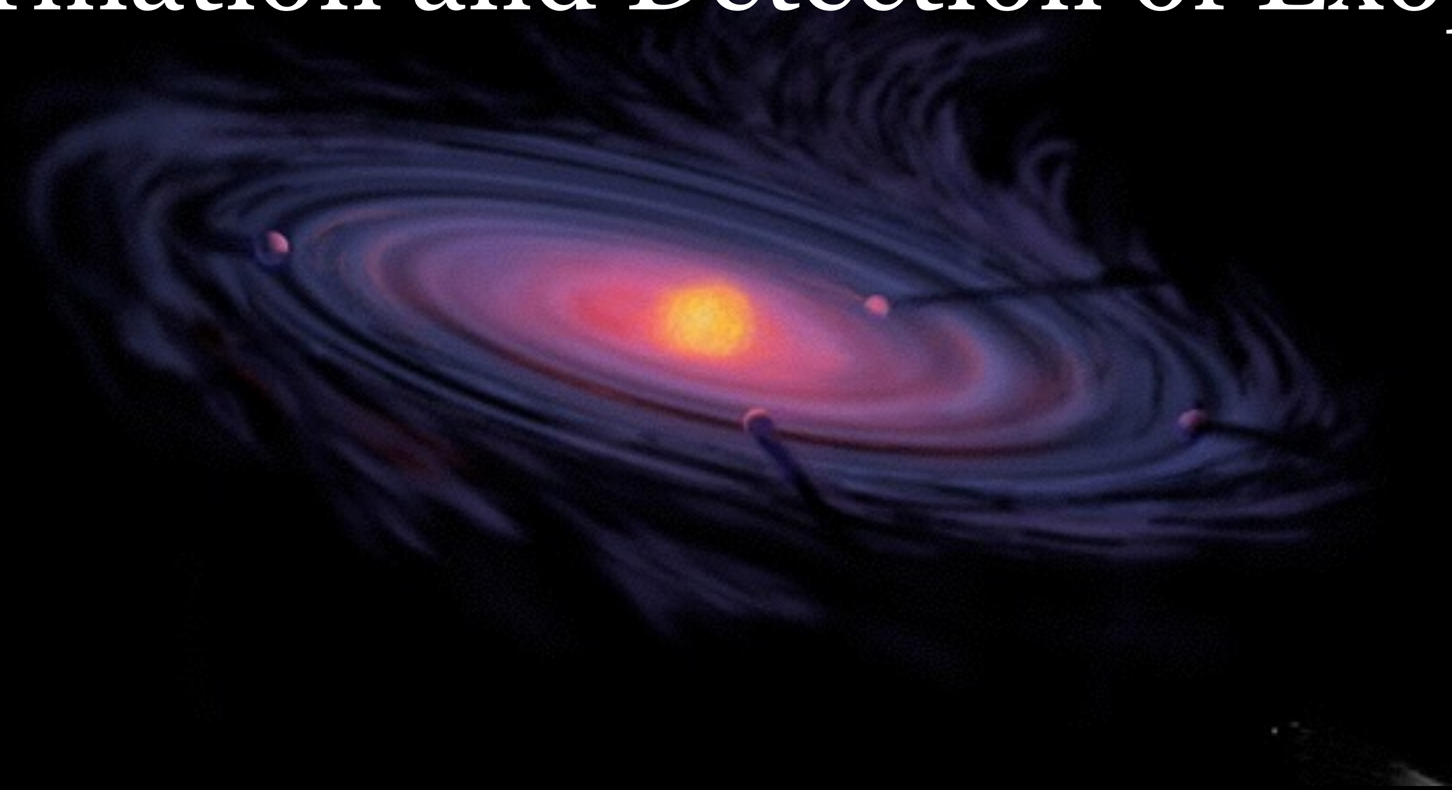
Critical Phenomenon, Nonlinear Dynamics & the Formation and Detection of Exoplanets



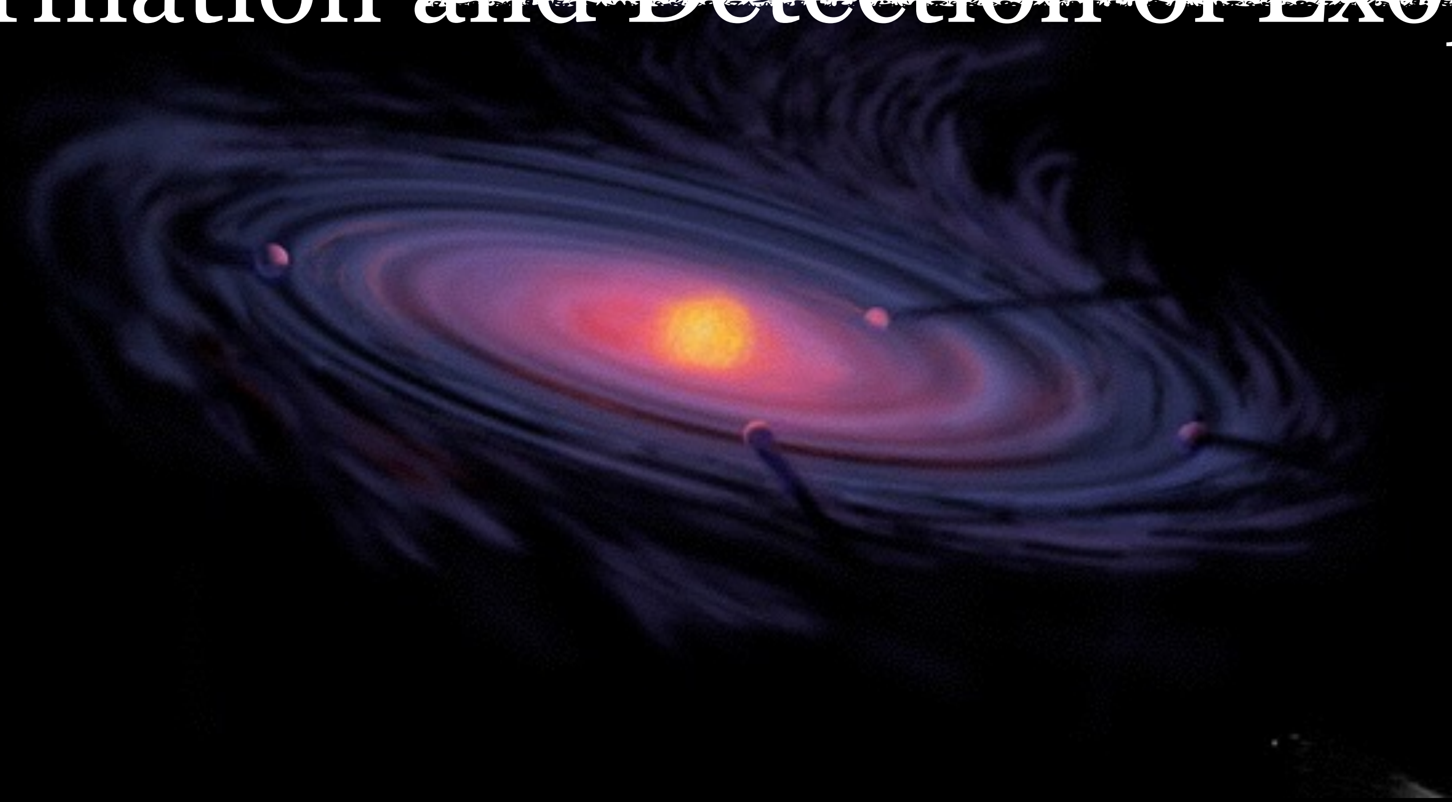
John Wettlaufer

Yale University

NORDITA

University of Oxford

Critical Phenomenon, ~~Nonlinear Dynamics~~ & the Formation ~~and Detection of Exoplanets~~



Critical Phenomenon, ~~Nonlinear Dynamics~~ & the Formation ~~and Detection of Exoplanets~~

What has happened in the past appears to be almost as vague and uncertain as what will happen in the future...Note that observers of automobile accidents almost invariably disagree as to the actual sequence of events. And so it is, even in science. We can reconstruct the history of the solar system with little more confidence than we can predict its future. Actually, we possess only a fragmentary knowledge of the system today and have inadequate theoretical tools to deal with many of the physical processes that have taken place.

F.L. Whipple, *PNAS* **52**, 565 (1964)

Critical Phenomenon, ~~Nonlinear Dynamics~~
& the Formation ~~and Detection of Exoplanets~~

HAT-P-26b: A Neptune-mass exoplanet with a well-constrained heavy element abundance

Hannah R. Wakeford,[1*†] David K. Sing,[2†] Tiffany Kataria,[3] Drake Deming,[4] Nikolay Nikolov,[2] Eric D. Lopez,[1,5] Pascal Tremblin,[6] David S. Amundsen,[7,8] Nikole K. Lewis,[9] Avi M. Mandell,[1] Jonathan J. Fortney,[10] Heather Knutson,[11] Björn Benneke,[11] Thomas M. Evans[2]

Science **356**, 628–631 (2017) 12 May 2017

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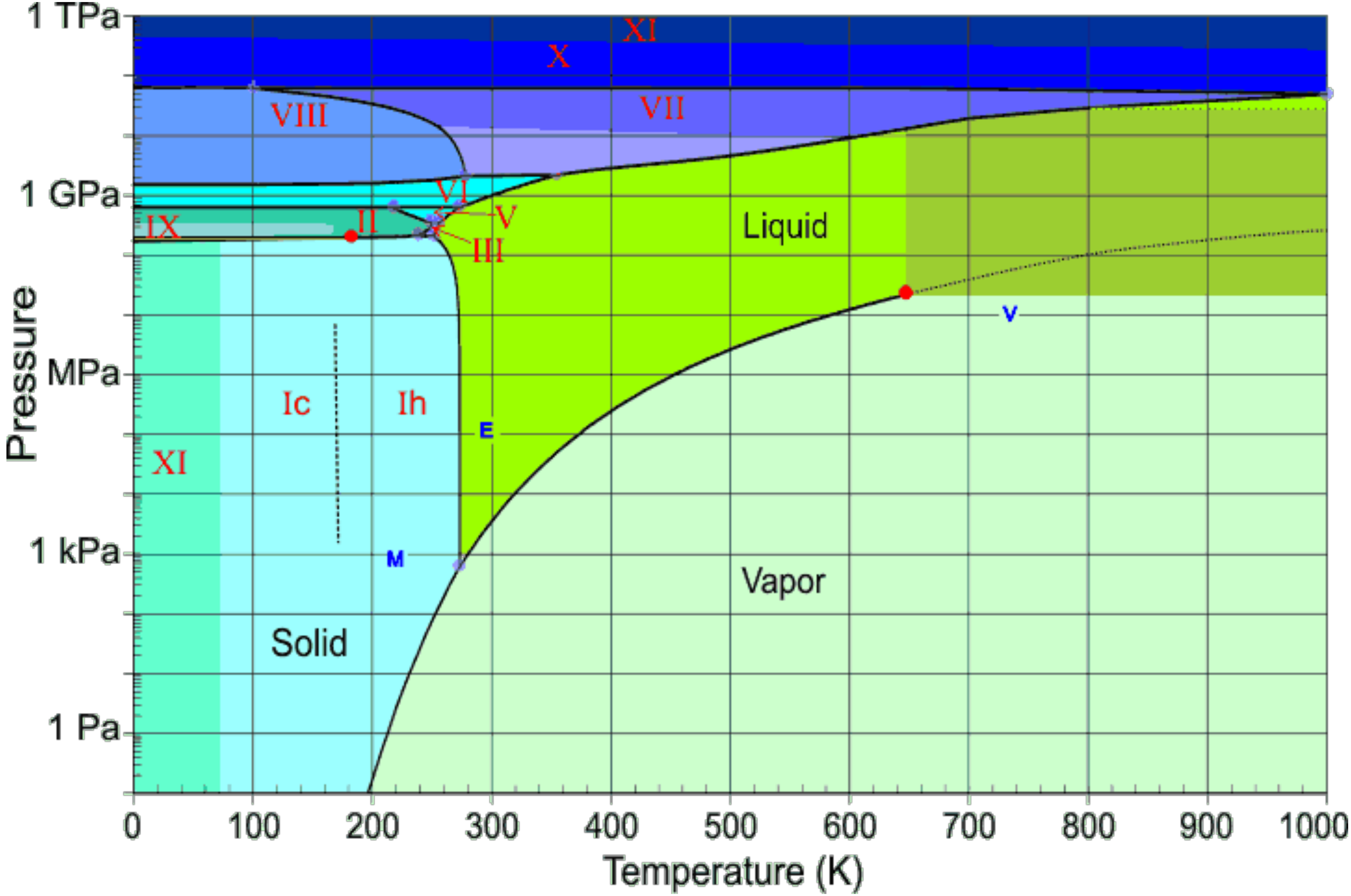
It's Friday—start with something we experience in a cocktail

Bulk Phases of Water

M. Chaplin, LSBU



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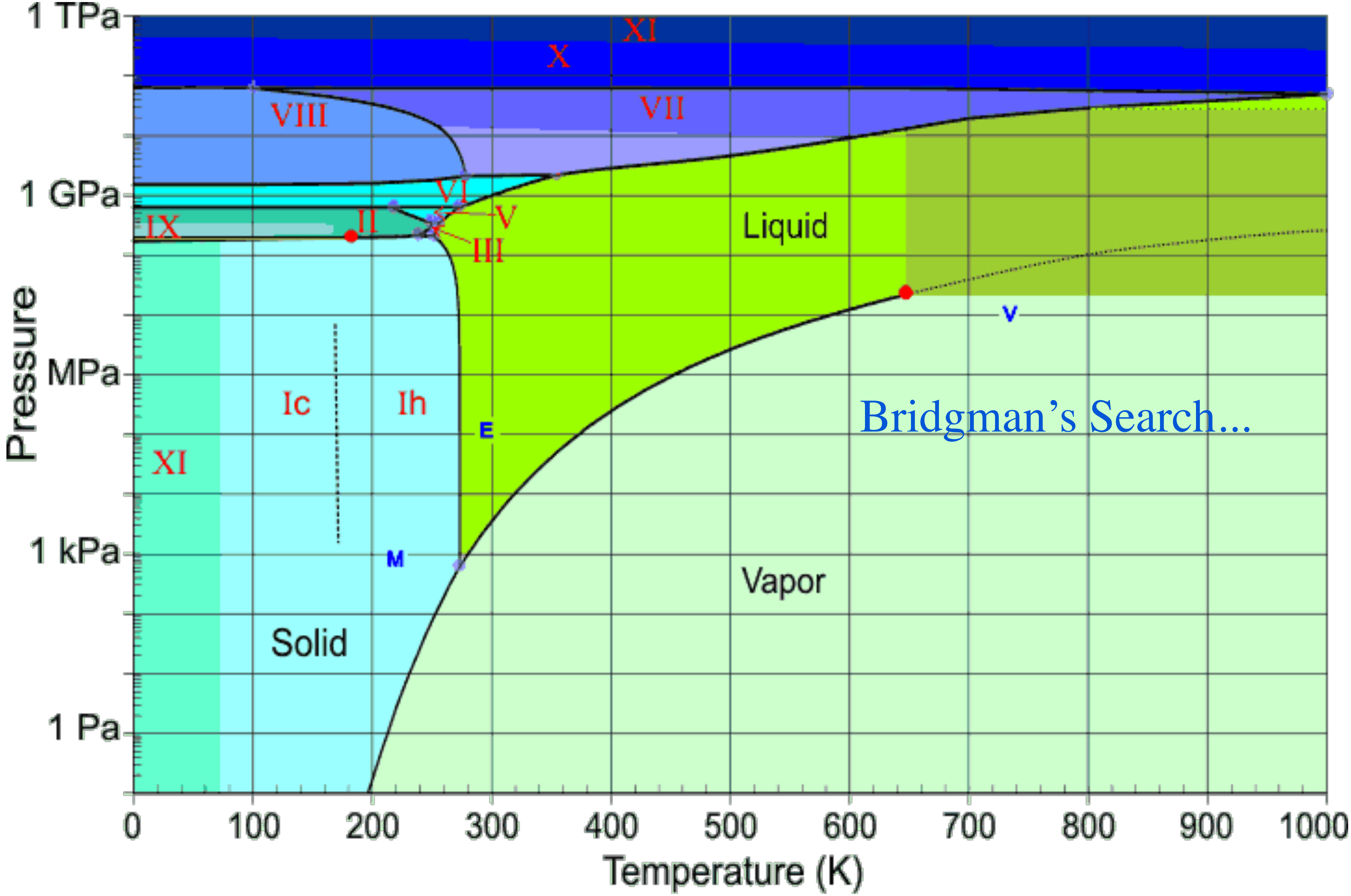

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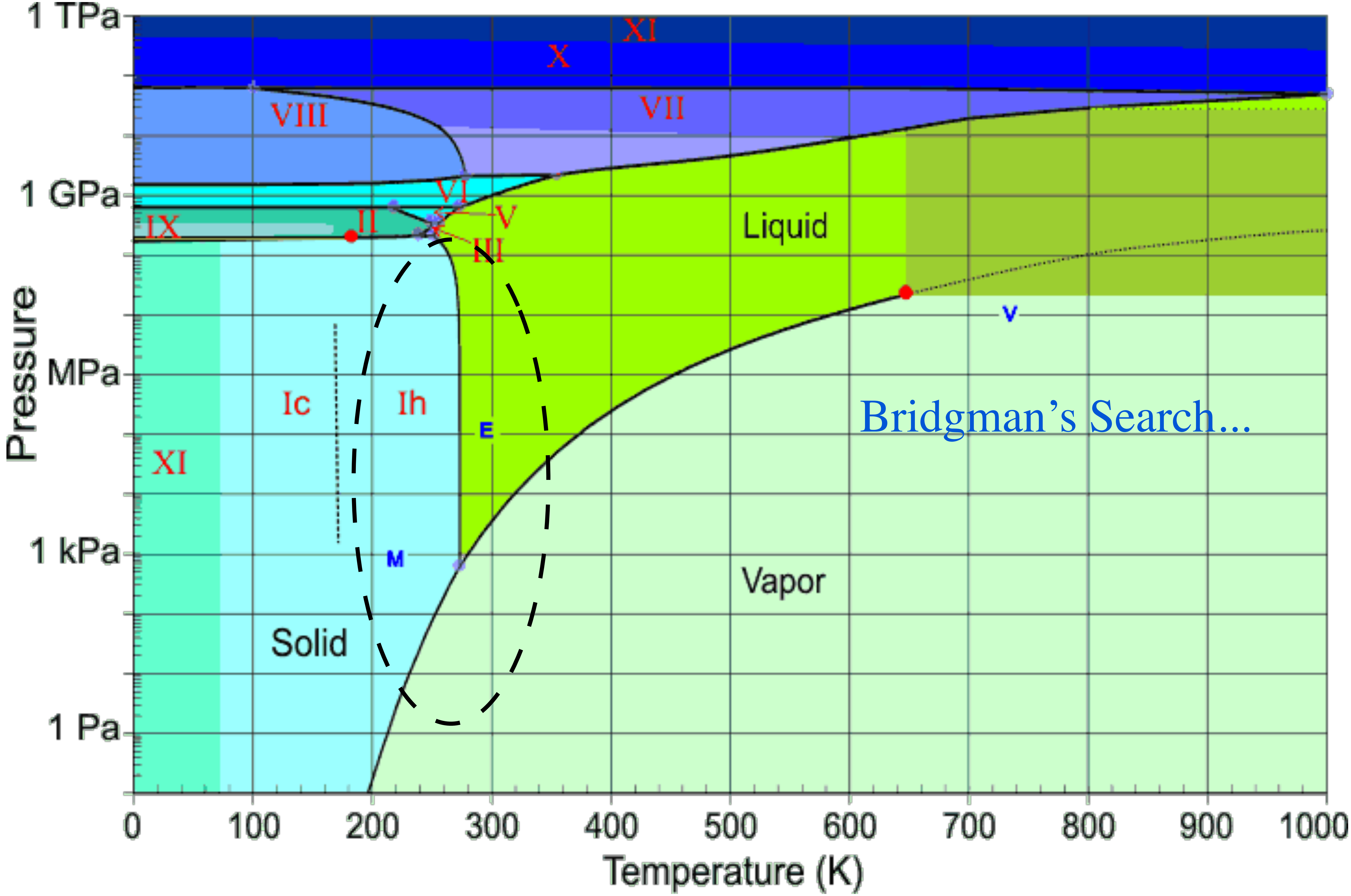

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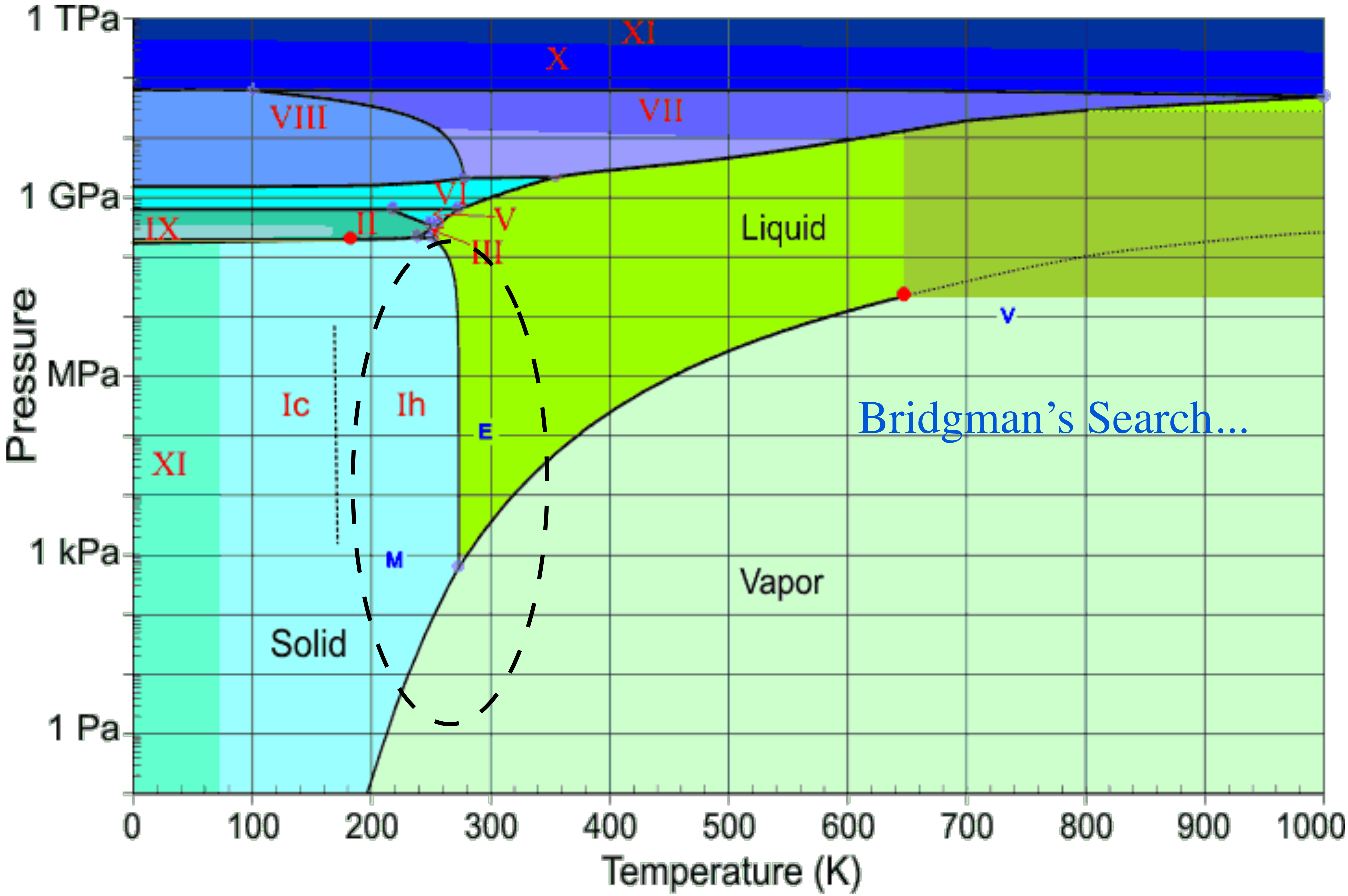


Let us move along a line of two-phase coexistence?

