

Clustering and dynamic decoupling of dust grains in turbulent molecular clouds

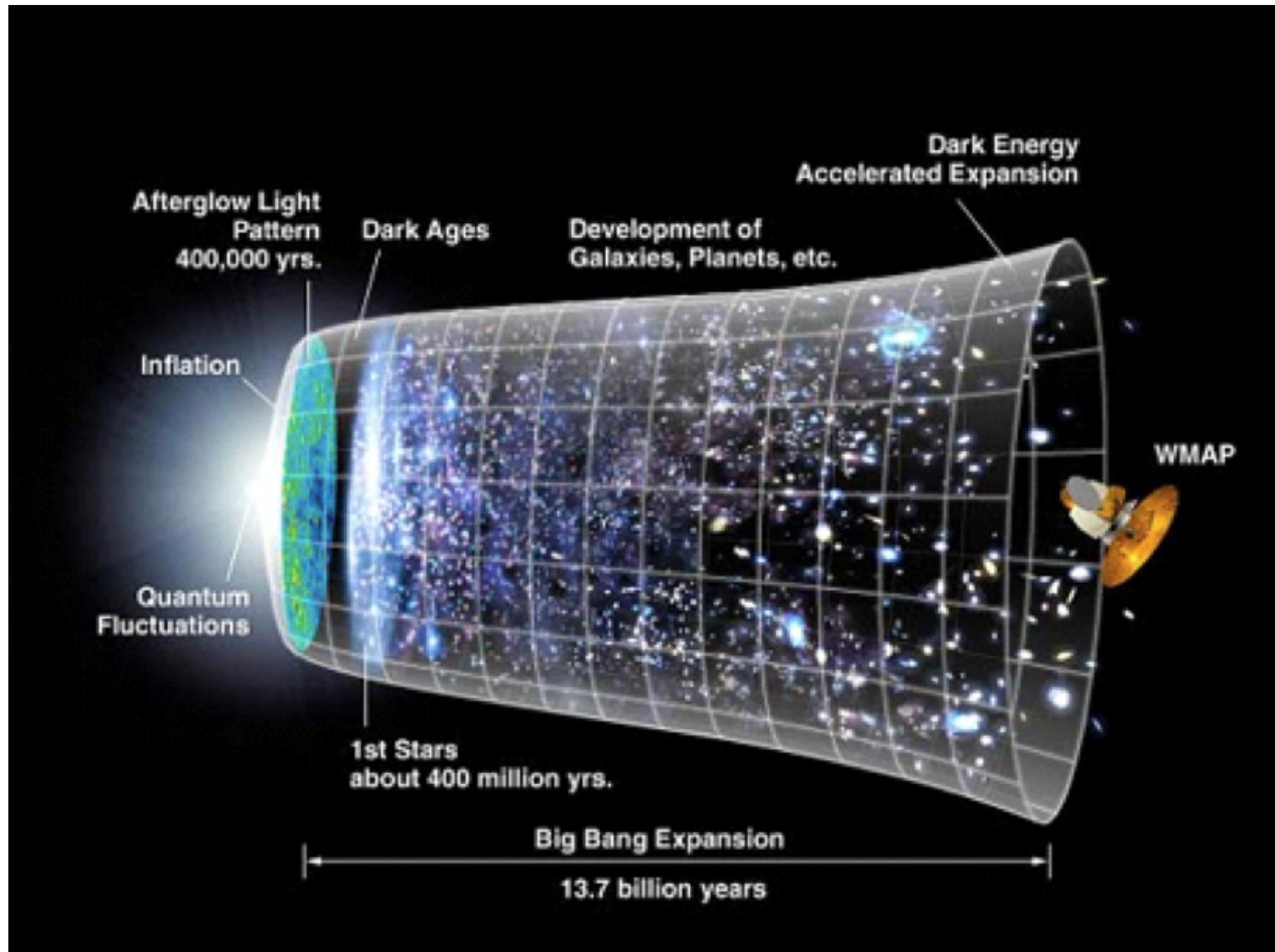
Phase transitions in astrophysics

Lars Mattsson

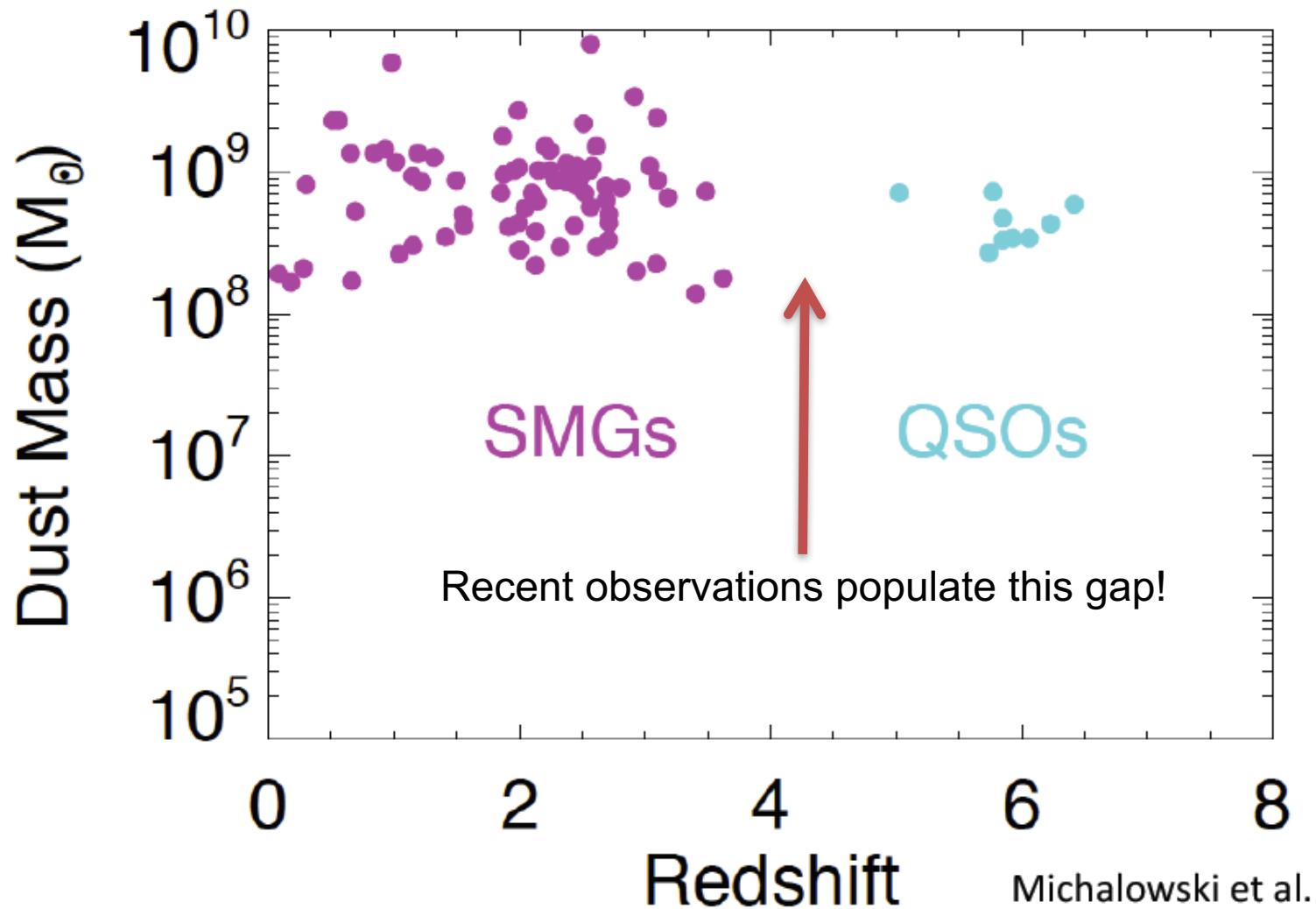
Nordita

NORDITA

Let's take it from the beginning...

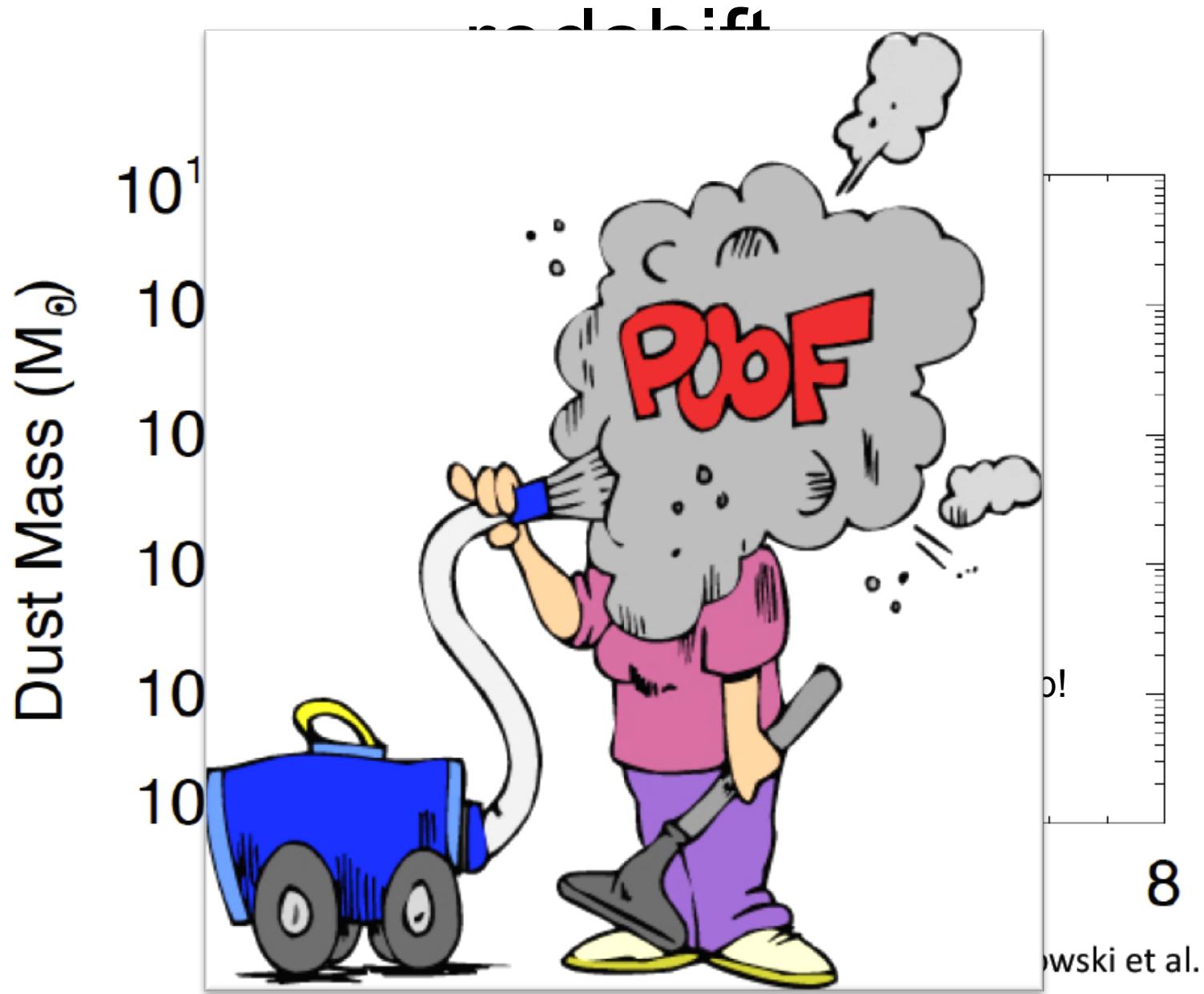


Large amounts of dust at high redshift



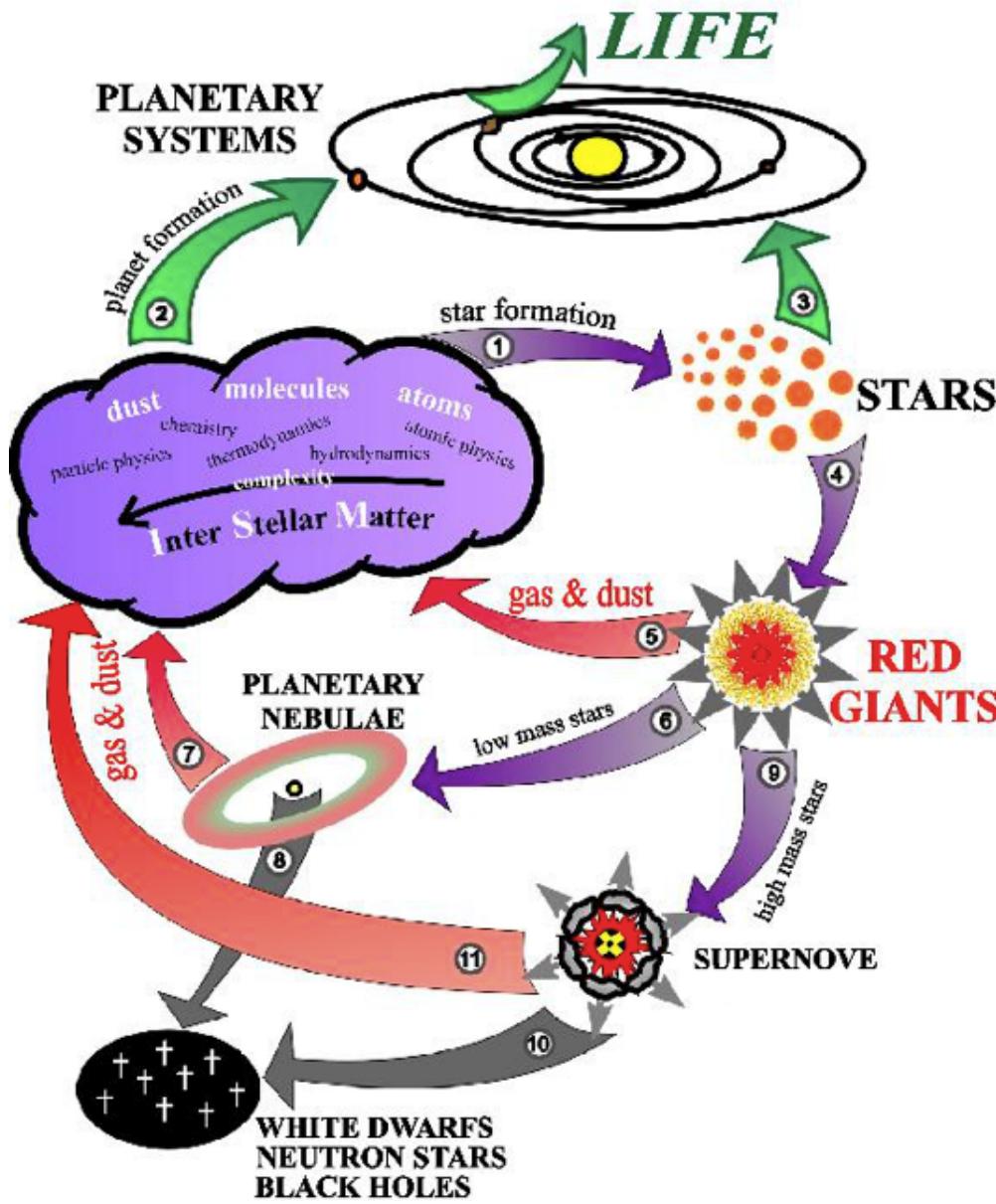
Bertoldi et al. (2003, A&A, 406, L55),
Michałowski et al. (2011) and many others....

