New Phenomena in Turbulent Transport: Astrophysics and Planetary Physics

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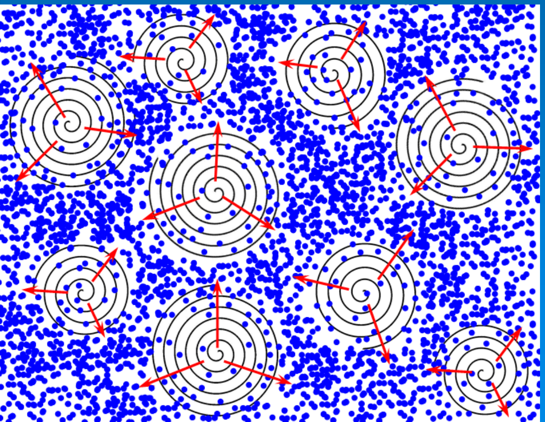
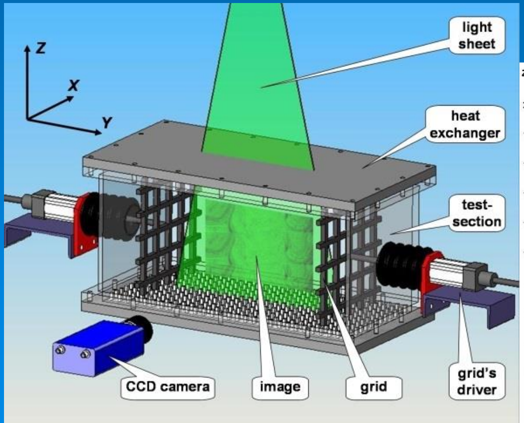
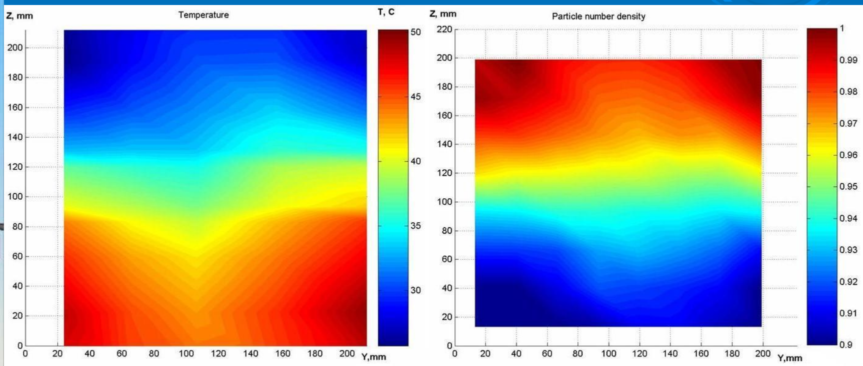






Effect of Chemical Reactions and Phase Transitions on Turbulent Diffusion

T. Elperin, N. Kleeorin, M. Liberman, I. Rogachevskii, Phys. Rev E 90, 053001 (2014);
T. Elperin, N. Kleeorin, M. Liberman, A. Lipatnikov, I. Rogachevskii, R. Yu, Phys. Rev E, submit. (2017)

Instantaneous particle number density of admixture:

$$\frac{\partial n_\beta}{\partial t} + \boldsymbol{\nabla}\cdot(n_\beta\, \boldsymbol{v}) = -\nu_\beta \hat{W}(n_\beta, T) + \hat{D}(n_\beta),$$

The source term:

$$-\nu_\beta \hat{W}(n_\beta, T)$$

The Arrhenius law:

$$\hat{W} = A \exp\left(-E_a/RT\right)\, \Pi_{\beta=1}^{m} \left(n_\beta\right)^{\nu_\beta},$$

Instantaneous fluid temperature field:

$$\frac{\partial T}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})T + (\gamma-1)T(\boldsymbol{\nabla}\cdot\boldsymbol{v}) = q\hat{W}(n_\beta, T) + \hat{D}(T),$$

Q is the reaction energy release; $q = Q/\rho c_p$

ν_β is the stoichiometric coefficient that is the order of the reaction;

Turbulent Diffusion of Gases

T. Elperin, N. Kleeorin, M. Liberman, I. Rogachevskii, Phys. Rev E 90, 053001 (2014);
T. Elperin, N. Kleeorin, M. Liberman, A. Lipatnikov, I. Rogachevskii, R. Yu, Phys. Rev E, submitted (2017)

$$\langle n'_\beta \, \boldsymbol{u} \rangle = -D^T_\beta \, \boldsymbol{\nabla} \overline{N}_\beta + \sum_{\lambda=1;\lambda\neq\beta}^{m} D^{\rm MTD}_\lambda(\beta) \, \boldsymbol{\nabla} \overline{N}_\lambda + \boldsymbol{V}_{\rm eff} \, \overline{N}_\beta,$$

$$\langle \theta \, \boldsymbol{u} \rangle = -D^T \, \boldsymbol{\nabla} \overline{T} - \sum_{\lambda=1}^{m} D^{\rm TDE}_\lambda \, \boldsymbol{\nabla} \overline{N}_\lambda.$$

the coefficient of turbulent diffusion:

$$D^T_\beta = \frac{D^T_0}{\rm Da} \left[1 - \frac{1}{2{\rm Da}\left[1 - {\rm Re}^{-1/2}\right]} \ln \frac{1 + 2{\rm Da}}{1 + 2{\rm Da}\,{\rm Re}^{-1/2}}\right] - 2D^T_0 \left(1 + {\rm Pr}^{-1}\right) \frac{\ln {\rm Re}}{\rm Re}$$

${\rm Da} = \tau_0/\tau_c$ is the turbulent Damköhler number

the simplest chemical reaction $A \to B$ $\qquad O_2 \to O + O$

${\rm Da} \gg 1$ $\qquad D^T_\beta = D^T_0 / {\rm Da} = \tau_c u_0^2/3$ $\qquad D^T_0 = \tau_0 u_0^2/3$

Concentration of reagent A decreases much faster during the chemical time, so that the usual turbulent diffusion based on the turbulent time does not contribute to the mass flux of a reagent A.

Comparison with Numerical Simulations (MFS)

A. Brandenburg, N. E. L. Haugen and N. Babkovskaia,
Phys. Rev. E 83, 016304 (2011)

Kolmogorov-Petrovskii-Piskunov-Fisher Equation
(advection-reaction-diffusion equation):

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\, \boldsymbol{v}) = \frac{n}{\tau_c} \left(1 - \frac{n}{n_0}\right) + D \Delta n,$$

Mean-Field KPPF-equation:

$$\frac{\partial \overline{N}}{\partial t} + \tau \frac{\partial^2 \overline{N}}{\partial t^2} = \frac{\overline{N}}{\tau_c} \left(1 - \frac{\overline{N}}{n_0}\right) + D_{\rm T} \Delta \overline{N},$$

the reaction speed (the front speed):

$$s_{\rm T} = (d/dt) \int (\overline{N}/n_0) \, dz$$

$$s_{\rm T} = 2(D_\beta^T/\tau_c)^{1/2}$$

T. Elperin, N. Kleeorin, M. Liberman,
I. Rogachevskii, Phys. Rev E 90, 053001 (2014)

$$\frac{D_\beta^{\rm T}}{D_0^T} = \frac{1}{{\rm Da}_{\rm T}} \left(1 - \frac{\ln(1 + 2{\rm Da}_{\rm T})}{2{\rm Da}_{\rm T}}\right)$$

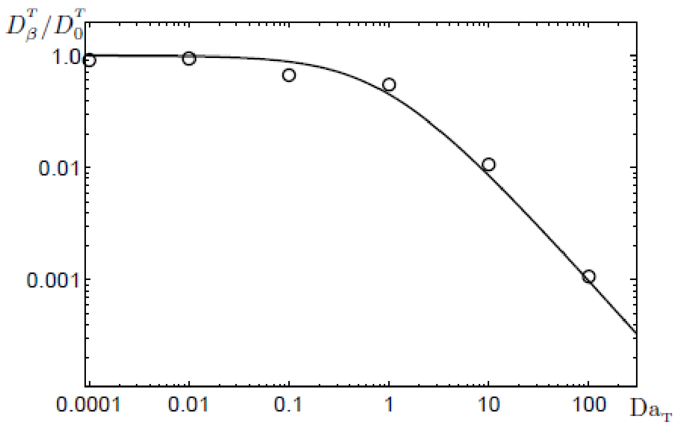


FIG. 1. Comparison of the theoretical dependence of turbulent diffusion coefficient D_β^T/D_0^T versus turbulent Damköhler number ${\rm Da}_{\rm T}$ with the corresponding results of MFS performed in [49].

Comparison with Direct Numerical Simulations

T. Elperin, N. Kleeorin, M. Liberman, A. Lipatnikov, I. Rogachevskii, R. Yu, Phys. Rev E, submitted (2017)

Direct numerical simulations of a finite thickness reaction wave propagation in forced, homogeneous, isotropic, and incompressible turbulence for the first-order chemical reactions.

$$\frac{\partial c}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}) c = W(c) + D\Delta c$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}) \boldsymbol{v} = -\frac{1}{\rho} \boldsymbol{\nabla} p + \nu \Delta \boldsymbol{v} + \boldsymbol{f}$$

TABLE I. DNS cases.

Case	Re	Re_λ	$\eta/\Delta x$	S_L/u_0	ℓ_{11}/δ_F	$\mathrm{Da}_{\mathrm{DNS}}$
1	50	18	0.68	0.1	2.1	0.2
2	50	18	0.68	0.2	2.1	0.4
3	50	18	0.68	0.5	2.1	1.0
4	50	18	0.68	1.0	2.1	2.1
5	50	18	0.68	2.0	2.1	4.1
6	100	30	0.86	0.1	3.7	0.4
7	100	30	0.86	0.2	3.7	0.7
8	100	30	0.86	0.5	3.7	1.9
9	100	30	0.86	1.0	3.7	3.7
10	100	30	0.86	2.0	3.7	7.5
11	200	45	1.06	0.1	6.7	0.7
12	200	45	1.06	0.2	6.7	1.3
13	200	45	1.06	0.5	6.7	3.4
14	200	45	1.06	1.0	6.7	6.7
15	200	45	1.06	2.0	6.7	13.5

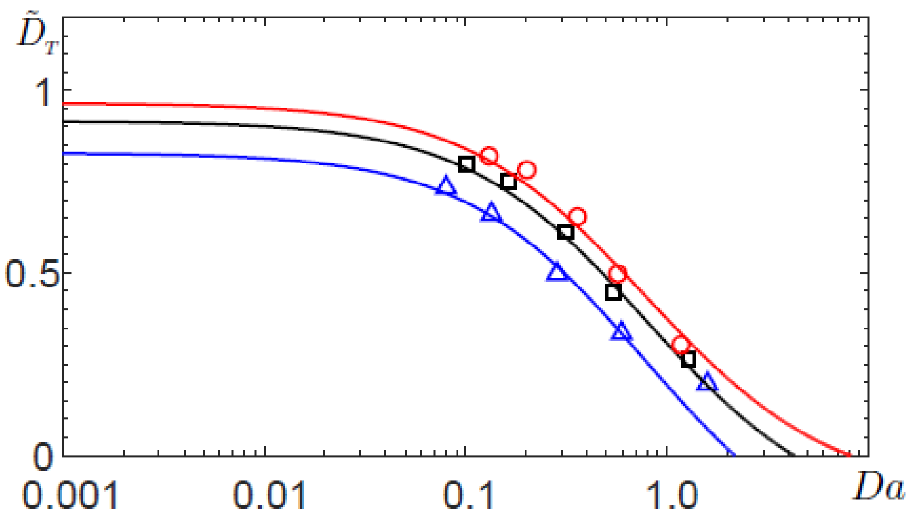


FIG. 5. Theoretical dependence $\tilde{D}_T \equiv D_T/D_0^T$ versus Damköhler number Da determined by Eq. (12) for different values of the Reynolds number Re = 50 (blue), 100 (black), 200 (red) at Pr = 1. The DNS data on $\langle D_T \rangle$ normalized using $u_0 l_{11}$ are shown in blue triangles (Re = 50), black squares (Re = 100), and red circles (Re = 200).

$$W = \frac{1-c}{\tau_R (1+\tau)} \exp\left(-\frac{E_a}{RT}\right)$$

Particle Clustering

- Large-scale clustering (large-scale inhomogeneous structures) in Stratified Turbulent Flows (with imposed mean temperature gradient)

$$L_c \gg \ell_0$$

- Small-Scale Tangling Clustering in Stratified Turbulent Flows

$$L_c \ll \ell_0$$

ℓ_0 is the integral (maximum) scale of turbulent motions

L_c is the characteristic size of clusters

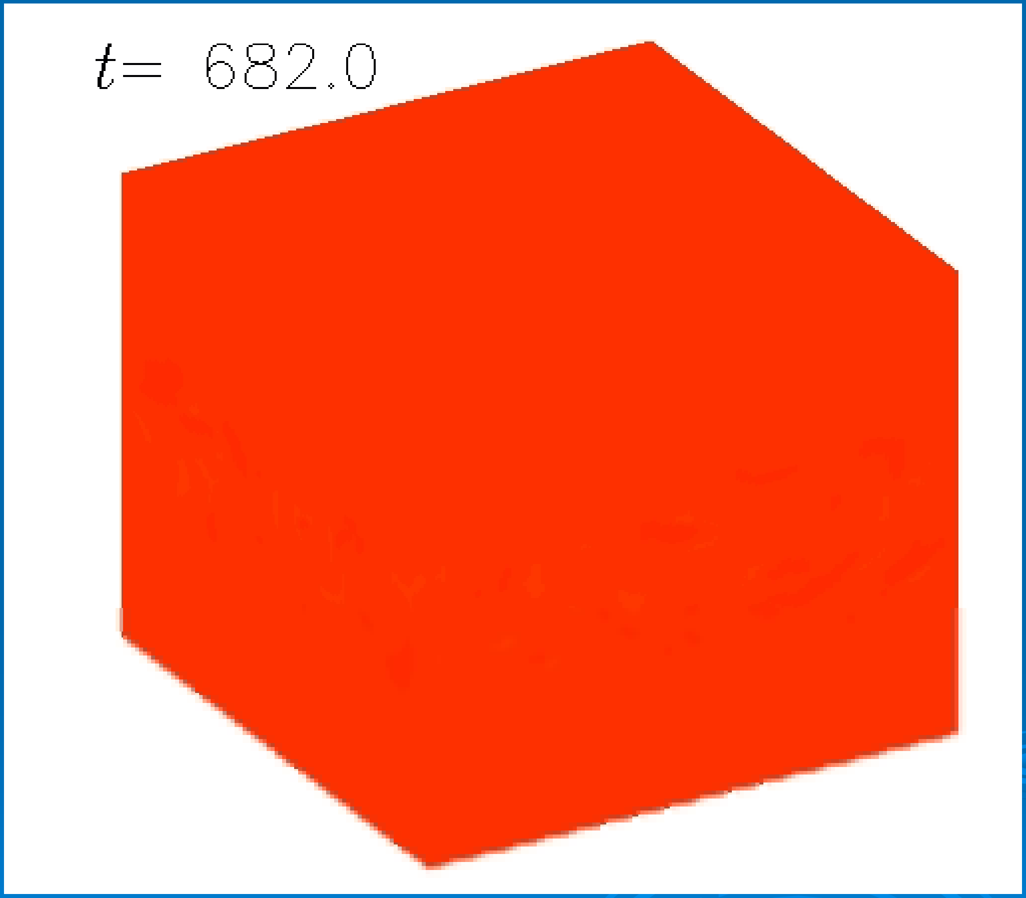
$$\frac{\partial \theta}{\partial t} \propto -(\mathbf{v}\cdot\boldsymbol{\nabla})T + ... \qquad \frac{\partial n'}{\partial t} \propto -(\mathbf{v}\cdot\boldsymbol{\nabla})N + ...$$

Large-Scale Clustering: DNS for Non-Inertial Particles in 3D Forced Turbulence

256^3

A white noise non-helical homogeneous and isotropic random forcing.

$$\frac{k_f}{k_1} = 5$$



All simulations are performed with the PENCIL CODE, which uses sixth-order explicit finite differences in space and a third-order accurate time stepping method.