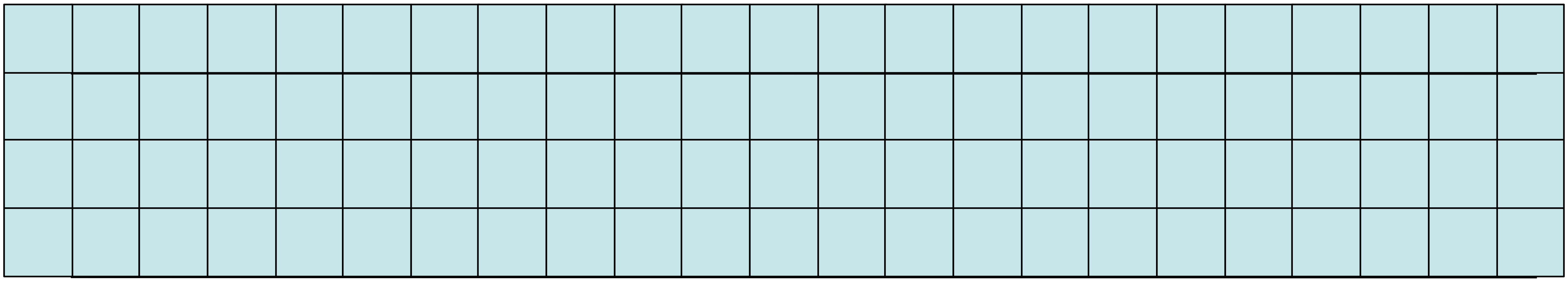
Some Surface Phases…

Equilibrium Transitions

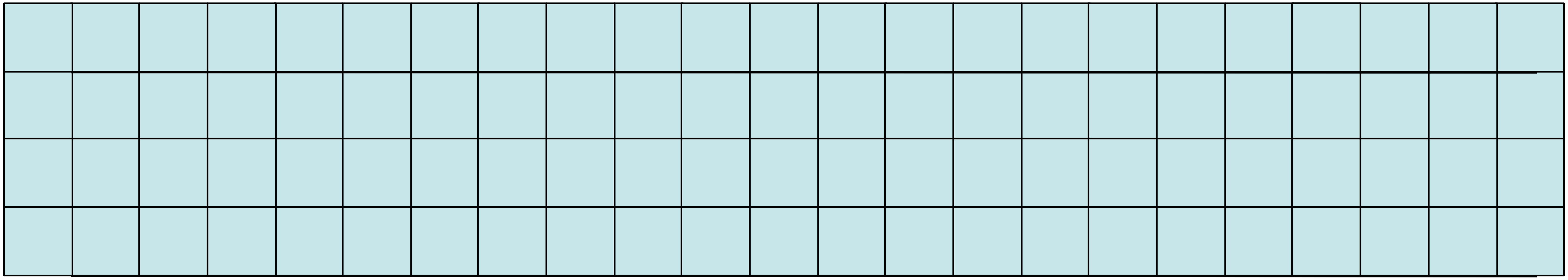
(1) Surface Roughening

(2) Surface Melting

(1) Faceted Orientations

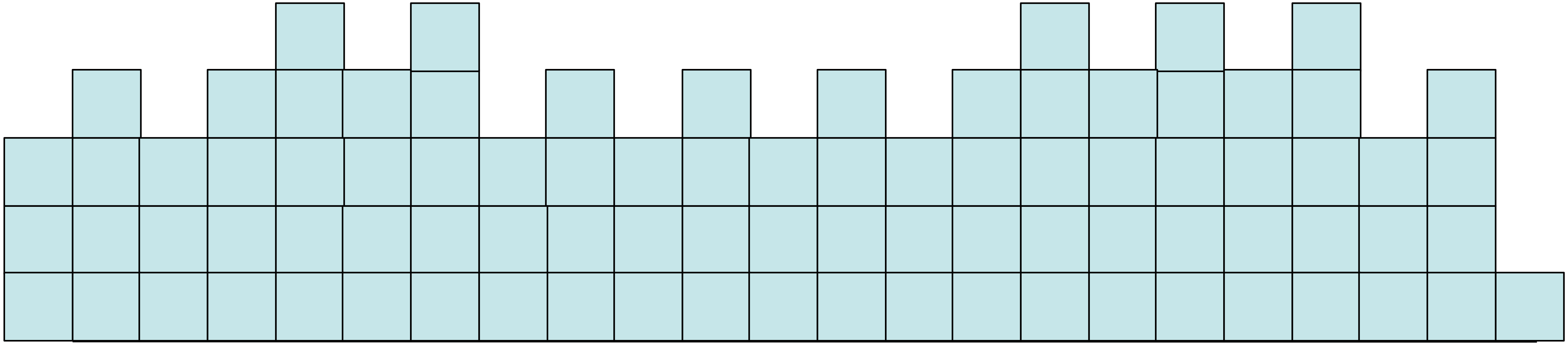


(1) Faceted Orientations

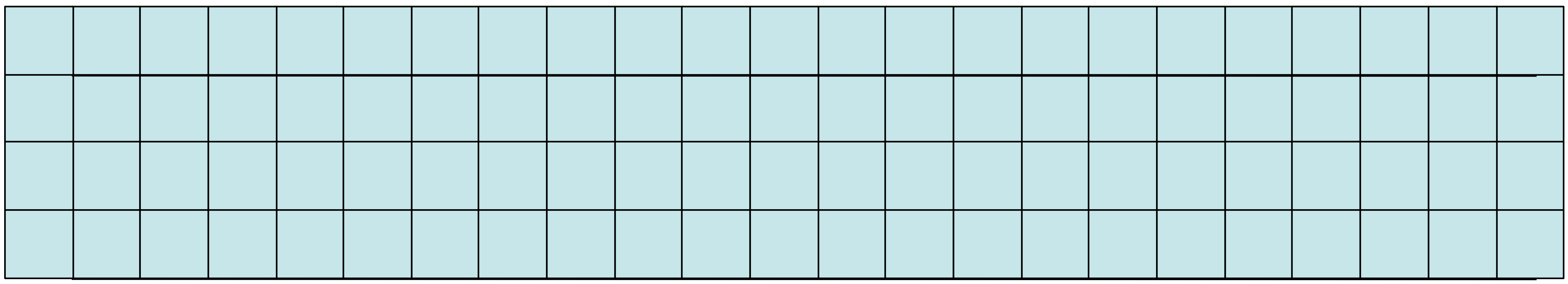


Can Become Rough Orientations

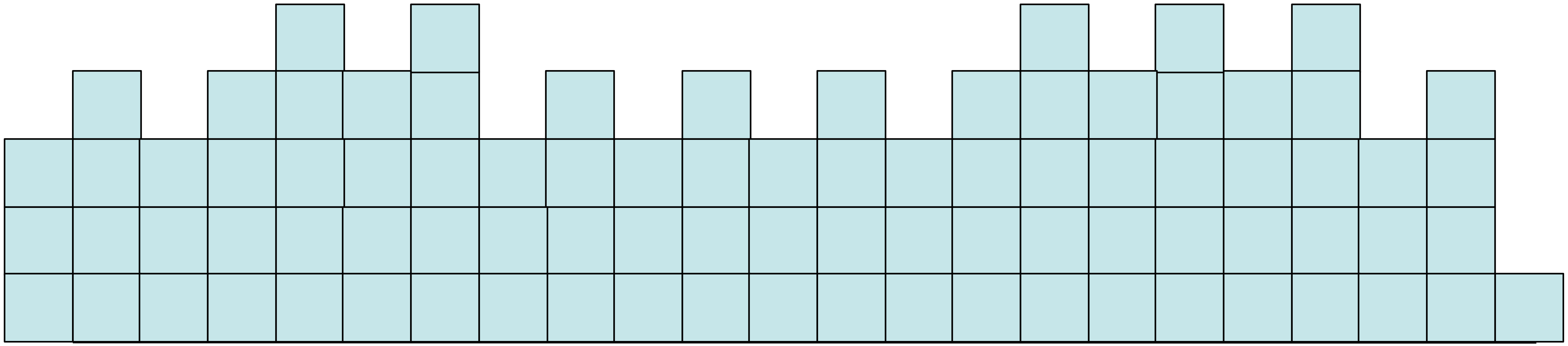
$$\xi = A \exp[c\ (T_R - T)^{-1/2}]$$



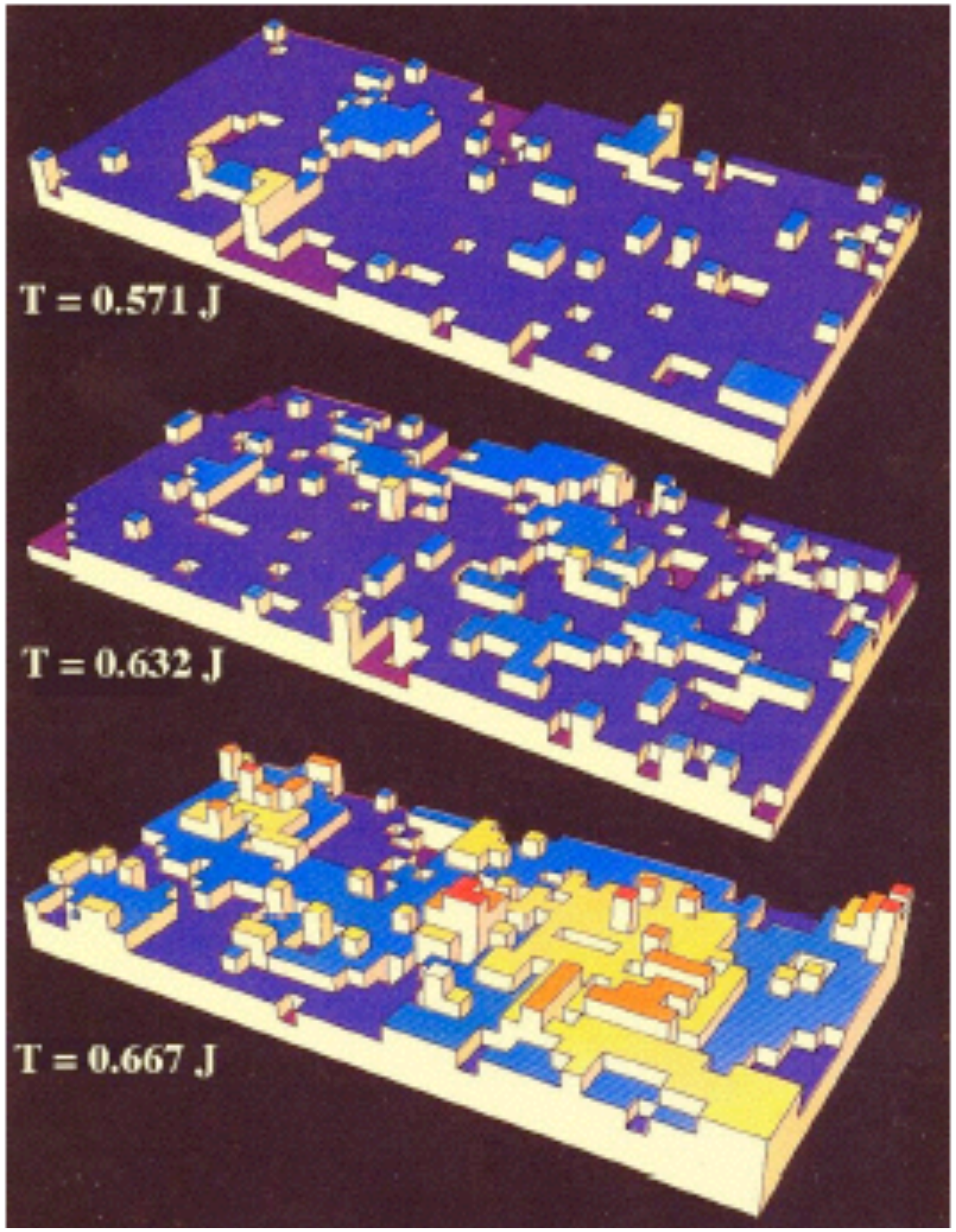
(2) Faceted Orientations



Can Coexist with Rough Orientations



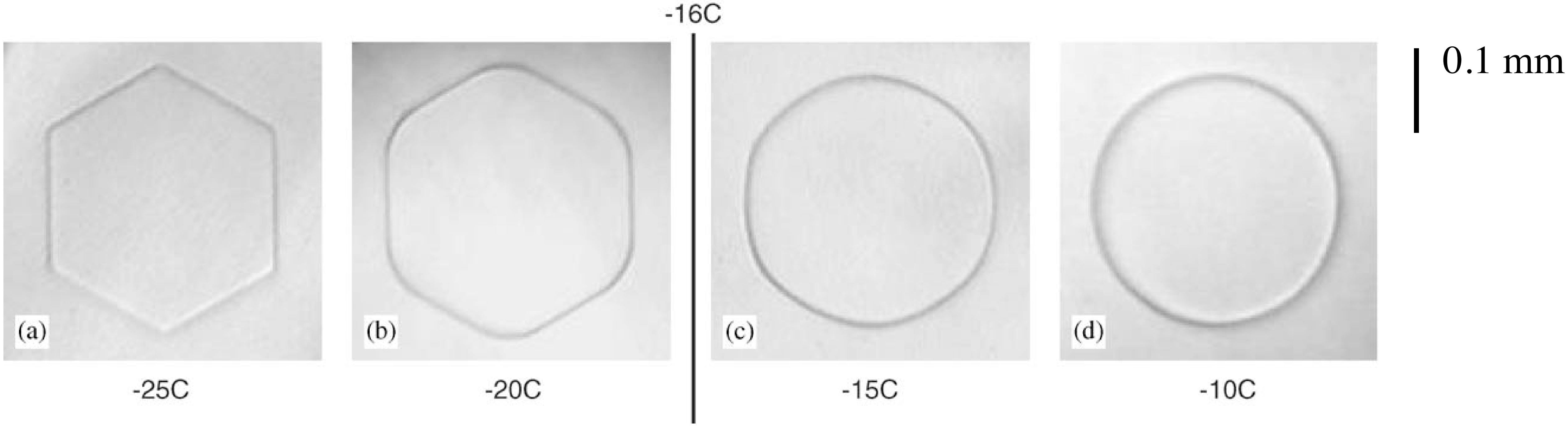
(1)

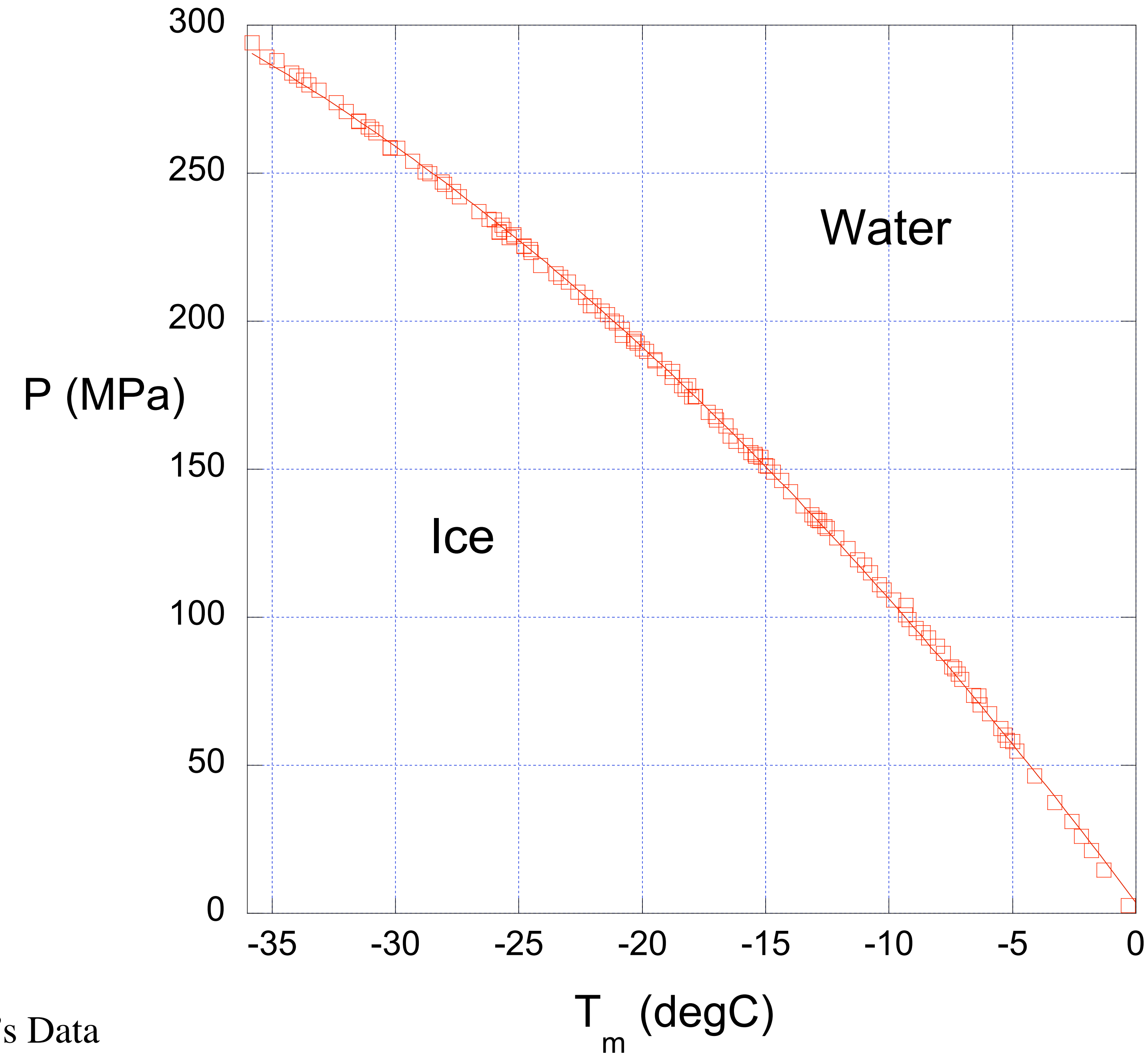


The Roughening Transition

Balibar, Alles & Parshin *Rev Mod Phys* **77**, 317 (2005)
Dash, Rempel & JSW *Rev Mod Phys* **78**, 695 (2006)

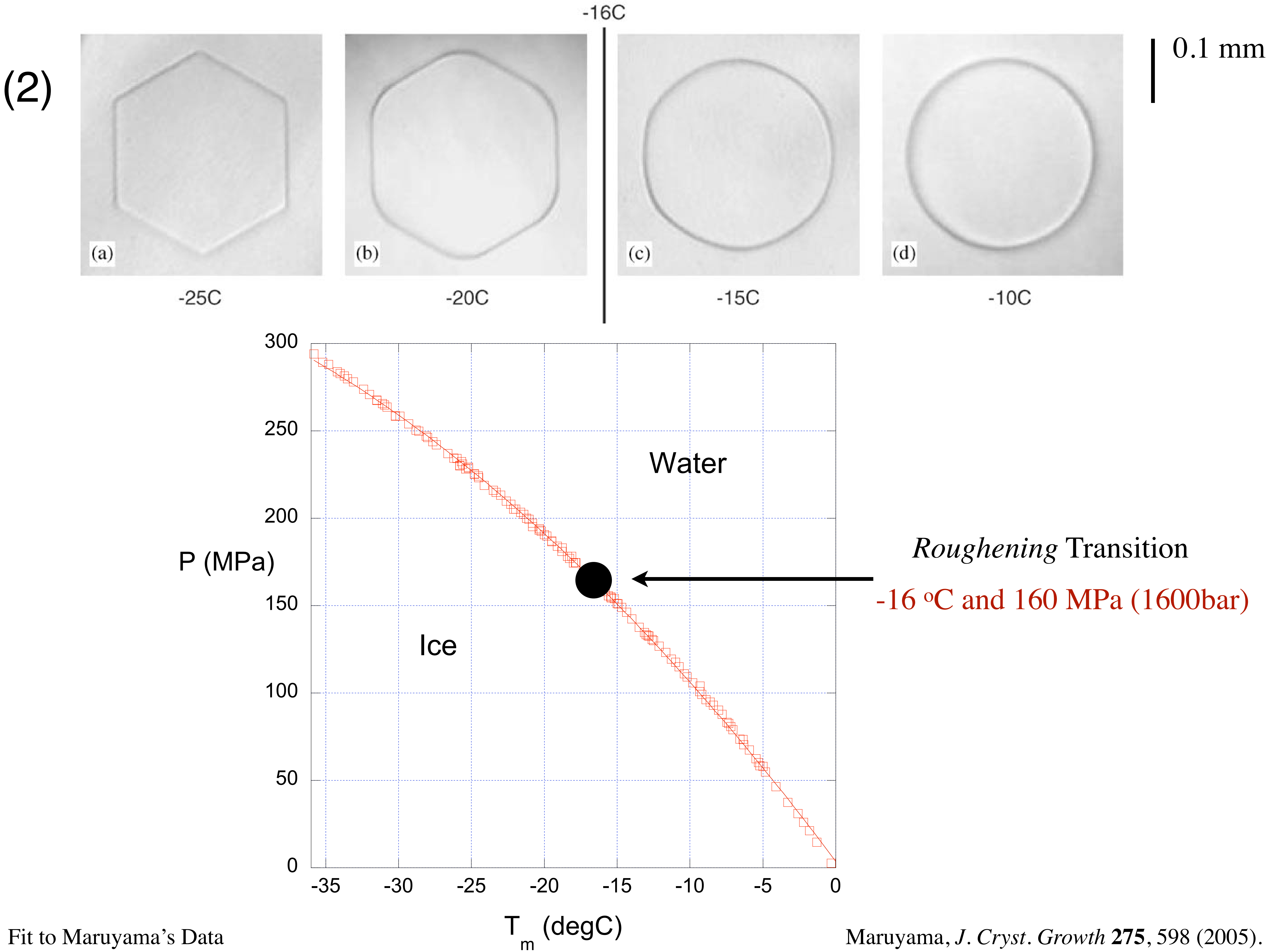
(2)





Fit to Maruyama's Data

Maruyama, *J. Cryst. Growth* **275**, 598 (2005).