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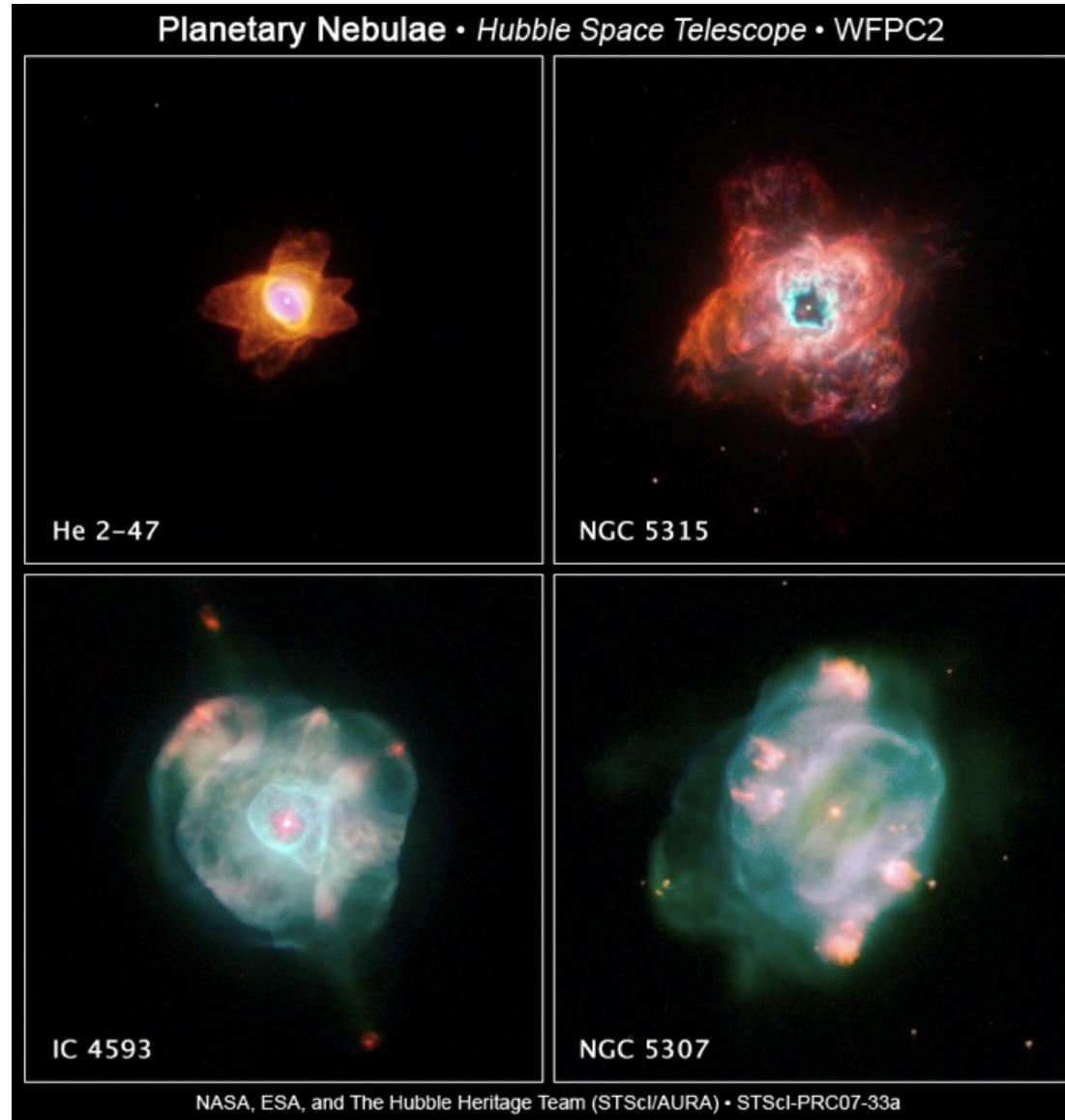
The cosmic matter cycle



Dust destruction

- Destruction may be induced by passage of SN shocks.
- Fragmentation by passage of SN shocks in combination with more efficient destruction of small grains (Slavin, Jones & Tielens 2004) may lead to a dust destruction timescale which is inversely proportional to the mass density of dust.
- Hydrodynamic instabilities and magnetic fields play an important role also here.
- What happens to the dust grains when a strong shock passes without destruction due to sputtering? Where do the grains end up due to instabilities and the decoupling between dust and gas?

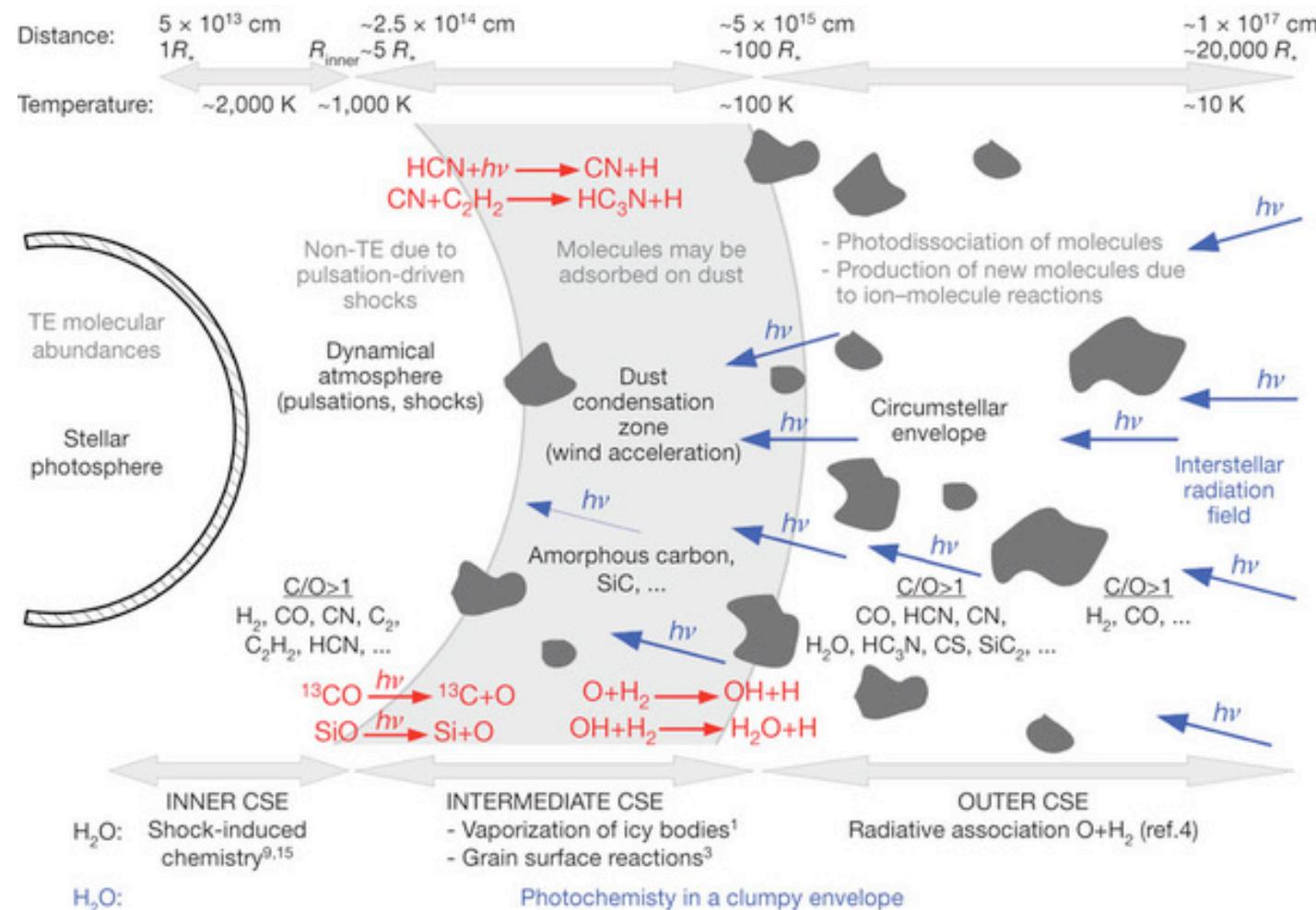
AGB evolution and dust formation



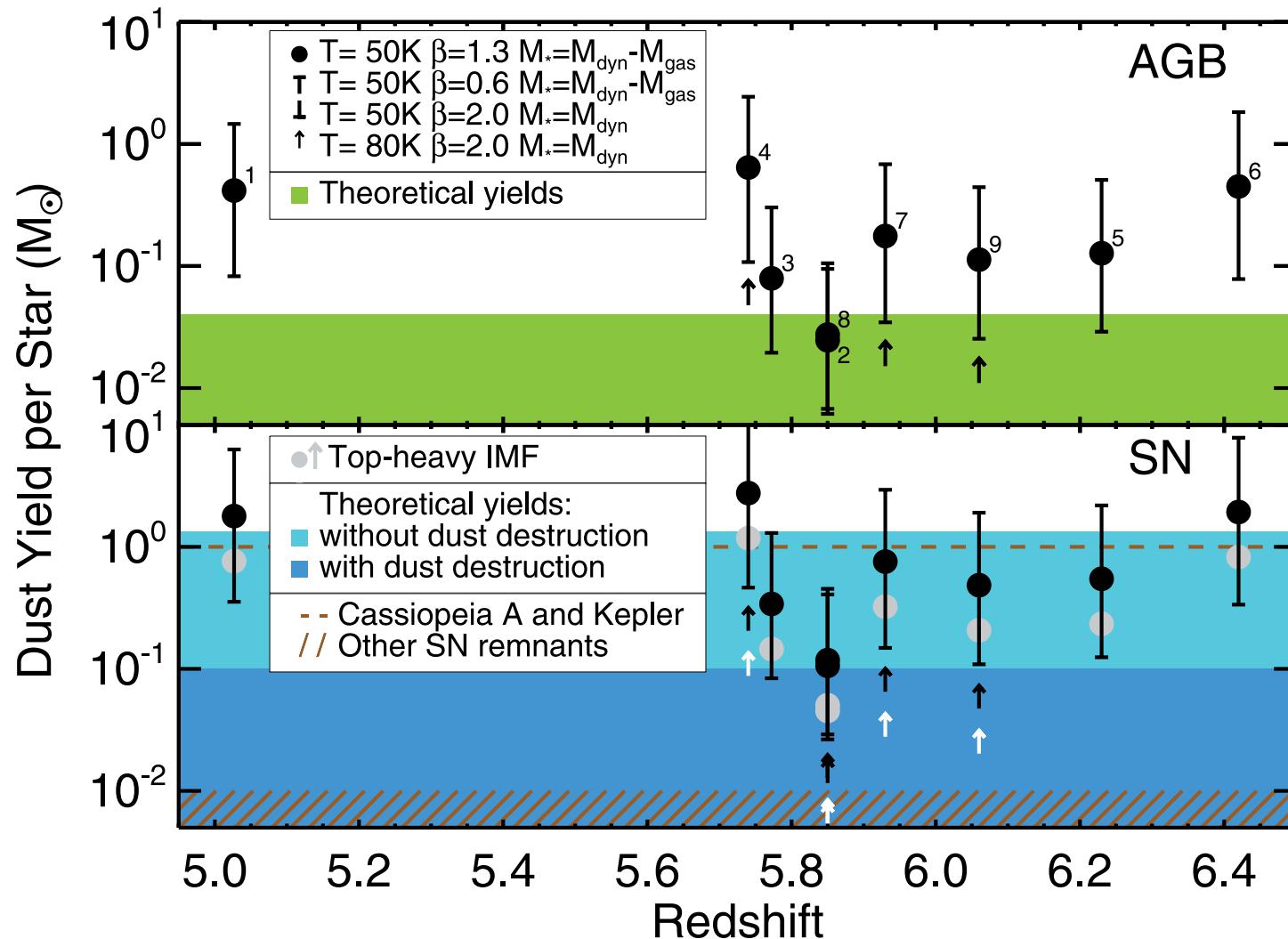
AGB evolution and dust formation

Grain
nucleation –
difficult!

Grain growth –
easier!

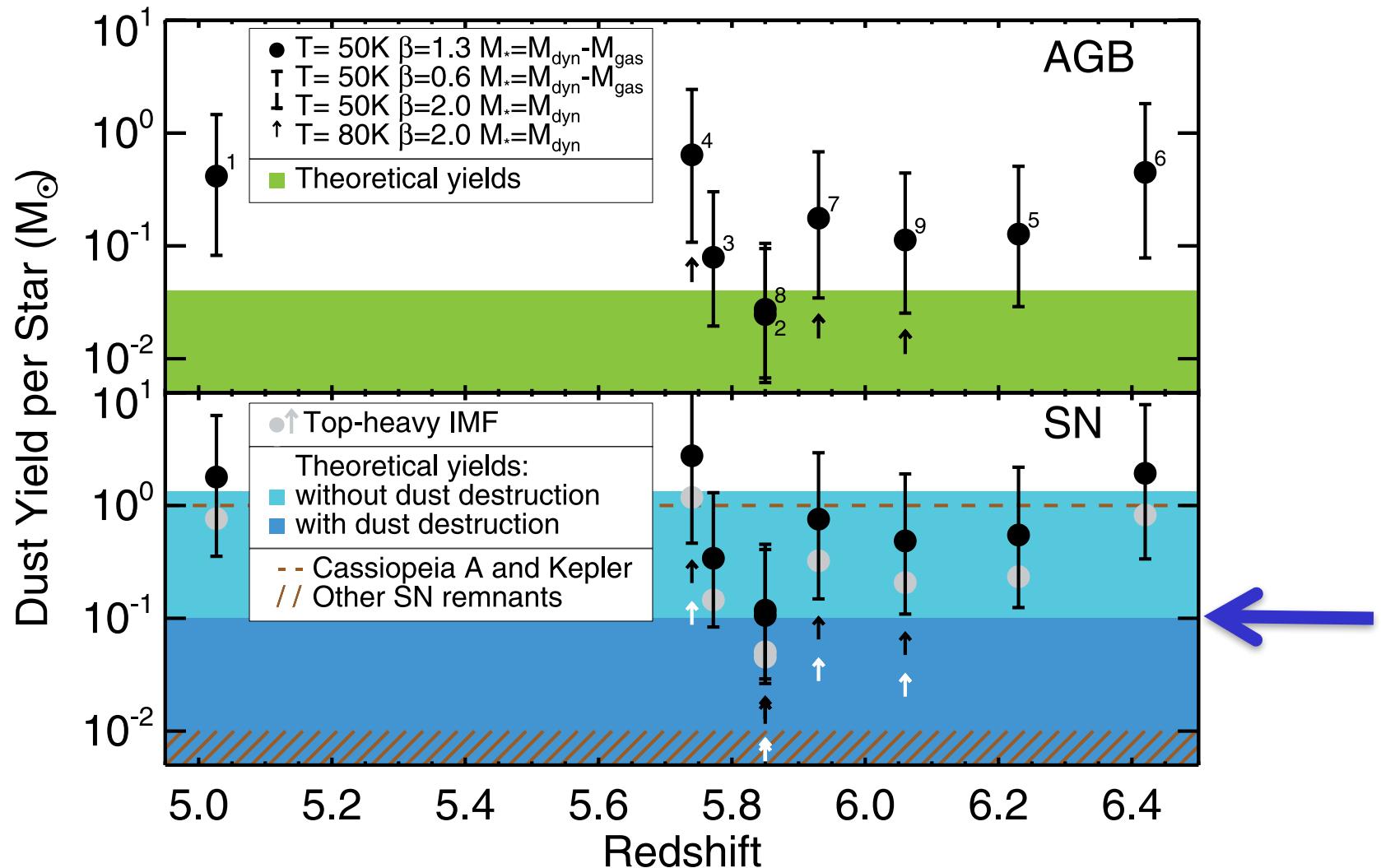


AGB stars? Nope!