Particle Flux in Turbulent Flow for Non-inertial Particles

Turbulent Flux of Particles: $$\langle \mathbf{u}\, n' \rangle = N\, \mathbf{V}^{\text{eff}} - D_T \boldsymbol{\nabla} N$$

Effective Pumping Velocity: $$\mathbf{V}^{\text{eff}} = D_T \frac{\boldsymbol{\nabla} \rho}{\rho} = -D_T \frac{\boldsymbol{\nabla} T}{T}$$

Turbulent Diffusion Coefficient:

$$Pe = \frac{u_0 \ell_0}{D_m} \ll 1 \qquad D_T = \frac{q-1}{q+1}\, \frac{u_0\, \ell_0}{3}\, \text{Pe}$$

$$Pe = \frac{u_0 \ell_0}{D_m} \gg 1 \qquad D_T = \frac{u_0\, \ell_0}{3}$$

Turbulent flux of particles

$$\frac{\partial n}{\partial t} + \mathrm{div}\left(n\,\mathbf{u}\right) = D\,\Delta\,n \qquad n = \overline{N} + n' \qquad \frac{\partial \overline{N}}{\partial t} + \mathrm{div}\left(\langle n'\mathbf{u}\rangle\right) = D\,\Delta\,\overline{N}$$

$$\bar{\mathbf{V}} = 0$$

Fluctuations of particles number density:

$$\frac{\partial n'}{\partial t} - D\,\Delta\,n' + \mathrm{div}\left(n'\mathbf{u} - \langle n'\mathbf{u}\rangle\right) = -\mathrm{div}\left(\overline{N}\,\mathbf{u}\right) \qquad \mathrm{LHS} \sim \frac{n'}{\tau}$$

$$n' \sim -\tau\,\overline{N}\,\mathrm{div}\,\mathbf{u} - \tau\,(\mathbf{u}\cdot\nabla)\overline{N}$$

$$\overline{\mathbf{J}}_T \equiv \langle \mathbf{u}\,n' \rangle \sim -\tau\,\overline{N}\langle \mathbf{u}\,\mathrm{div}\,\mathbf{u}\rangle - \tau\langle \mathbf{u}\,(\mathbf{u}\cdot\nabla)\rangle\overline{N}$$

$D_T \equiv D_{ij} = \tau\langle u_i\,u_j\rangle$ - turbulent diffusion tensor

$\mathbf{V}_{eff} = -\tau\langle\,\mathbf{u}\,\mathrm{div}\,\mathbf{u}\rangle$ - effective velocity

$\overline{\mathbf{J}}_T = \overline{N}\,\mathbf{V}_{eff} - D_T\nabla\overline{N}$ - turbulent flux of particles

Turbulent thermal diffusion of non-inertial particles

$$\mathbf{v}_p = \mathbf{u}$$

$$\rho \, \mathrm{div} \; \mathbf{u} + (\mathbf{u} \cdot \nabla) \rho \approx 0$$

$$\mathrm{div} \; \mathbf{u} \approx -\mathbf{u} \cdot \frac{\nabla \rho}{\rho}$$

Equation of state for ideal gas yields:

$$\frac{\nabla \bar{\rho}}{\bar{\rho}} \approx -\frac{\nabla \bar{T}}{\bar{T}}$$

$$\frac{\partial \bar{N}}{\partial t} + \mathrm{div} \left(\bar{N} \, \bar{\mathbf{V}} + \bar{N} \, \mathbf{V}_{eff} - D_T \nabla \bar{N} \right) = 0$$

$$\mathbf{V}_{eff} = -\tau \langle \mathbf{u} \, \mathrm{div} \; \mathbf{u} \rangle$$

$$-\tau \langle u_i \, \mathrm{div} \; \mathbf{u} \rangle = \tau \langle u_i \, u_j \rangle \frac{\nabla_j \bar{\rho}}{\bar{\rho}} = D_T \frac{\nabla_i \bar{\rho}}{\bar{\rho}}$$

$$\mathbf{V}_{eff} = D_T \frac{\nabla \bar{\rho}}{\bar{\rho}} = -D_T \frac{\nabla \bar{T}}{\bar{T}}$$

Turbulent thermal diffusion of inertial particles

M. R. Maxey, J. Fluid Mech. 174, 441 (1987)

$$\frac{d\,\mathbf{v}_p}{d\,t} = -\frac{\mathbf{v}_p - \mathbf{u}}{\tau_p}$$

$$\mathbf{v}_p = \mathbf{u} - \tau_p \frac{d\,\mathbf{u}}{d\,t} + O(\tau_p^2)$$

$$\mathbf{V}_{eff} = -\tau \langle \mathbf{v}_p \ \mathrm{div}\ \mathbf{v}_p \rangle$$

$$\mathrm{div}\ \mathbf{v}_p = \mathrm{div}\ \mathbf{u} + \tau_p \frac{\Delta P}{\rho} + O(\tau_p^2)$$

$$\boxed{\mathbf{V}_{eff} = -D_T \alpha \frac{\nabla \bar{T}}{\bar{T}}}$$

$$\alpha \approx 1 + \left(\frac{m_p}{m_\mu}\right)\left(\frac{\bar{T}}{T_*}\right)\frac{\ln(\mathrm{Re})}{\mathrm{Pe}}$$

$$\bar{\mathbf{J}}_T = -D_T\, k_T \frac{\nabla \bar{T}}{\bar{T}} - D_T \nabla \bar{N}$$ - turbulent flux of particles

$$k_T = \alpha\, \bar{N}$$ - turbulent thermal diffusion ratio

Derivation of Effect of Turbulent Thermal Diffusion

- **All known approaches (including dimensional reasoning)**
- **Path integral approach (finite correlation time);** $\mathrm{V}_{\mathrm{eff}} = -\tau \langle \mathbf{v}(\mathbf{x}) \, \nabla \cdot \mathbf{v}(\mathbf{x}) \rangle$
- **The spectral tau approximation; Quasi-linear approach, etc.**

T. Elperin, N. Kleeorin and I. Rogachevskii

- Physical Review Letters **76**, 224 (1996)
- Physical Review E **55**, 2713 (1997)
- Physical Review Letters **80**, 69 (1998)
- Intern. Journal of Multiphase Flow **24**, 1163 (1998)
- Atmospheric Research **53**, 117 (2000)

T. Elperin, N. Kleeorin, I. Rogachevskii and D. Sokoloff

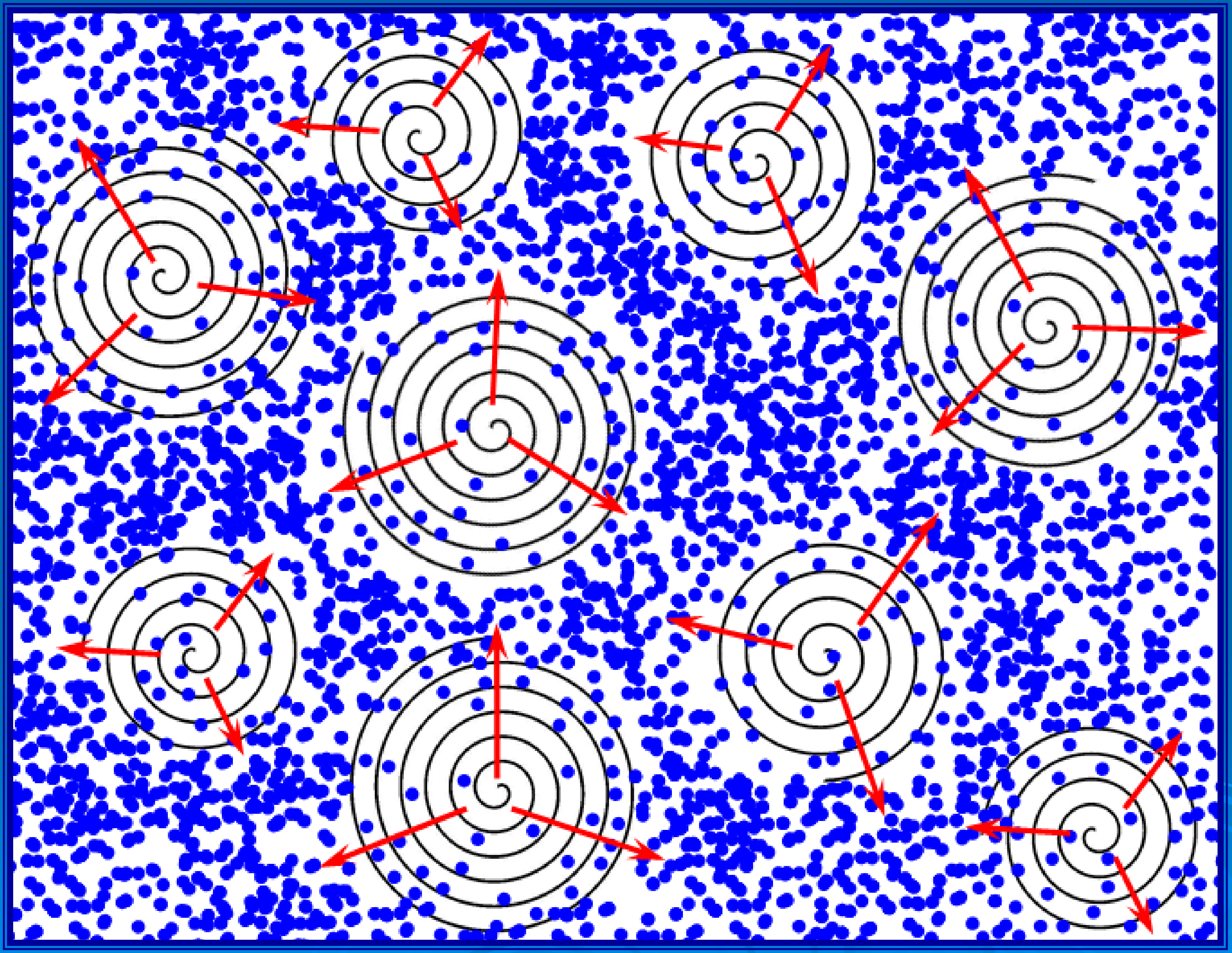
- Physical Review E **61**, 2617 (2000)
- Physical Review E **64**, 026304 (2001)

R.V.R. Pandya and F. Mashayek, Physical Review Letters **88**, 044501 (2002)

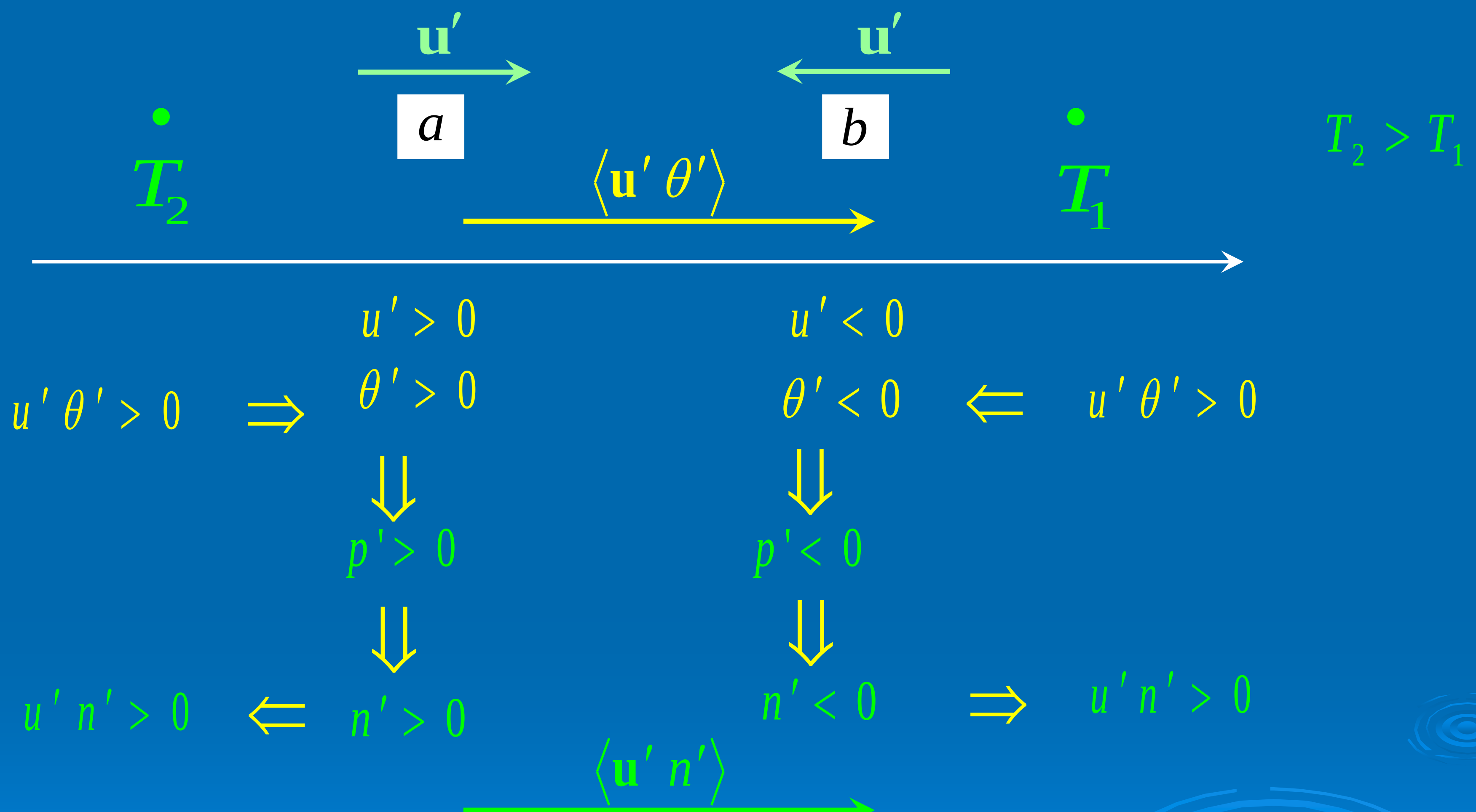
M.W. Reeks, Intern. Journal of Multiphase Flow **31**, 93 (2005)

M. Sofiev, V. F. Sofieva, T. Elperin, N. Kleeorin, I. Rogachevskii and S. Zilitinkevich, **J.** Geophys. Res. 114, D18209 (2009).

Particle Inertia Effect

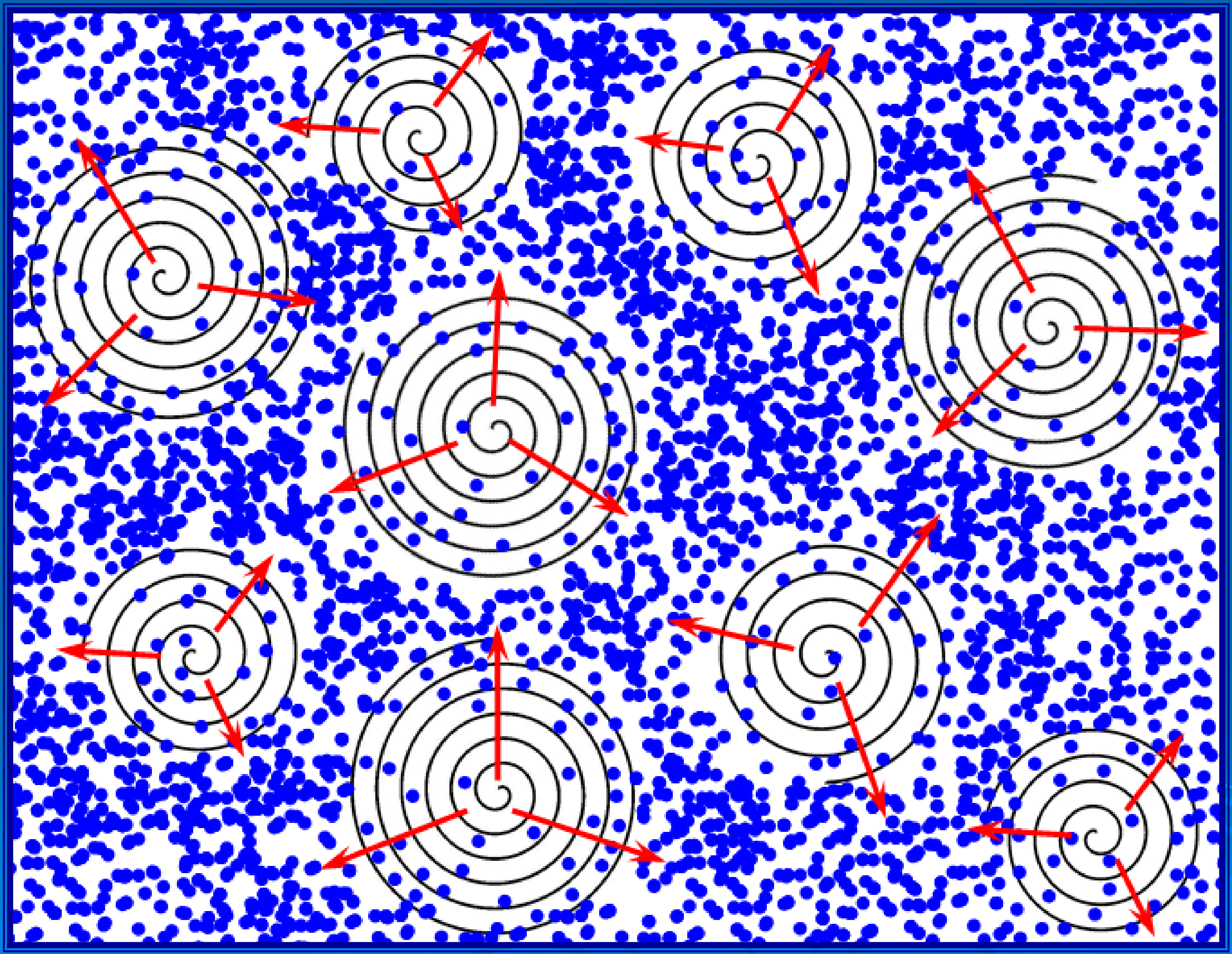


Turbulent Thermal Diffusion: Inertial Particles



Non-diffusive mean flux of particles is in the direction of the mean heat flux (i.e., in the direction of minimum fluid temperature).

