Effective Theory of Dark Energy

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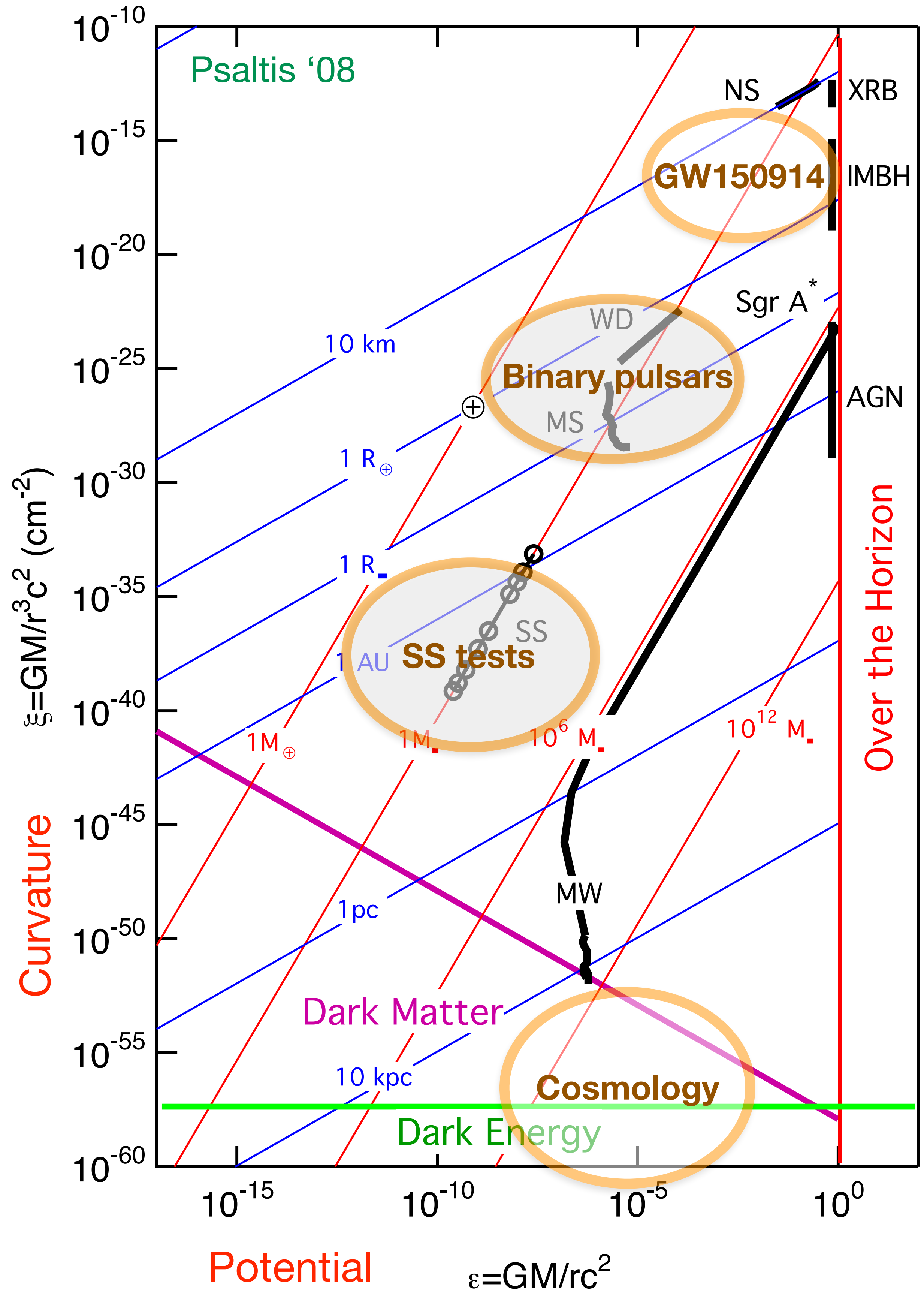
Motivations

- ✦ Gravity only been tested over specials ranges of scales and masses
- ✦ Cosmology is a window for testing gravity on very large distances



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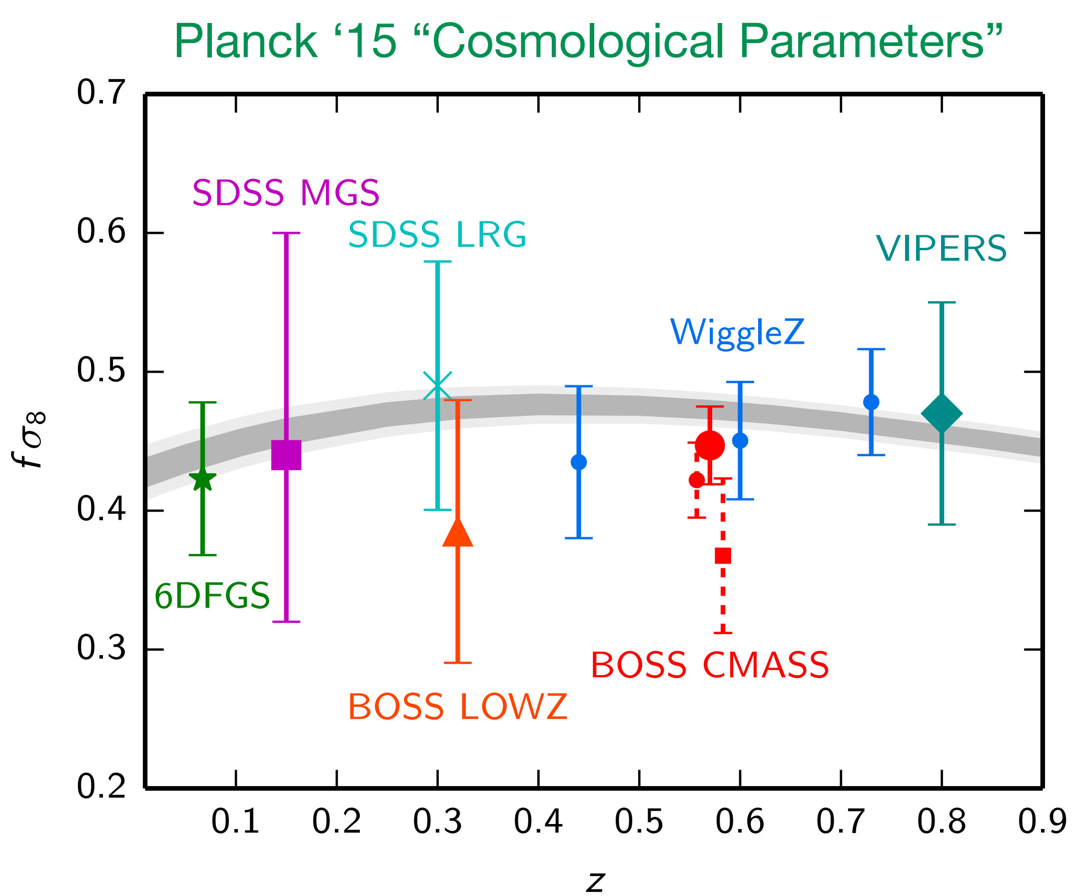

