On the different kinds of superfluid vortices in the interior of neutron stars

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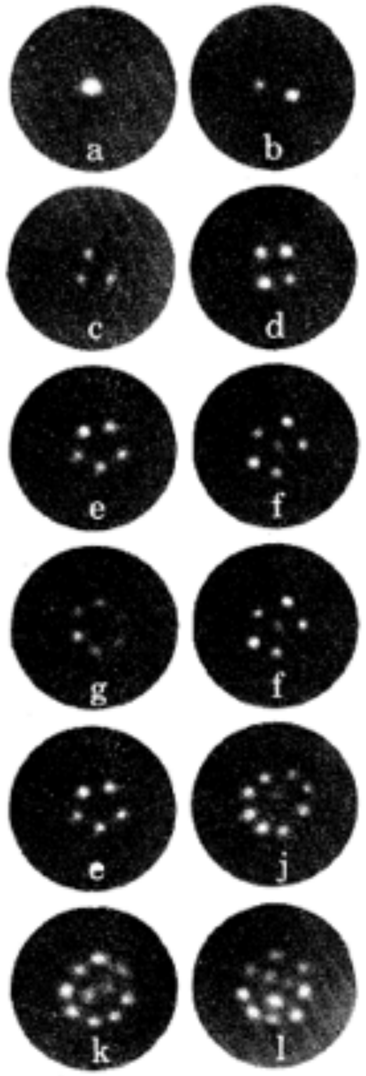
Outline

- Superfluid helium
- Bose-Einstein condensation and vortices
- BCS theory
- Superfluid vortices in neutron stars
- Questions

Superfluid helium

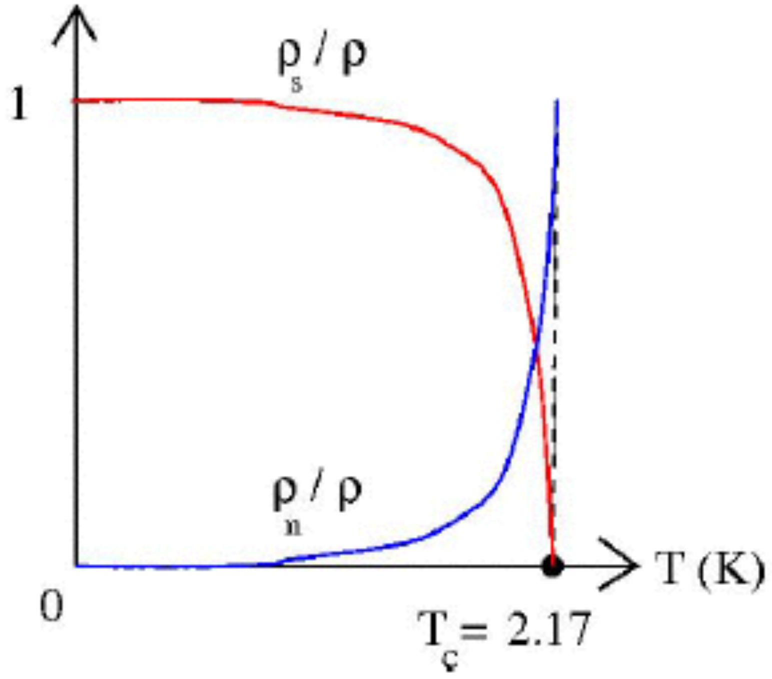
- Kapitza (1937): Flow through capillaries without friction when $T < 2.17$ K
- In a rotating vessel the superfluid component does not rotate with the vessel, so that the moment of inertia looks reduced [Hess-Fairbank experiment Phys. Rev. Lett. 19, 216 (1967)]

- Upon increasing the angular velocity, vortices start to nucleate in the sample
- If one rotates above the critical temperature, cools down below it and stops the vessel one has persistent flow of the superfluid component



Yarmchuk, Gordon, & Packard PRL43, 214 (1979)

Two-fluid model

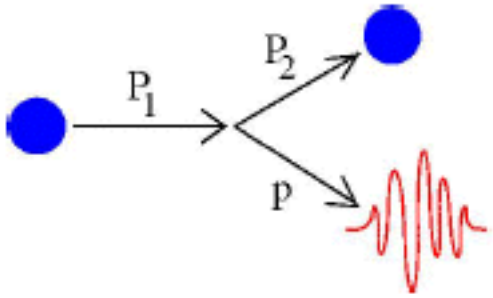


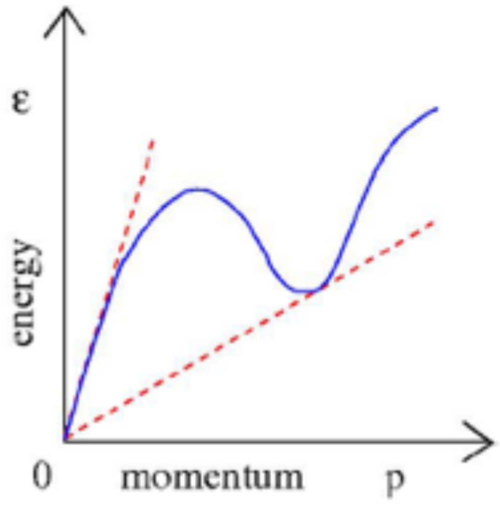
$$\rho = \rho_n + \rho_s$$

	Normal fluid	Superfluid
Density	ρ_n	ρ_s
Velocity	$\mathbf{v}_n$	$\mathbf{v}_s$
Entropy	S	0
Viscosity	η	0

- Landau (1941): The dispersion of excitations explains superfluidity. Excitations cannot be created unless the velocity satisfies

$$v_1 > \min \frac{\epsilon(p)}{p}$$





Dispersion of excitation in helium (phonons, rotons)

Bose-Einstein condensate (BEC)

- Intuitively, BEC occurs when a macroscopic number of particles occupy the lowest energy state and are coherently described by the same complex wave-function

$$\psi(\mathbf{r})$$

(which reduces simply to a complex number in the ideally uniform case).