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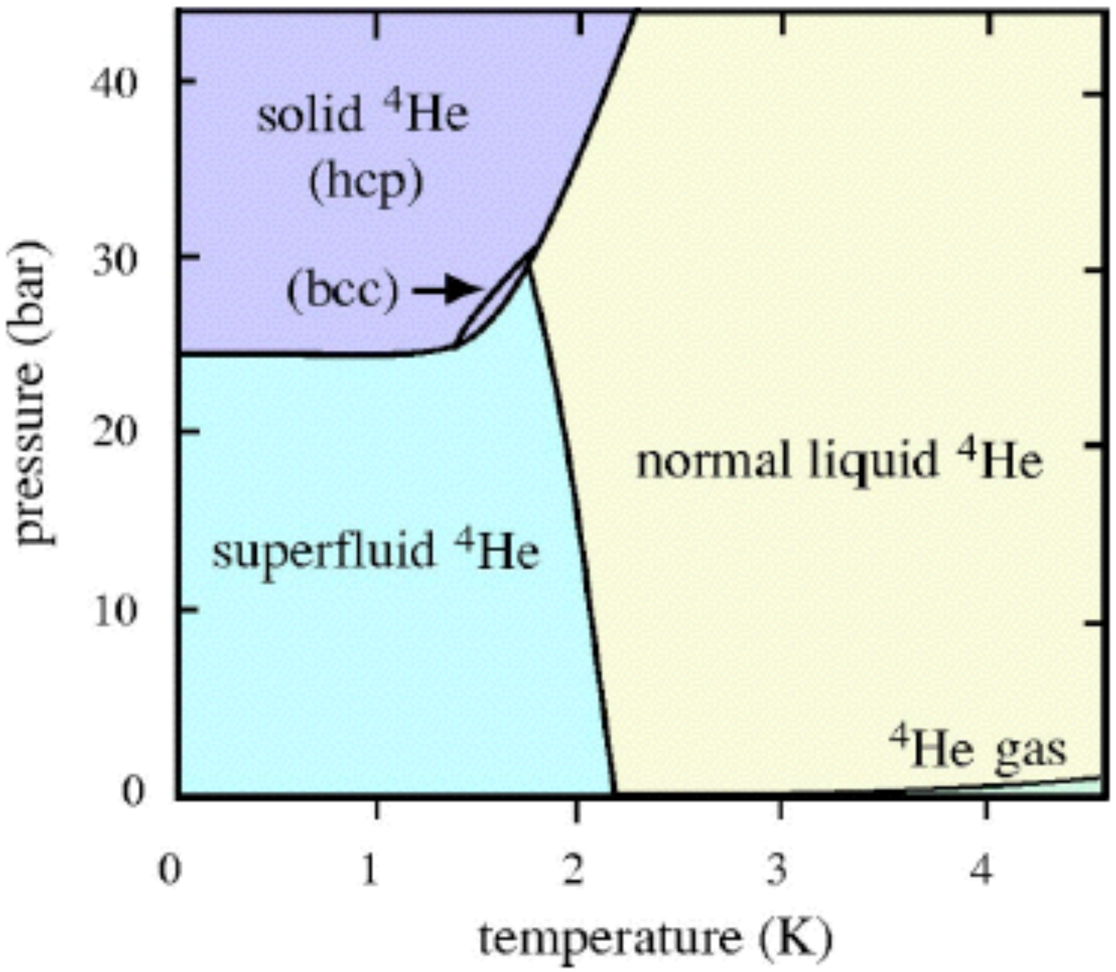
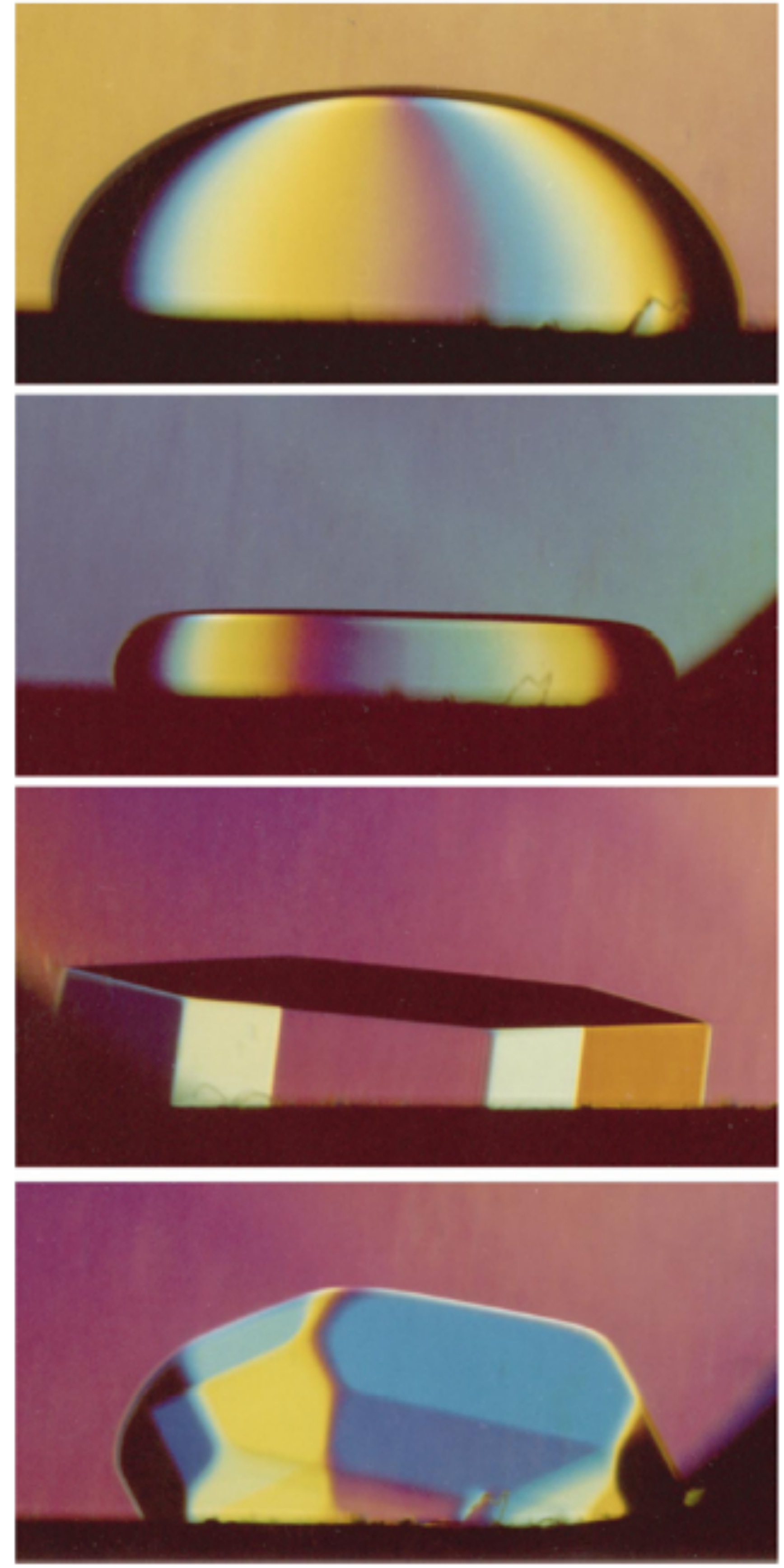
^{4}He; Best Test Bed





T= 1.4K

T= 1.0K

T= 0.4K

T= 0.1K

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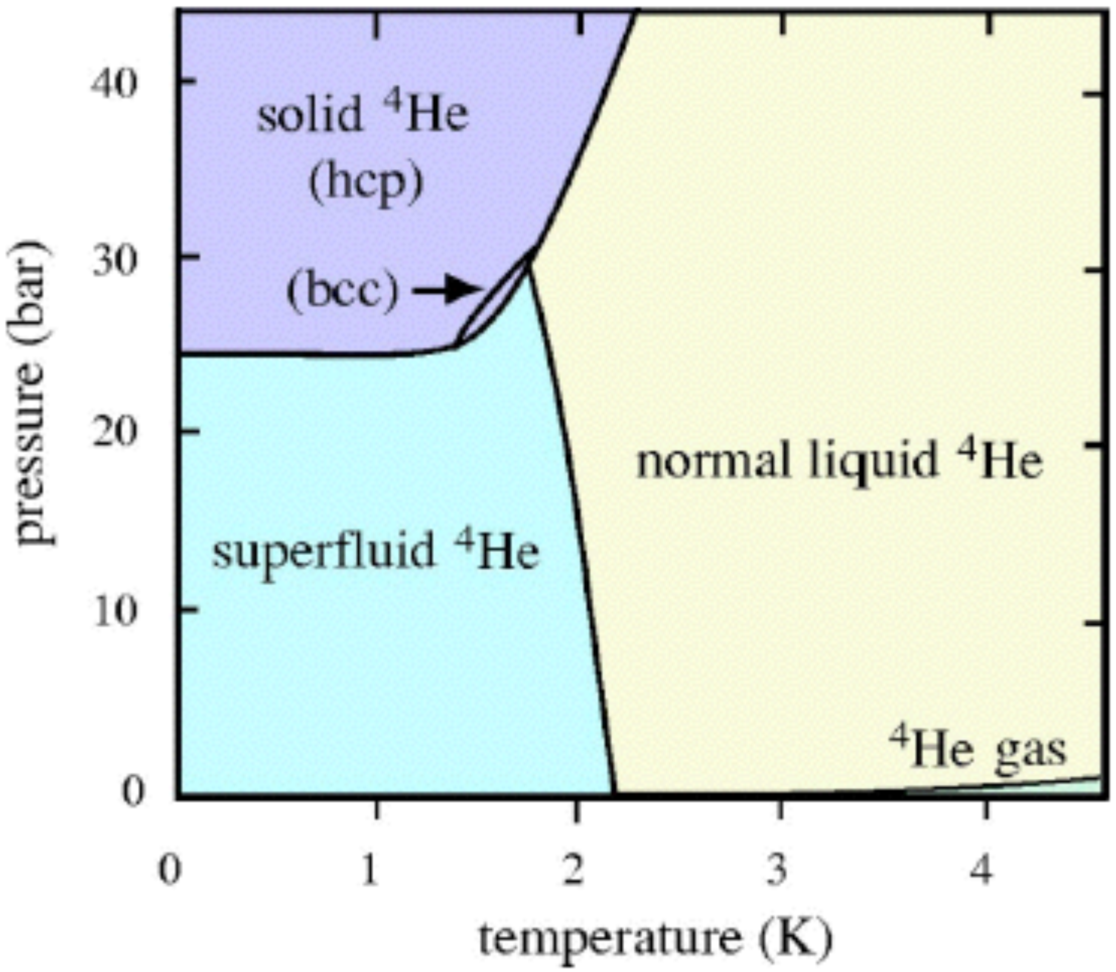
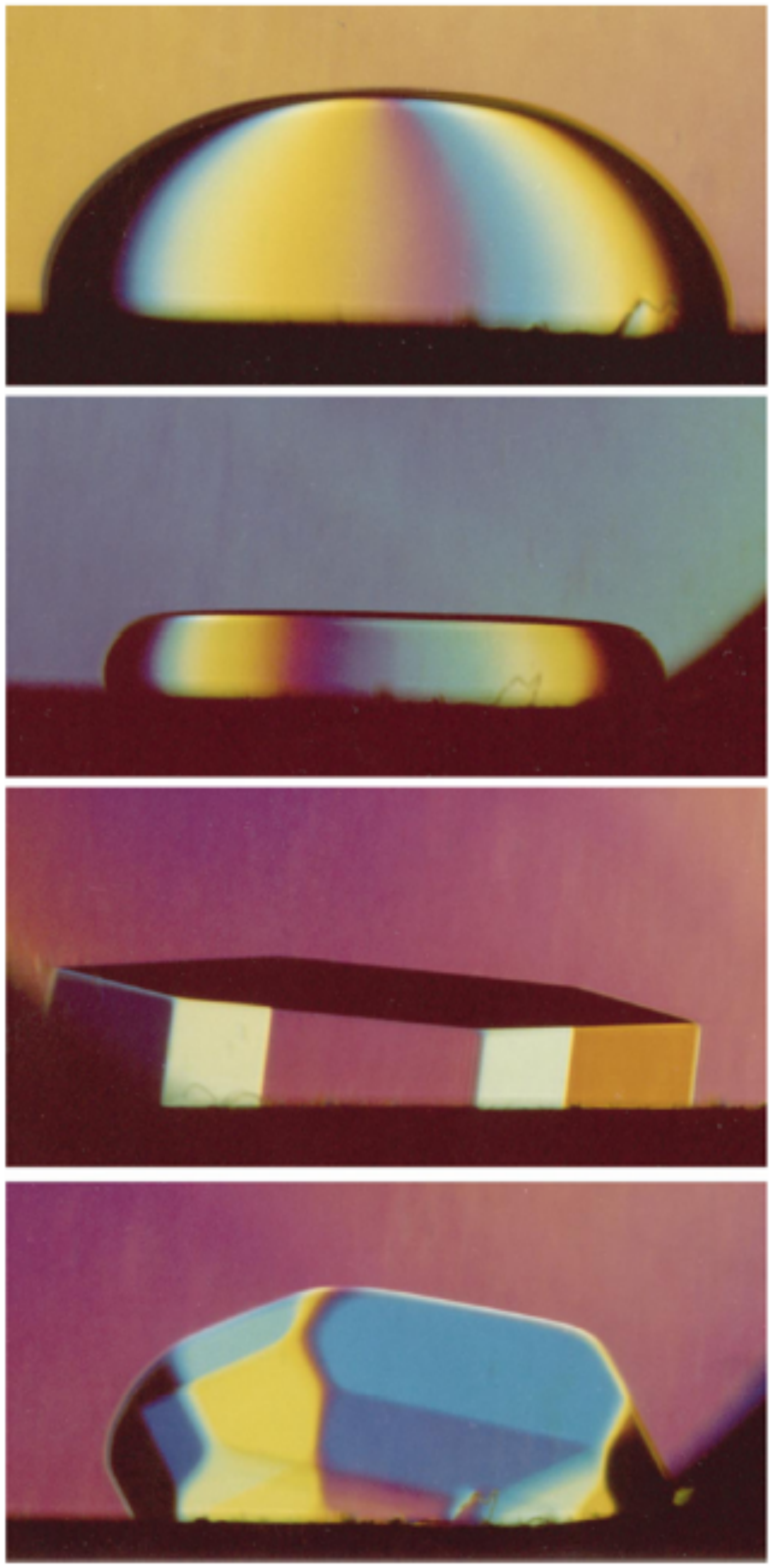


What about the Vapor Surface near the bulk melting point?



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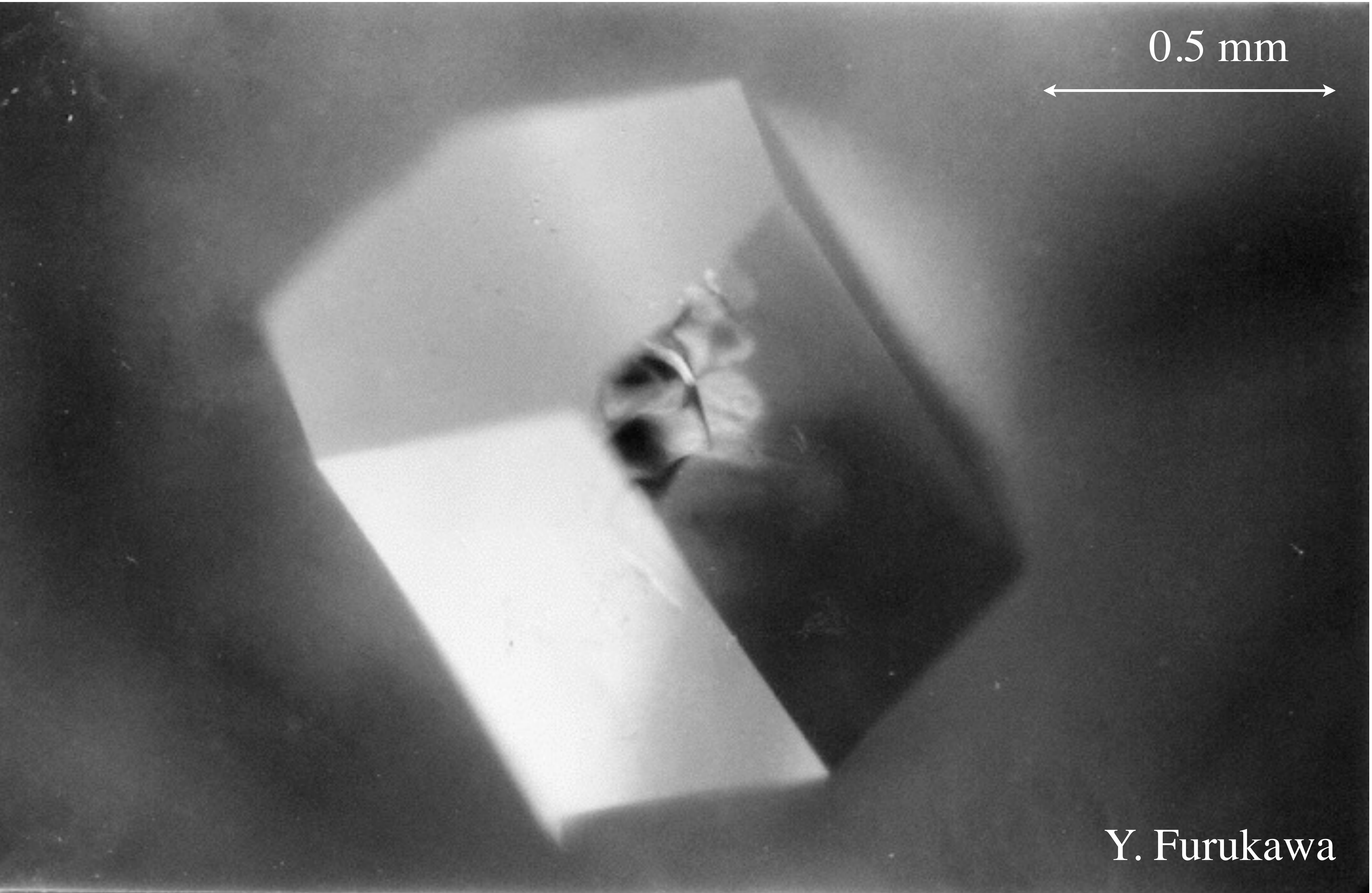
Solid-Vapor Coexistence Exhibits some Differences



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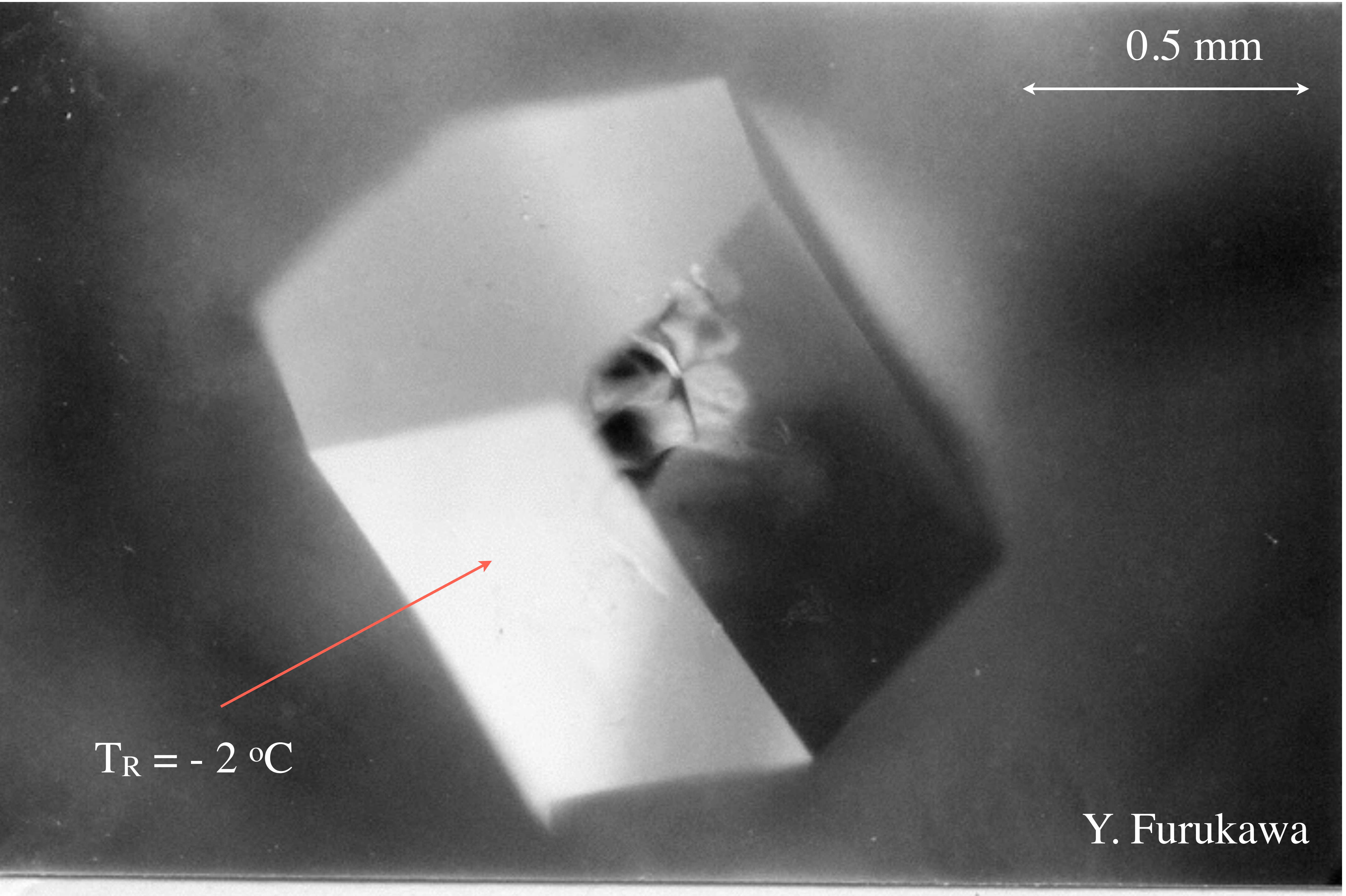

