Initial conditions and gravitational waves in large field inflation

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Related talk by Kallosh

Stockholm 2017

5 definitions of large field inflation, and the possibility to test it with B-modes

AL 1612.00020

A. *Global definition of large field inflation.* From the point of view of the foundations of inflationary theory, one of the main issues is whether the canonically normalized inflaton field φ may have a super-Planckian value $\varphi > 1$ *at any stage of the cosmological evolution.*

*One **cannot** rule out large field inflation defined above by **not** finding tensor modes with any r.*

B. *Large field inflation during the last 50-60 e-foldings.* From the point of view of the observational cosmology, one may want to know whether the canonically normalized field φ was large during the last 50-60 e-foldings of inflation.

One cannot rule out large field inflation during the last 50-60 e-foldings unless one does not find gravitational waves all the way down to $r = 2 \times 10^{-5}$

Garcia-Bellido, Roest, Scalisi, Zavala 1405.7399

AL 1612.00020

C. *Characteristic scale of the inflaton field.* One may wonder whether the characteristic scale of the inflaton field, describing a typical range $\Delta\varphi$ in which the inflaton potential changes in a significant way, can be super-Planckian. For example, the characteristic scale for the theory with a potential $V = V_0(1 - e^{-\varphi/M})$ is $\Delta\varphi = M$.

For many models of α–attractors, $r = 10^{-2} - 10^{-3}$, and $M = \sqrt{3\alpha/2}$. Models with $\alpha < 1$ ($r < 4 \times 10^{-3}$) correspond to $M < 1.2$, i.e. to sub-Planck characteristic scale). This does **not** mean that inflation with $r < 4 \times 10^{-3}$ is "**small field**".

Finding r and scale of inflation M is important for determining geometric properties of the inflationary theory: The curvature of the moduli space in the theory of α-attractors, to be discussed shortly, is $-1/M^2$.

D. *The energy scale of inflation.* There is an important relation between the inflaton potential V and r

$$V^{1/4} \sim 1.04 \times 10^{16} \text{ GeV } \left(\frac{r}{0.01}\right)^{1/4} .$$

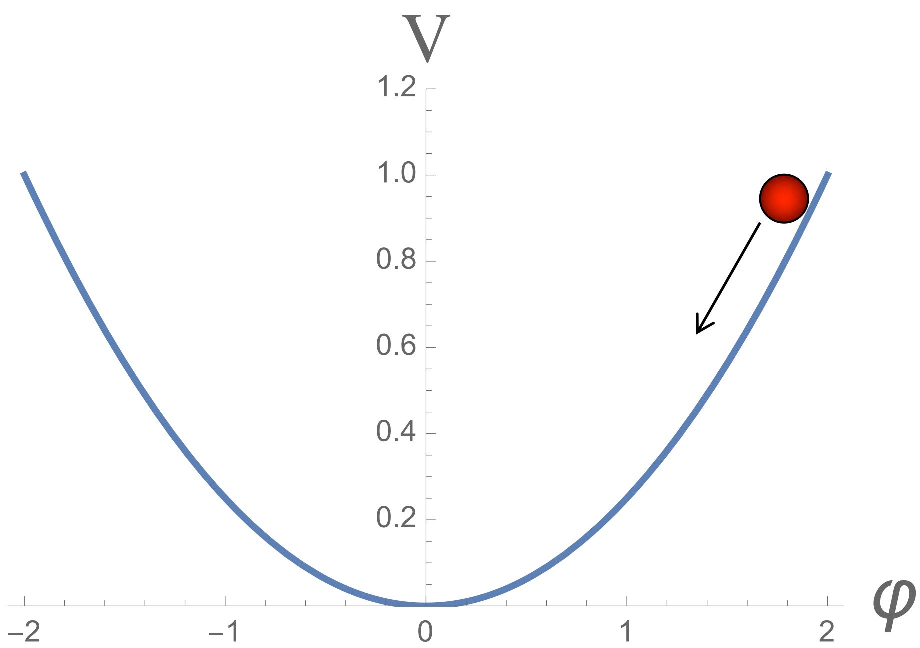
This suggests that the discovery of the tensor modes would give us access to physics at energy scales 10^{11} times higher than LHC. However, during inflation the universe did not have any particles with energies $V^{1/4} \sim 10^{16}$ GeV.

