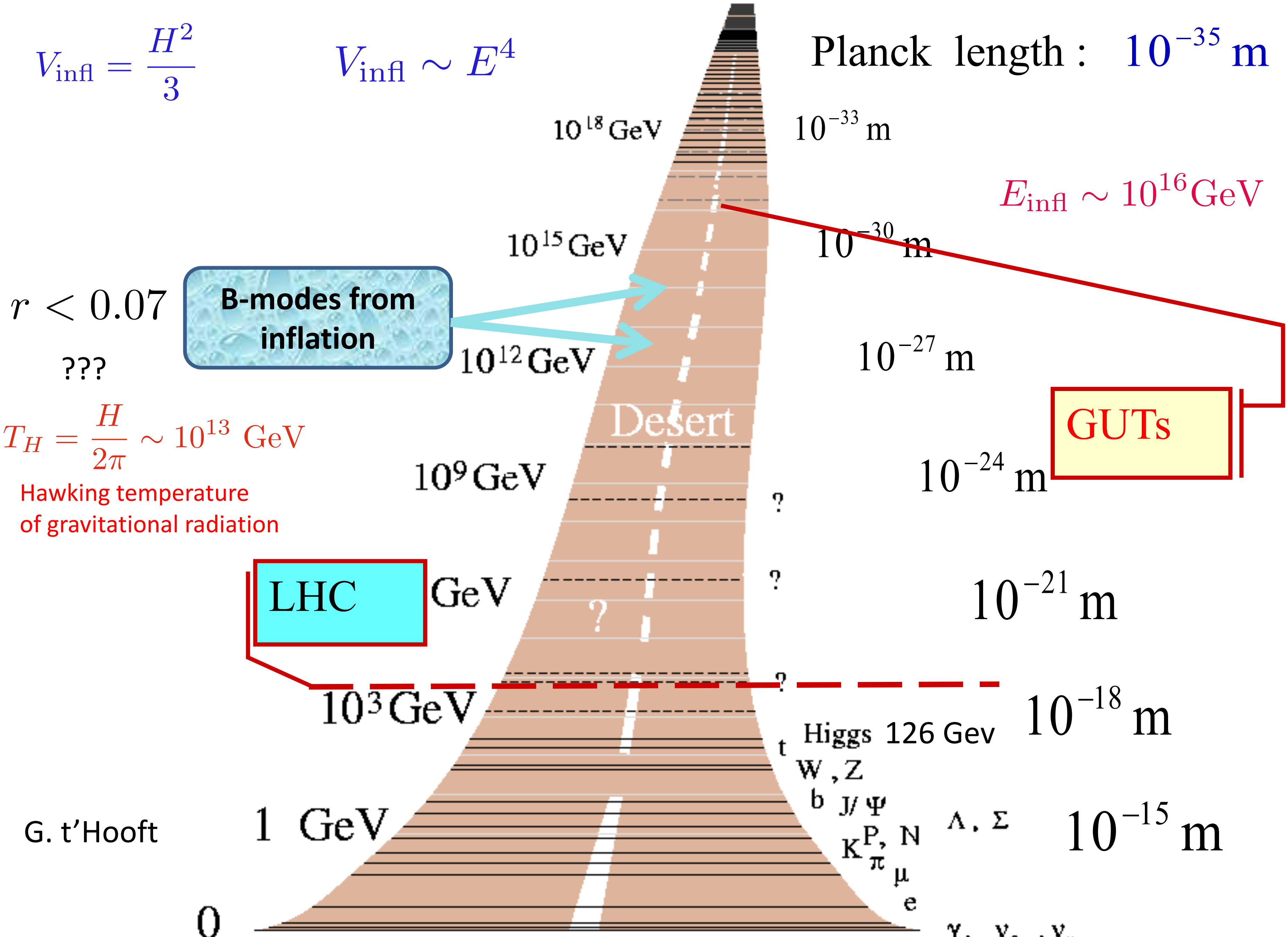
Primordial Gravitational Waves in String Theory/Supergravity

Renata Kallosh

Nordita, Stokholm, July 20, 2017

Related talk by Linde

Ferrara, RK, 2016
RK, Linde, Wrase,Yamada, 2017
RK, Linde, Roest, Yamada, 2017
RK, Linde, Roest, Westphal,Yamada, 2017

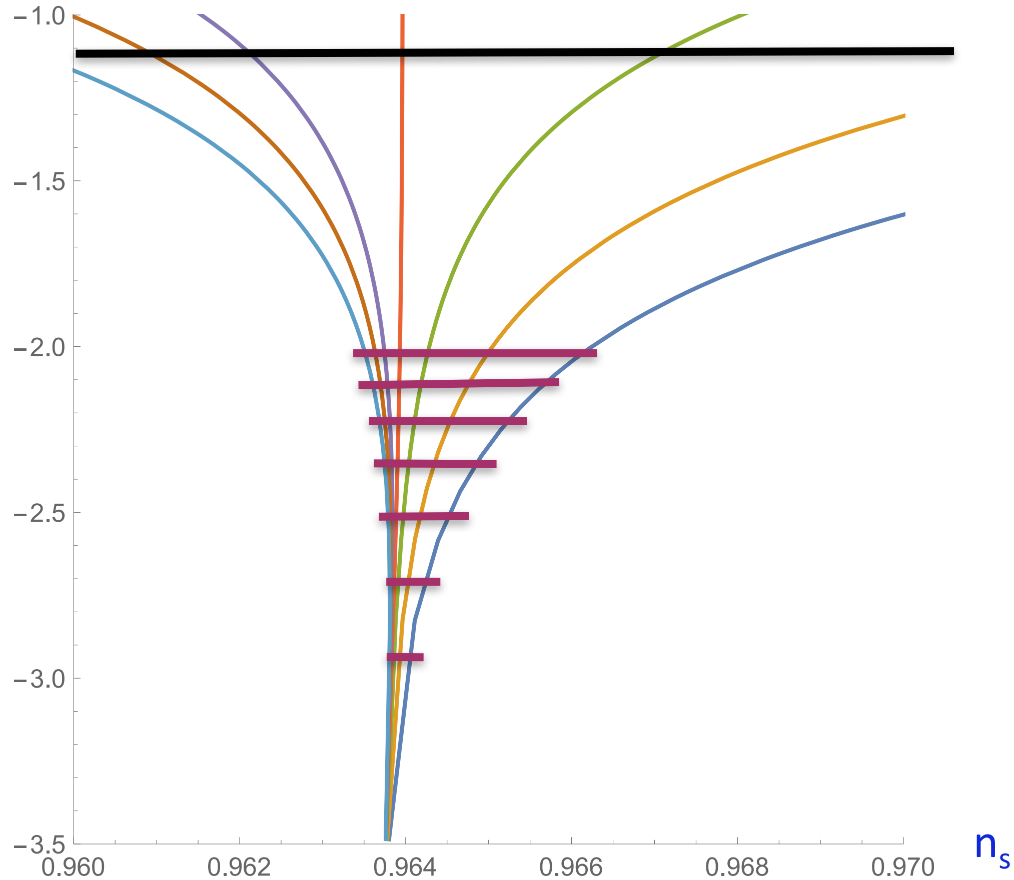


$V_{\text{infl}} = \frac{H^2}{3}$
$V_{\text{infl}} \sim E^4$
Planck length : 10^{-35} m
10^{18} GeV
10^{-33} m
$E_{\text{infl}} \sim 10^{16}$GeV
10^{15} GeV
10^{-30} m
$r < 0.07$
B-modes from inflation
???
10^{12} GeV
10^{-27} m
Desert
GUTs
$T_H = \frac{H}{2\pi} \sim 10^{13}$ GeV
Hawking temperature
of gravitational radiation
10^9 GeV
10^{-24} m
?
LHC
GeV
?
10^{-21} m
?
10^3 GeV
10^{-18} m
Higgs 126 Gev
t
W , Z
b J/ Ψ
Λ , Σ
K P, π N
μ
e
10^{-15} m
G. t'Hooft
1 GeV
0
γ , ν_e , ν_μ

Main new result:

New B-mode targets: from maximal supersymmetry to minimal supersymmetry

α-attractors $\log_{10} r$- n_s plane



r < 0.07 at 95% CL

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$$

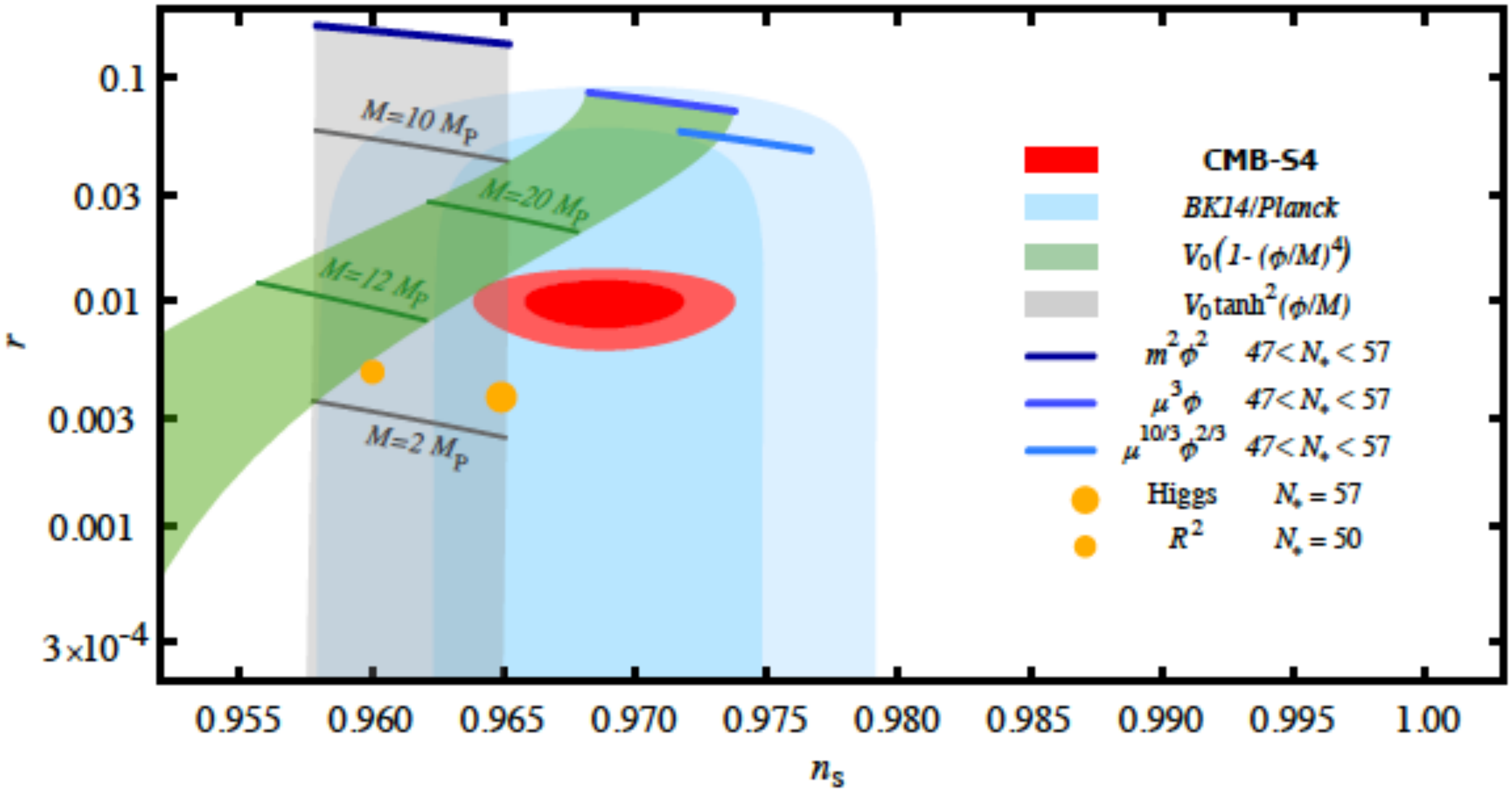
$R^2_{Escher} = 3\alpha = 7,6,5,4,3,2,1$

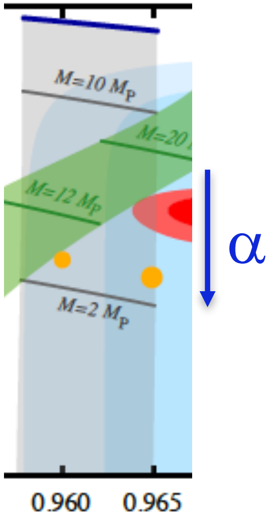
Ferrara, RK, Linde, Roest, Wrase, Yamada, 2016-2017

New inflationary **α-attractor** models describe inflation and dark energy and SUSY breaking. They provide **B-mode targets** for future B-mode detectors, with **r** between **10^{-2}** and **10^{-3}**

Alpha-Attractors and B-mode Targets

CMB-S4





$$n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha\,\frac{12}{N^2}$$

Meaning of the measurement of the curvature of the 3d space

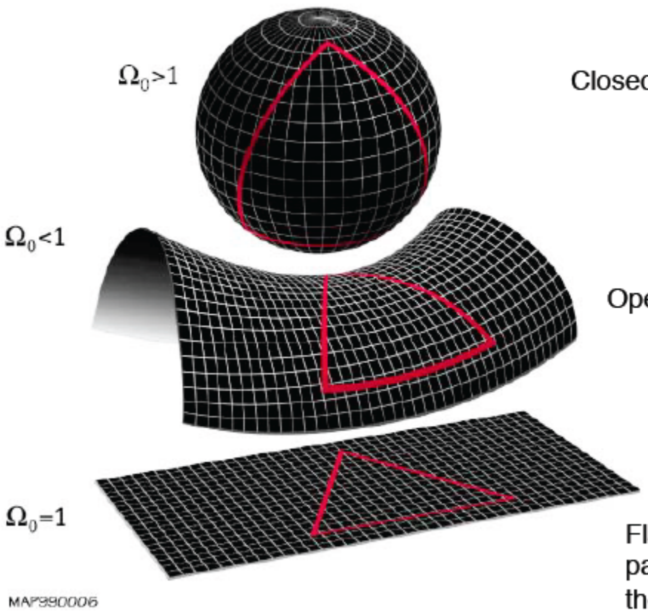
k=+1, k=-1, k=0

Spatial curvature parameter

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

$$\Omega_K = -0.0003 \pm 0.0026$$

Closed, open or flat universe



Closed universe. Parallel lines intersect

Open universe. Parallel lines diverge

Flat universe. Parallel lines remain parallel, but the distance between them grow with time

For α-attractors, measuring r means measuring the curvature of the hyperbolic geometry of the moduli space

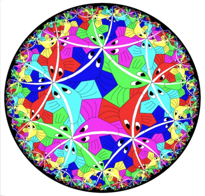
$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha \frac{12}{N^2}$$

$$R_K = -\frac{2}{3\alpha}$$

scalar fields are coordinates of the Kahler geometry

Decreasing r, decreasing α, increasing curvature R_K

$$3\alpha = R^2_{\rm Escher} \approx 10^3 r$$



Hyperbolic geometry of a Poincaré disk

What is the meaning of α-attractors?

Kallosh, AL 2013; Ferrara, Kallosh, AL, Porrati, 2013;
Kallosh, AL, Roest 2013; Galante, Kallosh, AL, Roest 2014

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

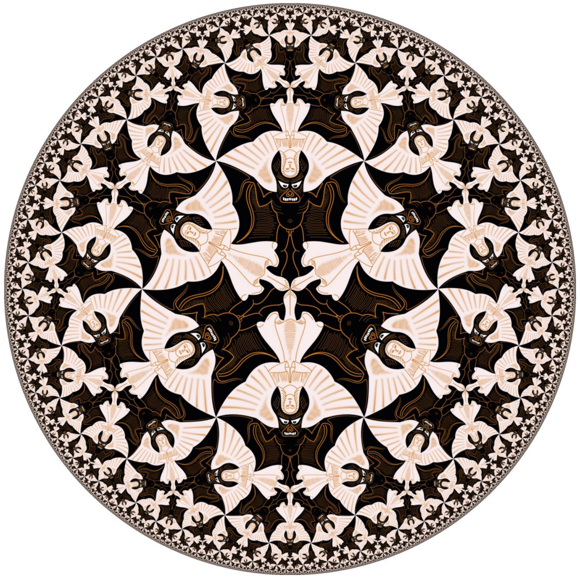
$$V = 3\alpha\, m^2 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}$$

Interpretation

(for a complex field Z)