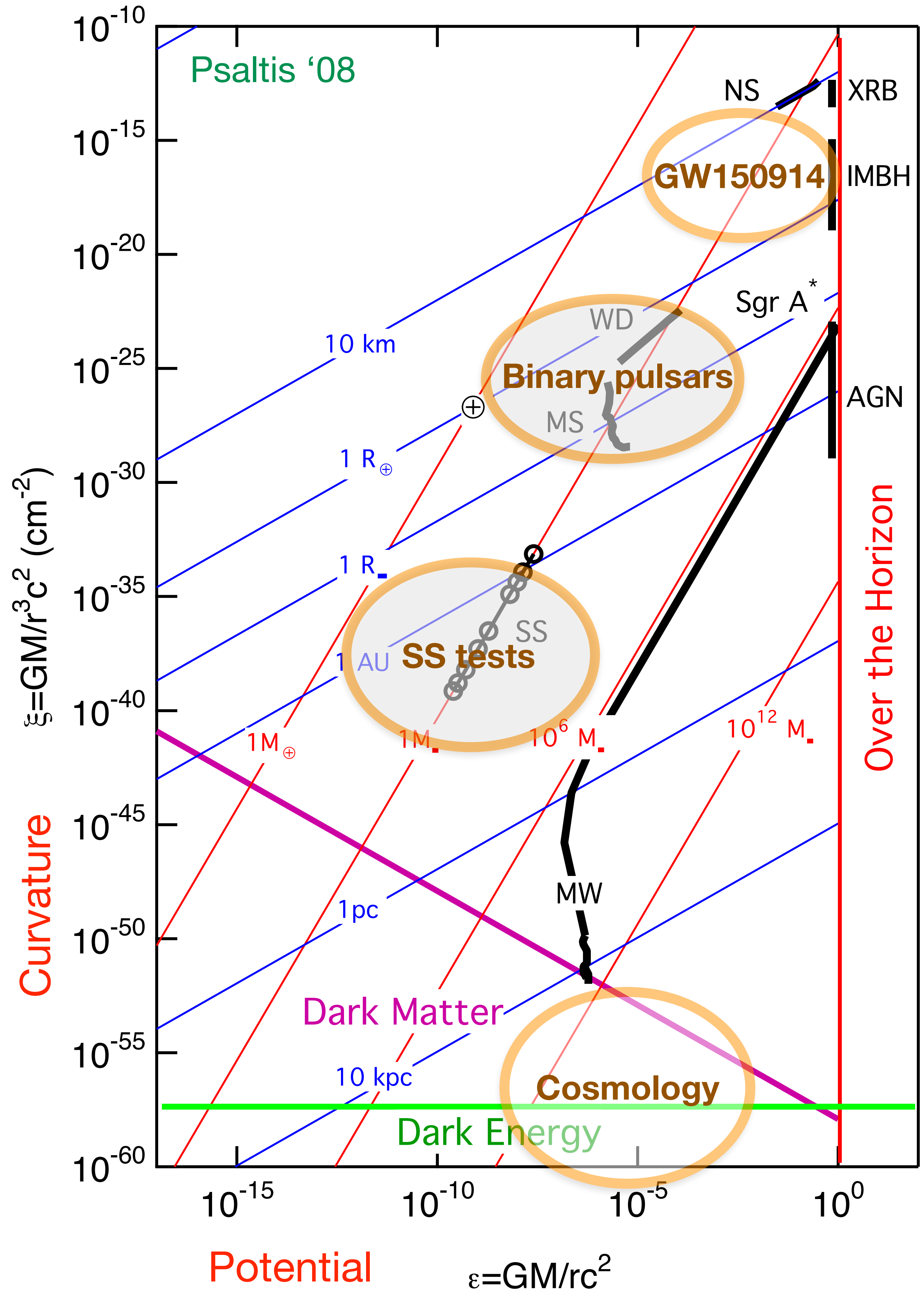

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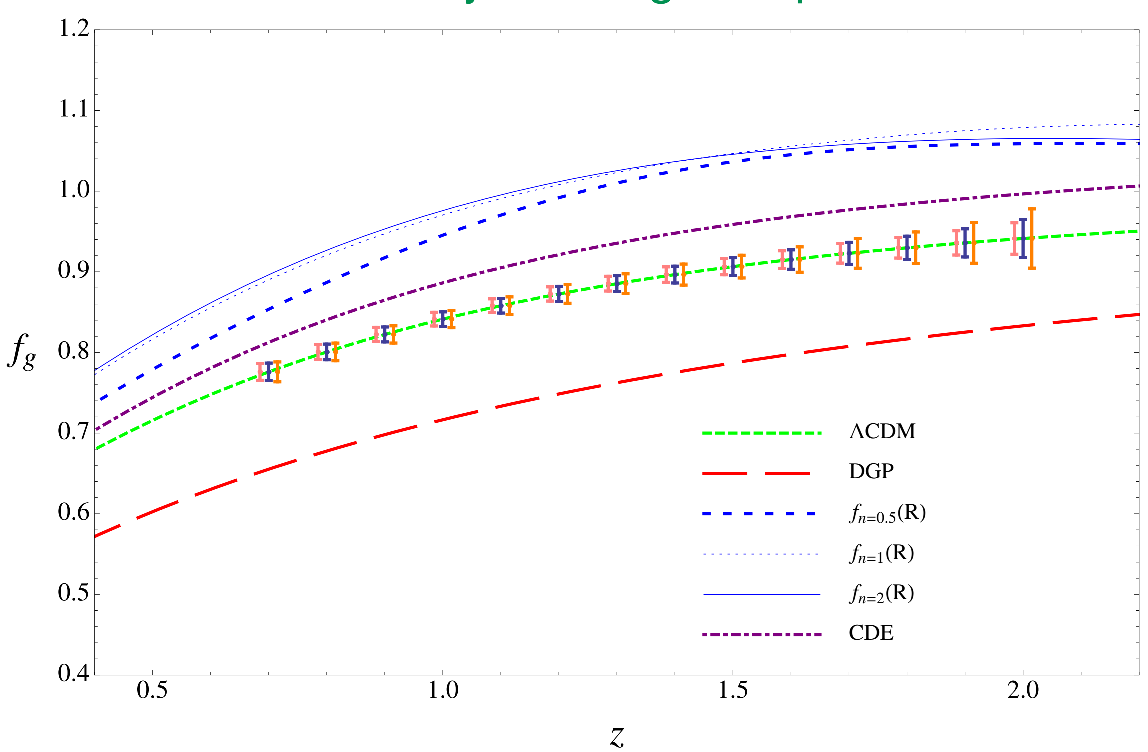
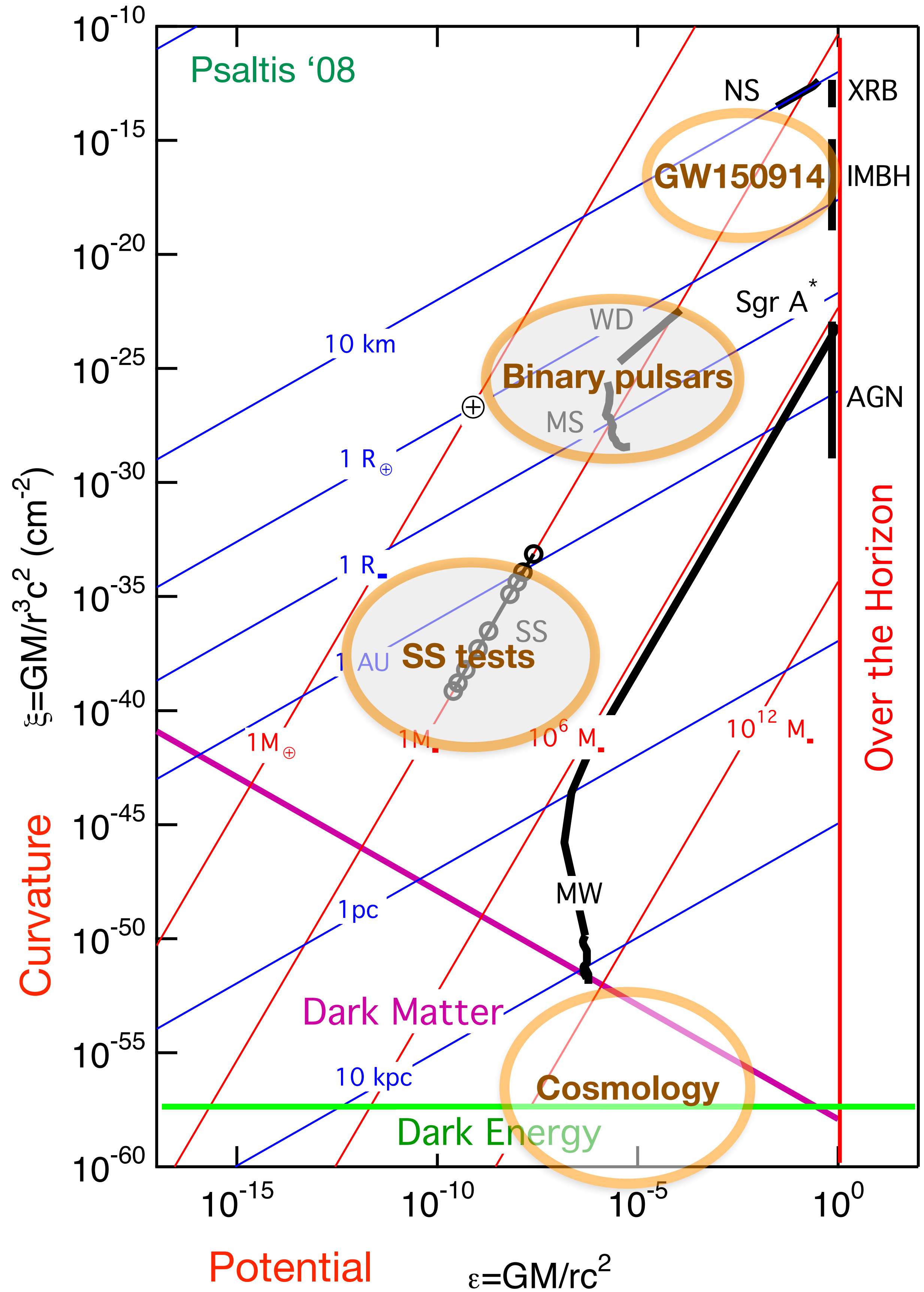
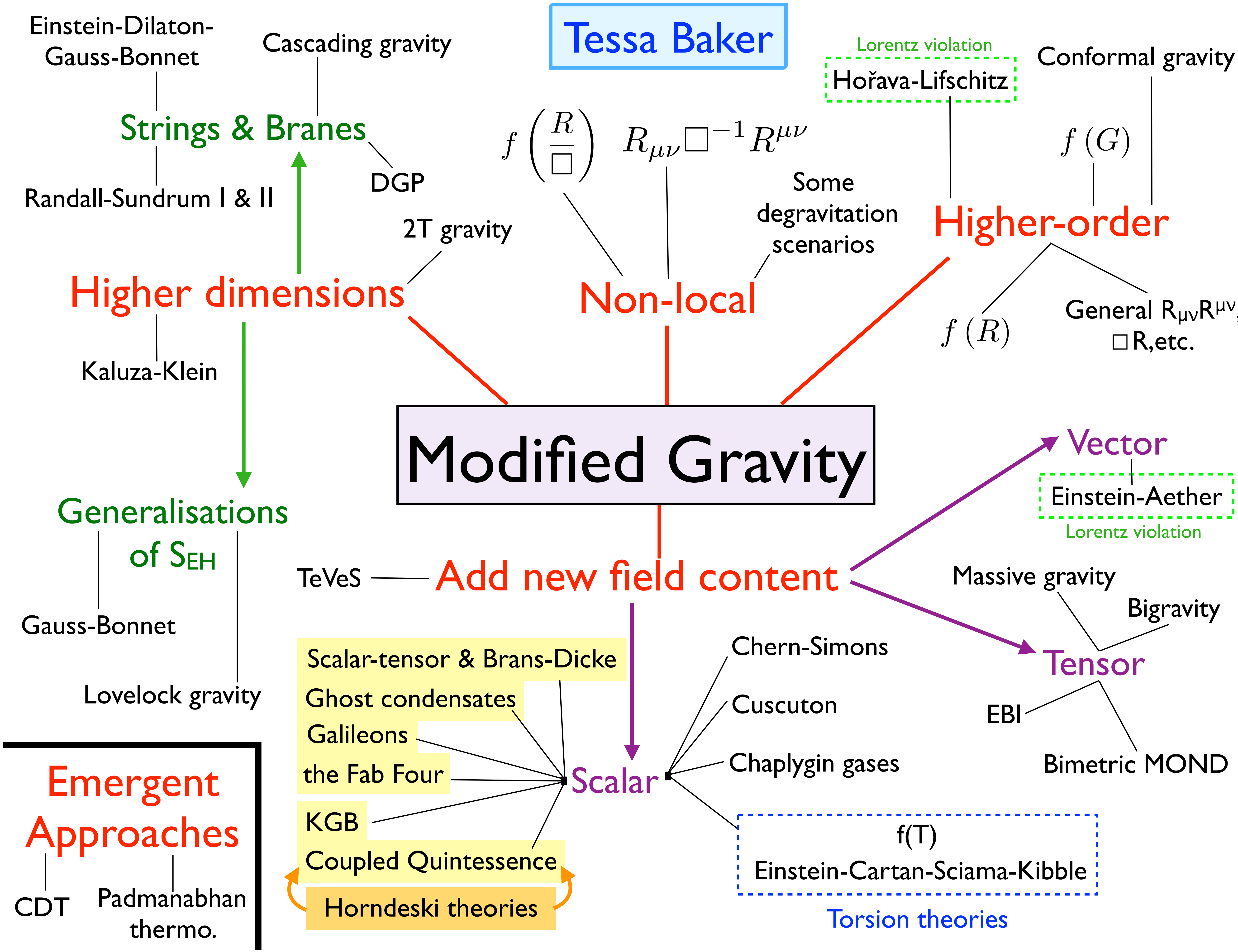
Euclid Theory Working Group '16