(Michalowski et al. 2010)

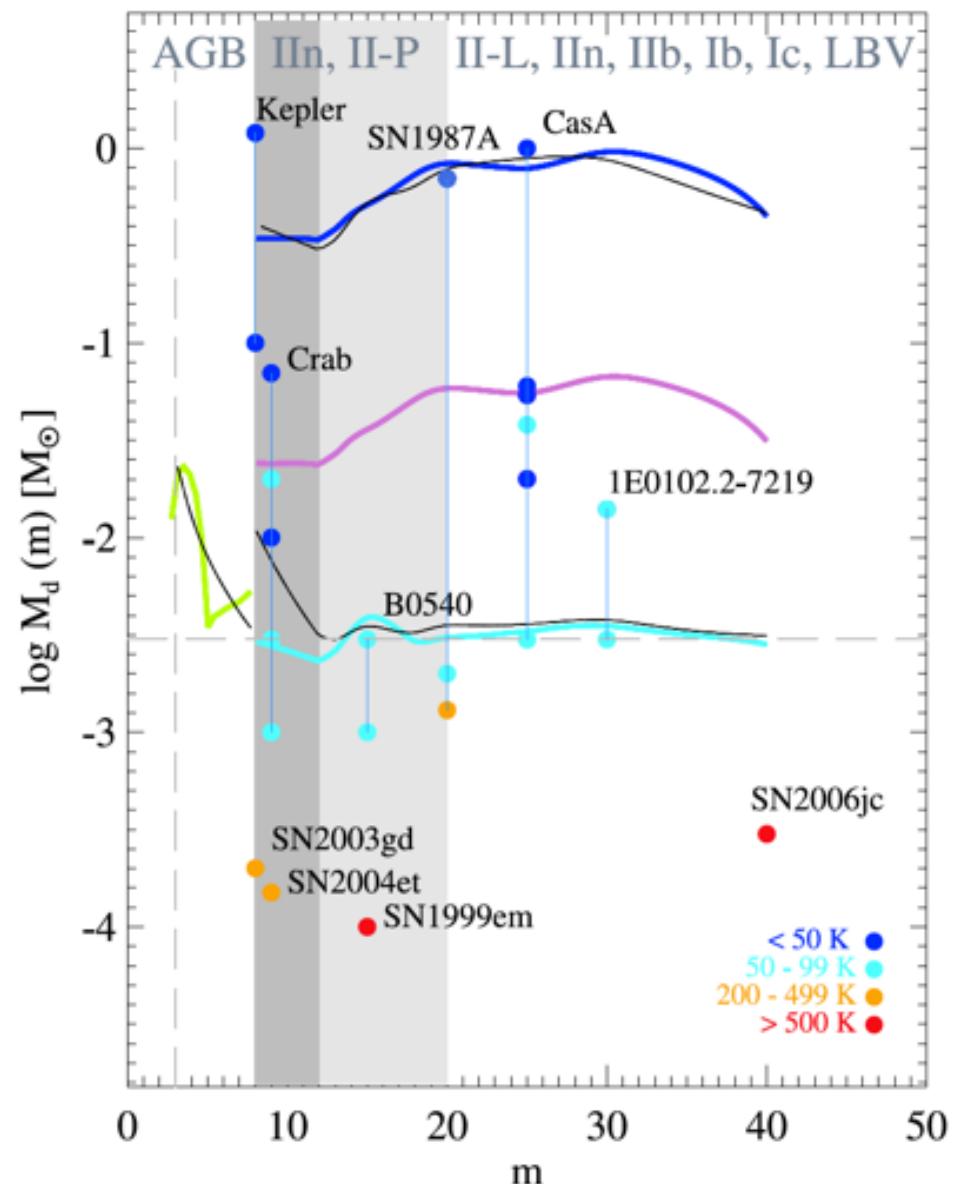
AGB stars? Nope!



(Michalowski et al. 2010)

SN dust works, but...

- Very little warm dust observed in SNe, $< 10^{-2} M_{\text{sun}}$ (e.g. Wooden et al. 1993; Elmhamdi et al. 2003; Kotak et al. 2009; Meikle et al. 2011)
- But still some controversy over large cold dust masses in SNRs...
- Suggest a constant or declining dust-to-metals ratio, which could be a problem (Mattsson 2011).

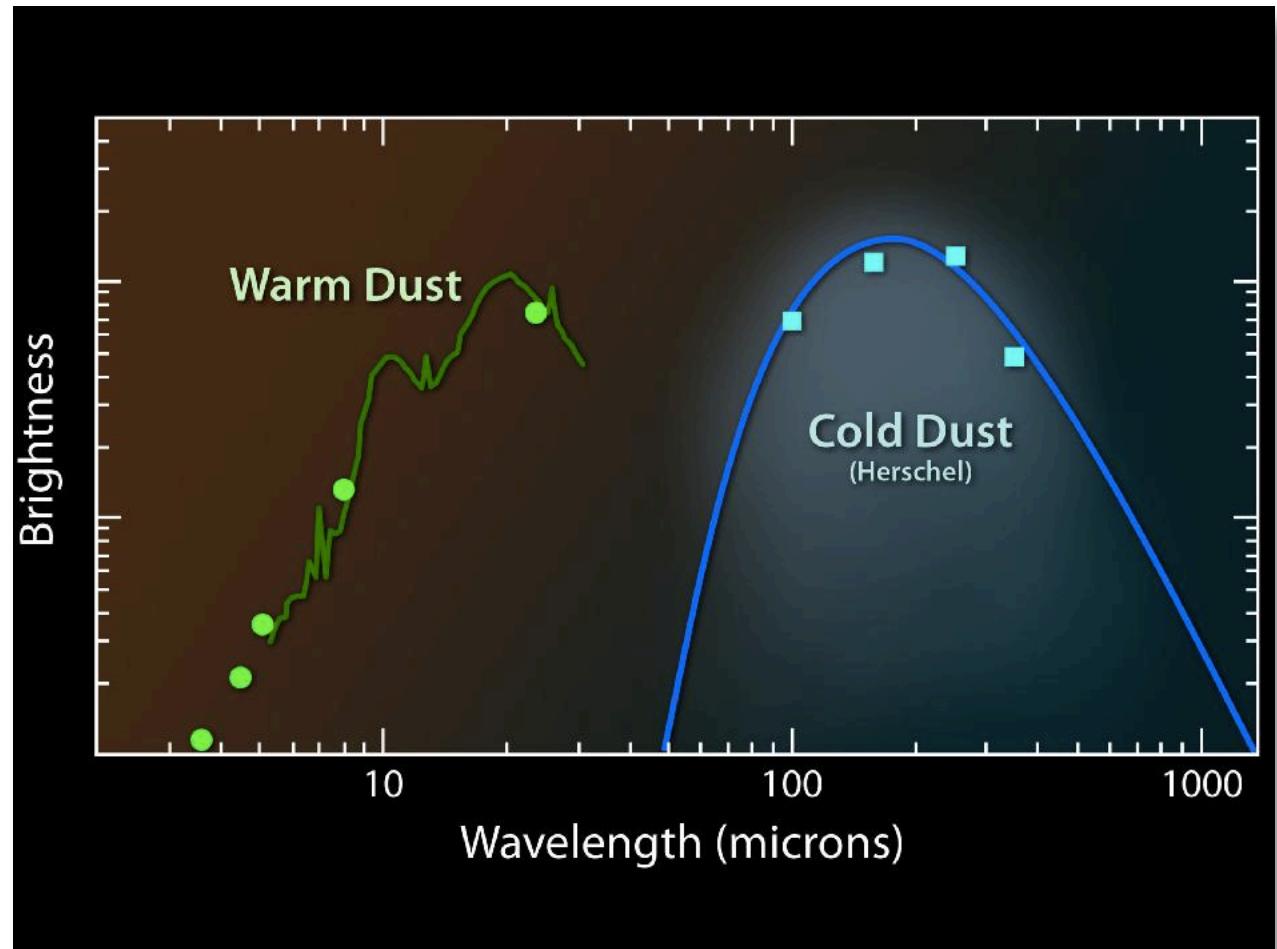
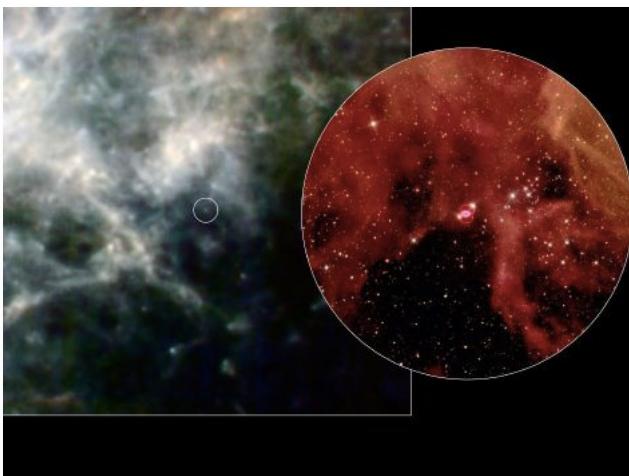


Gall et al. (2011, A&AR, 19, 43)

SN dust works, but...

SN1987A

- 100% dust efficiency?
- All metals are locked up in dust – no free metals to enter the ISM?



Matsuura et al. (2011, Science, 333, 1258)

SN 1987 A

- $0.5 - 0.7 M_{\text{sun}}$ of cold dust if there is C-dust,
 $2.4 M_{\text{sun}}$ if only silicates (Matsuura et al.
2011, Science, 333, 1258).
- The progenitor was a $15 - 20 M_{\text{sun}}$ star.
- An $18 M_{\text{sun}}$ star produces $0.13 M_{\text{sun}}$ of
silicon.
- $A_{\text{silicates}} = 121.41 \rightarrow M_{\text{silicates}} < 0.56 M_{\text{sun}}$
- $M_C = 0.22 M_{\text{sun}} \rightarrow M_{\text{c-dust}} < 0.22 M_{\text{sun}}$

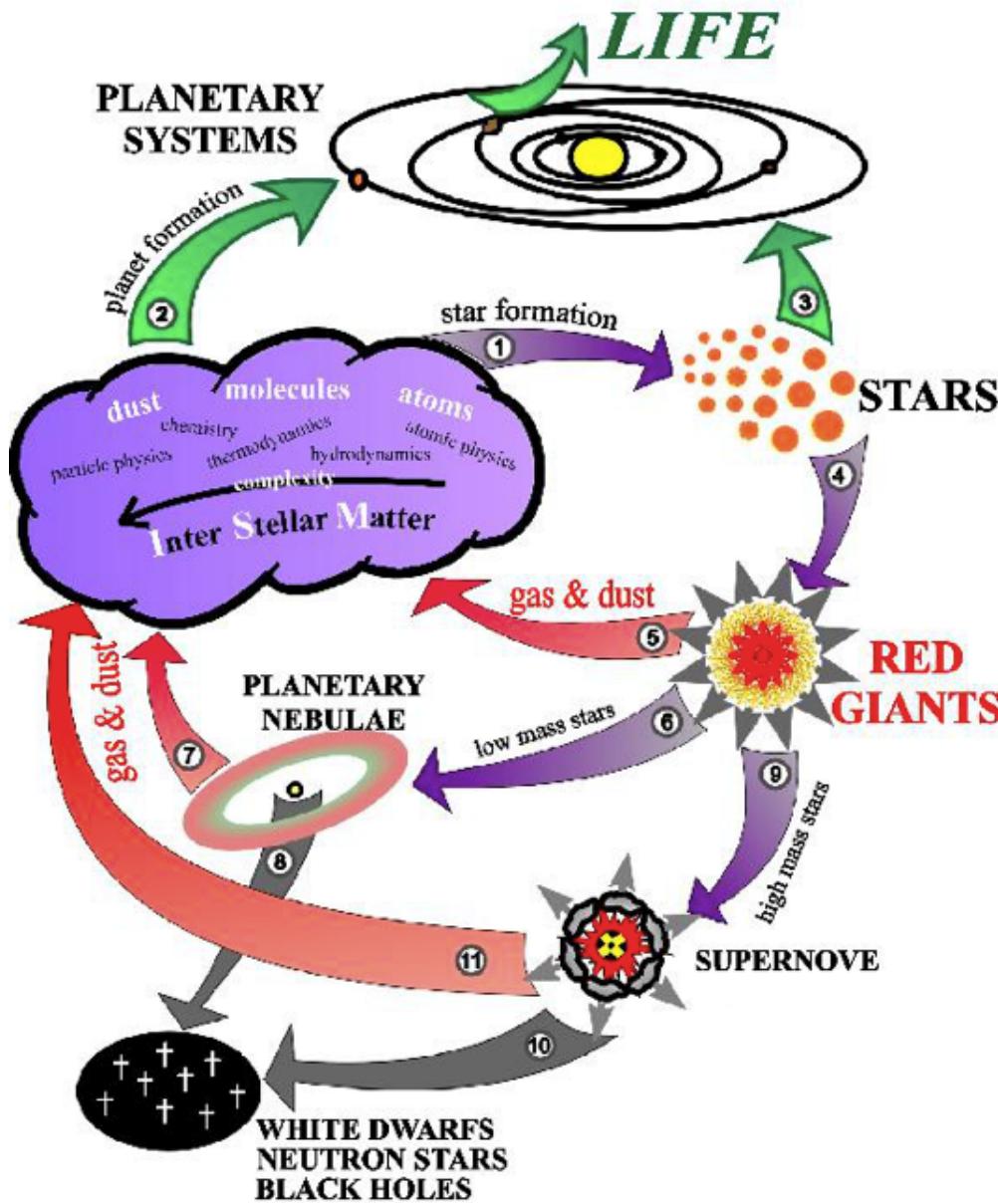
Anyway...

- Maximum time to build large dust masses:
 $< 400 - 500$ Myr.
- SNe can produce dust rapidly, but also destroy dust – A catch 22!
- The universe have been at least as dusty and possibly even more dusty at earlier epochs. But how?
- What source is compensating for the dust destruction? We NEED a replenishment mechanism!

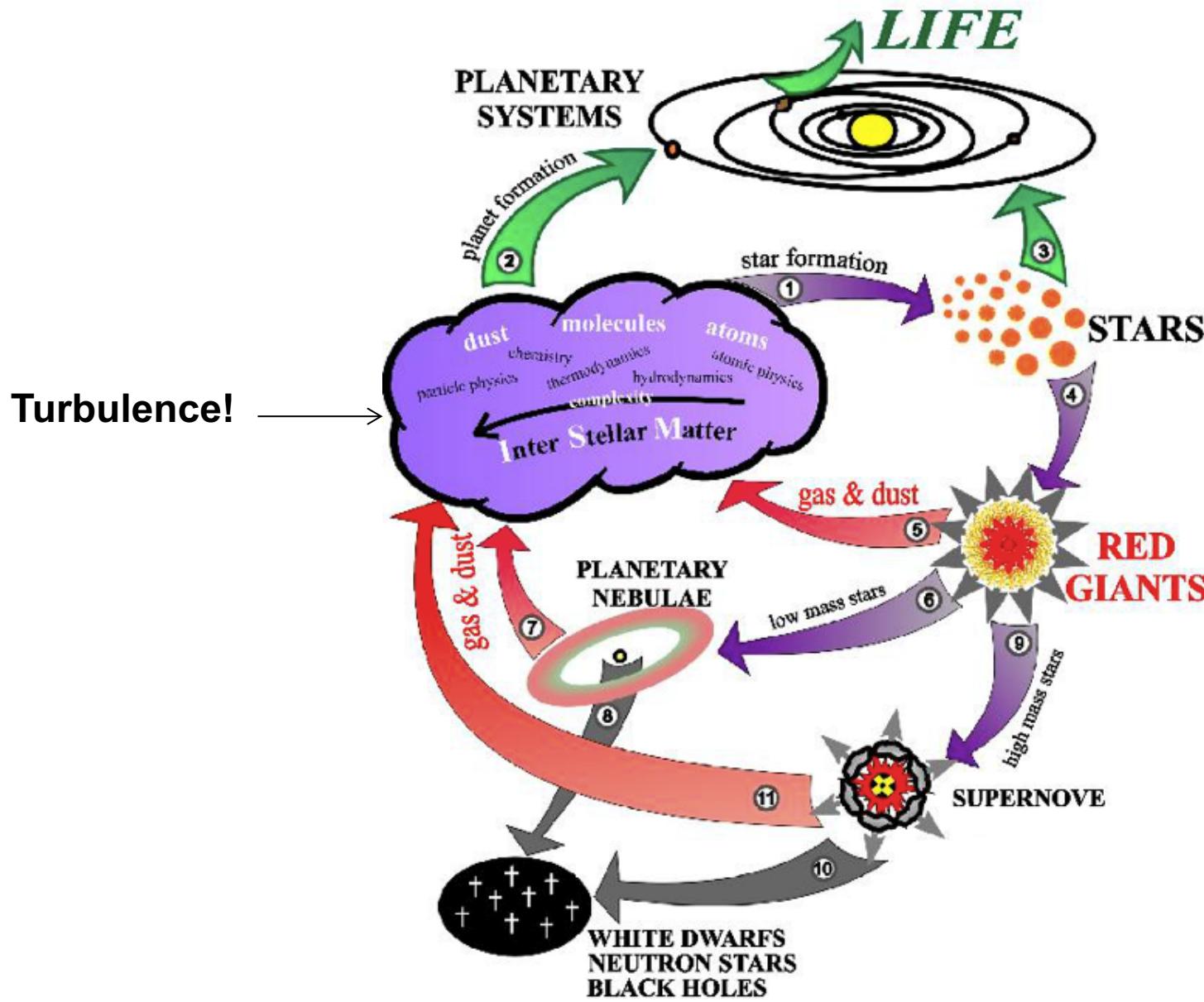
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The cosmic matter cycle



