Presentation of an all-Mach regime solver

Thomas PADIOLEAU

Advisor : Pascal TREMBLIN

MAISON DE LA SIMULATION

June 1st 2017

Plan

Plan

Simulations of convection

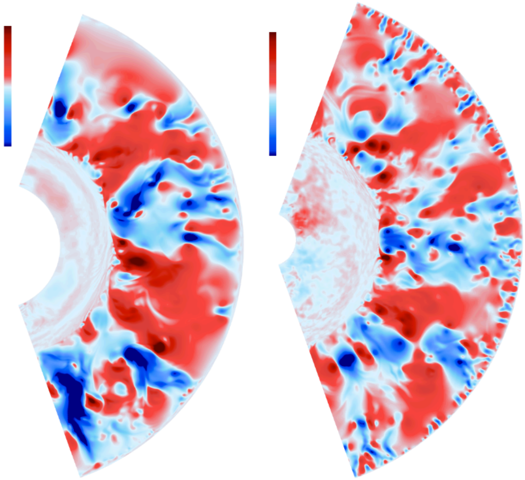
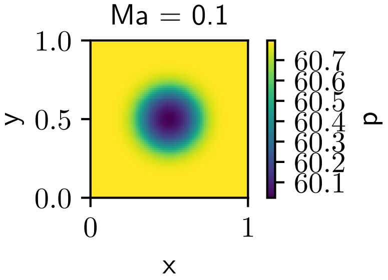


