GWs from first-order phase transitions

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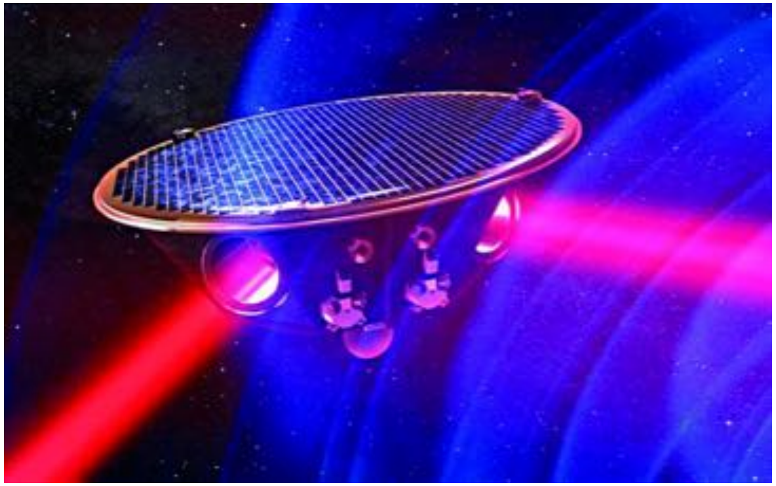
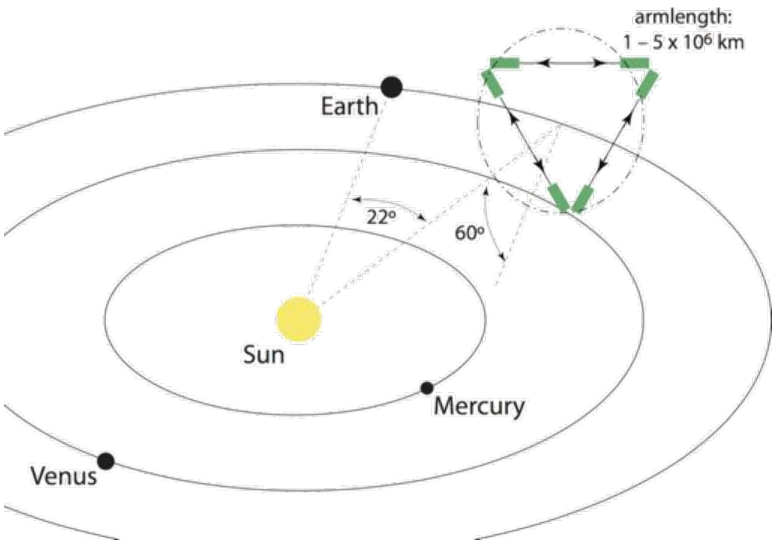
NORDITA, 5 July 2017

tinyurl.com/nordita-weir

arXiv:1705.01783 and references therein

What's next? LISA





- LISA: three arms (six laser links), 2.5 M km separation
- Launch as ESA's third large-scale mission (L3) in (or before) 2034
- Proposal officially submitted earlier this year arXiv:1702.00786
- Officially adopted on 20.6.2017

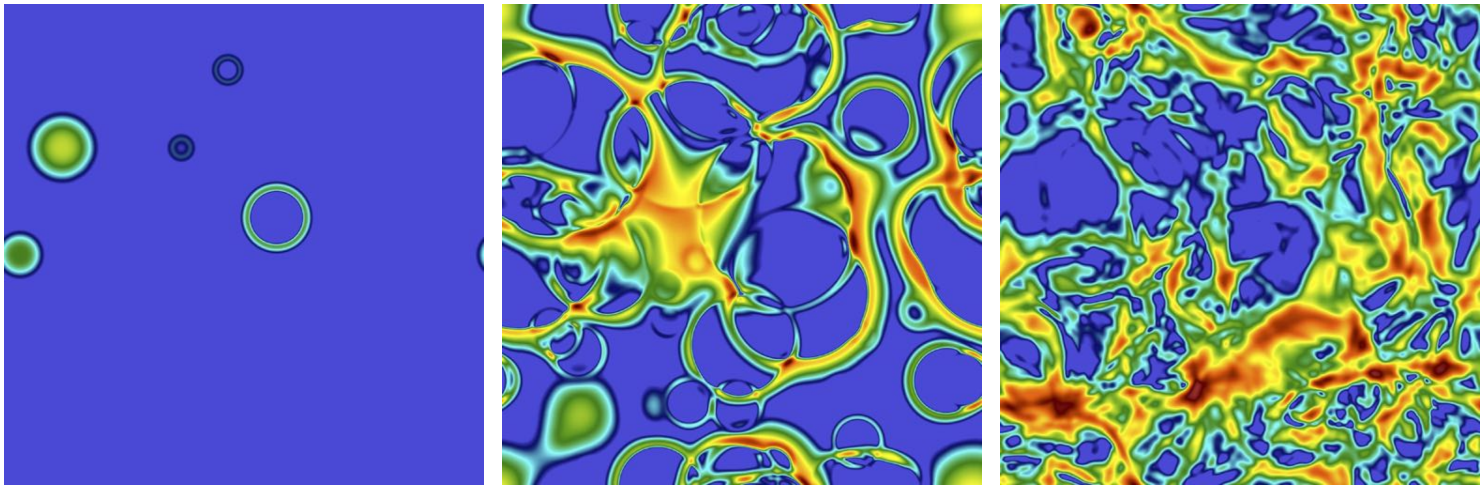
From the LISA proposal:

SI7.2 : Measure, or set upper limits on, the spectral shape of the cosmological stochastic GW background

OR7.2: Probe a broken power-law stochastic background from the early Universe as predicted, for example, by first order phase transitions [21] (other spectral shapes are expected, for example, for cosmic strings [22] and inflation [23]). Therefore, we need the ability to measure $\Omega = 1.3 \times 10^{-11} \left(f/10^{-4}\,\mathrm{Hz}\right)^{-1}$ in the frequency ranges $0.1\,\mathrm{mHz} < f < 2\,\mathrm{mHz}$ and $2\,\mathrm{mHz} < f < 20\,\mathrm{mHz}$, and $\Omega = 4.5 \times 10^{-12} \left(f/10^{-2}\,\mathrm{Hz}\right)^{3}$ in the frequency ranges $2\,\mathrm{mHz} < f < 20\,\mathrm{mHz}$ and $0.02 < f < 0.2\,\mathrm{Hz}$.