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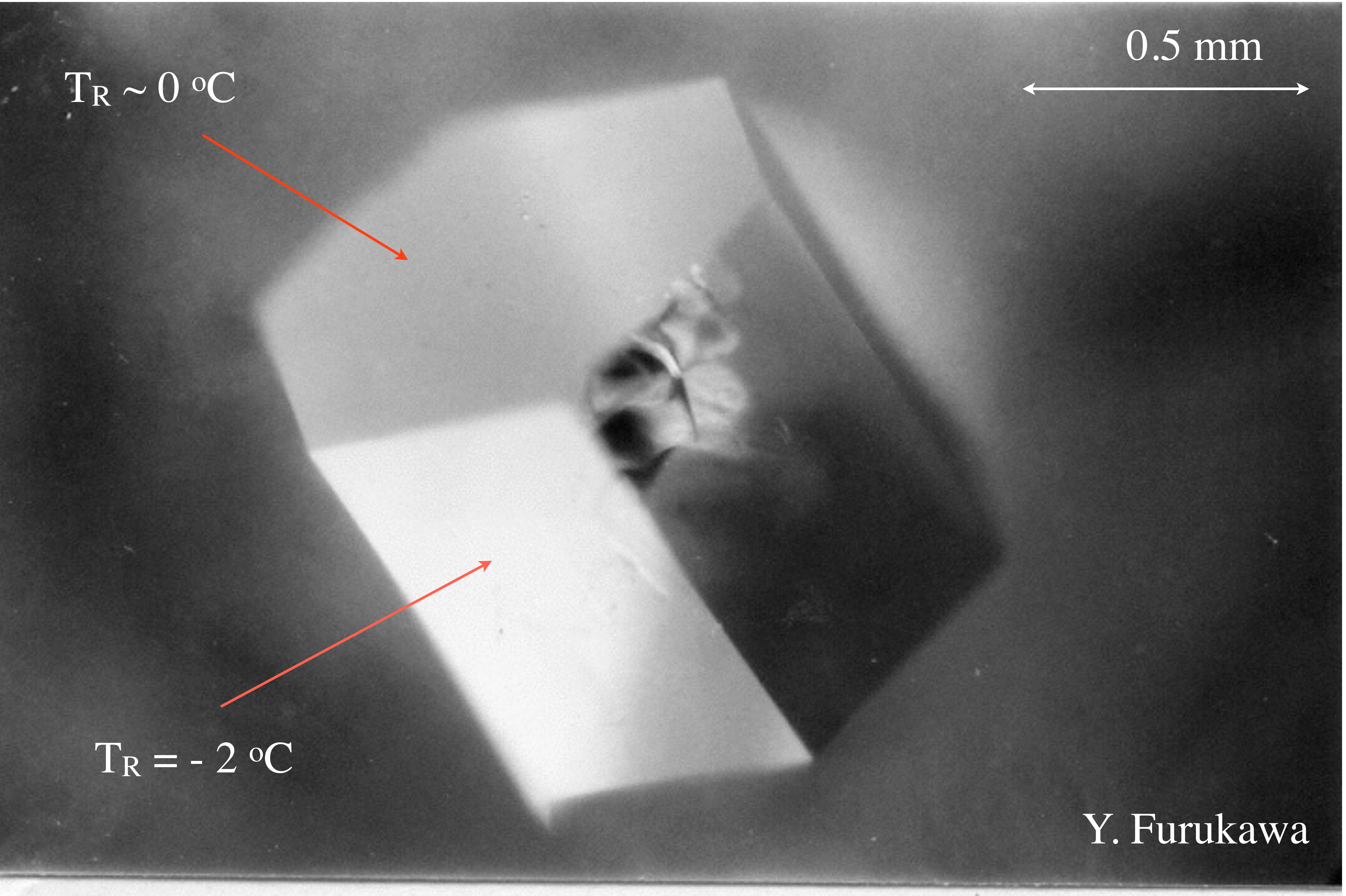

Surface Melting: Intuition

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$$\cos\theta = \frac{\gamma_{sa} - \gamma_{s\ell}}{\gamma_{\ell a}}$$

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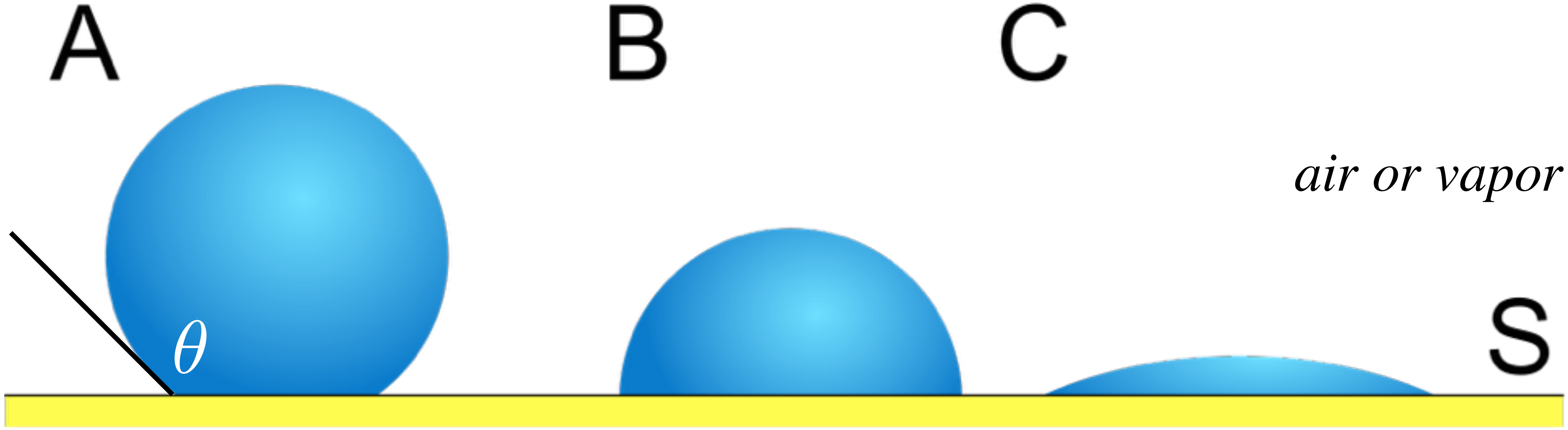
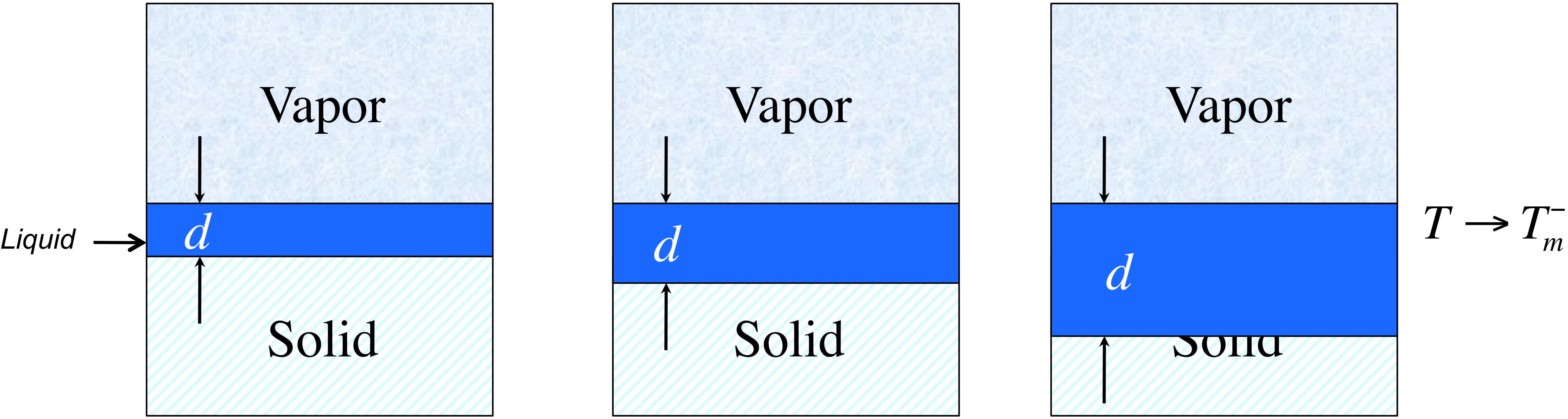
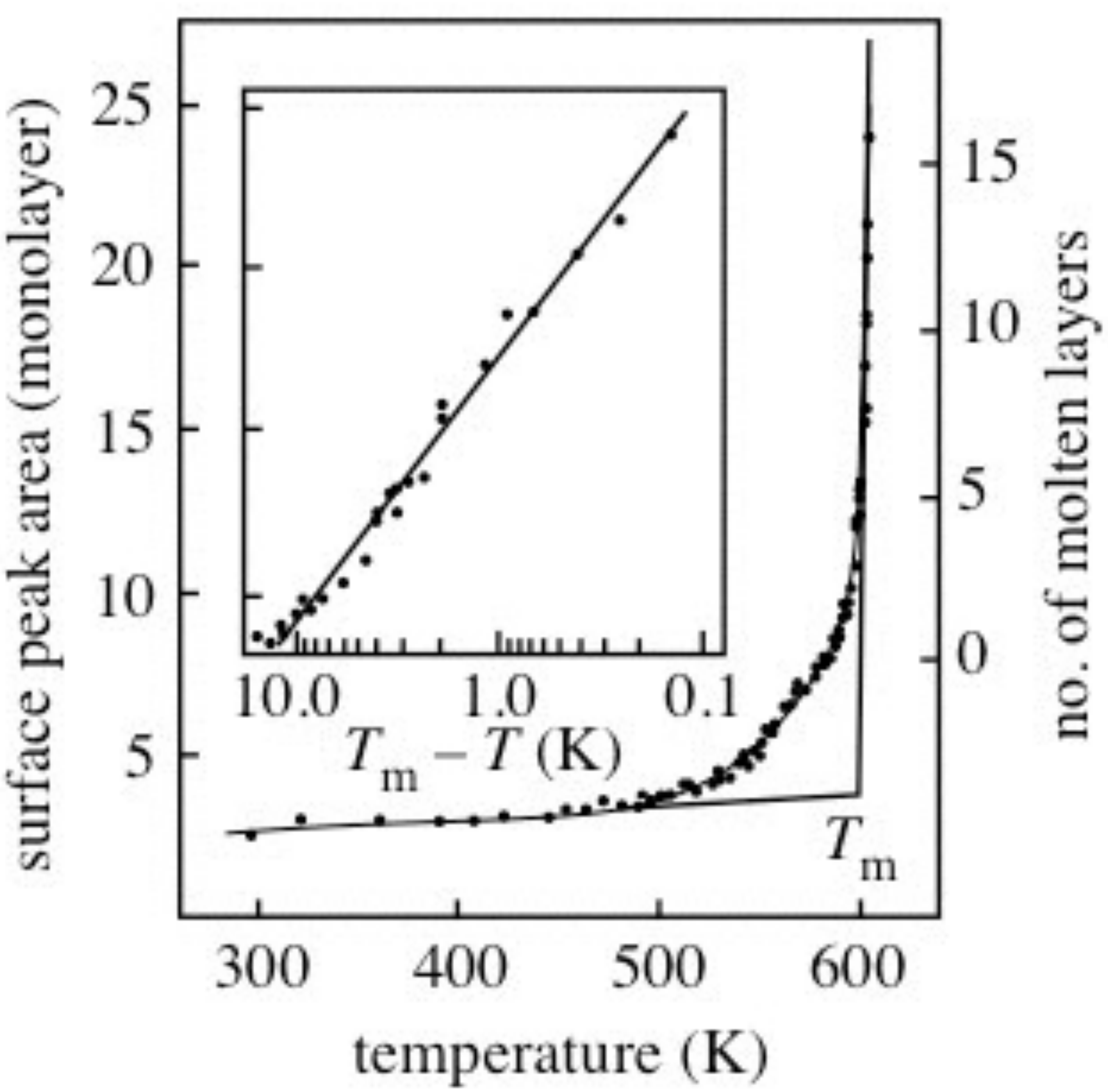
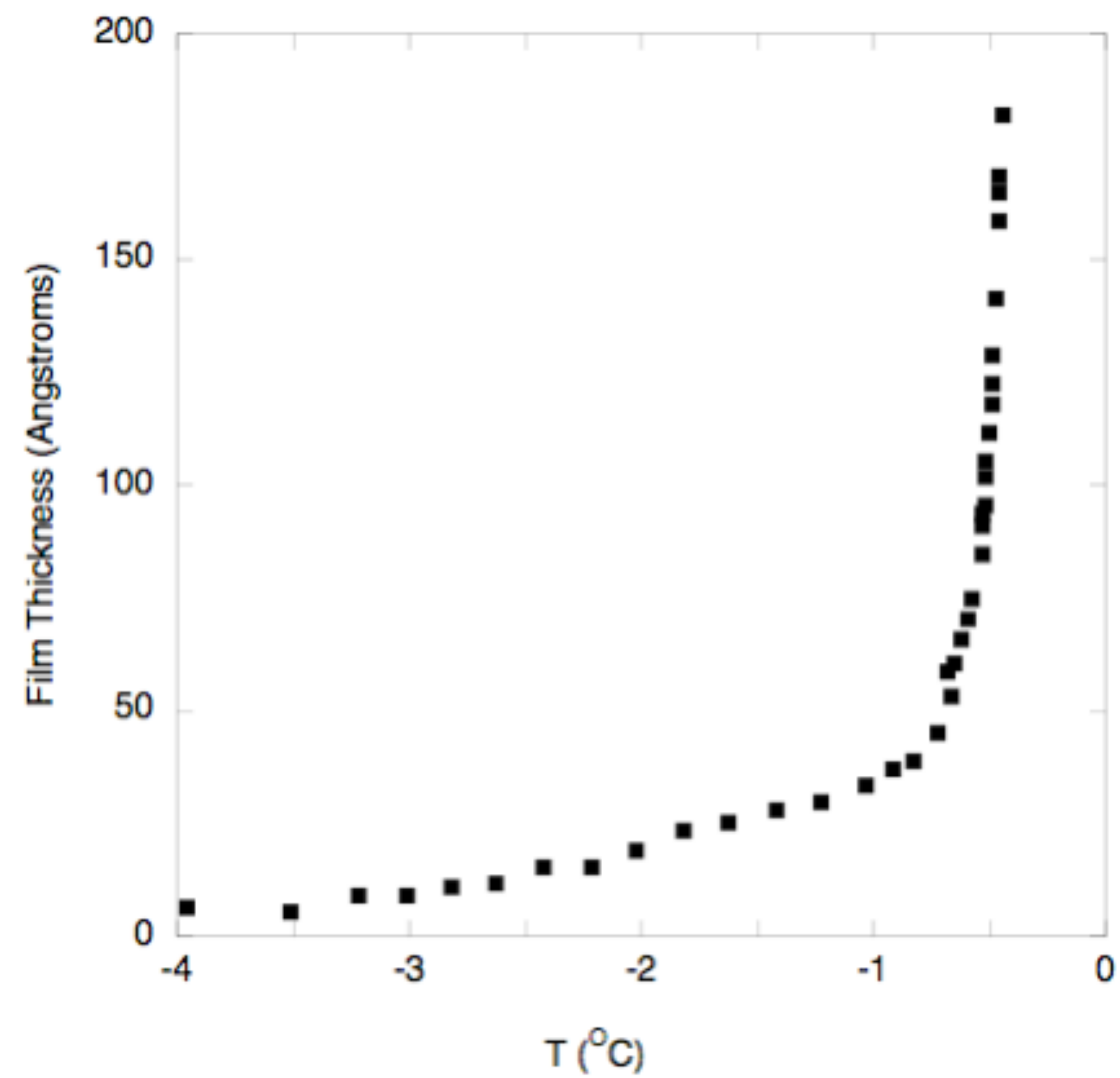


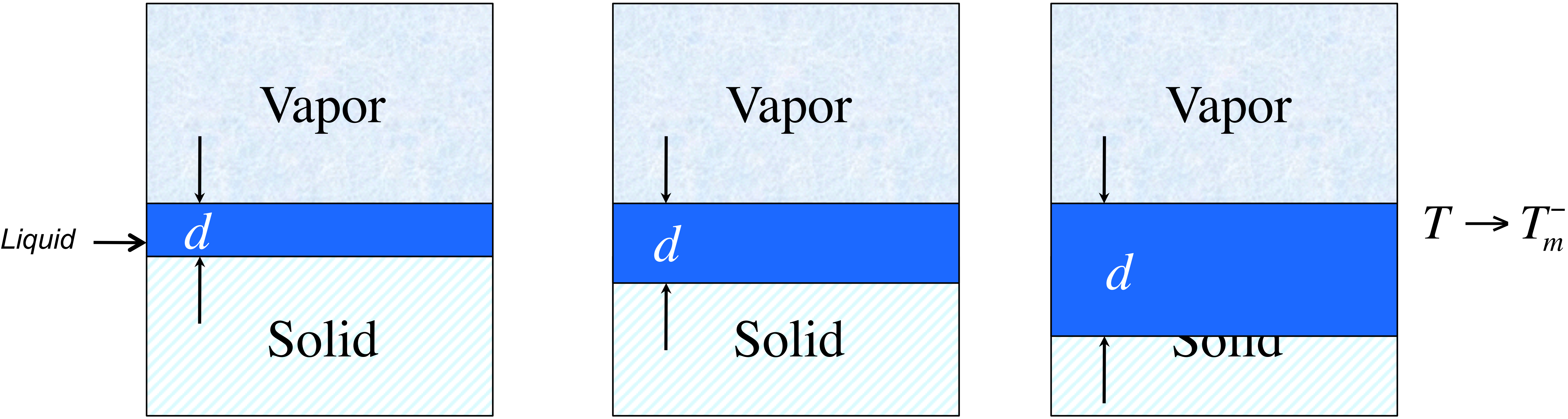


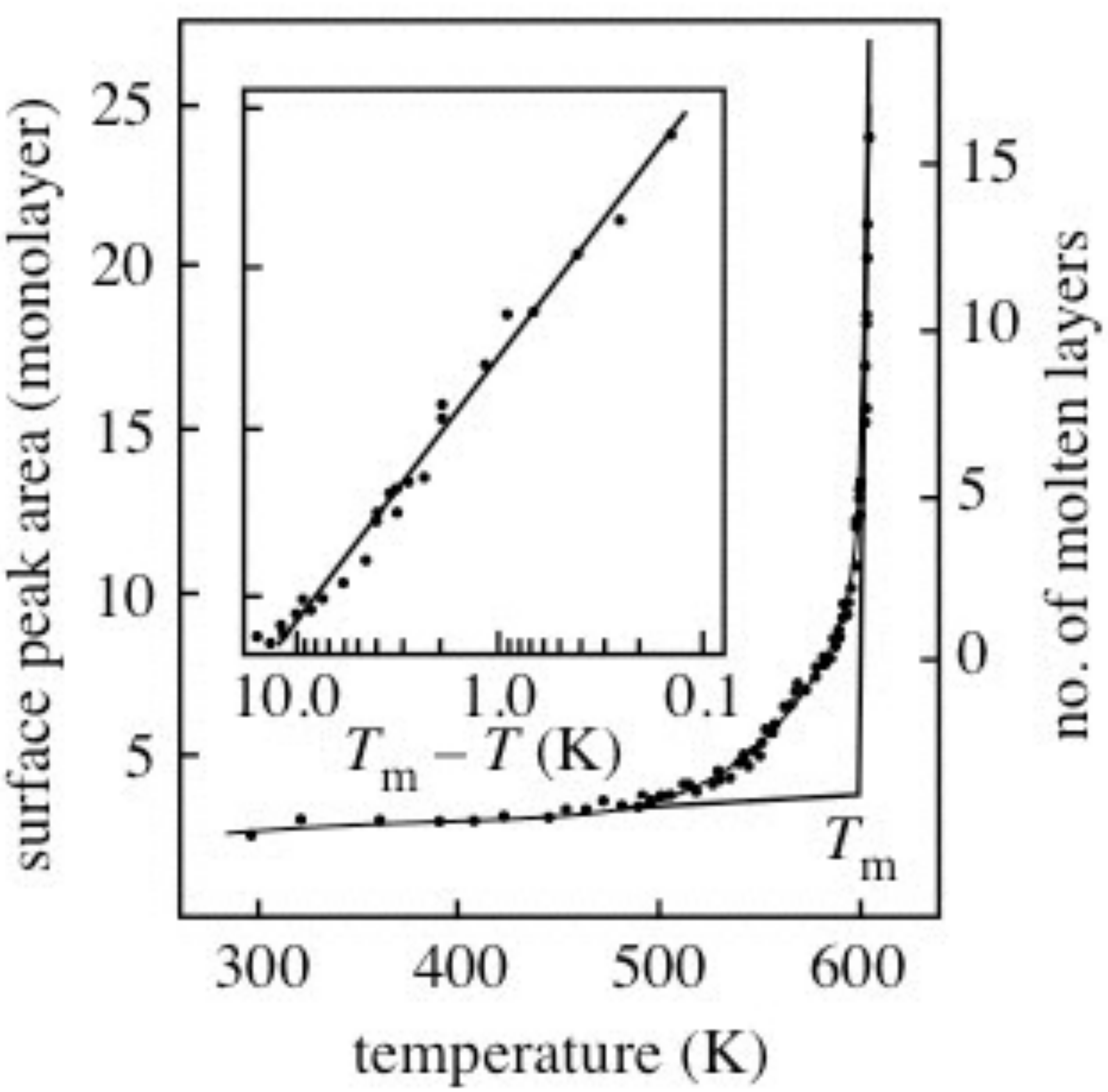


Frenken & van der Veen *Phys Rev Lett* **54**, 134 (1985)



Elbaum, Lipson & Dash *J Cryst Growth* **129**, 491 (1993)