The cosmic matter cycle



Epstein drag

EOM for a particle:

$$m_{\text{gr}} \frac{d\mathbf{v}}{dt} = \mathbf{F}_{\text{Epstein}} = -\frac{4\pi}{3} a^2 \rho v_{\text{th}} \Delta \mathbf{v}$$
$$m_{\text{gr}} = \frac{4\pi}{3} a^3 \rho_{\text{gr}}, \quad \tau_{\text{stop}} = \frac{\rho_{\text{gr}} a}{\rho v_{\text{th}}}$$

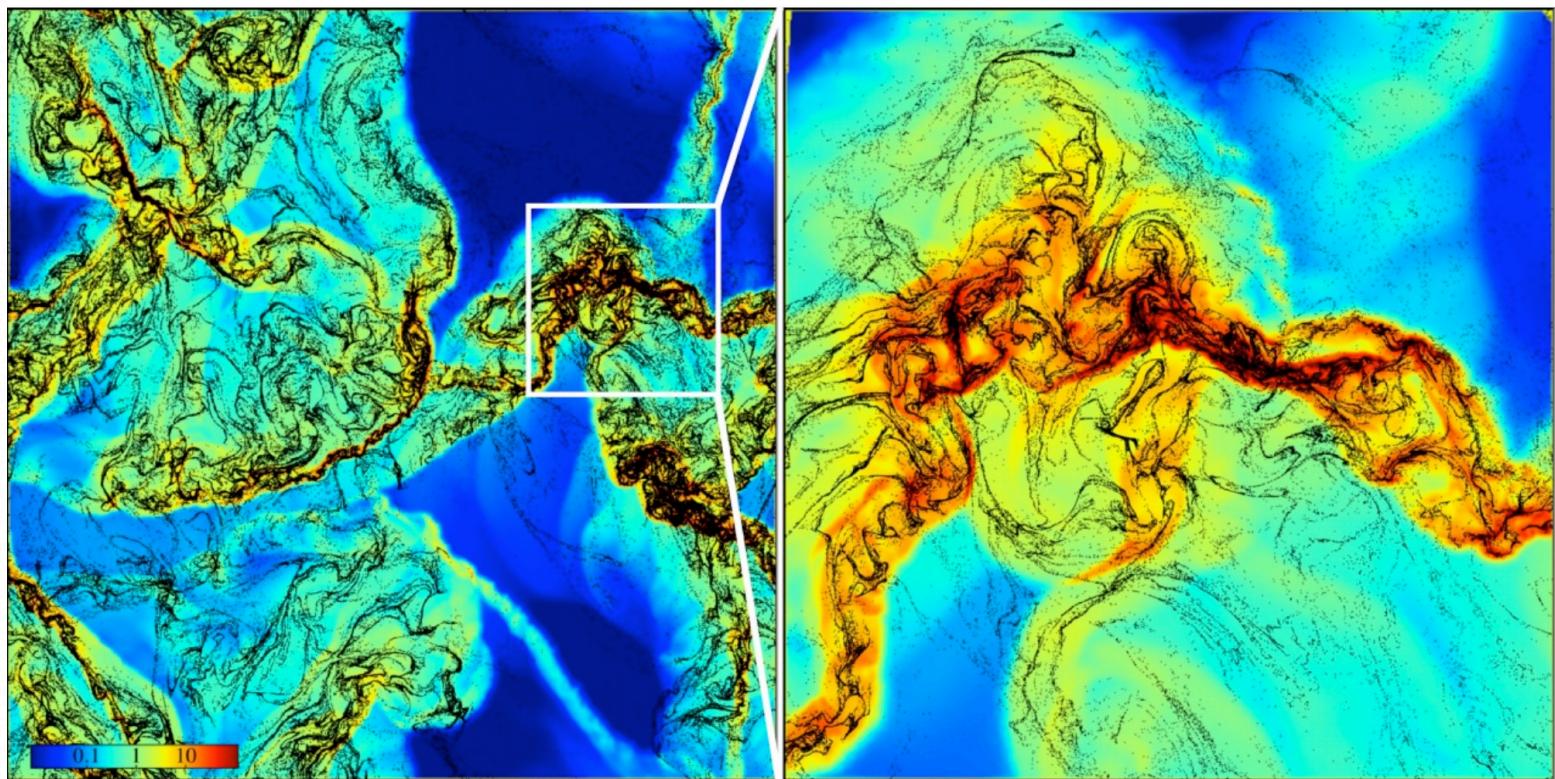
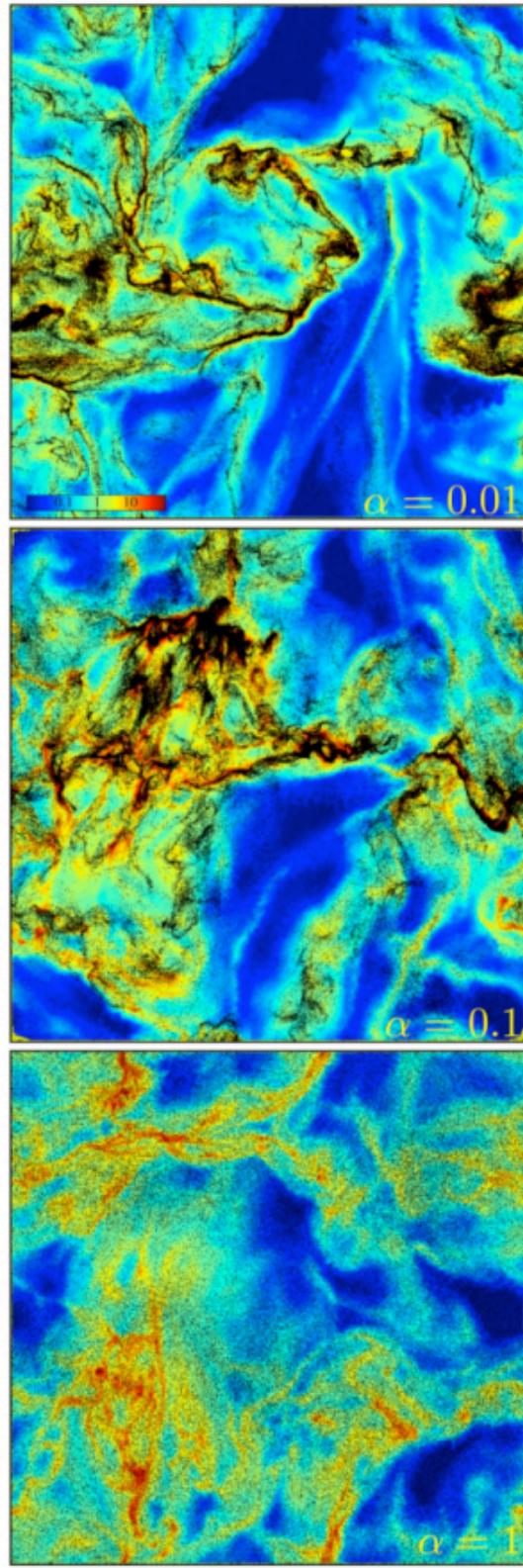
EOM for dust:

$$\frac{\partial \mathbf{v}_{\text{gr}}}{\partial t} + \mathbf{v}_d \cdot \nabla \mathbf{v}_{\text{gr}} = -\frac{\Delta \mathbf{v}}{\tau_{\text{stop}}}$$

Including a correct decoupling of the gas and dust dynamics is crucial!

In ISM simulations one can safely assume the drag is always in the Epstein limit.

Epstein drag



Hopkins & Lee (2016)

Epstein drag



“Swedish compass”

Simulation with PENCIL code

- Central region of cold ($T \sim 10K$) molecular gas cloud in ISM.
- Non-isothermal: entropy equation & temperature structure.
- A range of different grain sizes included in dust phase.
- Stochastically forced turbulence.
- “Only” 256^3 resolution because non-isothermal and spectrum of grain sizes.

Simulation with PENCIL code

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \nabla P + \mathbf{F}_{\text{visc}} + \mathbf{F}_{\text{force}}, \quad \mathbf{F}_{\text{visc}} = \nabla \cdot (2\nu \rho \mathbf{S}) \quad (2)$$

$$\rho T \frac{\partial s}{\partial t} + \rho T \mathbf{v} \cdot \nabla s = 2\nu \rho \mathbf{S}^2 + \mathcal{H} - \mathcal{L}, \quad (3)$$

$$\frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \mathbf{v}_d = \frac{\mathbf{v} - \mathbf{v}_d}{\tau_s}$$

$$\tau_s = \frac{\rho_{\text{gr}}}{\rho} \frac{a}{\langle v_{\text{th}} \rangle}$$

Simulation with PENCIL code

Table 1

Properties of Giant Molecular Clouds, Clumps, and Cores (Goldsmith 1987;
Cernicharo 1991).

Properties	GMC	Clump	Core
Size (pc)	20–60	3–20	0.5–3
Density (cm^{-3})	100–300	10^3 – 10^4	10^4 – 10^6
Mass (M_{\odot})	10^4 – 10^6	10^3 – 10^4	10 – 10^3
Linewidth (km s^{-1})	6–15	4–12	1–3
Temperature (K)	7–15	15–40	30–100

Simulation with PENCIL code

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Some definitions...

- Stokes number:

$$St = \frac{u_0 t_0}{\ell_0} = \frac{\tau_s}{\tau_\ell}$$

- Reynolds number:

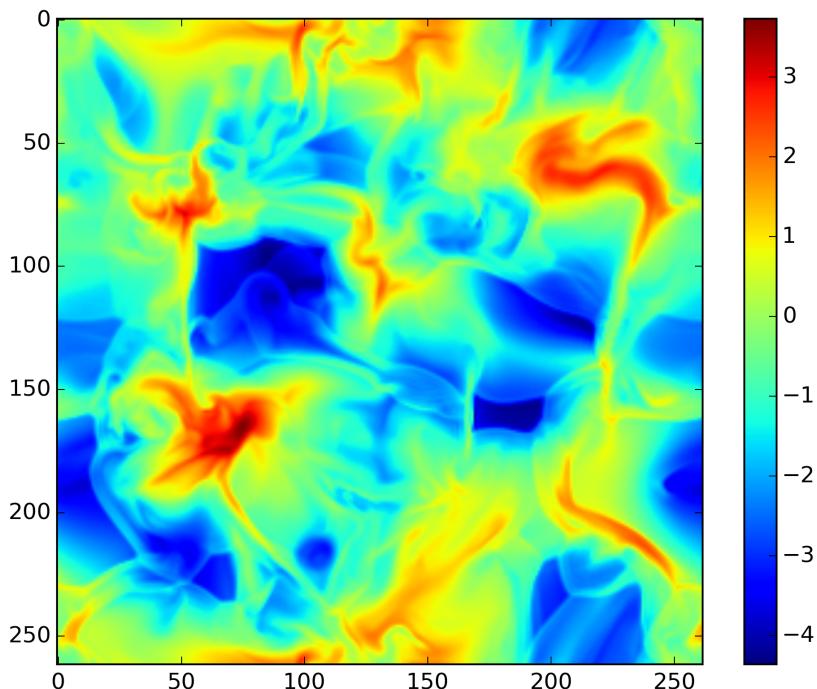
$$Re = \frac{u_0 \ell_0}{\nu}$$

- Gas-to-dust ratio:

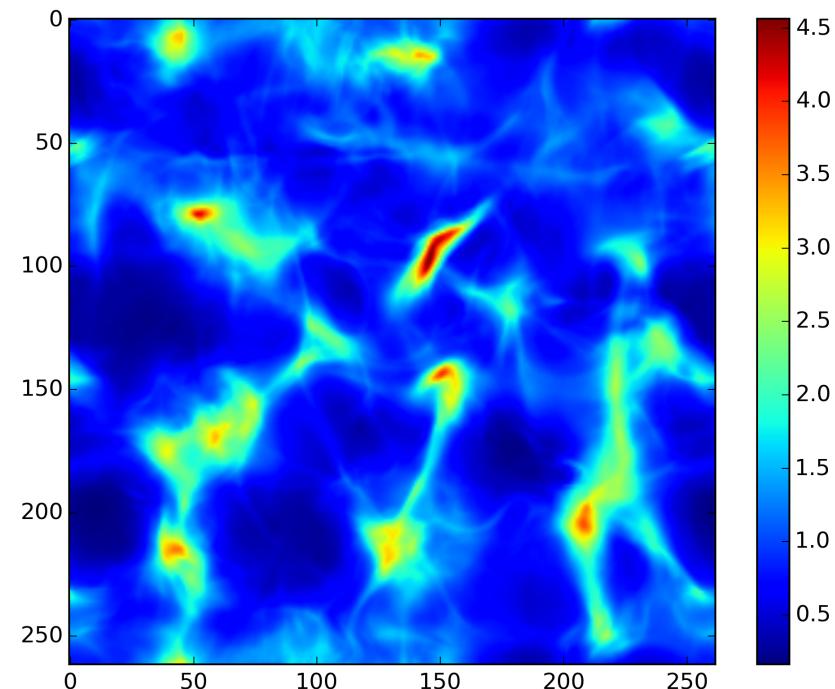
$$g_\ell = \frac{\rho}{\bar{n}_d^\ell} \left| \left\langle \frac{\rho}{\bar{n}_d^\ell} \right\rangle \right|$$

Turbulence in a box

Slice of $\log(\rho)$

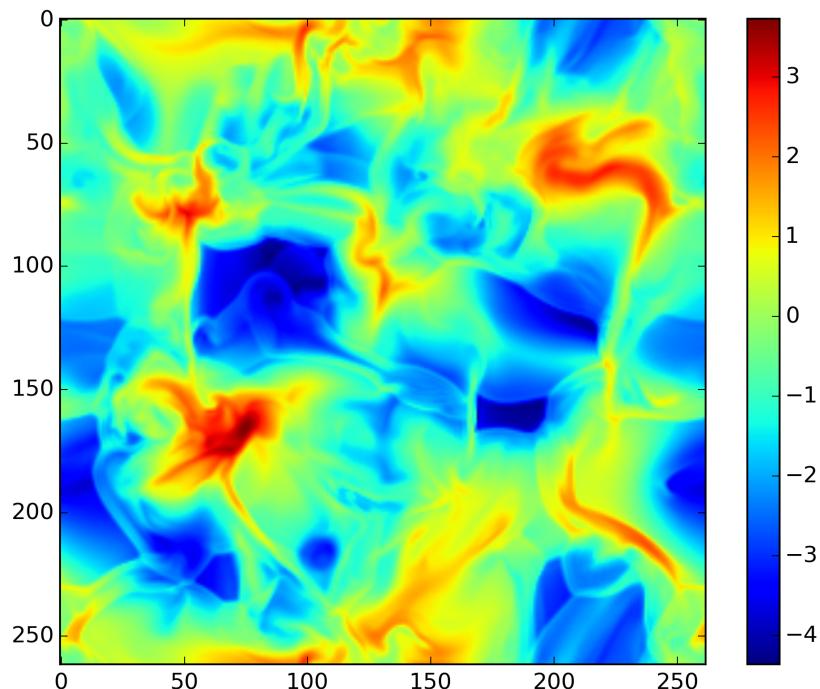


Column density (linear)