First order thermal phase transition:

1. Bubbles nucleate and grow
2. Expand in a plasma - create shock waves
3. Bubbles + shocks collide - violent process
4. Sound waves left behind in plasma
5. Turbulence; expansion



Thermal phase transitions

- Standard Model is a crossover

  Kajantie *et al.*; Karsch *et al.*; ...

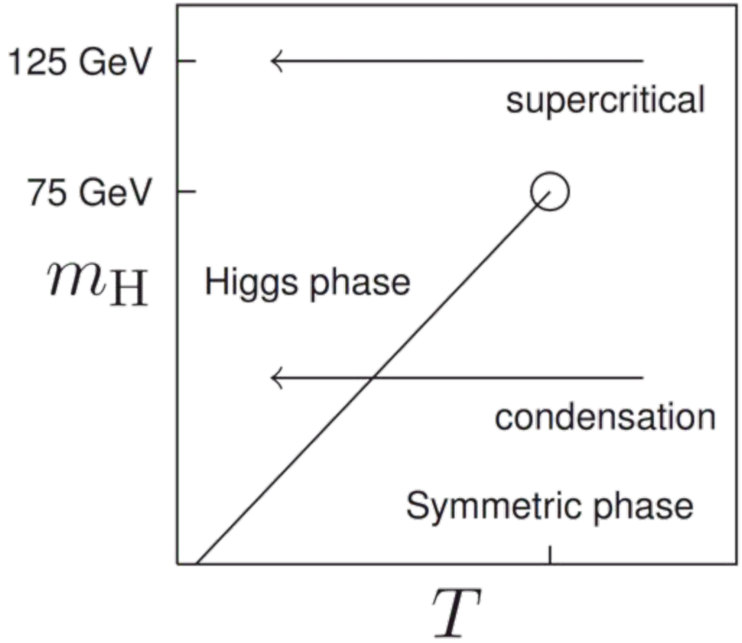
- First order possible in extensions (xSM, 2HDM, ...)

  Andersen *et al.*, Kozaczuk *et al.*, Carena *et al.*,

  Bödeker *et al.*, Damgaard *et al.*, Ramsey-Musolf *et al.*,

  Cline and Kainulainen, ...

- Baryogenesis?
- GW PS ⇔ model information?



What the metric sees at a thermal phase transition

- Bubbles nucleate and expand, shocks form, then:
  1. $h^2\Omega_\phi$: Bubbles + shocks collide - 'envelope phase'
  2. $h^2\Omega_{\rm sw}$: Sound waves set up - 'acoustic phase'
  3. $h^2\Omega_{\rm turb}$: [MHD] turbulence - 'turbulent phase'

- Sources add together to give observed GW power:

$$h^2\Omega_{\rm GW} \approx h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

- Equation of motion is (schematically)

  Liu, McLerran and Turok; Prokopec and Moore

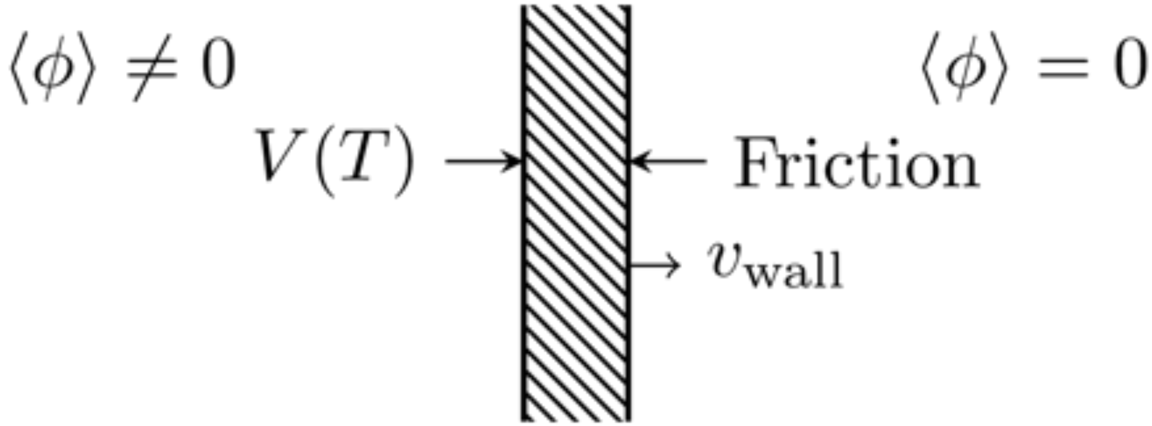
$$\partial_\mu \partial^\mu \phi + V'_{\text{eff}}(\phi, T) + \sum_i \frac{dm_i^2}{d\phi} \int \frac{\mathrm{d}^3 k}{(2\pi)^3 2E_i} \delta f_i(\mathbf{k}, \mathbf{x}) = 0$$

  - $V'_{\text{eff}}(\phi)$: gradient of finite-T effective potential
  - $\delta f_i(k, x)$: deviation from equilibrium phase space density of ith species
  - m_i: effective mass of ith species:
    - Leptons: $m^2 = y^2 \phi^2 / 2$
    - Gauge bosons: $m^2 = g_w^2 \phi^2 / 4$
    - Also Higgs and pseudo-Goldstone modes

Put another way:

$$\overbrace{\partial_\mu T^{\mu\nu}}^{\text{Force on }\phi} - \overbrace{\int \frac{d^3 k}{(2\pi)^3} f(\mathbf{k}) F^\nu}^{\text{Force on particles}} = 0$$

This equation is the realisation of this idea:



Yet another interpretation:

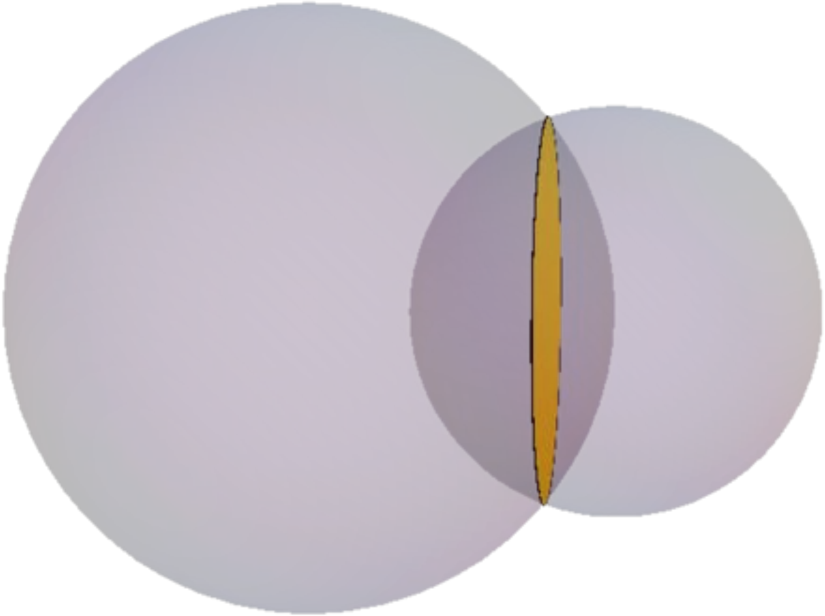
$$\overbrace{\partial_\mu T^{\mu\nu}}^{\text{Field part}} - \overbrace{\int \frac{d^3k}{(2\pi)^3} f(\mathbf{k}) F^\nu}^{\text{Fluid part}} = 0$$

i.e.:

$$\partial_\mu T^{\mu\nu}_{\phi} + \partial_\mu T^{\mu\nu}_{\text{fluid}} = 0$$

We will return to this later!

Envelope approximation



Envelope approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kosowsky and Turner

- Thin, hollow bubbles, no fluid
- Stress-energy tensor $\propto R^3$ on wall
- Solid angle: overlapping bubbles $\rightarrow$ GWs
- Simple power spectrum:
  - One length scale (average radius R_*)
  - Two power laws (ω^3, $\sim \omega^{-1}$)
  - Amplitude

$\Rightarrow$ 4 numbers define spectral form

NB: Used to be applied to shock waves (fluid KE), now only use for bubble wall (field gradient energy)

Envelope approximation

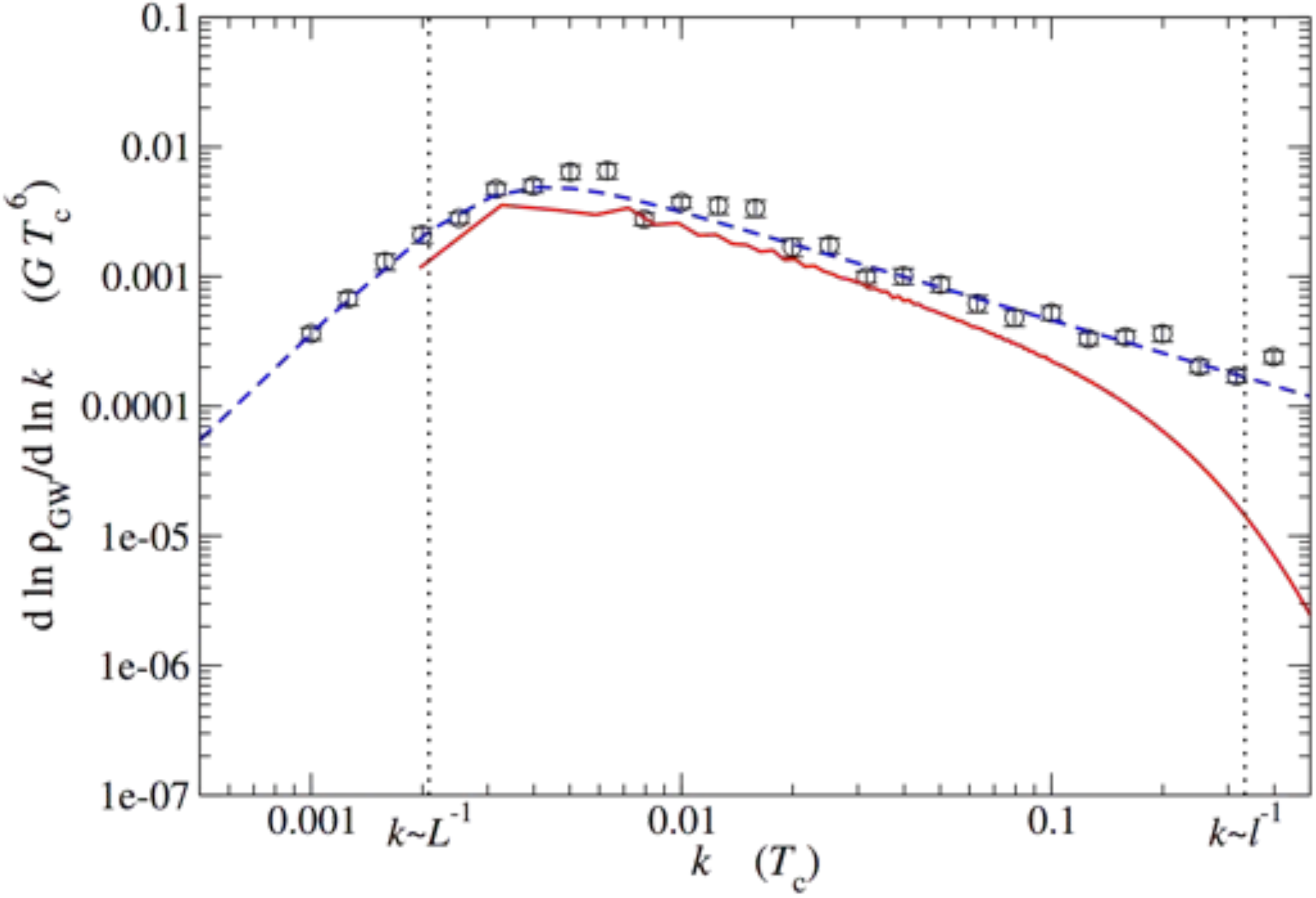
4-5 numbers parametrise the transition:

- α_{T_*}, vacuum energy fraction
- v_{w}, bubble wall speed
- κ_ϕ, conversion 'efficiency' into gradient energy $(\nabla\phi)^2$
- Transition rate:
  - H_*, Hubble rate at transition
  - β, bubble nucleation rate

$\rightarrow$ ansatz for $h^2\Omega_\phi$

[only matters for near-vacuum/runaway transitions]