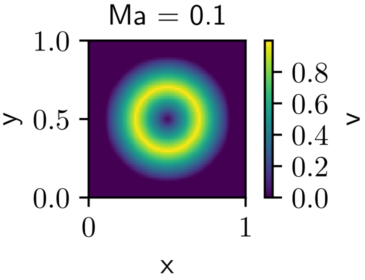
Figure: Pratt et al 2016

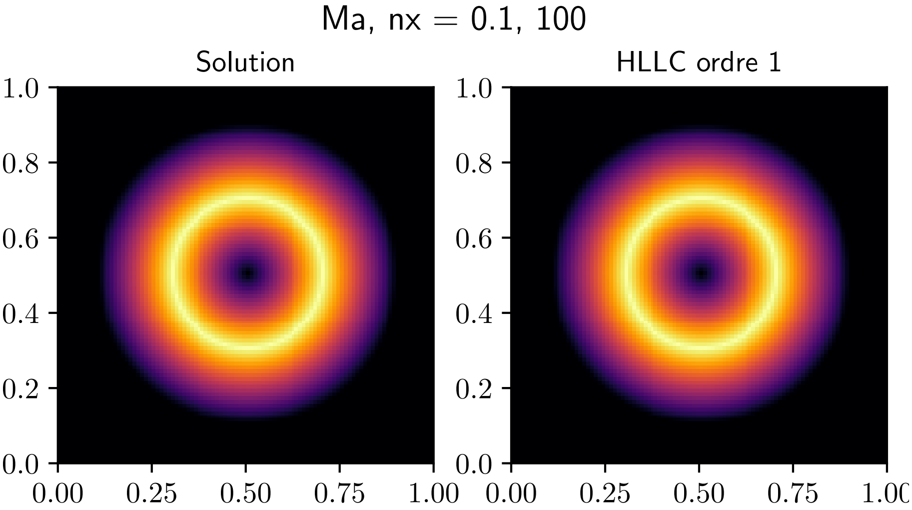
The Gresho test case

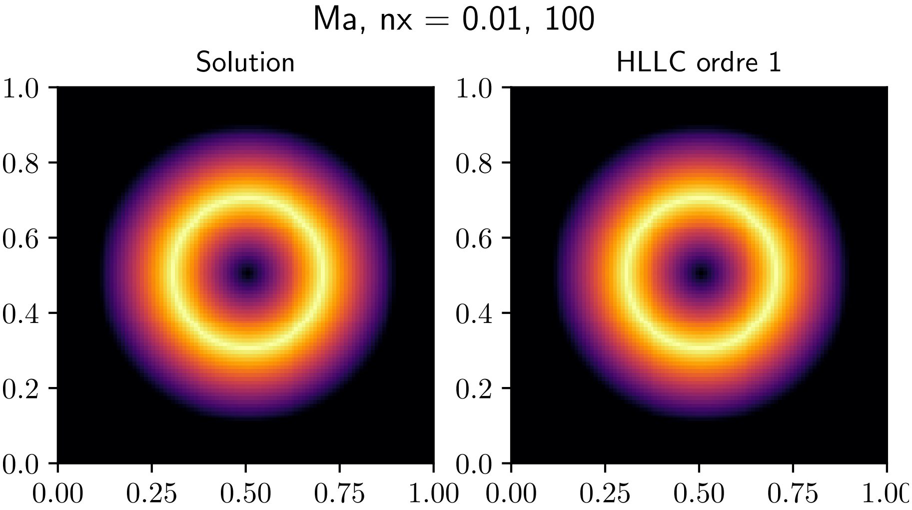
$$p_0 = \frac{\rho}{\gamma \mathrm{Ma}^2}, \gamma = 1.666 \approx \frac{5}{3}$$

$$(u_\theta(r), p(r)) = \begin{cases} \left(5r, p_0 + \frac{25}{2} r^2\right) & 0 \leq r \leq 0.2 \\ \left(2 - 5r, p_0 + \frac{25}{2} r^2 + 4\left(1 - 5r - \ln 0.2 + \ln r\right)\right) & 0.2 \leq r \leq 0.4 \\ (0, p_0 - 2 + 4 \ln 2) & 0.4 \leq r \leq 0.5 \end{cases}$$






Ma, nx = 0.1, 100
Solution
HLLC ordre 1
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


Ma, nx = 0.01, 100
Solution
HLLC ordre 1
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

Plan

The all regime solver verifies the following properties (Chalons, Girardin, Kokh 2014):

- splitting of acoustic and transport terms :
  - explicit / implicit treatment of the acoustic step
  - explicit treatment of the transport step
- "all regime" accuracy, meaning :
  - conservative scheme to capture shocks
  - correction of the low Mach regime to have less diffusion
- treatment of non-linear equations of state
- treatment of unstructured grids

Splitting

1D Euler system :

$$\partial_t \rho + \partial_x(\rho u) = 0$$
$$\partial_t(\rho u) + \partial_x(\rho u^2 + p) = 0$$
$$\partial_t(\rho E) + \partial_x(u(\rho E + p)) = 0$$

Splitting

1D Euler system :

$$\begin{aligned} \partial_t \rho + \partial_x(\rho u) = 0 \\ \partial_t(\rho u) + \partial_x(\rho u^2 + p) = 0 \\ \partial_t(\rho E) + \partial_x(u(\rho E + p)) = 0 \end{aligned}$$

$$\begin{aligned} \partial_t \rho + \rho \partial_x u + u \partial_x \rho = 0 \\ \partial_t(\rho u) + \rho u \partial_x u + u \partial_x(\rho u) + \partial_x p = 0 \\ \partial_t(\rho E) + \rho E \partial_x u + u \partial_x(\rho E) + \partial_x(p u) = 0 \end{aligned}$$

Splitting

1D Euler system :

$$\begin{aligned}
\partial_t \rho + \rho \partial_x u + u \partial_x \rho &= 0 \\
\partial_t (\rho u) + \rho u \partial_x u + u \partial_x (\rho u) + \partial_x p &= 0 \\
\partial_t (\rho E) + \rho E \partial_x u + u \partial_x (\rho E) + \partial_x (pu) &= 0
\end{aligned}$$

Acoustic system

$$\begin{aligned}
\partial_t \rho + \rho \partial_x u &= 0 \\
\partial_t (\rho u) + \rho u \partial_x u + \partial_x p &= 0 \\
\partial_t (\rho E) + \rho E \partial_x u + \partial_x (pu) &= 0
\end{aligned}$$