$$ds^2 = \frac{3\alpha}{(1 - Z\bar{Z})^2} dZ d\bar{Z}$$

Hyperbolic geometry of a Poincaré disk



For $Z = \bar{Z}$, i.e. for the real inflaton field

$$ds^2 = \frac{1}{2} \frac{(\partial\phi)^2}{(1 - \frac{\phi^2}{6\alpha})^2}$$

$$3\alpha = R^2_{\rm Escher} \approx 10^3 r$$

α-attractors in $\mathcal{N}=1$ supergravity

$SL(2,\mathbb{R})$ symmetry

$$ds^2 = \frac{3\alpha}{(1-Z\bar{Z})^2} dZ d\bar{Z}$$

$$ds^2 = \frac{3\alpha}{(T+\bar{T})^2} dT d\bar{T}$$

$$\mathcal{R}_K = -\frac{2}{3\alpha}$$

Curvature of the moduli space in Kahler geometry

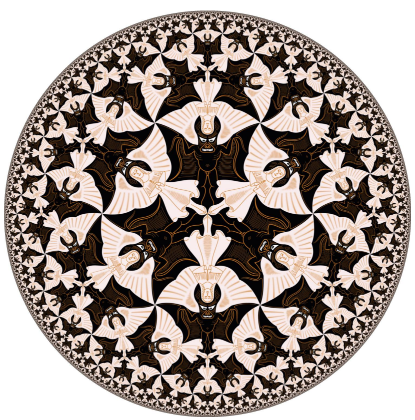
$Z\bar{Z} < 1$

Hyperbolic geometry
of a Poincaré disk

Disk or **half-plane**

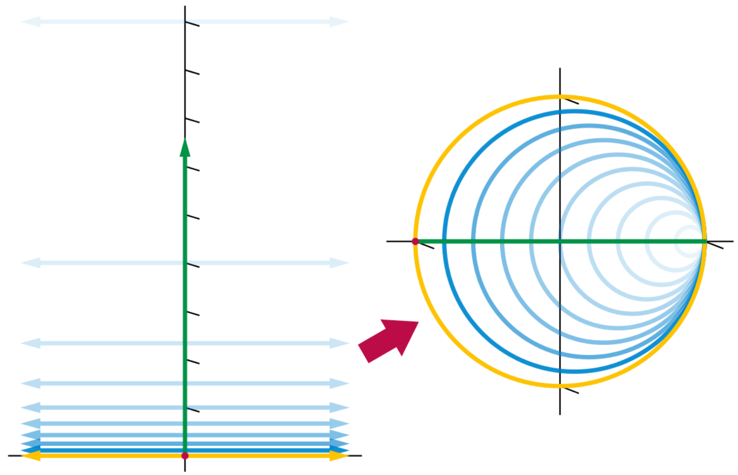
$T + \bar{T} > 0$

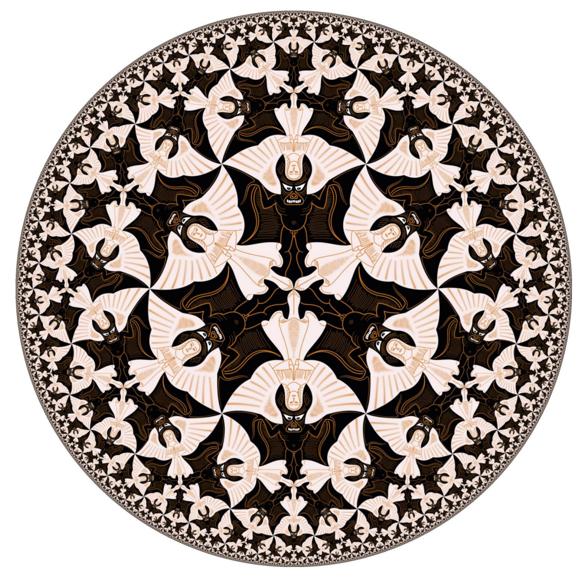
Escher in the Sky, RK, Linde 2015



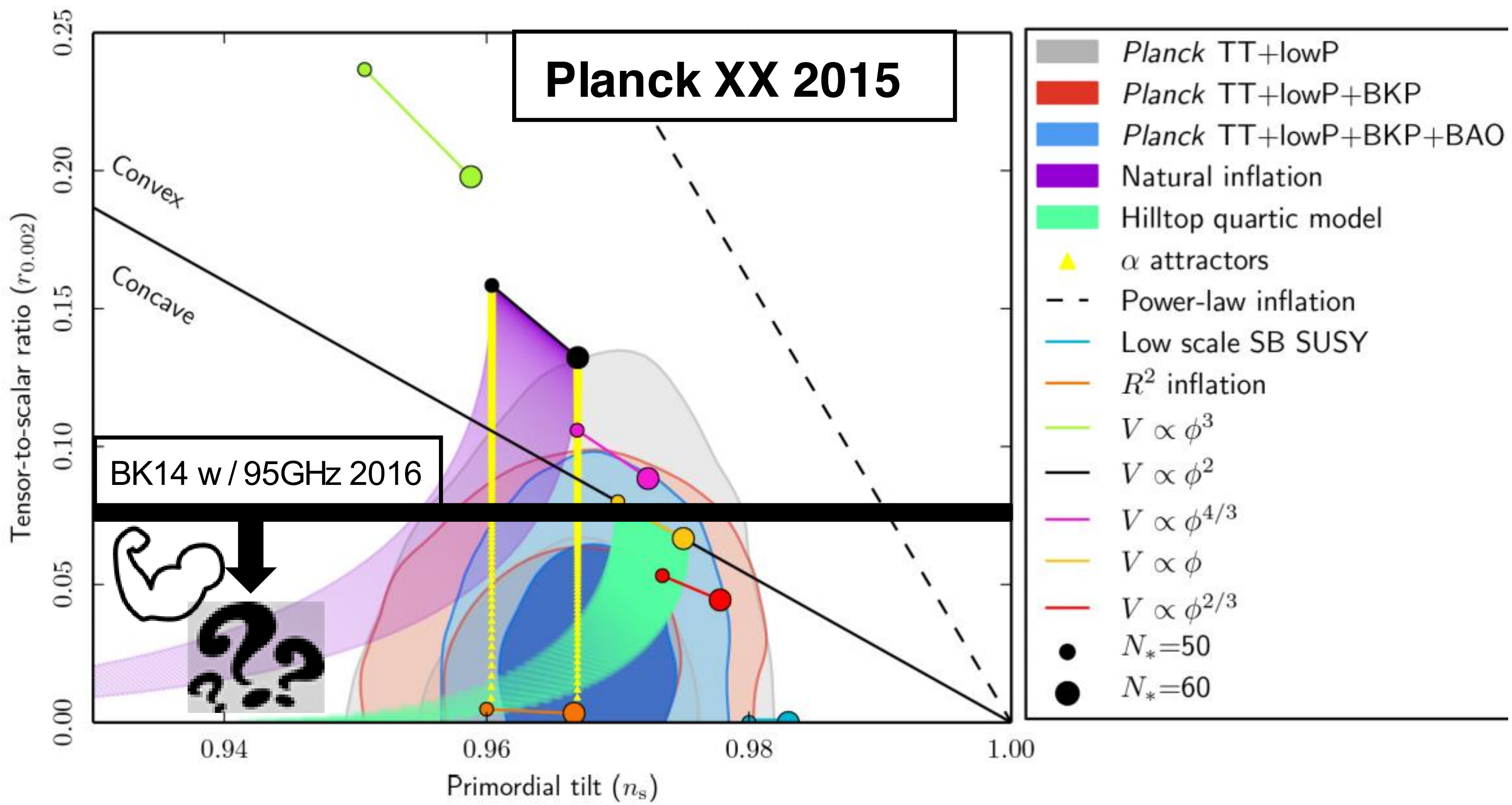
$$3\alpha = R^2_{\rm Escher} \approx 10^3 r$$

From the disk to a half-plane (the Cayley transform)





$$T = \frac{1+Z}{1-Z}$$



Next in CMB cosmology:

Relentless observation

If B-modes will be discovered soon, $r > 10^{-2}$
natural inflation models, axion monodromy models, α-attractor models,…, will be validated
No need to worry about log scale r

