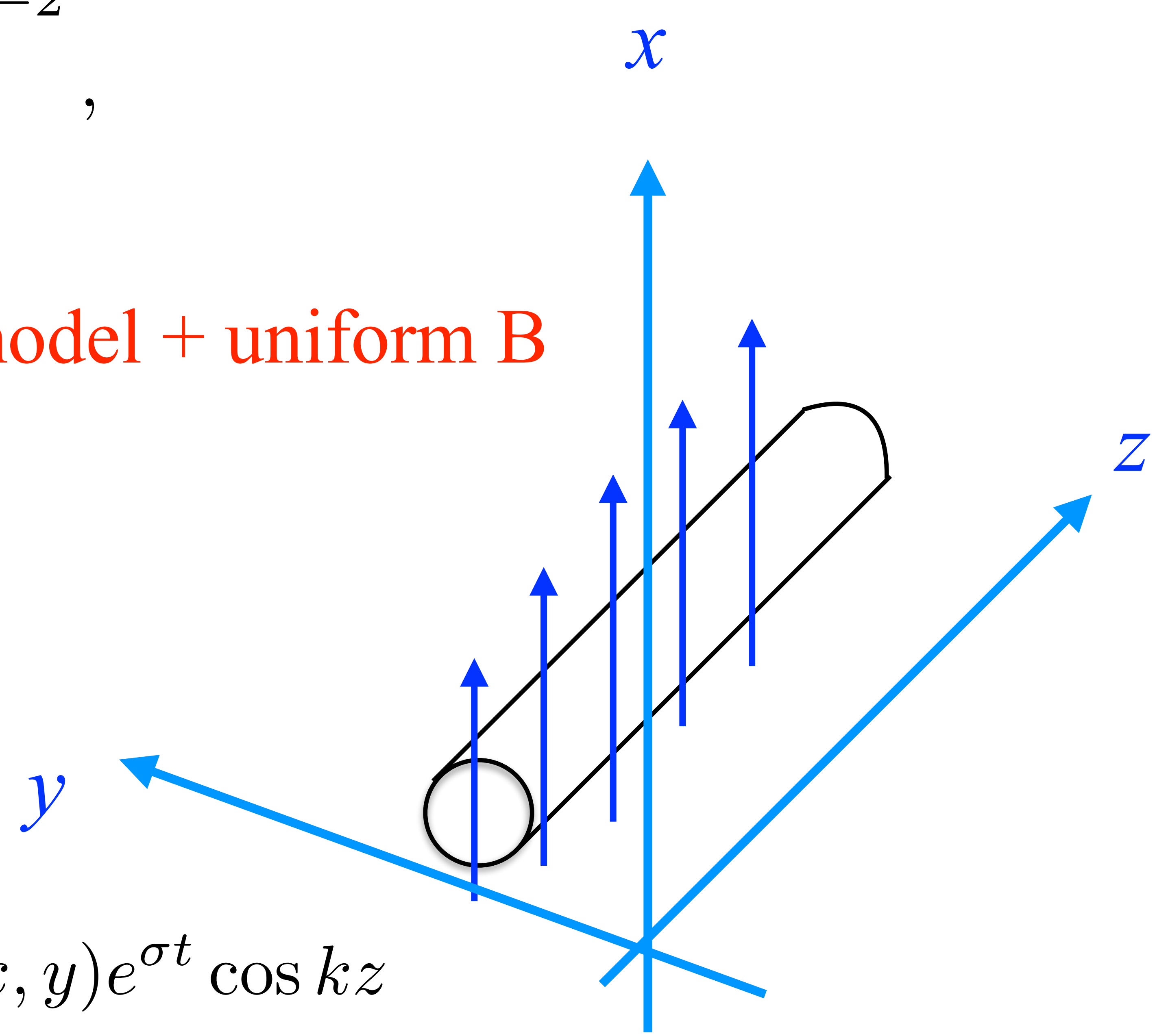
$$\rho_0 = \rho_c \left(1 + \frac{x^2 + y^2}{8H^2}\right)^{-2},$$

$$H^2 = \frac{c_s^2}{4\pi G \rho_c},$$

$$\boldsymbol{B}_0 = B_0 \boldsymbol{e}_x,$$

1D model + uniform B

isothermal cloud



3D perturbation

$$\rho(x, y, z, t) = \rho_0(x, y) + \varrho(x, y) e^{\sigma t} \cos kz$$

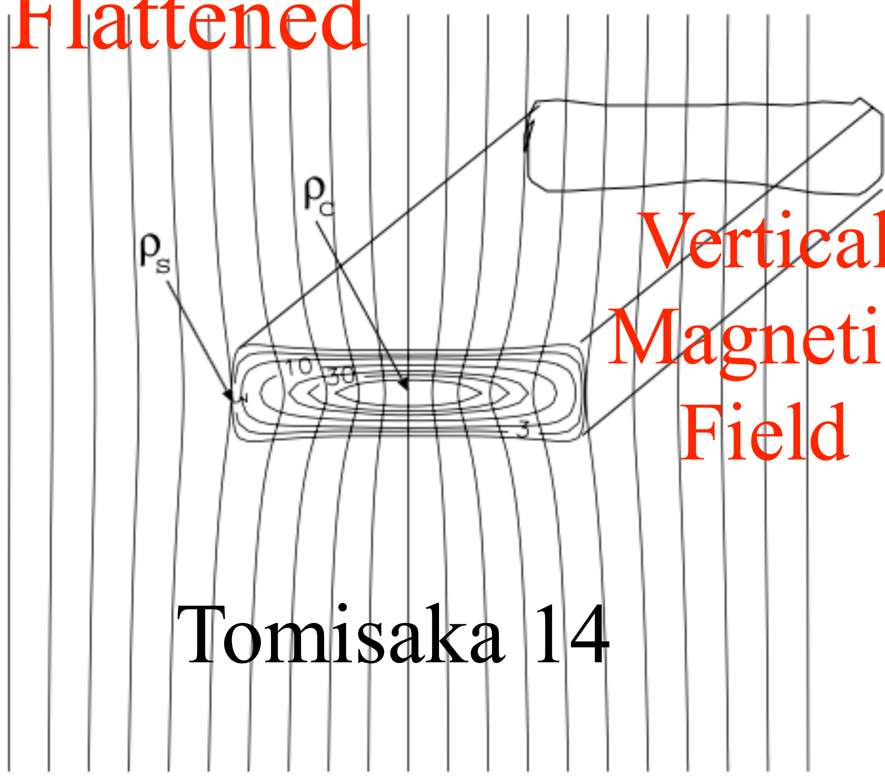
$$\beta = \frac{8\pi \rho_c c_s^2}{B_0^2}$$

Caveats

2D

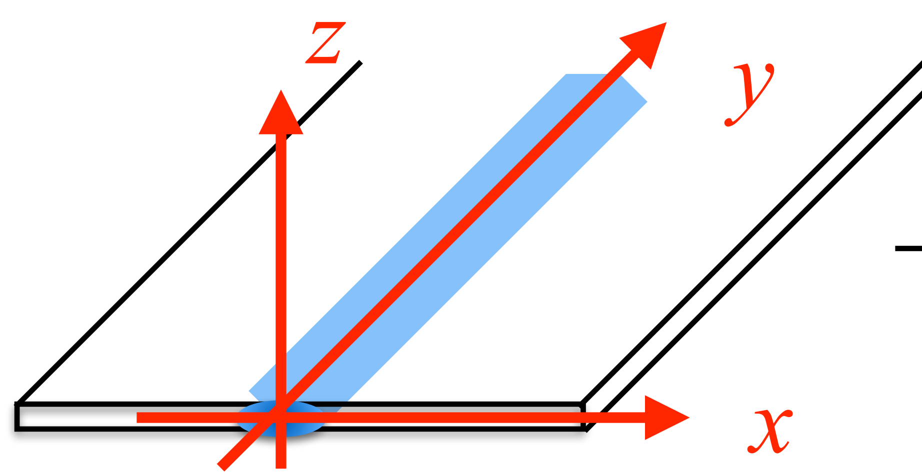
magnetohydrostatic configuration

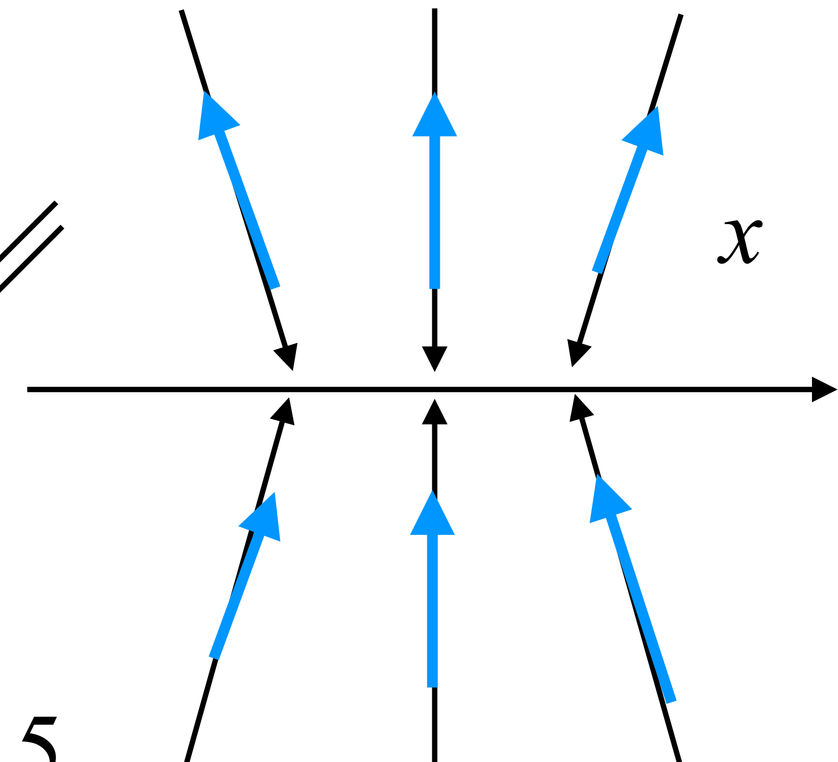
Flattened



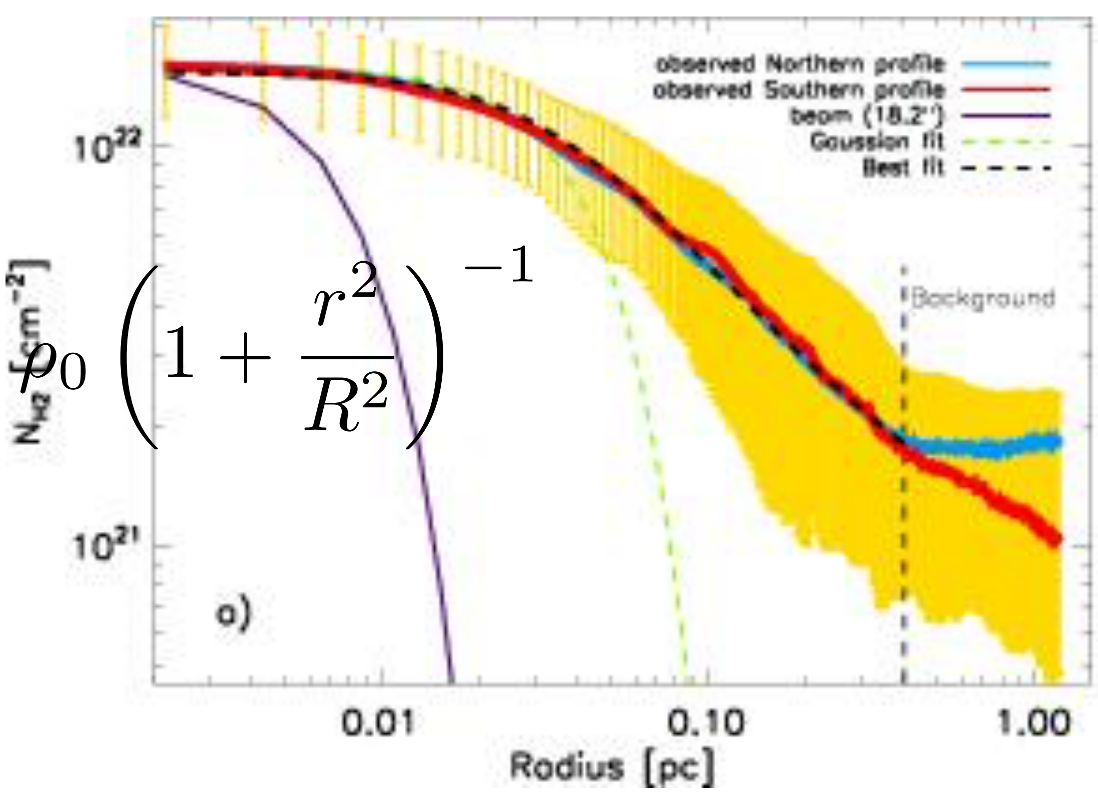
Vertical Magnetic Field

Tomisaka 14

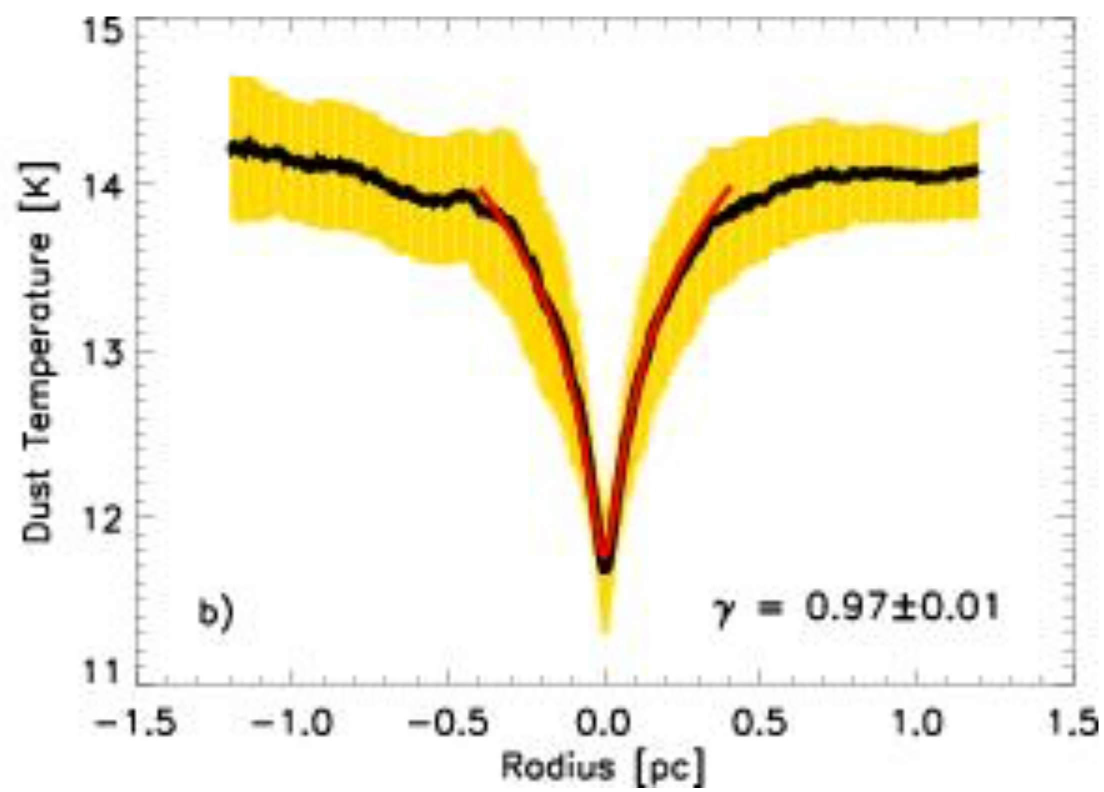




Hanawa+Tomisaka 15



$$\rho \simeq \rho_0 \left(1 + \frac{r^2}{R^2}\right)^{-1}$$



Equilibrium

$$\rho_0 = \rho_c \left(1 + \frac{x^2 + y^2}{8H^2}\right)^{-2},$$

$$H^2 = \frac{c_s^2}{4\pi G \rho_c},$$

$$\boldsymbol{B}_0 = B_0 \boldsymbol{e}_x,$$

x: magnetic field, z: filament axis

c_s: sound speed

Ideal MHD Eq.