Transport / advection system

$$\begin{aligned}
\partial_t \rho + u \partial_x \rho &= 0 \\
\partial_t (\rho u) + u \partial_x (\rho u) &= 0 \\
\partial_t (\rho E) + u \partial_x (\rho E) &= 0
\end{aligned}$$

Splitting

1D Euler system :

$$
\begin{aligned}
\partial_t \rho + \rho \partial_x u + u \partial_x \rho = 0 \\
\partial_t (\rho u) + \rho u \partial_x u + u \partial_x (\rho u) + \partial_x p = 0 \\
\partial_t (\rho E) + \rho E \partial_x u + u \partial_x (\rho E) + \partial_x (p u) = 0
\end{aligned}
$$

Acoustic system

$$
\begin{aligned}
\partial_t \rho + \rho \partial_x u = 0 \\
\rho \partial_t u + u \partial_t \rho + \rho u \partial_x u + \partial_x p = 0 \\
\rho \partial_t E + E \partial_t \rho + \rho E \partial_x u + \partial_x (p u) = 0
\end{aligned}
$$

Transport / advection system

$$
\begin{aligned}
\partial_t \rho + u \partial_x \rho = 0 \\
\partial_t (\rho u) + u \partial_x (\rho u) = 0 \\
\partial_t (\rho E) + u \partial_x (\rho E) = 0
\end{aligned}
$$

Splitting

1D Euler system :

$$\begin{aligned}\partial_t \rho + \rho \partial_x u + u \partial_x \rho = 0 \\ \partial_t (\rho u) + \rho u \partial_x u + u \partial_x (\rho u) + \partial_x p = 0 \\ \partial_t (\rho E) + \rho E \partial_x u + u \partial_x (\rho E) + \partial_x (pu) = 0\end{aligned}$$

Acoustic system

$$\begin{aligned}\partial_t \rho + \rho \partial_x u = 0 \\ \rho \partial_t u + \partial_x p = 0 \\ \rho \partial_t E + \partial_x (pu) = 0\end{aligned}$$

Transport / advection system

$$\begin{aligned}\partial_t \rho + {\color{red}u} \partial_x \rho = 0 \\ \partial_t (\rho u) + {\color{red}u} \partial_x (\rho u) = 0 \\ \partial_t (\rho E) + {\color{red}u} \partial_x (\rho E) = 0\end{aligned}$$

Splitting

1D Euler system :

$$\begin{aligned} \partial_t \rho + \rho \partial_x u + u \partial_x \rho = 0 \\ \partial_t(\rho u) + \rho u \partial_x u + u \partial_x(\rho u) + \partial_x p = 0 \\ \partial_t(\rho E) + \rho E \partial_x u + u \partial_x(\rho E) + \partial_x(pu) = 0 \end{aligned}$$

Acoustic system

$$\begin{aligned} \partial_t \tau - \tau \partial_x u = 0 \\ \partial_t u + \tau \partial_x p = 0 \\ \partial_t E + \tau \partial_x(pu) = 0 \end{aligned}$$

Transport / advection system

$$\begin{aligned} \partial_t \rho + {\color{red}u} \partial_x \rho = 0 \\ \partial_t(\rho u) + {\color{red}u} \partial_x(\rho u) = 0 \\ \partial_t(\rho E) + {\color{red}u} \partial_x(\rho E) = 0 \end{aligned}$$

Splitting

1D Euler system :

$$\begin{aligned}\partial_t \rho + \rho \partial_x u + u \partial_x \rho &= 0\\ \partial_t(\rho u) + \rho u \partial_x u + u \partial_x(\rho u) + \partial_x p &= 0\\ \partial_t(\rho E) + \rho E \partial_x u + u \partial_x(\rho E) + \partial_x(p u) &= 0\end{aligned}$$