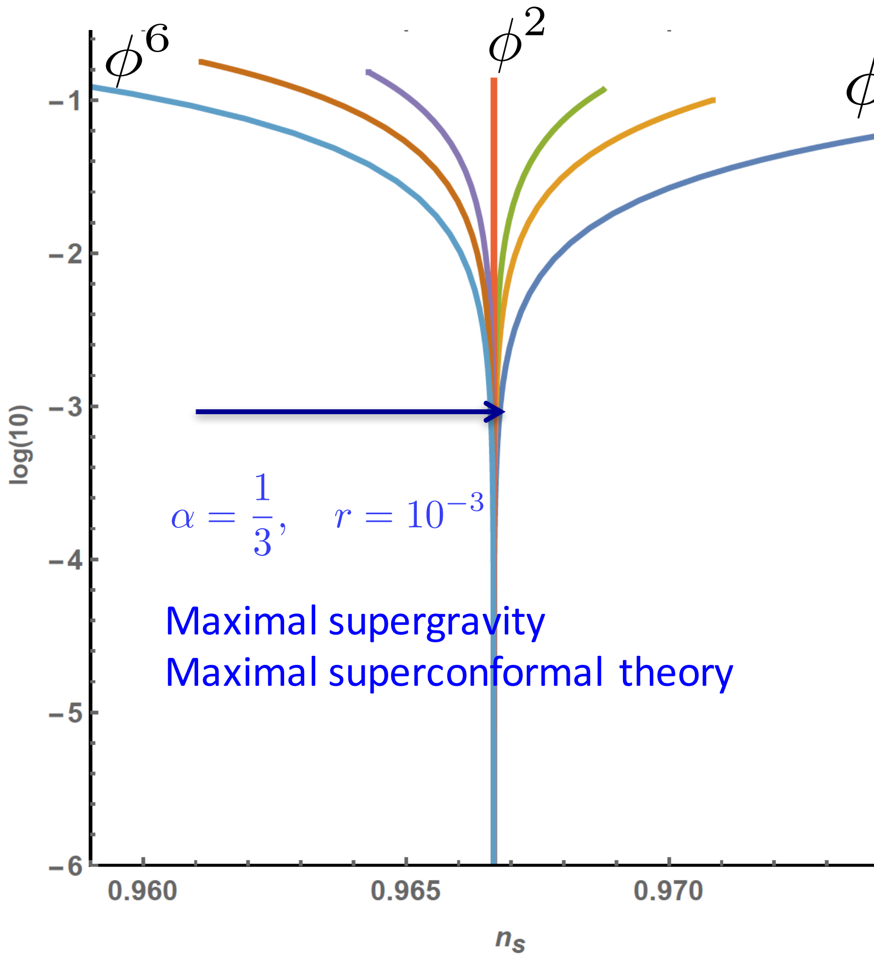
Otherwise, we switch to log r

A majority of string theory inflation models have very small r

2015

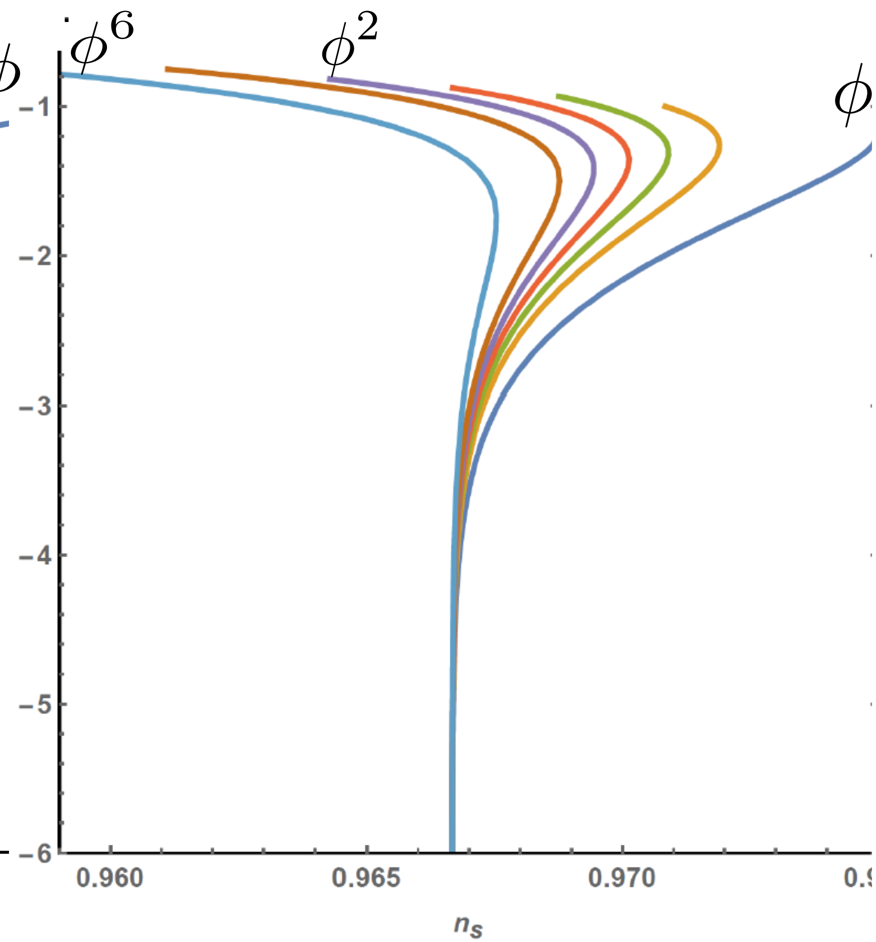
α-attractors

Simple Fanned T-models



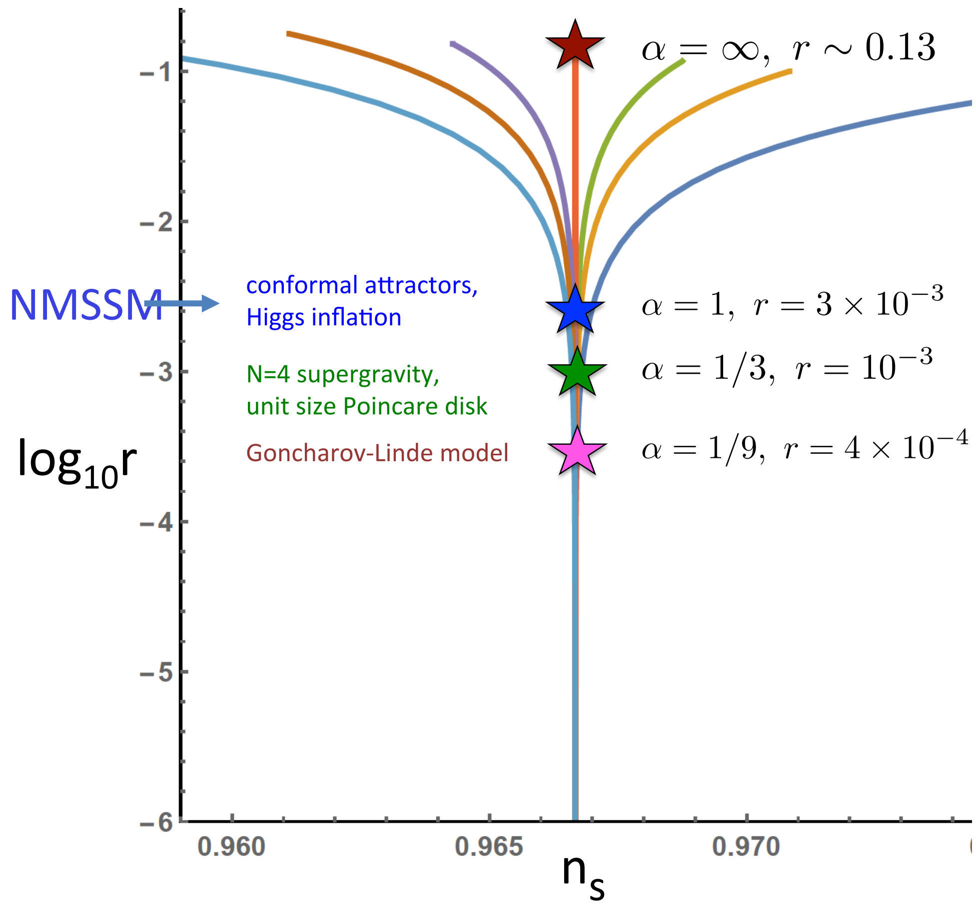
$$\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^{2n}$$

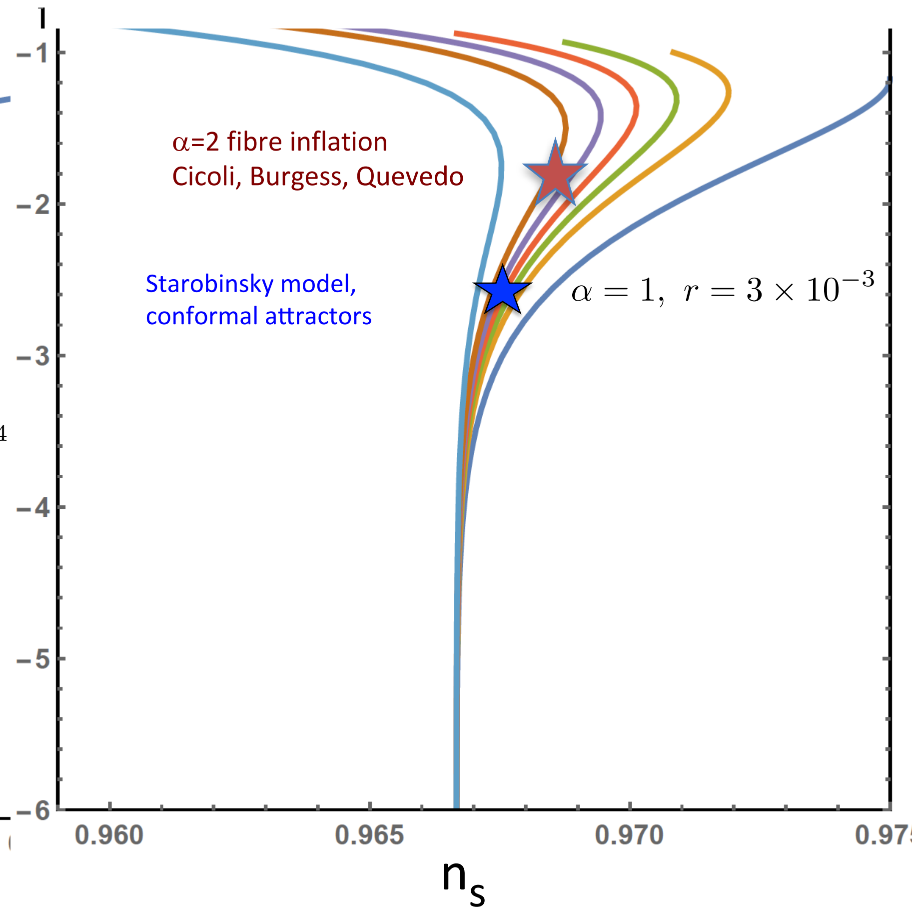
Simple Fanned E-models



$$\left(1-e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^{2n}$$

2015



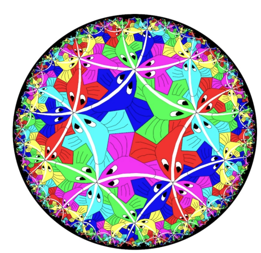


Any $\alpha < 20$ $r < 0.07$

$$3\alpha = R^2_{\rm Escher} \approx 10^3 r$$

Generic $\mathcal{N}$=1 supergravity

Any α



Hyperbolic geometry of a Poincaré disk

Anti-D3 Brane Induced Geometric Inflation

Kahler function defines the geometric kinetic term as well as the potential

KKLT construction of de Sitter vacua in string theory: Positive energy from anti-D3 brane

Basic idea: D-brane and anti-D-brane are extended objects in superstring theory. Like strings, they have various possible descriptions.