Particle Inertia Effect

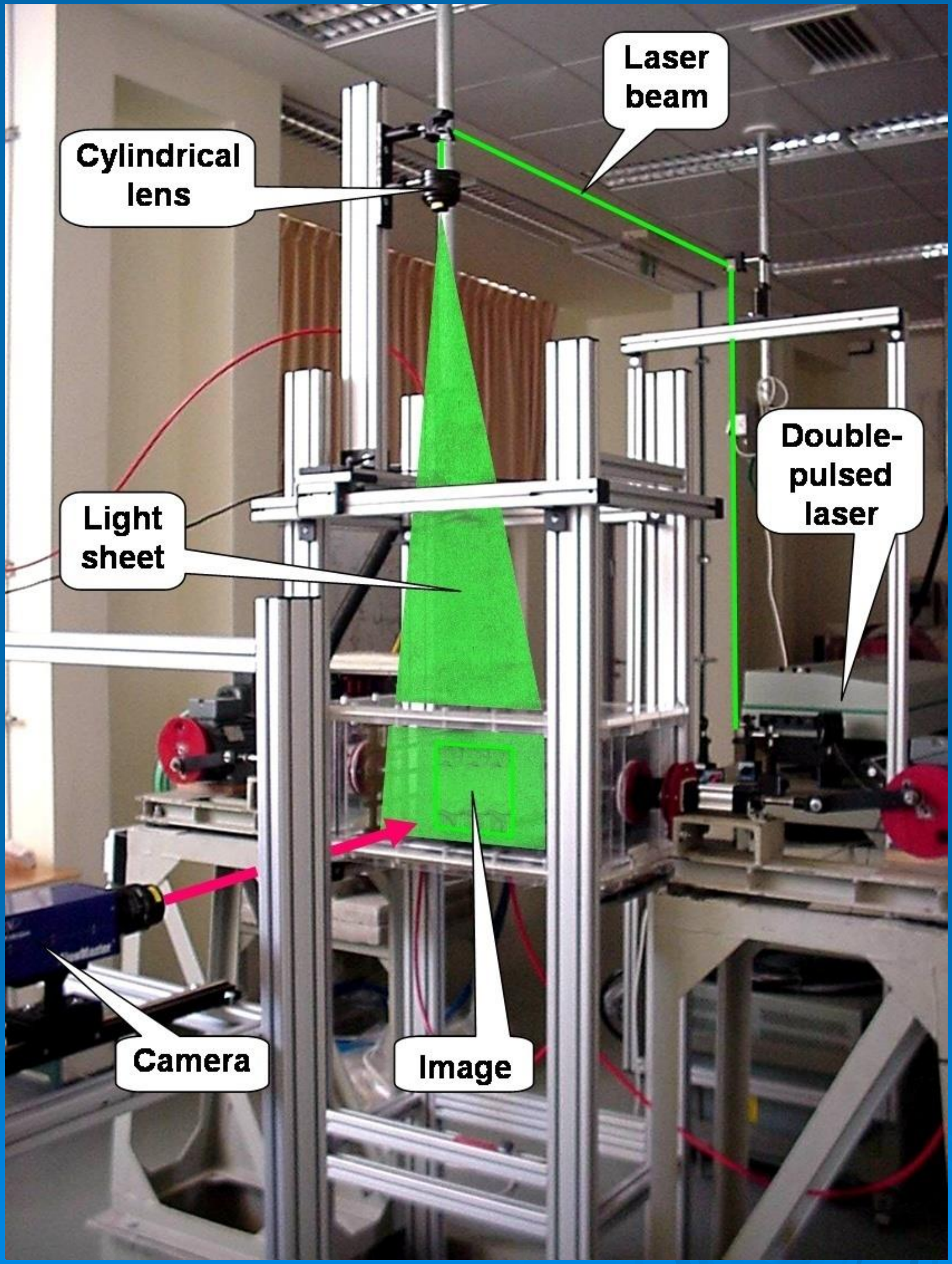


Non-inertial Particles

$$\longrightarrow \boldsymbol{\nabla}\overline{\rho}$$

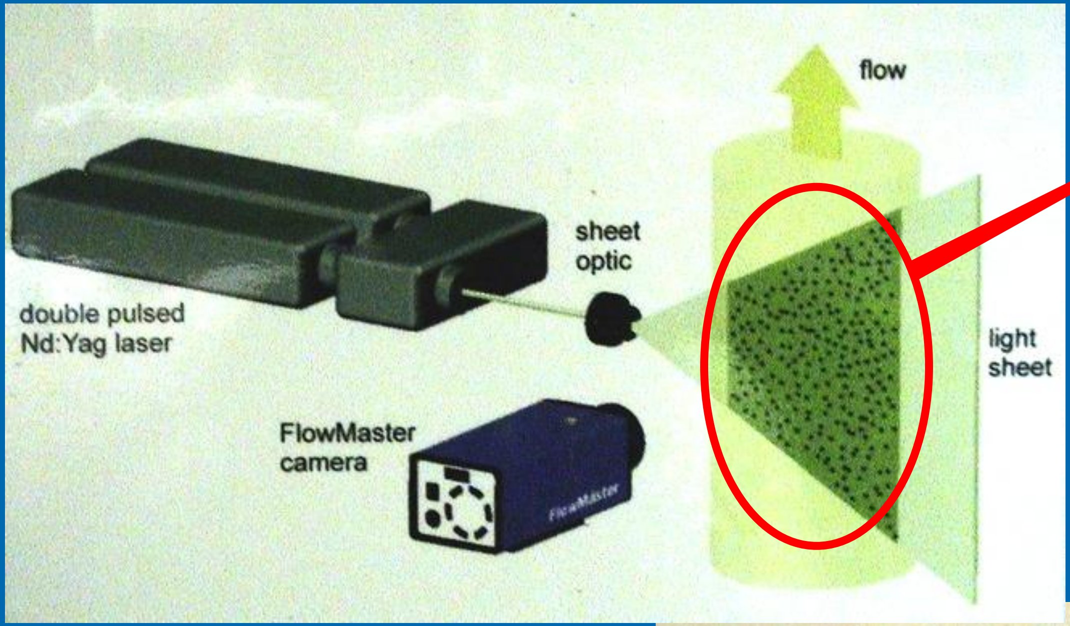
a	b
$U_x > 0$	$U_x < 0$
$\boldsymbol{\nabla}\cdot\boldsymbol{U} < 0$	$\boldsymbol{\nabla}\cdot\boldsymbol{U} > 0$
$n > 0$	$n < 0$
$n\,U_x > 0$	$n\,U_x > 0$

$$\longrightarrow \overline{n\boldsymbol{U}}$$



Experimental set - up: oscillating grids turbulence generator and particle image velocimetry system

Particle Image Velocimetry System

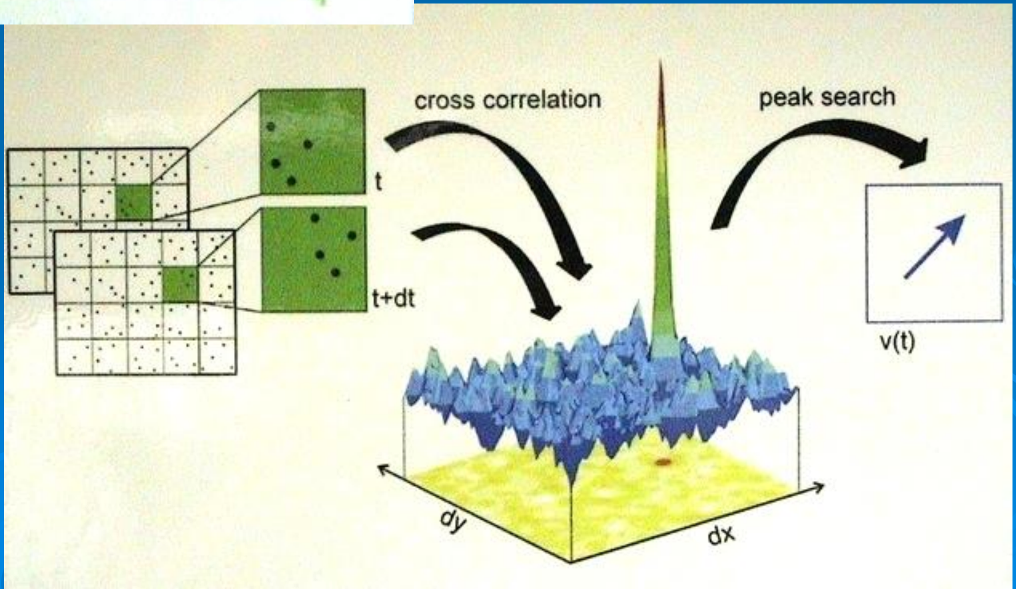


Raw image of the incense smoke tracer particles in oscillating grids turbulence

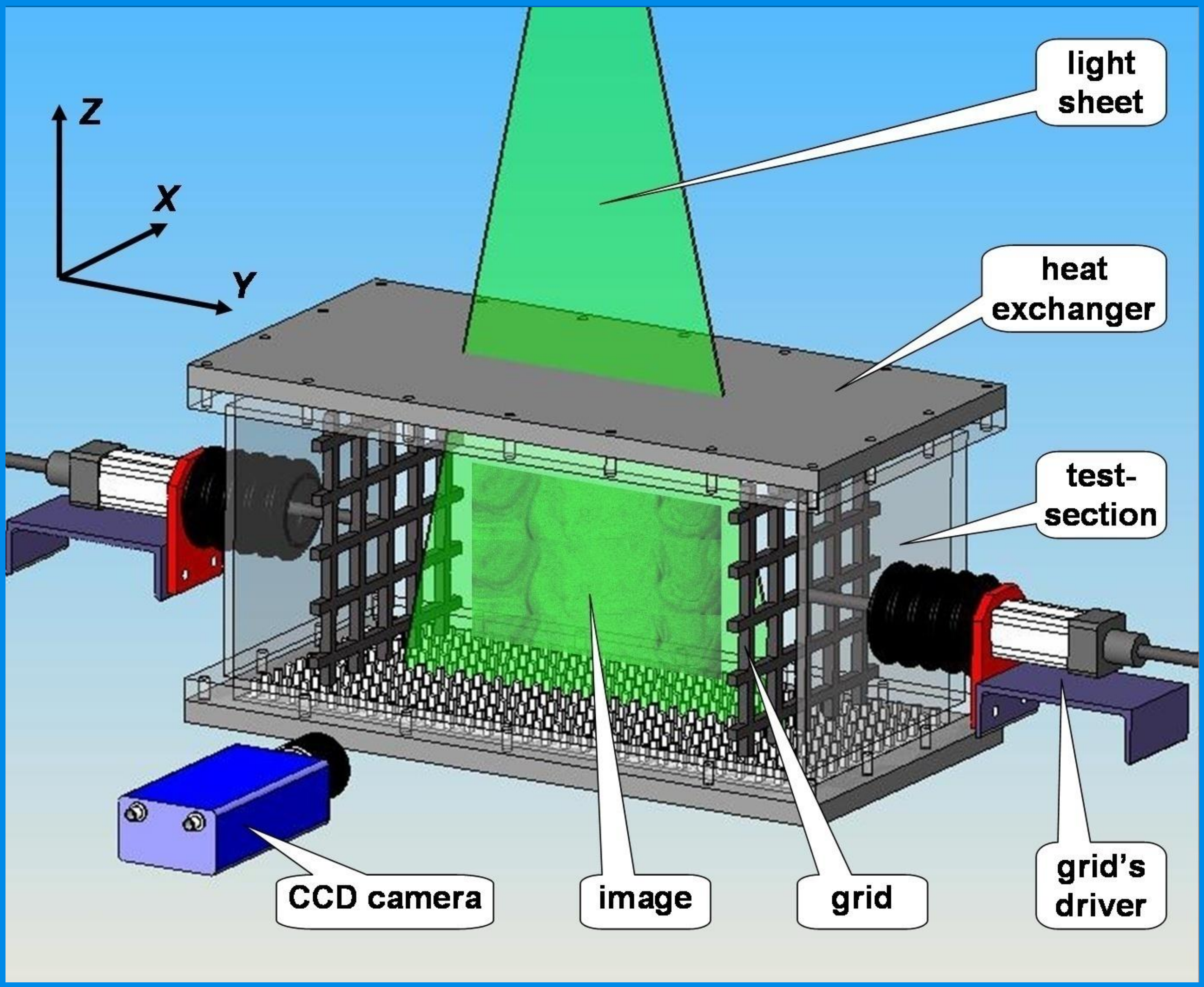
Particle Image

Velocimetry Data

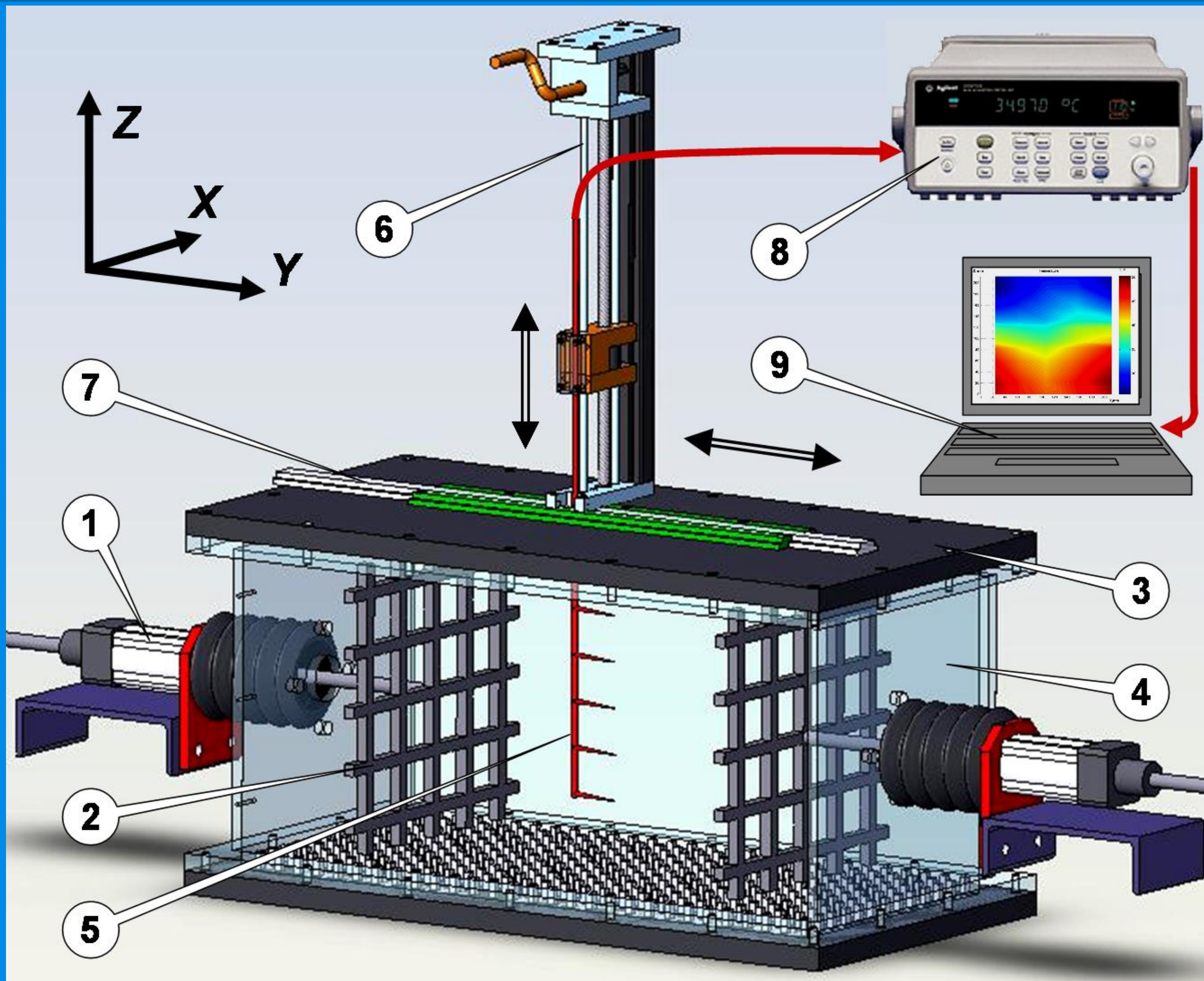
Processing



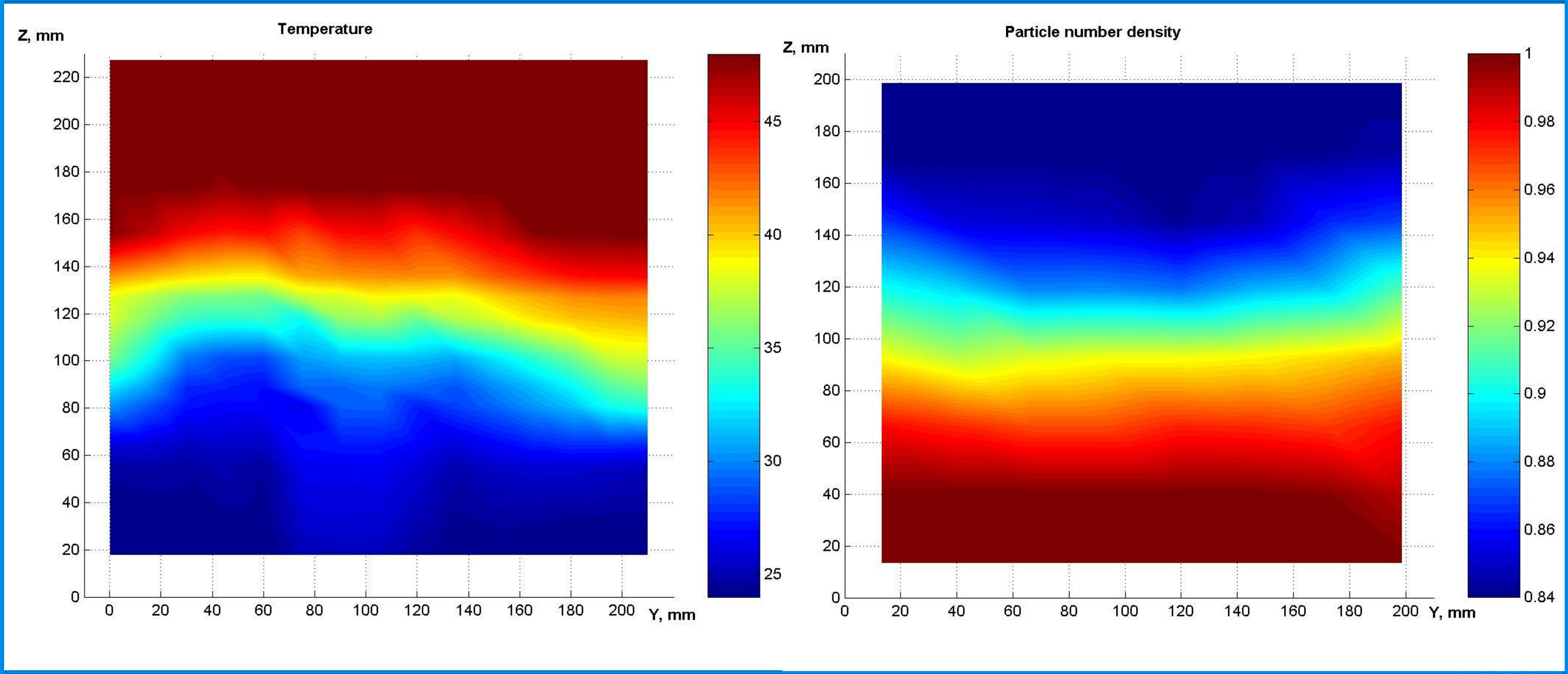
Experimental Set–up



Experimental Set–up for Temperature Measurements

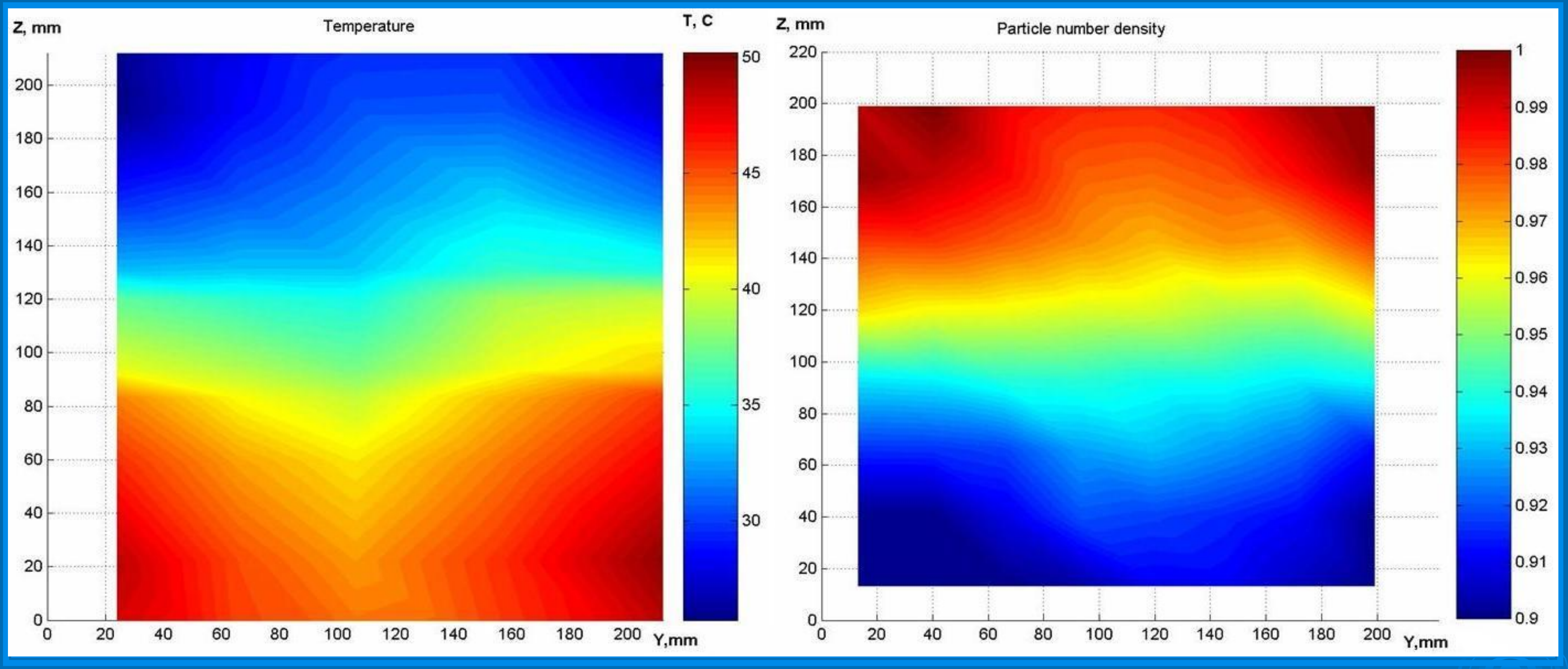


Mean Temperature and Particle Number Density (Stable Stratification , f = 10.5 Hz)