E. *The energy scale of inflationary perturbations.* The energy scale of these perturbations at the moment of their production is $k \sim H = \sqrt{V/3}$

$$k \sim H = \sqrt{V/3} \sim 2.6 \times 10^{13} \text{ GeV } \left(\frac{r}{0.01}\right)^{1/2} .$$

Simplest inflationary model: $V = \frac{m^2\phi^2}{2}$

Inflation can start at the Planck density if there is **a single Planck size domain** with a potential energy V of the same order as kinetic and gradient density. This is the minimal requirement, compared to standard Big Bang, where initial homogeneity is requires across **10^{90} Planck size domains**.

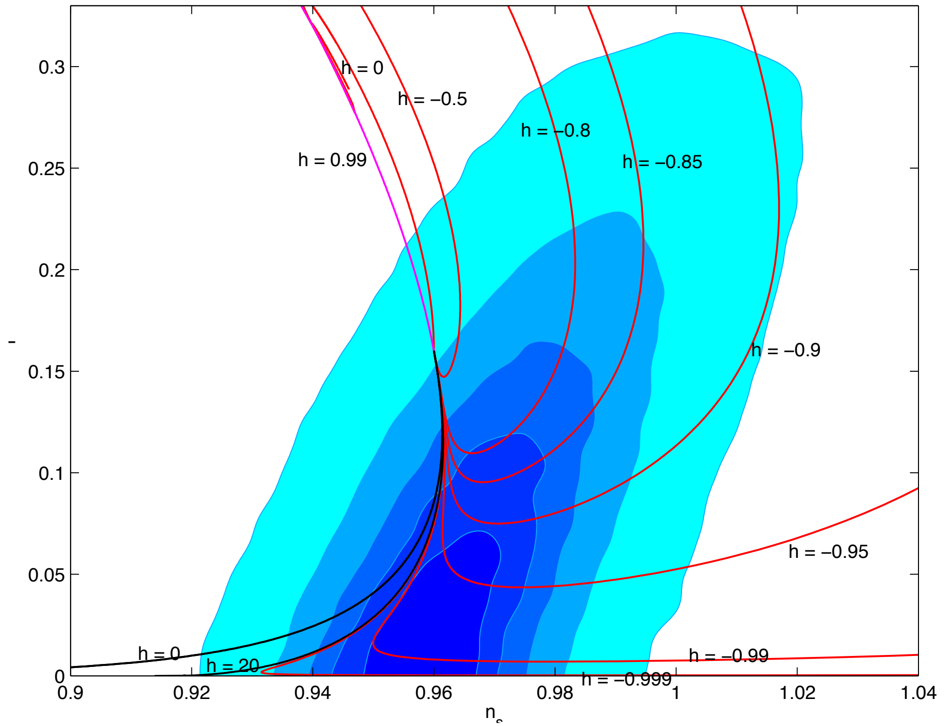


Polynomial inflation:

Simplest quadratic model predicts too large amount of the gravitational waves. However, it can be trivially generalized to avoid this problem, while still offering the possibility of inflation beginning at the Planck density

$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

Destri, de Vega, Sanchez, 2007



One can fit all Planck data by a polynomial, with inflation starting at the Planck density