1. They are solutions of d=10 supergravity

2. They have their own world-volume action

Anti-D-brane: Volkov-Akulov geometric construction

$$E = dX - \bar{\theta}\Gamma^m d\theta$$

ultimate spontaneously broken supersymmetry: Majorana goldstino

$$S^{\overline{\mathrm{D3}}} = -2T_3 \int d^4\sigma \, \det E$$

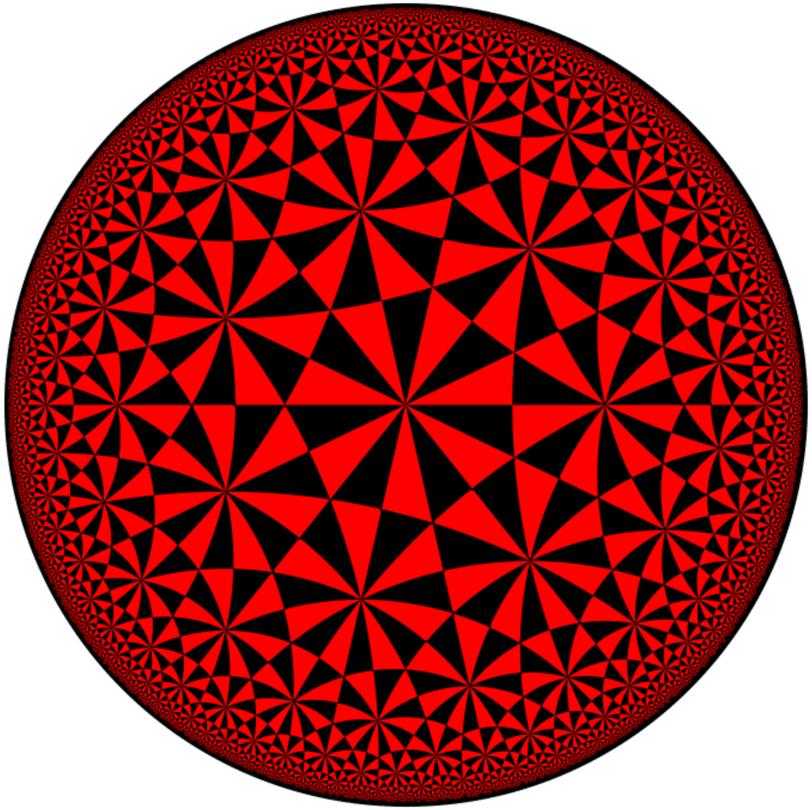
From Geometry to Dynamics

From geometry of anti-D3 brane interacting with CY moduli to effective supergravity models of inflation with the following features

- Fit to data
- Allow an exit to de Sitter vacua
- Models are simple and falsifiable by observations
- Include advanced α-attractor models and new ones
- Hyperbolic disk mergers with discrete $3\alpha= 1,2,3,4,5,6,7$ as B-mode targets
- Simple version of fibre inflation with $3\alpha=6$, paper in hep-th today

Inflationary dynamics including the exit to de Sitter space is fully defined by the geometric Kahler function in underlying supergravity

Möbius transformations applied to hyperbolic tilings allowed to produce animation in disk variables



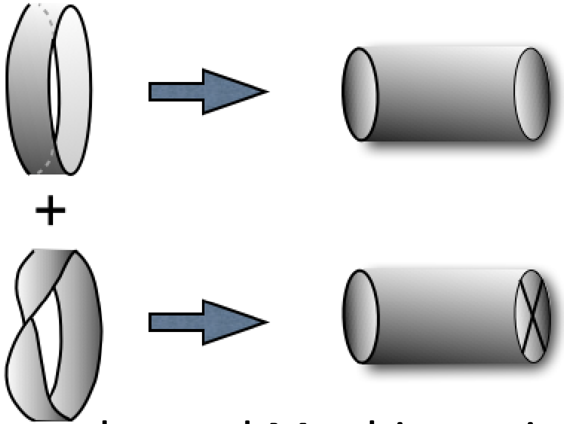
$$K_{\text{disk}} = -\frac{1}{2}\log\frac{(1-Z\bar{Z})^2}{(1-Z^2)(1-\bar{Z}^2)}$$

String Theory Realizations of the Nilpotent Goldstino

RK, Quevedo, Uranga 2015

A technical tool for the string theory landscape construction and for inflationary model building

$$S^2(x, \theta) = 0$$



The one-loop open string annulus and Moebius strip diagrams turn into closed string channel diagrams describing tree level exchange of NSNS and RR states between two boundaries (branes or antibranes), or between one boundary and one crosscap (O3-plane)

Non-linear supersymmetry: not of the kind that was not found at LHC

Volkov-Akulov, 1972

Allows de Sitter vacua in supergravity without scalars

String theory D-branes: supersymmetric KKLT

RK, Wrase, 2014
Bergshoeff, Dasgupta, RK, Wrase, Van Proeyen 2015
RK, Quevedo, Uranga 2015
RK, Vercnocke , Wrase 2016

Cosmological Models with nilpotent stabilizer

Antoniadis, Dudas, Ferrara and Sagnotti, 2014
Ferrara, RK, Linde 2014 Dall'Agata, Zwirner 2014
RK, Linde, Scalisi, 2014
Carrasco, RK, Linde, Roest, 2015
McDonough, Scalisi, 2016
Ferrara, RK, 2016
RK, Linde, Wrase,Yamada, 2017

The positive contribution to the vacuum energy which converts the AdS minumum into dS minimum due to the presence of the non-perturbative anti-D3 brane can be effectively described in supergravity by the presence of the nilpotent superfield S

$$S^2 = 0$$

RK, Linde, Roest, Yamada, 2017

Anti-D3 Induced Geometric Inflation

In current literature on de Sitter vacua this function is called Kahler function

$\mathcal{G}$

Cremmer, Ferrara, Girardello, Julia, Scherk,
van Nieuwenhuizen, Van Proeyen, from 1978
Binetruy, Gaillard, from 1985
Gomez-Reino et al, Achucarro et al, Covi et al, from 2007

We are interested in anti-D3 brane interaction with Calabi-Yau moduli T_i. In supergravity we expect some interaction between the nilpotent superfield S and Calabi-Yau moduli T_i

$$\mathcal{G}(T^i, \bar{T}^i; S, \bar{S})$$

$$\mathcal{G} \equiv K + \log W + \log \bar{W} \,, \qquad \mathbf{V} = e^{\mathcal{G}}(\mathcal{G}^{\alpha\bar{\beta}}\mathcal{G}_\alpha \mathcal{G}_{\bar{\beta}} - 3)$$

Erich Kahler noticed in 1933 and Moroianu suggested in 2004, that once the hermitian Kahler function is introduced

"a long list of miracles occur then"

May 2017

RK, Linde, Roest, Yamada

Model Building Paradise

now confirmed in the cosmological context

In models with Hermitian Kähler function of the form

$$\mathcal{G} = \mathcal{G}_0(T_i, \bar{T}_i) + S + \bar{S} + \mathcal{G}_{S\bar{S}}(T_i, \bar{T}_i) S\bar{S}$$

If during inflation,
as in α-attractor models,
or 7-disk geometries

$$T_i = \bar{T}_i\,, \qquad S = 0$$

$$e^{\mathcal{G}} = m_{3/2}^2 \qquad \mathcal{G}_i = 0$$

one finds the following simple relation between the potential and the nilpotent field geometry

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

From the sky to fundamental physics

Easy to establish stability during and after inflation with the exit into de Sitter vacuum: sectional and bisectional curvature associated with our Kähler function play important role in the stability analysis

The models are relatively simple if the T-moduli have hyperbolic geometry of a combination of Poincare disks. In half-flat geometry variables

$$\mathcal{G}\Big|_{S=0} = -\frac{1}{2}\sum_{i=1}\log\Big[\frac{(T_i+\bar{T}_i)^2}{4T_i\bar{T}_i}\,\frac{1}{m_{3/2}^4}\Big]$$

The Kahler function is invariant under **inversion and scaling part of the Mobius symmetry**

$$T_i \to \frac{1}{T_i}, \qquad T_i \to a^2\, T_i$$

Inflaton shift symmetry is broken only via interaction with the anti-D3 brane, via the S-field geometry

$$\mathcal{G}_{S\bar{S}}(T_i,\bar{T}_i)S\bar{S}$$

Inflaton shift symmetry in a hyperbolic geometry with inversion/scaling symmetry

$$-\frac{1}{2}\log\left[\frac{(T+\bar{T})^2}{4T\bar{T}}\frac{1}{m_{3/2}^4}\right] \quad =$$

$$-\frac{1}{2}\log\left[\frac{(\mathrm{Re}T)^2}{(\mathrm{Re}T)^2+(\mathrm{Im}T)^2}\frac{1}{m_{3/2}^4}\right]$$

If during inflation Im T =0 is a minimum, which is valid in our models, we find

$$e^{\mathcal{G}} = m_{3/2}^2 \qquad\qquad \frac{\partial\mathcal{G}}{\partial T} = 0$$

for $\qquad S = T - \bar{T} = 0$

Model-building Paradise

Why do we use these words? Let us look again at this equation from the previous page:

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

This equation shows incredible simplicity of inflationary model construction in this approach. One can take any function V(X) of a real inflaton field X, replace X, e.g., by $(T + \overline{T})/2$, and put the resulting function $V(T, \overline{T})$ to the equation above. The resulting potential evaluated by usual SUGRA rules automatically has the desired shape V(X) in the inflaton direction. The only thing remaining to check is stability with respect to the imaginary part of the field T, which is usually not a problem.