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}),$$

$$\frac{d\boldsymbol{v}}{dt} = -c_s^2 \boldsymbol{\nabla} \ln \rho - \boldsymbol{\nabla} \psi + \boldsymbol{j} \times \boldsymbol{B},$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} (\boldsymbol{v} \times \boldsymbol{B}),$$

$$\boldsymbol{j} = \frac{\boldsymbol{\nabla} \times \boldsymbol{B}}{4\pi},$$

$$\Delta \psi = 4\pi G \rho.$$

$$\begin{aligned}
\rho &= \rho_0 + \delta\varrho(x,y) e^{\sigma t} \cos kz, \\
\boldsymbol{\xi} &= e^{\sigma t} \left[\xi_x(x,y) \cos kz \boldsymbol{e}_x + \xi_y(x,y) \cos kz \boldsymbol{e}_y + \xi_z(x,y) \sin kz \boldsymbol{e}_z\right], \\
\boldsymbol{B} &= B_0 \boldsymbol{e}_x + e^{\sigma t} \left[b_x(x,y) \cos kz \boldsymbol{e}_x + b_y(x,y) \cos kz \boldsymbol{e}_y + b_z(x,y) \sin kz \boldsymbol{e}_z\right], \\
\boldsymbol{j} &= e^{\sigma t} \left[j_x(x,y) \sin kz \boldsymbol{e}_x + j_y(x,y) \sin kz \boldsymbol{e}_y + j_z(x,y) \cos kz \boldsymbol{e}_z\right], \\
\psi &= \psi_0 + e^{\sigma t} \delta\psi(x,y) \cos kz,
\end{aligned}$$

Numerical Methods

Displacement vector

$$\boldsymbol{\xi} = \int \boldsymbol{v} dt$$

$$\delta\varrho = -\frac{\partial}{\partial x}\left(\rho_0 \xi_x\right) - \frac{\partial}{\partial y}\left(\rho_0 \xi_y\right) - k\rho_0 \xi_z,$$

$$b_x = -B_0 \left[\frac{\partial}{\partial y}\xi_y(x, y) + k\xi_z\right],$$

$$b_y = B_0 \frac{\partial \xi_y}{\partial x},$$

$$b_z = -B_0 \frac{\partial \xi_z}{\partial x},$$

$$j_x = \frac{1}{4\pi}\left(\frac{\partial b_z}{\partial y} + k b_y\right),$$

$$j_y = -\frac{1}{4\pi}\left(k\delta b_x + \frac{\partial b_z}{\partial x}\right),$$

$$j_z = \frac{1}{4\pi}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right).$$

$$\delta\psi(\boldsymbol{r}) = \int \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{r}')\varrho(\boldsymbol{r}')d\boldsymbol{r}'$$

$$\rho_0 \frac{d^2\boldsymbol{\xi}}{dt^2} = \boldsymbol{F}\left(\boldsymbol{\xi}\right),$$

$$\rho_0 \sigma^2 \boldsymbol{\xi} = \left(\boldsymbol{A} + \frac{B_0^2}{4\pi}\boldsymbol{C}\right)\boldsymbol{\xi}.$$

Force is proportional to ξ.

generalized eigenvalue problem

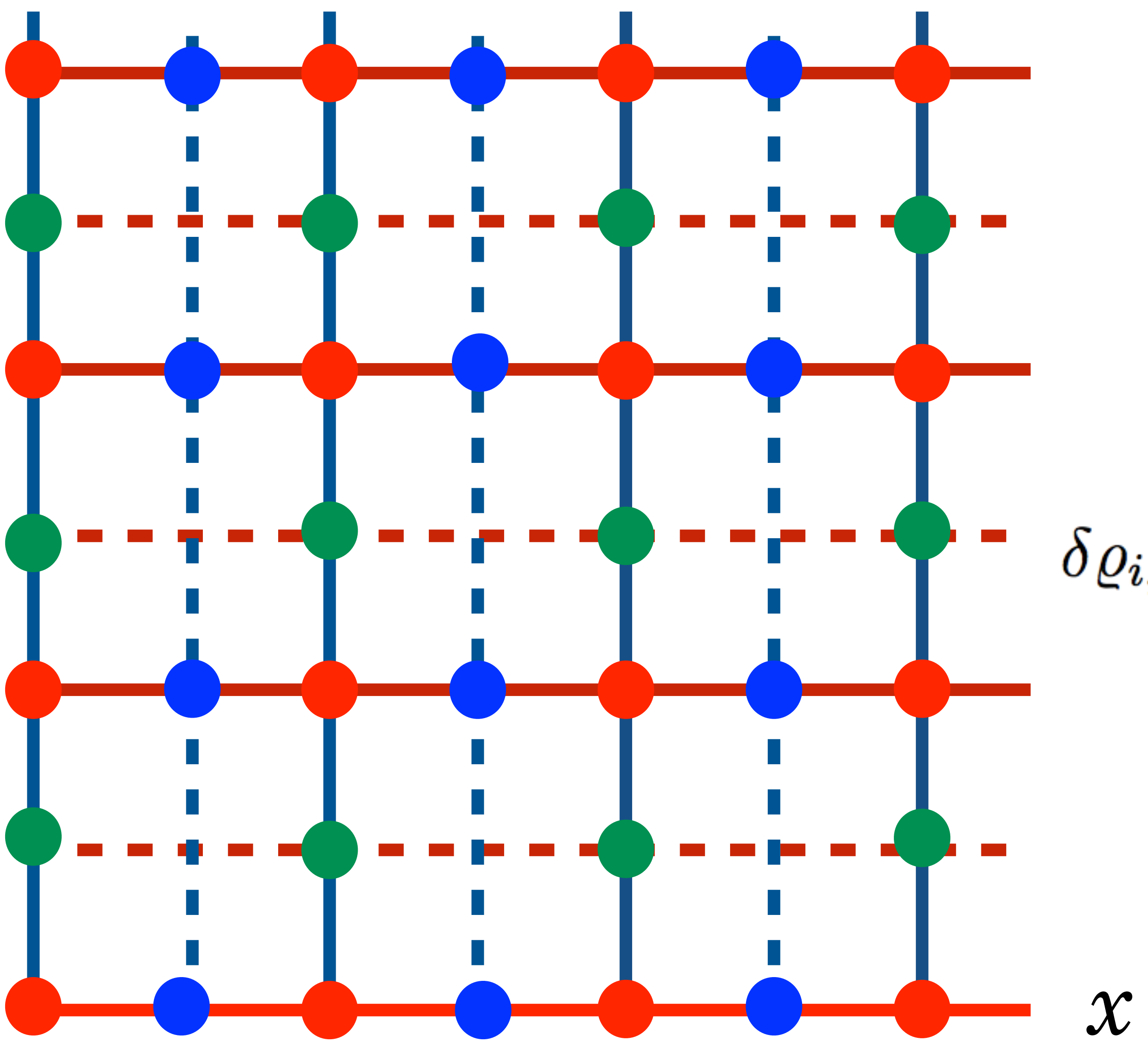
$$\left|\boldsymbol{A} + \frac{B_0^2}{4\pi}\boldsymbol{C} - \rho_0 \boldsymbol{I}\right| = 0$$

LAPACK
Numerical Library

A perturbed quantity is expressed as a function ofξ.

Finite Difference Eqs.

y staggered mesh



- (red) $\xi_z,\ \varrho,\ \delta\psi,\ b_x,\ j_y$ — x, y sym
- (blue) $\xi_x,\ b_z$ — x anti, y sym
- (green) $\xi_y,\ j_z$ — x sym, y anti

$$
\begin{aligned}
\delta\varrho_{i,j} = & -\frac{\rho_{0,i+1/2,j}\xi_{x,i+1/2,j} - \rho_{0,i-1/2,j}\xi_{x,i-1/2,j}}{\Delta x} \\
& -\frac{\rho_{0,i,j+1/2}\xi_{x,i,j+1/2} - \rho_{0,i,j-1/2}\xi_{x,i,j-1/2}}{\Delta y} \\
& -k\rho_{0,i,j}\xi_{z,i,j}
\end{aligned}
$$

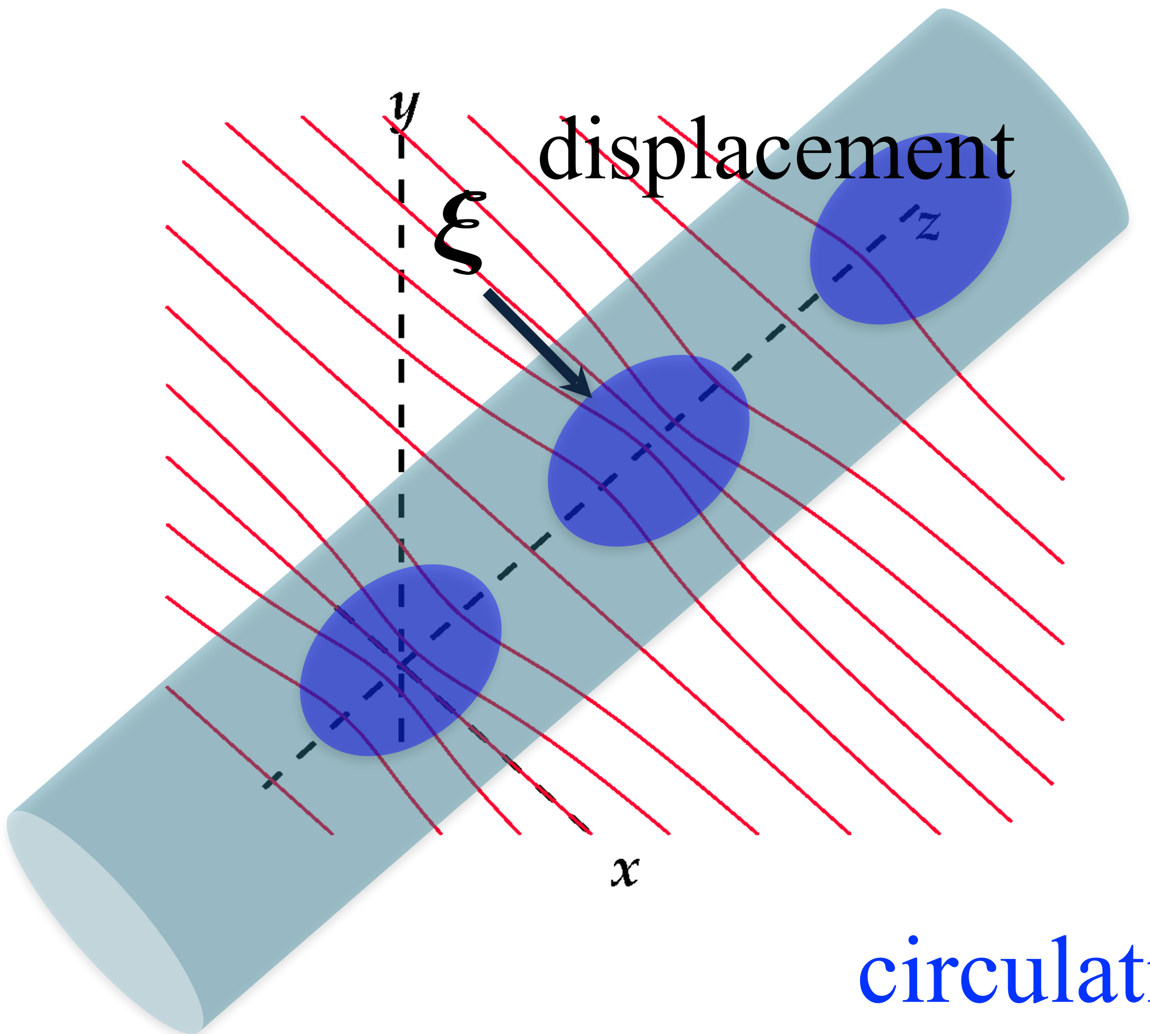
2nd order accuracy

Boundary (1) Fixed $\xi_x, \xi_y, \xi_z = 0$ for $x > n_x\Delta x$ or $y > n_y\Delta y$

(2) Free $\dfrac{\partial \boldsymbol{\xi}}{\partial x} = 0$ for $x > n_x\Delta x$