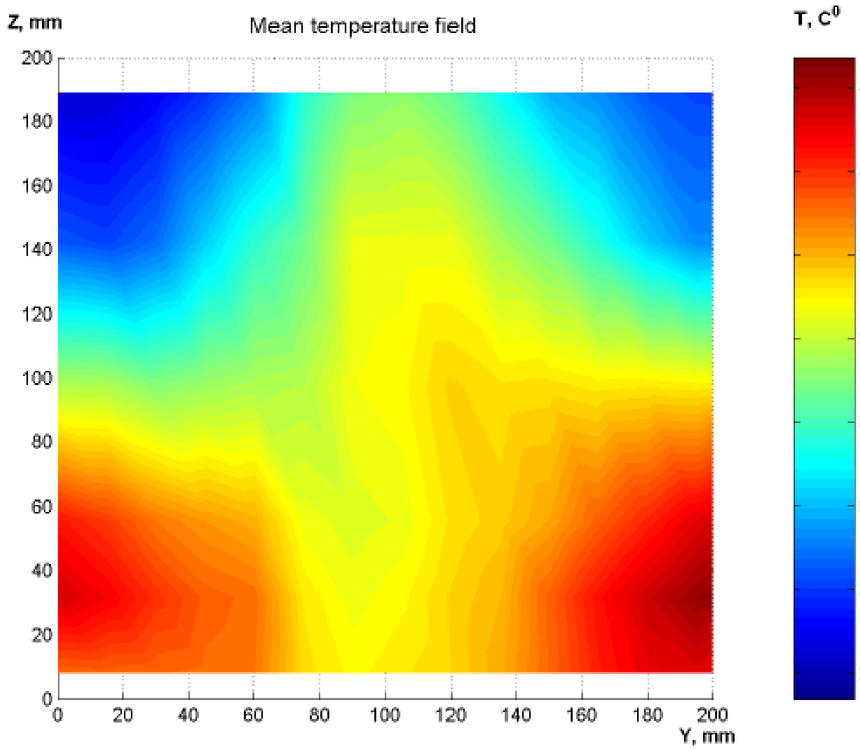
$\bar{T}(z)$ $\bar{N}(z)$

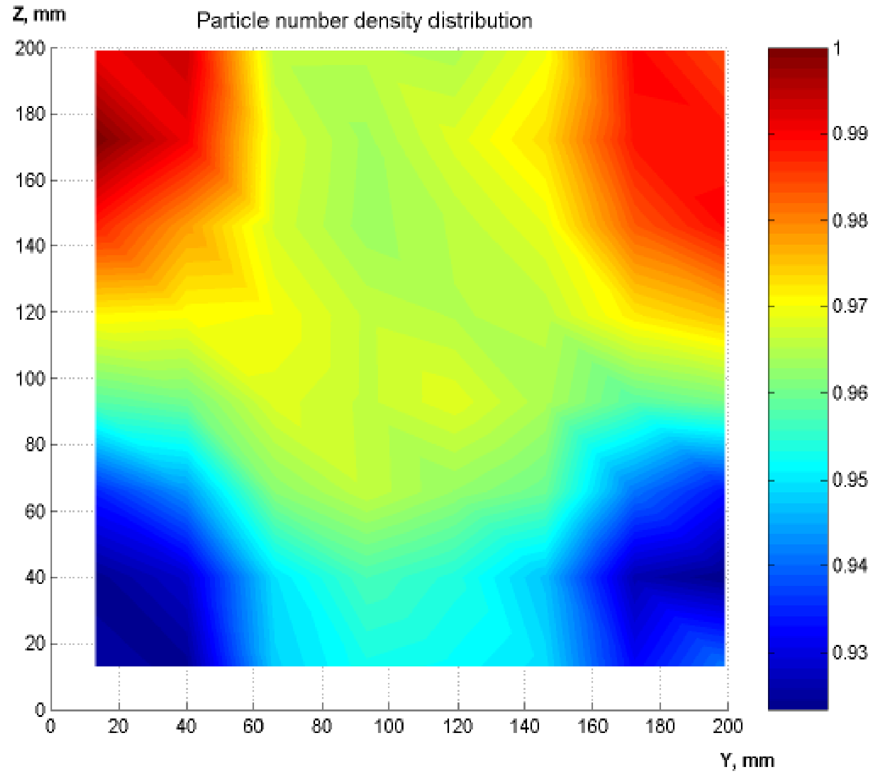
Mean Temperature and Particle Number Density (Unstable Stratification, f = 10.5 Hz)



$\bar{T}(z)$ $\bar{N}(z)$

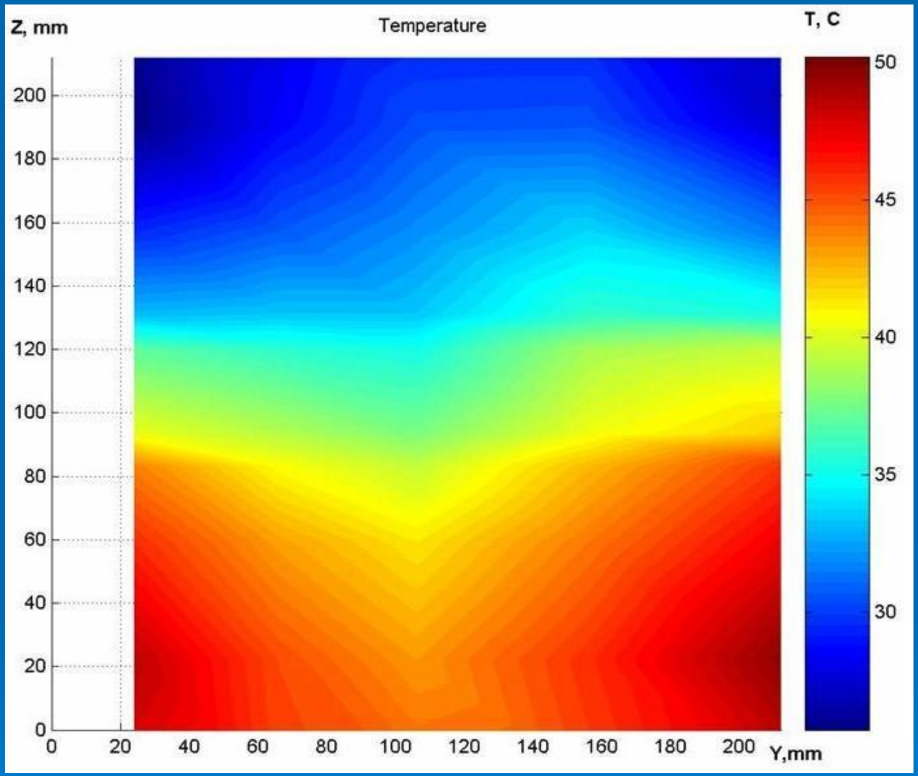
Mean Temperature and Particle Number Density (Unstable Stratification, f = 6 Hz)



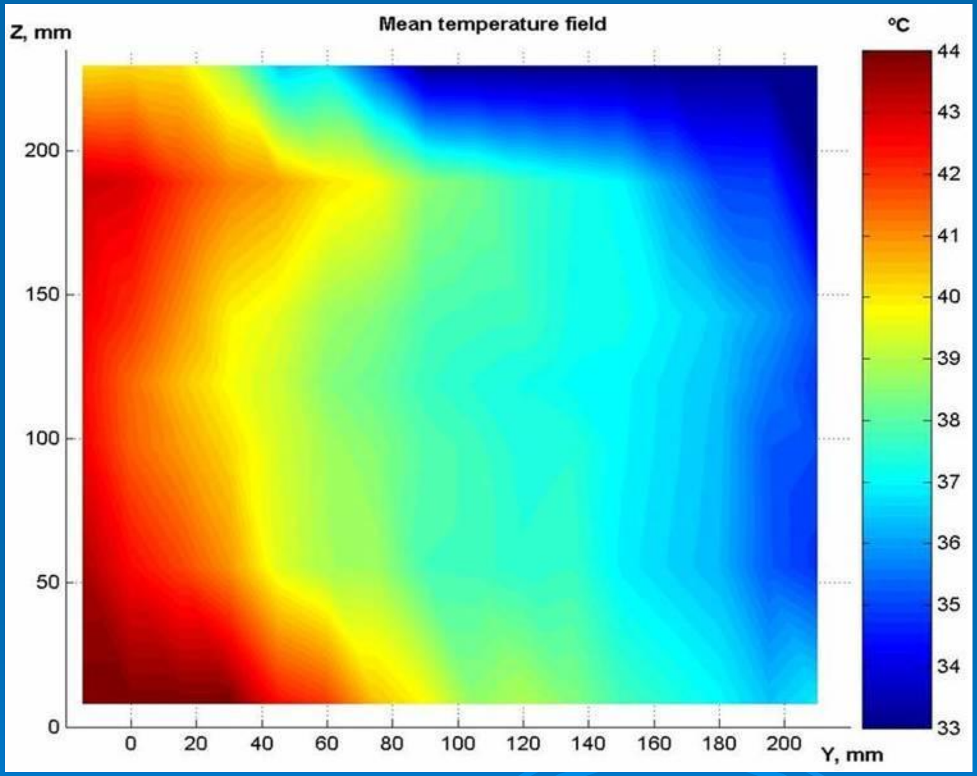


$\bar{T}(z)$ $\bar{N}(z)$

Mean Temperature Fields in Forced and Unforced Turbulent Convection

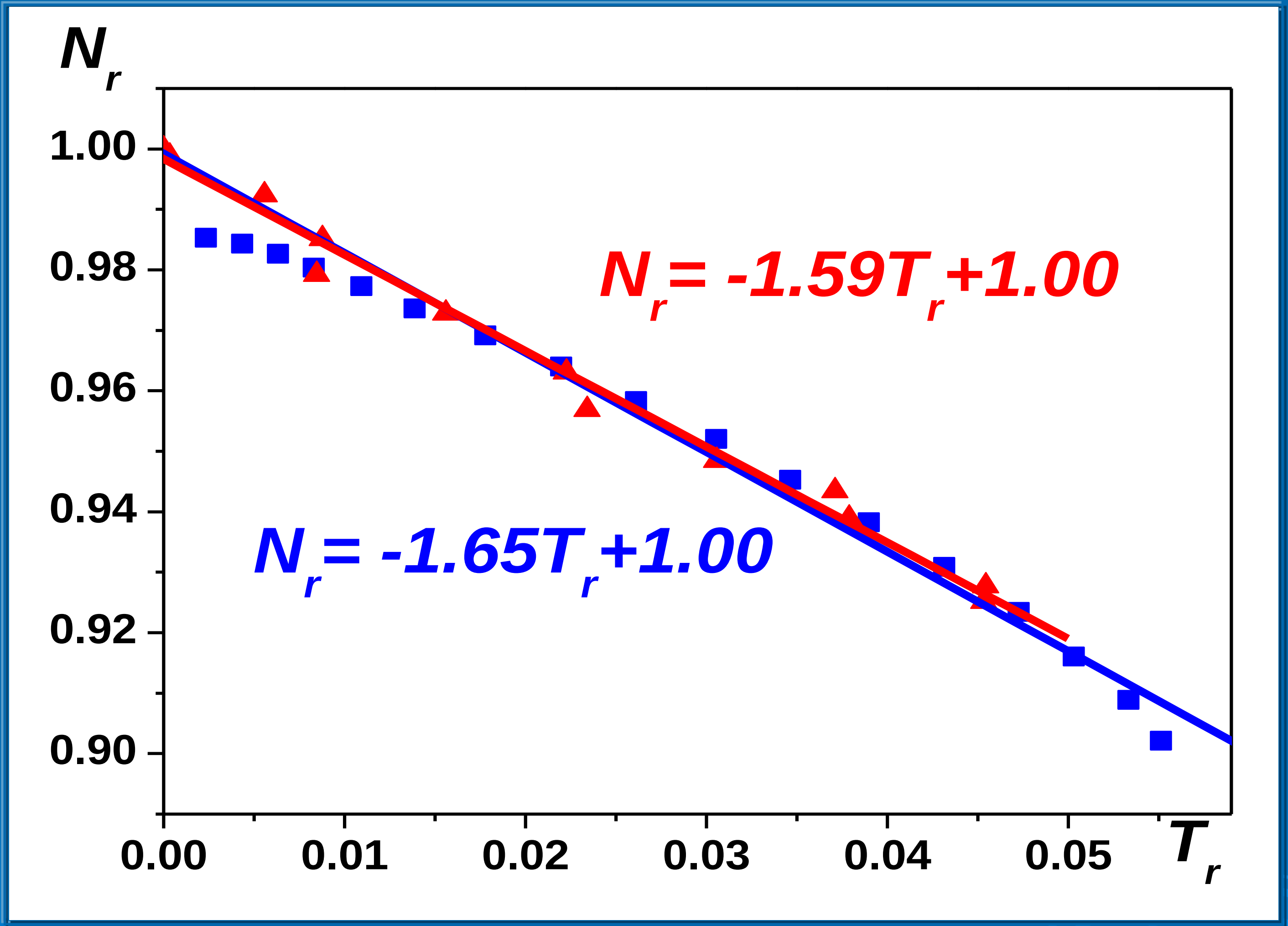


Forced turbulent convection (two oscillating grids)