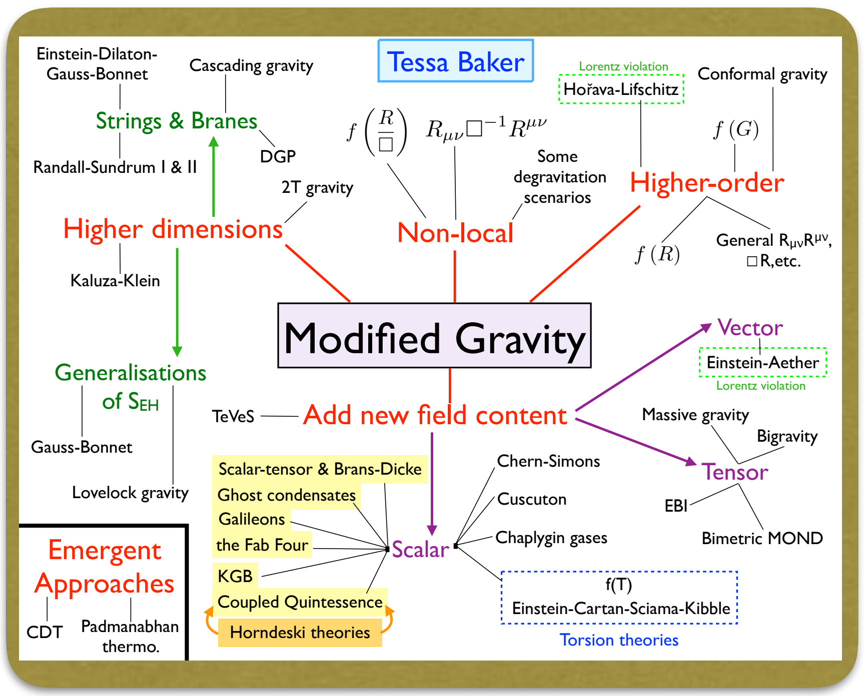





Tessa Baker
Modified Gravity
Higher dimensions
Strings & Branes
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
DGP
Randall-Sundrum I & II
2T gravity
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Non-local
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Higher-order
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$,etc.
Add new field content
TeVeS
Vector
Einstein-Aether
Lorentz violation
Tensor
Massive gravity
Bigravity
EBI
Bimetric MOND
Scalar
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories
Emergent Approaches
CDT
Padmanabhan thermo.

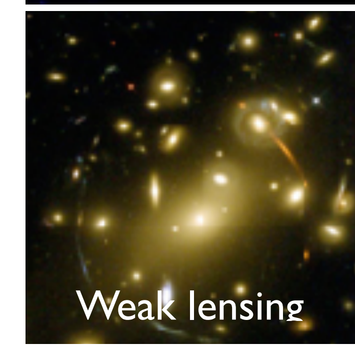
Motivations

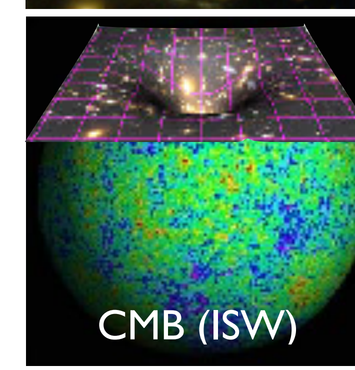
Models

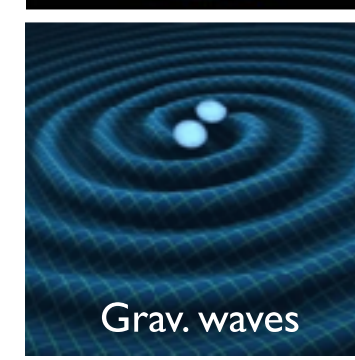


Many theoretical models of modified gravity, each with their own motivation and phenomenology

Observations







Motivations

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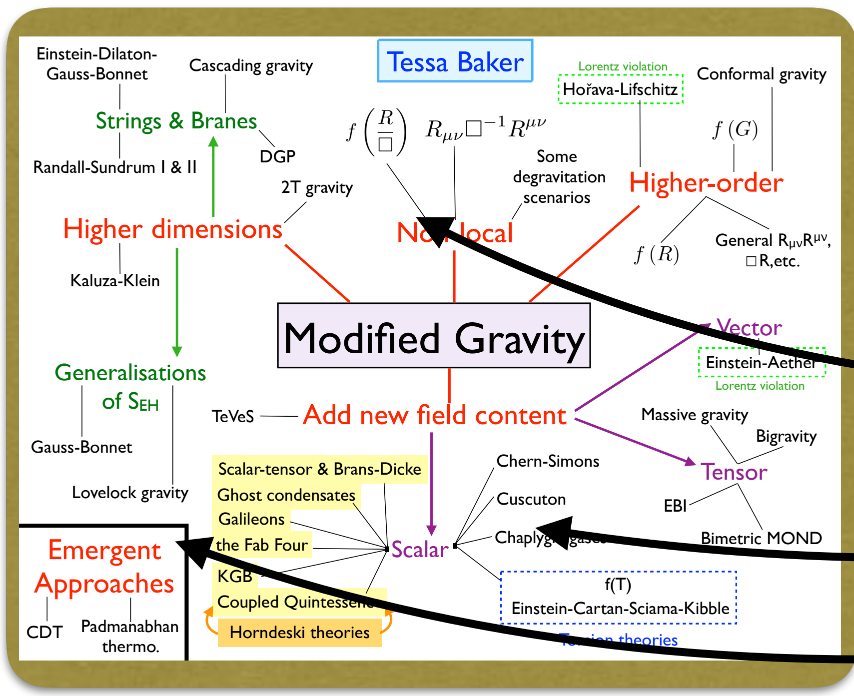

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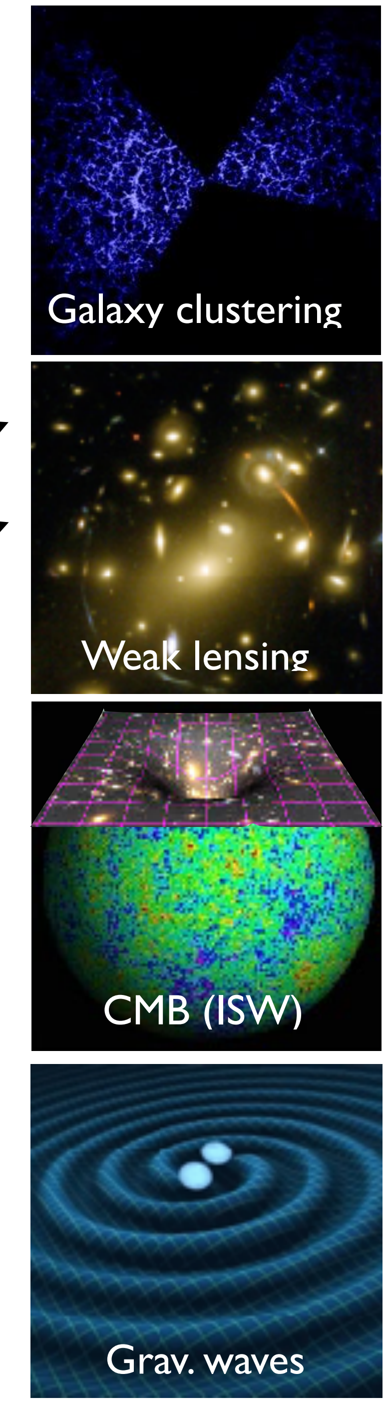

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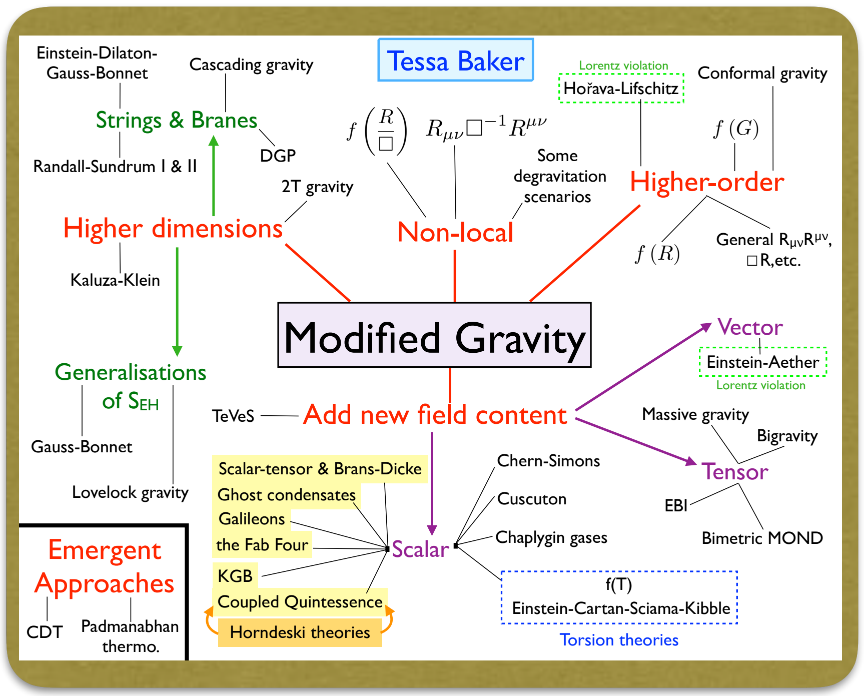

ETofDE

$\alpha_K(t),\ \alpha_B(t),\ \alpha_M(t),$
$\alpha_T(t),\ \alpha_T(t),\ \ldots$

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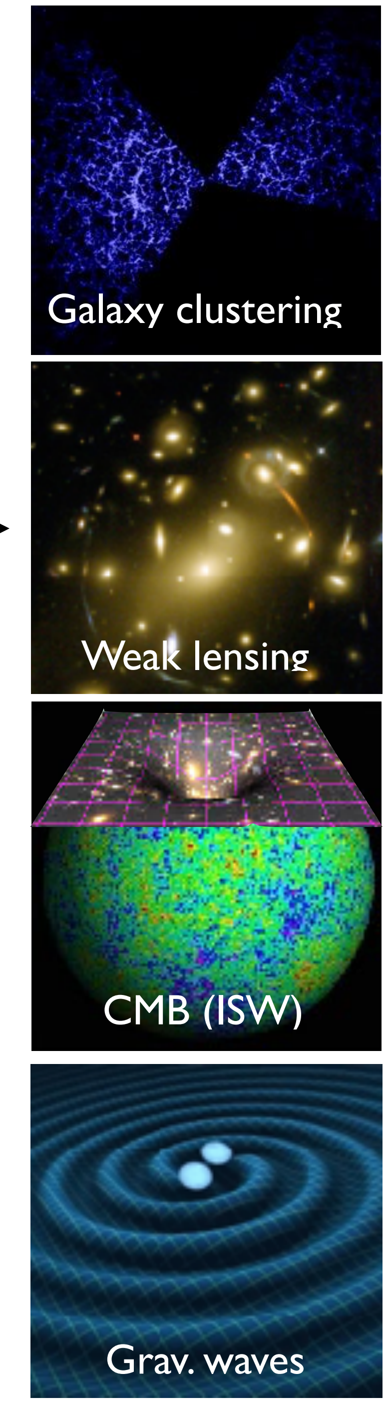