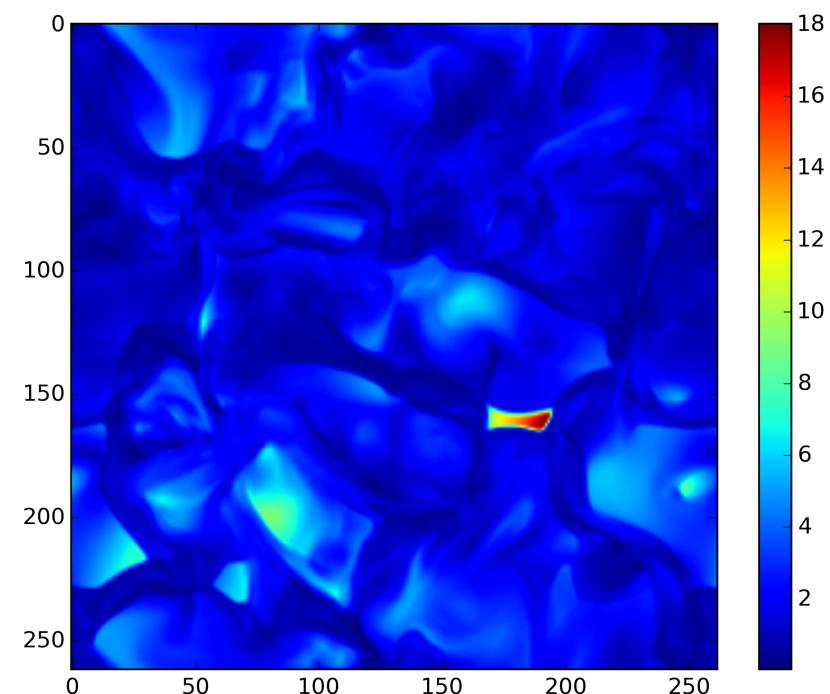


Turbulence in a box

Slice of $\log(\rho)$

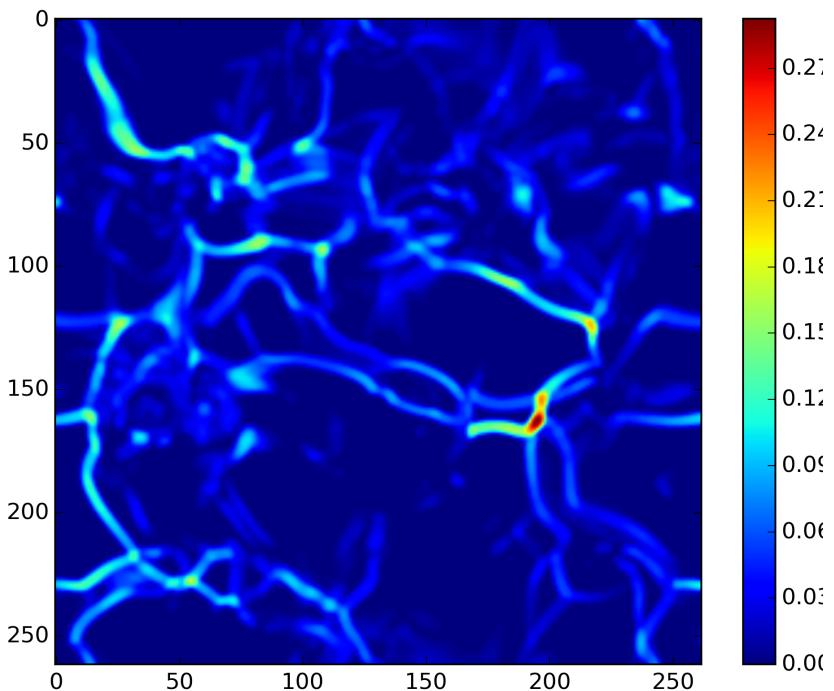


Mach number

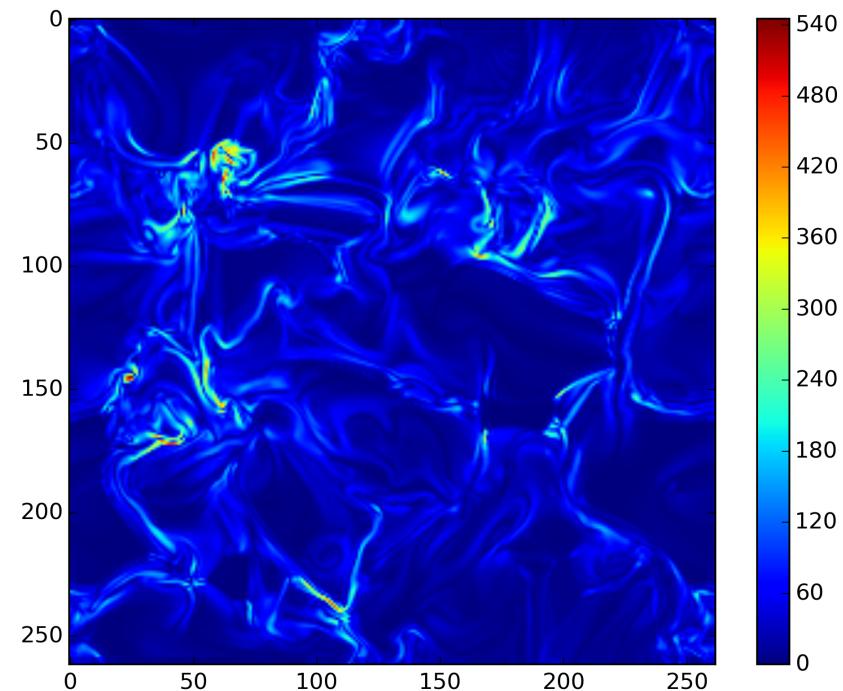


Turbulence in a box

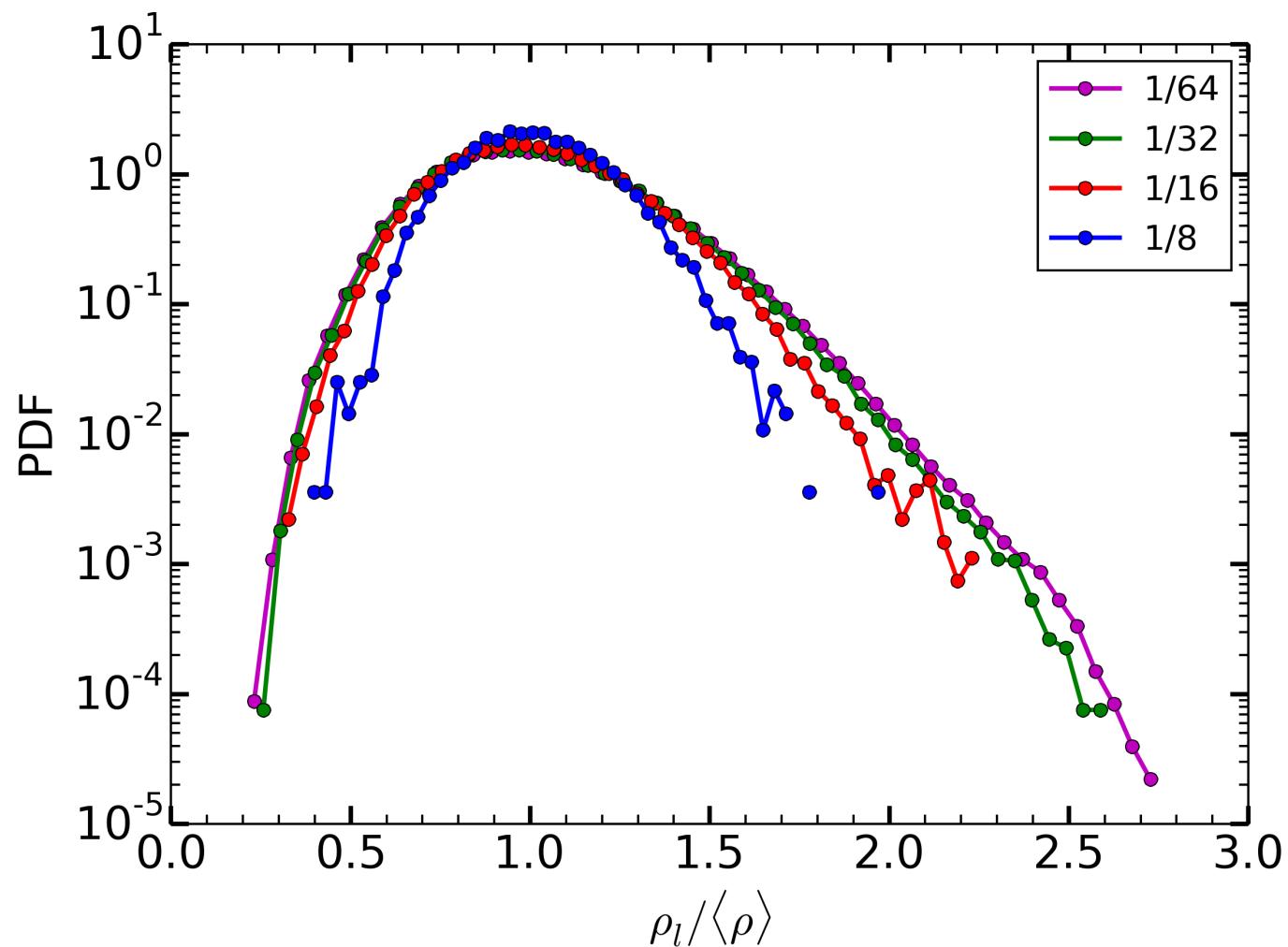
- $\text{div}(\mathbf{v})$



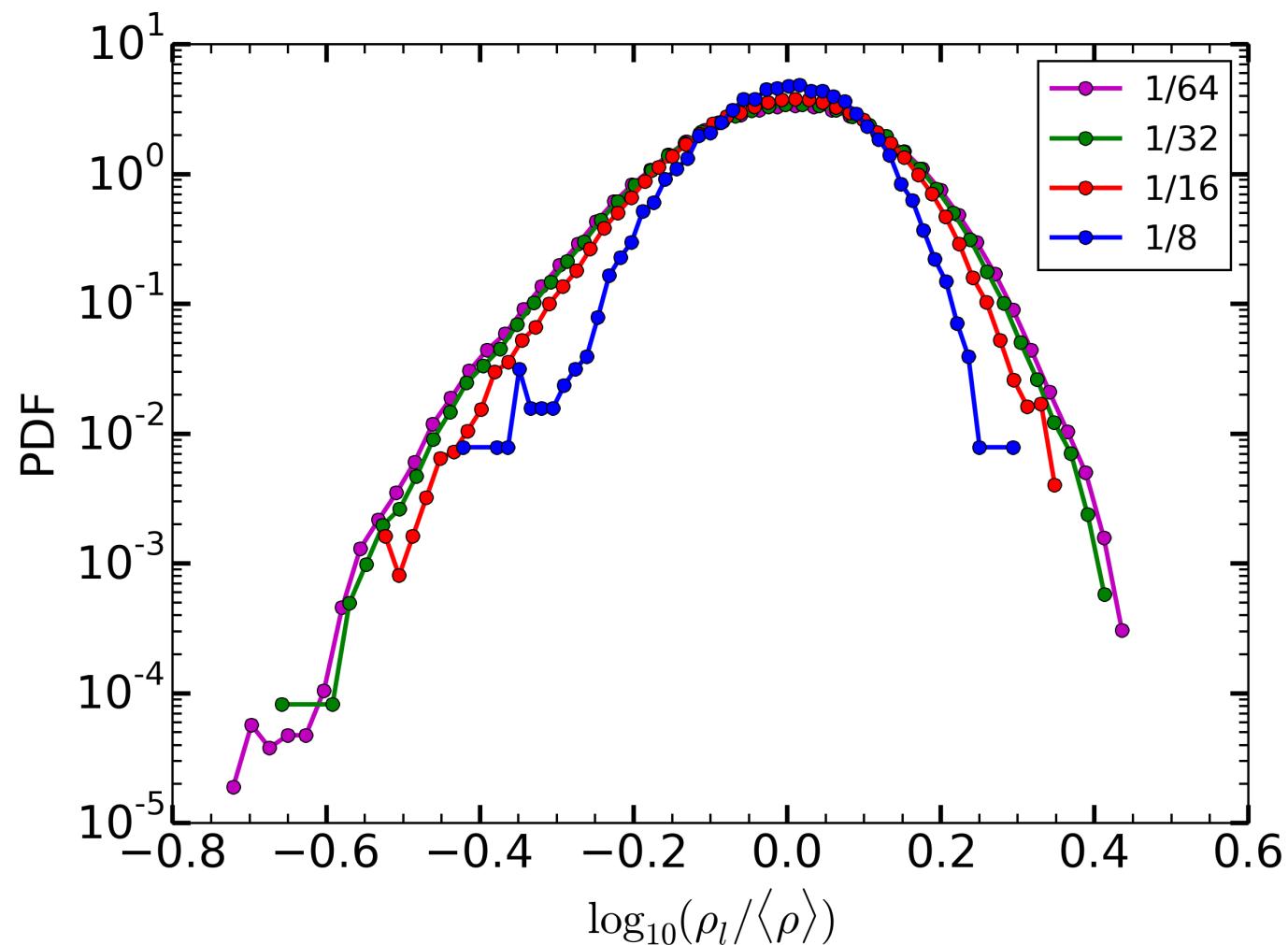