- The above quantity is nothing but the expectation value of the bosonic field operator appearing in the Hamiltonian of the system

$$\hat{H} = \int d\mathbf{r}\, \frac{\hbar^2}{2m} \nabla \hat{\Psi}^\dagger \nabla \hat{\Psi} + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}'\, \hat{\Psi}^\dagger \hat{\Psi}^{\dagger\prime} V(\mathbf{r} - \mathbf{r}') \hat{\Psi}' \hat{\Psi}$$

- The Hamiltonian has a *U(1)* symmetry, corresponding to the conservation of the mass current (Noether theorem), which is spontaneously broken by the condensate.

- One can quantize the fluctuations above the condensate and obtain the Bogoliubov spectrum

$$\epsilon(p) = \left[\frac{\mu}{m} p^2 + \left(\frac{p^2}{2m} \right)^2 \right]^{\frac{1}{2}}$$

(cf. Landau’s criterion for superfluidity)

- Focusing on the condensate, at weak coupling one can in the first approximation substitute the bosonic operator with its expectation value, which will be a solution of the Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + g|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t)$$

time dependent

$$\left(-\frac{\hbar^2\nabla^2}{2m} - \mu + g|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t) = 0$$

stationary

- Superfluid current conservation

$$\psi(\mathbf{r},t) = \sqrt{\rho(\mathbf{r},t)}e^{iS(\mathbf{r},t)}$$

$$\mathbf{j}(\mathbf{r},t) = \rho\frac{\hbar}{m}\nabla S$$

$$\Rightarrow \frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0$$

- Superfluid velocity and vorticity

$$\mathbf{v}(\mathbf{r},t) = \frac{\hbar}{m}\nabla S$$

$$\nabla\times\mathbf{v}(\mathbf{r},t) = 0$$

except at singular points

Vortex solution

- Consider the cylindrically symmetric ansatz $\psi(\mathbf{r}) = f(r)e^{in\phi}$ $\quad n \in \mathbb{Z}$

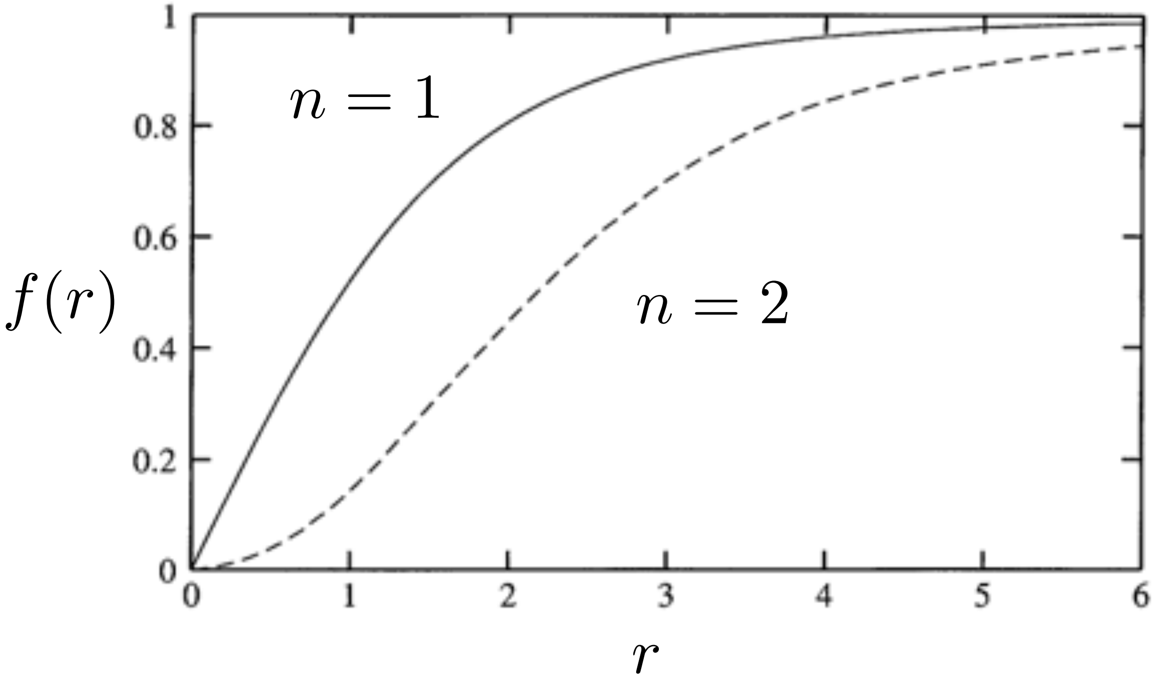
i) total angular momentum $L_z = Nn\hbar$

ii) velocity $v_\phi = \frac{\hbar}{m}\frac{n}{r}$

iii) quantized circulation

$$\oint \mathbf{v} \cdot d\mathbf{l} = 2\pi n \frac{\hbar}{m}$$

(independent of the radius of the contour)



vorticity $\quad \nabla \times \mathbf{v} = 2\pi n \frac{\hbar}{m} \delta^{(2)}(\mathbf{r}_\perp)\,\hat{\mathbf{z}}$