Effective approach to bridge models with observations in a minimal and systematic way

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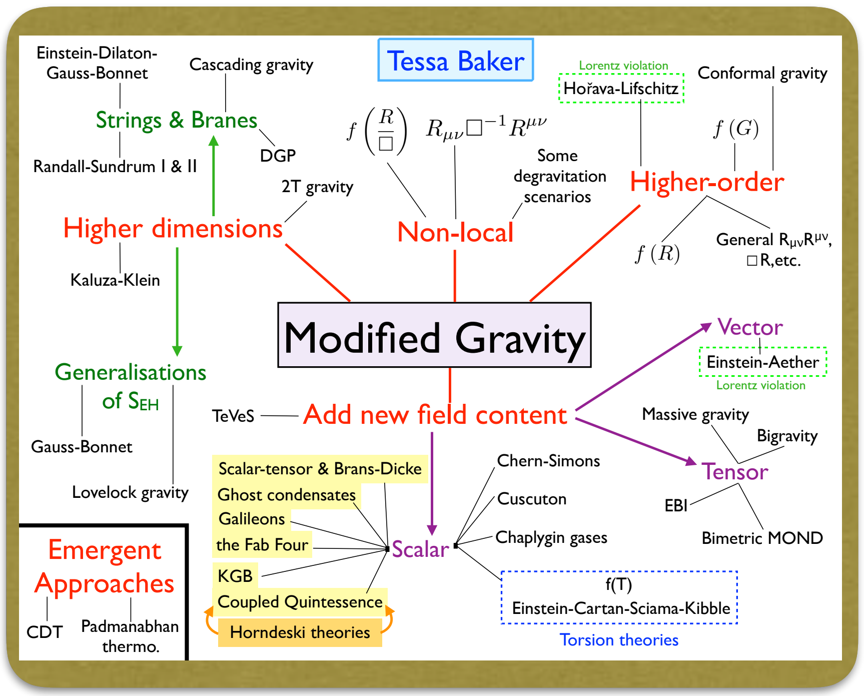

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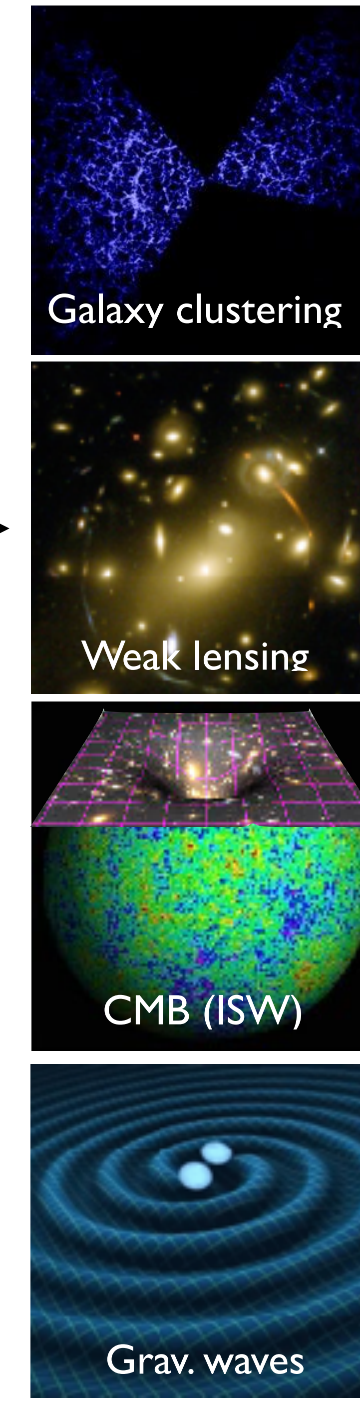
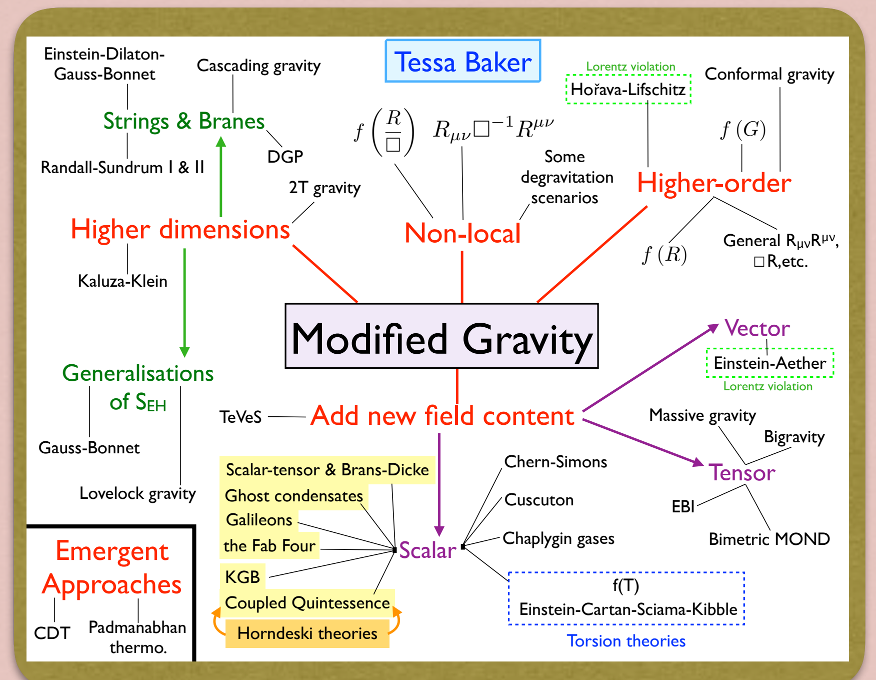


Effective approach to bridge models with observations in a minimal and systematic way

Theoretically motivated: locality, causality, diff invariance, unitarity, etc…

Models



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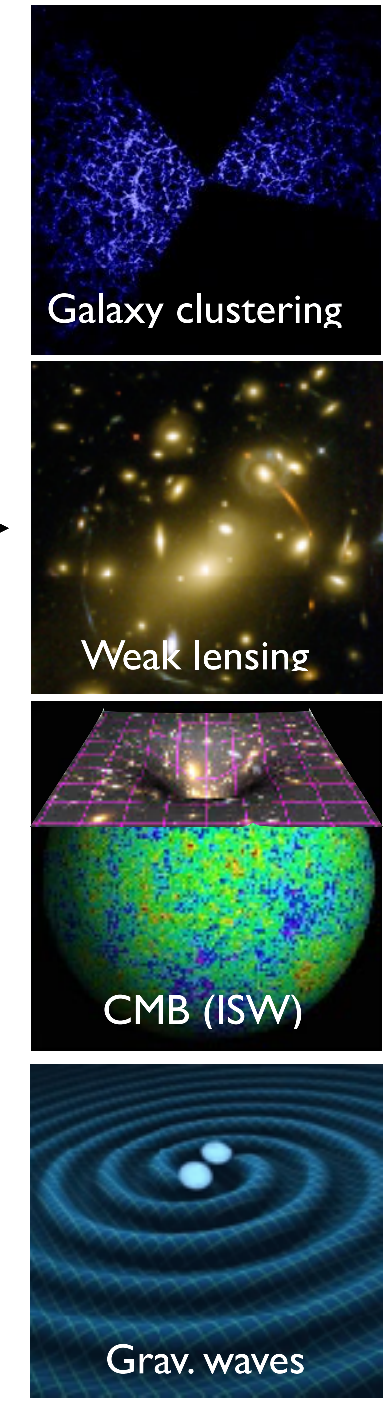

Scalar-tensor theories

- Simplest models of modified gravity are base on single scalar field

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second-order
equations of motion

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- Horndeski theories:

Horndeski '73, see also Deffayet et al.'11

$$X \equiv \phi_{;\mu}\phi^{;\mu} \equiv \nabla_\mu \phi \nabla^\mu \phi$$

$$\begin{aligned} L_H = & \, G_2(\phi, X) + G_3(\phi, X)\Box\phi + \\ & + G_4(\phi, X)^{(4)}R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right] \\ & + G_5(\phi, X)^{(4)}G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\nu\lambda}\phi_{;\lambda}^{\ ;\mu}\right] \end{aligned}$$

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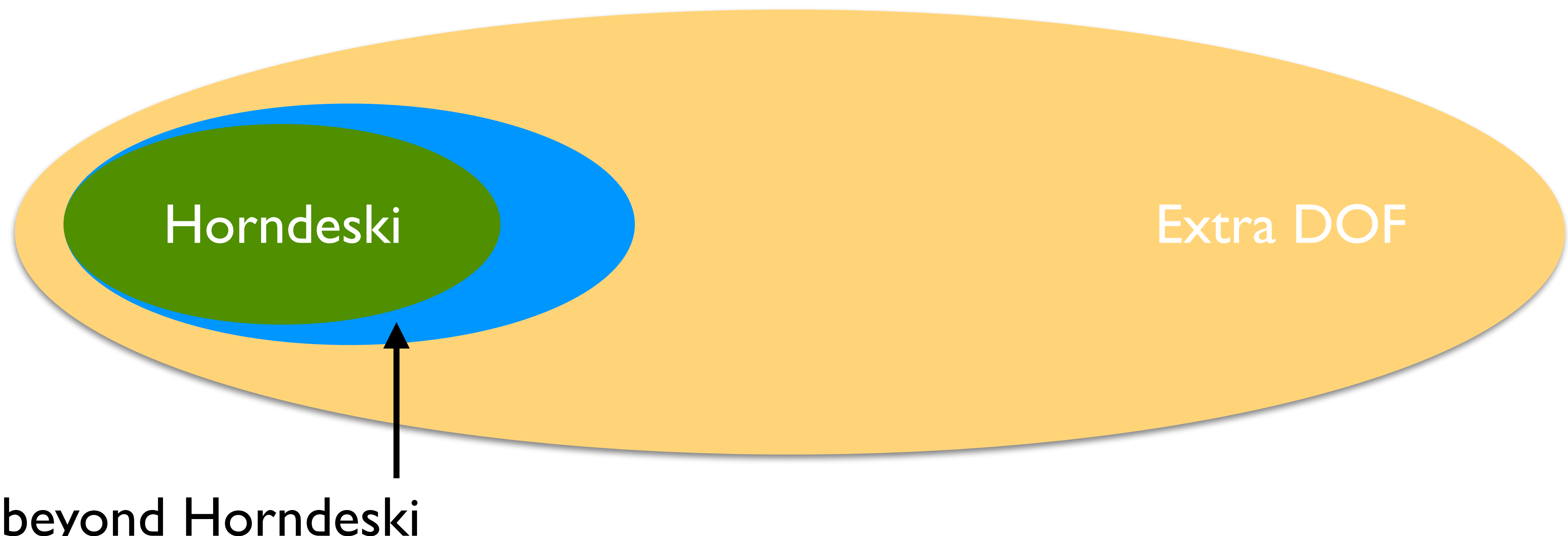