Envelope approximation



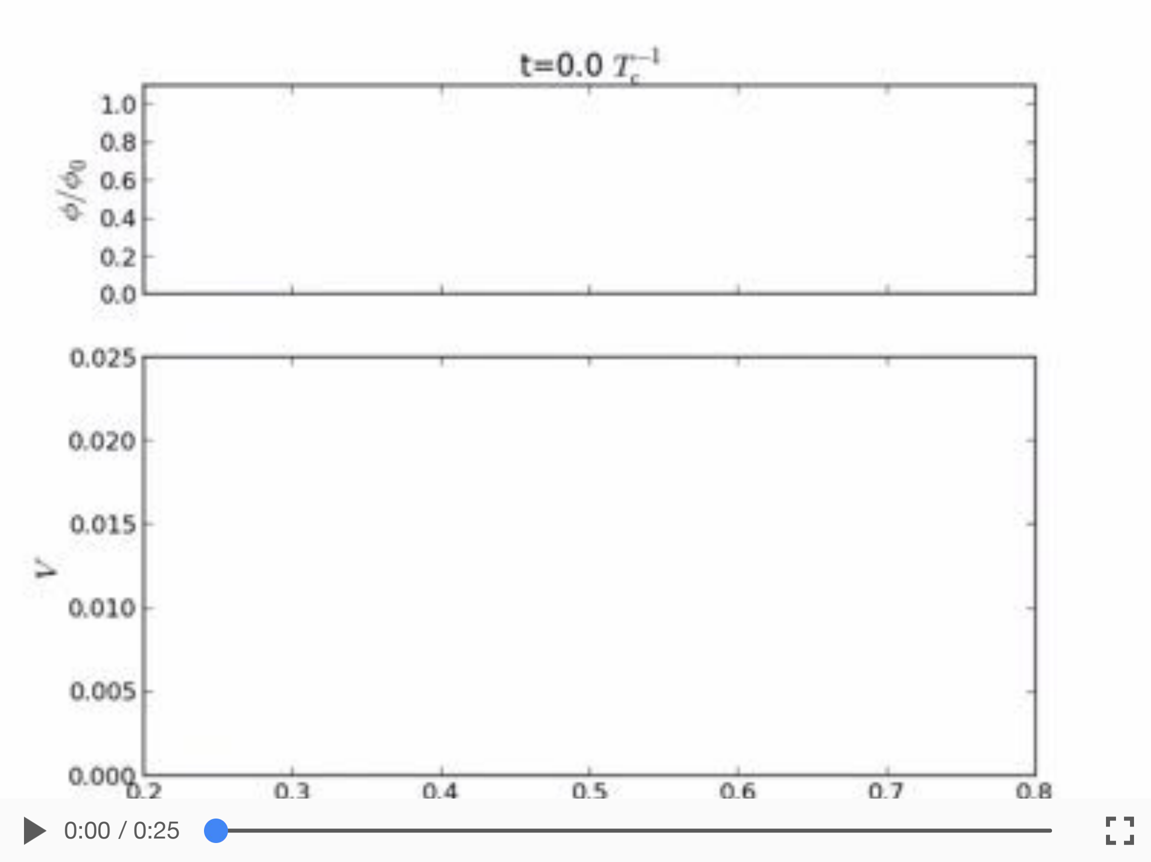
Coupled field and fluid system

Ignatius, Kajantie, Kurki-Suonio and Laine

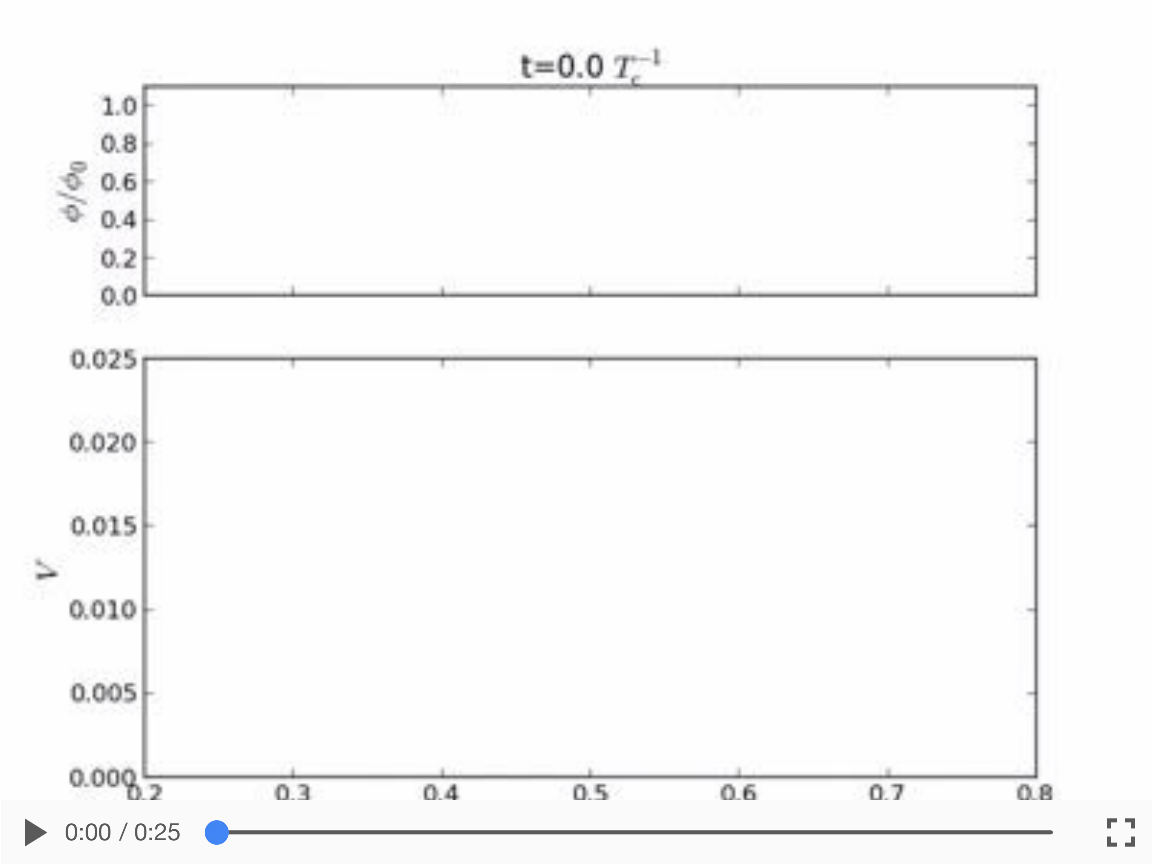
- Scalar ϕ and ideal fluid u^μ:
  - Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits
$$\partial_\mu T^{\mu\nu} = \partial_\mu (T^{\mu\nu}_\phi + T^{\mu\nu}_{\text{fluid}}) = 0$$
  - Parameter η sets the scale of friction due to plasma
$$\partial_\mu T^{\mu\nu}_\phi = \tilde{\eta} \frac{\phi^2}{T} u^\mu \partial_\mu \phi \partial^\nu \phi \qquad \partial_\mu T^{\mu\nu}_{\text{fluid}} = -\tilde{\eta} \frac{\phi^2}{T} u^\mu \partial_\mu \phi \partial^\nu \phi$$
  - $V(\phi, T)$ is a 'toy' potential tuned to give latent heat $\mathcal{L}$
  - $\beta \leftrightarrow$ number of bubbles; $\alpha_{T_*} \leftrightarrow \mathcal{L}$, $v_{\text{wall}} \leftrightarrow \tilde{\eta}$

Begin in spherical coordinates:
what sort of solutions does this system have?