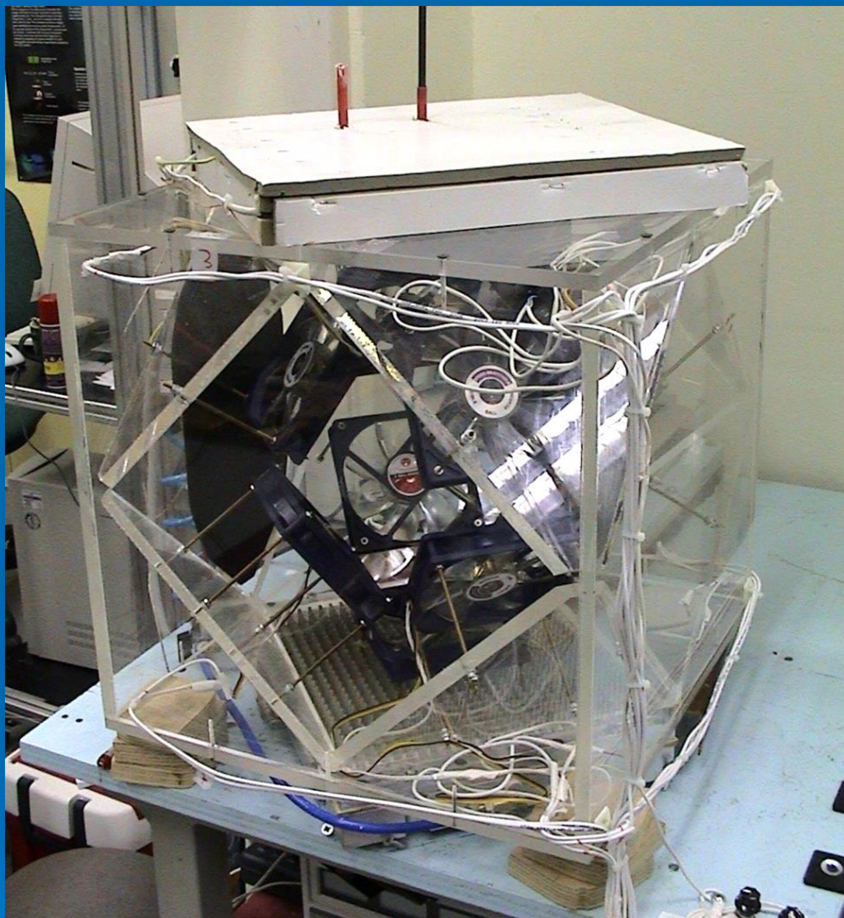
Unforced convection

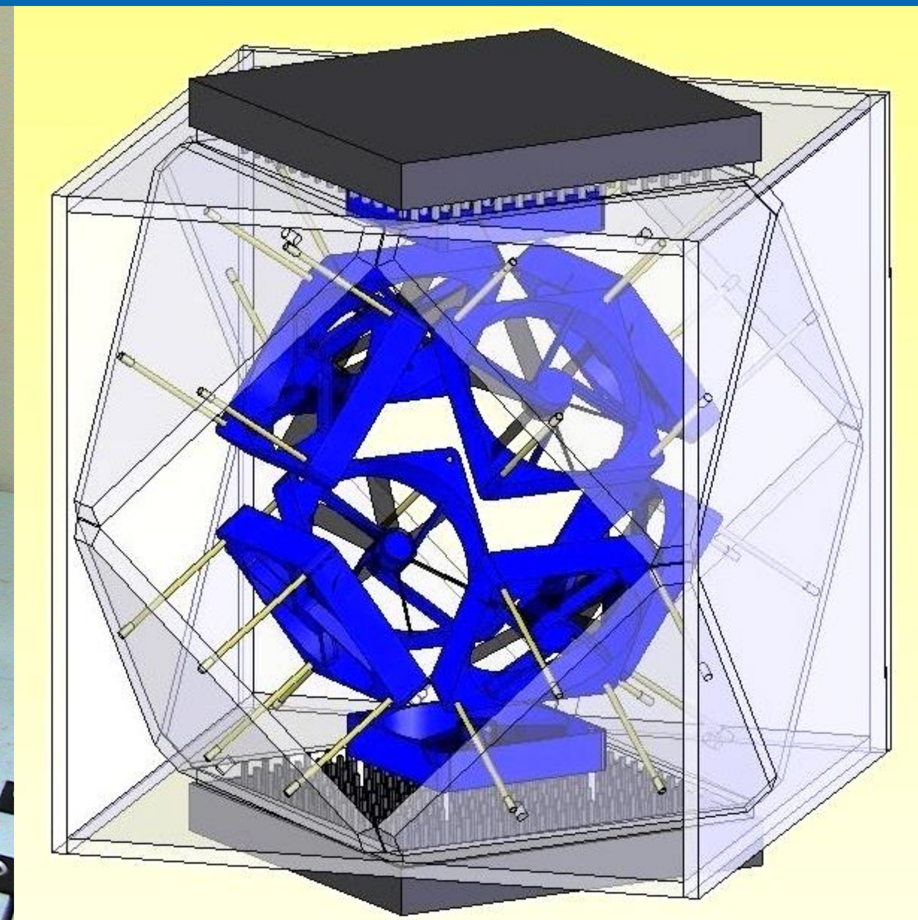
Turbulent Thermal Diffusion



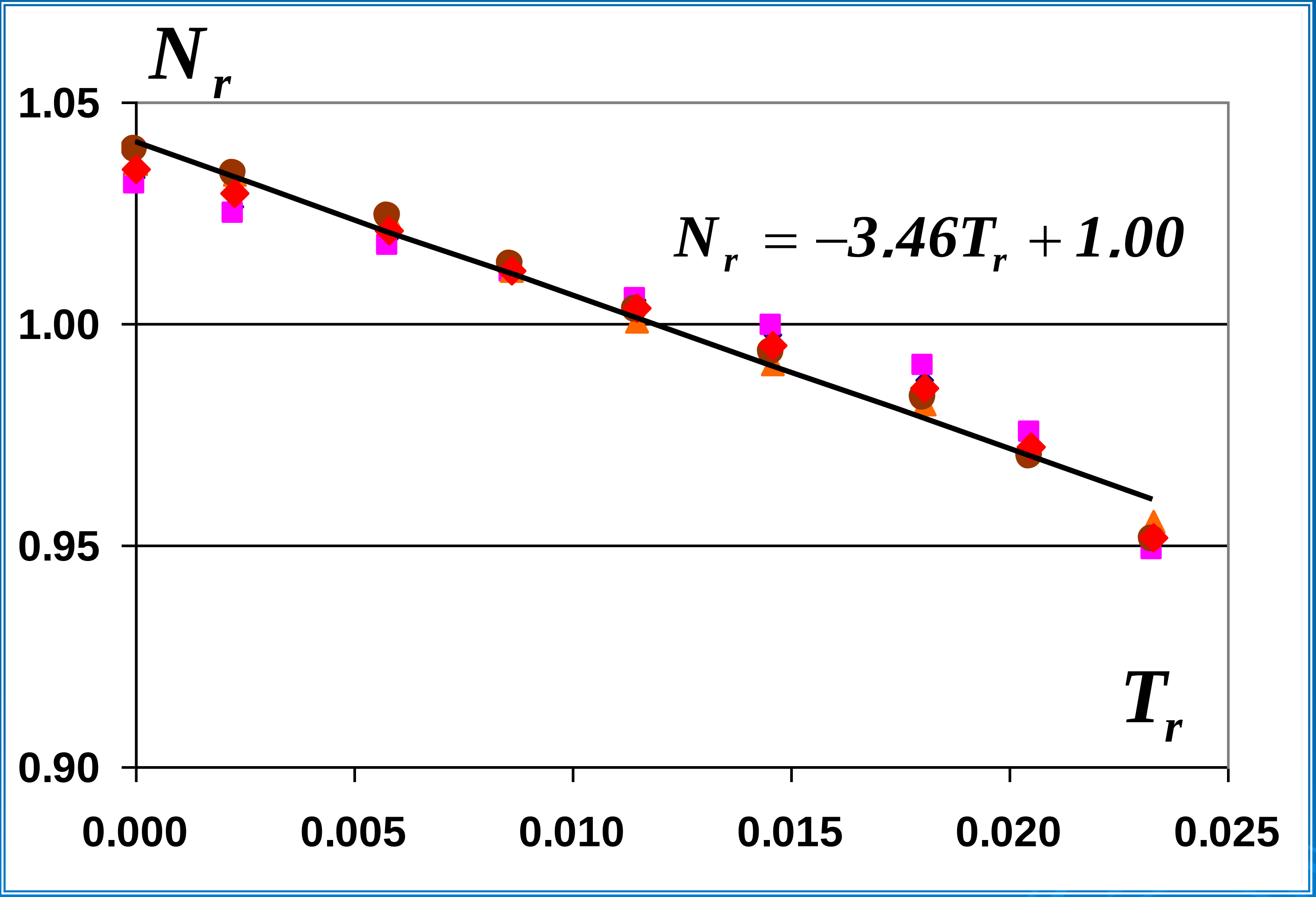
Normalized mean particle number density vs. normalized temperature gradient:
■ - stable stratification, ▲ - unstable stratification.

Experimental set–up with ten fans





Turbulent Thermal Diffusion



Normalized mean particle number density vs. normalized temperature

DNS for Non-Inertial Solid Particles in 3D Forced Turbulence

$$\frac{\partial n}{\partial t} + \mathrm{div}\,(n\,\mathbf{u} - D\,\boldsymbol{\nabla} n) = 0,$$

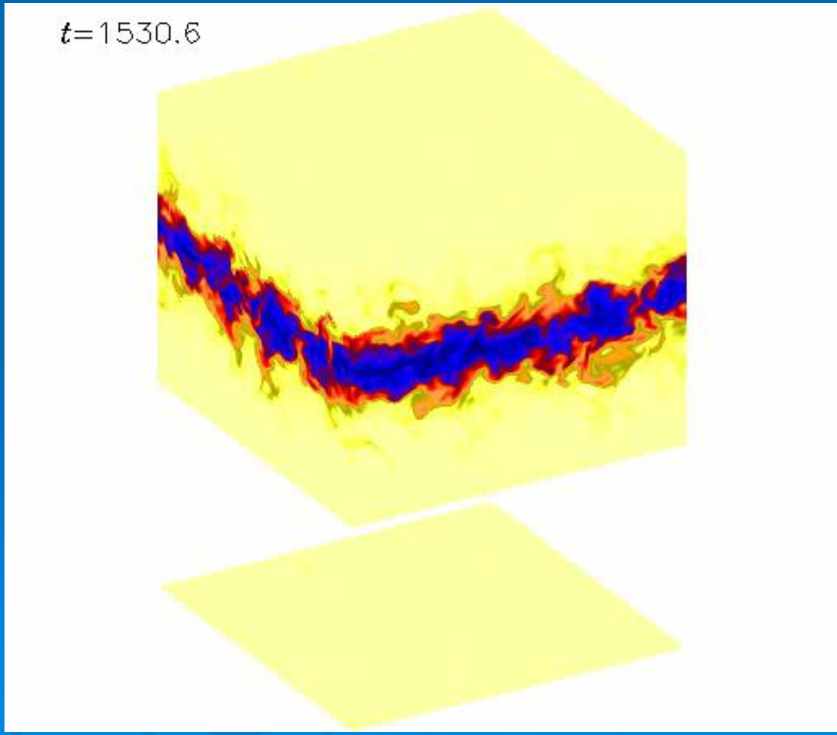
$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right)\mathbf{u} = -\frac{\boldsymbol{\nabla} p}{\rho} + \mathbf{f}_\nu + \mathbf{f},$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho\mathbf{u}) = 0,$$

$$T\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla}\right) s = \frac{1}{\rho}\mathrm{div}\,(\kappa\,\boldsymbol{\nabla} T) + I_\nu - I_{\mathrm{cooling}},$$

All simulations are performed with the PENCIL CODE, which uses sixth-order explicit finite differences in space and a third-order accurate time stepping method.

Entropy distribution:



BOUNDARY CONDITIONS are periodic in 3D.

$$\mathrm{Re} = \mathrm{Pe}_D = \frac{u_f}{D\,k_f} = 75$$

A white noise non-helical homogeneous and isotropic random forcing.

$$k_f = 5k_1$$

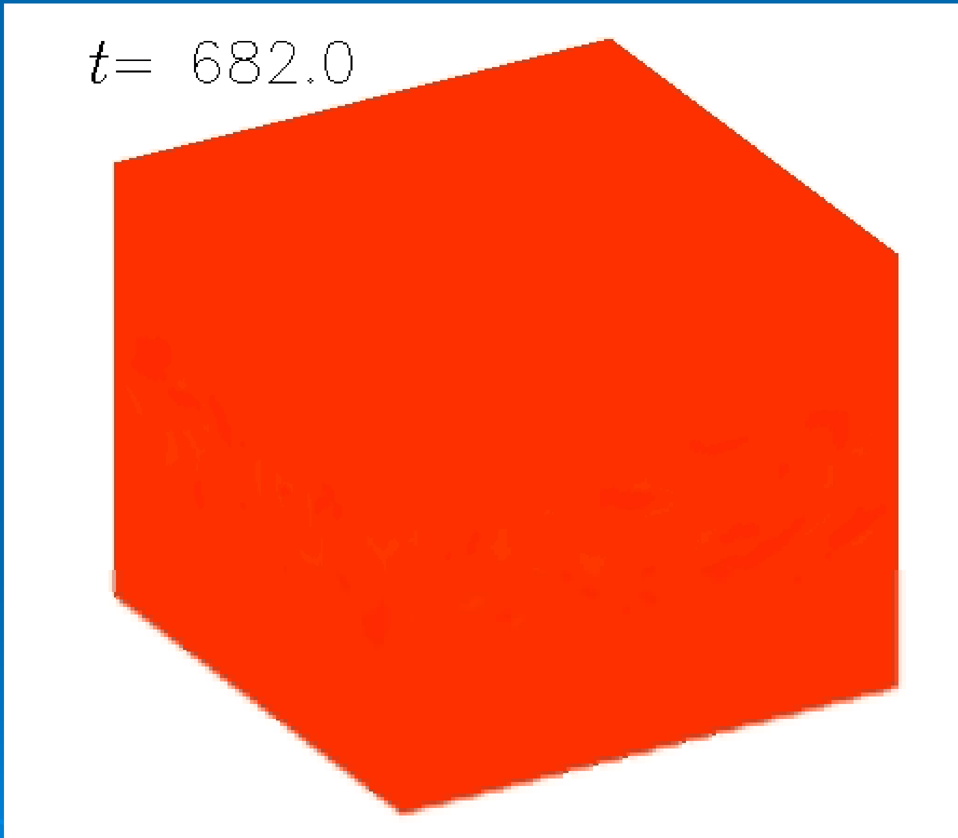
DNS for Non-Inertial Solid Particles in 3D Forced Turbulence

N.E.L. Haugen, N. Kleeorin, I. Rogachevskii and A. Brandenburg, Phys. Fluids 24, 075106 (2012).

$$\mathrm{Re} = \mathrm{Pe}_D = \frac{u_f}{D\,k_f} = 75$$

A white noise non-helical homogeneous and isotropic random forcing.

$$\frac{k_f}{k_1} = 5$$



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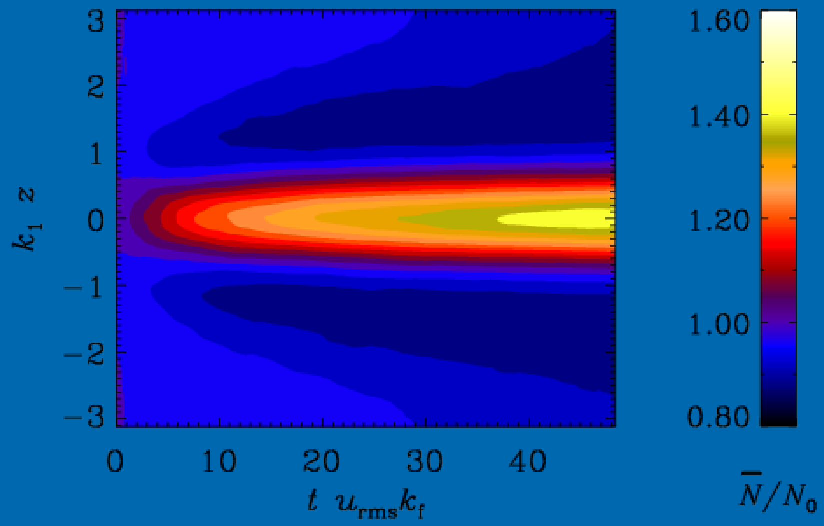
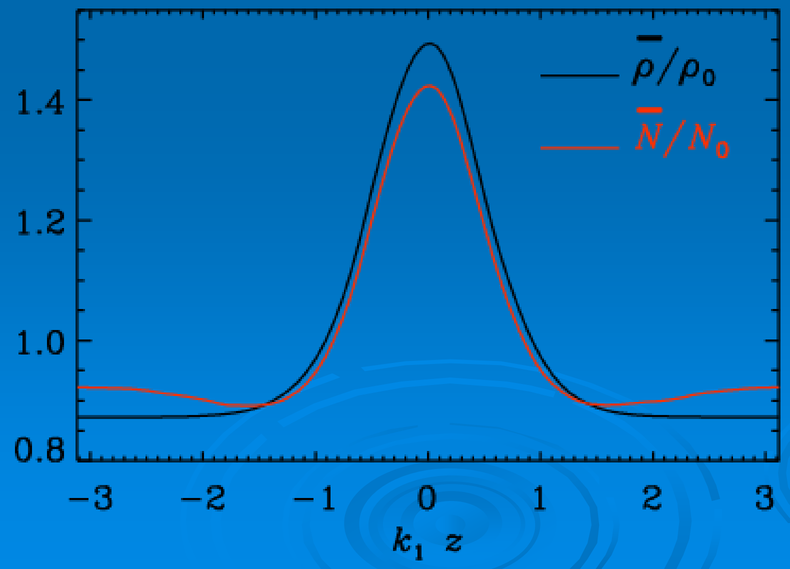
DNS for Non-Inertial Solid Particles in 3D Forced Turbulence





DNS for Inertial Solid Particles in 3D Forced Turbulence

Fluid: DNS in an Eulerian framework

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\boldsymbol{\nabla}\cdot\boldsymbol{U},$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = -\frac{1}{\rho}\left[\boldsymbol{\nabla}p - \boldsymbol{\nabla}\cdot(2\rho\nu\mathbf{S})\right] + \boldsymbol{f},$$

$$T\frac{\mathrm{D}s}{\mathrm{D}t} = \frac{1}{\rho}\boldsymbol{\nabla}\cdot K\boldsymbol{\nabla}T + 2\nu\mathbf{S}^2 - c_{\mathrm{P}}(T - T_{\mathrm{ref}}),$$

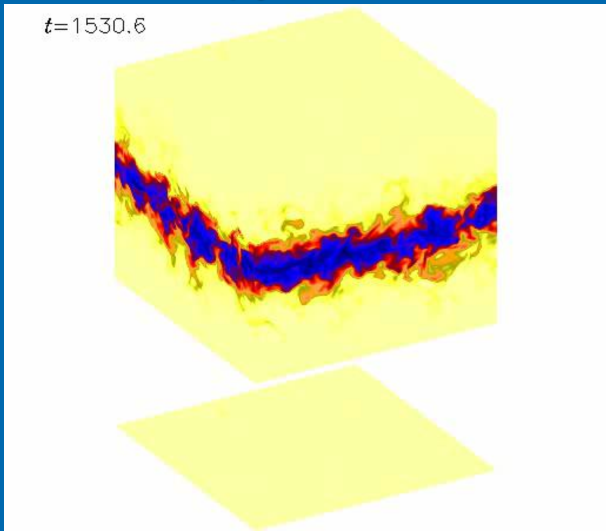
$$T_{\mathrm{ref}} = T_0 - \delta T\,\exp(-z^2/2\sigma^2), \qquad \sigma = 0.5$$

Particles: Lagrangian framework

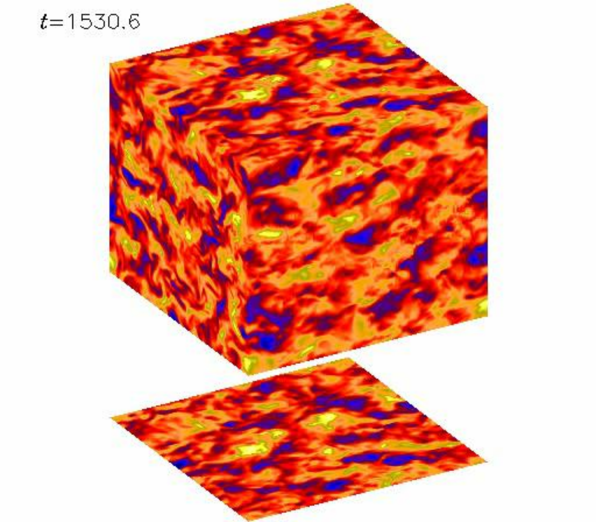
$$\frac{\mathrm{d}\boldsymbol{U}_{\mathrm{p}}}{\mathrm{d}t} = \boldsymbol{g} - \tau_{\mathrm{p}}^{-1}(\boldsymbol{U}_{\mathrm{p}} - \boldsymbol{U})$$

BOUNDARY CONDITIONS are periodic in 3D.
In the presence of gravity, particles are made elastically reflecting from the vertical boundaries.