- Energy of the vortex

$$E_v = L\pi\rho_\infty \frac{\hbar^2}{m} \ln\left(\frac{1.46R}{\xi}\right)$$

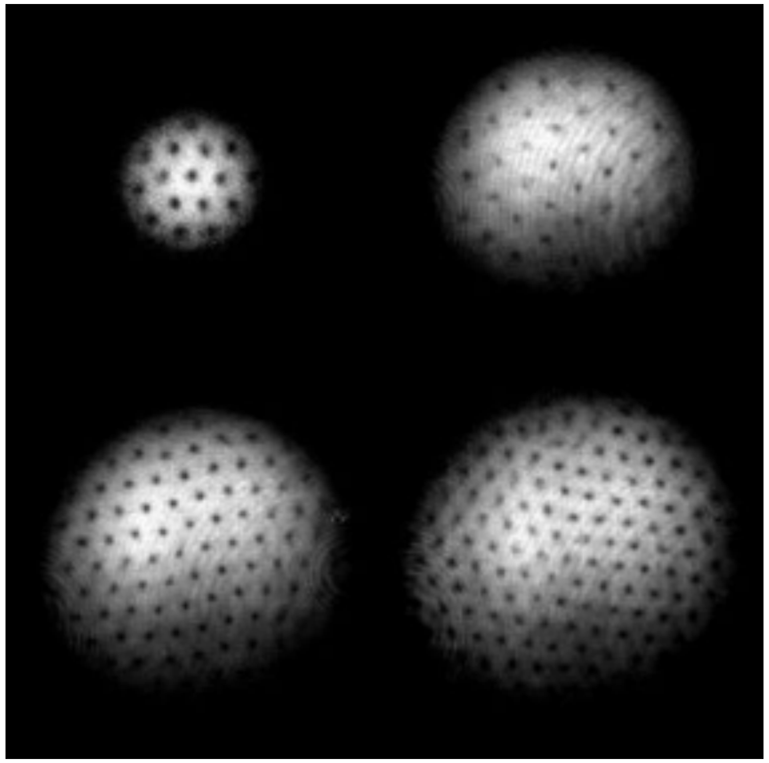
Vortex nucleation and vortex lattice

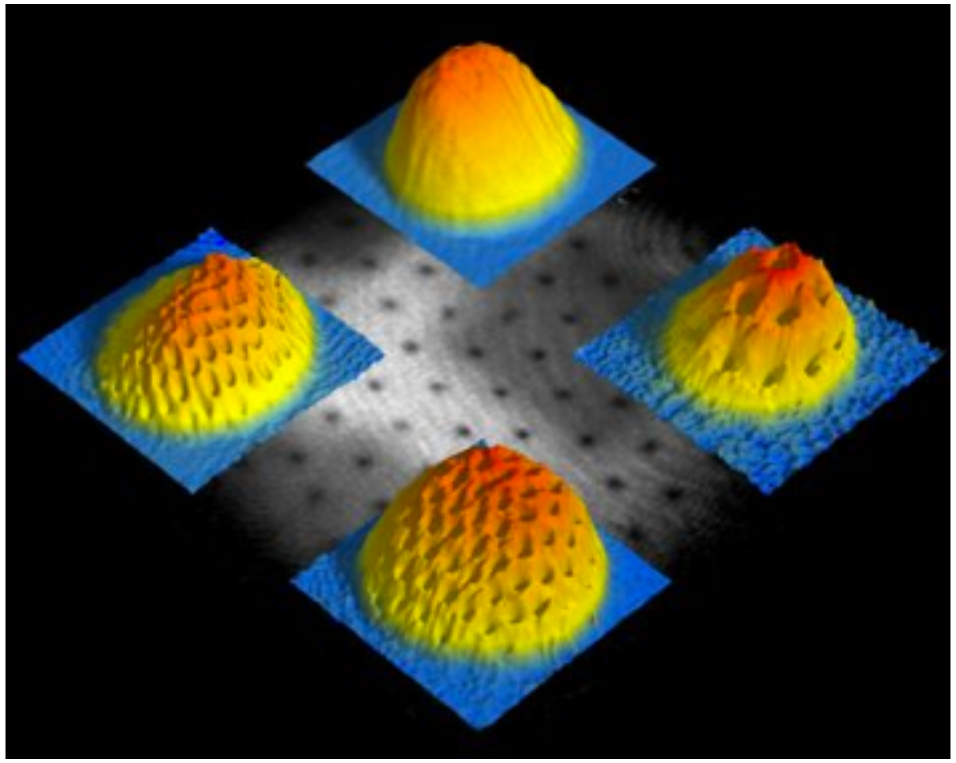
- Energy of the vortex in rotating frame

$$E'_v = E_v - \Omega L_z$$

A single vortex becomes stable at the critical frequency $\Omega_c = \dfrac{E_v}{N\hbar}$