Acoustic system

$$\begin{aligned}\partial_t \tau - \partial_m u &= 0\\ \partial_t u + \partial_m p &= 0\\ \partial_t E + \partial_m (p u) &= 0\end{aligned}$$

Transport / advection system

$$\begin{aligned}\partial_t \rho + u \partial_x \rho &= 0\\ \partial_t(\rho u) + u \partial_x(\rho u) &= 0\\ \partial_t(\rho E) + u \partial_x(\rho E) &= 0\end{aligned}$$

Conservativity

Notations :

- Acoustic step : $\rho_j^n \to \rho_j^{n+1-}$
- Transport step : $\rho_j^{n+1-} \to \rho_j^{n+1}$
- Full step : $\rho_j^n \to \rho_j^{n+1}$

The entire step must be conservative, i.e. :

$$\rho_j^{n+1} = \rho_j^n - \frac{dt}{dx}\left(h_{j+1/2} - h_{j-1/2}\right)$$

Acoustic step...

$$\partial_t \tau - \partial_m u = 0$$

Acoustic step...

$$\partial_t \tau - \partial_m u = 0$$

$$\tau_j^{n+1-} = \tau_j^n + \frac{dt}{dm_j}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

Acoustic step...

$$\partial_t \tau - \partial_m u = 0$$

$$\tau_j^{n+1-} = \tau_j^n + \frac{dt}{dm_j}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

$$\frac{1}{\rho_j^{n+1-}} = \frac{1}{\rho_j^n} + \frac{dt}{dx}\frac{1}{\rho_j^n}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

Acoustic step...

$$\partial_t \tau - \partial_m u = 0$$

$$\tau_j^{n+1-} = \tau_j^n + \frac{dt}{dm_j}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

$$\frac{1}{\rho_j^{n+1-}} = \frac{1}{\rho_j^n} + \frac{dt}{dx}\frac{1}{\rho_j^n}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

$$\frac{1}{\rho_j^{n+1-}} = \frac{1}{\rho_j^n}\left(1 + \frac{dt}{dx}\left[f^{\#}\right]_j\right)$$

Acoustic step...

$$\partial_t \tau - \partial_m u = 0$$

$$\tau_j^{n+1-} = \tau_j^n + \frac{dt}{dm_j}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

$$\frac{1}{\rho_j^{n+1-}} = \frac{1}{\rho_j^n} + \frac{dt}{dx}\frac{1}{\rho_j^n}\left(f^{\#}_{j+\frac{1}{2}} - f^{\#}_{j-\frac{1}{2}}\right)$$

$$\frac{1}{\rho_j^{n+1-}} = \frac{1}{\rho_j^n}\left(1 + \frac{dt}{dx}\left[f^{\#}\right]_j\right)$$

$$L_j \rho_j^{n+1-} = \rho_j^n \text{ with } L_j = 1 + \frac{dt}{dx}\left[f^{\#}\right]_j$$

...Transport step...

$$\partial_t \rho + u \partial_x \rho = 0$$

...Transport step...

$$\partial_t \rho + u \partial_x \rho = 0$$

$$\partial_t \rho + \partial_x(\rho u) - \rho \partial_x u = 0$$

...Transport step...

$$\partial_t \rho + u \partial_x \rho = 0$$

$$\partial_t \rho + \partial_x(\rho u) - \rho \partial_x u = 0$$

$$\rho_j^{n+1} = \rho^{n+1-} - \frac{dt}{dx} \left[\rho^{n+1-} u \right] + \{\rho \partial_x u\}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \rho^{n+1-} - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{L_j}{L_j}\rho^{n+1-} - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{L_j \rho^{n+1-}}{L_j} - \frac{dt}{dx}\left[\rho^{n+1-} u\right] + \{\rho \partial_x u\}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j} \rho^n - \frac{dt}{dx} \left[\rho^{n+1-} u \right] + \{ \rho \partial_x u \}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j}\rho^n - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