$\dfrac{\partial \boldsymbol{\xi}}{\partial y} = 0$ for $y > n_y\Delta y$

Boundary Condition



1) Fixed

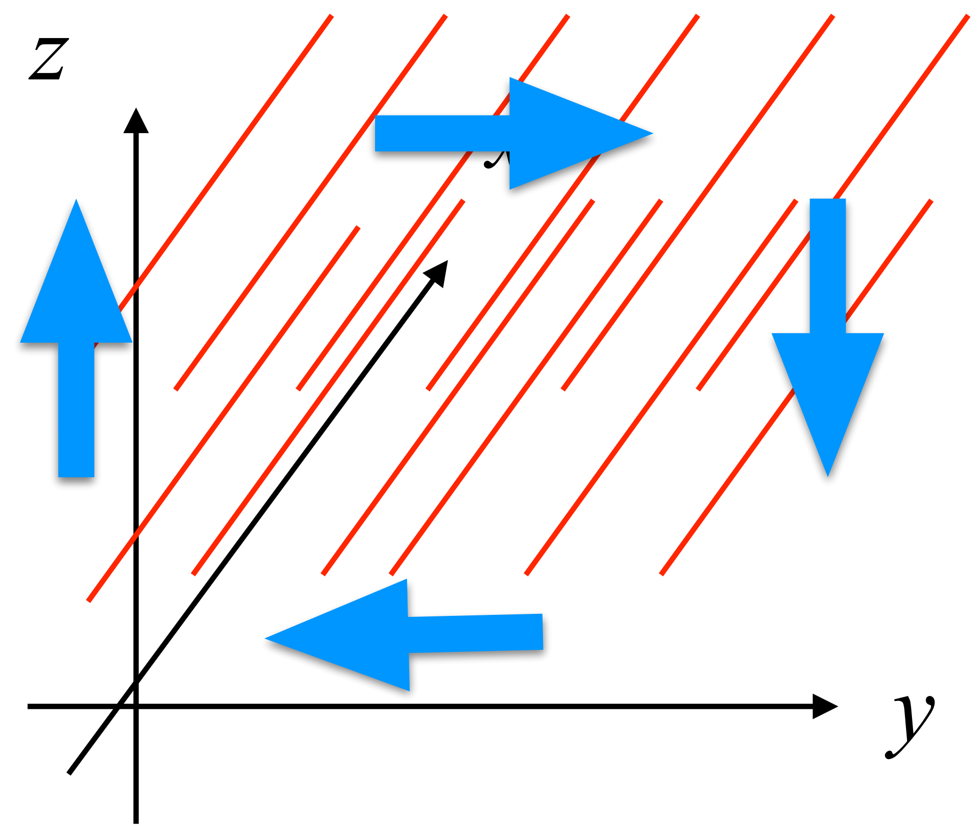
   $\xi = 0$

2) Free

   $\partial\xi/\partial \boldsymbol{x} = 0$

   keep straight

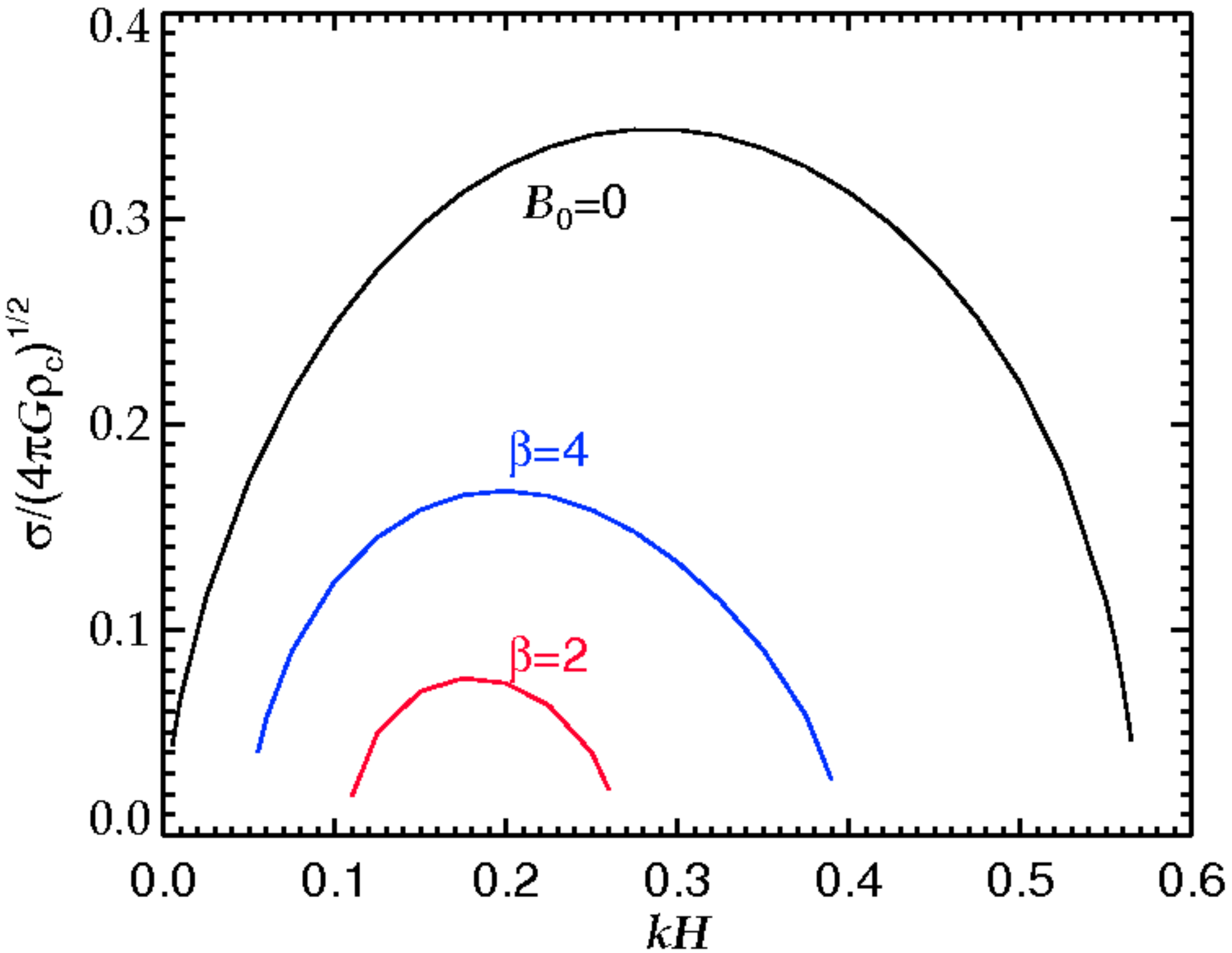
circulation permitted

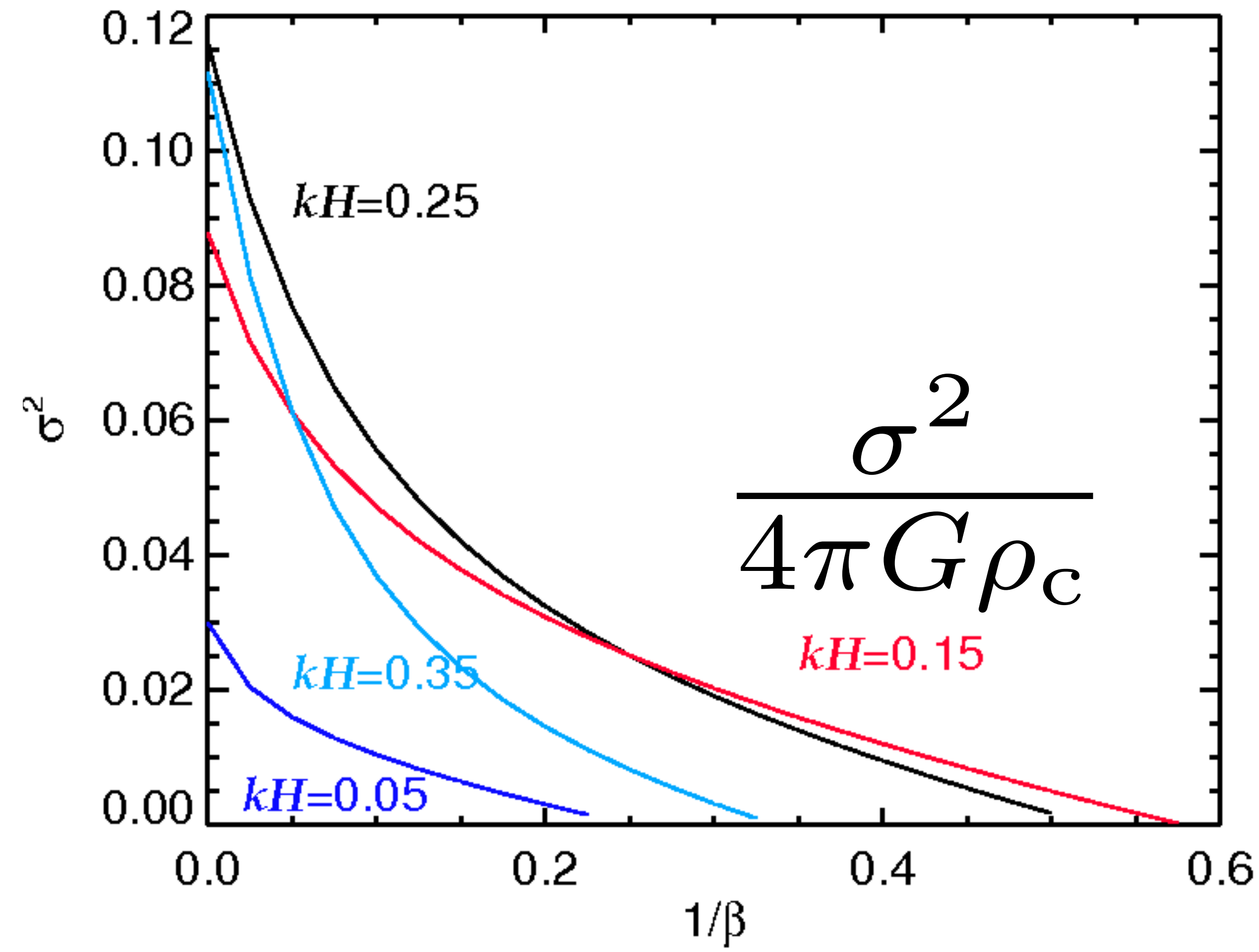


Fixed Boundary

Growth rate

$\Delta x = \Delta y = 0.6\ H,\ \ n_x = n_y = 40$





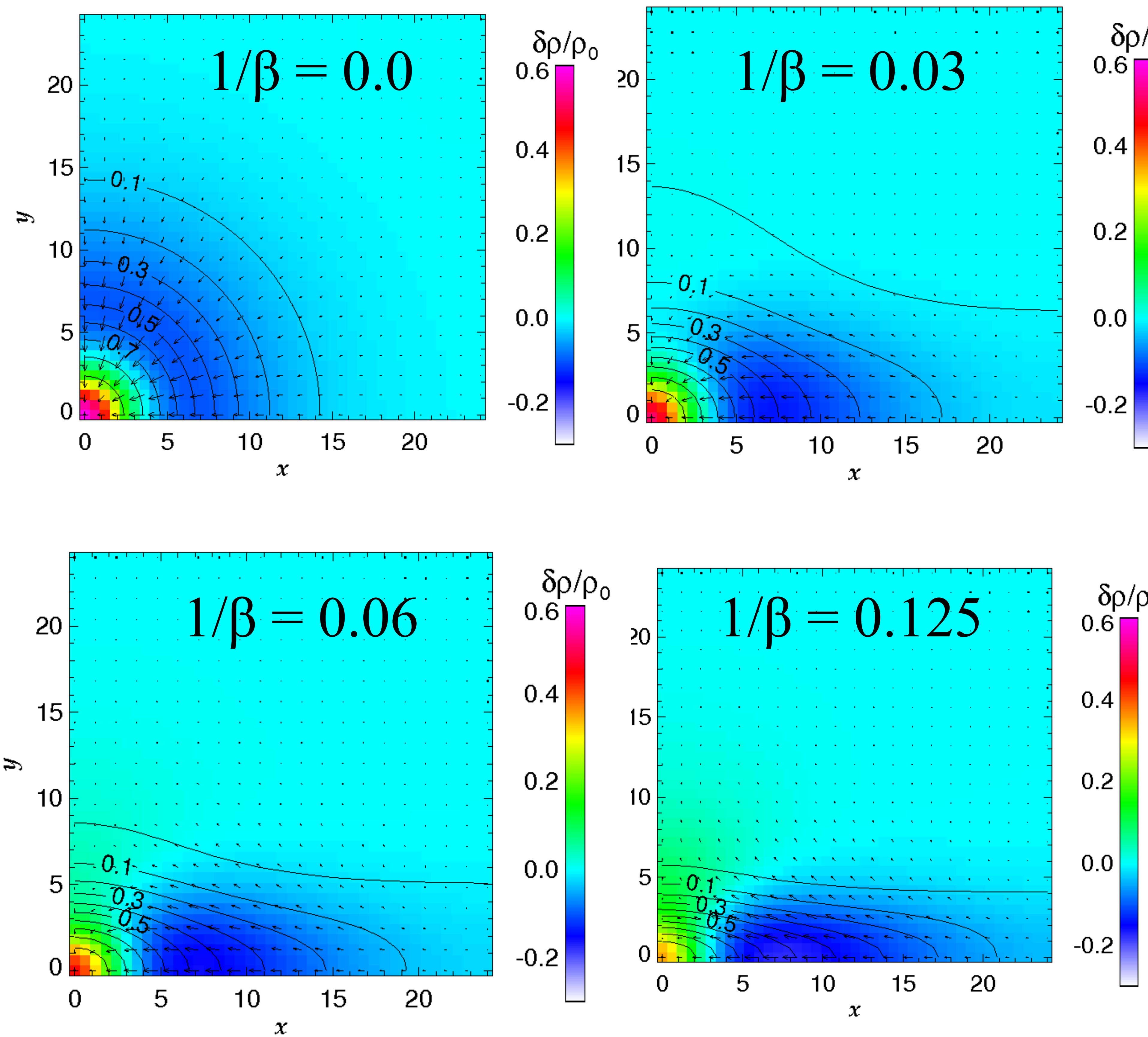
$$\frac{\sigma^2}{4\pi G\rho_c}$$

Critical mass to flux ratio

$$\frac{2\pi\sqrt{G}\Sigma_c}{B_0} = \sqrt{\frac{\beta}{0.405}}$$

$$kH = 0.3 \quad \rightarrow \quad \lambda = 5.8\ d_{\mathrm{FWHM}}$$

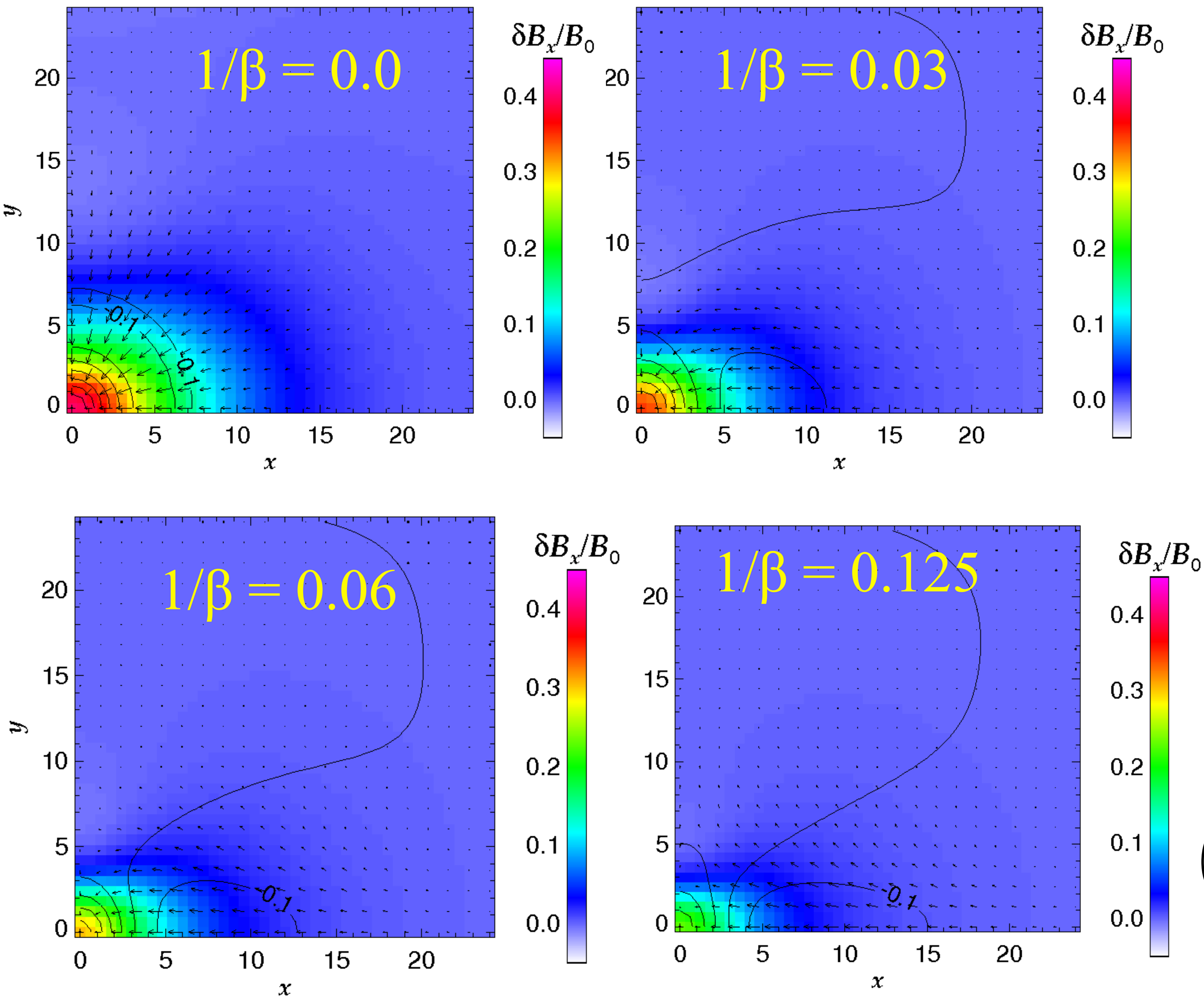
Eigen function $kH = 0.2$ normalization $\xi_z\,(0, 0) = -\,H$