Velocity profile development: small $\tilde{\eta} \Rightarrow$ detonation (supersonic wall)

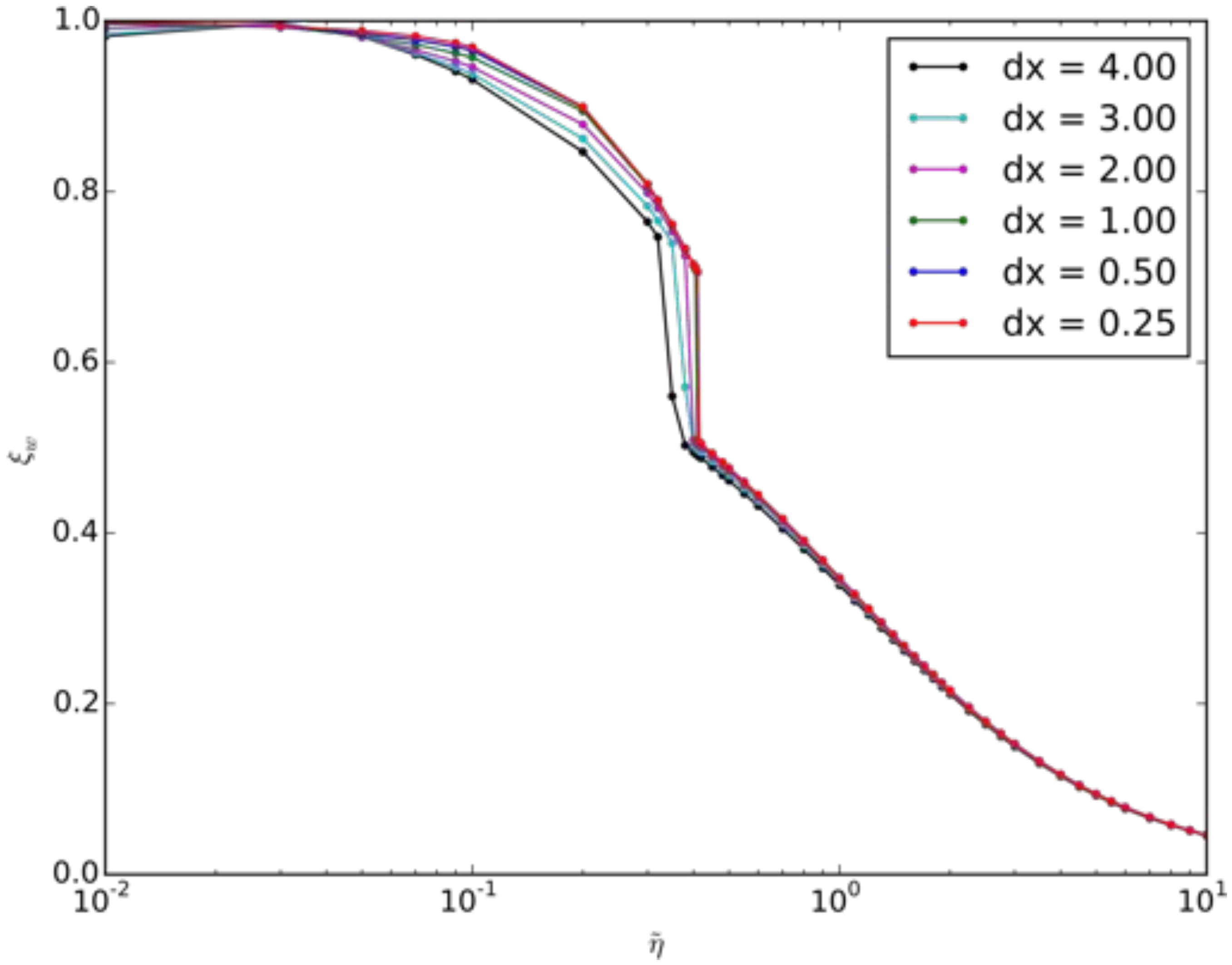


Velocity profile development: large $\tilde{\eta} \Rightarrow$ deflagration (subsonic wall)