To identify to :

$$\rho_j^{n+1} = \rho_j^n - \frac{dt}{dx}\left[h\right]_j$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j}\rho^n - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

To identify to :

$$\rho_j^{n+1} = \rho_j^n - \frac{dt}{dx}\left[h\right]_j$$

A sufficient condition is to set :

$$\frac{1}{L_j}\rho_j^n + \{\rho\partial_x u\} = \rho_j^n$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j}\rho^n - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

To identify to :

$$\rho_j^{n+1} = \rho_j^n - \frac{dt}{dx}\left[h\right]_j$$

A sufficient condition is to set :

$$\{\rho\partial_x u\} = \frac{L_j - 1}{L_j}\rho_j^n$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j}\rho^{n} - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

To identify to :

$$\rho_j^{n+1} = \rho_j^{n} - \frac{dt}{dx}\left[h\right]_j$$

A sufficient condition is to set :

$$\{\rho\partial_x u\} = (L_j - 1)\rho_j^{n+1-}$$

...Recombining the two steps

By recombining the two steps we obtain :

$$\rho_j^{n+1} = \frac{1}{L_j}\rho^n - \frac{dt}{dx}\left[\rho^{n+1-}u\right] + \{\rho\partial_x u\}$$

To identify to :

$$\rho_j^{n+1} = \rho_j^n - \frac{dt}{dx}[h]_j$$

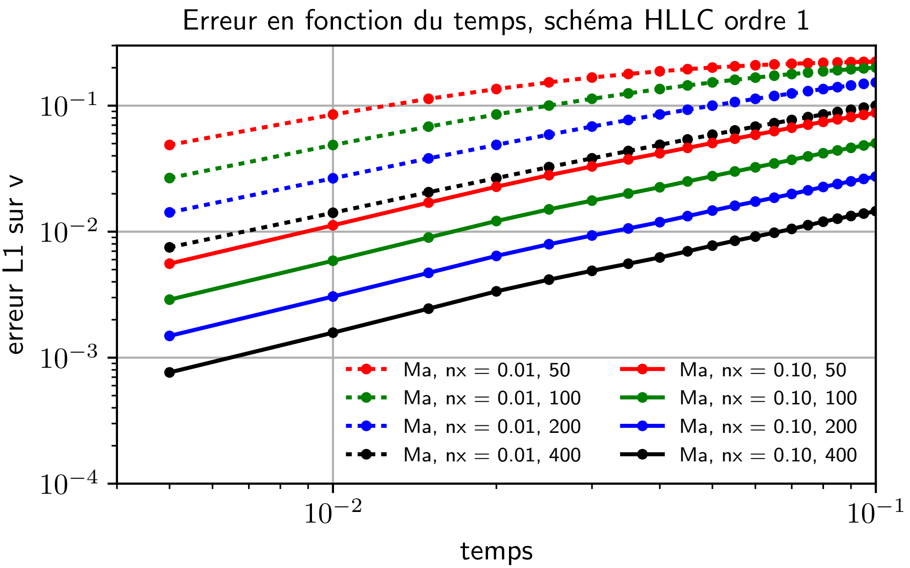
A sufficient condition is to set :