$$\left(\frac{\delta\varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_x}{\partial x} - \frac{\partial \xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_y}{\partial y} - k\xi_z$$

Change in B $\quad kH = 0.2 \quad \xi_z\,(0, 0) = -\,H$

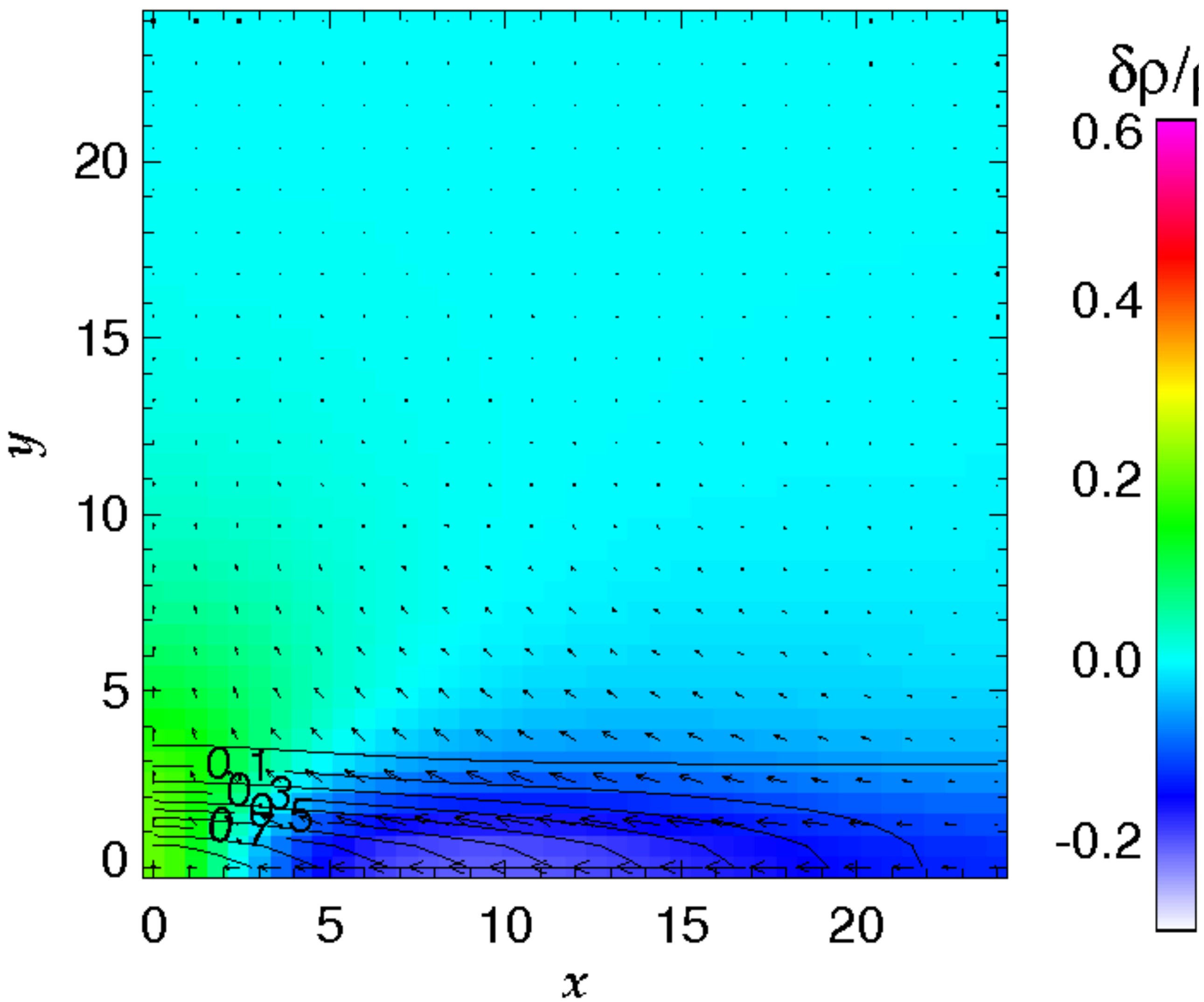


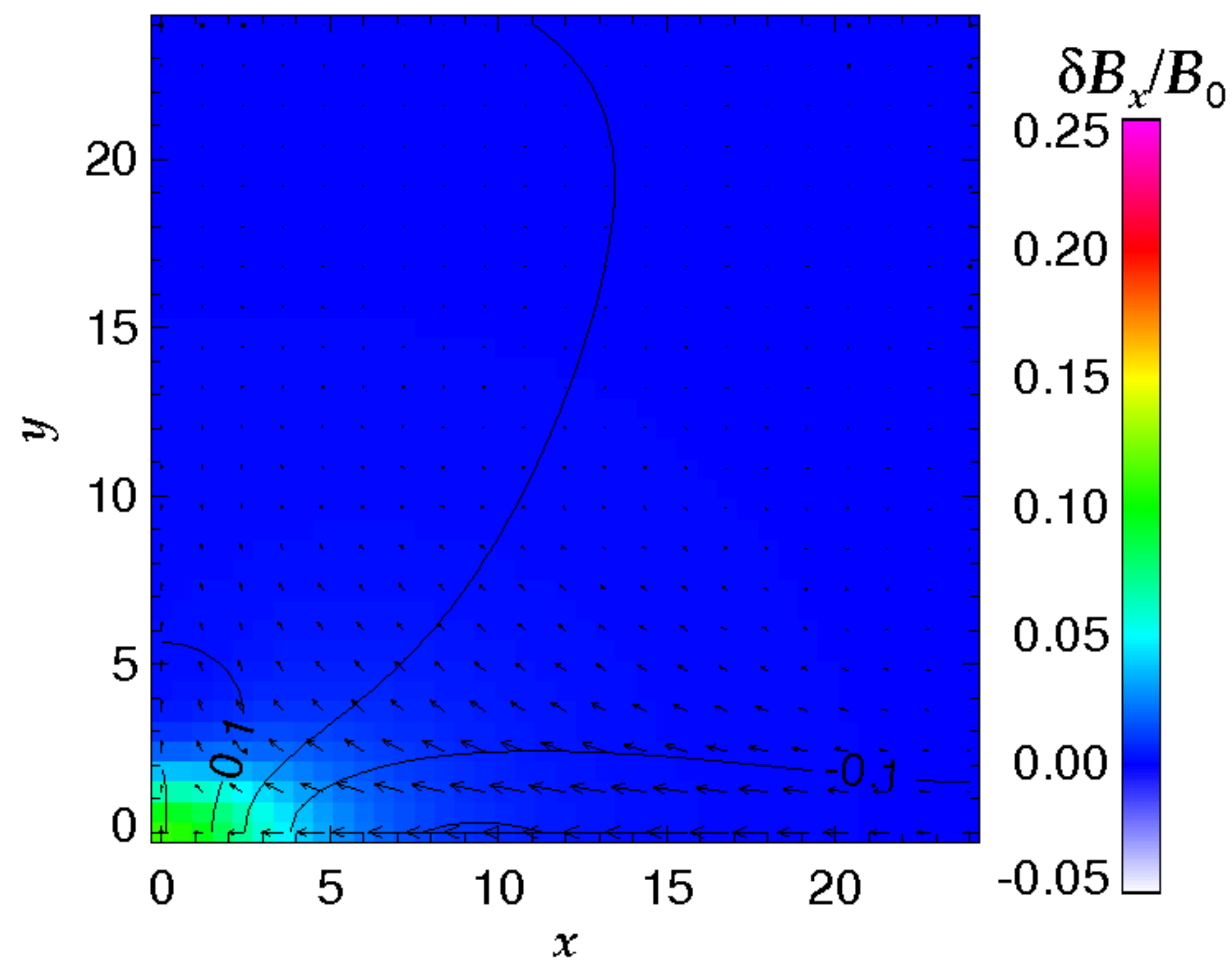
$$\left(\frac{\delta\varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_x}{\partial x} - \frac{\partial \xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_y}{\partial y} - k\xi_z$$

Further strong magnetic field ($kH = 0.2$)

$1/\beta = 0.375$





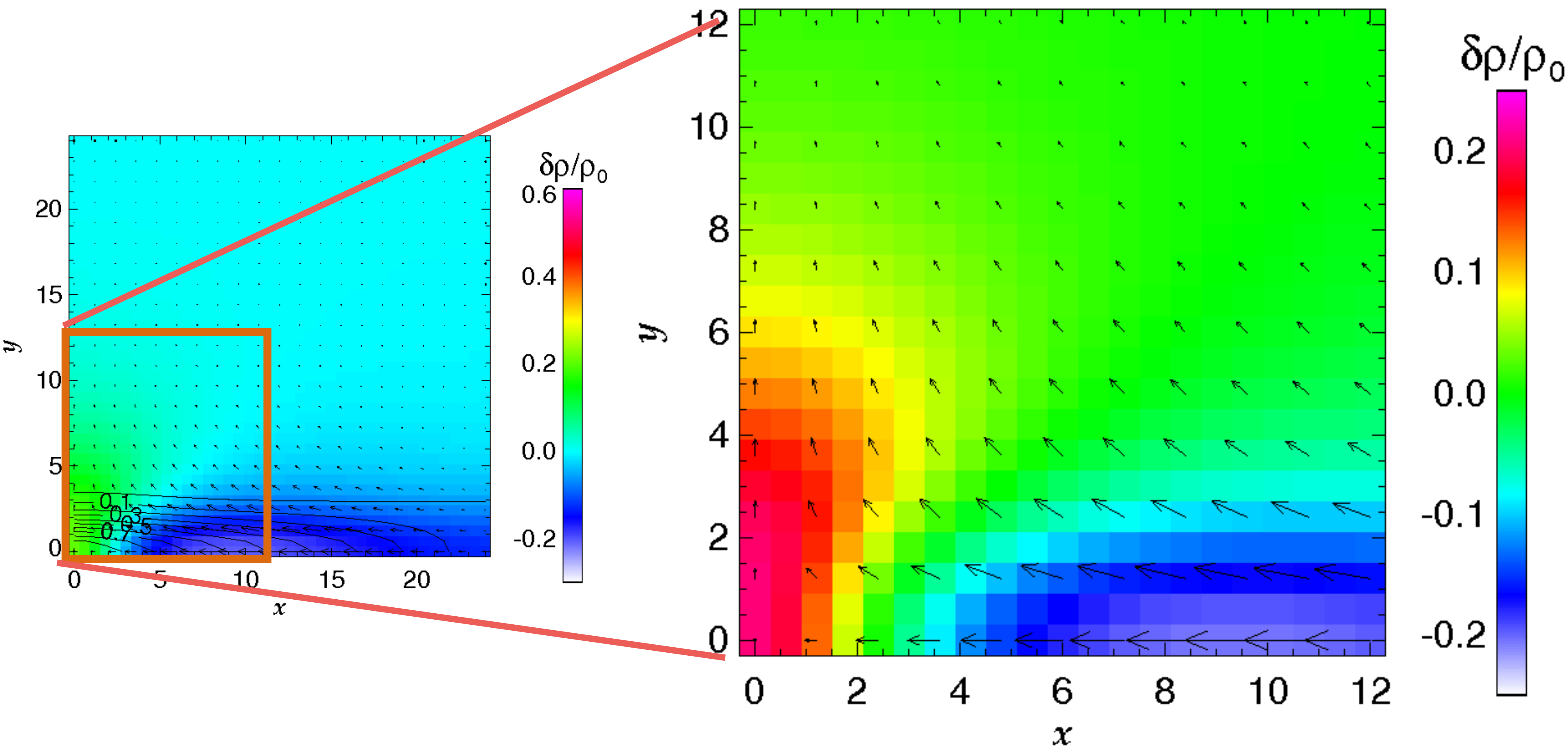
$\xi_z\,(0, 0) = -\,H$

$$\left(\frac{\delta\varrho}{\rho_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_x}{\partial x} - \frac{\partial \xi_y}{\partial y} - k\xi_z$$

$$\left(\frac{\delta B_x}{B_0}\right)_{\boldsymbol{r}=0} = -\frac{\partial \xi_y}{\partial y} - k\xi_z$$