By this method we easily reproduced and improved many previously known models, and generalized them in a way allowing to have an arbitrary cosmological constant and SUSY breaking after inflation.

Why so simple?

- One nilpotent multiplet (representing anti-D3 brane)
- Supersymmetry is broken only in the nilpotent goldstino direction due to inversion/scaling symmetry of the Kahler function. It is unbroken in absence of anti-D3 brane
- Only the nilpotent multiplet geometry breaks inversion/scaling symmetry of the moduli geometry

Möbius transformations
and inflaton shift symmetry

The set of *Möbius transformations* of the upper half-plane is
a fractional linear transformations

$$\left\{ T : z \mapsto \frac{az+b}{cz+d} \;\middle|\; a,b,c,d \in \mathbb{R},\ ad-bc>0 \right\}$$

Möbius transformations come in three basic types:

(1) Translations of the form $T_\alpha : z \mapsto z + \alpha$ for α in $\mathbb{R}$

(2) Dilations of the form $D_r : z \mapsto rz$ for positive r in $\mathbb{R}$,
(3) Inversion $I : z \mapsto -\frac{1}{z}$.

Inflaton-axion models:
(1) symmetry provides the inflaton shift symmetry of the S-independent Kahler function
It is broken only by the S-geometry

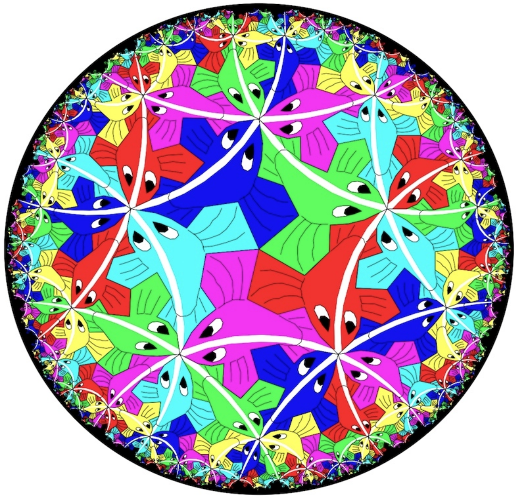
Dilaton-inflaton models:
(2), (3) symmetries provide the inflaton shift symmetry of the S-independent Kahler function
It is broken only by the S-geometry

Isometries:

Mobius Transform

$$Z' = \frac{\beta Z + \gamma}{\bar{\gamma} Z + \bar{\beta}}$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$



The choice of the Kahler frame is suggested by the tessellation of the hyperbolic geometry

A tessellation is the tiling of a plane or hyperbolic plane, or a hyperbolic disk using one or more geometric shapes, called tiles, with no overlaps and no gaps

It leads to an improved stability of inflationary trajectory since the moduli dependent part of the geometry is flat in the inflaton direction due to inversion and scaling symmetry of the Kahler fuction

7-disk cosmological model

$3\alpha=7$ example

Ferrara, RK, Linde, Roest, Wrase, Yamada

1. Start with M-theory, or String theory, or $\mathcal{N}$=8 supergravity

2. Perform a consistent truncation to $\mathcal{N}$=1 supergravity in d=4 with a 7-disk manifold

$$\mathcal{G} = \log W_0^2 - \frac{1}{2}\sum_{i=1}^{7} \log \frac{(1 - Z_i \overline{Z}_i)^2}{(1 - Z_i^2)(1 - \overline{Z}_i^2)} + S + \overline{S} + \mathcal{G}_{S\overline{S}} S\overline{S},$$

Tessellation

$$\mathcal{G}^{S\overline{S}} = \frac{1}{W_0^2}(3W_0^2 + \mathbf{V}).$$

corresponding to the merger of seven disks of unit size

The scalar potential defining geometry is

$$\mathbf{V} = \Lambda + \frac{m^2}{7}\sum_i |Z_i|^2 + \frac{M^2}{7^2}\sum_{1\leq i\leq j\leq 7}\Big((Z_i + \overline{Z}_i) - (Z_j + \overline{Z}_j)\Big)^2,$$

De Sitter exit

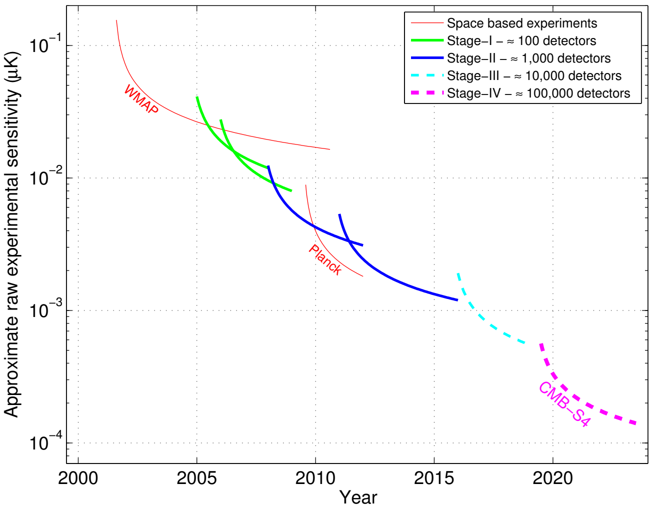
During inflation $\quad \mathbf{V}(\varphi) = \Lambda + m^2 \tanh^2 \frac{\varphi}{\sqrt{14}},$

$$r \approx 10^{-2}$$

Primordial Gravity Waves

CMB-S4 Science Book

October 2016

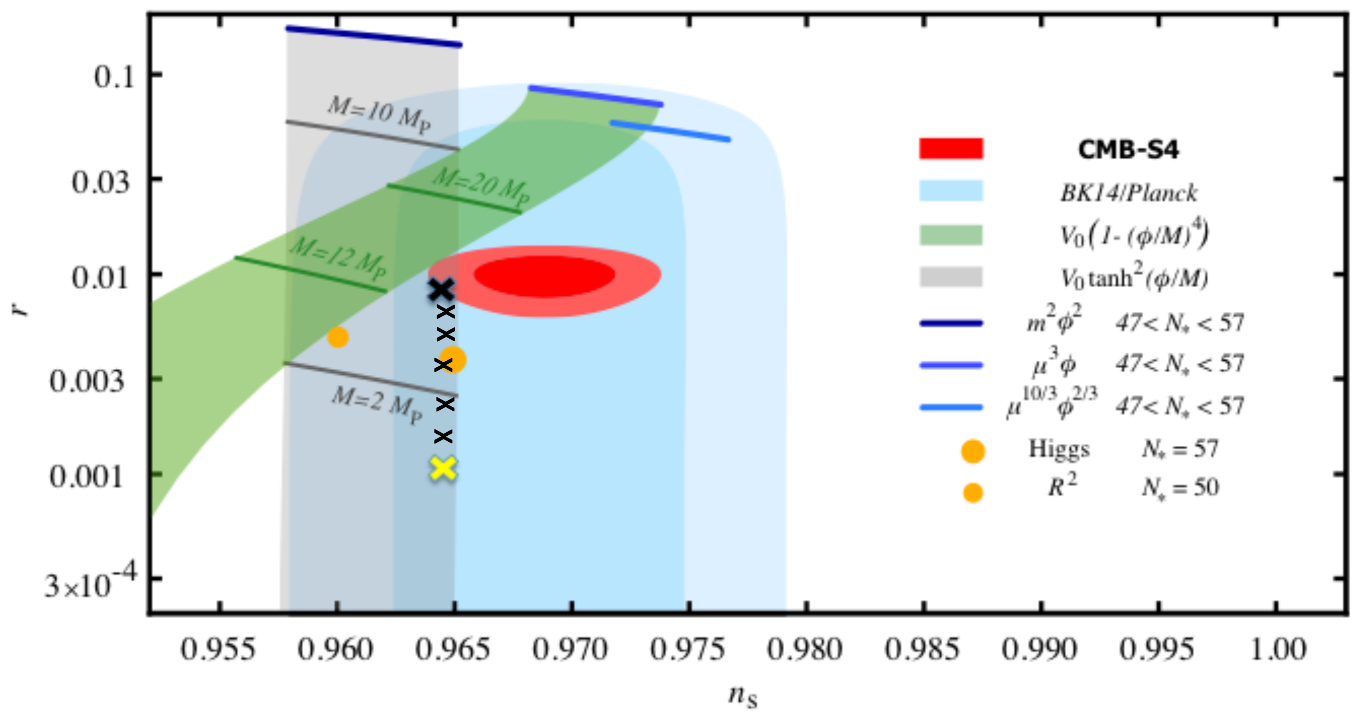


Ferrara, RK, 2016,
RK, Linde, Wrase, Yamada, 2017
RK, Linde, Roest, Yamada, 2017
α–attractor models