Zumalacarregui, Garcia-Bellido '13
with Gleyzes, Langlois, Piazza '14;

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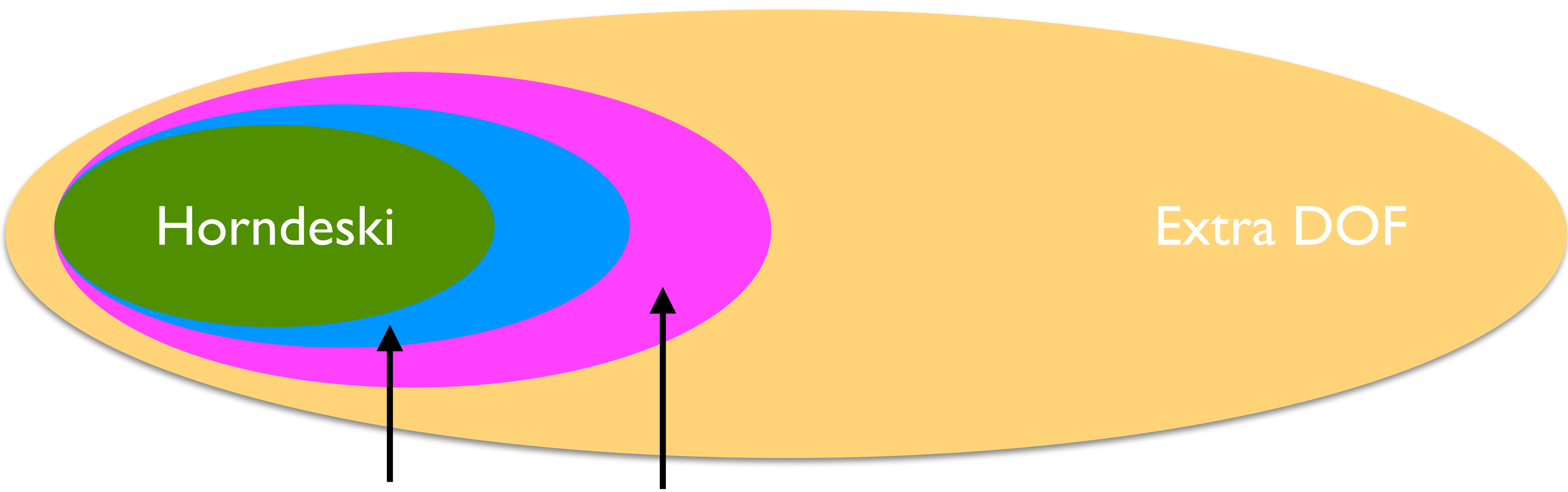
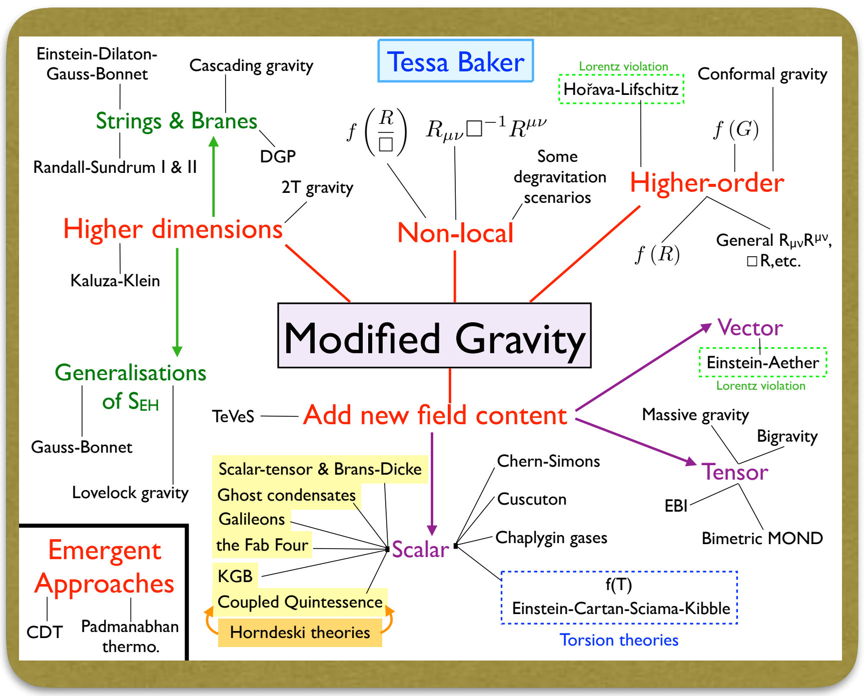


beyond Horndeski
Zumalacarregui, Garcia-Bellido '13
with Gleyzes, Langlois, Piazza '14;

Degenerate Higher-Order Scalar-Tensor theories
Langlois, Noui '15, '16;
Crisostomi, Hull et al. '16;
Crisostomi, Koyama, Tasinato '16;
Achour et al. '16

Models



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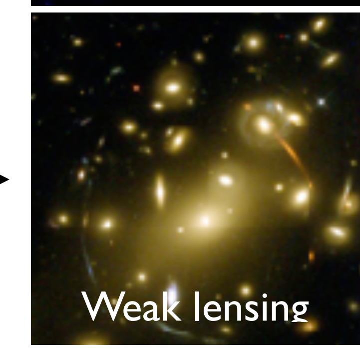
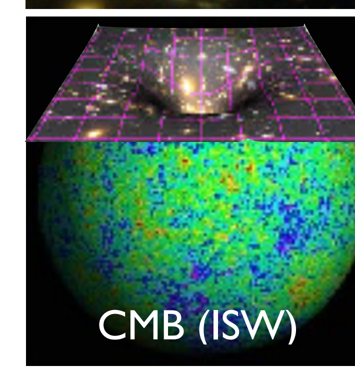
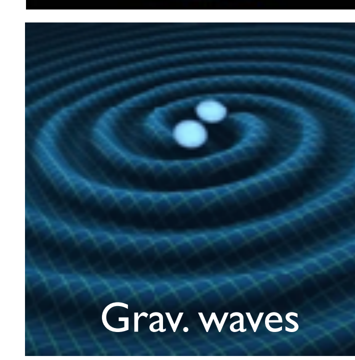
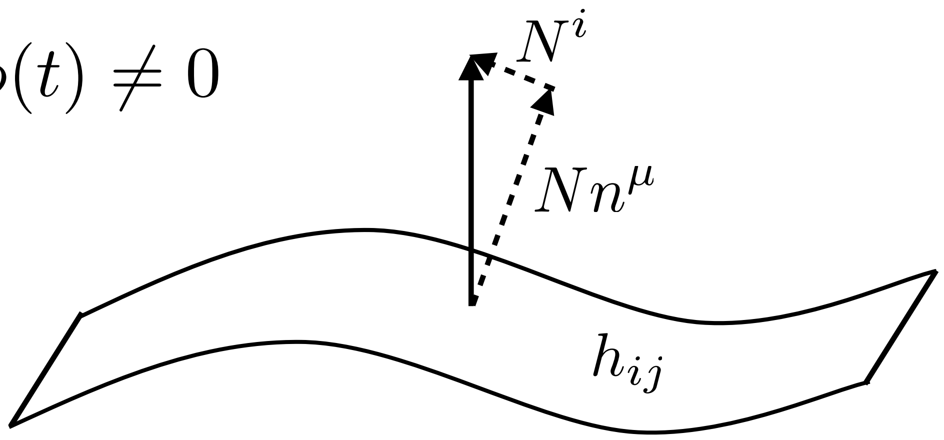






Constructing the action

- Use metric quantities in uniform scalar field slicing $\dot{\phi}(t) \neq 0$



  - ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

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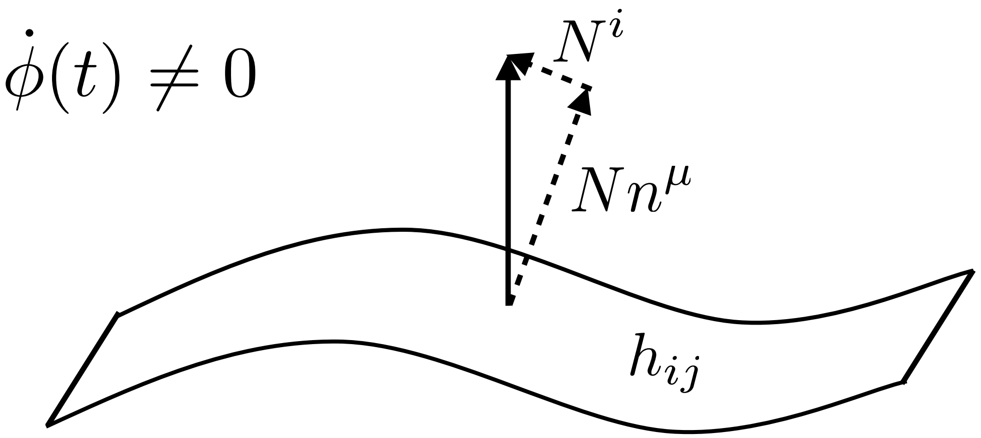

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$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

- Lagrangian contains all possible scalars under spatial diffs, ordered by number of perturbations and derivatives Cheung et al. \`07

$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

Lapse	N	$\sim \dot{\phi}$	$(\partial\phi)^2 = -\dot{\phi}_0^2(t)/N^2$
Extrinsic curvature	K_{ij}	$\sim \partial_t g_{ij}$	$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
Intrinsic curvature	${}^{(3)}R_{ij}$	$\sim \partial^2 g_{ij}$	