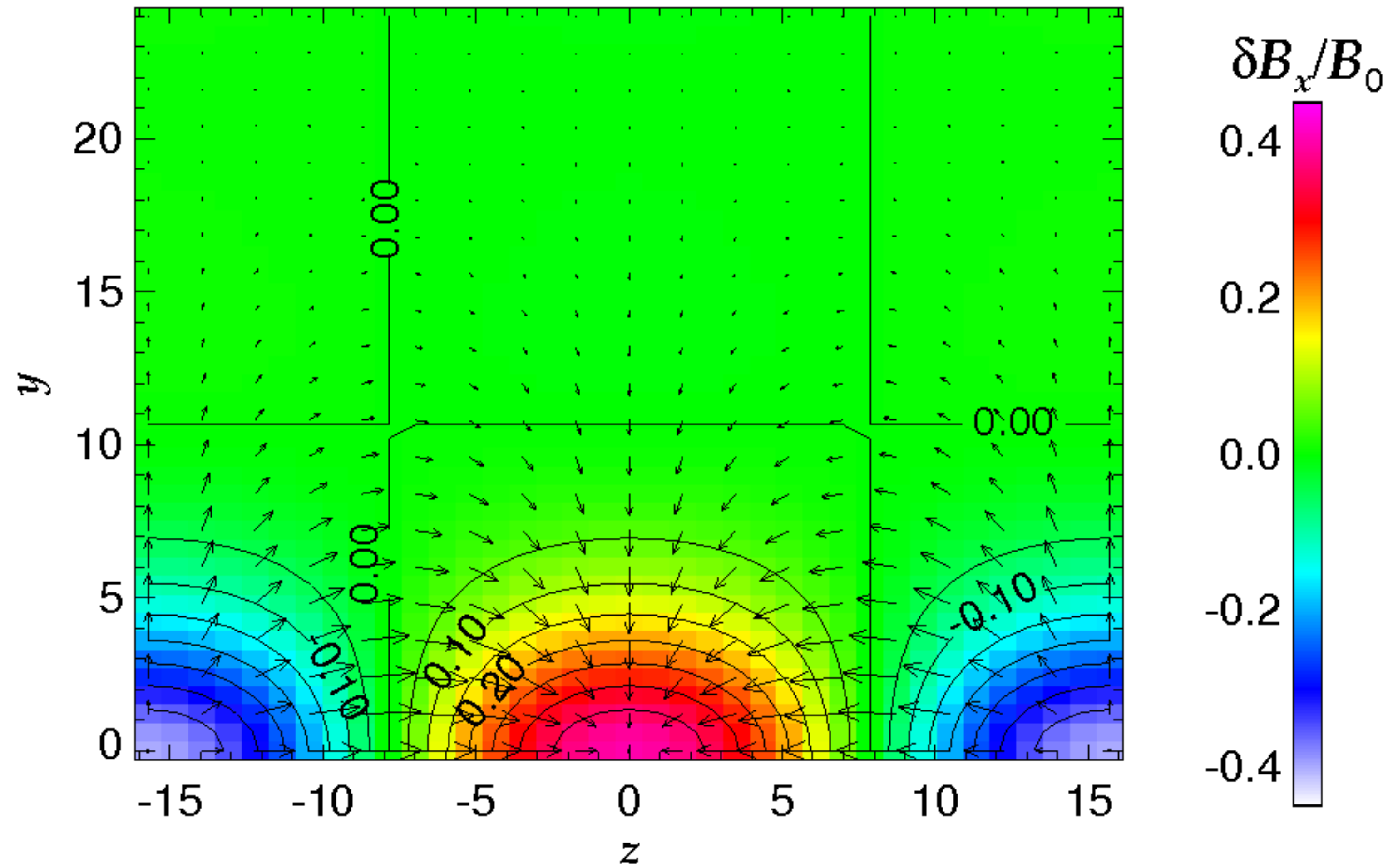
Enlargement

$$1/\beta = 0.375, \quad kH = 0.2$$

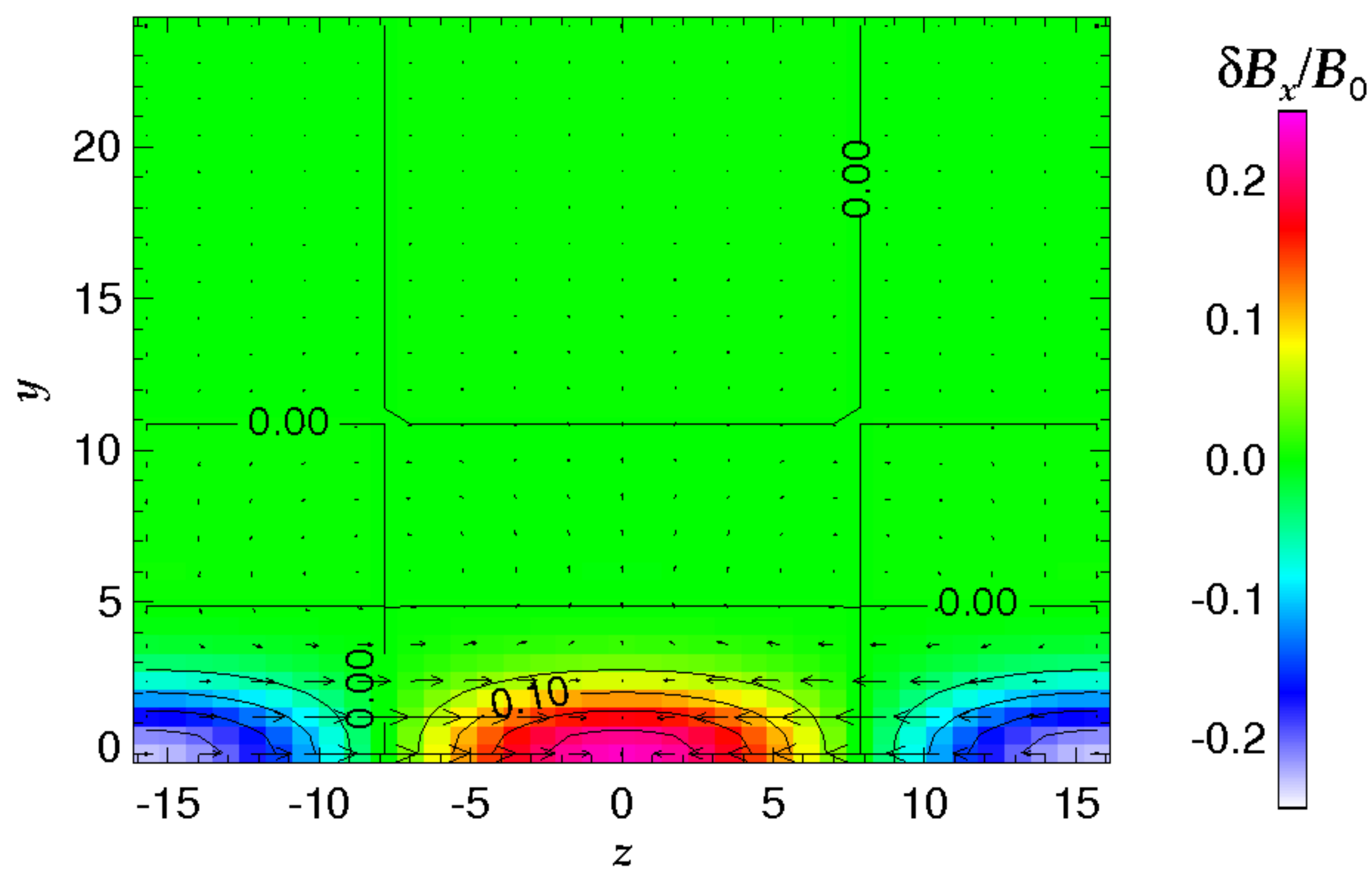


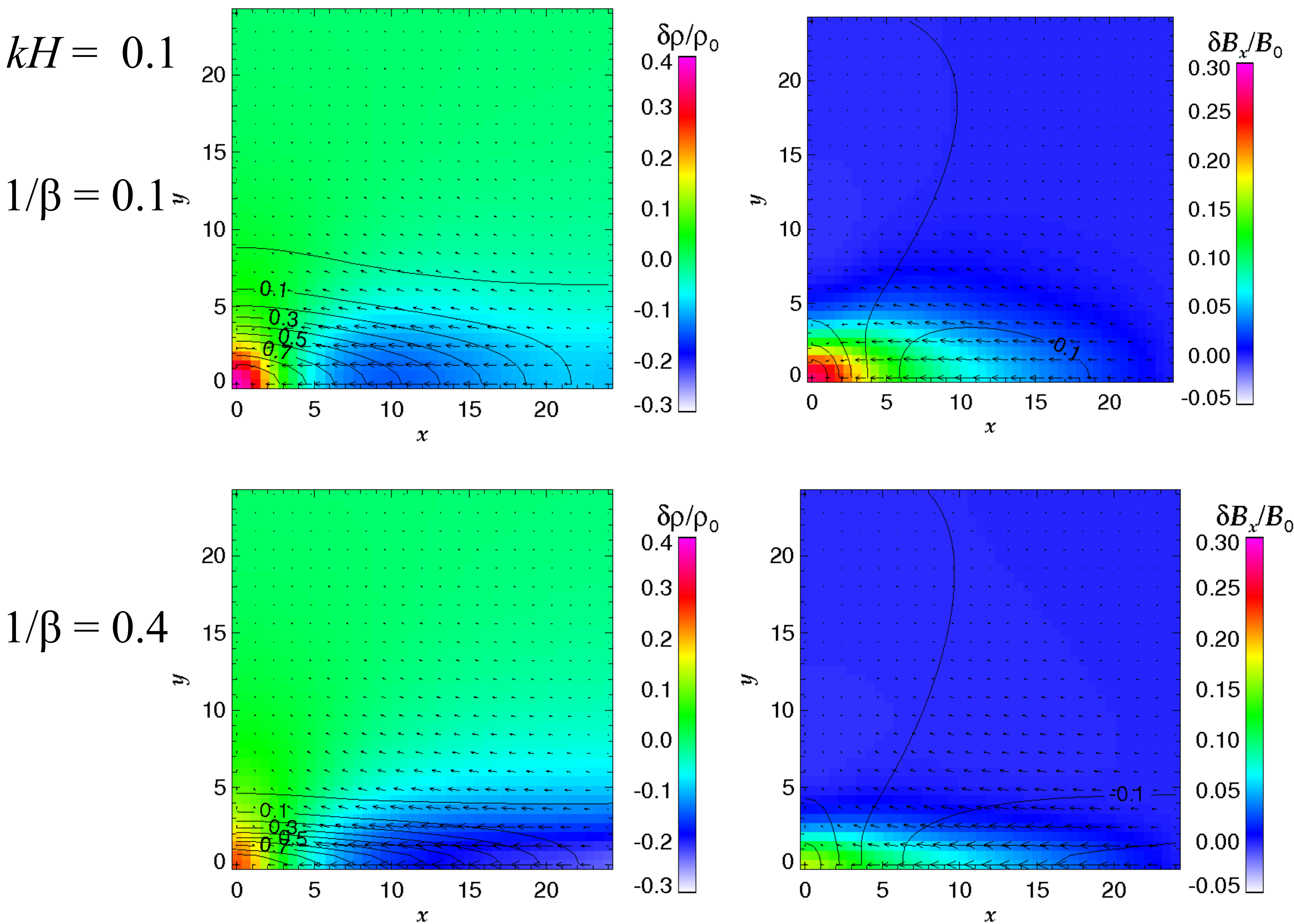
Flow in the *yz*-plane ($x = 0$)