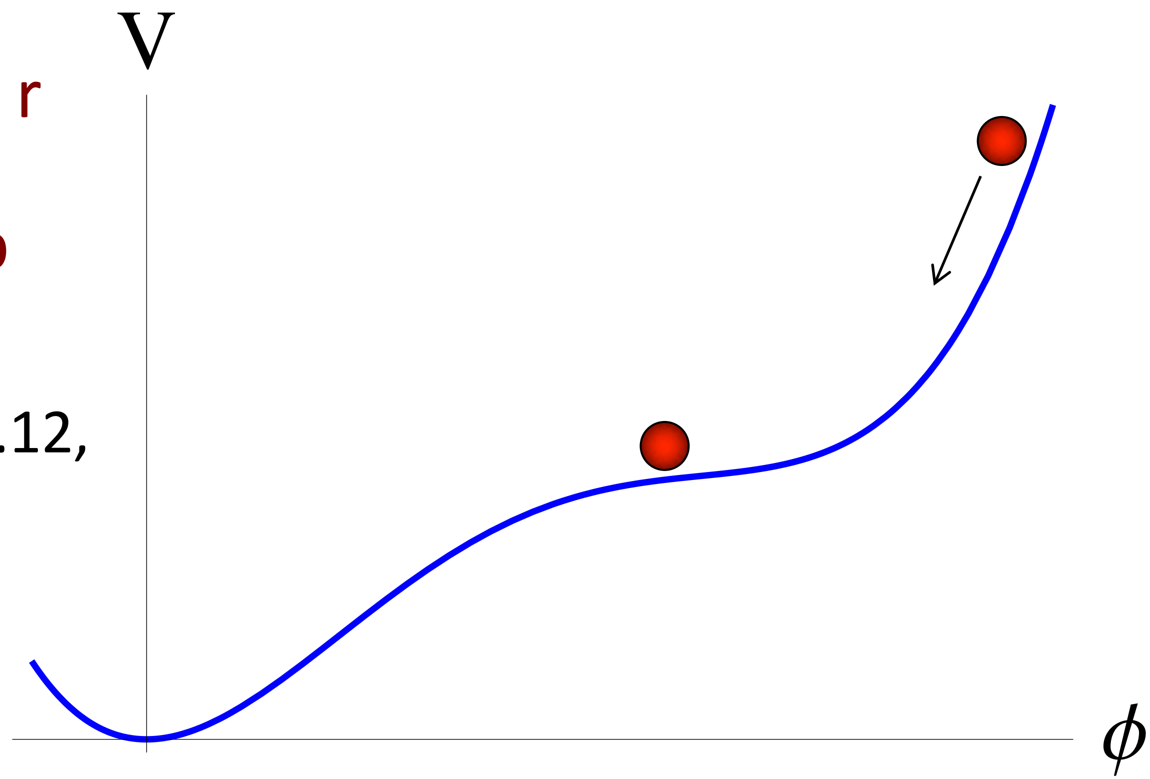
$$V = \frac{m^2\phi^2}{2}\left(1 - a\phi + b\phi^2\right)$$

Destri, de Vega, Sanchez, 2007
Nakayama, Takahashi and Yanagida, 2013
Kallosh, AL, Westphal 2014
Kallosh, AL, Roest, Yamada **1705.09247**