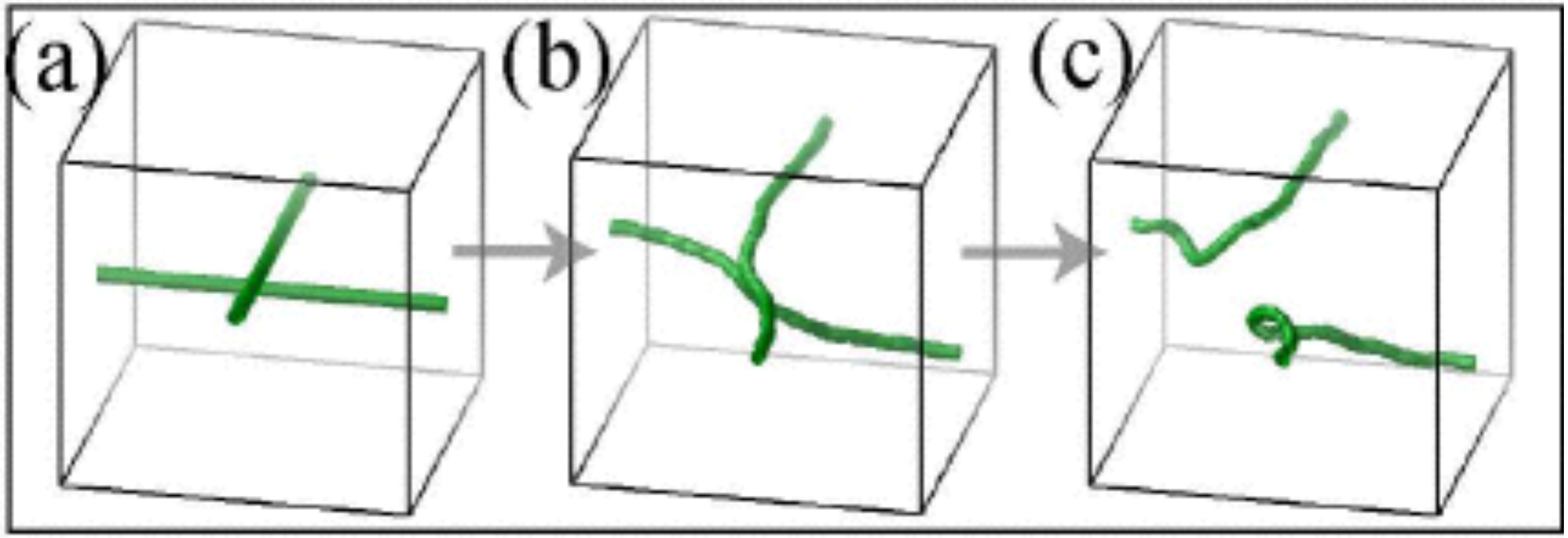
- At increasing rotation frequency there is the formation of a triangular vortex lattice





MIT, Ketterle group

U(1) vortex reconnection



[J. Koplik and H. Levine, Phys. Rev. Lett. 71, 1375 (1993); 76, 4745 (1996); M. Leadbeater, T. Winiecki,D. C. Samuels, C. F. Barenghi, and C. S. Adams, Phys. Rev. Lett. 86, 1410 (2001)]

[M.Kobayashi et al. Phys.Rev.Lett.103:115301; figure by M.Kobayashi based on time-dependent Gross-Pitaevskii]

Homotopy argument

- First homotopy group $\pi_1(M)$

$$\{f : S^1 \to M\}/\text{continuous deformations}$$

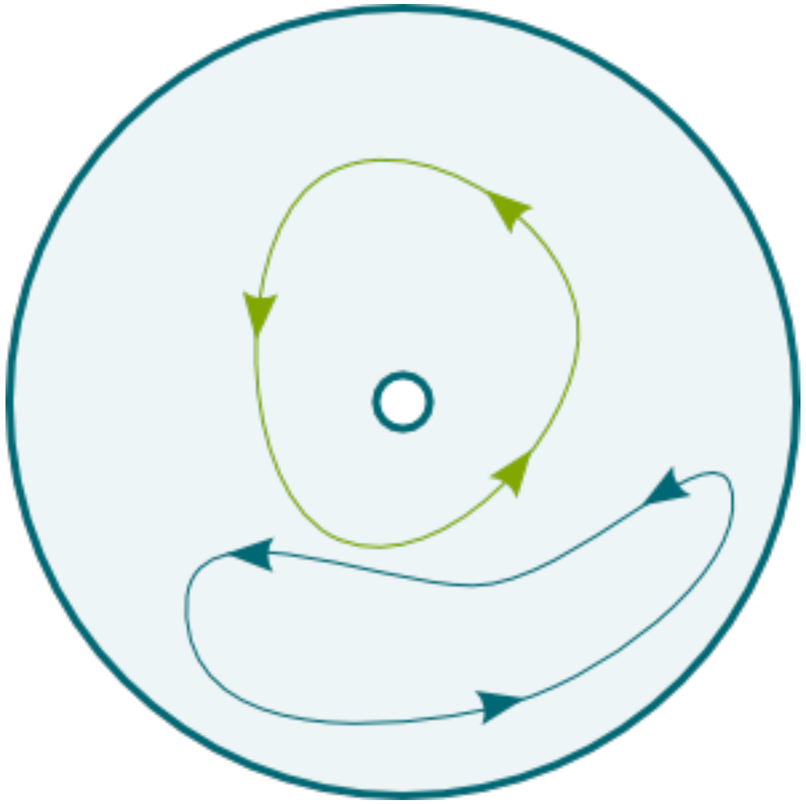
- Spontaneous symmetry breaking $G \to H$

Order parameter space G/H

Vortices are classified by

$$\boxed{\pi_1(G/H)}$$

Note: it does not need to be Abelian.