$kH = 0.2,\ 1/\beta = 0.0$



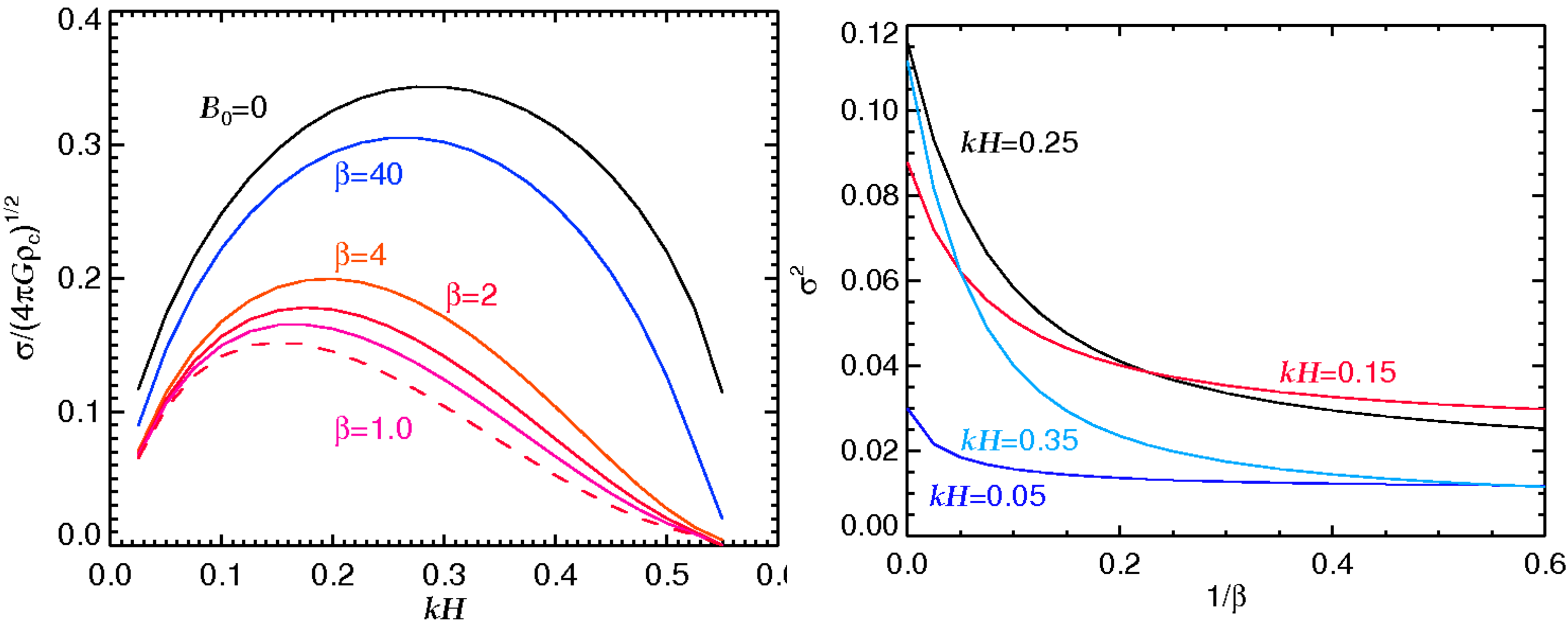
$kH = 0.2,\ 1/\beta = 0.125$





kH = 0.1
1/β = 0.1
1/β = 0.4
δρ/ρ0
δBx/B0
x
y
0.1
0.3
0.5
0.7
-0.1

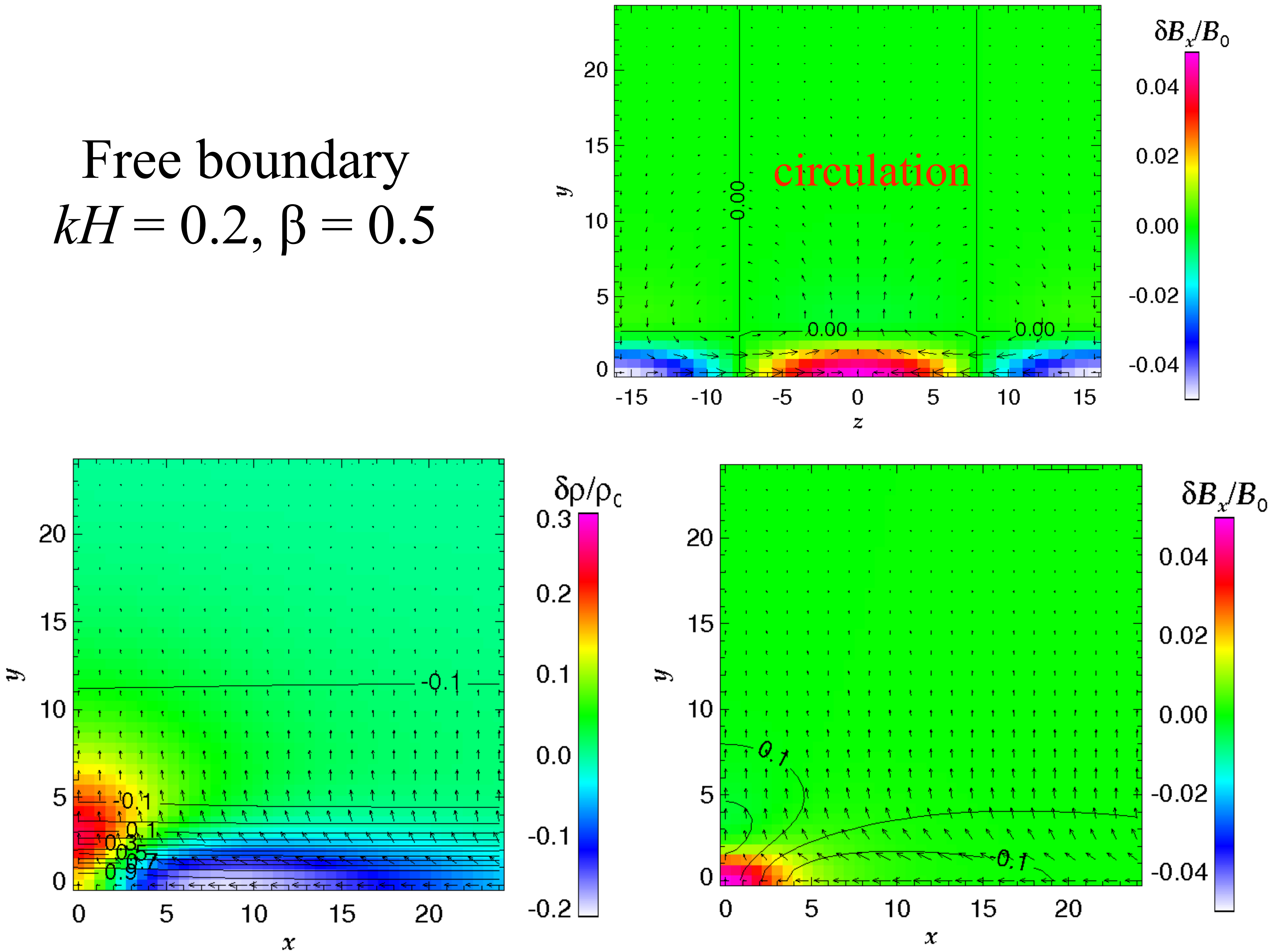
Free boundary



incompressible mode

(cf. Nagai+98, Fiege & Pudritz 00)

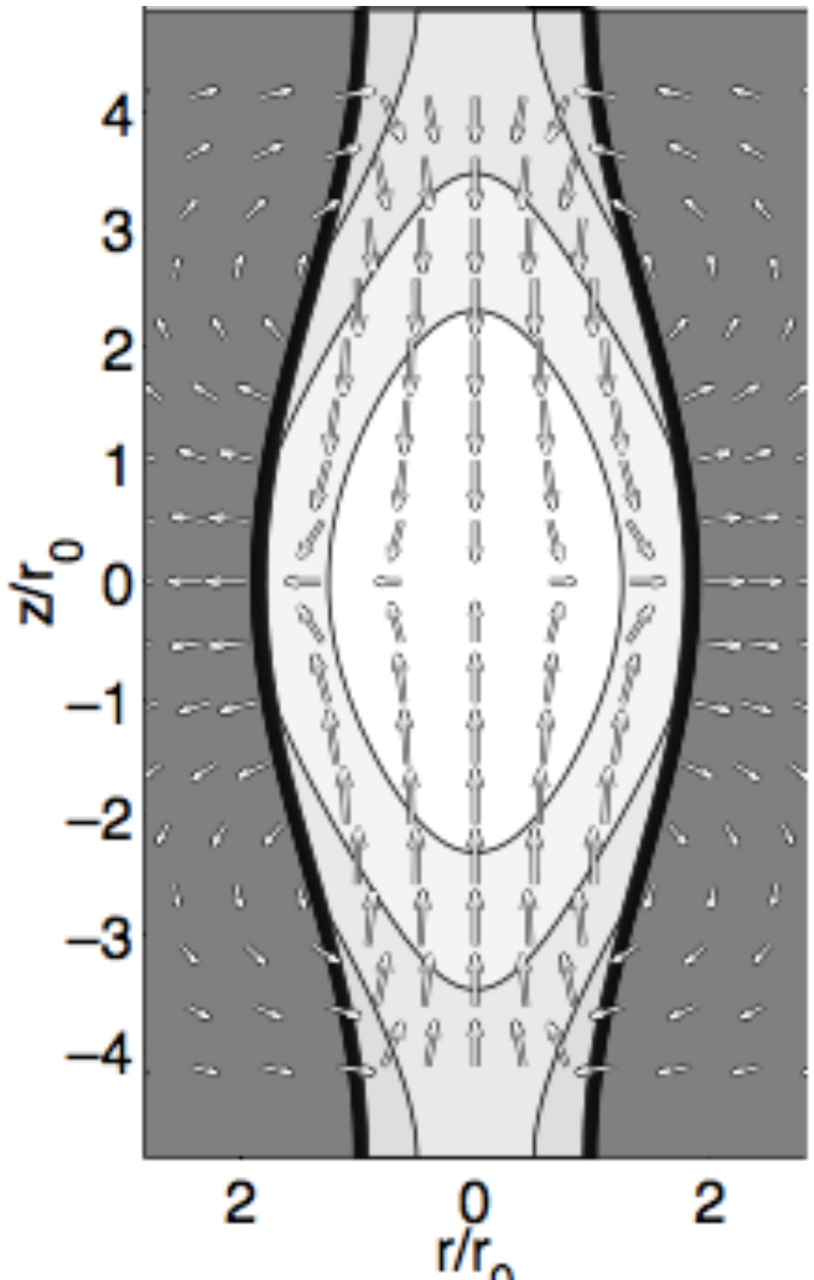
Free boundary
$kH = 0.2$, $\beta = 0.5$



Similarity to truncated filament model.