Entropy



Fluid Velocity



DNS for Inertial Solid Particles in 3D Forced Turbulence

Particles: Lagrangian framework

$$\frac{\mathrm{d}\boldsymbol{U}_\mathrm{P}}{\mathrm{d}t} = \boldsymbol{g} - \tau_\mathrm{P}^{-1}(\boldsymbol{U}_\mathrm{P} - \boldsymbol{U})$$

$$\mathrm{d}\boldsymbol{X}/\mathrm{d}t = \boldsymbol{U}_\mathrm{P}$$

1. Particles are treated as point particles (point-particle approximation).
2. One-way coupling approximation, i.e., there is an effect of the fluid on the particles only, while the particles do not influence the fluid motions.
3. BOUNDARY CONDITIONS are periodic in 3D. In the presence of gravity, particles are made elastically reflecting from the vertical boundaries.

Stokes time

$$\tau_p = \frac{\rho_p}{\rho} \frac{d^2}{18\nu\,(1 - f_c)}$$

$$f_\mathrm{C} = 0.15\mathrm{Re}_p^{0.687}$$

$$\mathrm{Re}_p = \frac{|\mathbf{U}_p - \mathbf{U}|\,d}{\nu}$$

Stokes number

$$\mathrm{St} = \frac{\tau_p}{\tau_k}$$

$$\tau_k = \frac{\tau_f}{\sqrt{\mathrm{Re}}},$$

DNS for Inertial Particles

N.E.L. Haugen, N. Kleeorin, I. Rogachevskii and A. Brandenburg,
Phys. Fluids 24, 075106 (2012).

$\mathrm{Re} = 240$

$\mathrm{St} = 0.9$

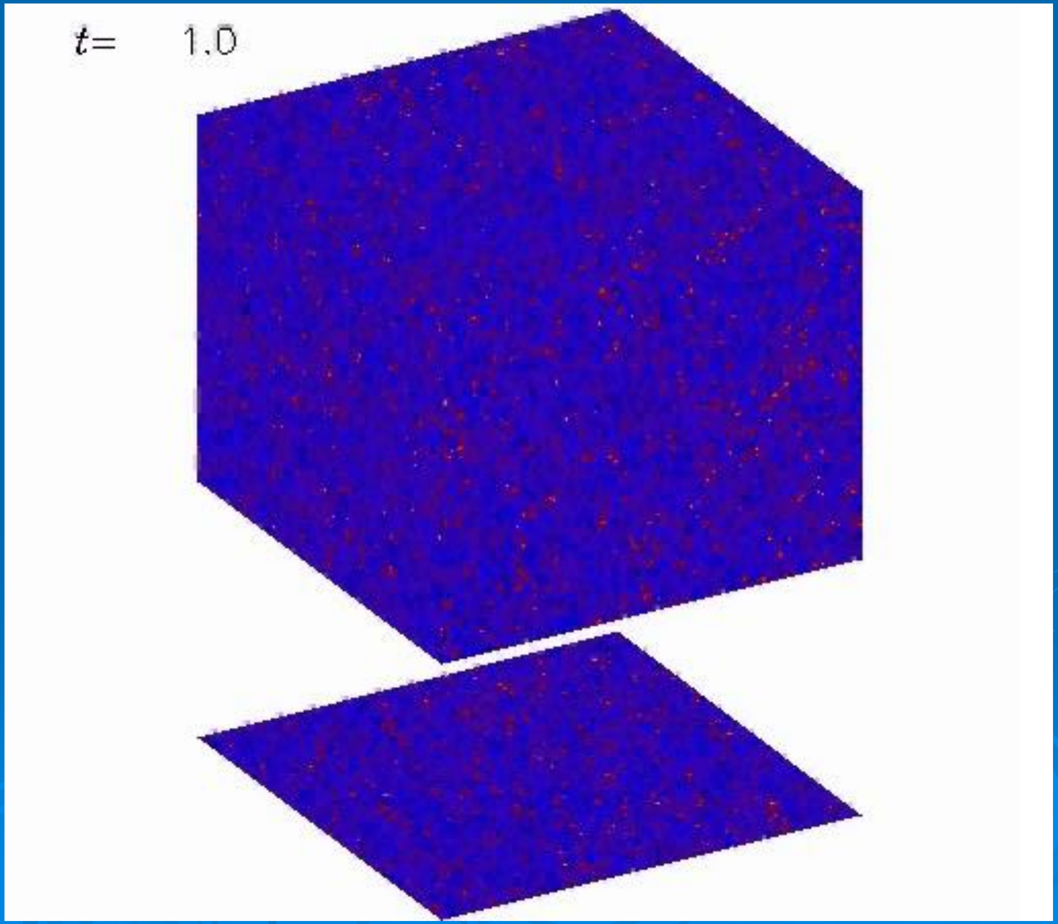
$k_f = 5$

without gravity

bottom-plane: $z = -3$

$$\mathrm{St} = \frac{\tau_p}{\tau_k},$$

$$\tau_k = \frac{\tau_f}{\sqrt{\mathrm{Re}}},$$



DNS for Inertial Particles

$\mathrm{Re} = 240$

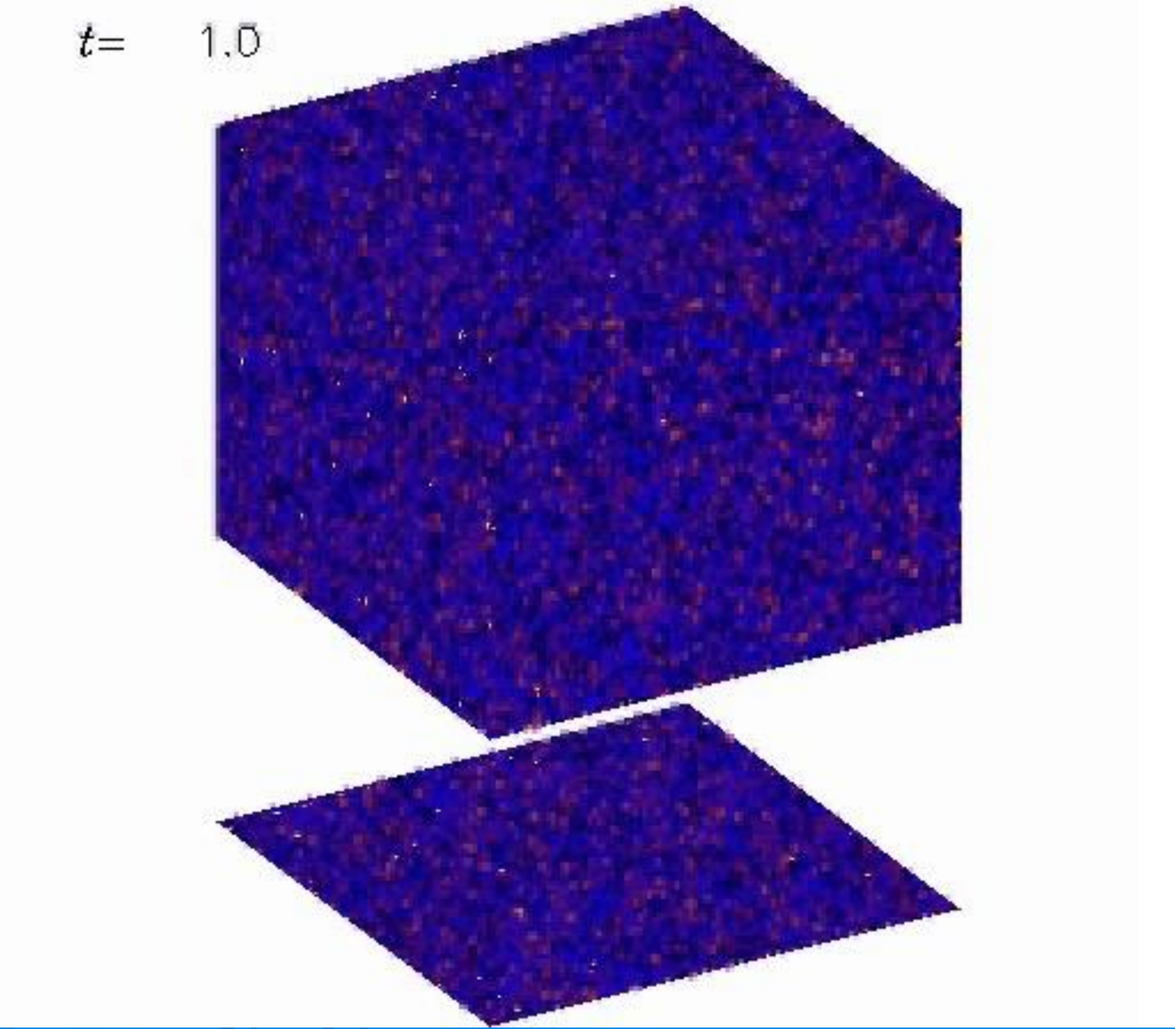
$\mathrm{St} = 0.9$

$k_f = 5$

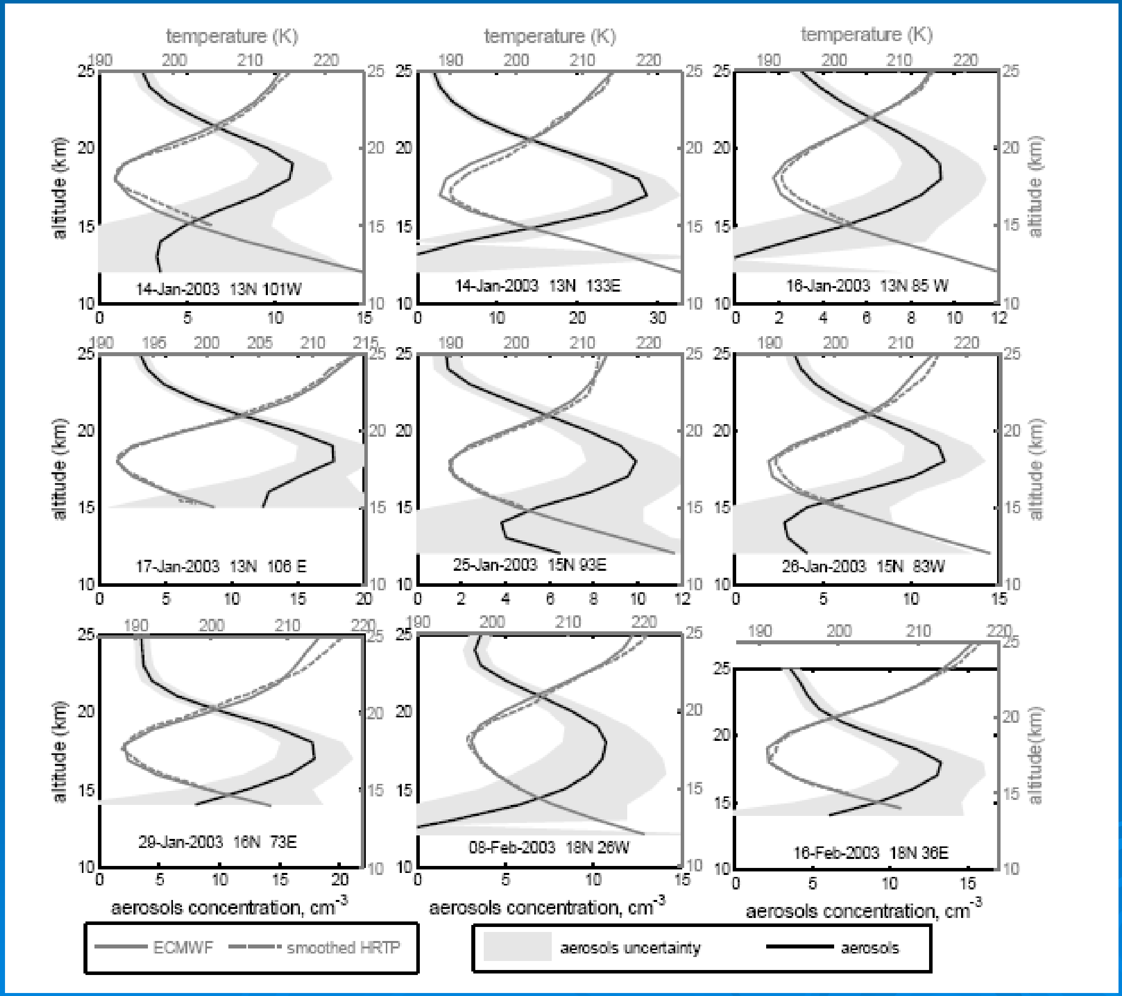
without gravity

middle-plane: $z = 0$

$\mathrm{St} = \frac{\tau_p}{\tau_k}, \qquad \tau_k = \frac{\tau_f}{\sqrt{\mathrm{Re}}},$



Distribution of Number Density of Aerosols (black) and Mean Temperature Distribution (gray) (Satellite Gomos Data)



M. Sofiev, V. F. Sofieva, T. Elperin, N. Kleeorin, I. Rogachevskii and S. Zilitinkevich, J. Geophys. Res. 114, D18209 (2009).

The ratio $\left| V_{eff} / W \right|$ for typical atmospheric parameters (different temperature gradients and different particle sizes)

a_*	1 K / 100 m	1 K / 200 m	1 K / 300 m
$1\,\mu$m	13	6.5	4.33
$5\,\mu$m	3.4	1.7	1.13
$10-20\,\mu$m	3	1.5	1
$30\,\mu$m	2.7	1.35	0.9

Time of Formation of Aerosol Layers

	1 K/100 m	1 K/200 m
$a_* = 30\ \mu m$	11 min	105 min
$a_* = 100\ \mu m$	1 min	120 min

$$t_T \propto \frac{L_T}{|\mathbf{V}_{eff} - \mathbf{W}|}$$

T. Elperin, N. Keeorin, I. Rogachevskii, *Atmospheric Research*, **53**, 117 (2000).

New Development in Theory and Experiments

G. Amir, A. Eidelman, T. Elperin, N. Kleeorin, I. Rogachevskii, Phys. Rev Fluids, in press (2017)

Turbulent Flux of Particles: $$\langle \mathbf{u}\, n' \rangle = N\, \mathbf{V}^{\rm eff} - D_T \boldsymbol{\nabla} N$$

Effective Pumping Velocity:

$$V^{\rm eff} = -\frac{2 D_T\, \alpha}{\sqrt{3}\,(B\,\delta_{_T})^{4/3}} \left[\frac{\pi}{6} + \arctan\left(\frac{2(B\,\delta_{_T})^{2/3} - 1}{\sqrt{3}}\right) - \frac{\sqrt{3}}{6} \ln \frac{\left[1 + (B\,\delta_{_T})^{2/3}\right]^3}{1 + (B\,\delta_{_T})^2}\right] \frac{\boldsymbol{\nabla} T}{T}.$$

$$V^{\rm eff} = -\alpha\, D_T \left[1 - \frac{1}{4}(B\,\delta_{_T})^{2/3}\right] \frac{\boldsymbol{\nabla} T}{T}, \qquad B\,\delta_{_T} \ll 1$$

$$V^{\rm eff} = -\frac{4\pi A\, D_T}{3^{3/2}}\, (B\,\delta_{_T})^{-4/3} \frac{\boldsymbol{\nabla} T}{T}, \qquad B\,\delta_{_T} \gg 1$$

$$\alpha^{\rm eff} \equiv \delta_{_N} / \delta_{_T} \qquad \delta_{_N} \equiv \ell_0 |\boldsymbol{\nabla} N| / N \qquad \delta_{_T} = \ell_0 \frac{|\boldsymbol{\nabla} T|}{T}$$

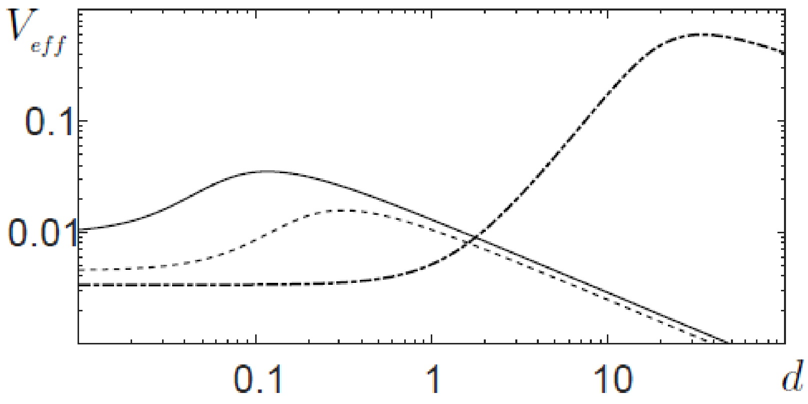
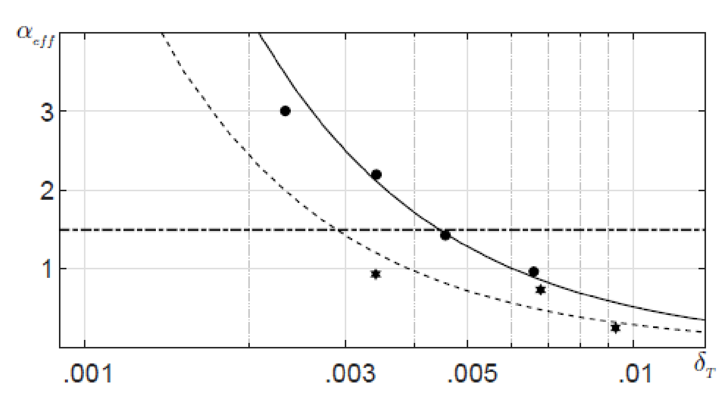


FIG. 2. Effective velocity $V^{\rm eff}$ measured in the units of the r.m.s turbulent vertical velocity, $u_z^{\rm (rms)}$, versus the particle diameter d (μm) for atmospheric conditions where the parameter $B/\alpha = 1$ (dashed-dotted) and for laboratory experiments conditions: the oscillating grid turbulence where the parameter $B/\alpha = 30$ (solid) and the multi-fan produced turbulence where the parameter $B/\alpha = 18$ (dashed).