Well motivated new models originating in string theory, M-theory, maximal supergravity

Ground based experiments

Future B-mode satellite missions

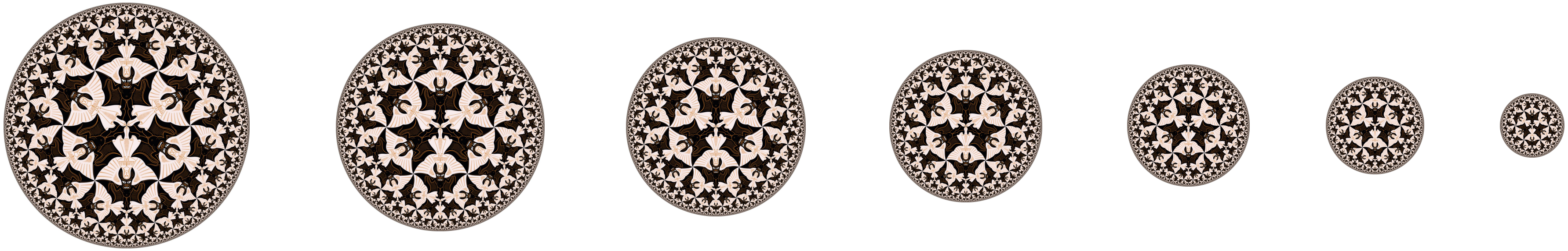


Why do we find it useful to talk about the size of the Escher's disks in discussions of the CMB future B-mode targets ?

Based on CMB data on the value of the tilt of the spectrum n_s as a function of N, we found that hyperbolic geometry of a Poincaré disk  suggests a way to explain the experimental formula

$$n_s \approx 1 - \frac{2}{N}$$

Using a consistent reduction from maximal $\mathcal{N}$=8 supersymmetry theories: M-theory in d=11, String theory in d=10, maximal supergravity in d=4, to the minimal $\mathcal{N}$=1 supersymmetry, we found favorite models with hyperbolic geometry with R^2_{Escher} = 7,6,5,4,3,2,1



$r \approx 0.9 \times 10^{-2}$

B-mode targets

$r \approx 1.3 \times 10^{-3}$

In contrast with $\mathcal{N}$=1 supersymmetry models where R^2_{Escher} is arbitrary

Back up slides

The construction of de Sitter vacua in string theory as well as building inflationary models is facilitated by the concept of an upliting anti-D3 brane. Supersymmetry is spontaneously broken during inflation as well as at the exit from inflation, and never restored in models we described

anti-D3 brane induced geometric inflationary models

Subject to specific assumptions about the geometry of T- moduli one finds a simple relation between the desirable inflationary potential and geometry of the anti-D3 brane in the background of the T-moduli

Geometry ⟷ **Dynamics**

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

We start with a geometry,

$$\mathcal{G} = \mathcal{G}_0(T_i, \bar{T}_i) + S + \bar{S} + \mathcal{G}_{S\bar{S}}(T_i, \bar{T}_i) S\bar{S}$$

Any phenomenological potential $\mathbf{V}_{pheno}$ can be reconstructed by choosing the metric of a nilpotent superfield S:

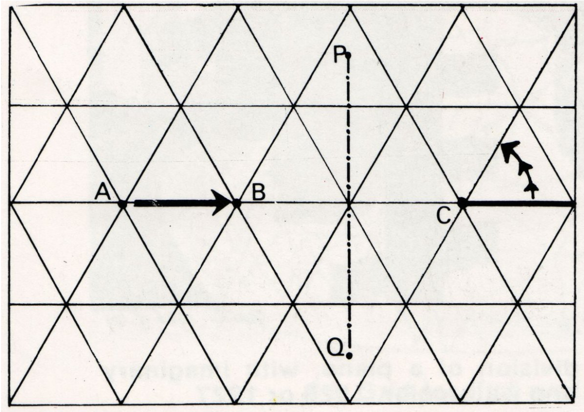
$$\mathbf{V}_{pheno} = e^{\mathcal{G}_0}(\mathcal{G}^{S\bar{S}} + \mathcal{G}_i \mathcal{G}^{i\bar{j}} \mathcal{G}_{\bar{j}} - 3)$$

Relation between a general potential and S-geometry

$$\mathcal{G}^{S\bar{S}} = \left(e^{-\mathcal{G}_0}\mathbf{V}_{pheno} - \mathcal{G}_i \mathcal{G}^{i\bar{j}} \mathcal{G}_{\bar{j}} + 3\right)$$

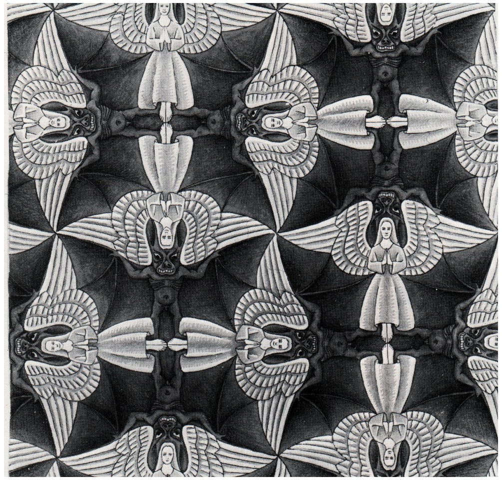
But the metric becomes quite complicated.

Principles of Plane Tessellations

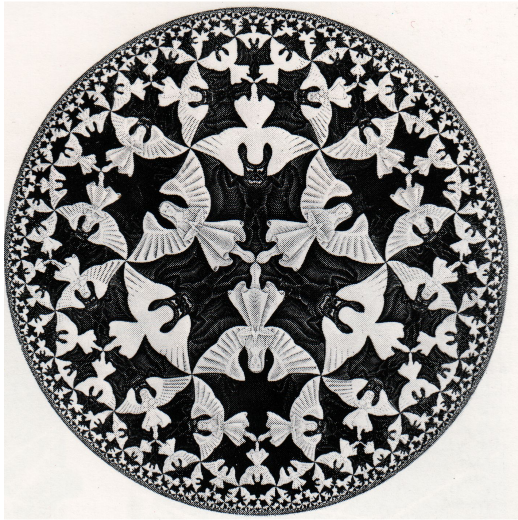


The whole surface is covered with equilateral triangles. If we shift the whole plane over the distance **AB**, it will cover the underlying pattern once again. This is a **translation** of the plane. We can also turn the duplicate through 60 degrees about the point **C**, and we notice that again it covers the original pattern exactly. This is a **rotation**. Also if we do a **reflection** about the line **PQ**, the pattern remains the same.

Periodic space filling for "Angels and Devils" (1941)



The tessellation and the final result for the hyperbolic tiling for "Angels and Devils", "Circle Limit IV" (1960)



Special choices of α and future data

2015

Ferrara and RK

New in2016

7/3

6/3 $\alpha = 2$ $r \approx 6 \times 10^{-3}$ Fibre inflation

5/3

4/3

3/3 $\alpha = 1$ $r \approx 3 \times 10^{-3}$ Critical point of superconformal $\mathcal{N}$=1 attractors, Higgs inflation, R^2 …

2/3

1/3 $\alpha = 1/3$ $r \approx 10^{-3}$ Maximal superconformal $\mathcal{N}$=4 model, maximal supergravity $\mathcal{N}$=8

$\alpha = 1/9$ $r \approx 3 \times 10^{-4}$ 1984 model of Goncharov-Linde

Any $\alpha < 20$ $r < 0.07$ Generic $\mathcal{N}$=1 supergravity

New in 2017

RK, Linde, Roest, Wrase, Yamada

Working dynamical models for 7-disk manifold

All of these models fit the current data

Complex scalar fields in supergravity and string theory

$$Z(t,\vec{x})\,,\bar{Z}(t,\vec{x})$$

are coordinates of some geometric space: **MODULI SPACE**

$$ds^2 = g_{Z\bar{Z}}\,dZ\,d\bar{Z}$$

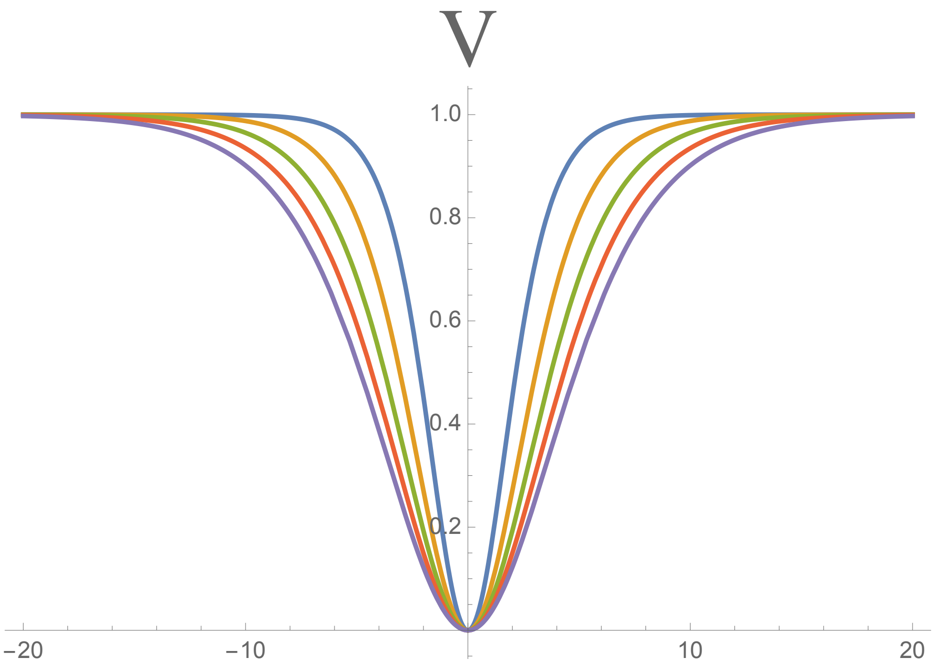
The metric of the moduli space is defined by a second derivative of the Kahler potential