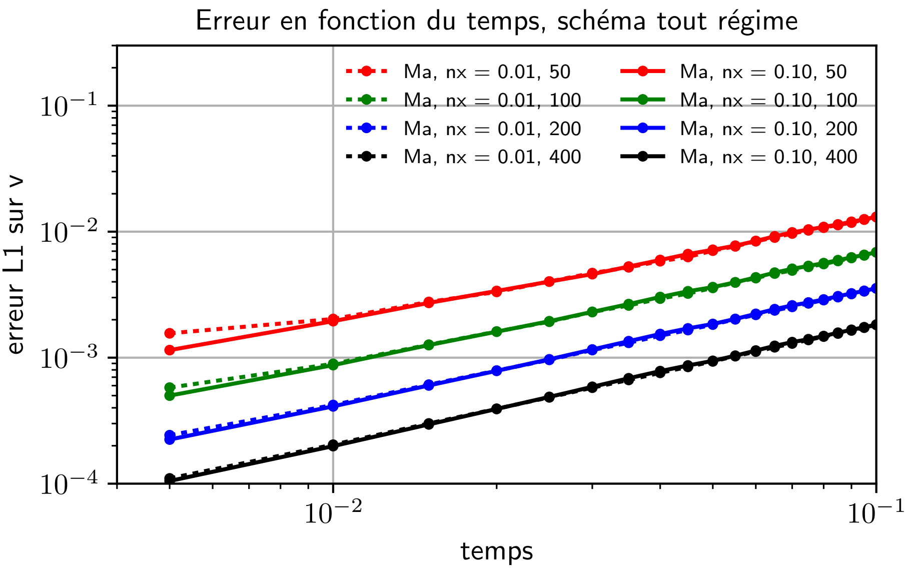
$$\{\rho\partial_x u\} = (L_j - 1)\rho_j^{n+1-}$$

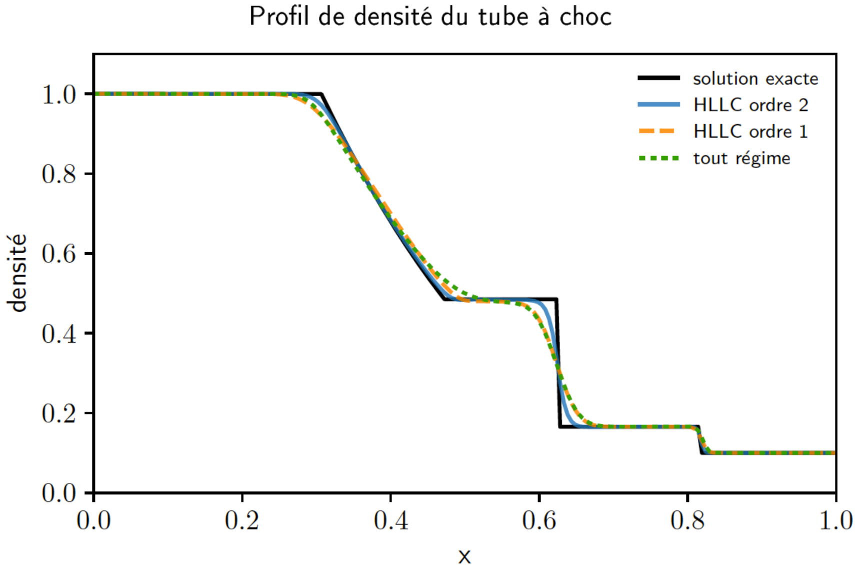
Thus we have :

$$\rho_j^{n+1} = \rho^n - \frac{dt}{dx}\left[\rho^{n+1-}u\right]$$

Plan


Erreur en fonction du temps, schéma HLLC ordre 1
erreur L1 sur v
temps
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Ma, nx = 0.01, 50
Ma, nx = 0.01, 100
Ma, nx = 0.01, 200
Ma, nx = 0.01, 400
Ma, nx = 0.10, 50
Ma, nx = 0.10, 100
Ma, nx = 0.10, 200
Ma, nx = 0.10, 400


Erreur en fonction du temps, schéma tout régime
Ma, nx = 0.01, 50
Ma, nx = 0.01, 100
Ma, nx = 0.01, 200
Ma, nx = 0.01, 400
Ma, nx = 0.10, 50
Ma, nx = 0.10, 100
Ma, nx = 0.10, 200
Ma, nx = 0.10, 400
erreur L1 sur v
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-2}
10^{-1}
temps


Profil de densité du tube à choc
solution exacte
HLLC ordre 2
HLLC ordre 1
tout régime
densité
x
0.0
0.2
0.4
0.6
0.8
1.0