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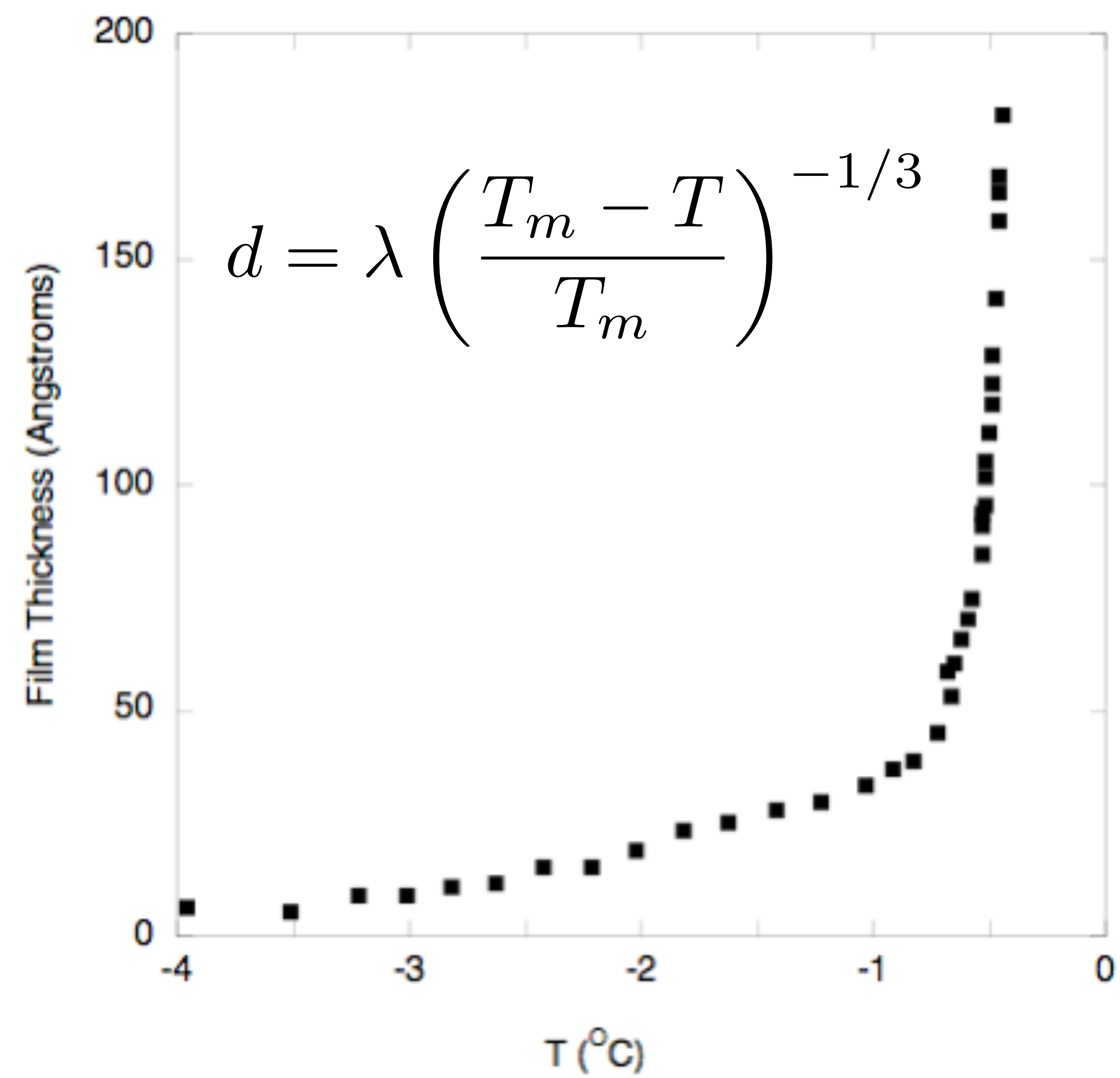

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Interfacial premelting–Warm Up

Equilibrium in an external field; T and μ are constant

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$$d\mu(T,p,z) = \left[\frac{\partial \mu(T,p,z)}{\partial p}\right]_{T,z} dp + \left[\frac{\partial \mu(T,p,z)}{\partial z}\right]_{p,T} dz = 0$$

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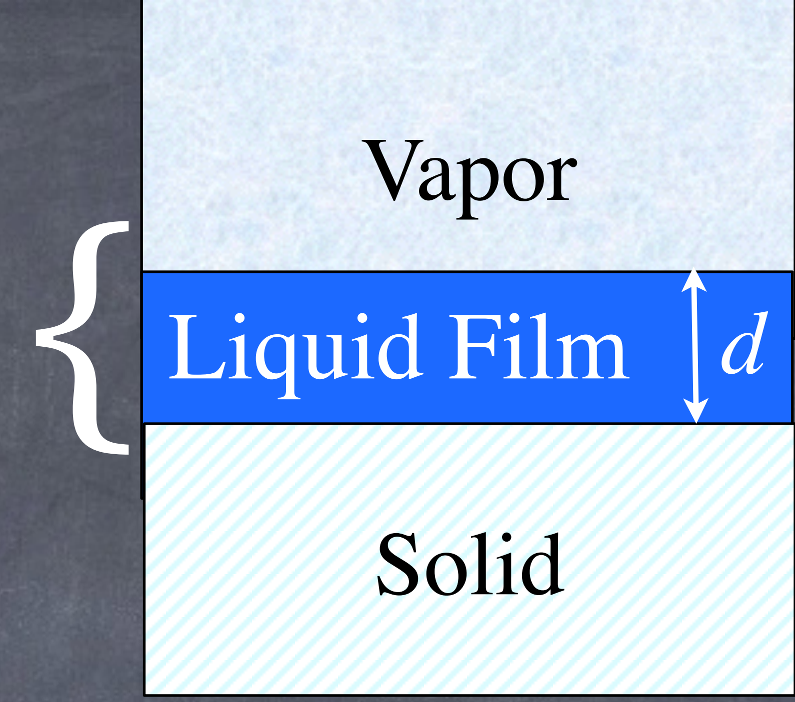
$$\Longrightarrow dp = -\ \frac{m}{v} g dz \equiv -\rho g dz$$

$$\mu_s(T,p) = \mu_\ell(T,p)$$ Bulk Coexistence



$$\mu_f(T,p,d) \equiv \mu_\ell(T,p) + U(d) = \mu_s(T,p)$$

$$\mu_s(T,p) - \mu_\ell(T,p) \equiv \Delta\mu = U(d)$$

$$U(d) = -\,\frac{2|\Delta\gamma|\sigma^2}{\rho_\ell d^3} = -\,\frac{|\mathcal{A}|}{6\pi\rho_\ell d^3}$$

$$\Delta\mu \approx \left[\frac{\partial\Delta\mu}{\partial T}\right]_{T_m,P_m}(T - T_m) + \left[\frac{\partial\Delta\mu}{\partial P}\right]_{T_m,P_m}(P - P_m) + \text{h.o.t.}$$

$$\Delta\mu \approx -\,q_m\frac{T_m - T}{T_m} \equiv -\,q_m t \qquad \Longrightarrow \boxed{d \;\propto t^{-1/3}}$$

The Truth…for Solid/Liquid/X

$$U(d) = -\frac{|\mathcal{A}|}{6\pi\rho_\ell d^3} \qquad \text{where} \qquad \mathcal{A} \equiv \lim_{d\to 0}\left[-12\pi d^2 I(d)\right]$$

$$I(d) = \frac{kT}{8\pi d^2}{\sum_{n=0}^{\infty}}' \int_{r_n}^{\infty} dx\, x \left\{ \ell n \left[1 - \frac{(x-x_i)(x-x_s)}{(x+x_i)(x+x_s)} e^{-\ x}\right] + \ell n \left[1 - \frac{(\epsilon_s x - \epsilon_w x_s)(\epsilon_i x - \epsilon_w x_i)}{(\epsilon_s x + \epsilon_w x_s)(\epsilon_i x + \epsilon_w x_i)} e^{-\ x}\right]\right\}$$

$$x_j = \left[x^2 - {r_n}^2\left(1 - \frac{\epsilon_j}{\epsilon_w}\right)\right]^{1/2} \qquad (j = i, s)$$

$$r_n = 2d(\epsilon_\ell)^{1/2}\xi_n/c$$

$\epsilon(i\xi)$ required in the integral is evaluated at $i\xi_n = i(2\pi kT/\hbar)n$ and obtained by analytic continuation of the material dielectric function $\epsilon(\omega)$ to imaginary frequencies using

$$\epsilon(\omega) = 1 + \sum_j \frac{f_j}{e_j^2 - i\hbar\omega g_j - (\hbar\omega)^2}$$

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Casimir-Polder…Long Story in and of itself

…Some More Surface Physics

Disequilibrium Transitions

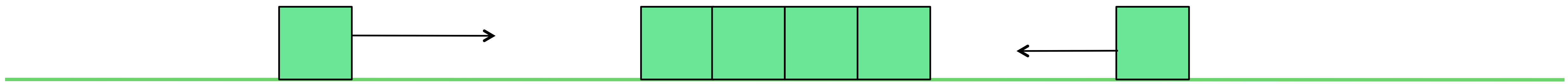
(1) Kinetic Roughening

(2) Damage Assisted Surface Melting

Growth & kinetic roughening

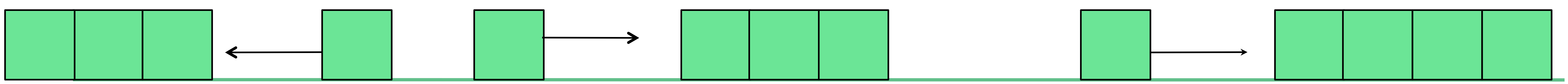
$$\frac{\Delta\mu}{k_b T} \ll 1$$

Faceted and Slow



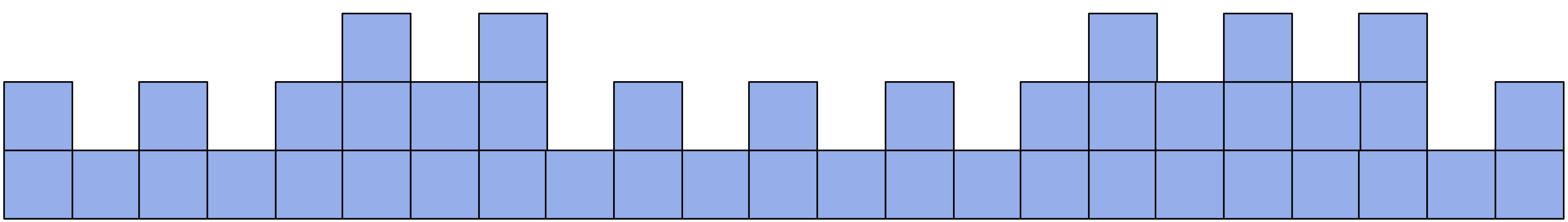
$$\frac{\Delta\mu}{k_b T} < 1$$

Faceted and Slow

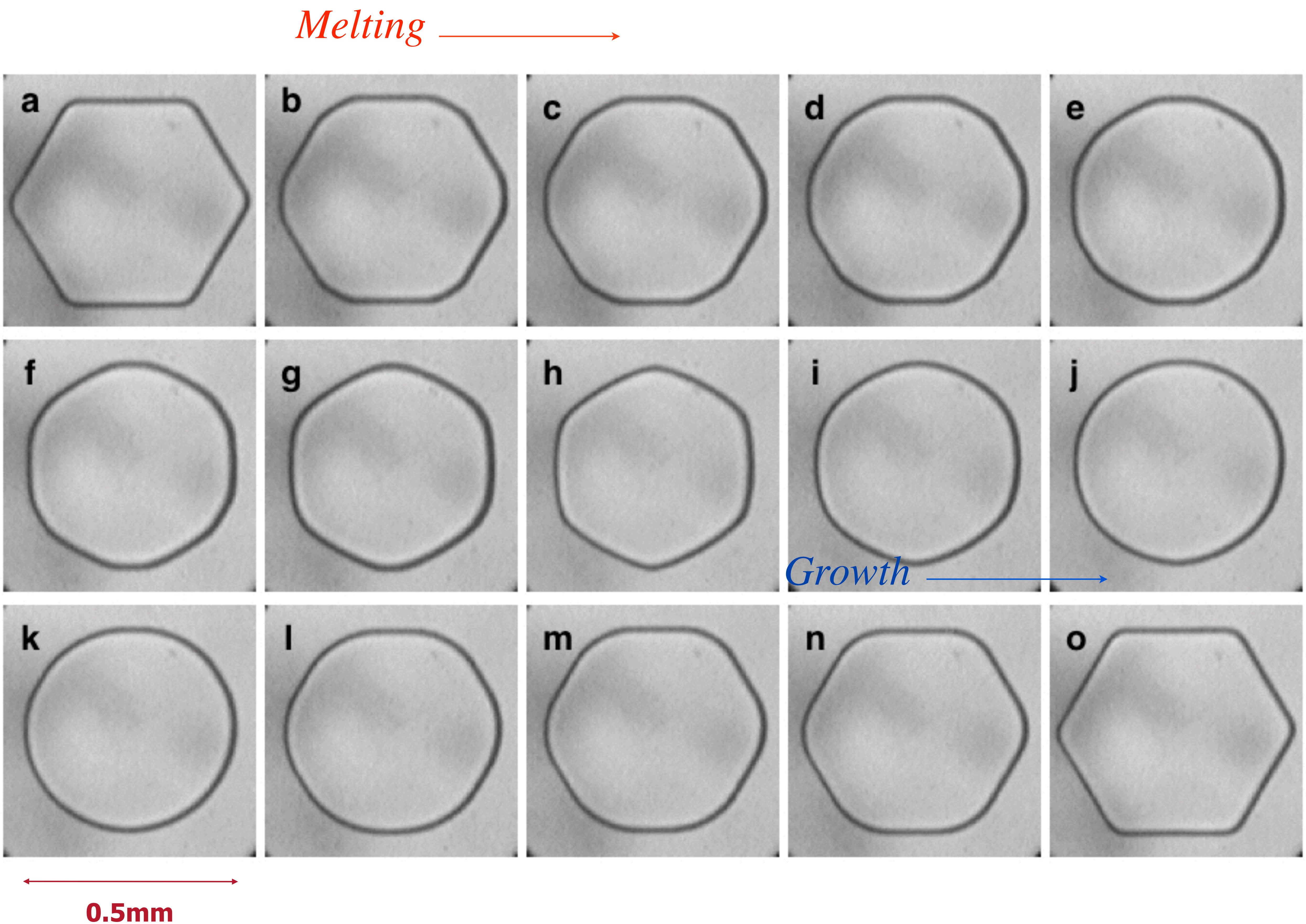


$$\frac{\Delta\mu}{k_b T} > \left[\frac{\Delta\mu}{k_b T}\right]_c$$

Rough & Fast



Ice In Water (T = -22 ^{o}C, P = 2000 bar)



A. Cahoon, M. Maruyama, and W. *Phys. Rev. Lett.* **96** 255502 (2006).

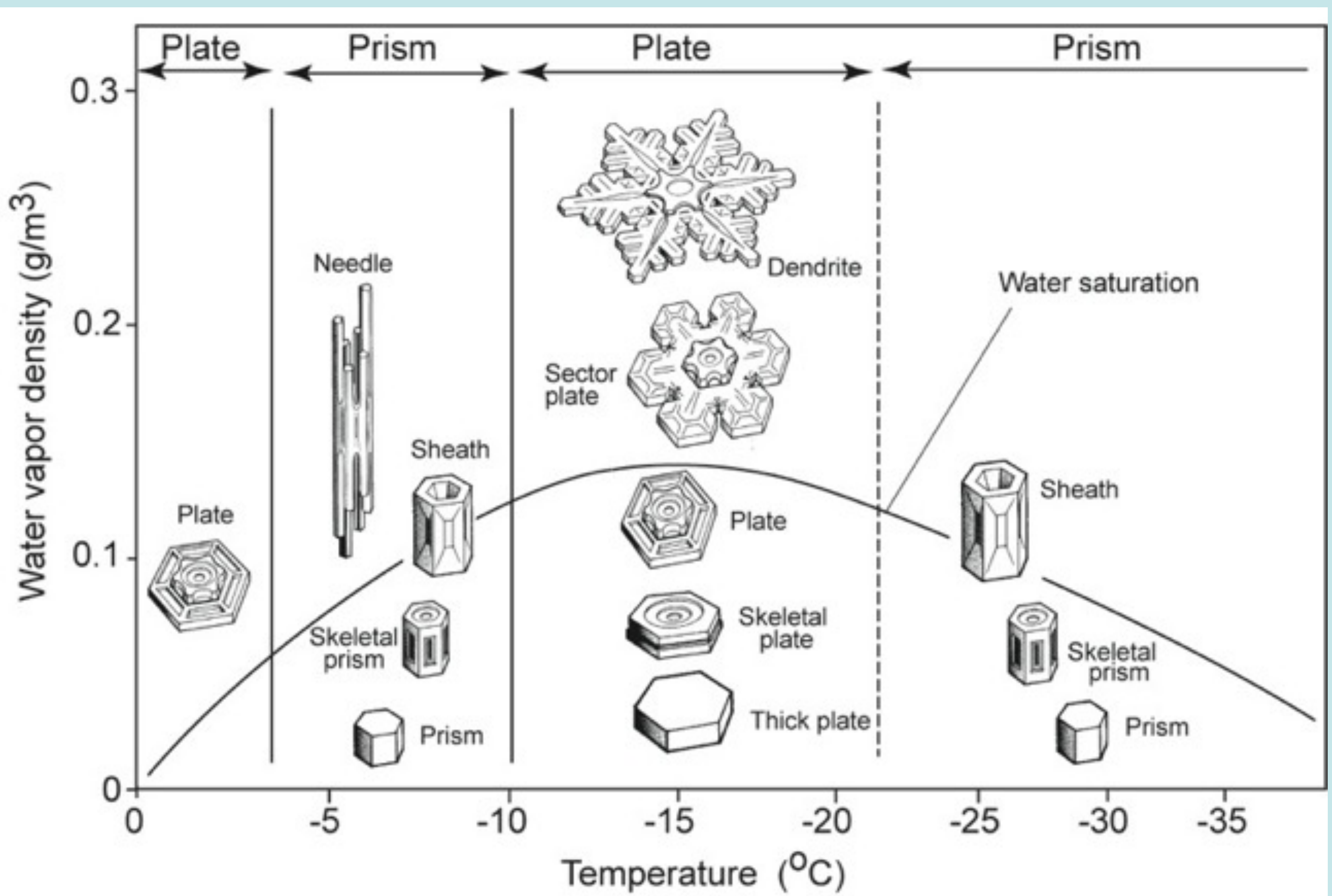
Physics Today

quick study

Snow and ice crystals

Yoshinori Furukawa and John S. Wettlaufer

Highly dendritic snowflakes are an aesthetic source of wonder, but the greater challenge for physicists lies in understanding the "simpler" prismatic and planar forms.



Letters from the Sky

What happens in a high speed collision?

Damage Assisted Melting: Some fraction of the collision energy breaks bonds and lowers the chemical potential of the liquid phase

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$$|\mu_{\mathcal{I}}(d)| + |\mu_{\mathcal{D}}(V_c)| = q_m \left(\frac{T_m - T}{T_m}\right) + \left(\frac{\rho_\ell - \rho_s}{\rho_\ell \rho_s}\right)(P_m - P)$$

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$$d = \left[\frac{|\mathcal{A}_H|}{\rho_\ell \frac{q_m}{T_m}(T_m - T) + \left(\frac{\rho_\ell - \rho_s}{\rho_s}\right)(P_m - P) - u_\mathcal{D}}\right]^{1/3}$$

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$$u_{\mathcal{D}} \equiv \rho_\ell |\mu_{\mathcal{D}}(V_c)|$$

So What has this got to do with Astro-anything?

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Every solid is finite and hence has a surface

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- These phenomena define and hence control material behavior

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- Every solid is finite and hence has a surface
- These phenomena define and hence control material behavior
- Examples abound…protoplanetary discs are filled with solids…

Collisions and Cosmogony

Phil Armitage, JILA

Collisions and Cosmogony

Phil Armitage, JILA

Collisions and Cosmogony

Phil Armitage, JILA

Collapse ~ 0.1 Myr

Collisions and Cosmogony

Phil Armitage, JILA

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Accretion & Primary Planetesimals Form ~ 1 Myr

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Phil Armitage, JILA

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$\therefore$ Need to make planets fast!