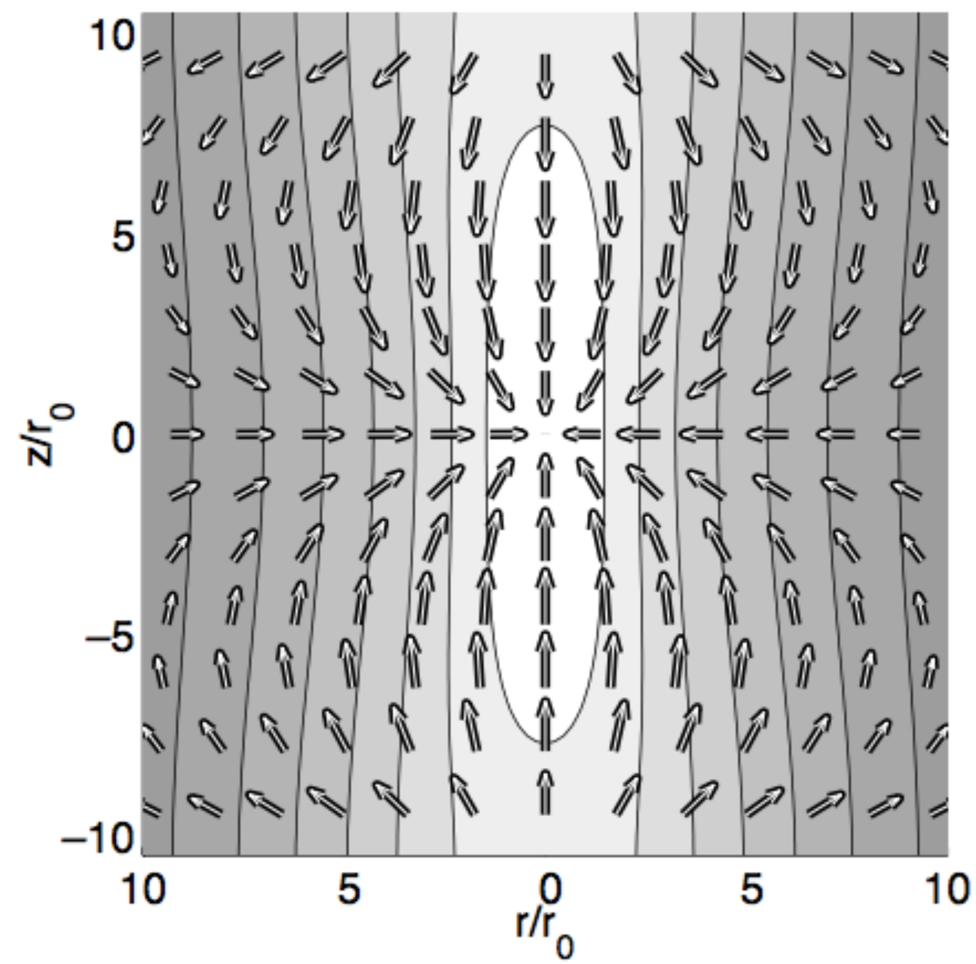
Fiege & Pudritz 00

truncated filament



circulation

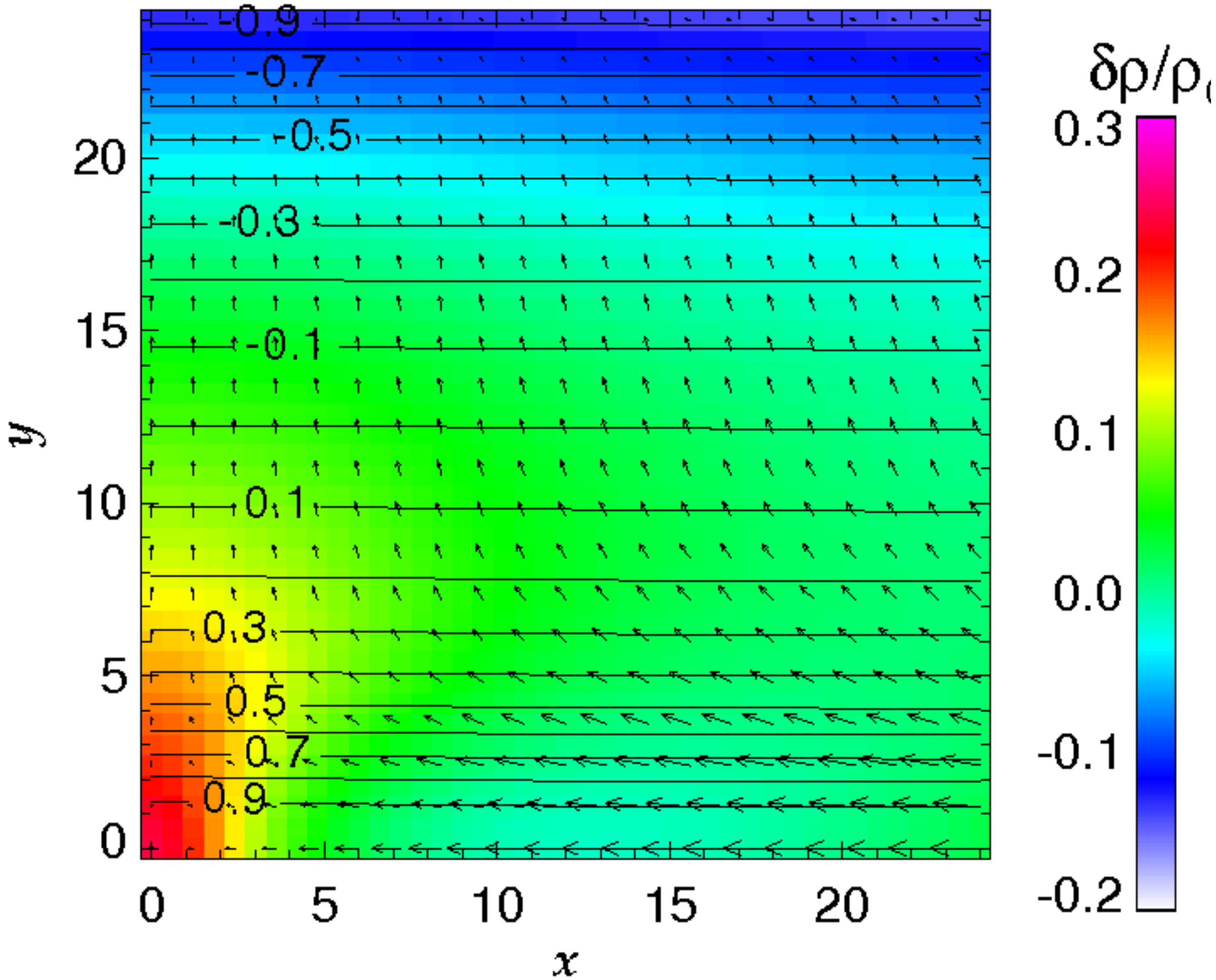
compression

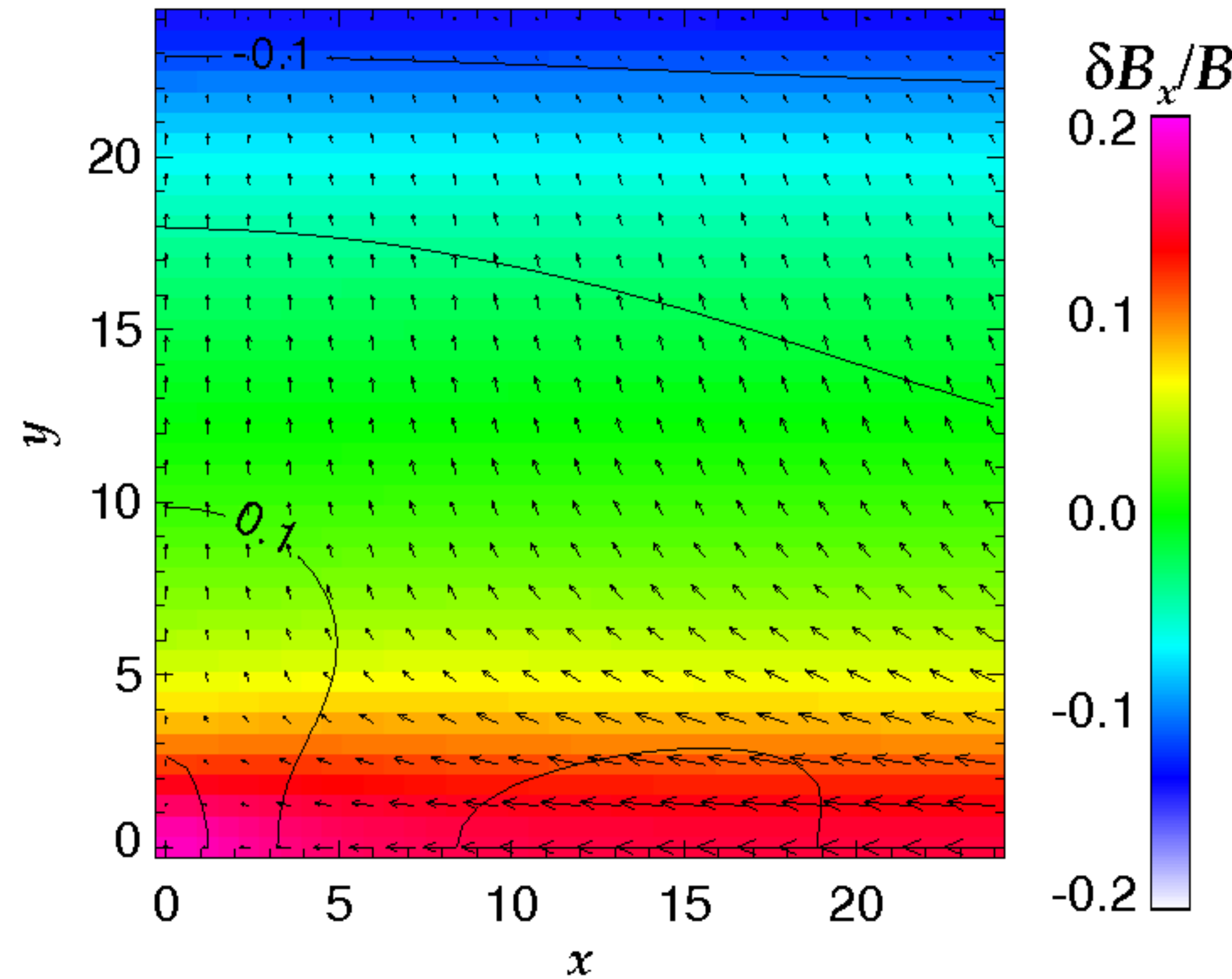


low β plasma ≒ high T

cf. Nagai+98 (truncated sheet)

Free boundary
$kH = 0.05$, $\beta = 2$



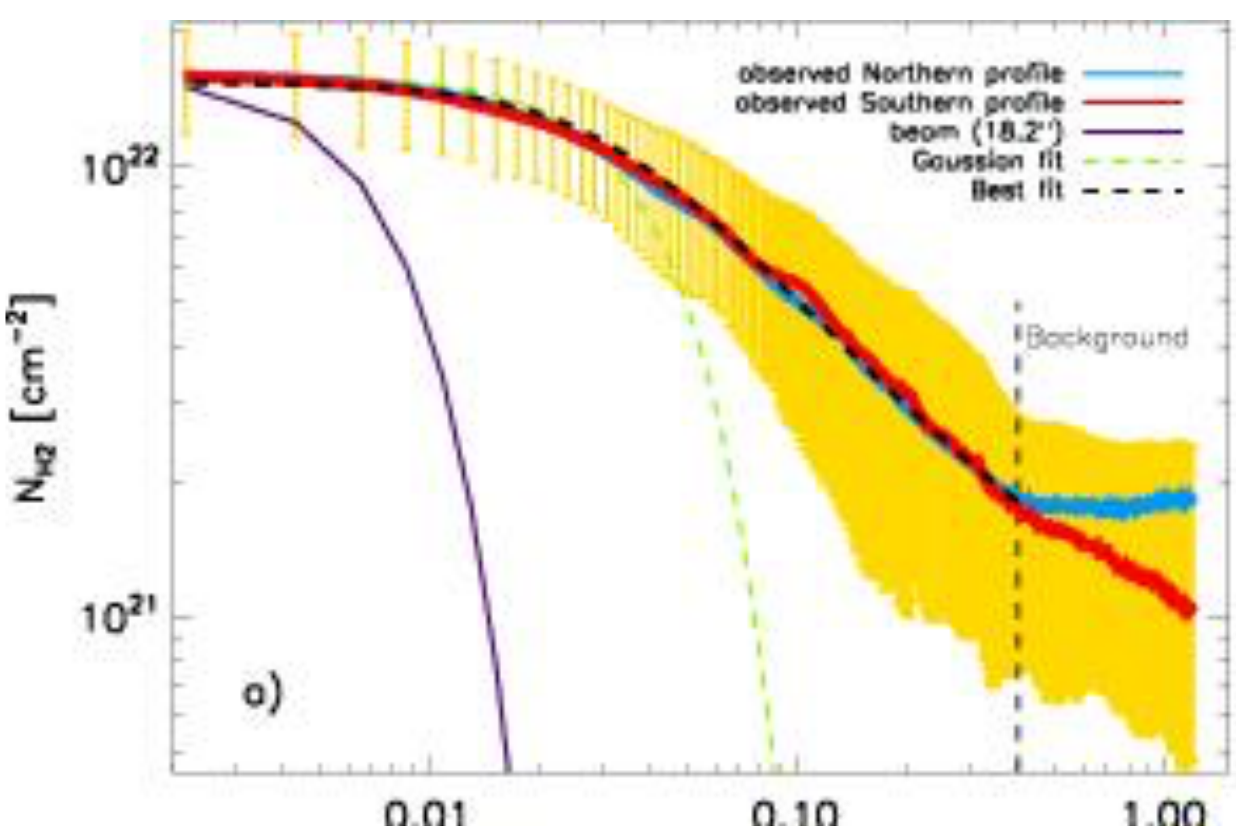


Why the growth rate depends on the boundary?

Alfvén transit time

$$\tau_{\mathrm{A}} = \int \frac{ds}{v_{\mathrm{A}}} = \int \frac{\sqrt{4\pi\rho}}{B} ds$$

Magnetic tension propagates at v_{A}.



τ_{A} is finite in our simple model, since B = const and $\rho \propto r^{-4}$.

How about in reality?

Empirical EOS

$$\rho = \rho_c \left(1 + \frac{r^2}{4H^2}\right)^{-1},$$

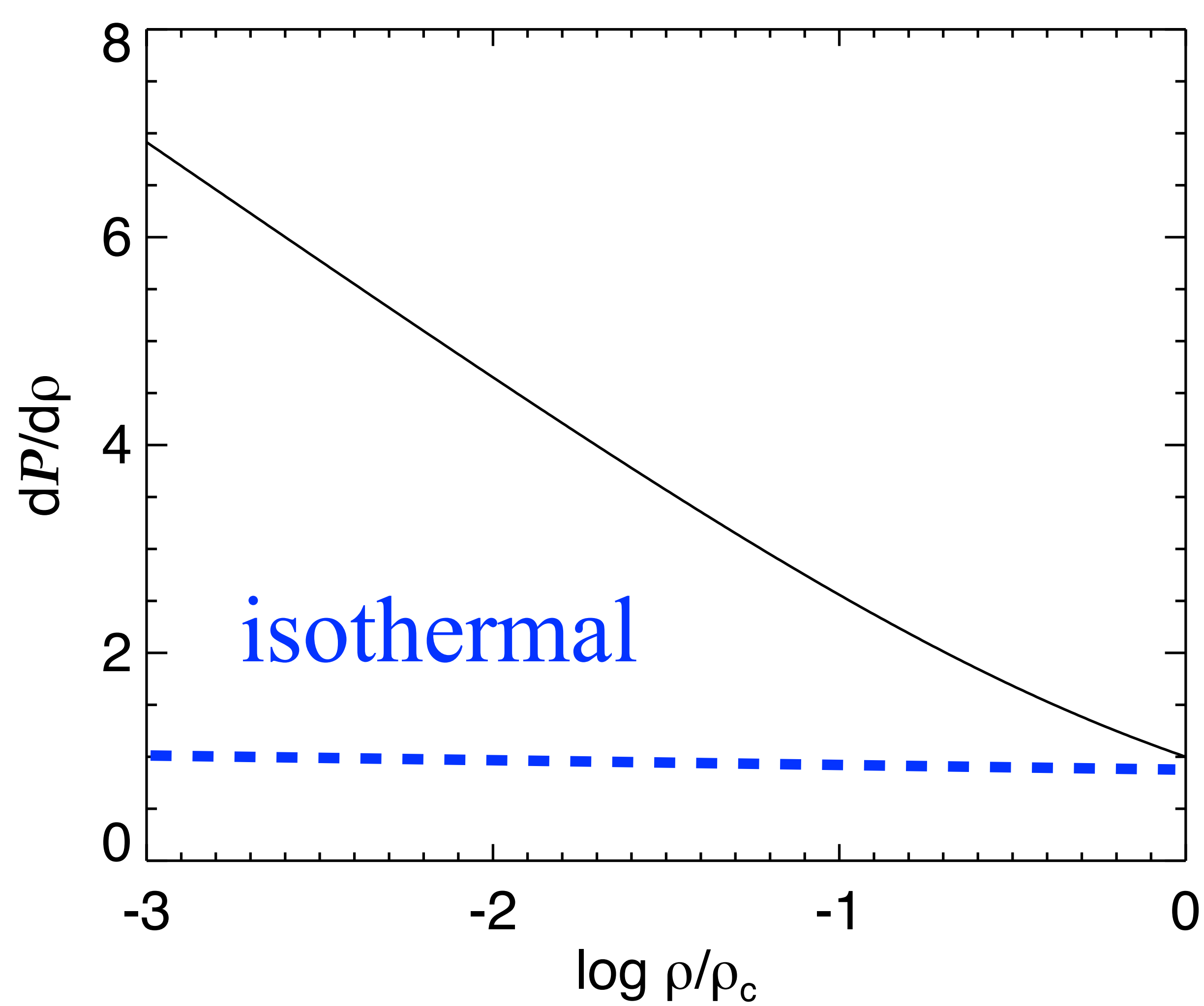
$$\lambda_r = \int_0^r 2\pi r' \rho(r') dr'$$
$$= 4\pi\rho_c H^2 \ln\left(1 + \frac{r^2}{4H^2}\right)^{-1},$$

$$g_r = -\frac{2G\lambda_r}{r}$$
$$= -\frac{8\pi G\rho_c H^2}{r} \ln\left(1 + \frac{r^2}{4H^2}\right),$$

$$\frac{dP}{dr} = \rho g_r$$
$$= -\frac{8\pi G\rho_c H^2}{r} \ln\left(1 + \frac{r^2}{4H^2}\right),$$

$$\frac{d\rho}{dr} = -\frac{r\rho_c}{2H^2}\left(1 + \frac{r^2}{4H^2}\right)^{-2},$$

$$\frac{dP}{d\rho} = \frac{dP/dr}{d\rho/dr}$$
$$= \frac{4\pi G\rho_c H^2}{r^2}\left(1 + \frac{r^2}{4H^2}\right) \ln\left(1 + \frac{r^2}{4H^2}\right)$$
$$= -4\pi G\rho_c \left(1 - \frac{\rho}{\rho_c}\right) \ln\left(\frac{\rho}{\rho_c}\right)$$