MNRAS 456, 3079–3089 (2016) doi:10.1093/mnras/stv2895

Turbulent thermal diffusion: a way to concentrate dust in protoplanetary discs

Alexander Hubbard★
Department of Astrophysics, American Museum of Natural History, New York, NY 10024-5192, USA

Turbulence acting on mixes of gas and particles generally diffuses the latter evenly through the former. However, in the presence of background gas temperature gradients, a phenomenon known as turbulent thermal diffusion appears as a particle drift velocity (rather than a diffusive term). This process moves particles from hot regions to cold ones. We re-derive turbulent thermal diffusion using astrophysical language and demonstrate that it could play a major role in protoplanetary discs by concentrating particles by factors of tens. Such a concentration would set the stage for collective behaviour such as the streaming instability and hence planetesimal formation.

$$\alpha - 1 \sim 1.7 \frac{St}{\alpha_{SS}} \ln St^{-1}$$

For $St = 3 \times 10^{-3}$ and $\alpha_{SS} = 10^{-3}$, equation (41) estimates $\alpha \sim 30.6$, which through equation (25) would imply extreme con-

For $St = 3 \times 10^{-3}$ and $\alpha_{SS} = 10^{-3}$, this becomes

$$|V_{TTD}| \sim 0.01 c_s \simeq 0.3 u_0$$

For $St = 3 \times 10^{-3}$ and $\alpha_{SS} = 10^{-3}$, equation (41) estimates $\alpha \sim 30.6$, which through equation (25) would imply extreme concentrations of particles in cold regions through TTD. Indeed, a temperature perturbation with amplitude $f_T T$ over a length ℓ_T has a corresponding $L_T = \ell_T / f_T$, so $f_T = 0.2$, $\ell_T = 0.6H$ satisfies $L_T = 3H \simeq 0.1R \ll R$ and stronger temperature gradients, at quasi-constant pressure, have been seen in simulations of MHD turbulence in protoplanetary discs (McNally et al. 2014). With those values, equation (26) implies a particle concentration by a factor of

$$T^{\alpha-1} \simeq (1 + f_T)^{\alpha-1} \simeq (1 + 0.2)^{30.6-1} \simeq 221: \qquad (43)$$

This is easily large enough to have strong effects on the behaviour of dust in protoplanetary discs and so TTD could act as a trigger for

Inertial Clustering of Small Solid Particles

- Inertia causes particles inside the turbulent eddies to drift out to the boundary regions between the eddies (i.e. regions with low vorticity or high strain rate and maximum of fluid pressure).

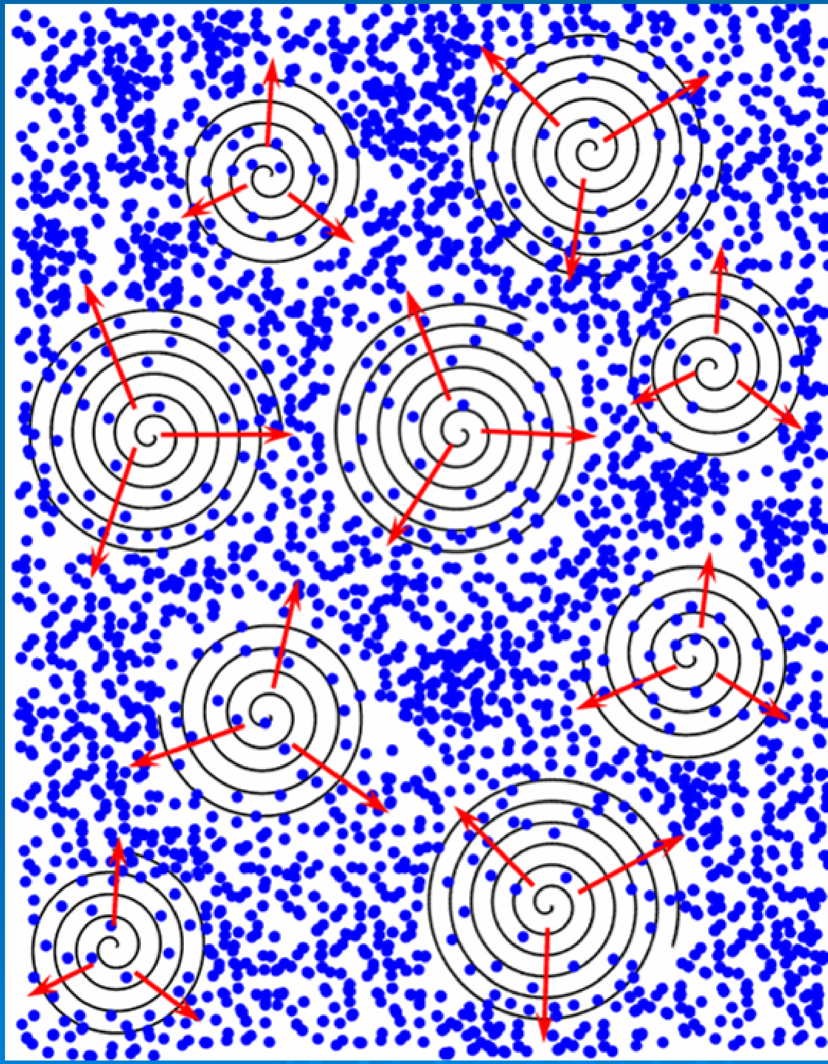
- This mechanism acts in a wide range of scales of turbulence.

- Scale-dependent turbulent diffusion causes relaxation of particle clusters.

- In small scales

$$D_T(\ell) \to D_m$$

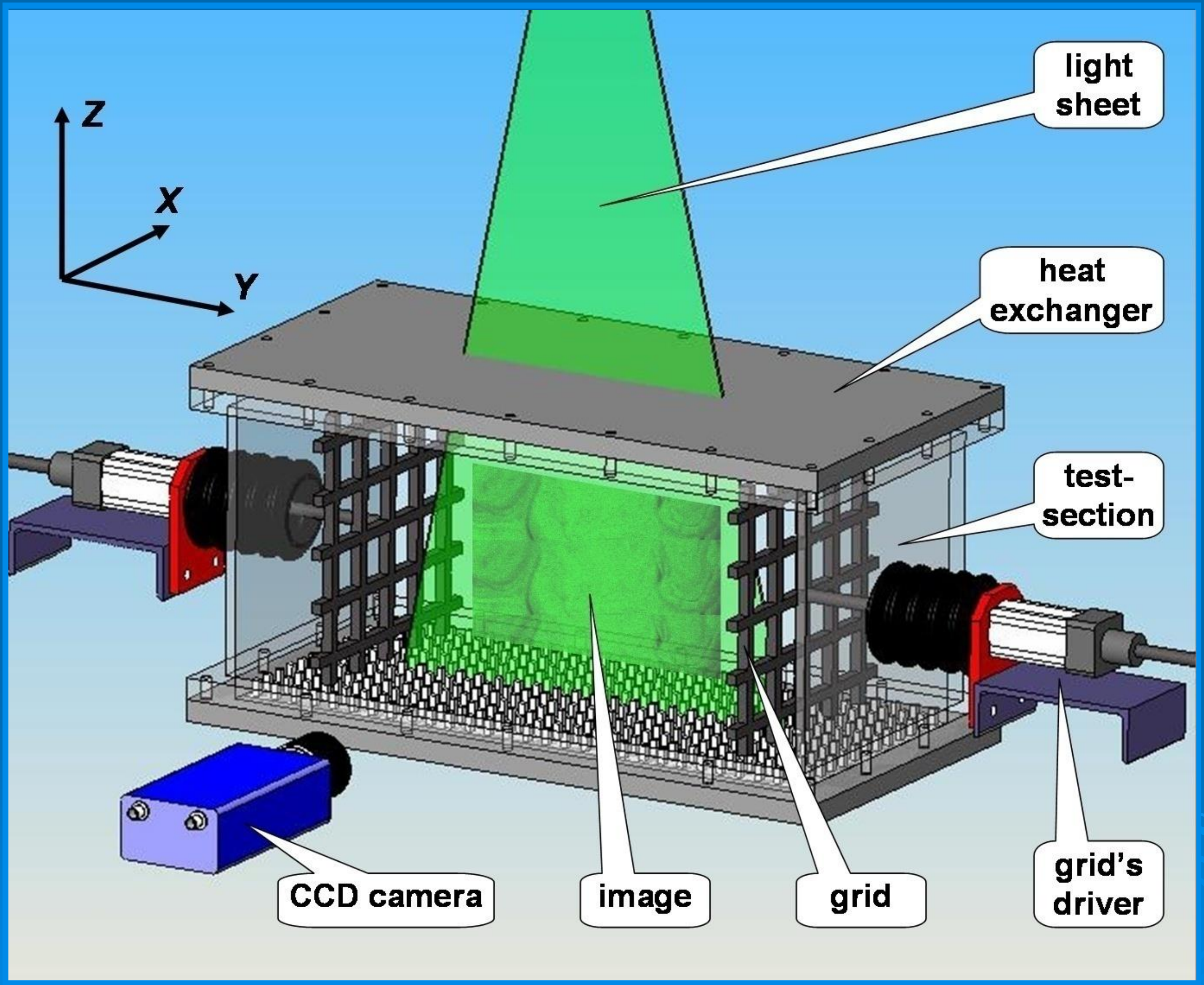
- Thus, clusters of particles are localized in small scales.



M. R. Maxey, J. Fluid Mech. 174, 441 (1987).

J.K. Eaton and J.R. Fessler, Int. J. Multiphase Flow 20, 169 (1994).

Experimental Set–up for Tangling Clustering



Parameters of Turbulence and Solid Particles in Experimental Study of Tangling Clustering in Air

$u_0 = \sqrt{\langle \mathbf{u}^2 \rangle} = 12\,\mathrm{cm/s}$ **is the r.m.s. velocity;**

$\ell_0 = 3.2\,\mathrm{cm}$ **is the integral (maximum) scale of turbulence;**

$\mathrm{Re} = u_0\,\ell_0/\nu = 250$ **is the Reynolds numbers;**

$\ell_\eta = \ell_0/\mathrm{Re}^{3/4} = 510\,\mu\mathrm{m}$ **is the Kolmogorov length scale;**

$\tau_\eta = \tau_0/\mathrm{Re}^{1/2} = 1.7 \times 10^{-2}\,\mathrm{s}$ **is the Kolmogorov time scale;**

$d_p = 10\,\mu\mathrm{m}$ **is the particle diameter;**

$\tau_s = 10^{-3}\,\mathrm{s}$ **is the Stokes time for the particles;**

$\mathrm{St} = \tau_s/\tau_\eta = 6 \times 10^{-2}$ **is the Stokes number for the particles;**

$\mathrm{Pe} = u_0\,\ell_0/D_m = 3 \times 10^9$ **is the Peclet number for the particles;**

$D_m = 1.4 \times 10^{-8}\,\mathrm{cm^2/s}$ **is the coefficient of molecular diffusion;**

Experimental Study of Tangling Clustering

- **Two-point correlation function of particle number density:**

$$\Phi(t,\mathbf{R}) = \langle n'(t,\mathbf{x})\, n'(t,\mathbf{x}+\mathbf{R})\rangle = N^2\,[G(t,\mathbf{R})-1]$$

- **Radial distribution function can be estimated as follows:**

$$G(\mathbf{R}) \approx \frac{N_{\Delta S}^{(p)}/\Delta S}{N_S^{(p)}/S}$$

$$n = N + n',$$
$$N = \langle n \rangle$$

$N_{\Delta S}^{(p)}$ **is the number of particle pairs separated by a distance:** $R \pm \frac{1}{2}\Delta R$

ΔS **is the area of the annular domain located between:** $R \pm \frac{1}{2}\Delta R$

S **is the area of the part of the image with the radius:** $R_{\text{max}} = 0.8\,\text{cm}$

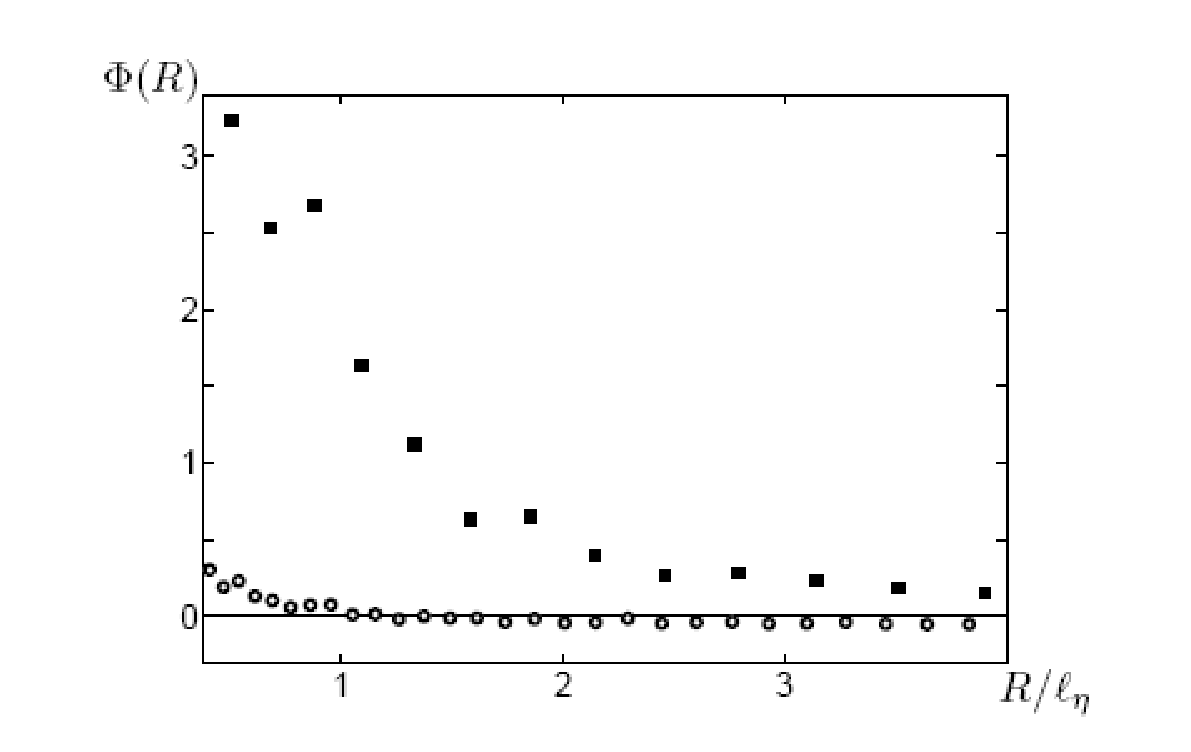
$N_S^{(p)} = \frac{1}{2}M(M-1)$ **is the total number of pairs in the area:** S

$M \sim 10^3$ **is the total number of particles in the area:** S

We perform the double averaging (i) over all particles in the image and (ii) over ensemble of 50 images.

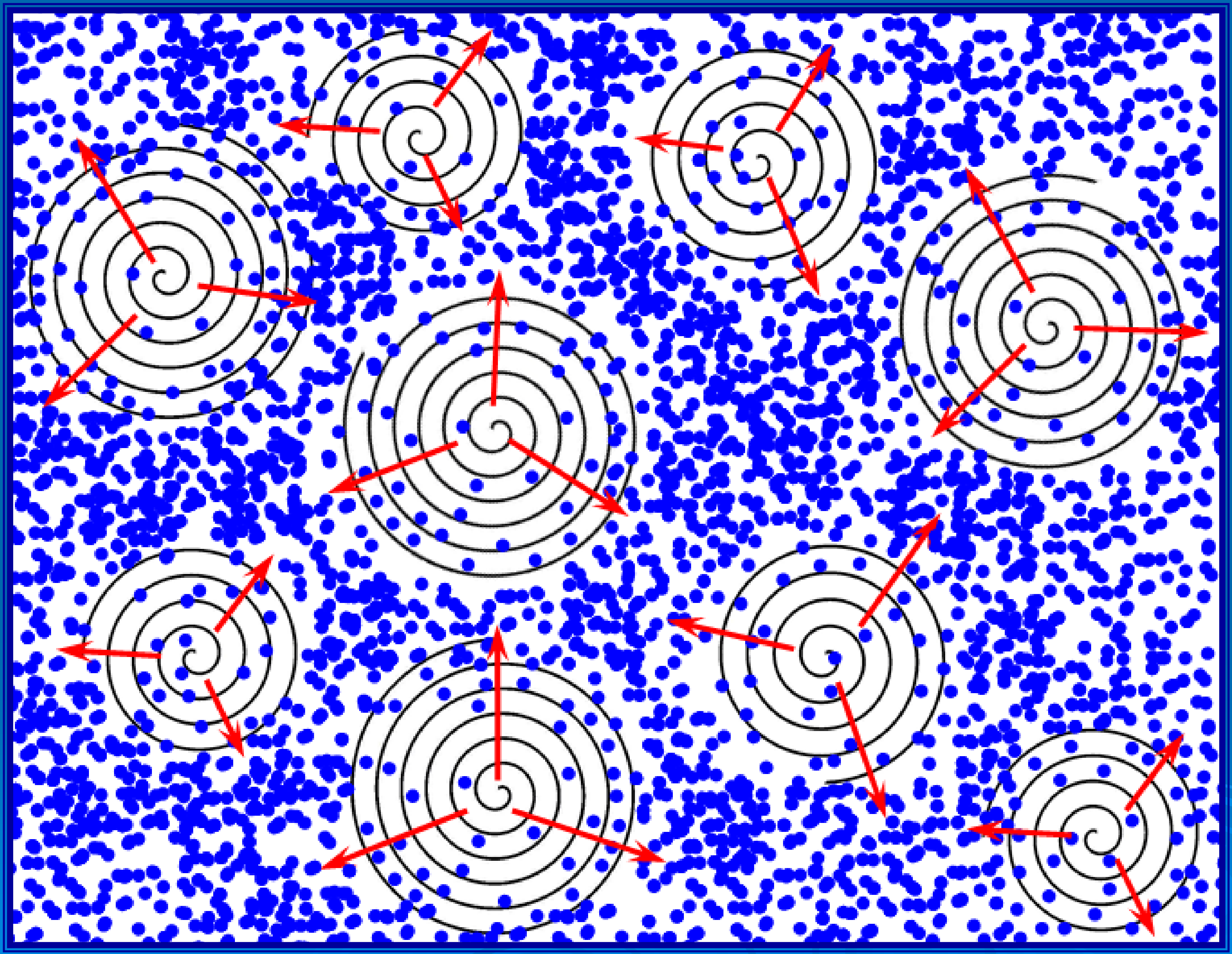
Normalized second-order correlation function determined in our experiments for