Vorticity



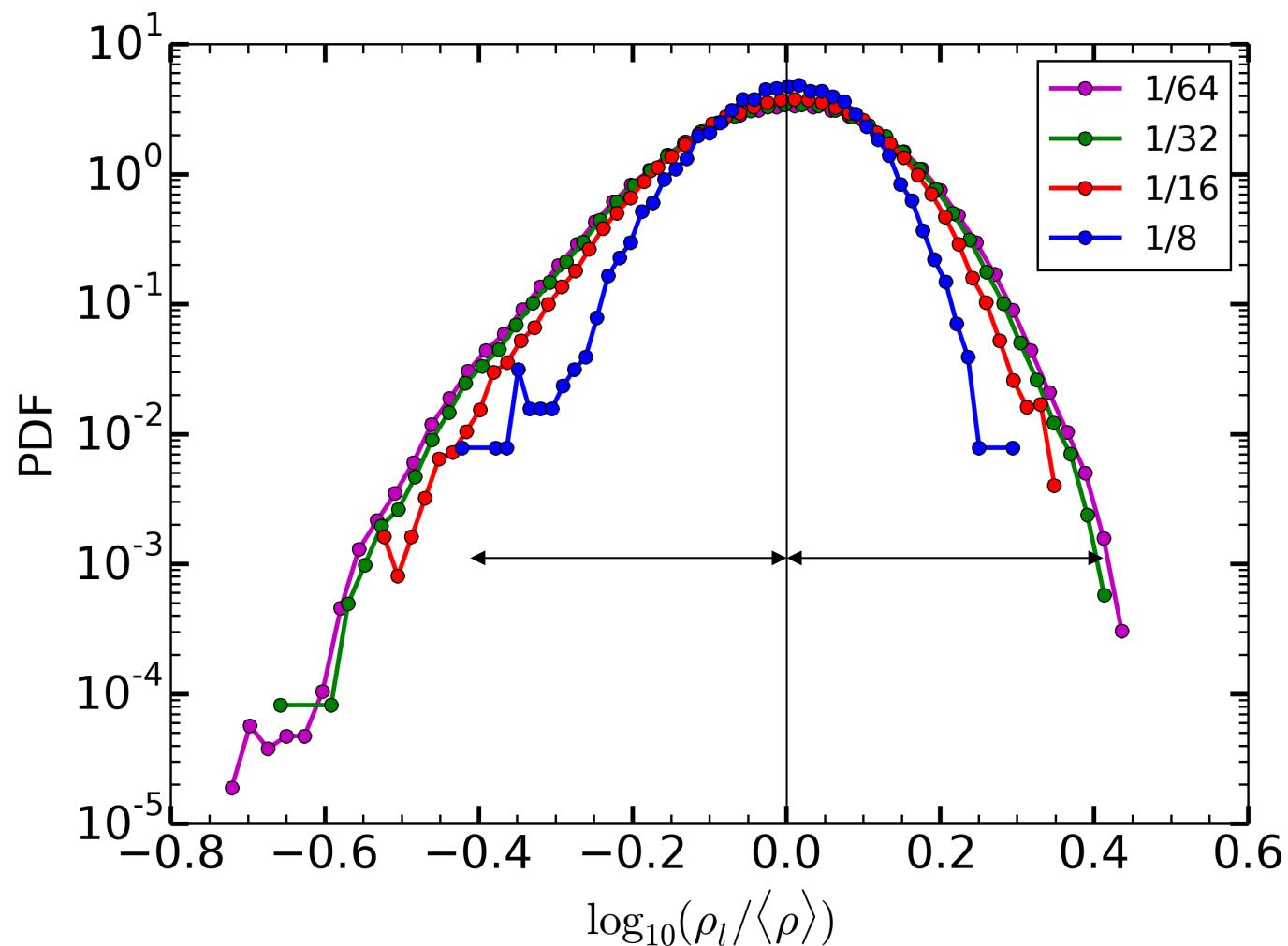
Gas-density PDF



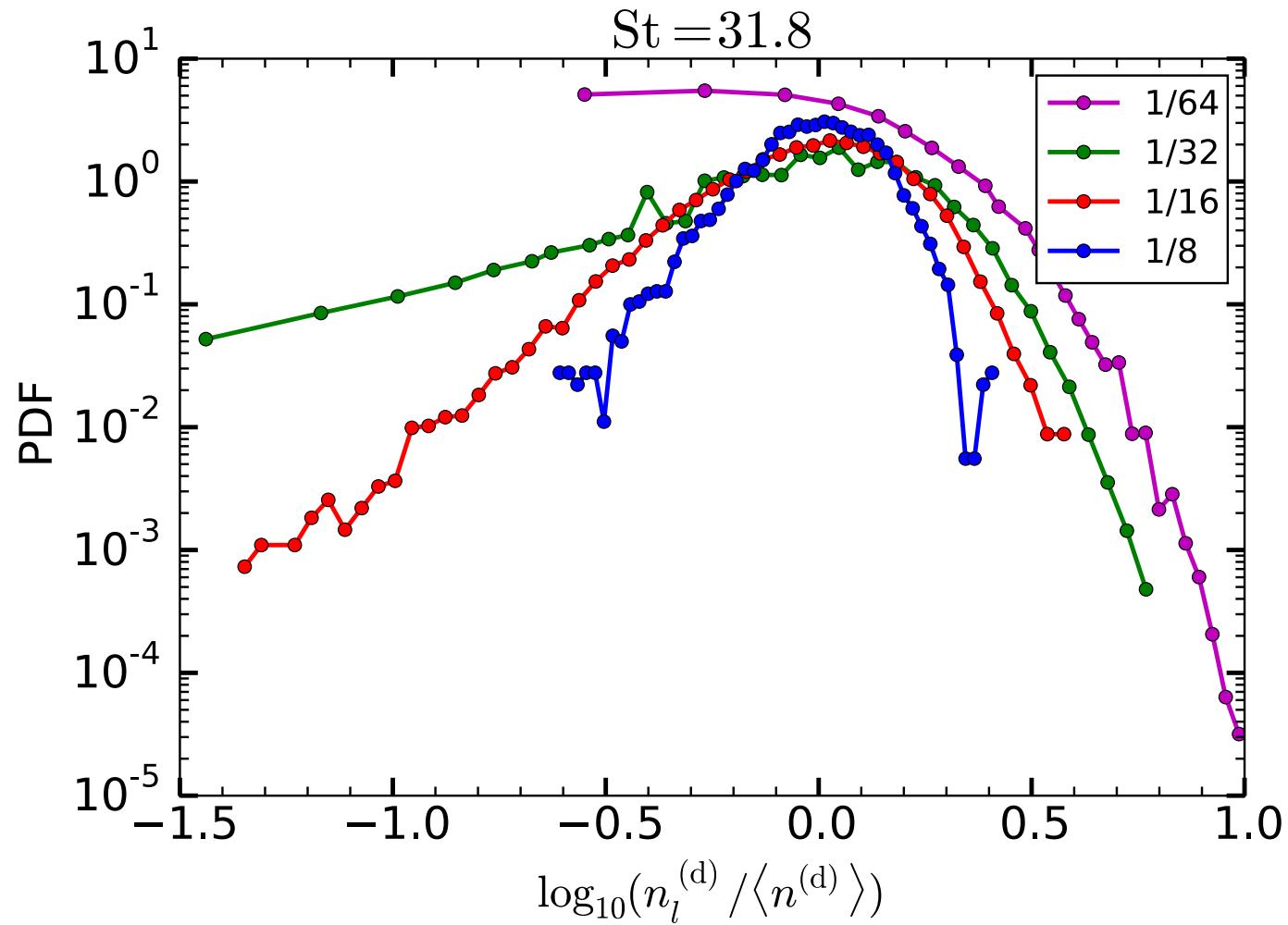
Gas-density PDF



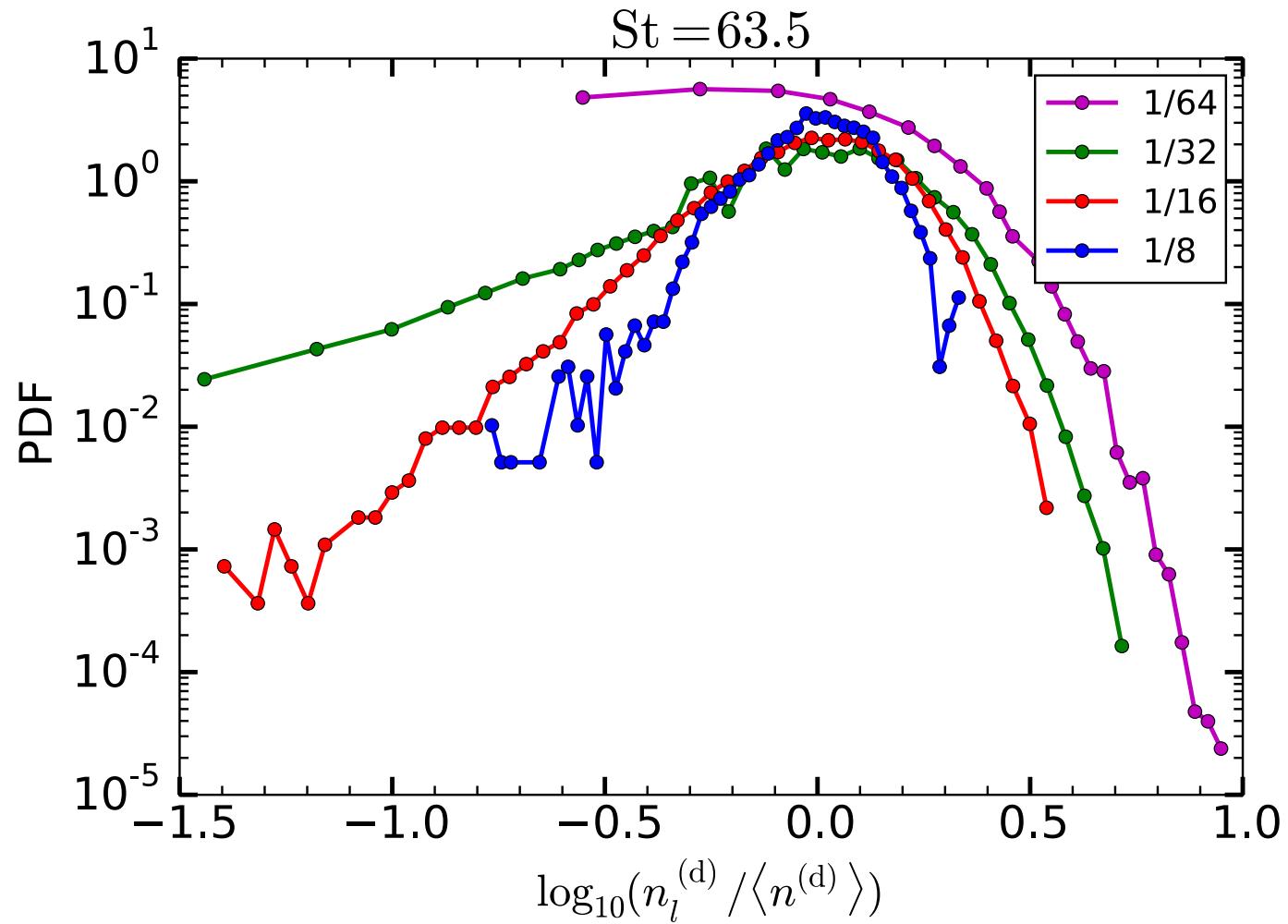
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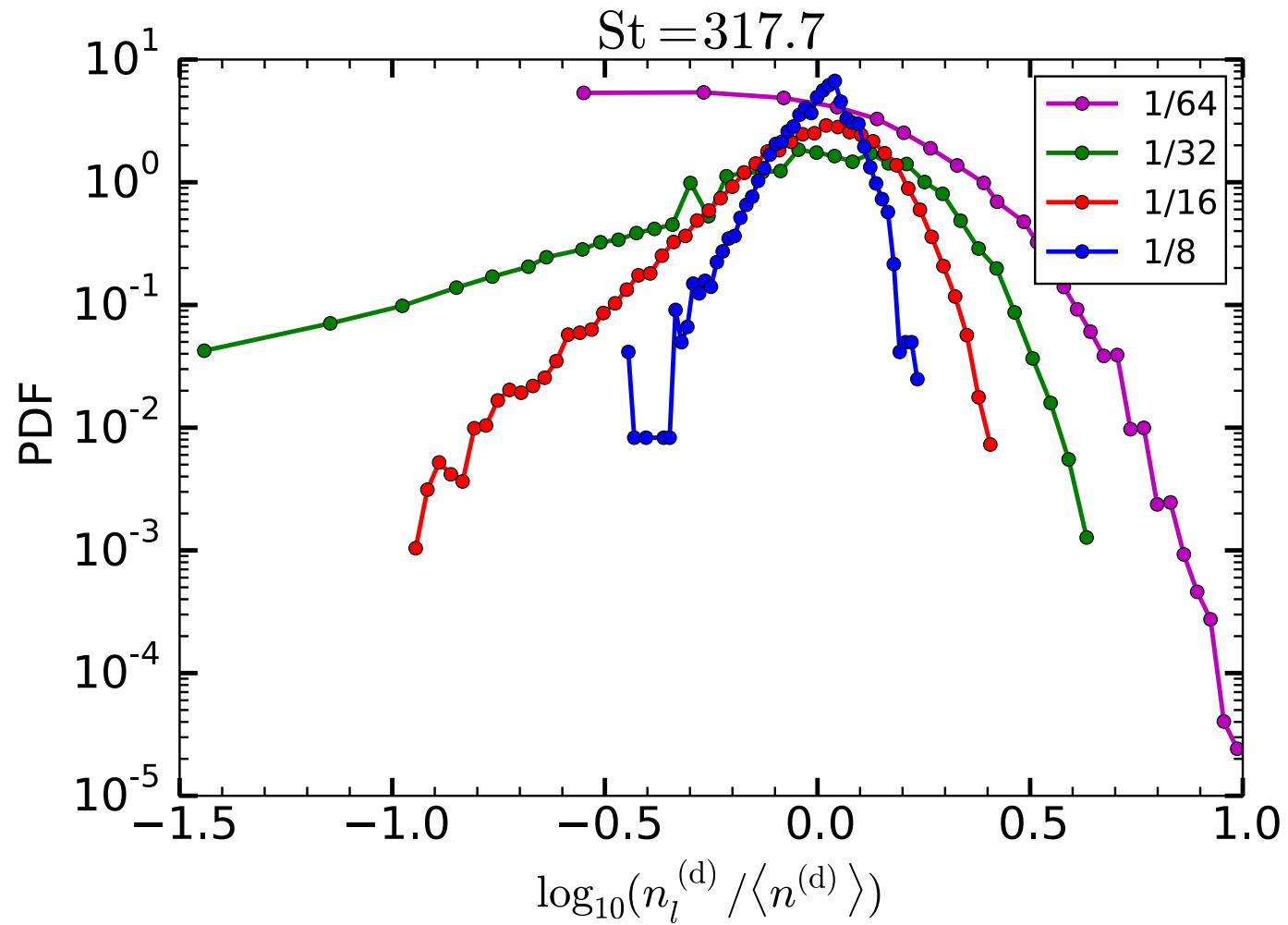
Dust-density PDF