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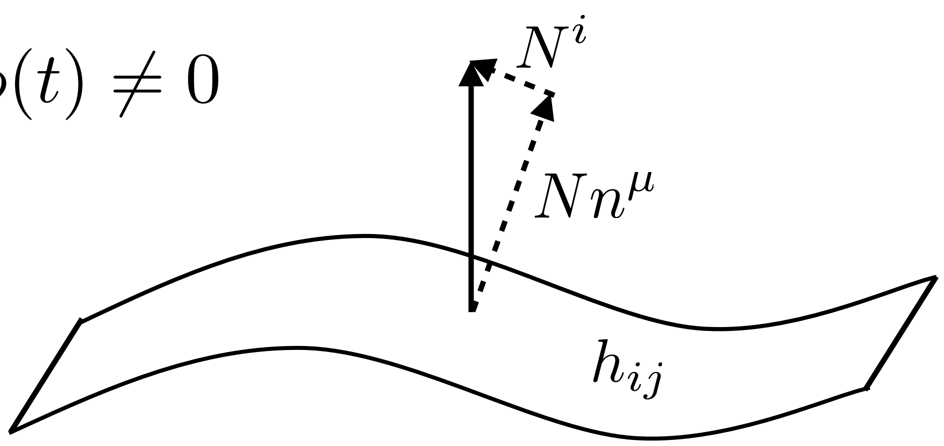

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✦ Expand the action

$$\delta N \equiv N - 1 \,, \qquad \delta K_{ij} \equiv K_{ij} - H h_{ij} \,, \qquad {}^{(3)}R_{ij}$$

$$L(N, K^i_j, R^i_j, \ldots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j} \delta K^i_j + \frac{\partial L}{\partial R^i_j} \delta R^i_j + L^{(2)} + \ldots$$

Building blocks of linear perts

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots) \right]$$

with Gleyzes, Langlois, Piazza '13 (see also Bloomfield '13)

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✦ Deviation from GR (LCDM) parameterized by time-dependent functions independent from background evolution

Notation of Bellini, Sawicki '14 for the alphas

α_i	α_K	α_B	α_M	α_T	α_H
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N {}^{(3)}R$
quintessence, k-essence	✓				
Cubic Galileon	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

5 functions of time instead of 5 functions of $\phi, (\partial\phi)^2$; minimal number of parameters

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- ✦ Deviation from GR (LCDM) parameterized by time-dependent functions independent from background evolution

- ✦ Locality, diff invariance preserved. Stability:

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2$	$M^2 > 0$
No gradient instability	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}\!R + \delta N {}^{(3)}\!R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

✦ All operators up to two derivatives

with Langlois, Mancarella, Noui '17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}\!R$	$\delta N {}^{(3)}\!R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$
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$\mathcal{O}_i^{(2)}$	δN^2	$\delta N\delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N\,{}^{(3)}R$	δK^2	$\delta\dot{N}^2$	$\delta\dot{N}\delta K$	$(\partial_i\delta N)^2$

✦ Generic scalar dispersion relation: $\mathcal{E}_1\omega^4 + \mathcal{E}_2\omega^2k^2 + \mathcal{E}_3\omega^2 + \mathcal{E}_4k^4 + \mathcal{E}_5k^2 = 0$

✦ Two types of degeneracy conditions lead to $\omega^2 - c_s^2k^2 = 0$

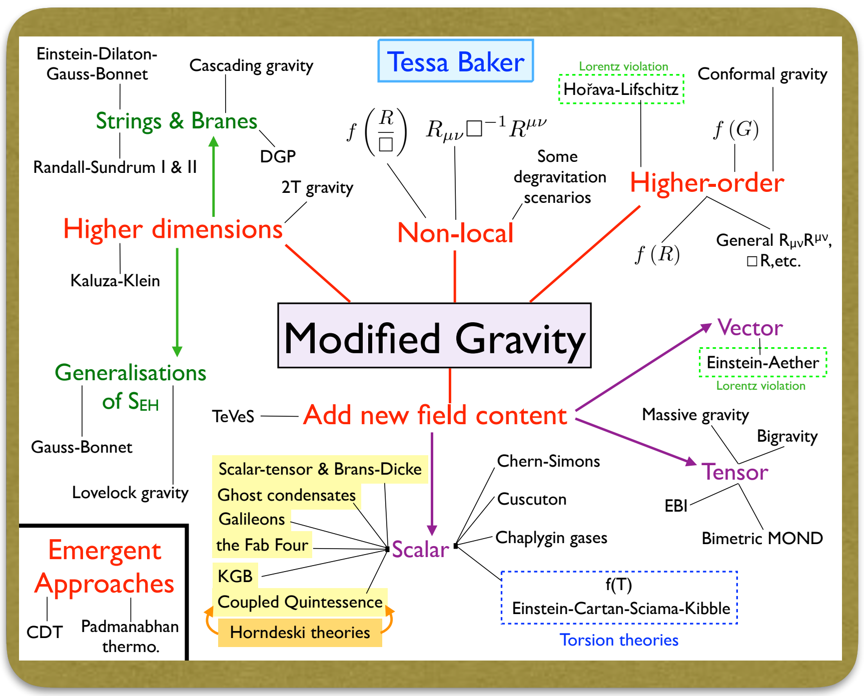
$$\mathcal{C}_{\rm I}: \quad \alpha_L = 0\,, \qquad \beta_2 = f_2(\beta_1)\,, \qquad \beta_3 = f_3(\beta_1)$$

related to Horndeski by metric redefinitions (that change the matter couplings)

$$\mathcal{C}_{\rm II}: \quad \beta_1 = f_1(\alpha_T,\alpha_H,\alpha_L)\,, \quad \beta_2 = f_2(\alpha_T,\alpha_H,\alpha_L)\,, \quad \beta_3 = f_3(\alpha_T,\alpha_H,\alpha_L)$$

$c_s^2 \propto -c_T^2$ ruled out!

Models



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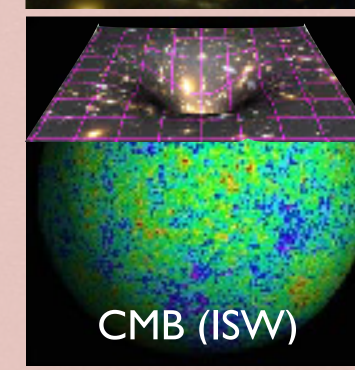
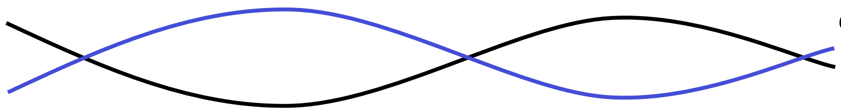


Phenomenology

- Undo unitary gauge: $t \rightarrow t + \pi(t, \vec{x})$



- Newtonian gauge (scalar flucts): $dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

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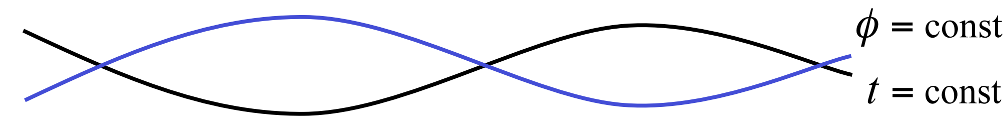

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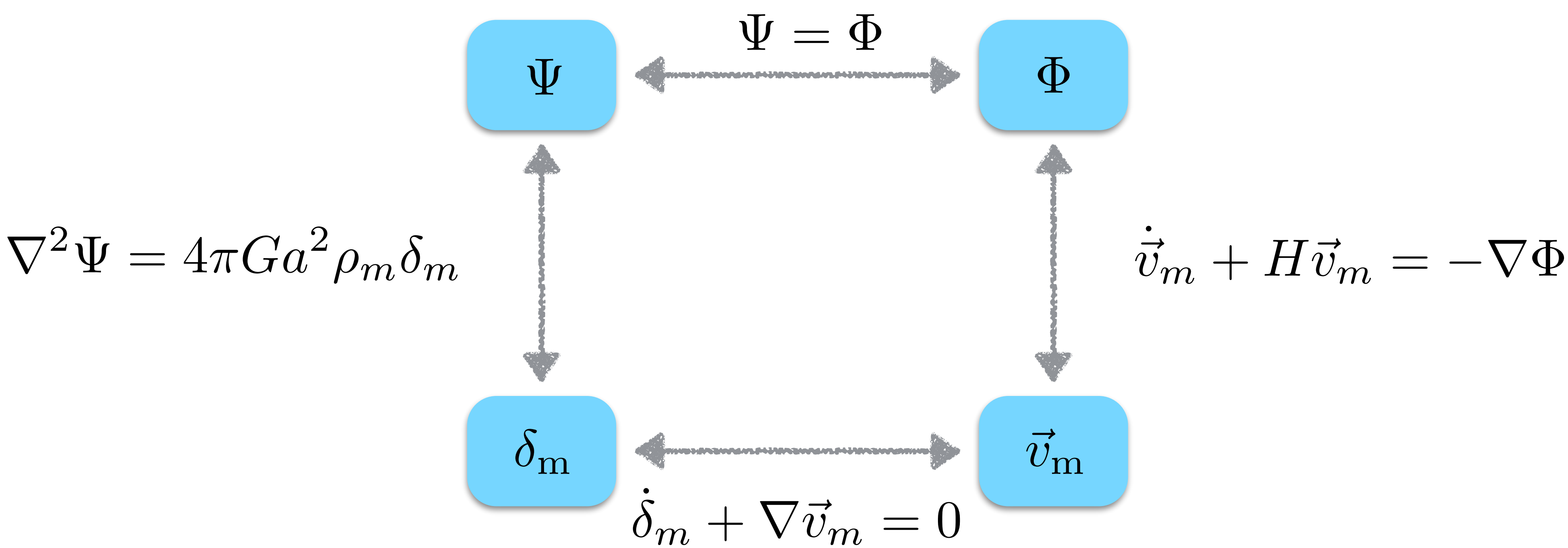
- Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15

E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

$$\nabla^2(\Psi + \Phi) = 8\pi G a^2 \rho_m \delta_m$$



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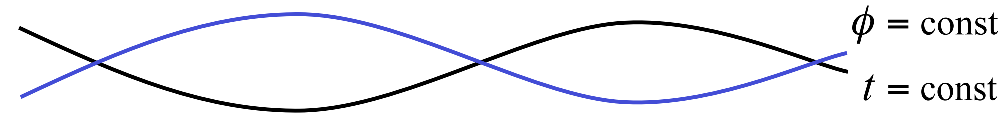
- Undo unitary gauge: $t \to t + \pi(t, \vec{x})$