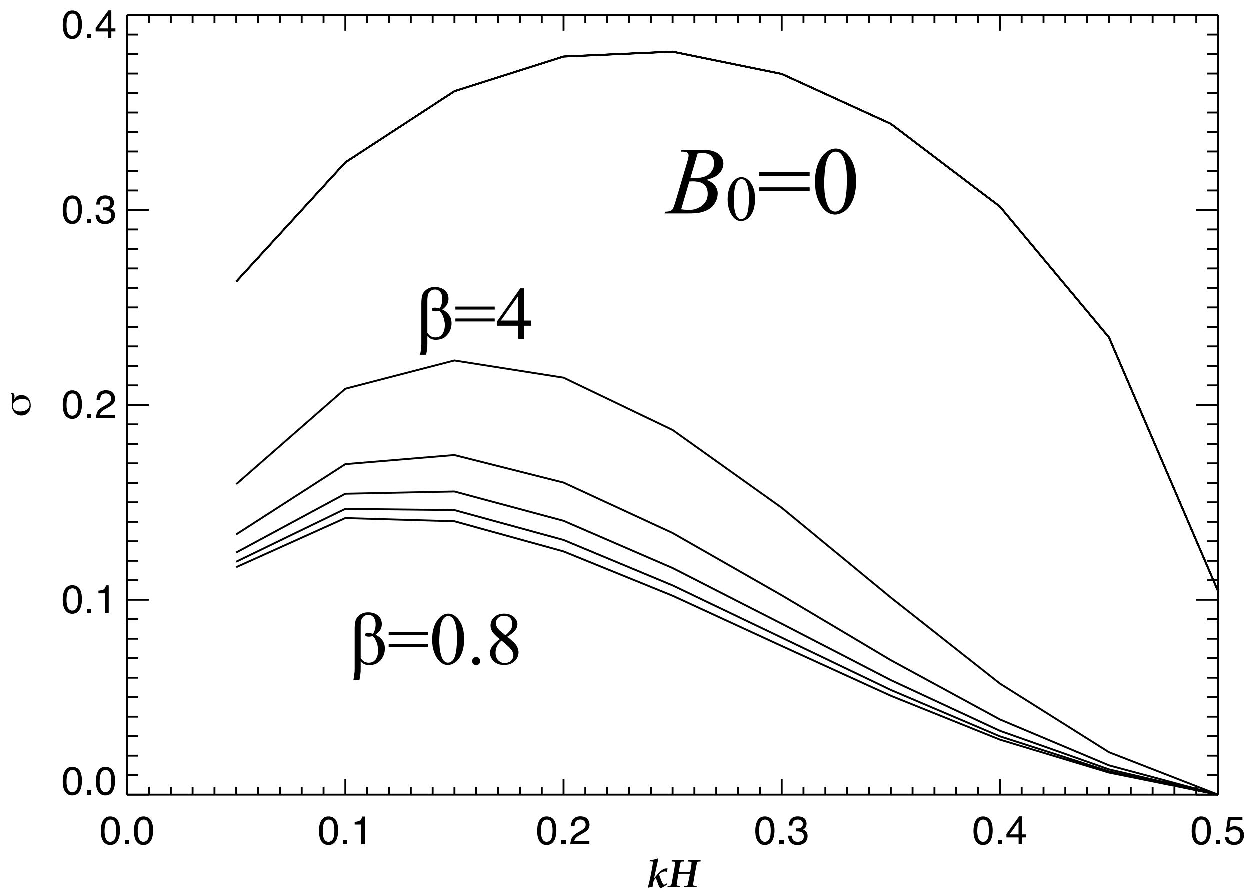


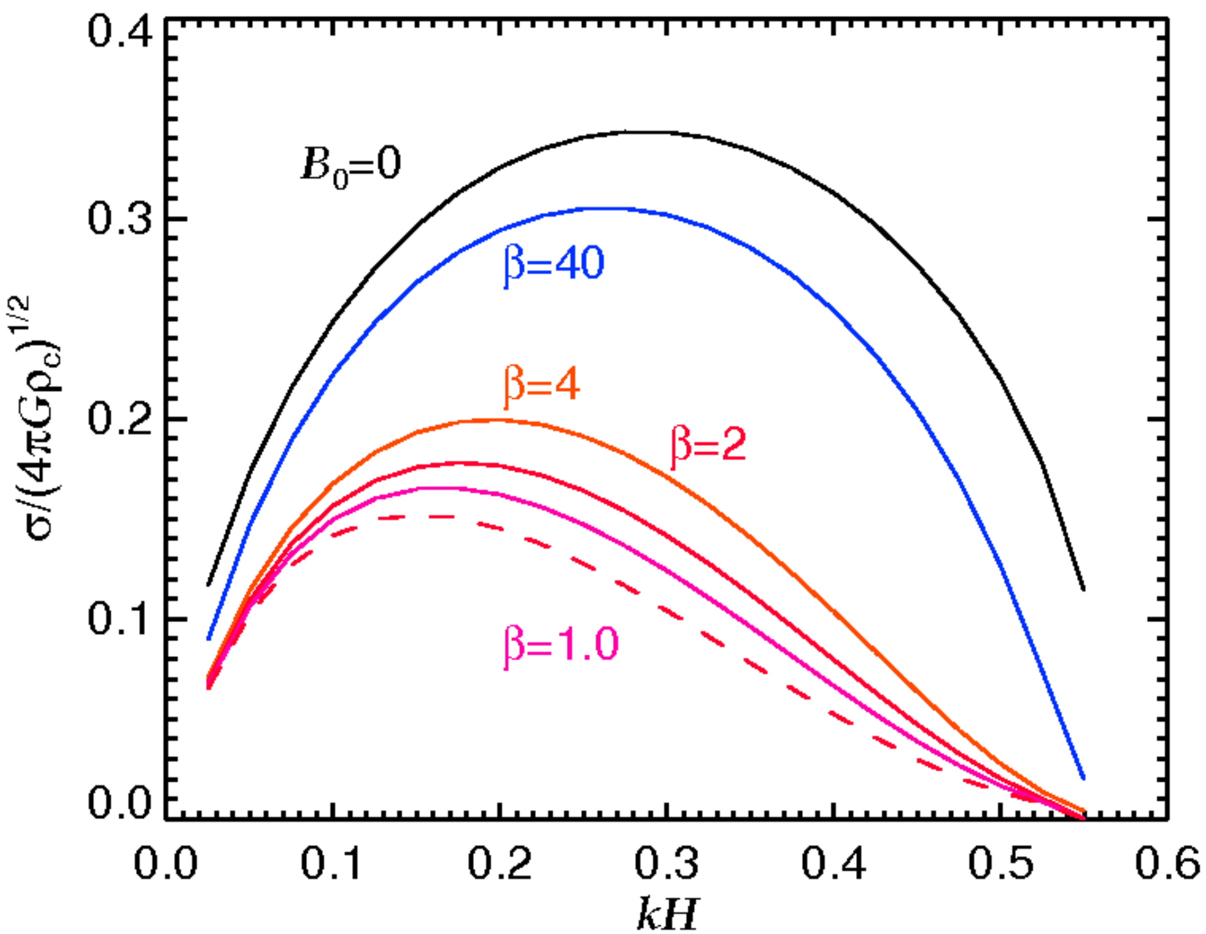
Stability Analysis

$$\rho \;=\; \rho_0 + \varrho(x,y)e^{\sigma t}\cos kz$$

$$\rho_0 = \rho_c\left(1+\frac{r^2}{4H^2}\right)^{-1} \qquad \frac{2\pi\sqrt{G}\Sigma_0}{B_0} = \sqrt{\frac{\beta}{0.203}}$$

$$\rho_0 = \rho_c\left(1+\frac{r^2}{8H^2}\right)^{-2}$$



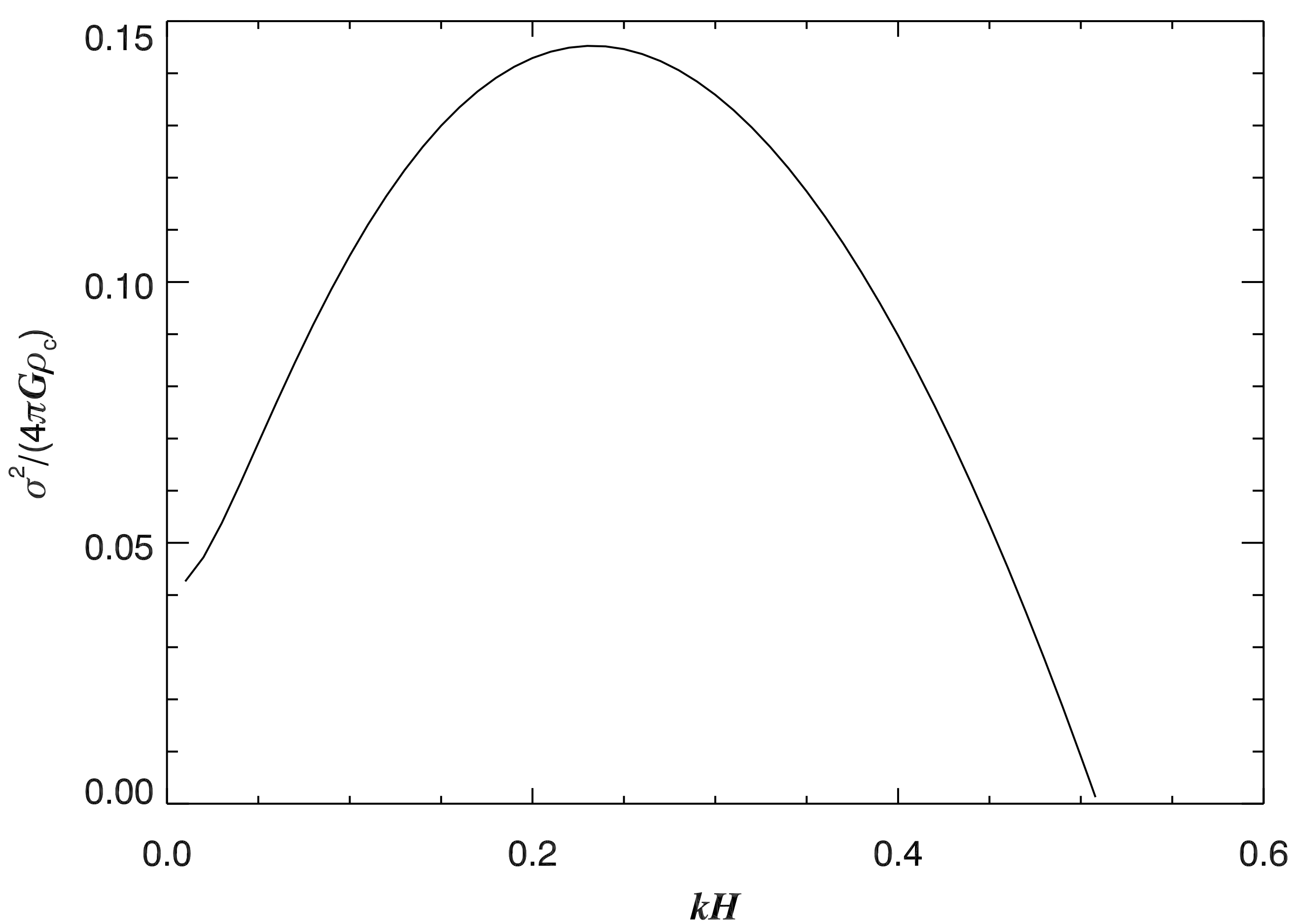


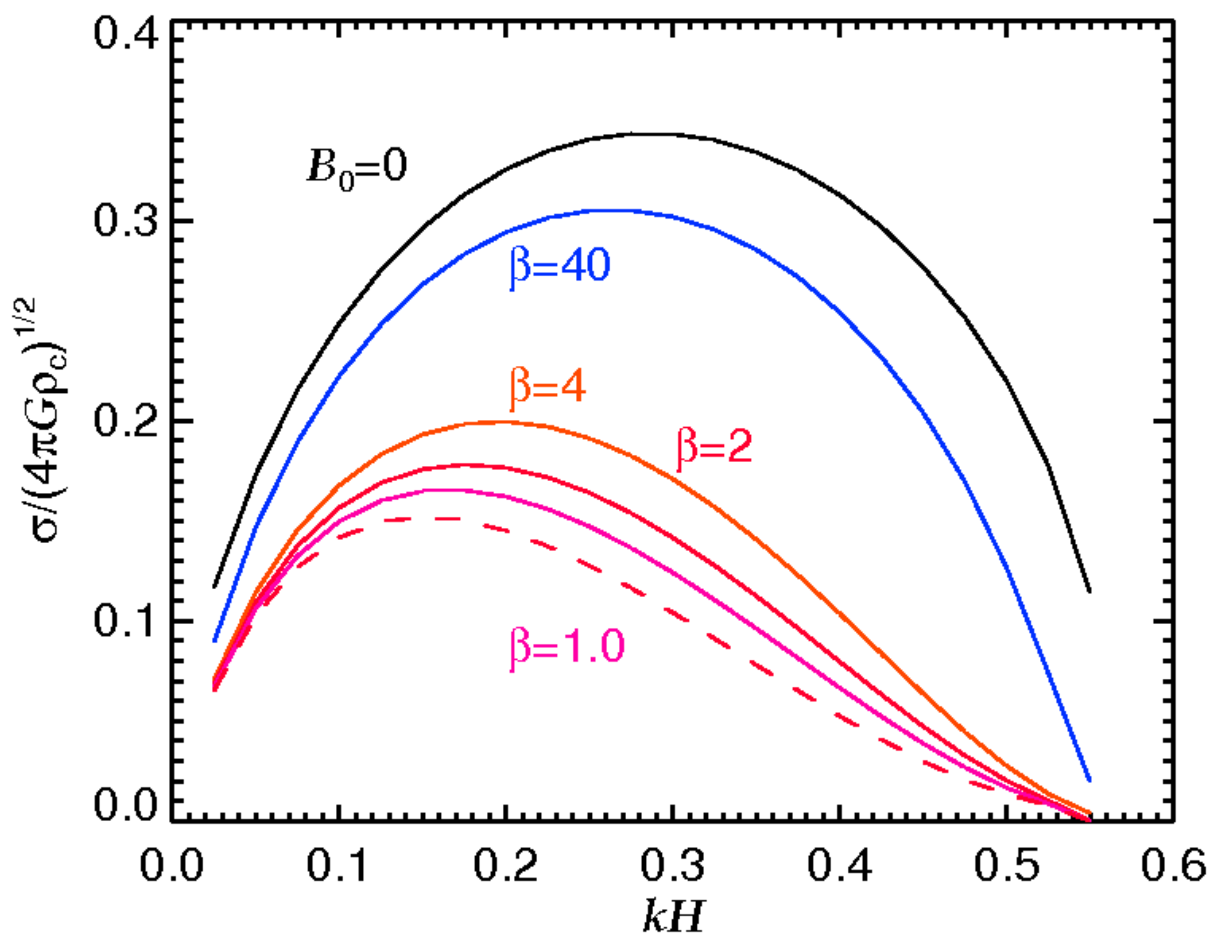
dx = 0.6H, n_x=n_y=100

Dispersion Relation

$$\rho \;=\; \rho_0 + \varrho(x,y)e^{\sigma t}\cos kz$$

$$\rho_0 = \rho_c\left(1+\frac{r^2}{4H^2}\right)^{-1} \qquad \text{cf.} \qquad \rho_0 = \rho_c\left(1+\frac{r^2}{8H^2}\right)^{-2}$$





unstable for radial collapse

Summary

- Vertical (uniform) magnetic field works against fragmentation.
- Compressible mode is suppressed by rather weak magnetic field.
- Incompressible mode survives even when B is extremely strong, if the magnetic field is not fixed on the boundary.
- Weak magnetic field affects flow in the low density region.