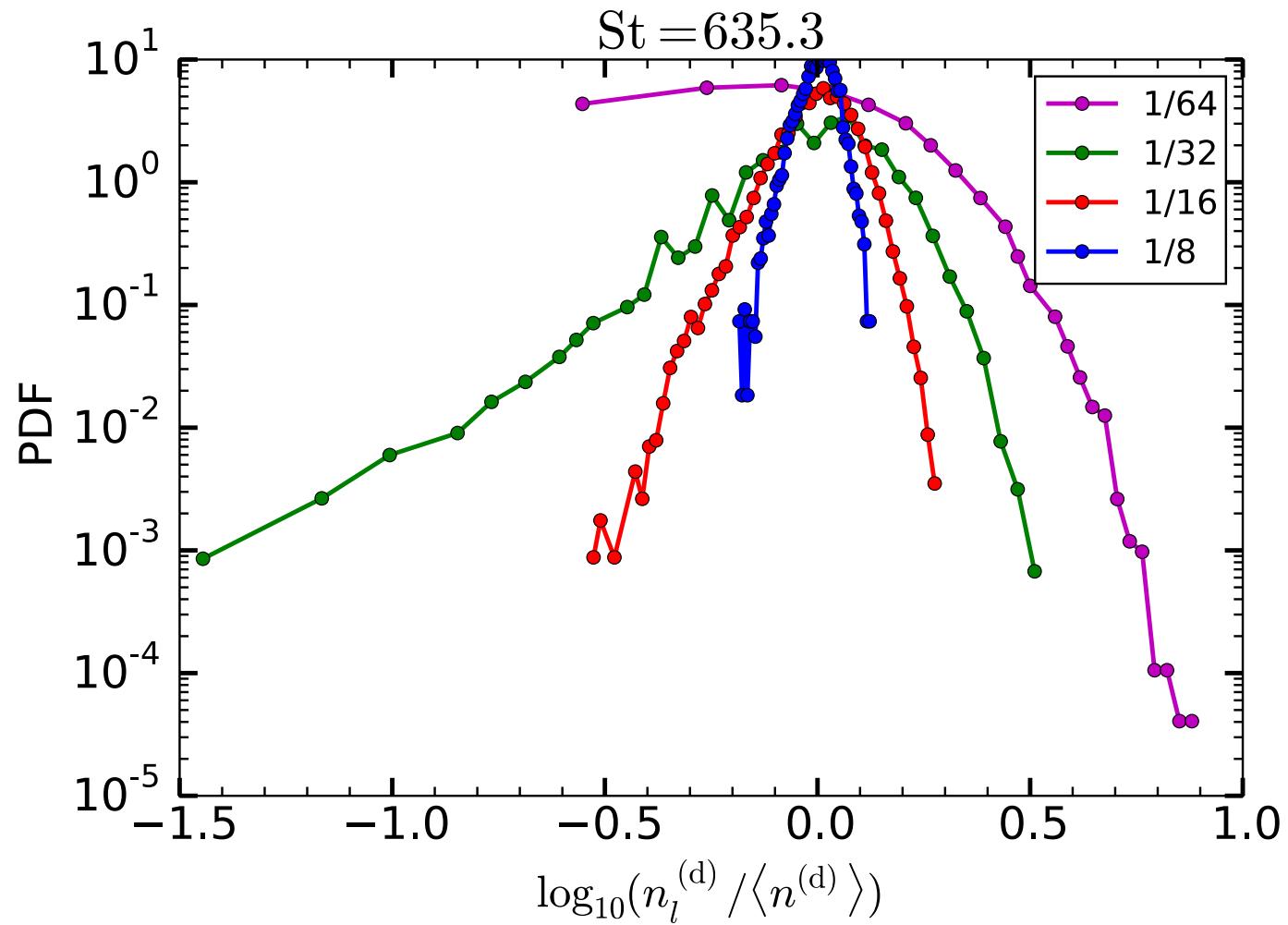
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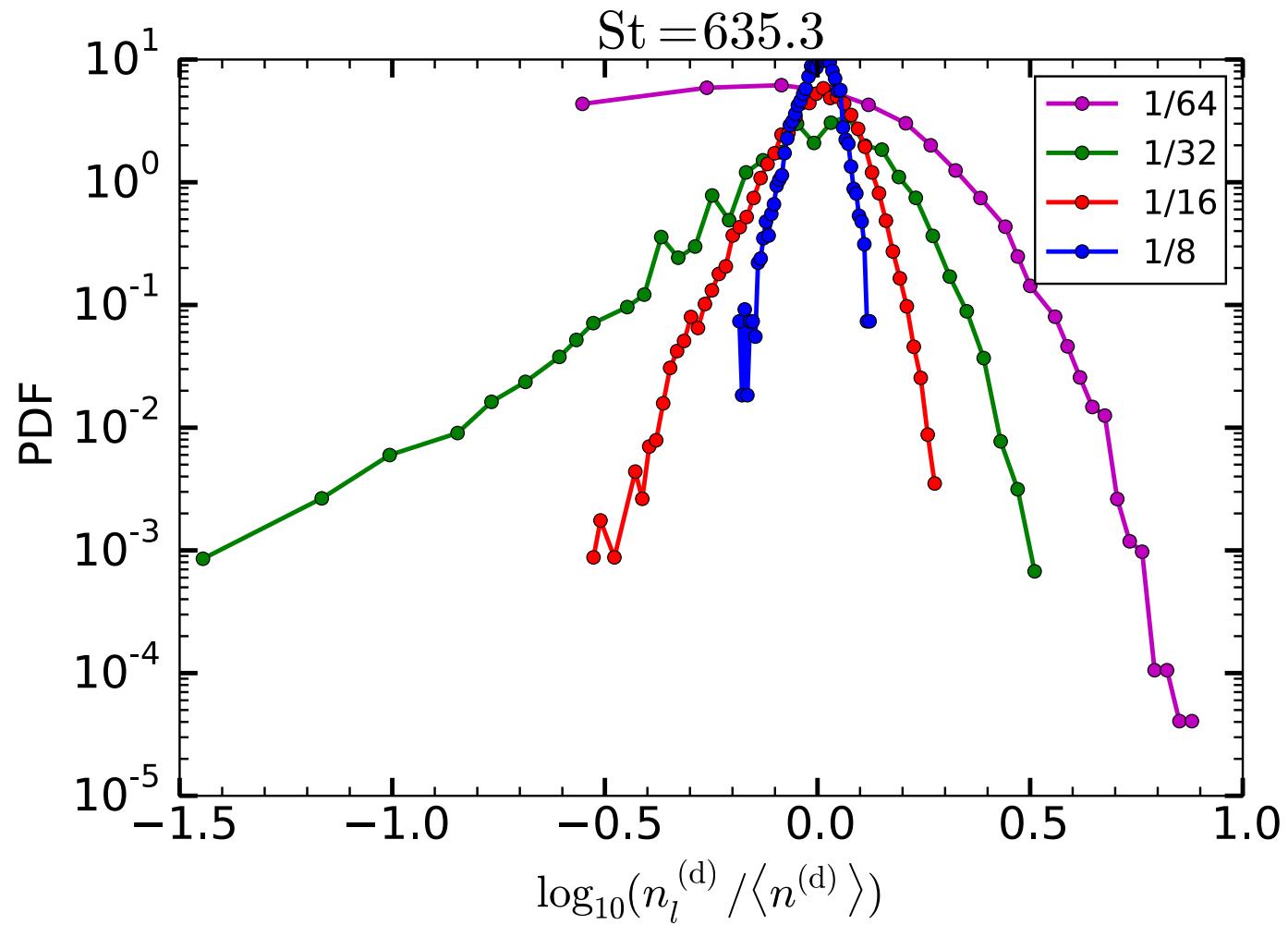
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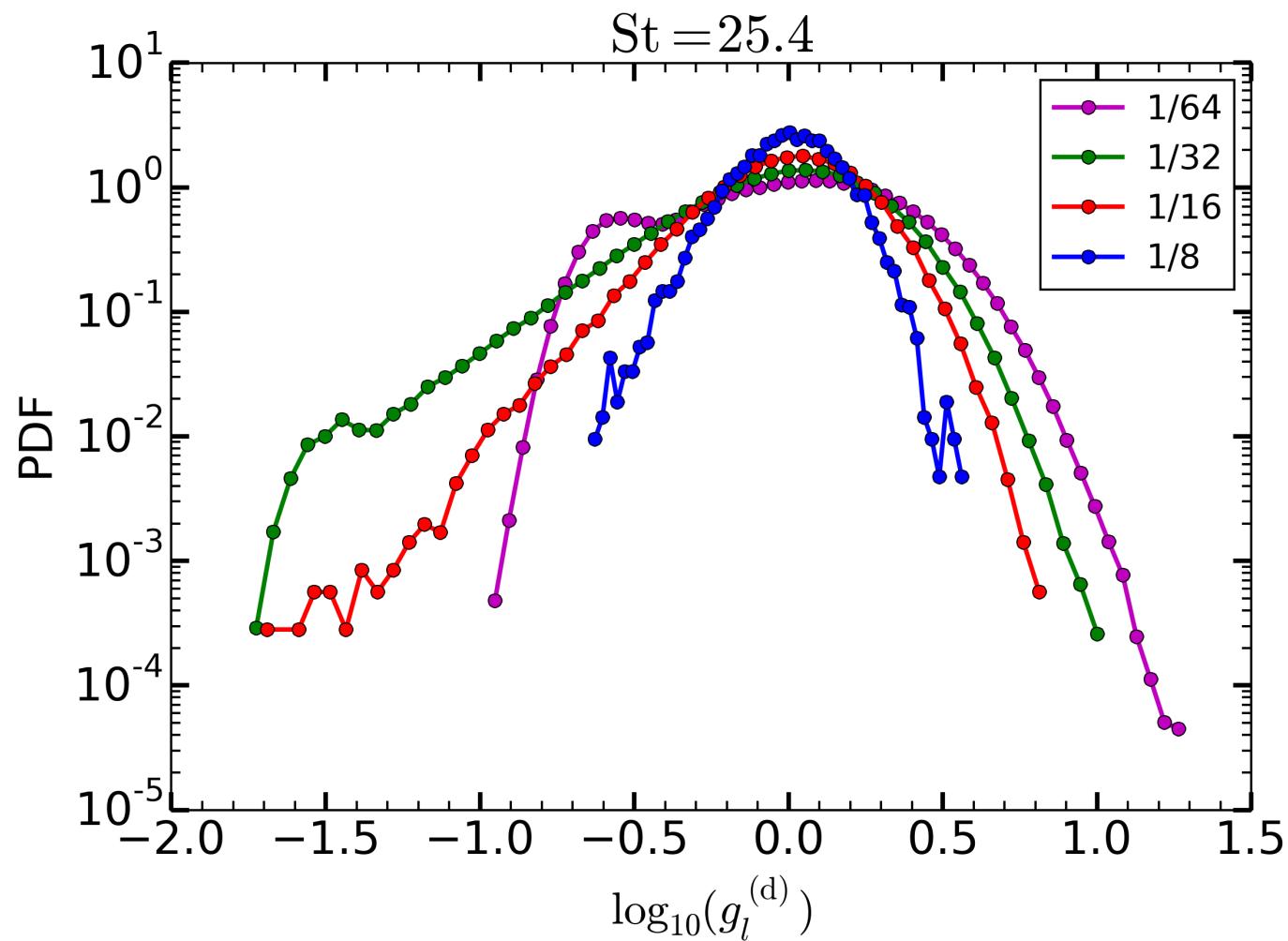
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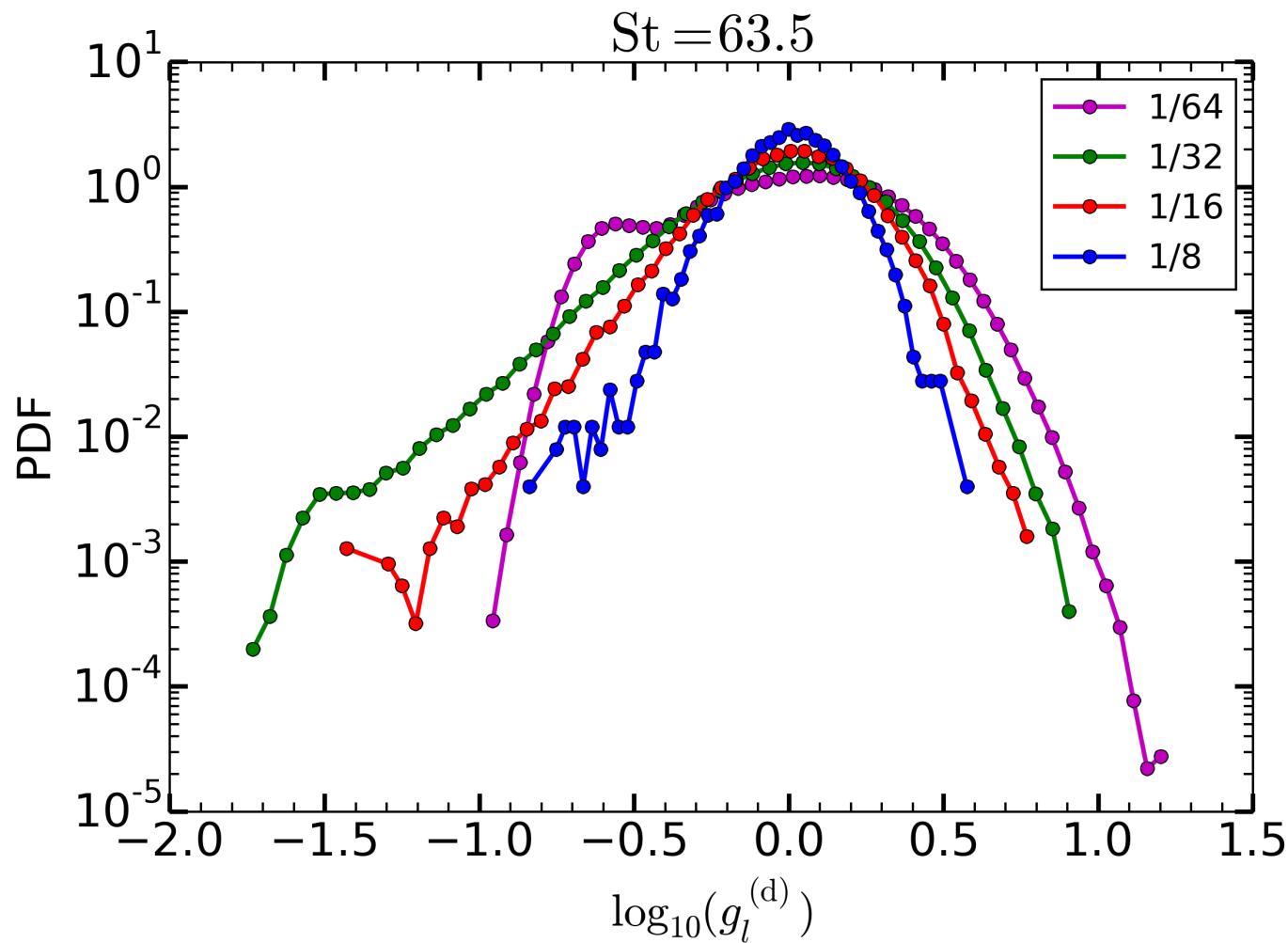
Dust-density PDF



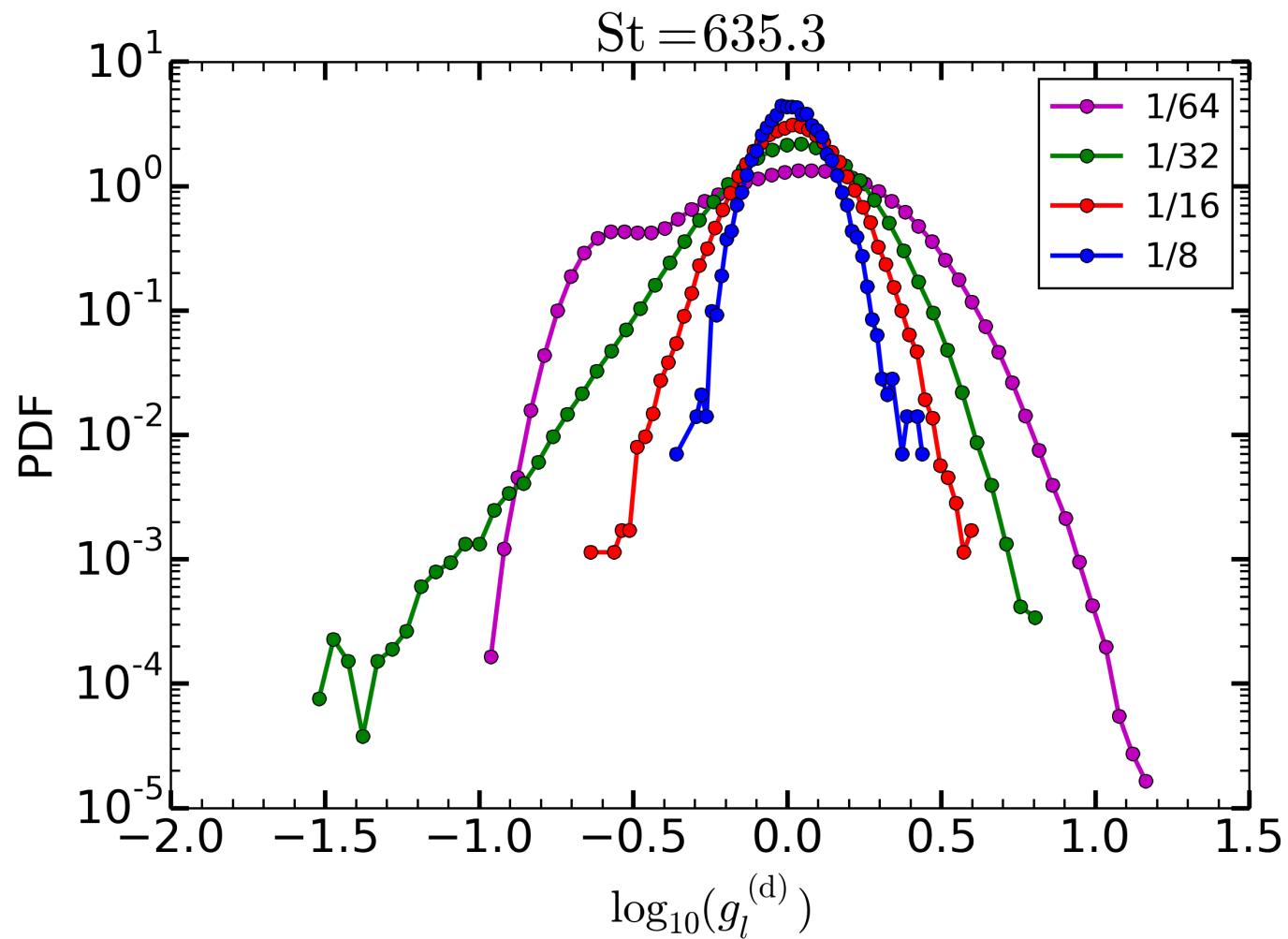
Dust-to-gas PDF



Dust-to-gas PDF



Dust-to-gas PDF



Clustering

Bec et al. (2007, PRL, 98, 084502):

Below the Kolmogorov length scale η where the velocity field is differentiable, the motion of inertial particles is governed by the fluid strain, and the dissipative dynamics leads their trajectories to converge to a dynamically evolving attractor. For any given response time of the particles, their mass distribution is singular and generically scale invariant with fractal properties at small scales [8,11]. To characterize particle clusters at these scales, we measured the correlation dimension \mathcal{D}_2 , which is estimated through the small-scale algebraic behavior of the probability to find two particles at a distance less than a given r : $P_2(r) \sim r^{\mathcal{D}_2}$.

Clustering

Bec et al. (2007, PRL, 98, 084502):

Below the velocity field is governed by the mutual interaction between particles, which leads them to cluster around their mass centers. This clustering is invariant under a global scaling transformation.