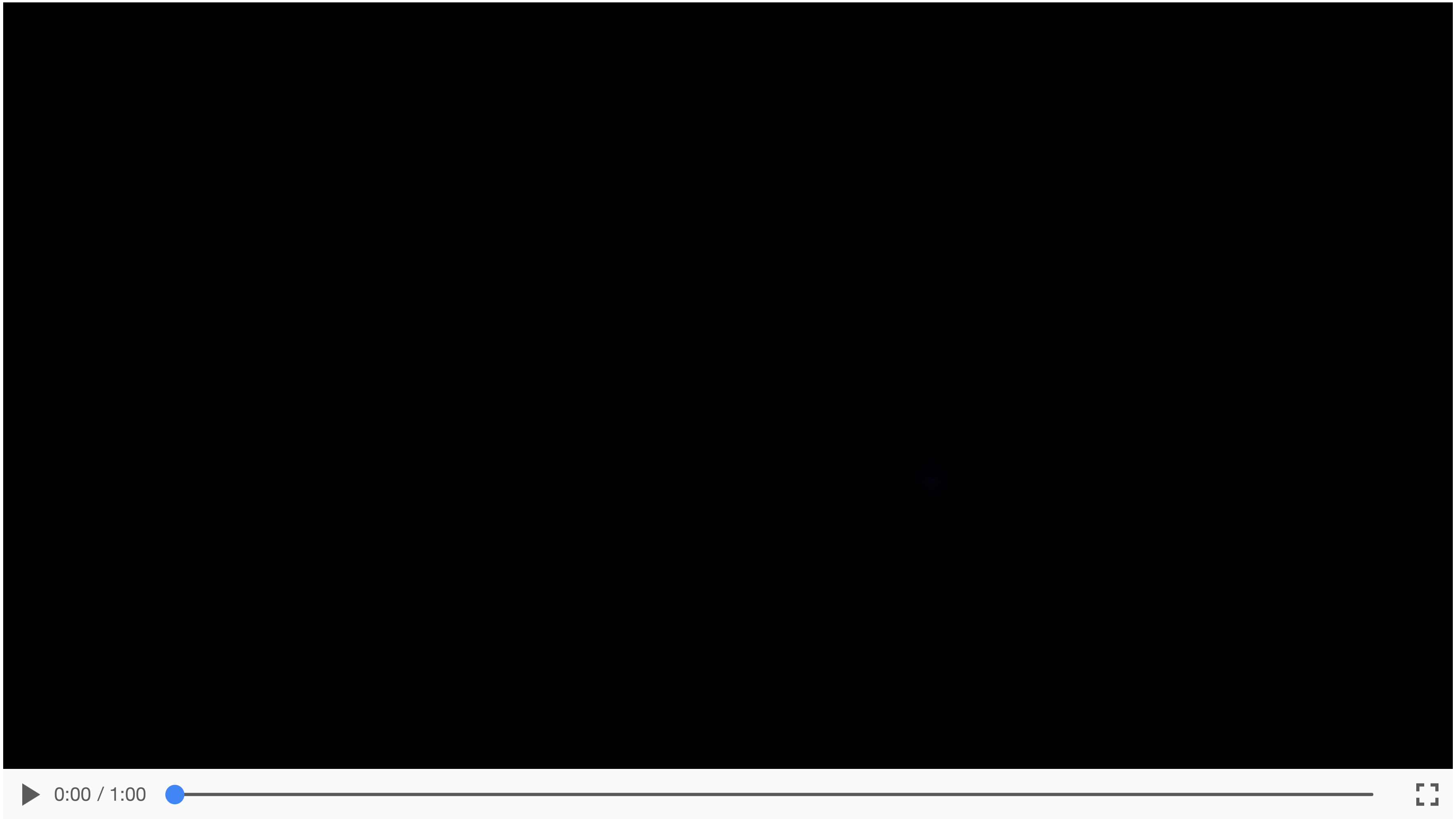


$v_{\rm w}$ as a function of $\tilde{\eta}$

Cutting [Masters dissertation]