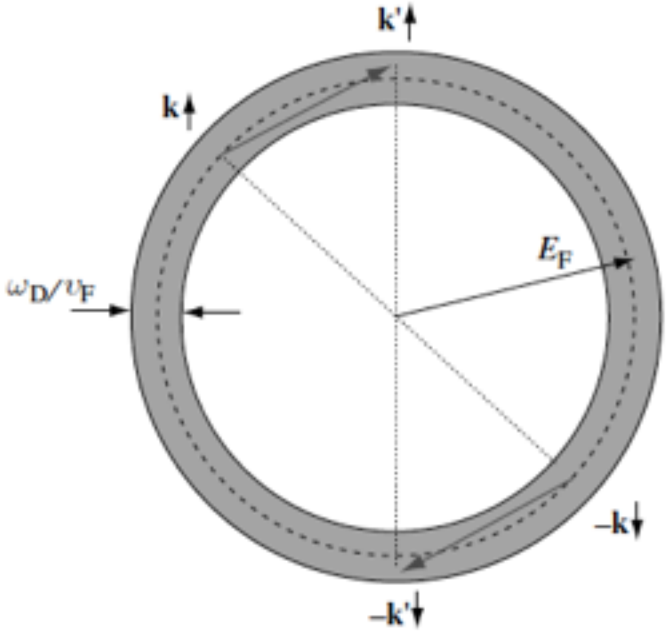
$\pi_1 = \mathbb{Z}$

- Example

$U(1) \to \mathbf{1}$ symmetry breaking

$\pi_1[U(1)] = \mathbb{Z}$ U(1) vortices have integer topological charge (winding number)

Fermionic superfluidity: BCS theory



- Fermions cannot condense like bosons. However, let us consider a Fermi surface at T~0. It is known to be unstable to Cooper pairing instability for any attractive interaction between fermions, however small.

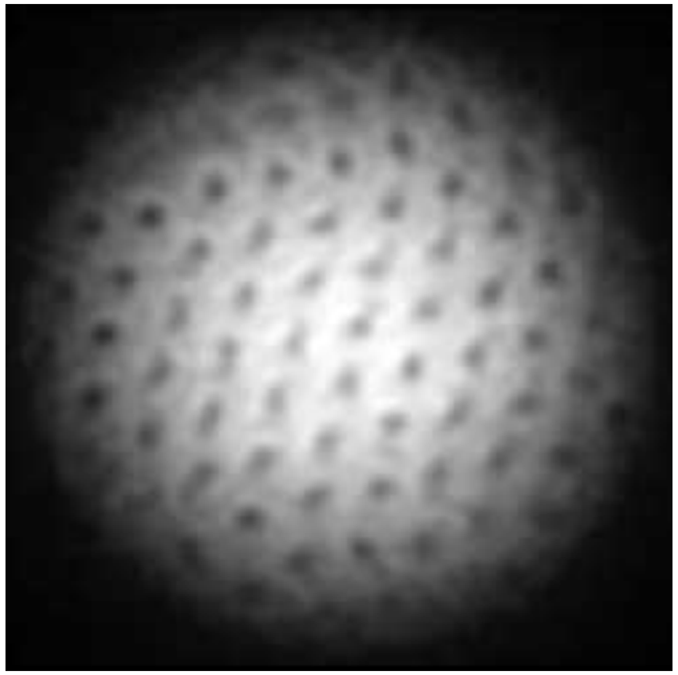
$$\Delta = \langle \psi_\uparrow(\mathbf{k})\psi_\downarrow(-\mathbf{k})\rangle$$

This fermion bilinear has bosonic statistics and can condense below a certain critical temperature.

- With a perturbative calculation at weak coupling it is possible to write down the Ginzburg-landau action in terms of the order parameter

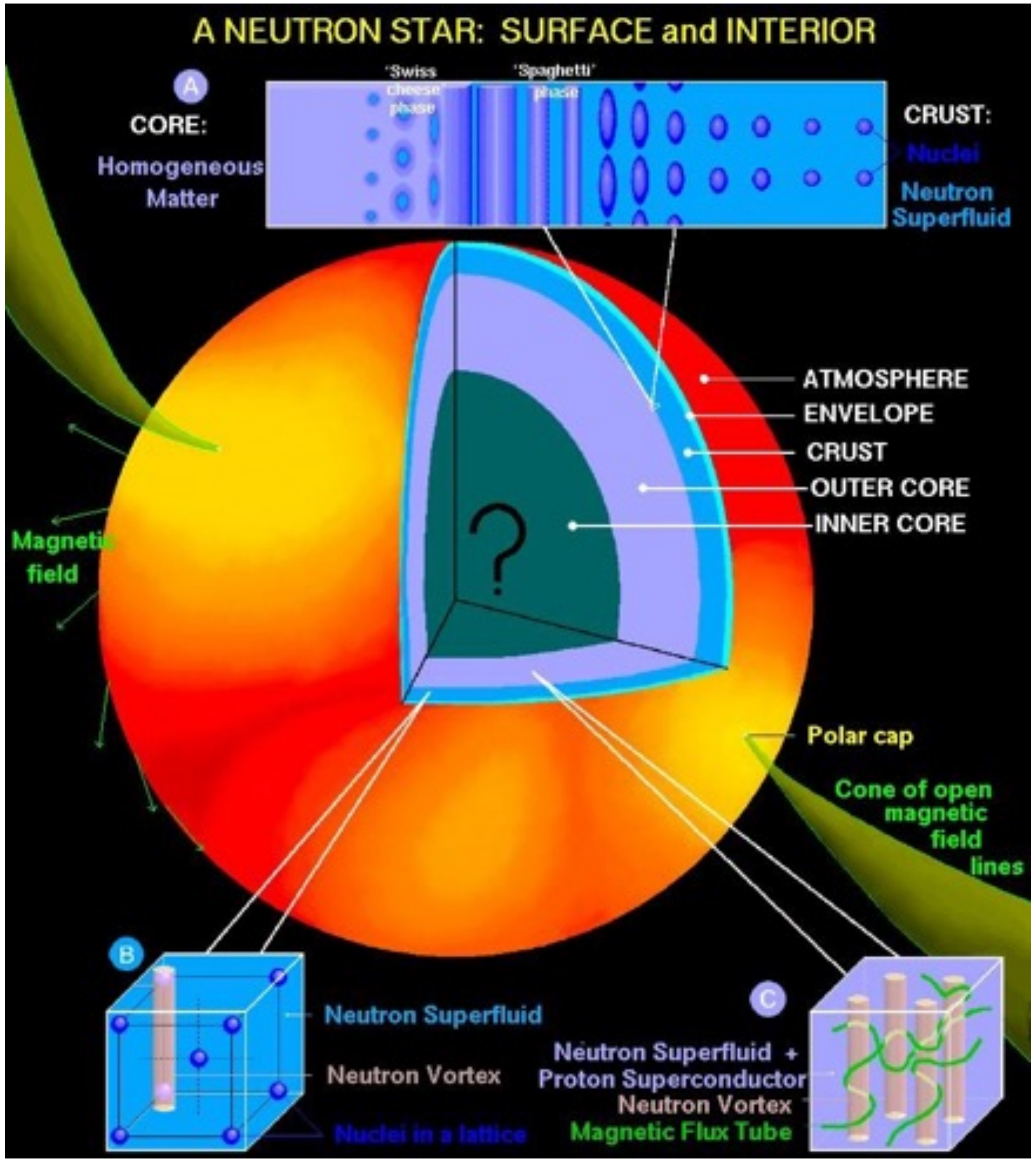
$$S_{\rm GL}[\Delta,\bar{\Delta}] = \beta\int d^dr\left[\frac{r}{2}|\Delta|^2 + \frac{c}{2}|\partial\Delta|^2 + u|\Delta|^4\right]$$