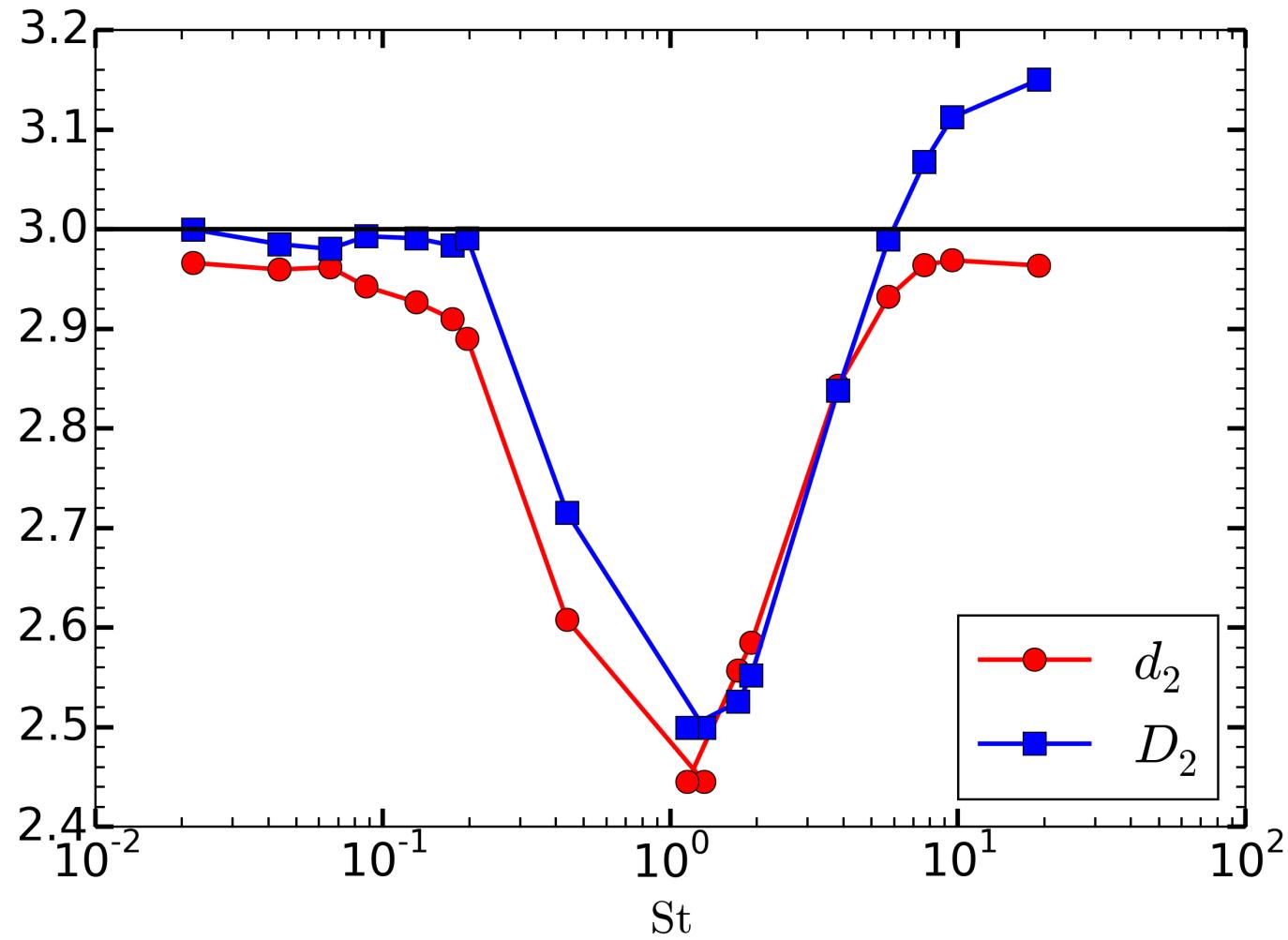
$$g(r) \propto r^{-2} \frac{dP}{dr} \propto r^{d_2}$$

$$w^2 \equiv \frac{x^2}{\eta^2} + \frac{v^2}{u_{\text{rms}}^2}, \quad g(w) \propto w^{D_2}$$

The velocity field is governed by the mutual interaction between particles, which leads them to cluster around their mass centers. This clustering is invariant under a global scaling transformation [10,11]. To

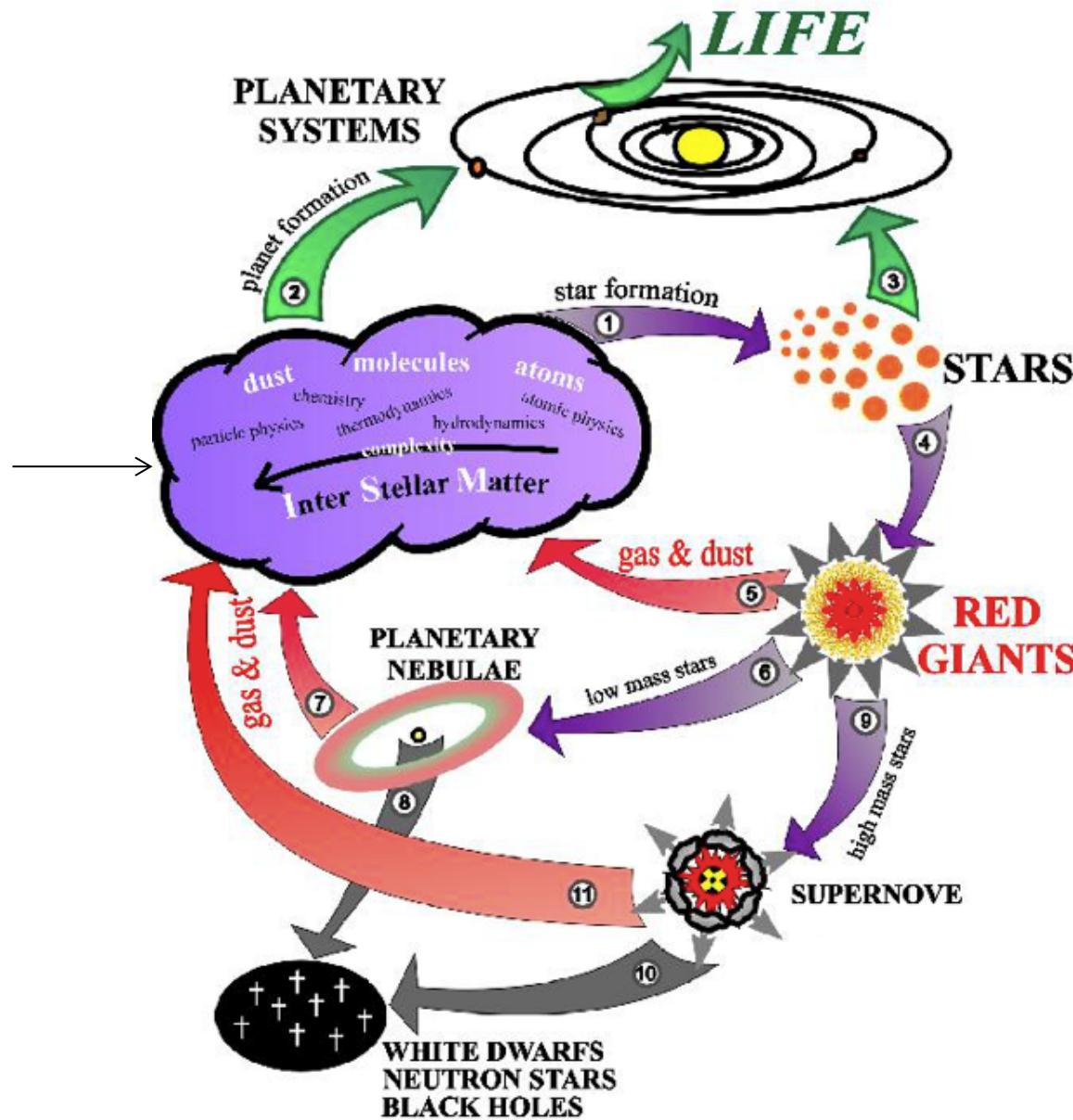
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Clustering



The cosmic matter cycle

Coagulation
&
condensation

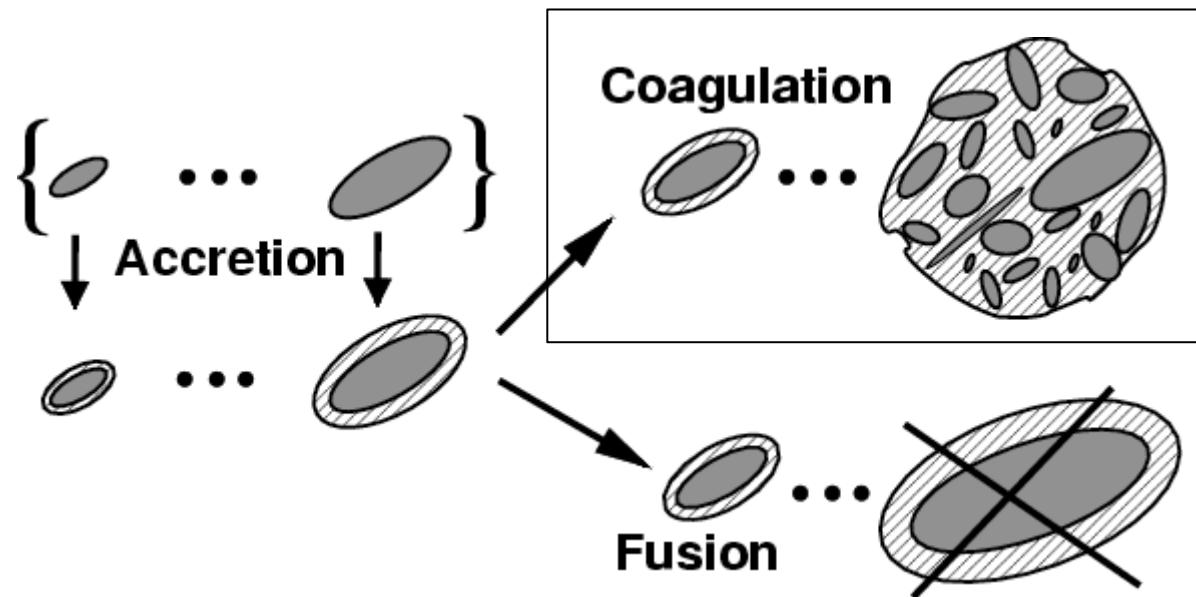


Coagulation

Smoluchowski (coagulation) equation:

$$\frac{\partial f}{\partial t} = \frac{1}{2} \sum_{j=1}^{i-1} C(m_i - m_j, m_j) f(m_i - m_j, t) f(m_j, t) - \sum_{j=1}^{\infty} C(m_i, m_j) f(m_i, t) f(m_j, t),$$

$$\frac{\partial f}{\partial t} = \frac{1}{2} \int_0^m C(m - m', m') f(m - m', t) f(m', t) dm' - f(m, t) \int_0^{\infty} C(m, m') f(m', t) dm',$$



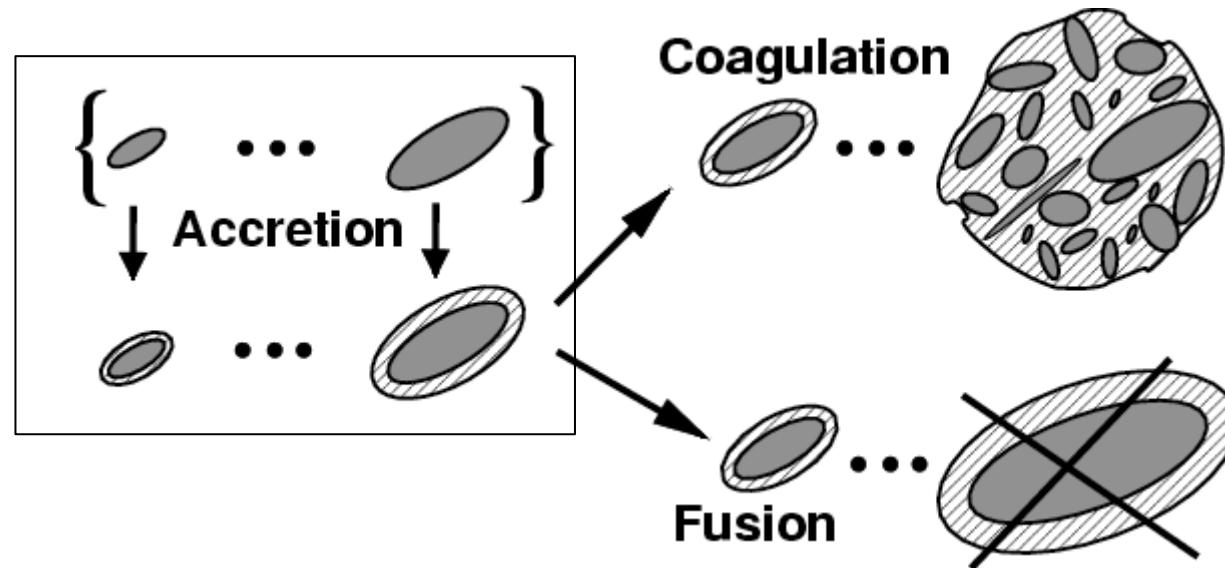
(+ fragmentation as a “reverse process”)

Condensation

Condensation equation:

$$\frac{dm}{dt} = 4\pi a^2 \alpha_s \langle v_{\text{mol}} \rangle \rho_{\text{mol}}(t),$$

$$\xi_{c,k}(t) = \frac{da}{dt} = \alpha_s \langle v_{\text{mol}} \rangle \frac{A_{\text{eff},j} \rho_k(t) - \rho_d(t)}{A_k \rho_{\text{gr}}},$$



Condensation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}_d) = [\text{cond. source term}]$$

Lagrangian frame:

$$(\nabla \cdot \mathbf{v})_L = \left(\frac{\partial v_1}{\partial x_1} \Big|_{\mathbf{r}(t)} + \frac{\partial v_2}{\partial x_2} \Big|_{\mathbf{r}(t)} + \frac{\partial v_3}{\partial x_3} \Big|_{\mathbf{r}(t)} \right)_L$$

Condensation

$$\mathcal{K}_\ell(t) = \int_0^\infty a^\ell f(a, t) da.$$

$$\frac{d\mathcal{K}_\ell}{dt} = \ell \xi(t) \mathcal{K}_{\ell-1}(t)$$

With dynamics (Lagrangian, dust-gas velocity coupling):

$$\frac{d\mathcal{K}_\ell}{dt} = \ell \xi(t) \mathcal{K}_{\ell-1}(t) - \mathcal{K}_\ell(t) (\nabla \cdot \mathbf{v})_{\text{L}}$$

$$\frac{d\langle a^\ell \rangle}{dt} = \frac{1}{\mathcal{K}_0} \left(\frac{d\mathcal{K}_\ell}{dt} - \frac{\mathcal{K}_\ell}{\mathcal{K}_0} \frac{d\mathcal{K}_0}{dt} \right)$$

$$\frac{d\langle a^\ell \rangle}{dt} = \ell \xi(t) \langle a^{\ell-1} \rangle$$

Condensation

$$\mathcal{K}_\ell = \bar{\mathcal{K}}_\ell + \mathcal{K}'_\ell, \quad \xi = \bar{\xi} + \xi'$$

$$\bar{Q}(t) = \frac{1}{2\tau} \int_{t-\tau}^{t+\tau} Q(t') dt'$$

Results in the following averaged equations:

$$\frac{d\bar{\mathcal{K}}_\ell}{dt} = \ell \bar{\xi} \bar{\mathcal{K}}_{\ell-1} + \ell \overline{\xi' \mathcal{K}'_{\ell-1}} - \overline{\mathcal{K}'_\ell (\nabla \cdot \mathbf{v})'_L}$$

$$\overline{(\nabla \cdot \mathbf{v})_L} = 0, \quad \overline{\mathcal{K}'_0 (\nabla \cdot \mathbf{v})'_L} = 0.$$

Condensation

$$\rho_d(t) \equiv \frac{4\pi\rho_{\text{gr}}}{3} \int_0^\infty a^3 f(a, t) da.$$

$$\mathbf{v}_d(t) \equiv \frac{4\pi}{3} \frac{\rho_{\text{gr}}}{\rho_d(t)} \int_0^\infty a^3 f(a, t) \mathbf{v}_d(a, t) da$$

$$\rho \equiv \rho_g + \rho_d, \quad \mathbf{v} \equiv \frac{\rho_g \mathbf{v}_g + \rho_d \mathbf{v}_d}{\rho}$$

$$\frac{\xi}{\xi_0} = \frac{X_i \rho - \rho_d}{\rho_0} = \left[\frac{X_i \bar{\rho}_g - (1 - X_i) \bar{\rho}_d}{\rho_0} \right] + \left[\frac{X_i \rho'_g - (1 - X_i) \rho'_d}{\rho_0} \right]$$

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$$\frac{\xi}{\xi_0} = \frac{X_i \rho - \rho_d}{\rho_0} = \boxed{\left[\frac{X_i \bar{\rho}_g - (1 - X_i) \bar{\rho}_d}{\rho_0} \right]} + \left[\frac{X_i \rho'_g - (1 - X_i) \rho'_d}{\rho_0} \right] > 0$$

Conclusions

- Stars produced the first dust grains, but most of the interstellar dust may have condensed in MCs.
- Under all circumstances, interstellar dust condensation is needed as a replenishment mechanism.
- Compressible turbulence leads to gas-dust separation and clustering of grains:
 - **Coagulation rate increases due to the clustering.**
 - **Condensation rate can be affected in various ways and may effectively decrease due to the separation.**