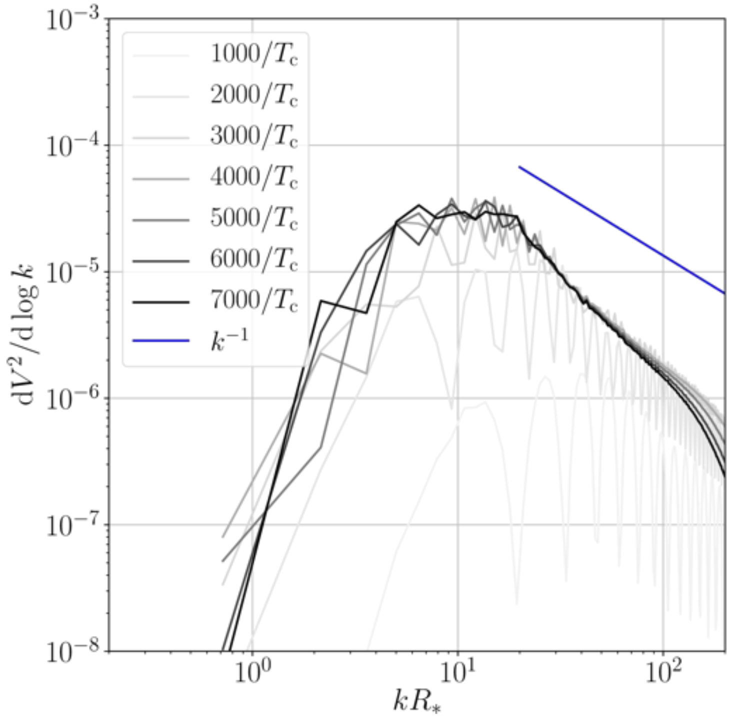


Simulation slice example

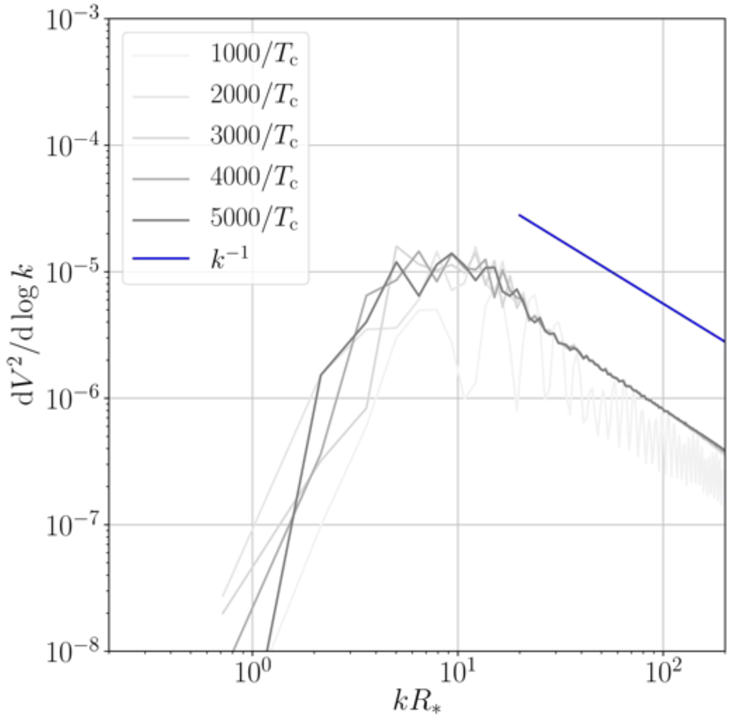


Velocity power spectra and power laws

Fast deflagration



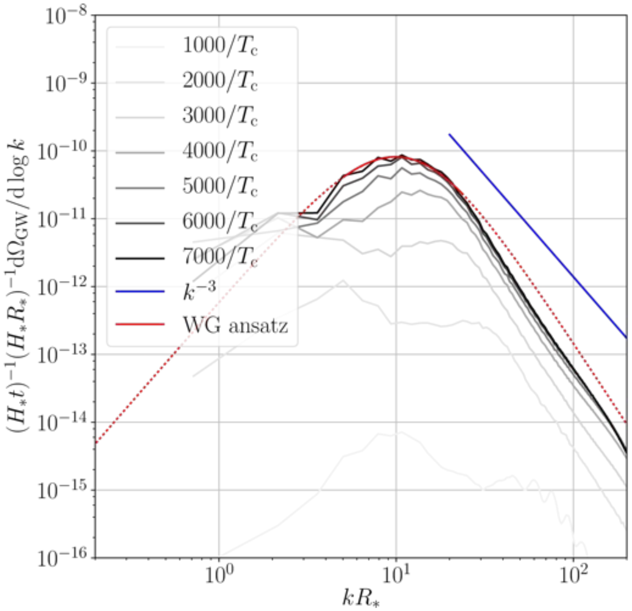
Detonation



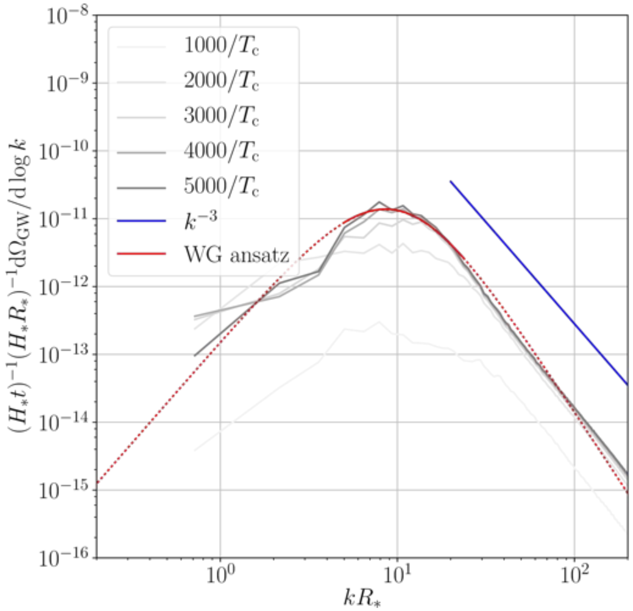
- Weak transition: $\alpha_{T_*} = 0.01$
- Power law behaviour above peak is between k^{-2} and k^{-1}
- "Ringing" due to simultaneous nucleation, unimportant

GW power spectra and power laws

Fast deflagration



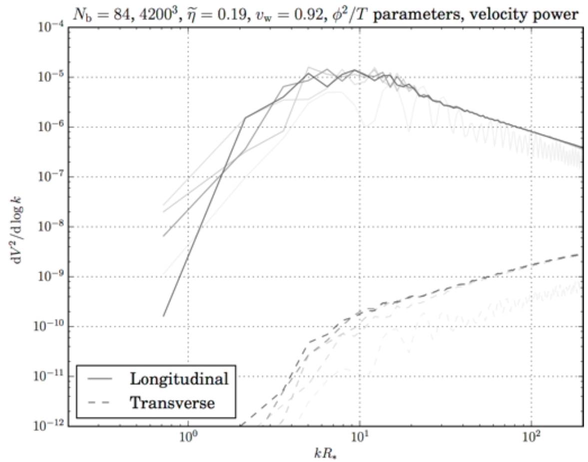
Detonation



- Causal k^3 at low k, approximate k^{-3} or k^{-4} at high k
- Curves scaled by t: source until turbulence / expansion

$\rightarrow$ power law ansatz for $h^2\Omega_{\mathrm{sw}}$

Transverse versus longitudinal modes – turbulence?



- Short simulation; weak transition (small α): linear; most power in longitudinal modes $\Rightarrow$ acoustic waves, turbulent
- Turbulence requires longer timescales $R_*/\overline{U}_f$
- Plenty of theoretical results, use those instead

Kahniashvili et al.; Caprini, Durrer and Servant; Pen and Turok; ...

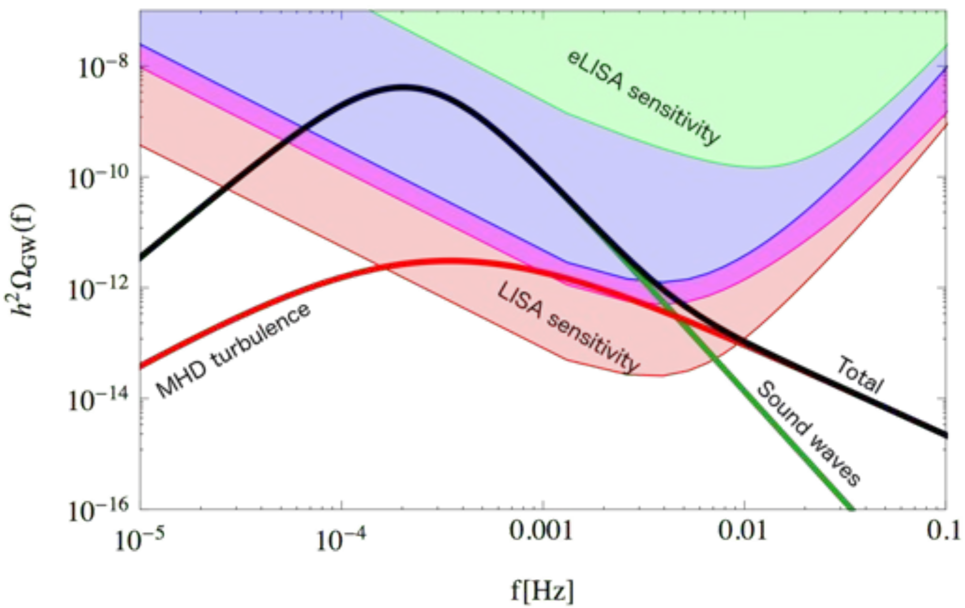
$\rightarrow$ power law ansatz for $h^2\Omega_{\rm turb}$

Putting it all together - $h^2\Omega_{\rm gw}$ arXiv:1512.06239

- Three sources, $\approx h^2\Omega_\phi, h^2\Omega_{\rm sw}, h^2\Omega_{\rm turb}$
- Know their dependence on $T_*, \alpha_T, v_{\rm w}, \beta$