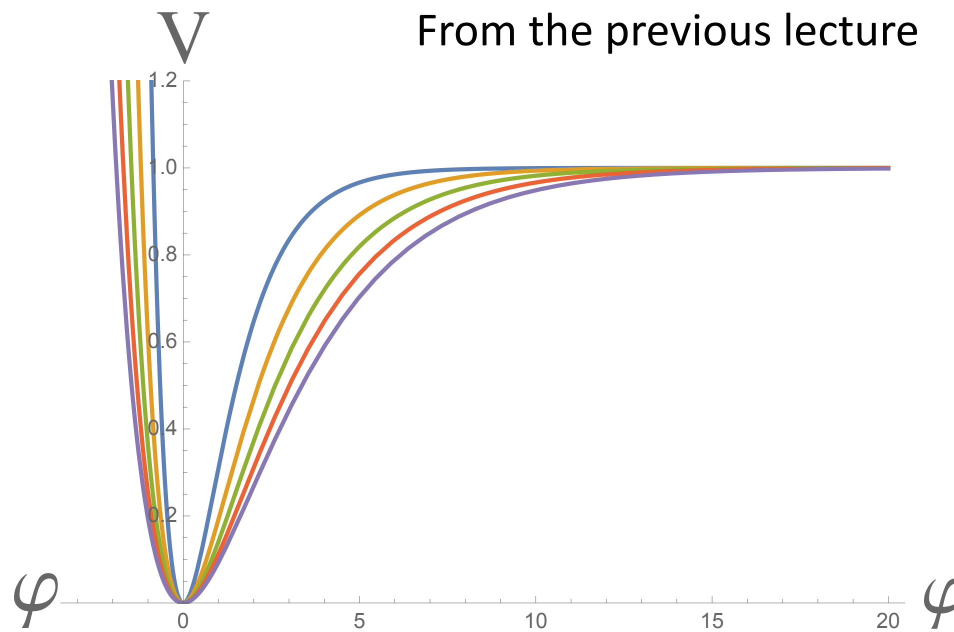
$$g_{Z\bar{Z}} = \partial_Z \partial_{\bar{Z}} K(Z,\bar{Z})$$

The curvature of the MODULI SPACE, Kahler curvature for our models is

$$\mathcal{R}_{\text{Kähler}} = -g^{-1}_{Z\bar{Z}} \partial_Z \partial_{\bar{Z}} \log g_{Z\bar{Z}} = -\frac{2}{3\alpha}$$

Plateau potentials α-attractors





From the previous lecture

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \quad - \alpha\mu^2\Big(\tanh\frac{\varphi}{\sqrt{6\alpha}}\Big)^2$$

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \quad - \alpha\mu^2\Big(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

Simplest T-model in canonical variables Simplest E-model

$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial\bar{Z}}{(1 - Z\bar{Z})^2} - V_0 Z\bar{Z}$$

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial\bar{T}}{(T + \bar{T})^2} - V_0(T - 1)^2$$

In geometric variables

Purely bosonic theory, what is the relation to fundamental theory ?

Details and assumptions underlying **3α=1,2,3,4,5,6,7** prediction

M-theory compactified on a 7-manifold with G_2 holonomy
special choice of Betti numbers $(b_0, b_1, b_2, b_3) = (1, 0, 0, 7)$
one can obtain d=4 N = 1 supergravity with rank 7 scalar coset

$$\left[\frac{SL(2,\mathbb{R})}{SO(2)}\right]^7$$

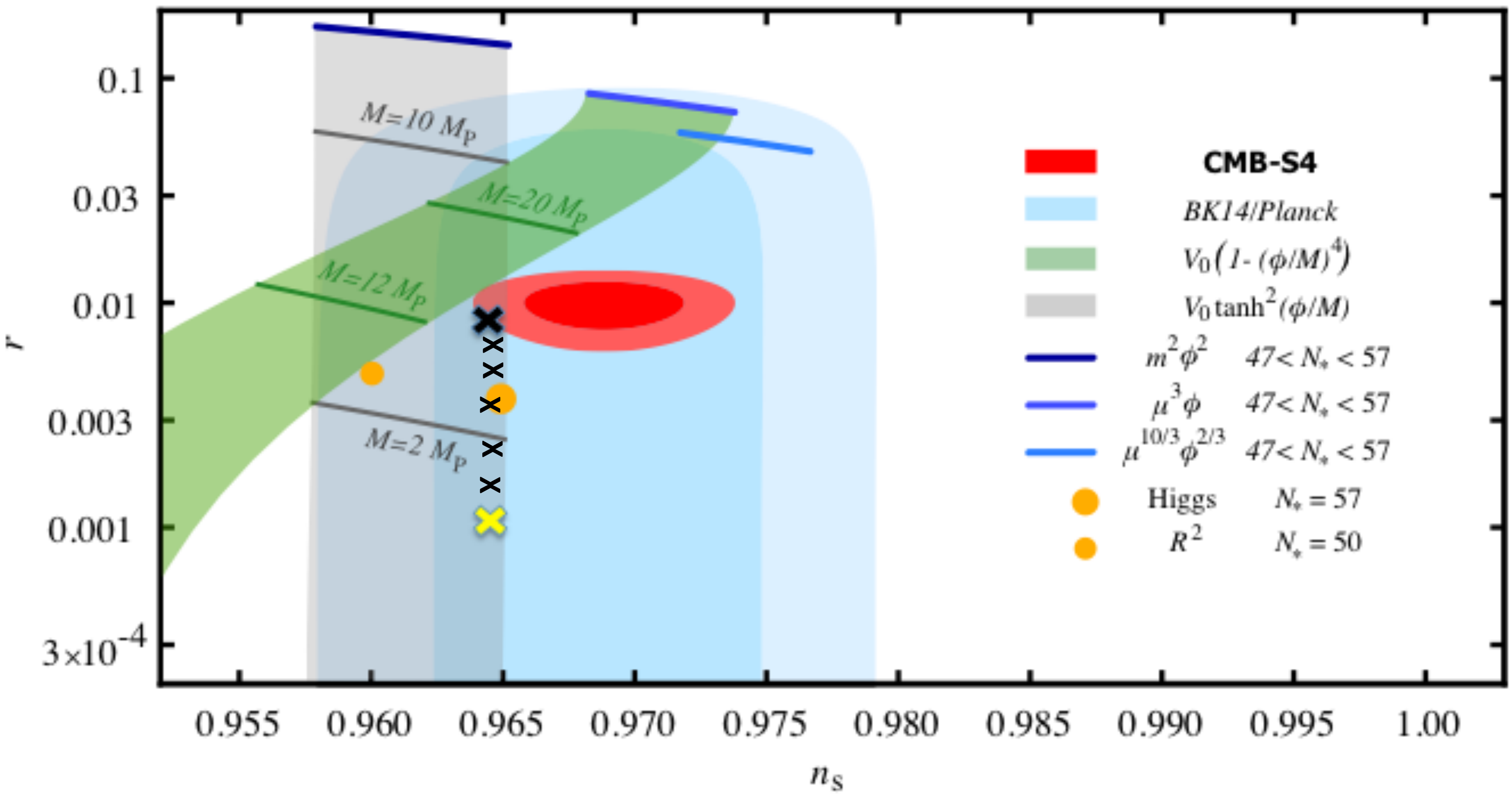
N=8 supergravity: consistent reduction to N=1 $E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$

$$K = -\sum_{i=1}^{7} \ln(\tau_i + \bar{\tau}_i)) \Rightarrow -7\ln(\tau + \bar{\tau})$$

3α=7

String theory compactified on $T_2 \times T_2 \times T_2 \subset T_6$

$$K = -\ln(S + \bar{S}) - 3\ln(U + \bar{U}) - 3\ln(T + \bar{T}) \Rightarrow -7\ln(\tau + \bar{\tau})$$



CMB-S4
BK14/Planck
V0(1- (φ/M)^4)
V0 tanh^2(φ/M)
m^2φ^2 47< N* < 57
μ^3φ 47< N* < 57
μ^10/3 φ^2/3 47< N* < 57
Higgs N* = 57
R^2 N* = 50
M=10 MP
M=20 MP
M=12 MP
M=2 MP
0.1
0.03
0.01
0.003
0.001
3×10^-4
r
0.955
0.960
0.965
0.970
0.975
0.980
0.985
0.990
0.995
1.00
nS