- This is in fact very similar to the Gross-Pitaevskii action and gives rise to similar physics, in particular the vortex lattice



6-Li [MIT]

Neutron stars

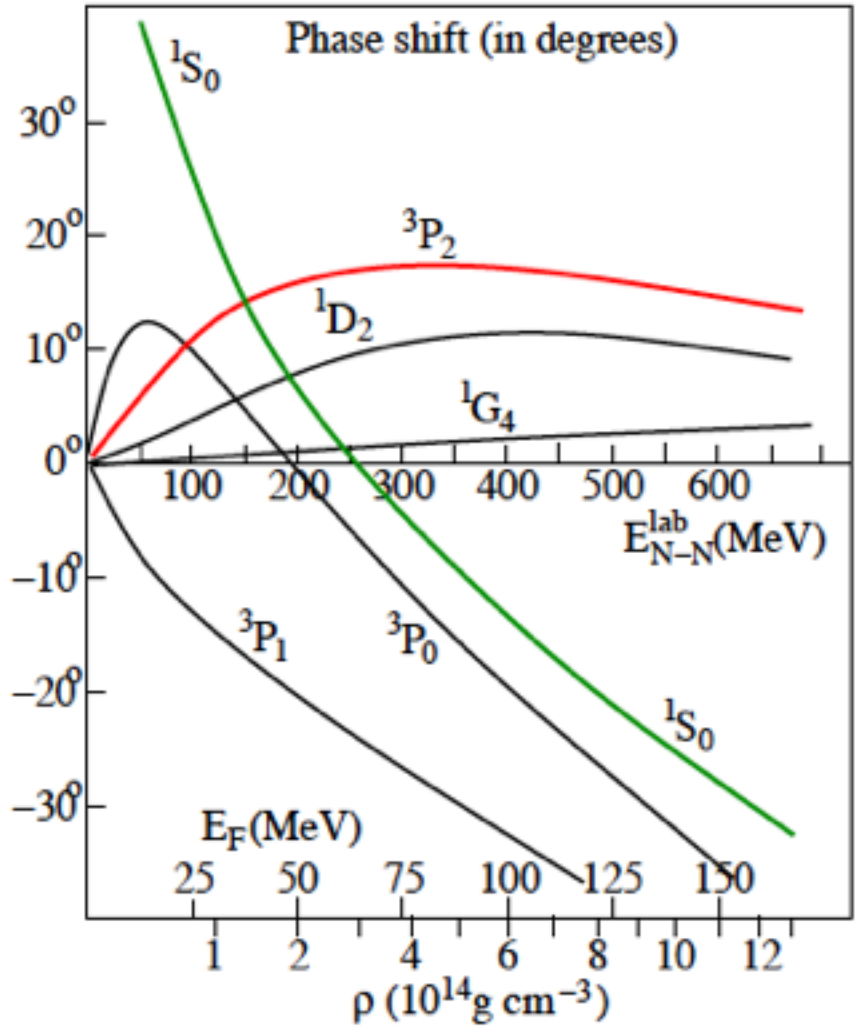


from http://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/NStar.html

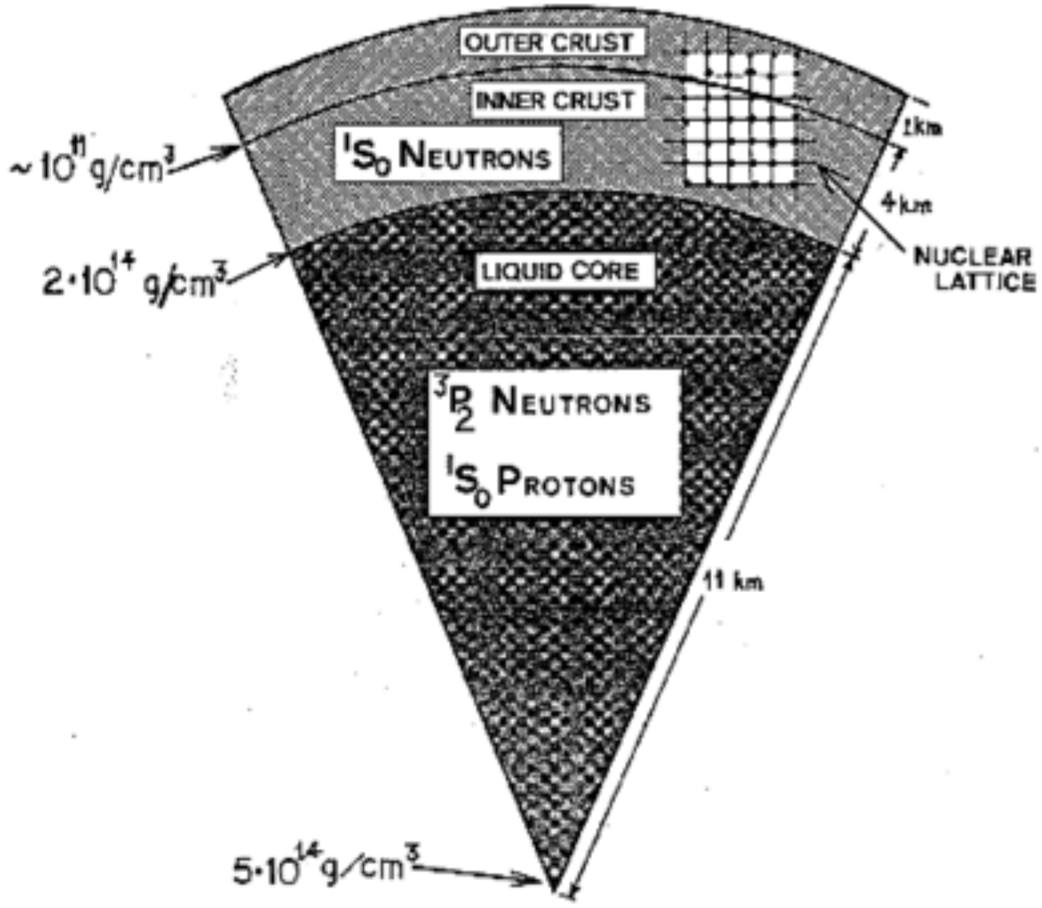
- Neutron stars are believed to host neutron superfluids.

- BCS theory predicts a pairing instability for any (however small) attractive interaction between fermions

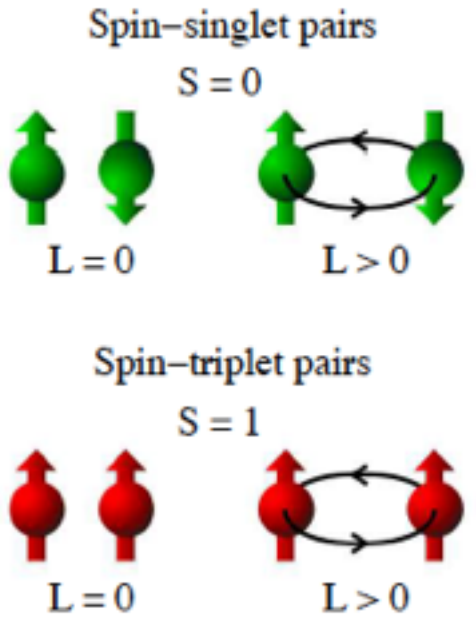
- The typical temperature of a neutron star is well below the predicted superfluid transition temperature



[Tamagaki, PTP 44 (1970), 905; adapted by Page et al. arXiv:1110.5116]



[Sauls, NATO ASI Series 262,457]



- $\rho > 4.3\text{x}10^{11}$ g/cm^3. Neutron leaking from nuclei. BCS pairing occurs in the usual s-wave channel and the $^1S_0$ superfluid is formed.
- $\rho > 2 \text{ x } 10^{14}$ g/cm^3. The $^3P_2$ channel becomes the most attractive and the $^3P_2$ superfluid is formed.

$^3P_2$ superfluid

$\mathbf{J} = \mathbf{L} + \mathbf{S}$ $\quad\quad$ $\mathbf{1} \otimes \mathbf{1} = \boxed{\mathbf{2}} \oplus \mathbf{1} \oplus \mathbf{0}$ $\quad\quad$ Total angular momentum

- The order parameter can be expressed as a traceless symmetric 3x3 complex matrix

$$A_{\mu i}$$

Spin

Orbital

- Transformation property under the symmetry group $G = U(1) \times SO(3)_{L+S}$

$$A \to e^{i\alpha} O A O^T$$

- Ginzburg-Landau free energy derived from BCS theory (weak coupling)