- Newtonian gauge (scalar flucts): $dt^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)d\vec{x}^2$

- Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15

E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

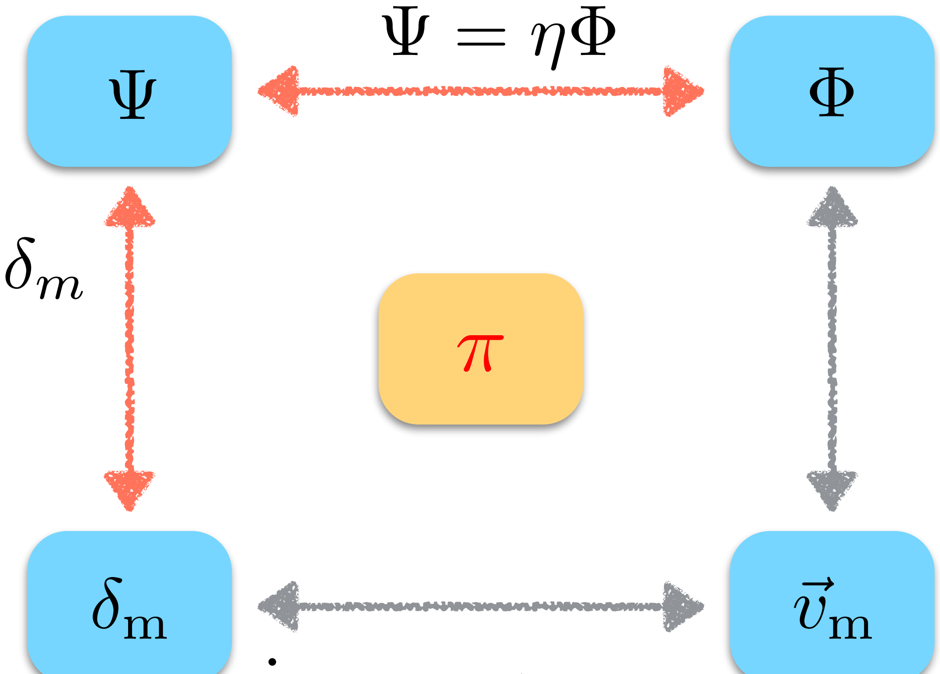
$$\nabla^2(\Psi + \Phi) = 8\pi G(1 + \Upsilon_{\rm lens})a^2\rho_m\delta_m$$

$$\Upsilon_{\rm lens} = \Upsilon_{\rm lens}(\alpha_i)$$

$$\Psi = \eta\Phi$$

$$\nabla^2\Psi = 4\pi G(1 + \Upsilon_\Psi)a^2\rho_m\delta_m$$

$$\Upsilon_\Psi = \Upsilon_\Psi(\alpha_i)$$

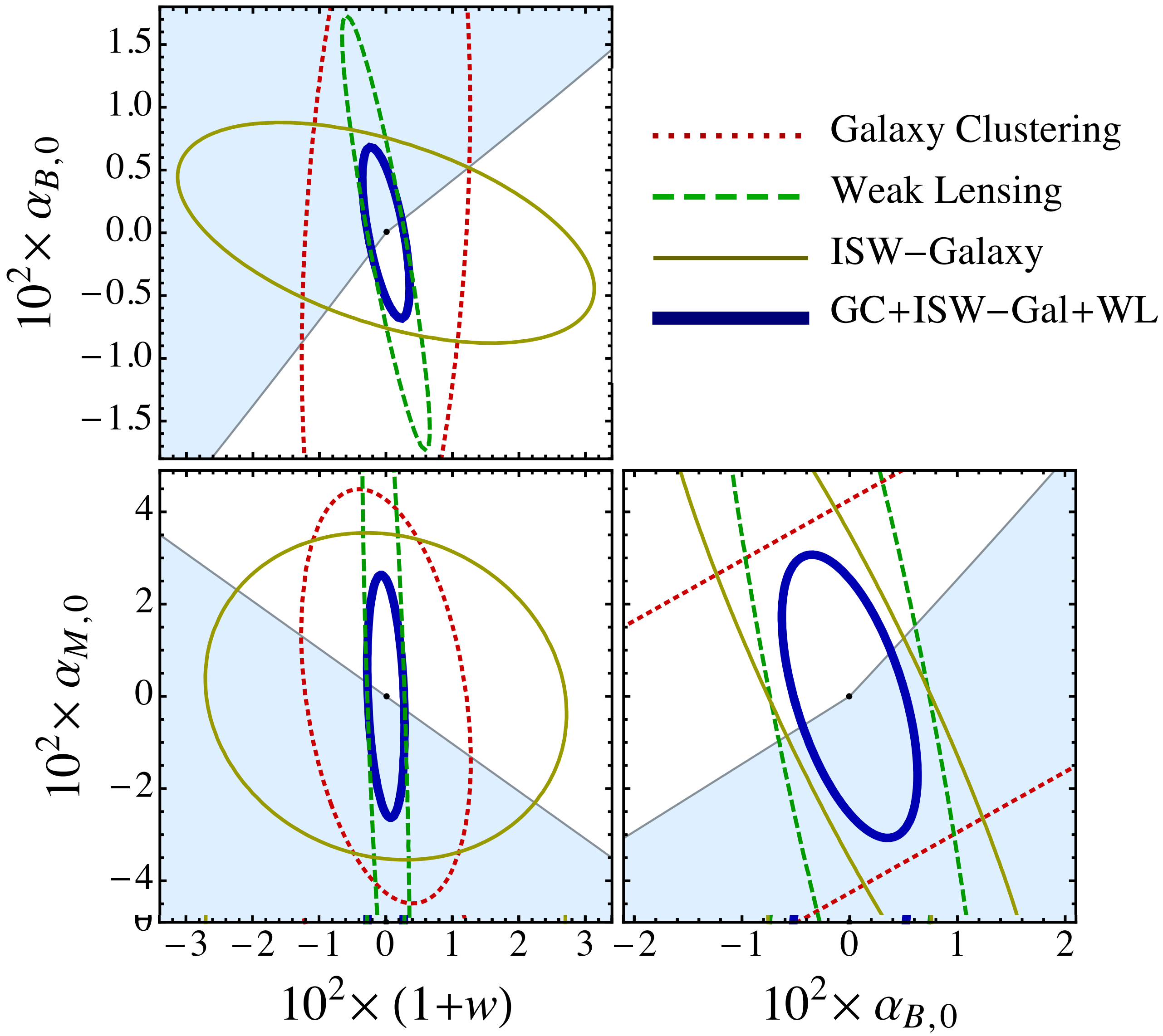


$$\dot{\vec{v}}_m + H\vec{v}_m = -\nabla\Phi$$

$$\dot{\delta}_m + \nabla\vec{v}_m = 0$$

Fisher matrix analysis

with Gleyzes, Langlois, Mancarella '15



Euclid specifications (LCDM fiducial)
Quasi-static approximation

✦ Background parametrization:

$$H^2 = H_0^2\left[\Omega_{\rm m0}a^{-3} + (1-\Omega_{\rm m0})a^{-3(1+w)}\right]$$

✦ Free functions parametrization:

$$\alpha_I(t) = \alpha_{I,0}\,\frac{1-\Omega_{\rm m}(t)}{1-\Omega_{\rm m,0}}$$

Piazza et al. '13, Bellini, Sawicki '14
see also Alonso et al. '16

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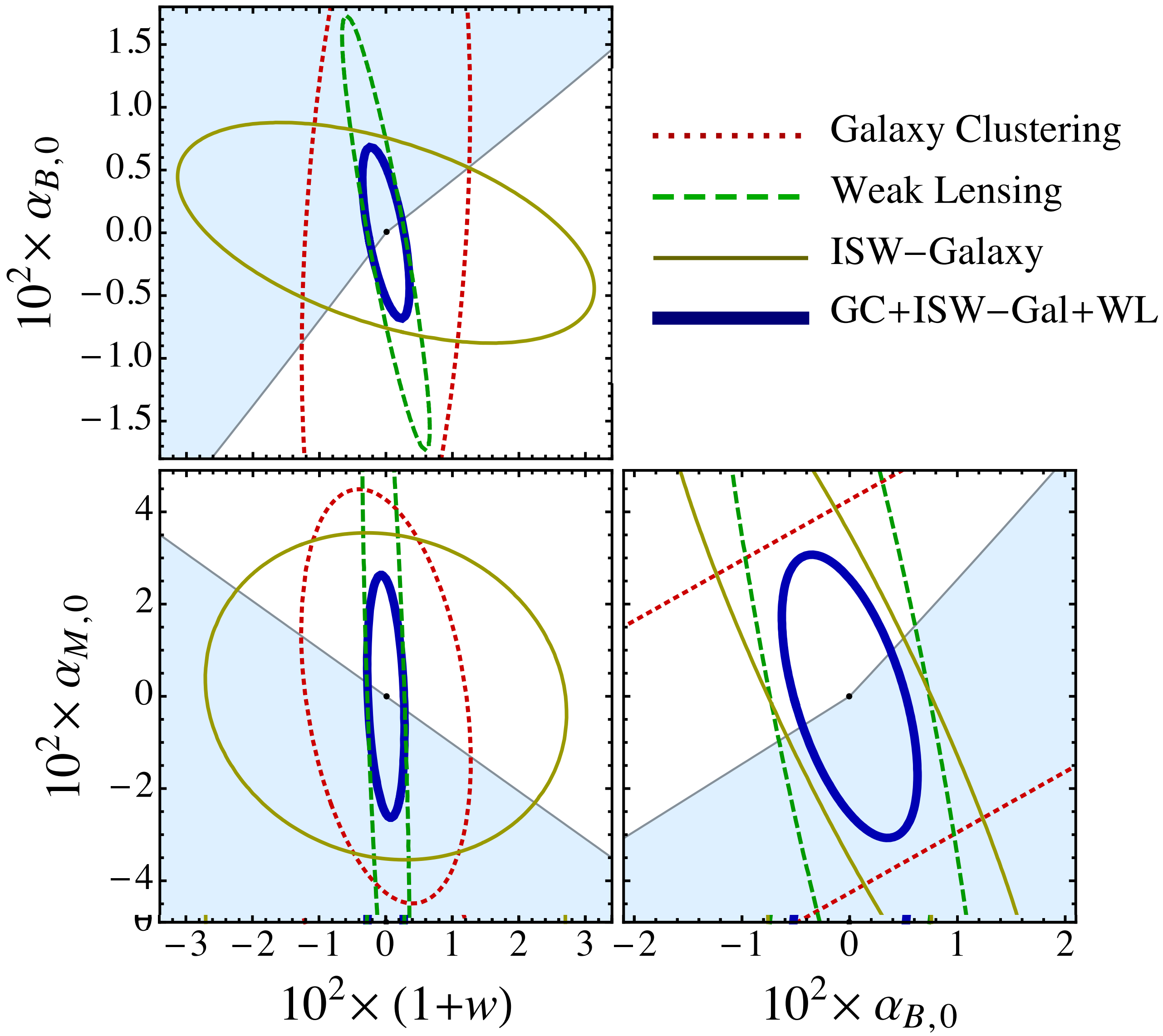