Resolution : 200^2

Ma = 0.01		
	hllc order 1	all regime
iterations	5040	5040
simulation time (in s)	40	58

time step : 2.010^{-5}

Ma = 0.1		
	hllc order 1	all regime
iterations	540	540
simulation time(in s)	6.6	9.1

time step : 1.910^{-4}

Plan

Given a gravitationnal potential ϕ :

$$\partial_t \rho + \partial_x(\rho u) = 0$$
$$\partial_t(\rho u) + \partial_x(\rho u^2 + p) = -\rho \partial_x \phi$$
$$\partial_t(\rho E) + \partial_x(u(\rho E + p)) = -\rho u \partial_x \phi$$

Given a gravitationnal potential ϕ :

$$\partial_t \rho + \partial_x(\rho u) = 0$$
$$\partial_t(\rho u) + \partial_x(\rho u^2 + p) = -\rho \partial_x \phi$$
$$\partial_t(\rho E) + \partial_x(u(\rho E + p)) = -\rho u \partial_x \phi$$

Putting the gravitationnal potential term in the acoustic step :

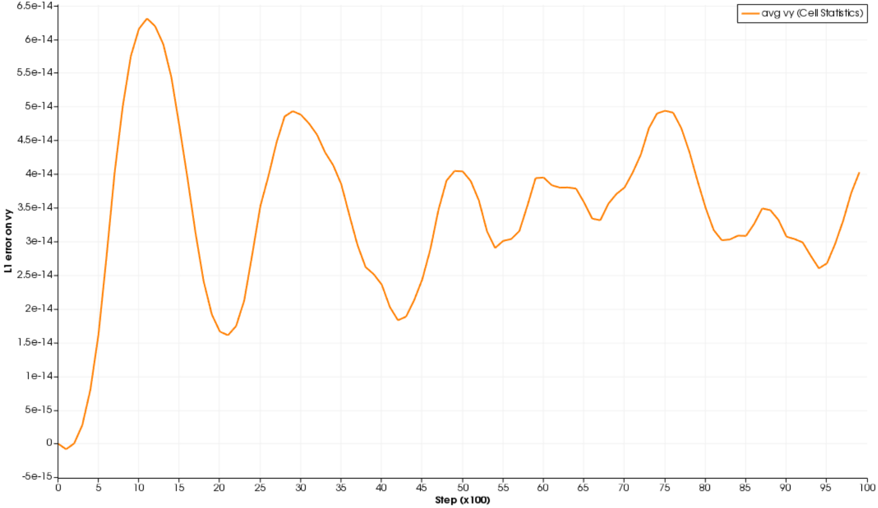
$$\partial_t \rho + \rho \partial_x u = 0$$
$$\partial_t(\rho u) + \rho u \partial_x u + \partial_x p = -\rho \partial_x \phi$$
$$\partial_t(\rho E) + \rho E \partial_x u + \partial_x(pu) = -\rho u \partial_x \phi$$

Given a gravitationnal potential ϕ :

$$\partial_t \rho + \partial_x(\rho u) = 0$$
$$\partial_t(\rho u) + \partial_x(\rho u^2 + p) = -\rho \partial_x \phi$$
$$\partial_t(\rho E) + \partial_x(u(\rho E + p)) = -\rho u \partial_x \phi$$

Putting the gravitationnal potential term in the acoustic step :

$$\partial_t \tau - \partial_m u = 0$$
$$\partial_t u + \partial_m p = -\frac{1}{\tau} \partial_m \phi$$
$$\partial_t E + \partial_m(pu) = -\frac{u}{\tau} \partial_m \phi$$


avg vy (Cell Statistics)
L1 error on vy
Step (x100)
6.5e-14
6e-14
5.5e-14
5e-14
4.5e-14
4e-14
3.5e-14
3e-14
2.5e-14
2e-14
1.5e-14
1e-14
5e-15
0
-5e-15
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100

Conclusion

Thank you for your attention !

MAISON DE LA SIMULATION