$$F = \int d^3\rho \left(f_{\mathrm{grad}} + f_{2+4} + f_6 + f_H \right)$$

Kinetic

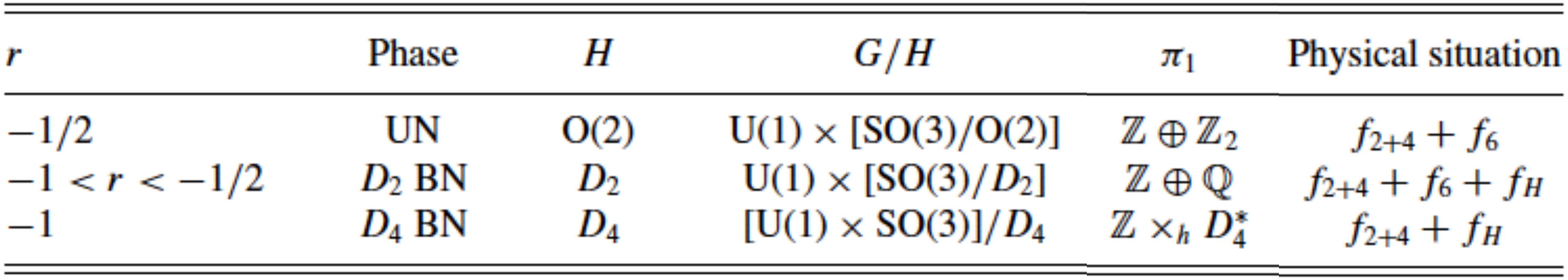
2nd & 4th order in A

6th order in A

coupling to magnetic field

- Various patterns of spontaneous symmetry breaking from the minimization of *F*

$$A \sim \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad -1 \le r \le -1/2$$



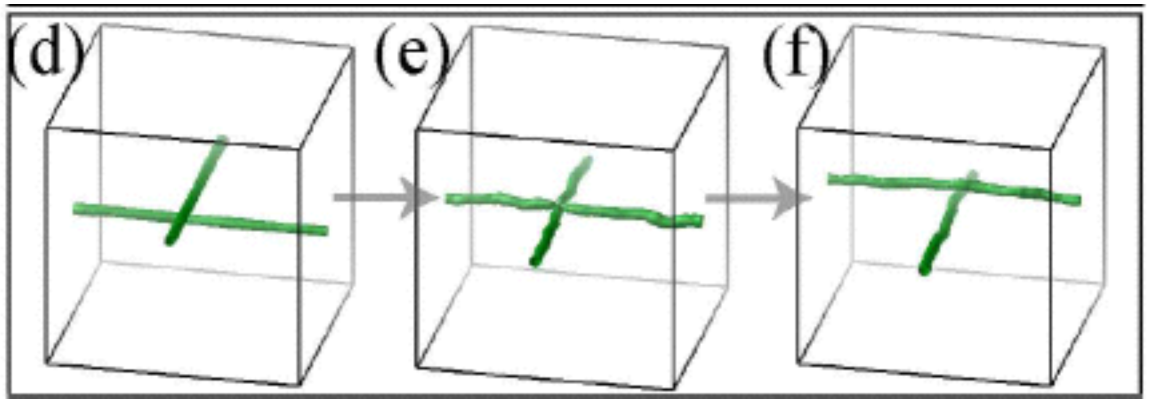
r	Phase	H	G/H	π_1	Physical situation
$-1/2$	UN	O(2)	$U(1) \times [SO(3)/O(2)]$	$\mathbb{Z} \oplus \mathbb{Z}_2$	$f_{2+4} + f_6$
$-1 < r < -1/2$	D_2 BN	D_2	$U(1) \times [SO(3)/D_2]$	$\mathbb{Z} \oplus \mathbb{Q}$	$f_{2+4} + f_6 + f_H$
-1	D_4 BN	D_4	$[U(1) \times SO(3)]/D_4$	$\mathbb{Z} \times_h D_4^*$	$f_{2+4} + f_H$

[K.Masuda and M.Nitta, Phys.Rev.C 93, 035804]

- Biaxial nematic states (BN) support non-Abelian first homotopy groups and therefore non-Abelian (non-commutative) vortices.

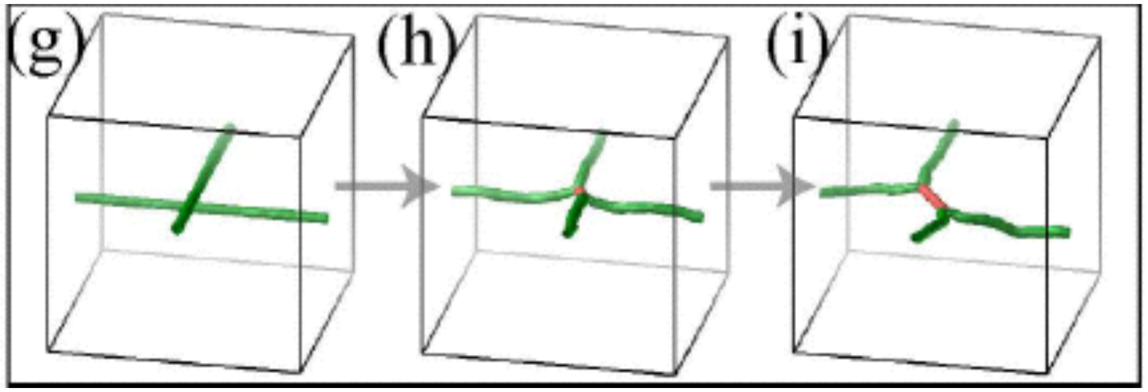
- The vortex topological charge is not just an integer number, but it can include an element of a non-commutative group, *e.g.* the quaternions $\mathbb{Q}$

Collision dynamics of non-Abelian vortices



$$[a, b] = 0$$

Passing through



$$[a, b] = c \neq 0$$

Rung formation

[M.Kobayashi et al. Phys.Rev.Lett.103:115301; figures by M.Kobayashi]

Questions

- Can we give more qualitative and quantitative characterization of the non-Abelian vortex network in neutron stars?
- Can we identify its possible signatures of the non-Abelian vortex network in neutron stars?