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- ✦ Time dependence of free functions α's still critical issue

see e.g. Linder '16, Gleyzes '17, Kennedy et al. '17

Boltzmann codes

✦ We want to go beyond the quasi-static approximation:

- Full Boltzmann solver:

$$\frac{df_I}{d\eta} = C_I[f_I]\,, \quad I = \gamma, \nu, b, \mathrm{CDM}$$

$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \qquad \& \qquad G_{ij}^{\mathrm{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

- EFTCAMB (Hu, Raveri, Frusciante, Silvestri et al.)
- hi_class (Zumalacarregui, Bellini, Sawicki, Lesgourgues et al.)
- COOP (Zhiqi Huang) (with D'Amico, Huang and Mancarella)

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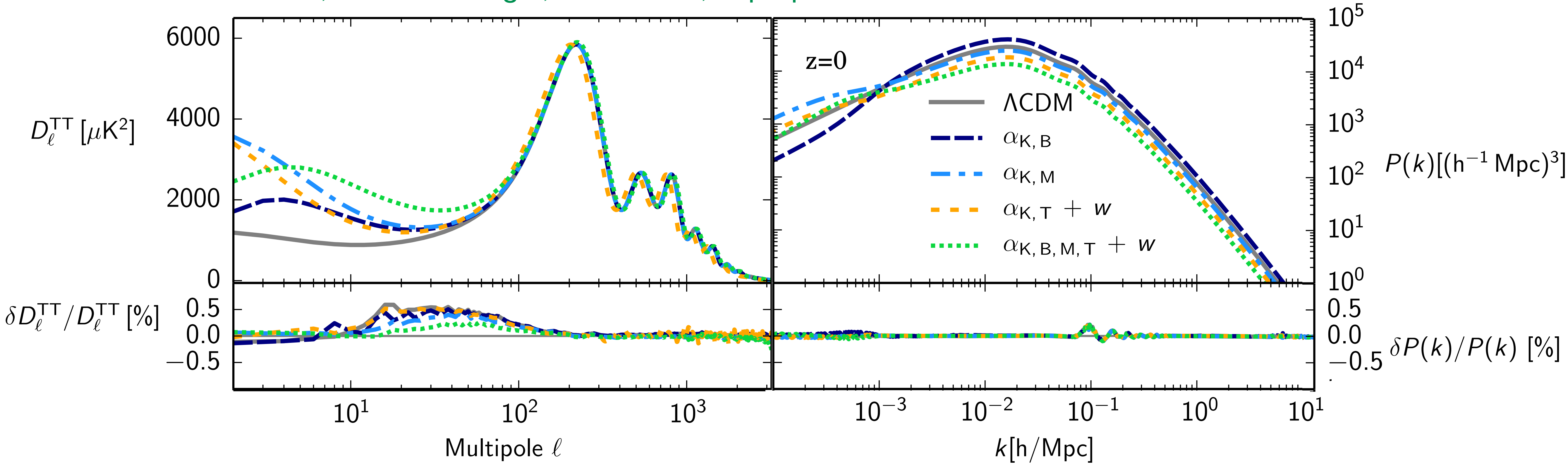
Bellini, Zumalacarregui, et multi alii, in prep.



Nonlinear ET of DE

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N\, {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

- In the Newtonian limit, a finite number of operators dominate

  Example: Horndeski has only 3 cubic operators and nothing more   Bellini, Jimenez, Verde '15

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  Example: Horndeski has only 3 cubic operators and nothing more — Bellini, Jimenez, Verde ‘15

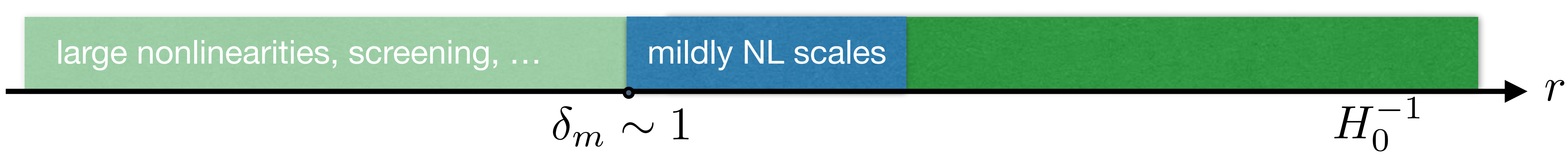
- Standard Perturbation Theory

$$\dot{\delta}_m + \nabla \left[(1+\delta_m) \vec{v}_m \right] = 0$$

$$\dot{\vec{v}}_m + H \vec{v}_m + \vec{v}_m \cdot \nabla \vec{v}_m = -\nabla \Phi$$

- Modifications of gravity encoded in Poisson-like equation

$$k^2 \Phi = -\frac{3}{2} a^2 H^2 \Omega_{\rm m} \mu_{\Phi,1} \delta_{\rm m} - \frac{9}{4} a^2 H^2 \Omega_{\rm m}^2 \mu_{\Phi,2}(\vec{k}_1, \vec{k}_2) \delta_{\rm m}(\vec{k}_1) \star \delta_{\rm m}(\vec{k}_2) + \dots$$



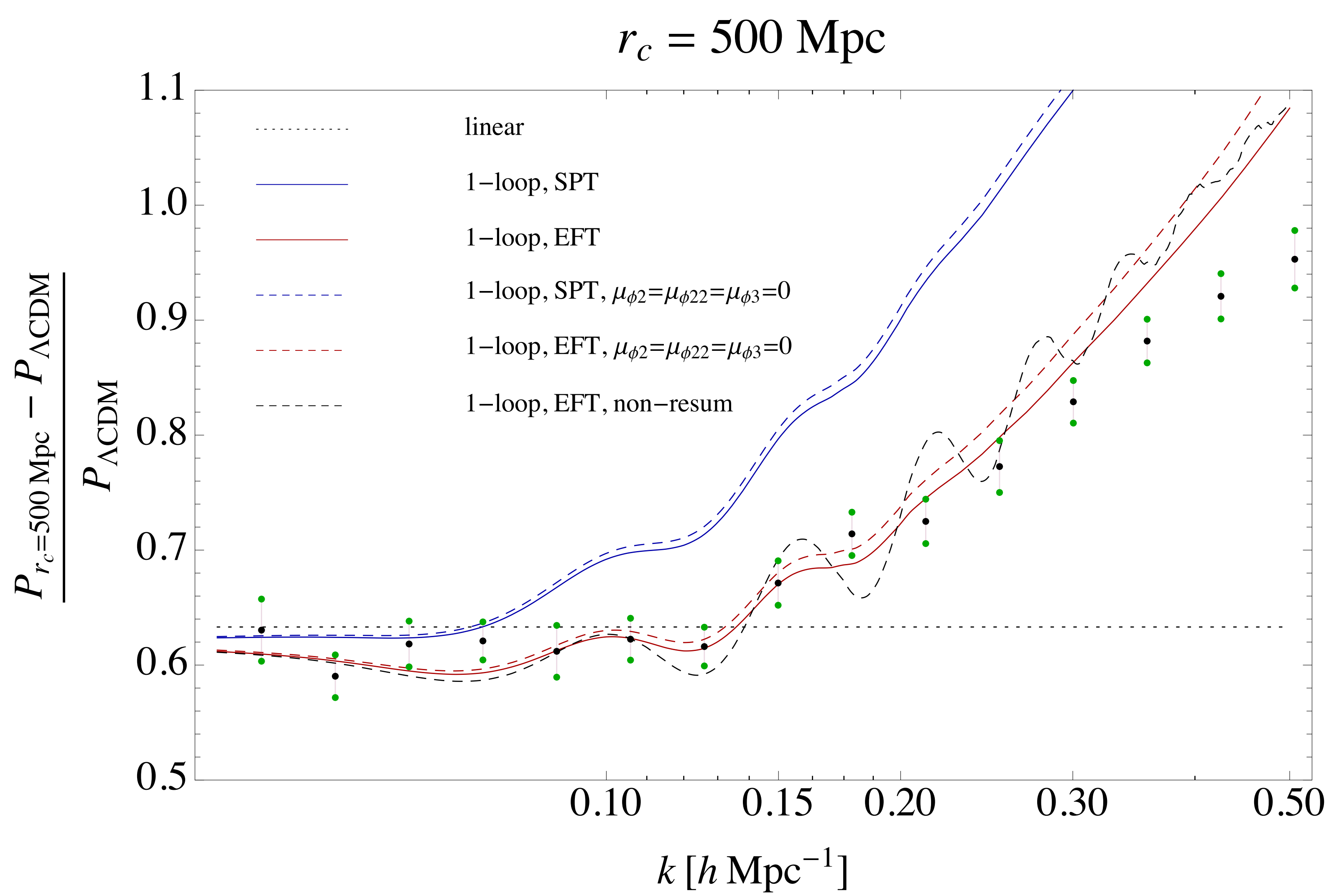
EFT of DE and LSS combined

- ✦ Naturally incorporated into the Effective Field Theory of Large Scale Structures

$$\dot{v}^i_m + H v^i_m + v^j_m \nabla_j v^i_m + \nabla^i \Phi = \frac{1}{\rho_m} \partial_j \tau^{ji} \sim \frac{c^2_{m,s}}{k^2_{\rm NL}} \nabla^i \delta_m$$

Baumann et al. ’10, Senatore et al. ’12, etc…

- ✦ Example: 1-loop Power Spectrum with IR resummation. Comparison with Fabian Schmidt ’09 simulations of nDGP, 3 realizations, from 400 to 128 Mpc/h box size:



Conclusions

- Unifying description for scalar-tensor theories, including higher-order ones (and more)
- Analysis of (degenerate higher-order) theories highly simplified
- Linear regime worked out! Issue of time dependence of α's
- Straightforward connection to mildly nonlinear and fully nonlinear regime