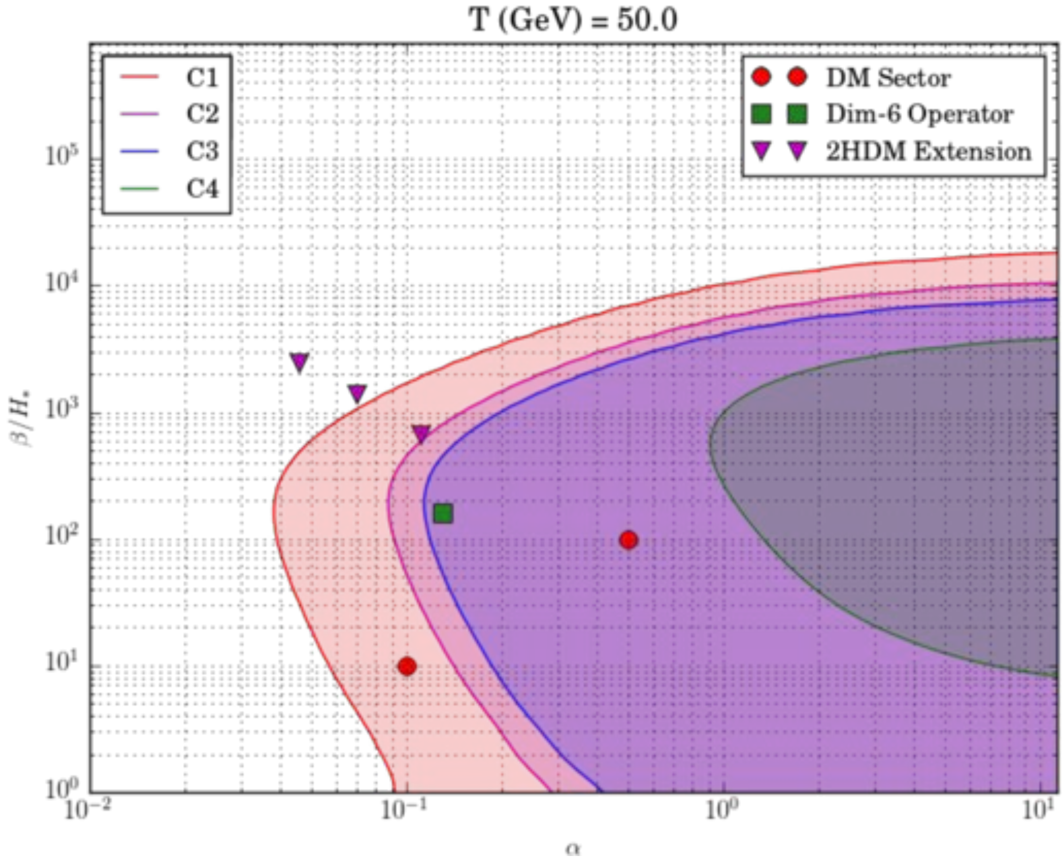
  Espinosa, Konstandin, No, Servant
- Know these for any given model, predict the signal...

(example, $T_* = 100\text{GeV}, \alpha_{T_*} = 0.5, v_{\rm w} = 0.95, \beta/H_* = 10$)



Putting it all together - physical models to GW power spectra

$$\text{Model} \longrightarrow (T_*, \alpha_{T_*}, v_\mathrm{w}, \beta) \longrightarrow \text{this plot}$$

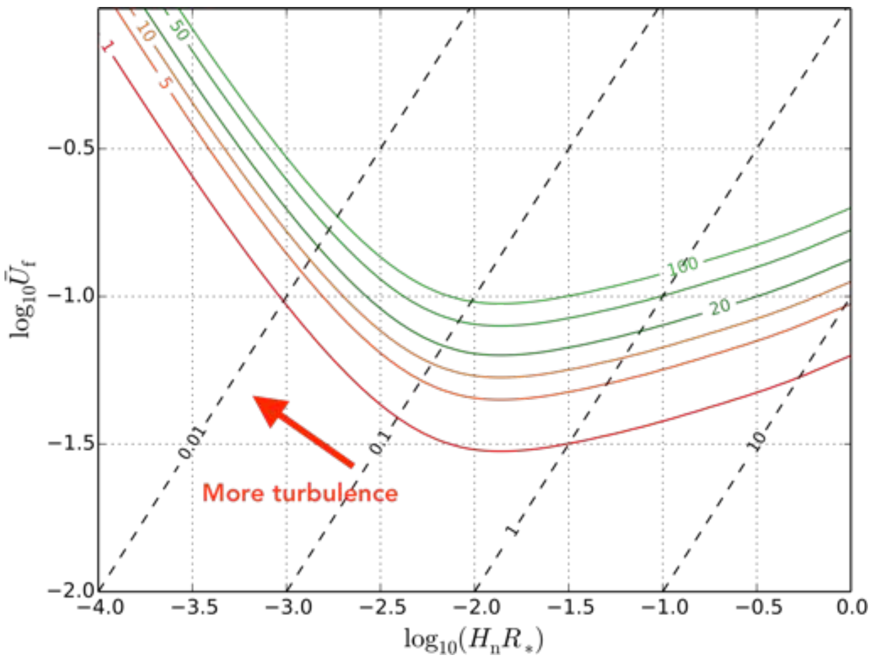


... which tells you if it is detectable by LISA (see arXiv:1512.06239)

Detectability from acoustic waves alone

- In many cases, sound waves dominant
- Parametrise by RMS fluid velocity $\overline{U}_{\mathrm{f}}$ and bubble radius R_* (quite easily obtained Espinosa, Konstandin, No and Servant)

Sensitivity plot:



The pipeline

1. Choose your model (e.g. SM, xSM, 2HDM, ...)
2. Dim. red. model Kajantie et al.
3. Phase diagram (α_{T_*}, T_*); lattice: Kajantie et al.
4. Nucleation rate (β); lattice: Moore and Rummukainen
5. Wall velocities ($v_{\rm wall}$) Moore and Prokopec; Kozaczuk
6. GW power spectrum $\Omega_{\rm gw}$
7. Sphaleron rate

Very leaky, even for SM!

Questions, requests or demands...

- Turbulence
  - MHD or no MHD?
  - Timescales $H_* R_* / \overline{U}_{\mathrm{f}} \sim 1$, sound waves and turbulence?
  - More simulations needed?

- Interaction with baryogenesis
  - Competing wall velocity dependence of BG and GWs?
  - Sphaleron rates in extended models?

- The best possible determinations for xSM, 2HDM, ΣSM, ...
  - What is the phase diagram?
  - Nonperturbative nucleation rates?