The Disc Gas Dynamics is Sub-Keplerian

$$\frac{v_\phi^2}{r} = \frac{GM_*}{r^2} + \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}$$

Estimating

$$\begin{aligned}\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} &\sim -\frac{1}{\rho}\frac{P}{r} \\ &\sim -\frac{1}{\rho}\frac{\rho c_s^2}{r} \\ &\sim -\frac{GM_*}{r^2}\left(\frac{h}{r}\right)^2 \equiv -\frac{{v_k}^2}{r}\left(\frac{h}{r}\right)^2\end{aligned}$$

$$h = c_s/\Omega$$

$$\Rightarrow \quad \underline{v_\phi^2 = v_K^2\left[1 - \mathcal{O}\left(\frac{h}{r}\right)^2\right]}$$

With $P = P_0\left(\frac{r}{r_0}\right)^{-n} \Rightarrow v_\phi = v_K(1-\eta)^{1/2}$ **with** $\eta = n\frac{c_s^2}{v_K^2}.$

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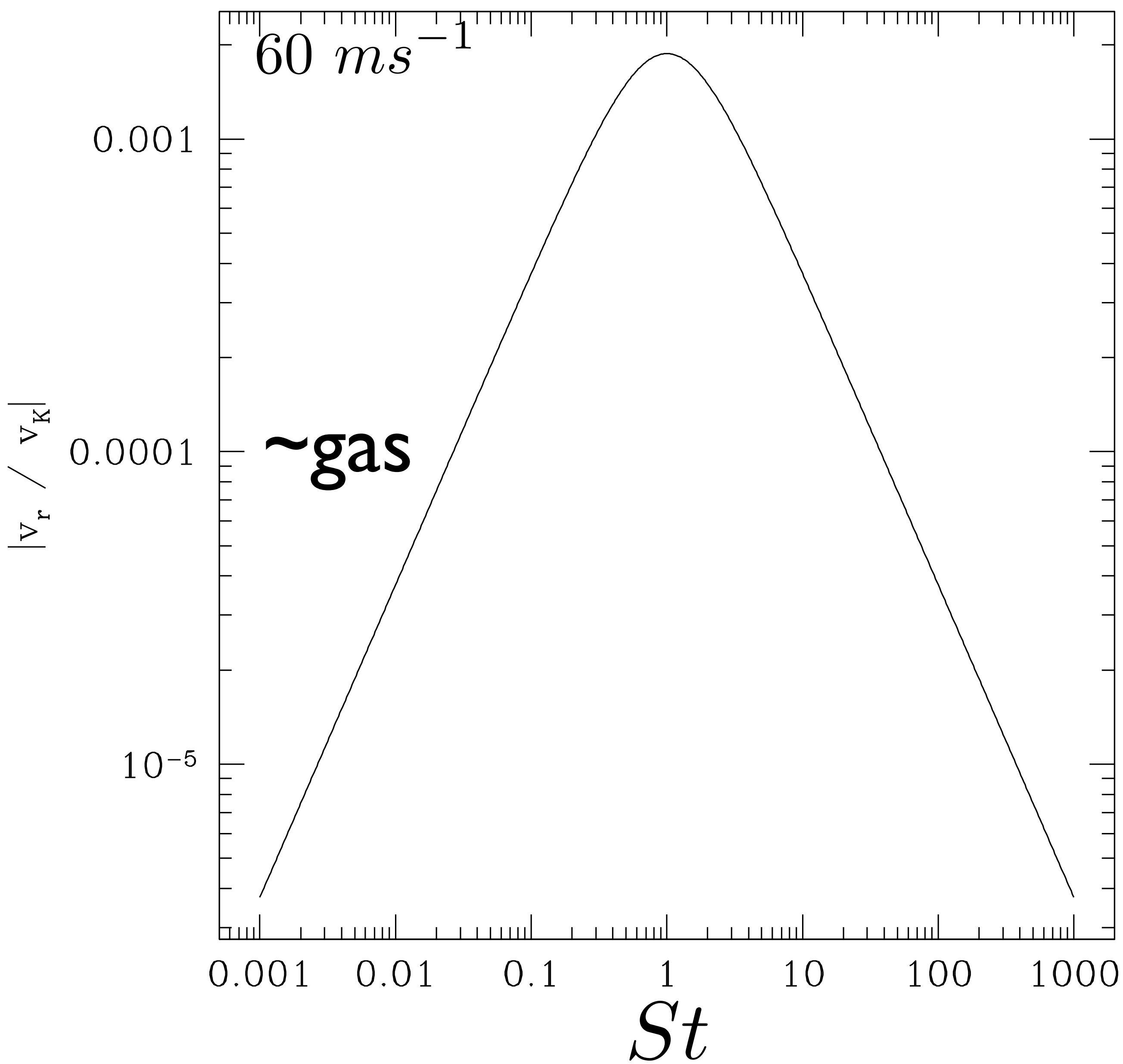
Particle Motion in the Disk:

$$\Rightarrow\Rightarrow \quad \boxed{\frac{v_r}{v_K} = \frac{-\ \eta}{St + St^{-1}}} \qquad St \equiv t_{\rm fric}\Omega_K$$



E.g., Papaloizou & Terquem, *Rep Prog Phys* **69,** 119 (2006)

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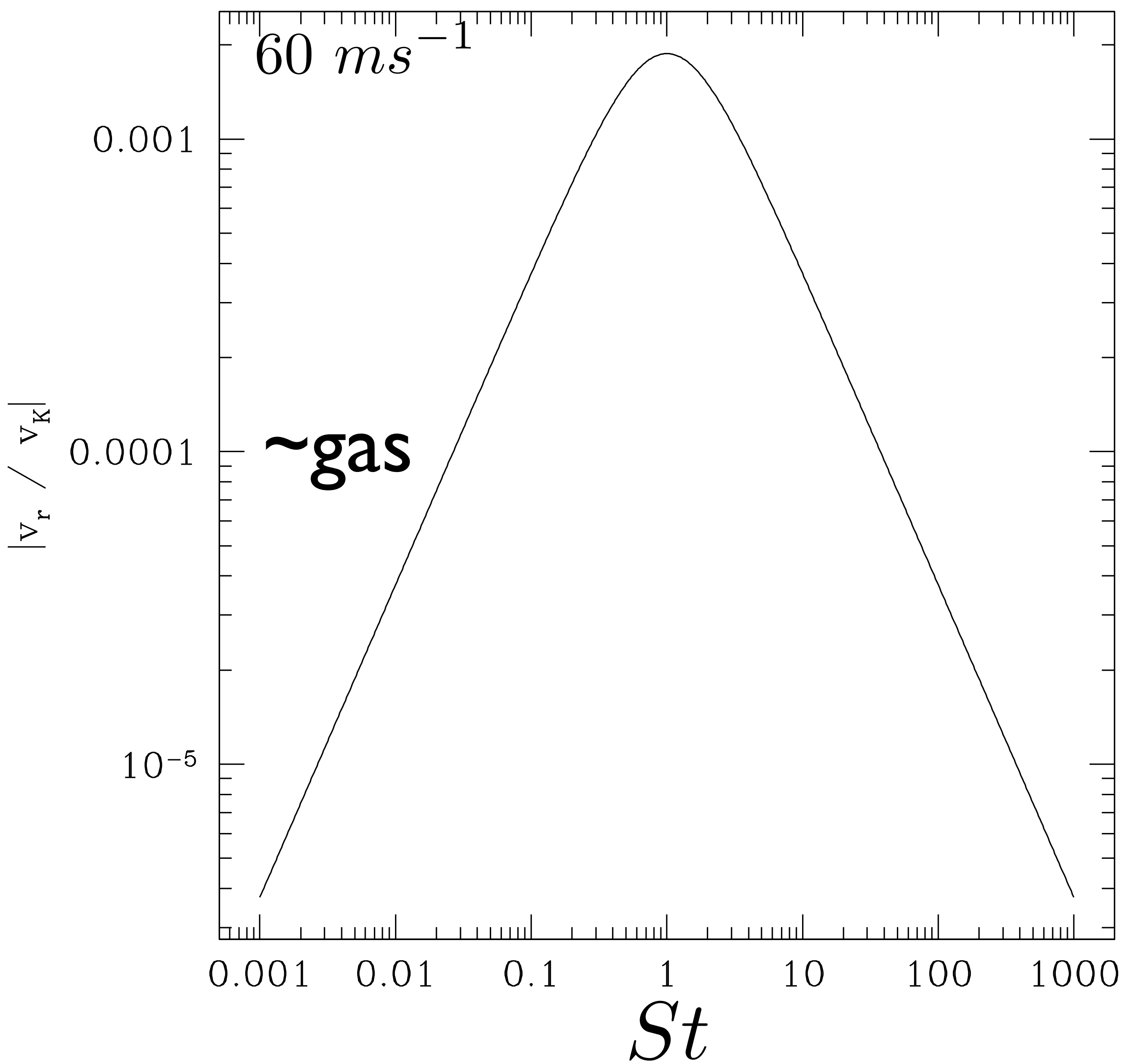

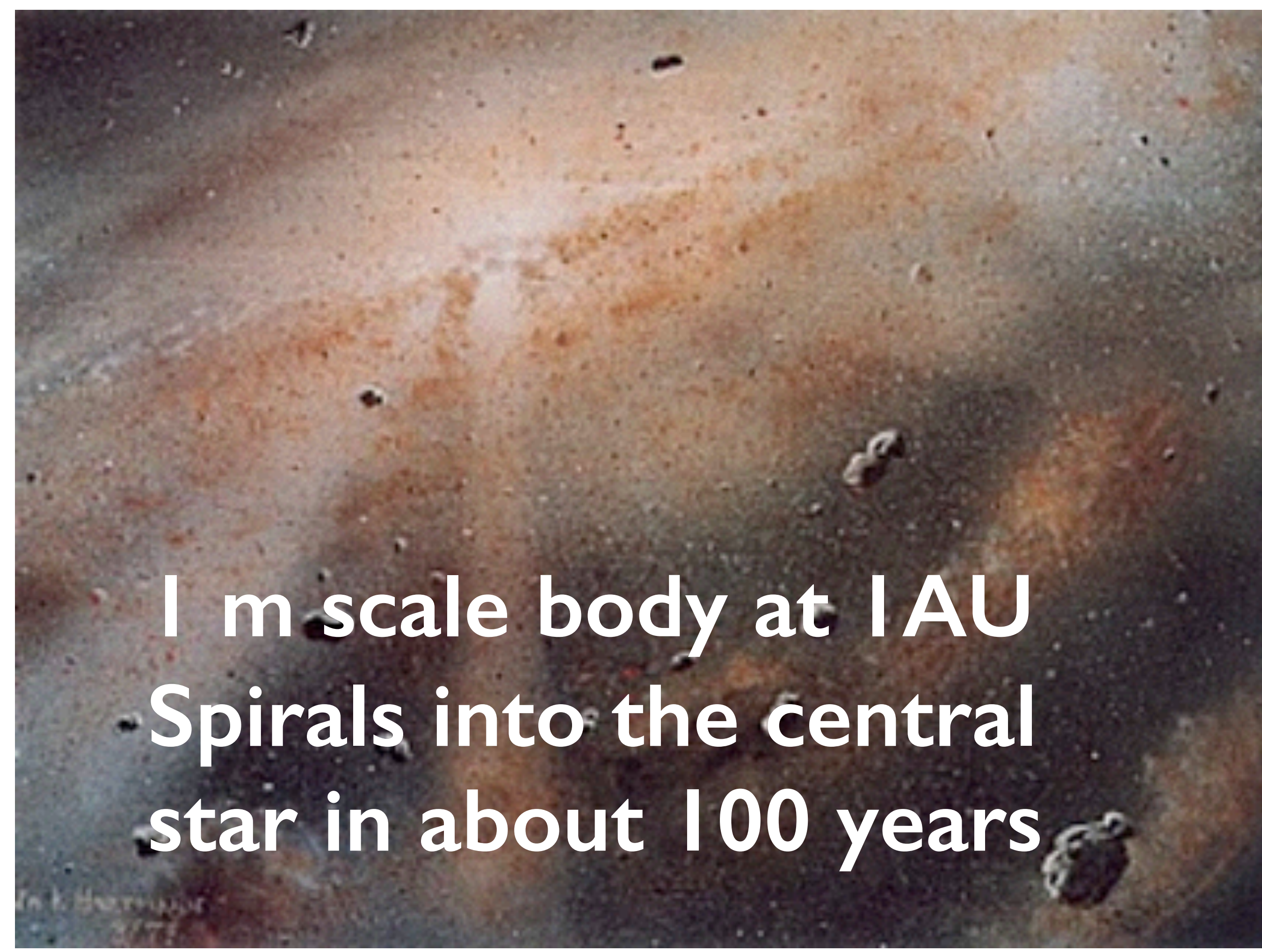

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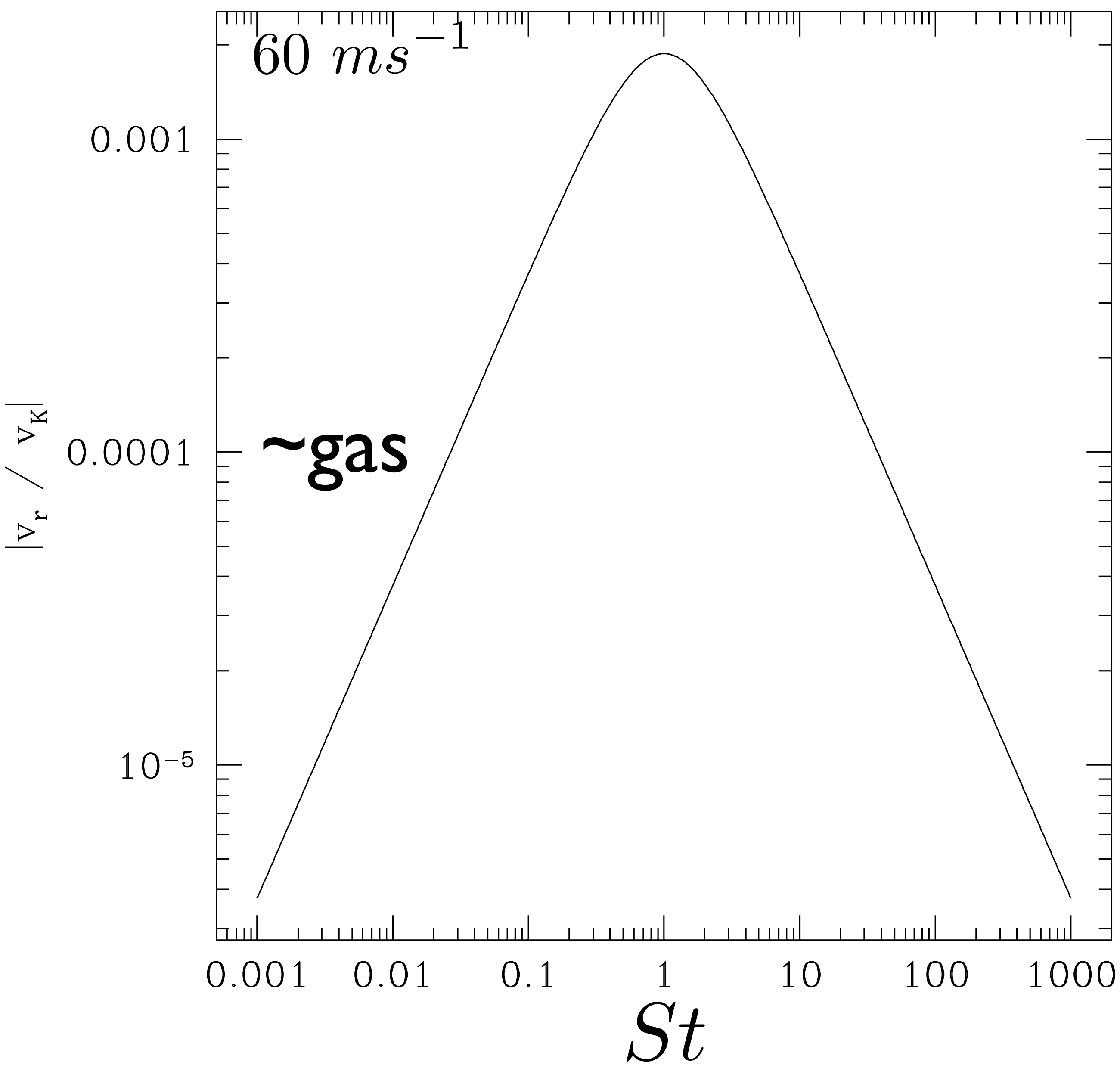

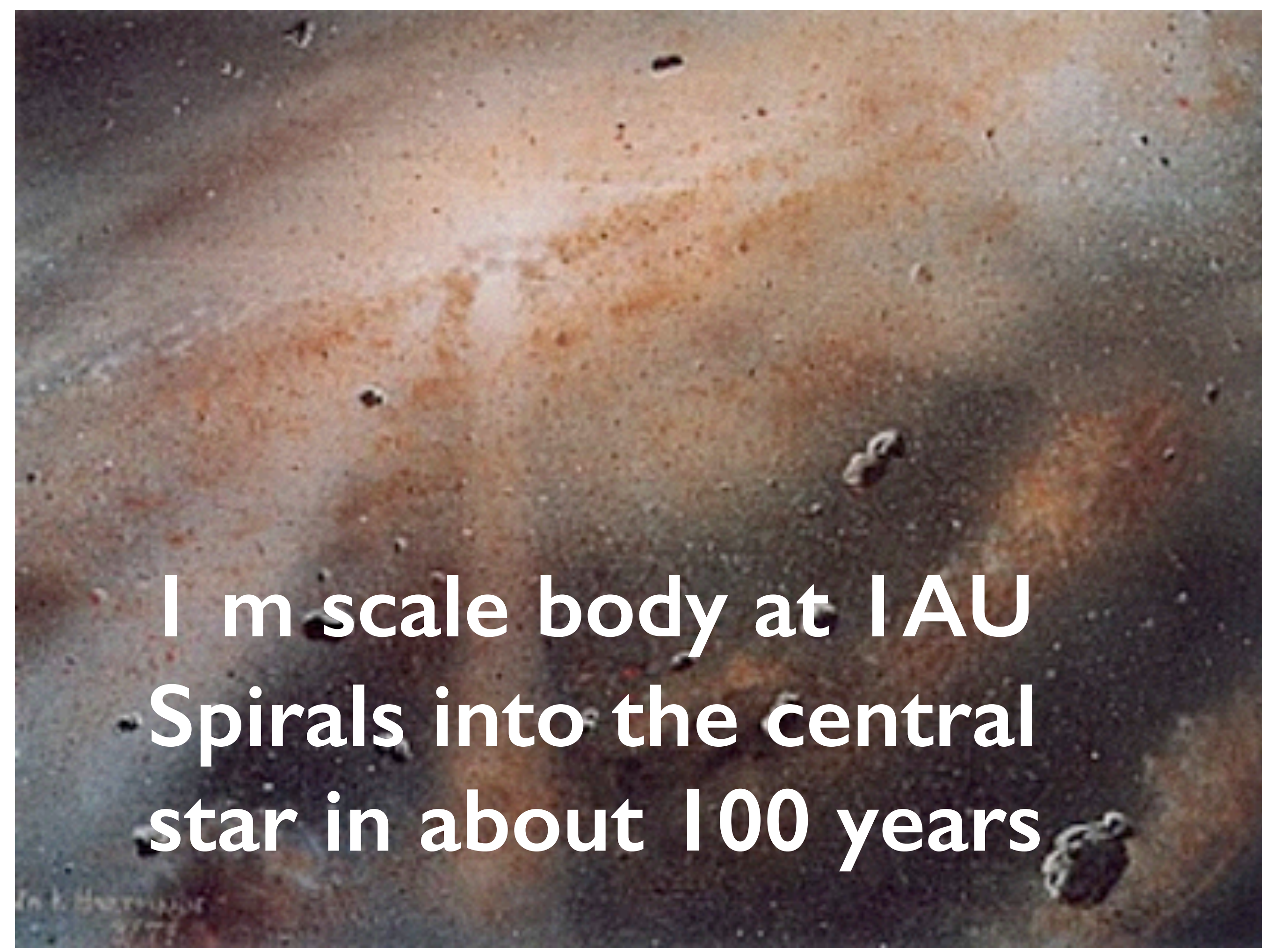

"1m *Bottleneck* for Accretion"

E.g., Papaloizou & Terquem, *Rep Prog Phys* **69**, 119 (2006)

WHAT HAPPENS IN A HIGH SPEED CENTRAL COLLISION?

Sticking together–Collisional Fusion

What happens in a high speed central collision?

Damage: Very near the divergent limit

$$d = \frac{\xi U_c}{\pi {r_c}^2 \left[\rho_\ell \frac{q_m}{T_m}(T_m - T) + \left(\frac{\rho_\ell - \rho_s}{\rho_s}\right)(P_m - P)\right]}$$

$$U_c = \frac{1}{2} M {V_c}^2 \qquad M = m_1 m_2/(m_1 + m_2) \qquad u_{\mathcal{D}} = \frac{\xi U_c}{\pi {r_c}^2 d}$$