(i) inertial clustering (isothermal turbulence, circles)
(ii) tangling clustering (non-isothermal turbulence, squares)

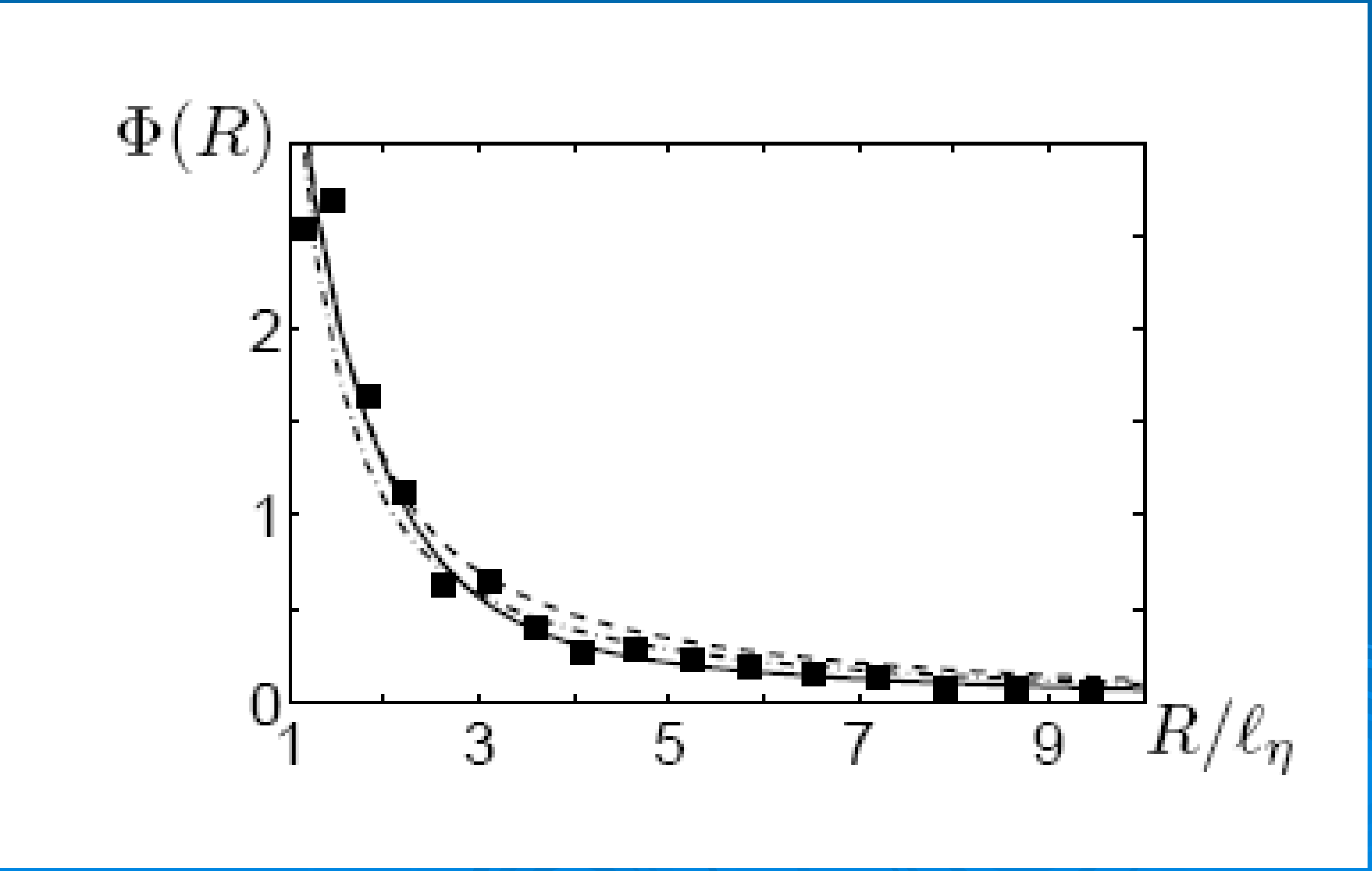


A. **Eidelman, T. Elperin, N. Kleeorin, B. Melnik, I. Rogachevskii**, Physical Review E **81**, 056313 (2010)

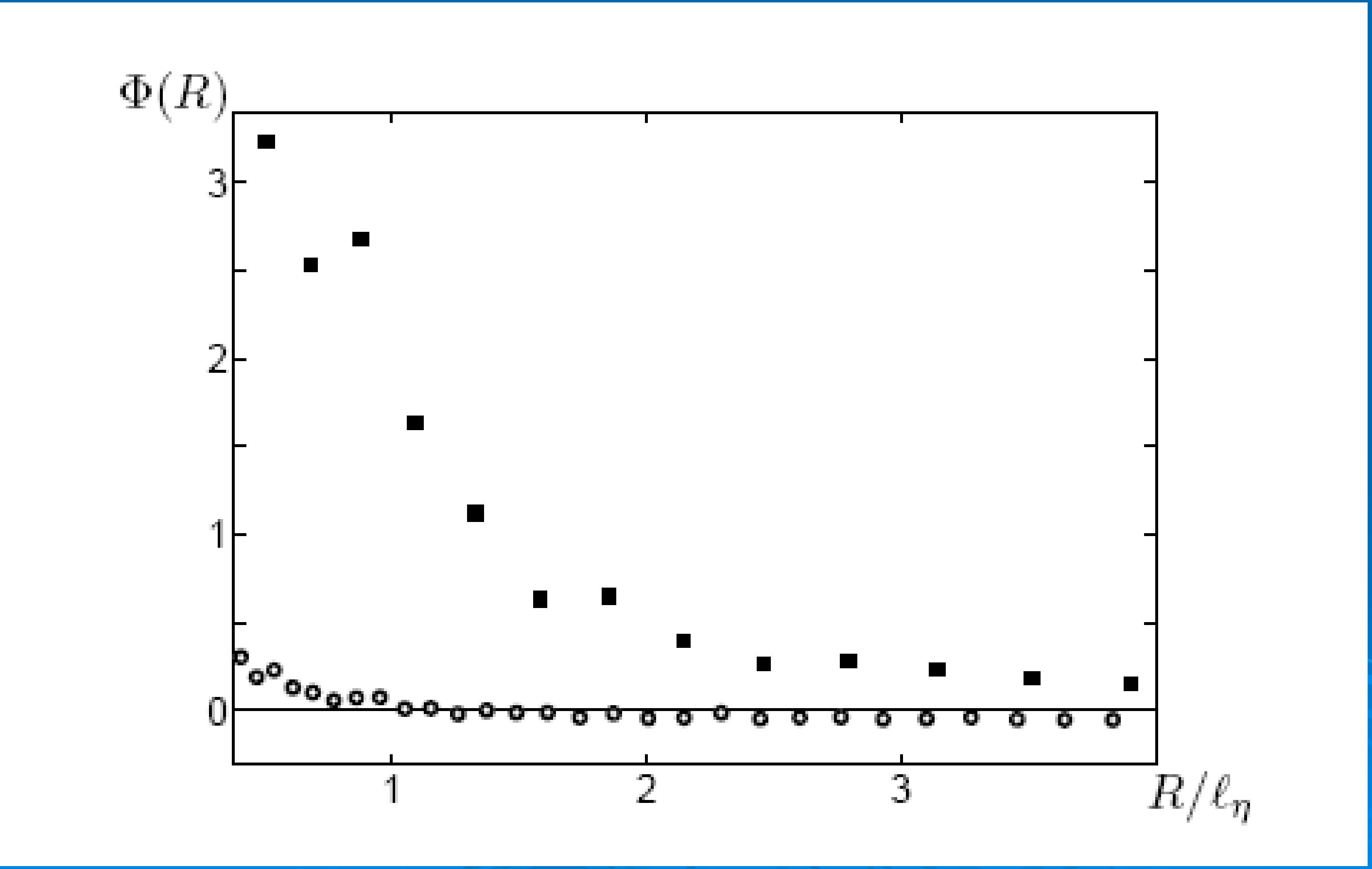
Particle Inertia Effect



Normalized second-order correlation function determined in our experiments (filled squares) and from our theoretical model (solid line)



Normalized second-order correlation function determined in our experiments for
(i) inertial clustering (isothermal turbulence, circles)
(ii) tangling clustering (non-isothermal turbulence, squares)



Tangling Clustering Instability in Temperature Stratified Turbulence

T. Elperin, N. Kleeorin, M. Liberman, I. Rogachevskii, Phys. Fluids 25, 085104 (2013)

$$\gamma_m = \frac{1}{3(1+3\sigma_T)}\left[\frac{200\sigma_v(\sigma_T-\sigma_v)}{(1+\sigma_v)^2} - \frac{(3-\sigma_T)^2}{2(1+\sigma_T)} - \frac{2\pi^2 m^2(1+3\sigma_T)^2}{(1+\sigma_T)\ln^2 \mathrm{Sc}}\right],$$

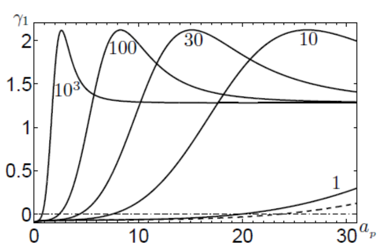


FIG. 1. The growth rate γ_1 of the tangling clustering instability (in units of $1/\tau_\eta$) of the first mode ($m = 1$) versus the particle radius a_p for different values of parameter Γ, and $\sigma_T = 1$, $\mathrm{Sc} = 10^6 a_p$. The particle radius a_p is given in μm. The dashed line corresponds to the inertial clustering instability ($\Gamma = 1$).

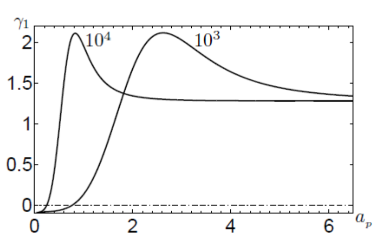


FIG. 2. The growth rate γ_1 of the tangling clustering instability of the first mode ($m = 1$) versus the particle radius a_p for larger values of parameter Γ, and $\sigma_T = 1$, $\mathrm{Sc} = 10^6 a_p$. The particle radius a_p is given in μm.

$$\sigma_v \equiv \frac{\langle(\boldsymbol{\nabla}\cdot\boldsymbol{v})^2\rangle}{\langle(\boldsymbol{\nabla}\times\boldsymbol{v})^2\rangle} = \frac{8}{3}\,\mathrm{St}_{\mathrm{eff}}^2$$

$$\sigma_T \equiv \frac{\nabla_i\nabla_j D_{ij}^{\mathrm{T}}(\boldsymbol{R})}{\nabla_i\nabla_j D_{mn}^{\mathrm{T}}(\boldsymbol{R})\epsilon_{imp}\epsilon_{jnp}} \approx \frac{\langle(\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{\xi}})^2\rangle}{\langle(\boldsymbol{\nabla}\times\tilde{\boldsymbol{\xi}})^2\rangle}$$

$$\mathrm{St}_{\mathrm{eff}} = \mathrm{St}\,\Gamma,$$

$$\Gamma(\mathrm{Ma},\mathrm{Re},\ell_0/L_T) = \left[1 + \frac{\mathrm{Re}^{1/2}}{81\,\mathrm{Ma}^4}\left(\frac{\ell_0\boldsymbol{\nabla}T}{T}\right)^2\right]^{1/2}$$

Tangling Clustering Instability in Temperature Stratified Turbulence

T. Elperin, N. Kleeorin, M. Liberman, I. Rogachevskii, Phys. Fluids 25, 085104 (2013)

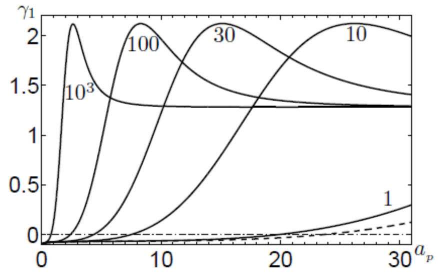


FIG. 1. The growth rate γ_1 of the tangling clustering instability (in units of $1/\tau_\eta$) of the first mode ($m = 1$) versus the particle radius a_p for different values of parameter Γ, and $\sigma_T = 1$, $\mathrm{Sc} = 10^6 a_p$. The particle radius a_p is given in μm. The dashed line corresponds to the inertial clustering instability ($\Gamma = 1$).

$$\sigma_\mathrm{v} \equiv \frac{\langle(\nabla\cdot v)^2\rangle}{\langle(\nabla\times v)^2\rangle} = \frac{8}{3}\,\mathrm{St}_\mathrm{eff}^2$$

$$B(\boldsymbol{R}) = \frac{20\,\sigma_\mathrm{v}}{\tau_\eta\,(1+\sigma_\mathrm{v})} \approx \frac{20\,\sigma_\mathrm{v}}{\tau_\eta} \approx \frac{160\,\mathrm{St}_\mathrm{eff}^2}{3\tau_\eta}$$

$$\mathrm{St}_\mathrm{eff} = \mathrm{St}\,\Gamma, \quad \Gamma(\mathrm{Ma},\mathrm{Re},\ell_0/L_T) = \left[1 + \frac{\mathrm{Re}^{1/2}}{81\,\mathrm{Ma}^4}\left(\frac{\ell_0\nabla T}{T}\right)^2\right]^{1/2}$$

$\mathrm{Re} = 10^7$, $u_0 = 1$ m/s and $\ell_0 = 100$m $\quad \Gamma \approx 2.5\times 10^3$

$\mathrm{Re} = 10^6$, $u_0 = 0.3$ m/s $\ell_0 = 30$ m $\quad \Gamma \approx 5\times 10^3$

Saturation of the Tangling Clustering Instability

$$\frac{\partial \Phi}{\partial t} = [B(\mathbf{R}) + 2\mathbf{U}^{(A)}(\mathbf{R})\cdot\boldsymbol{\nabla} + D_{ij}(\mathbf{R})\nabla_i\nabla_j]\,\Phi(t,\mathbf{R}) + I(\mathbf{R})$$

$$\Phi(t,\mathbf{R}) = \langle n'(t,\mathbf{x})\, n'(t,\mathbf{y})\rangle$$

$$\Gamma(\mathrm{Ma}, \mathrm{Re}, \ell_0/L_T) = \left[1 + \frac{\mathrm{Re}^{1/2}}{81\,\mathrm{Ma}^4}\left(\frac{\ell_0 \boldsymbol{\nabla} T}{T}\right)^2\right]^{1/2}$$

$$\frac{n_p^{\max}}{N} = \left(1 + \frac{e\,\lambda}{\pi}\,\mathrm{Sc}^{\lambda/2}\,\ln \mathrm{Sc}\right)^{1/2}$$

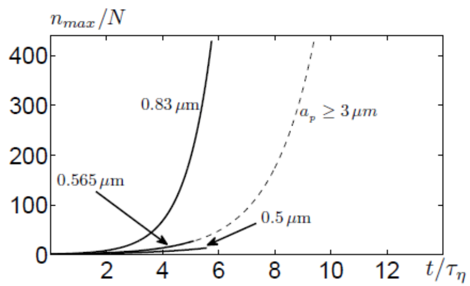


FIG. 5. The particle number density inside the cluster $n_{\max}/N$ as a function of time for different values of the particle radius a_p and $\Gamma = 10^4$, $\sigma_T = 1$, $\mathrm{Sc} = 10^6 a_p$. The particle radius a_p is given in μm. The dashed line is for $a_p \geq 3\mu$m.

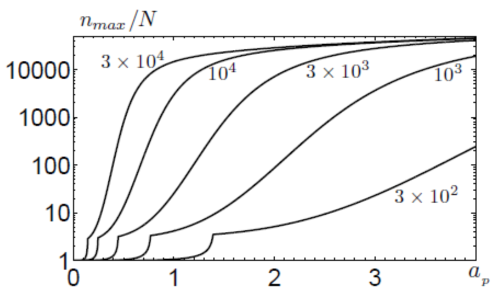


FIG. 3. The particle number density inside the cluster $n_{\max}/N$ versus the particle radius a_p for different values of parameter Γ, and $\sigma_T = 1$, $\mathrm{Sc} = 10^6 a_p$. The particle radius a_p is given in μm.

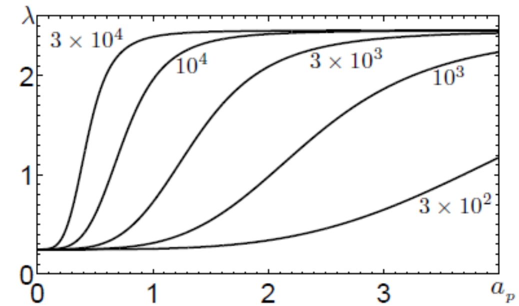


FIG. 4. The exponent λ versus the particle radius a_p for different values of parameter Γ and $\sigma_T = 1$. The particle radius a_p is given in μm.

Conclusions

- We have predicted theoretically and detected in laboratory experiments **a new type of particle clustering** (namely, **tangling clustering of inertial particles**) in a stably stratified turbulence with imposed mean vertical temperature gradient. The **tangling clustering** is much **more effective** than a **pure inertial clustering** that has been observed in isothermal turbulence. Our theoretical predictions are in a good agreement with the obtained experimental results.

- **Tangling clustering** can be significant in various industrial multi-phase turbulent flows (e.g. internal combustion engines), can also elucidate the mechanism of rain formation in turbulent clouds.

- We demonstrated a strong modification of turbulent transport in fluid flows with chemical reactions or phase transitions: turbulent diffusion of the reacting species can be strongly depleted by a large factor that is the ratio of turbulent and chemical times. This result is in a good agreement with that obtained in the numerical modelling of a reactive front propagating in a turbulent flow.

Conclusions

- A phenomenon of **turbulent thermal diffusion** associated with turbulent transport of particles, has been predicted theoretically and detected in laboratory experiments, in the atmosphere and in DNS.

- The essence of this phenomenon is the **appearance of a non-diffusive mean flux of particles in the direction of the mean heat flux,** which results in **the formation of large-scale inhomogeneous structures** in the spatial distribution of particles.

- Turbulent thermal diffusion is important in turbulent combustion systems, and it can also explain the large-scale aerosol layers that form inside atmospheric temperature inversions.

THE END