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- Phase Diagram for High P, Low T
- Constraint on fraction of damage ξ
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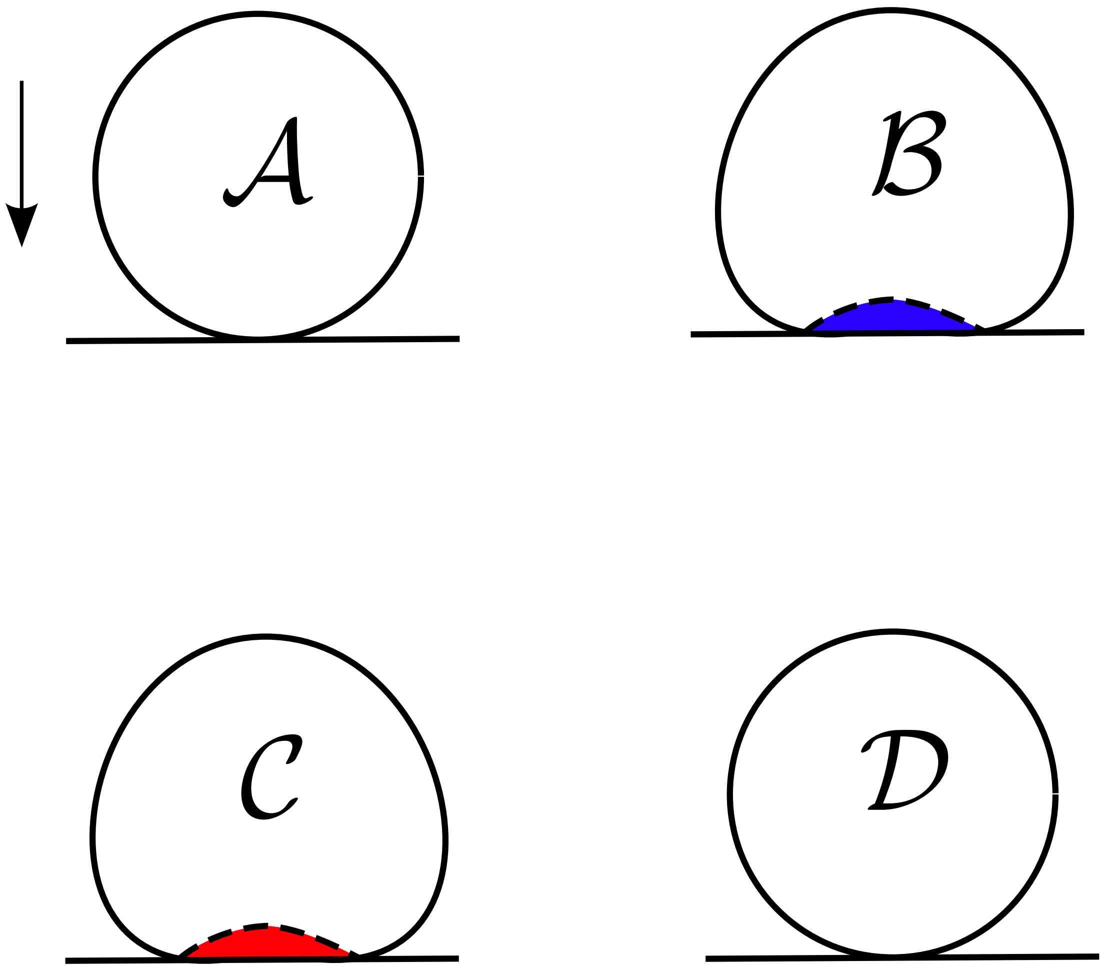
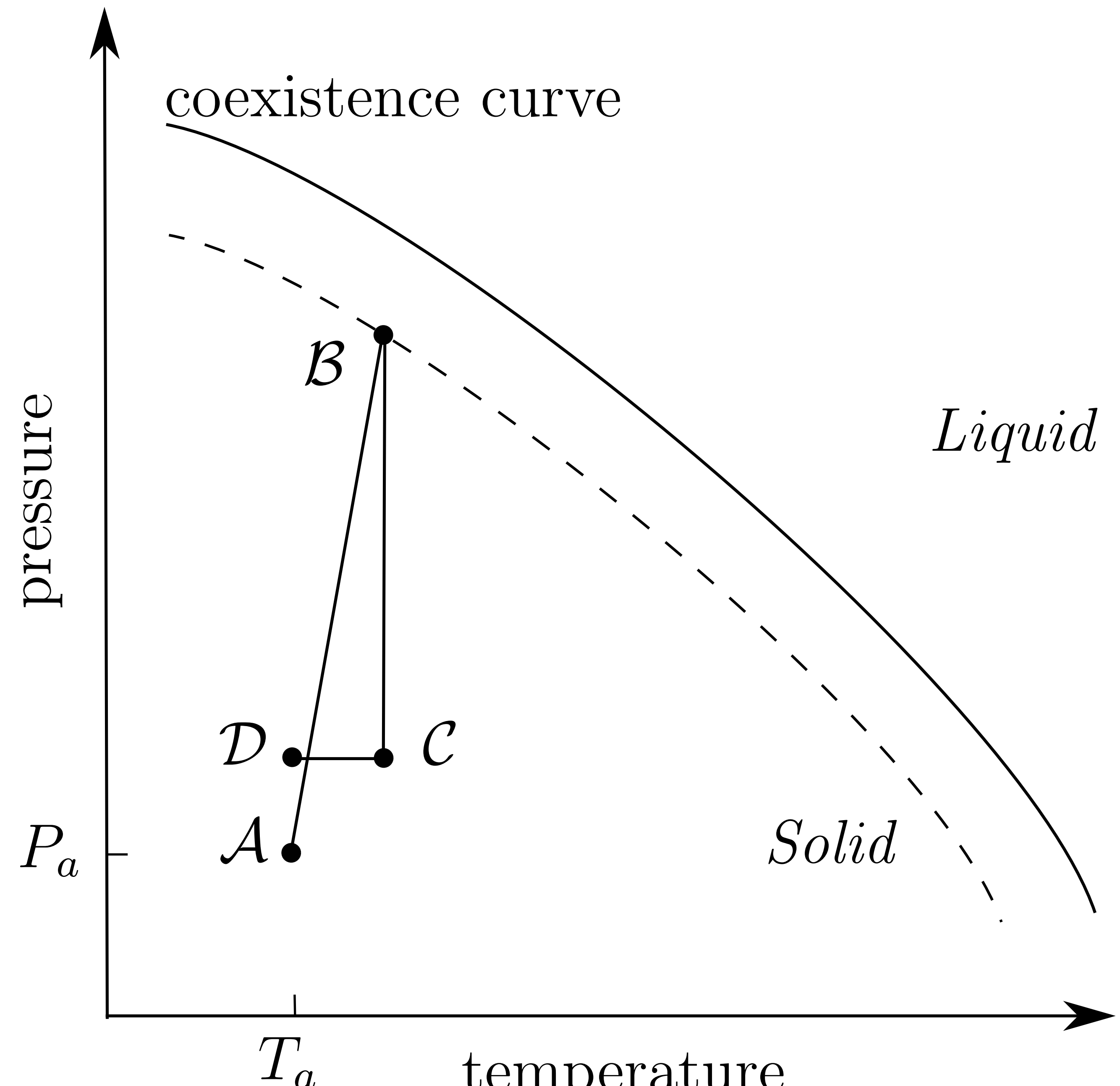
Ingredients:

- Phase Diagram for High P, Low T
- Constraint on fraction of damage ξ
- Collision Dynamics; bouncing/freezing
- Nebular Setting & Drifter growth

Combining it all...



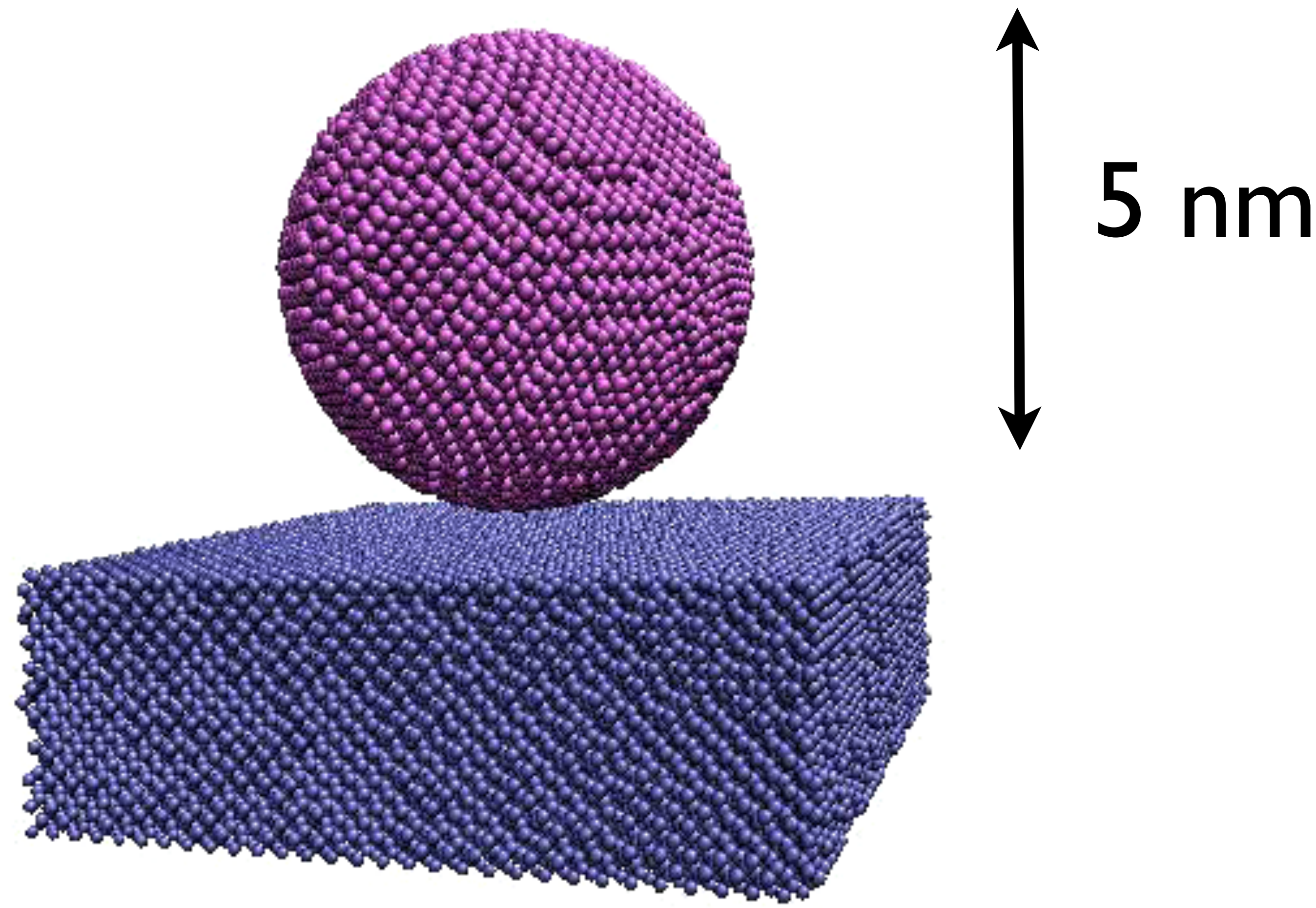


Hertz Beyond Belief

Andong He & JSW

Soft Matter, **10**, 2264 (2014)

T_m = 1694K Inner "Solar System"

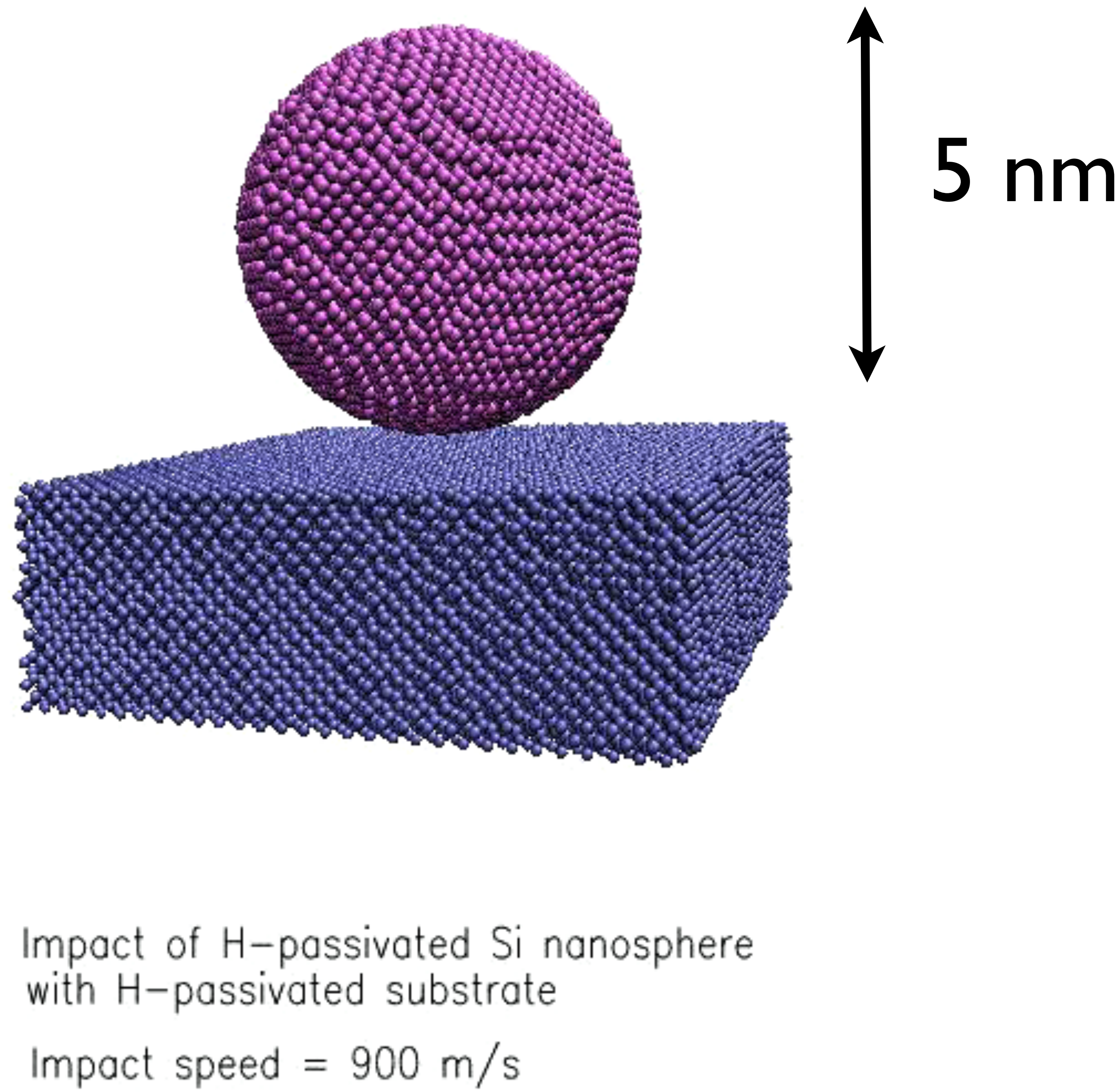


Impact of H−passivated Si nanosphere
with H−passivated substrate

Impact speed = 900 m/s

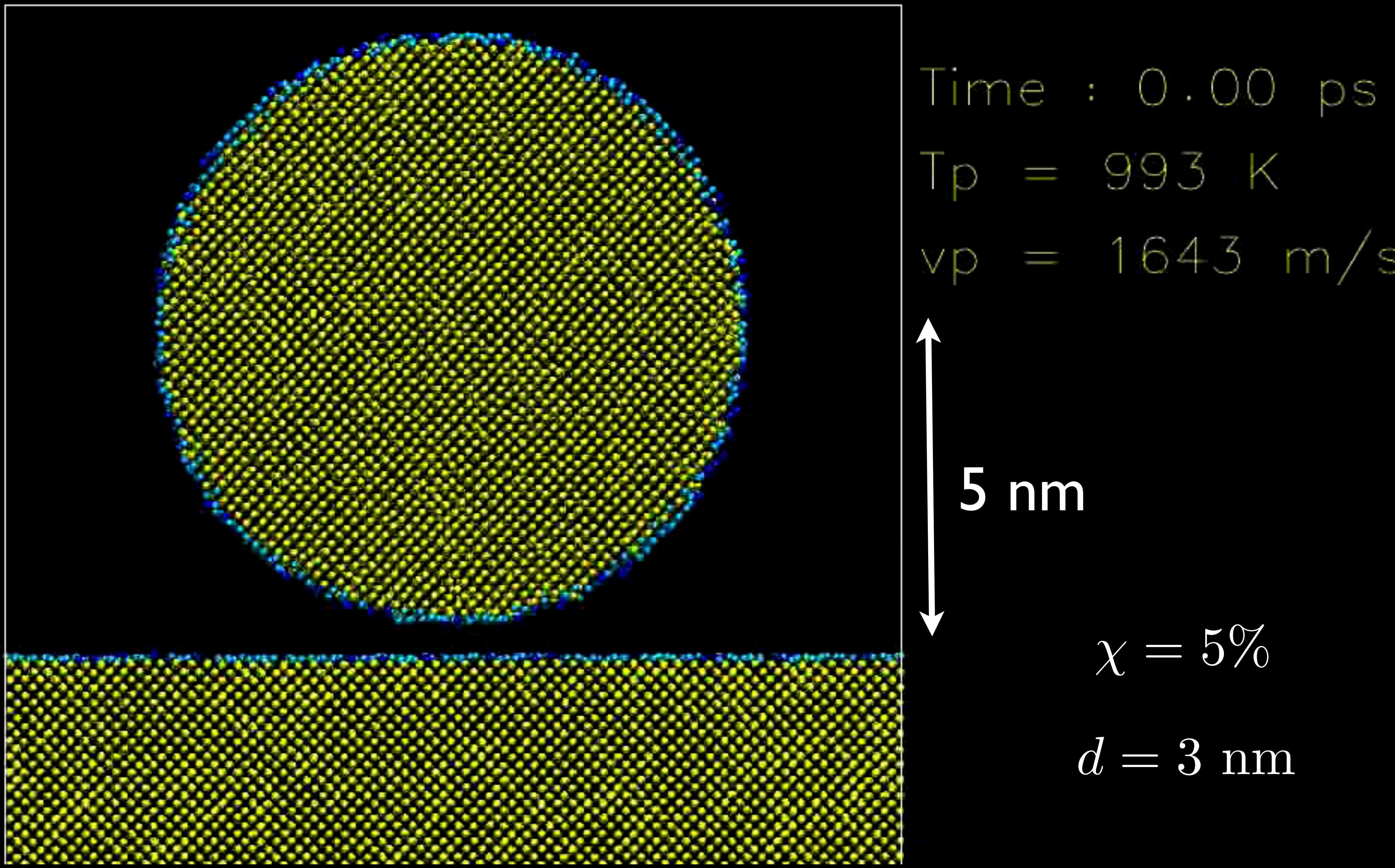
Nikiforov, Suri, Dumitrică, *Phys. Rev. B* **78**, 081405(R) (2008)

T_m = 1694K   Inner "Solar System"



Nikiforov, Suri, Dumitrică, *Phys. Rev. B* **78**, 081405(R) (2008)

Impact of a 5-nm radius Si nanoparticle, equilibrated at 1000 K and travelling at 1640 m/s, onto a crystalline Si substrate at 800 K.



$\chi = 5\%$

$d = 3\ \mathrm{nm}$

Suri and Dumitrică, *Phys. Rev. B* **78**, 081405(R) (2008); Valentini and Dumitrică, *Phys. Rev. B* **75**, 224106 (2007)

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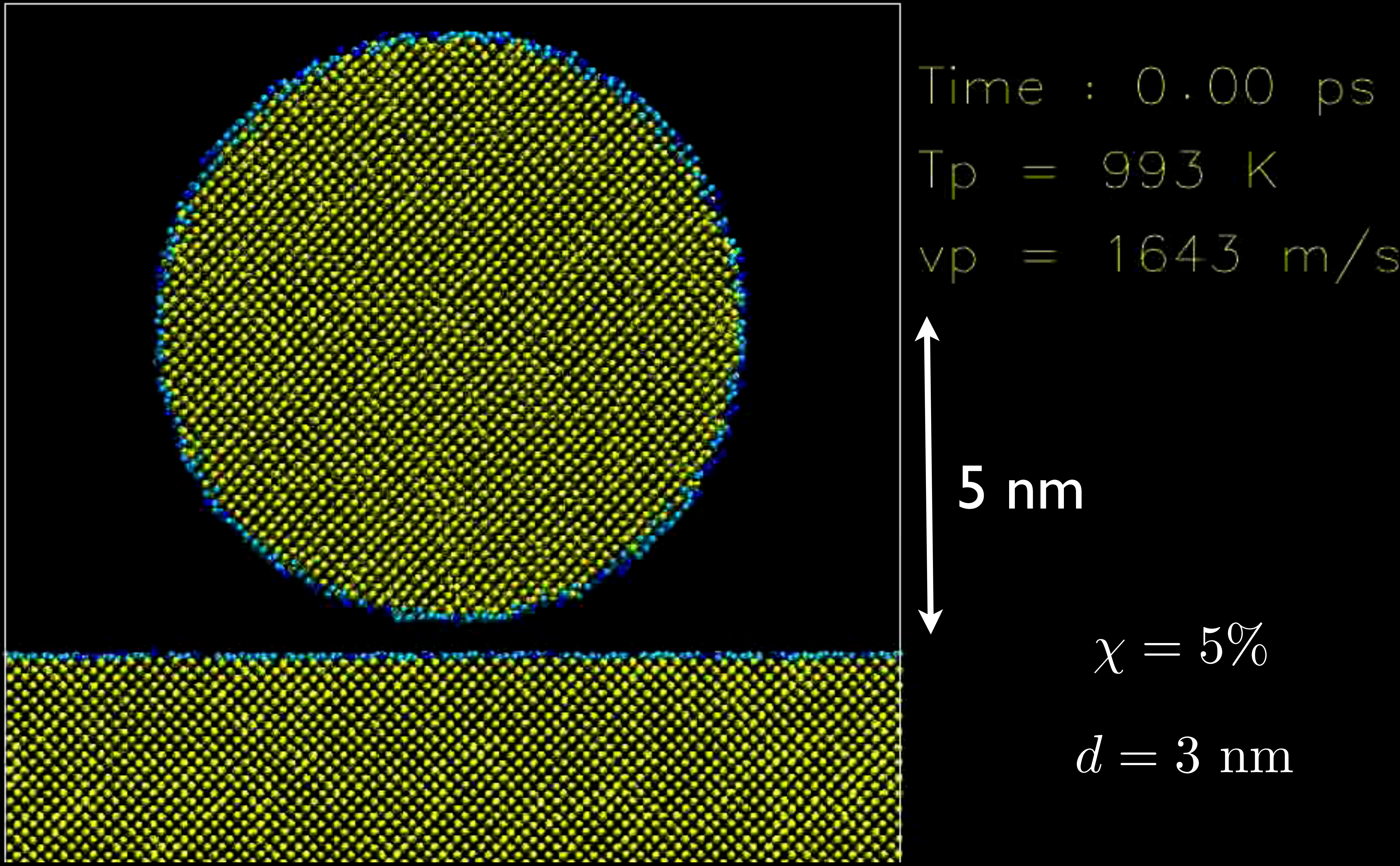

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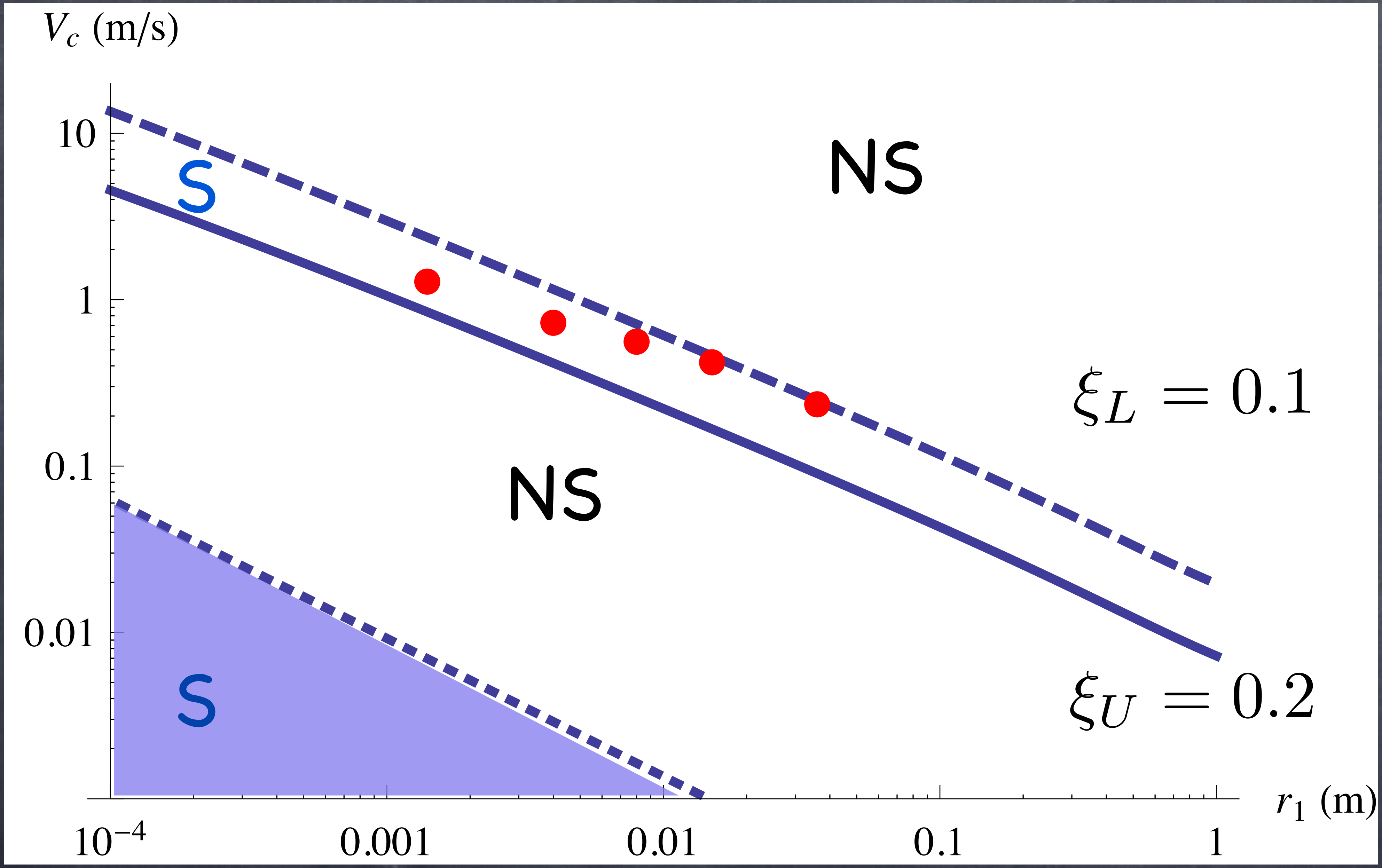
What happens in a high speed central collision?

Constraint on degree of damage ξ

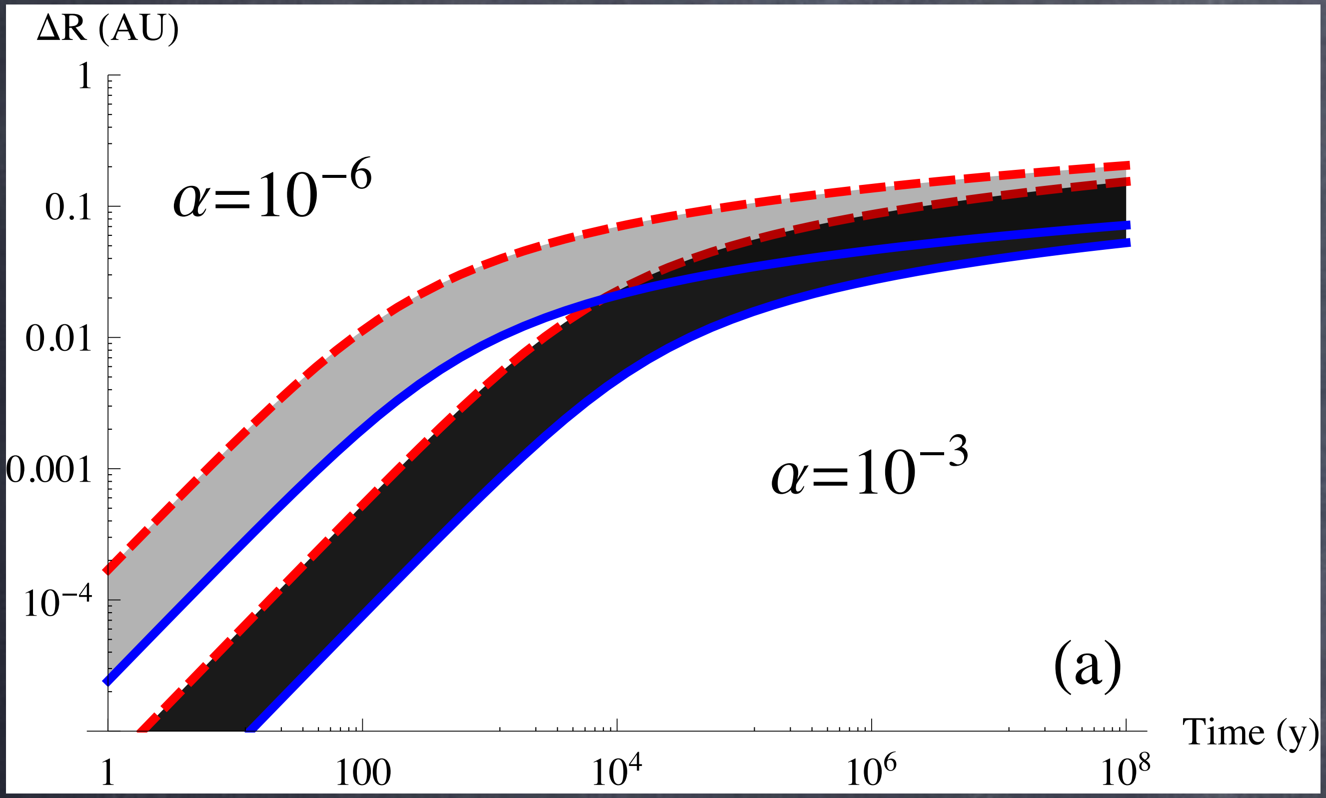


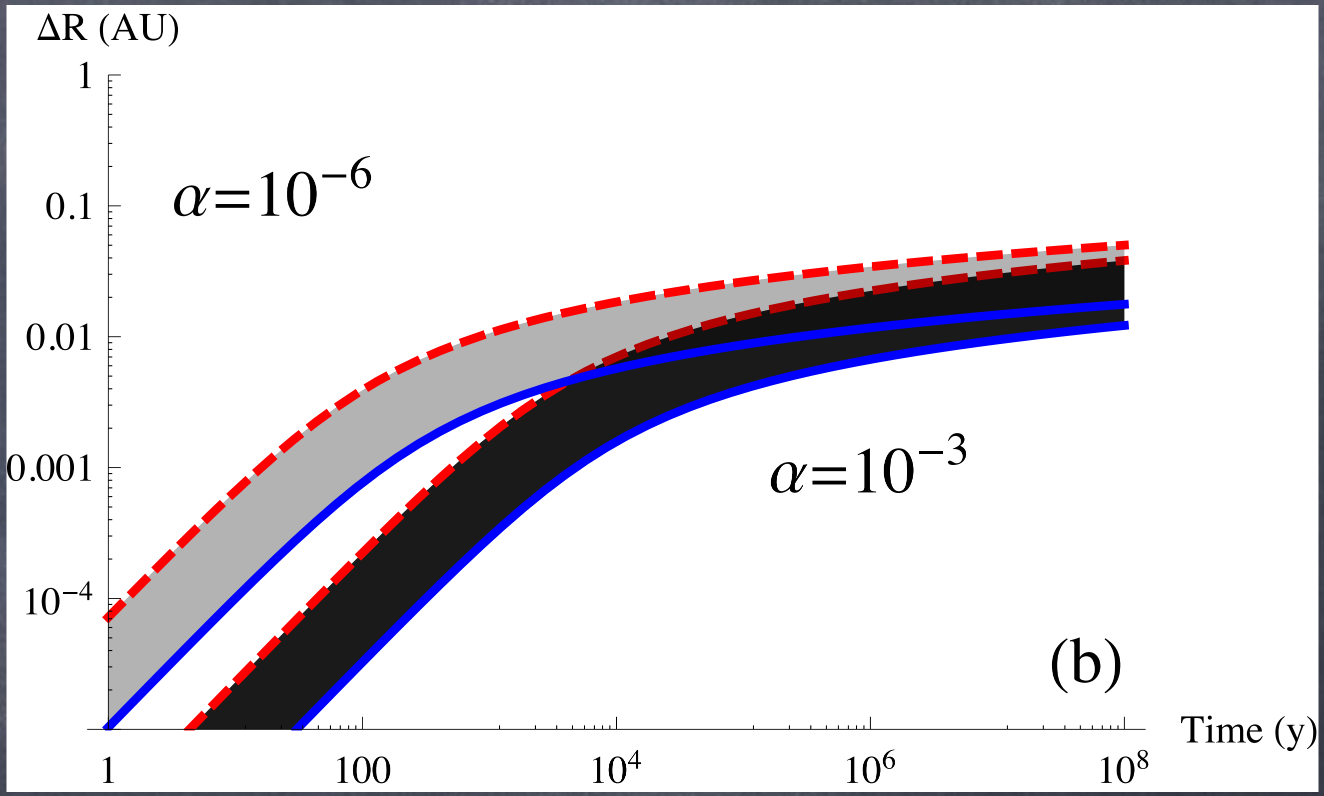
Data: Higa et al., Icarus 133, 310 (1998).

Chokshi et al., ApJ 407, 806 (1993).

Nebular Setting and Drifter Growth

Drifter Trajectory: $$\Delta R(t) = \frac{6V_c}{\Omega(R)} \ell n \left[\frac{r(t)}{r_{po}} \right]$$

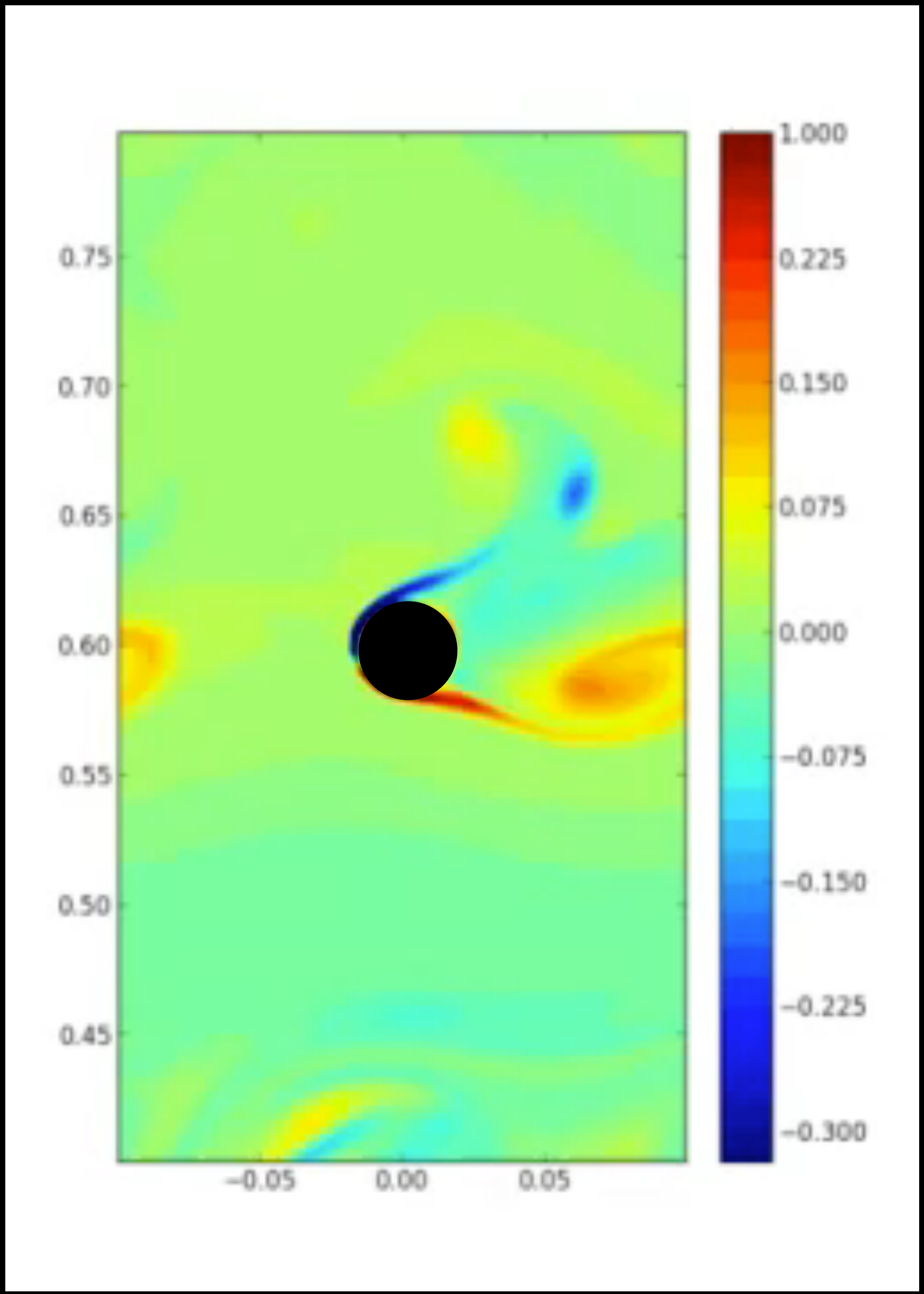




7 Au, T = 105 K

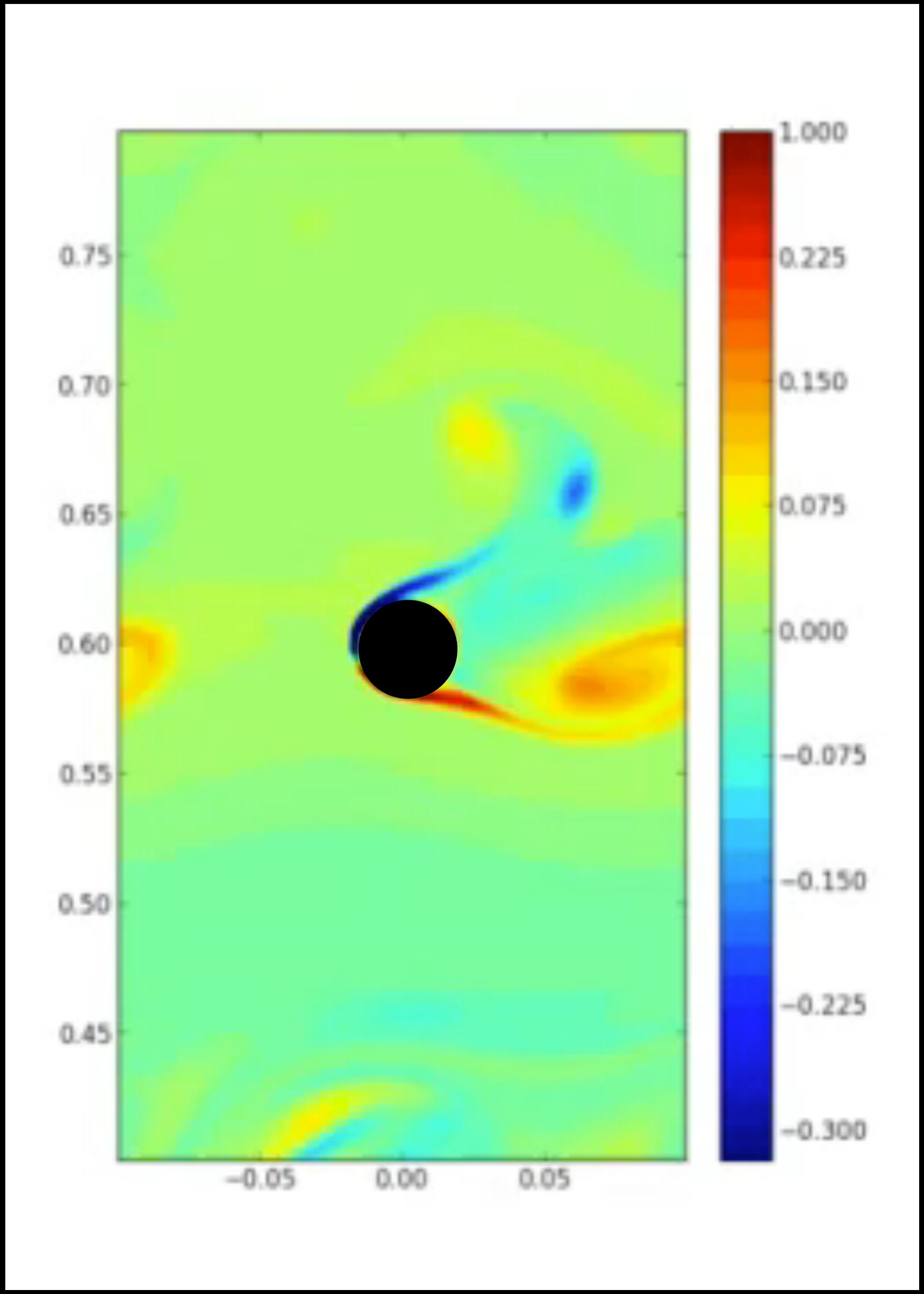
3 Au, T = 150 K

Slowing of Trajectory due to collisional accretion



Dhruba Mitra &
Axel Brandenburg, NORDITA

Mitra et al., *ApJ*, 773:120 (2013)



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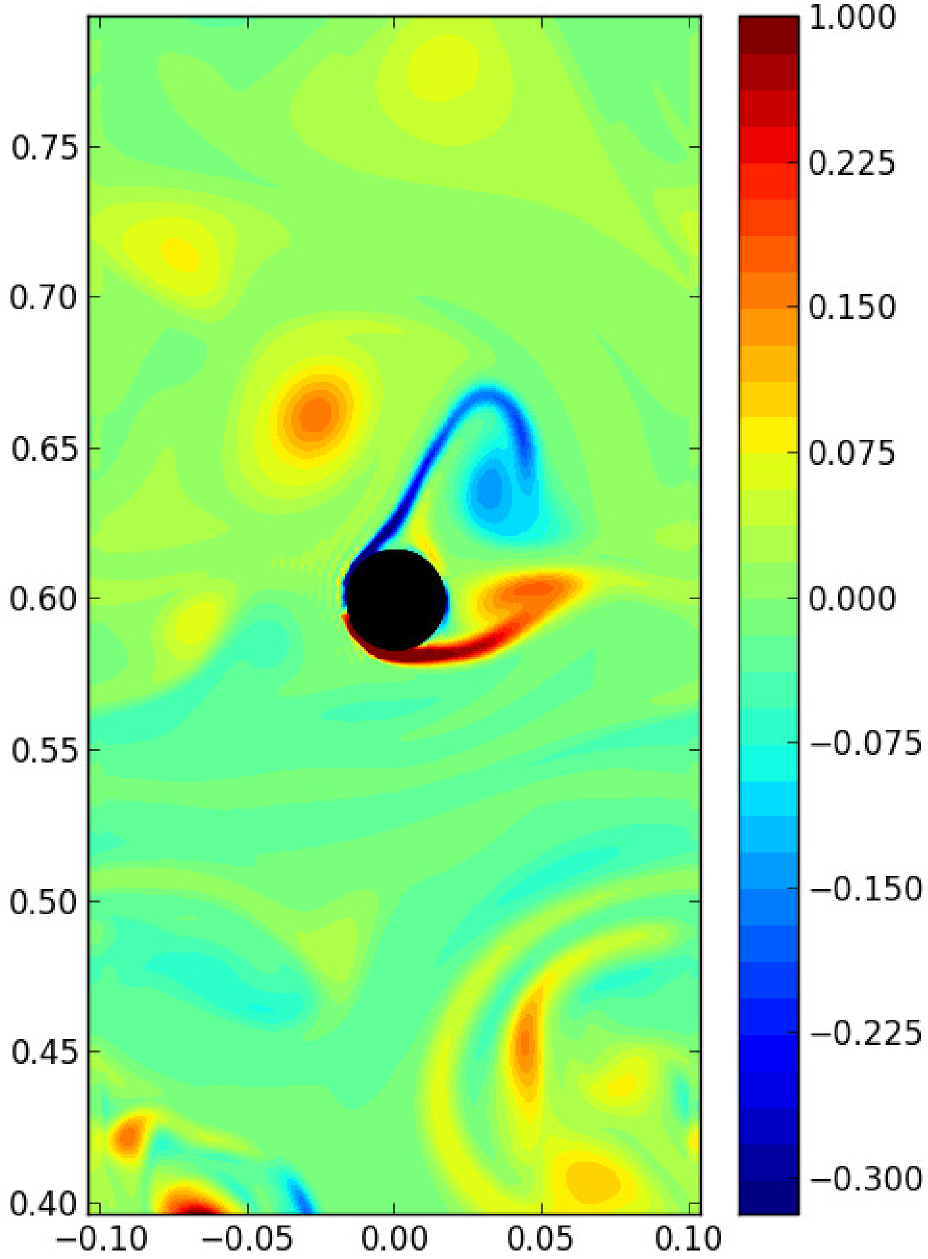
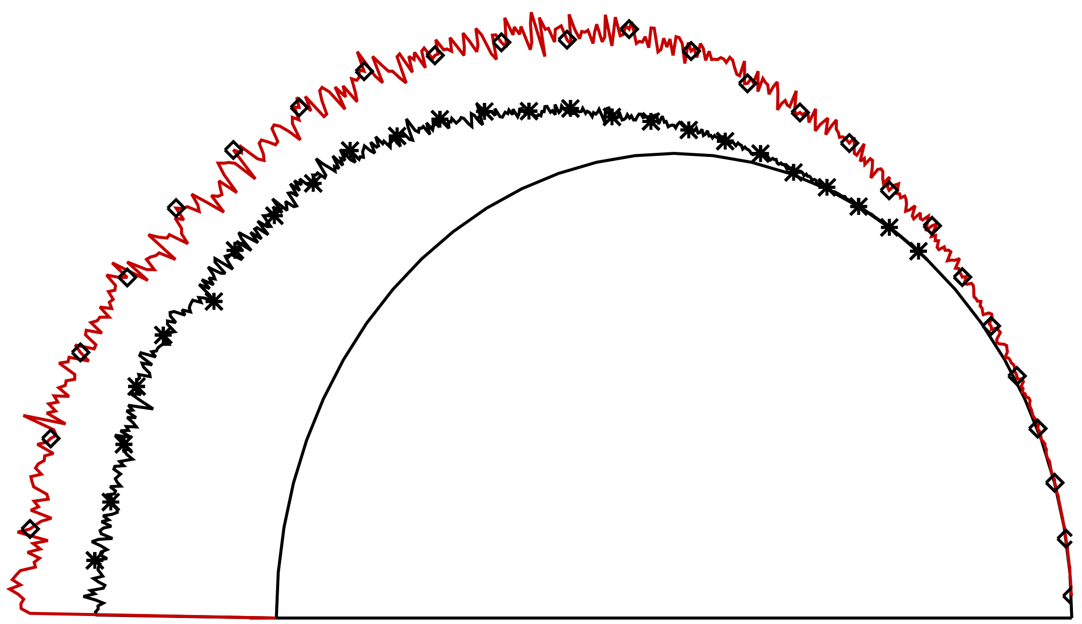


0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
−0.10
−0.05
0.00
0.05
0.10
1.000
0.225
0.150
0.075
0.000
−0.075
−0.150
−0.225
−0.300



Summary

(1) All Materials are finite and have surfaces where phase transitions are initiated.

(2) The meter bottleneck problem can be overcome by

Growth by particle accretion and ASSUMING perfect sticking for all collisional speeds

Weidenschilling & Cuzzi

(a) This assumption violated previously understood collisional physics

(b) The mechanism described here provides the collisional physics that underlies sticking

Support from Swedish Research Council

Eyjafjallajokul

Thank You

ApJ **719**, 540 (2010)
ApJ, **773**,120 (2013)
Rev. Mod. Phys. **78**, *695* (2006)
Rev. Mod. Phys. **82**, *1887* (2010)