3 observables: A_s, n_s, r

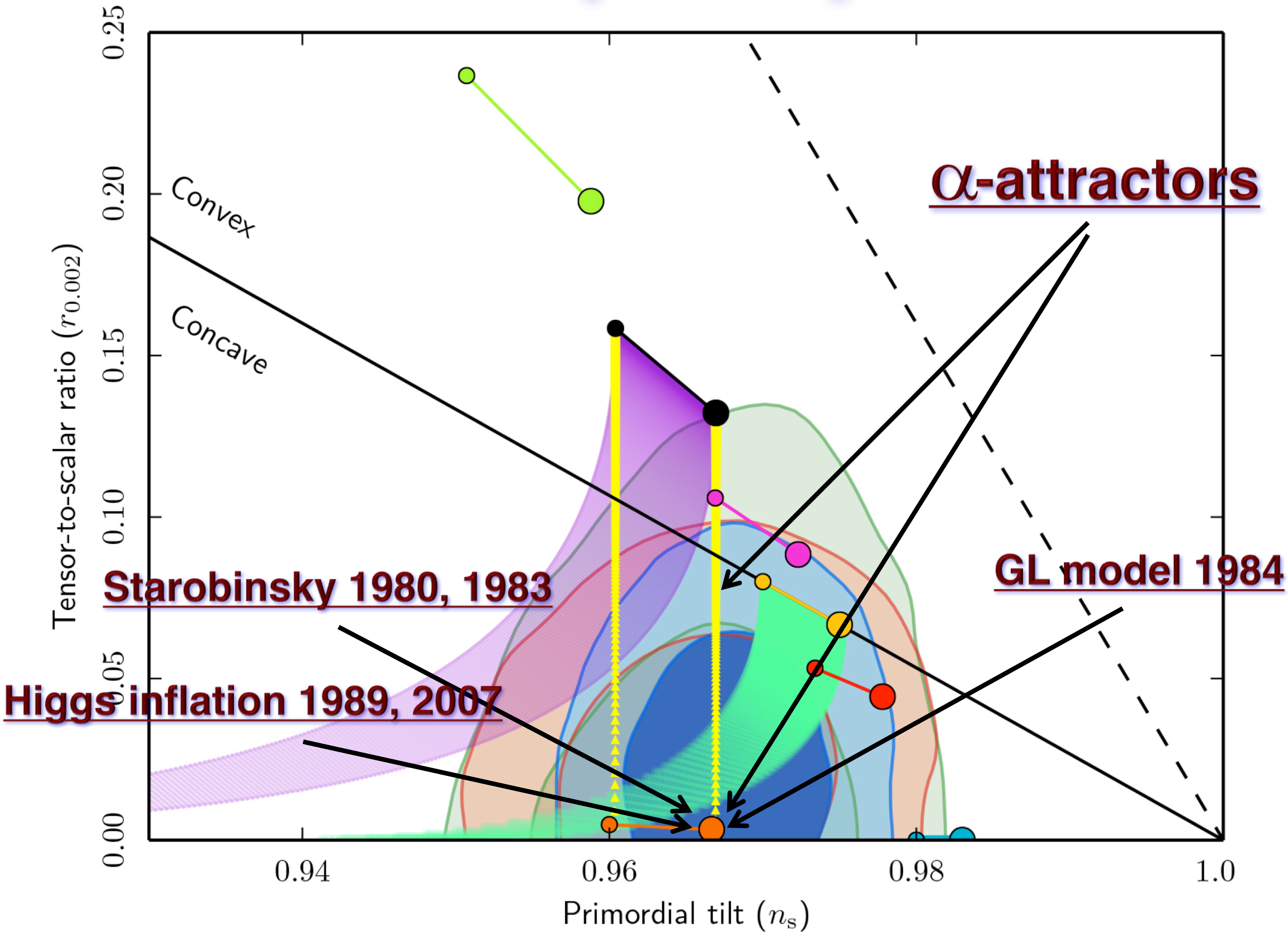
3 parameters: m, a, b

Example: $m = 10^{-5}$, $a = 0.12$, $b = 0.29$



But the best fit is provided by models with plateau potentials

The most natural fit is provided by models with plateau potentials



What is the meaning of α-attractors?

Kallosh, AL 2013; Ferrara, Kallosh, AL, Porrati, 2013;
Kallosh, AL, Roest 2013; Galante, Kallosh, AL, Roest 2014

Start with the simplest chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - \frac{1}{2}m^2\phi^2$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

General chaotic inflation model

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\phi^2 - V(\phi)$$

Modify its kinetic term

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{(1-\underline{\frac{\phi^2}{6\alpha}})^2} - V(\phi)$$

Switch to canonical variables $\phi = \sqrt{6\alpha}\tanh\frac{\varphi}{\sqrt{6\alpha}}$

The potential becomes

$$V = V(\tanh\frac{\varphi}{\sqrt{6\alpha}})$$

This is a **plateau potential** for any nonsingular $V(\phi)$

The essence of α-attractors

Galante, Kallosh, AL, Roest 1412.3797

$$\frac{1}{2}R - \frac{3}{4}\alpha\left(\frac{\partial t}{t}\right)^2 - V(t)$$

Suppose inflation takes place near the pole at t = 0, and V(0) > 0, V'(0) >0, and V has a minimum nearby. Then in canonical variables

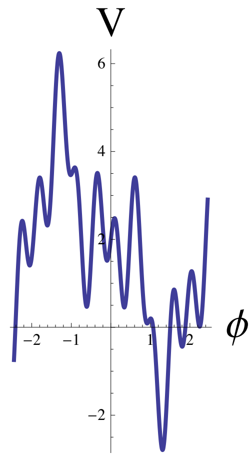
$$\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + \ldots)$$

Then in the leading approximation in 1/N, for any non-singular V

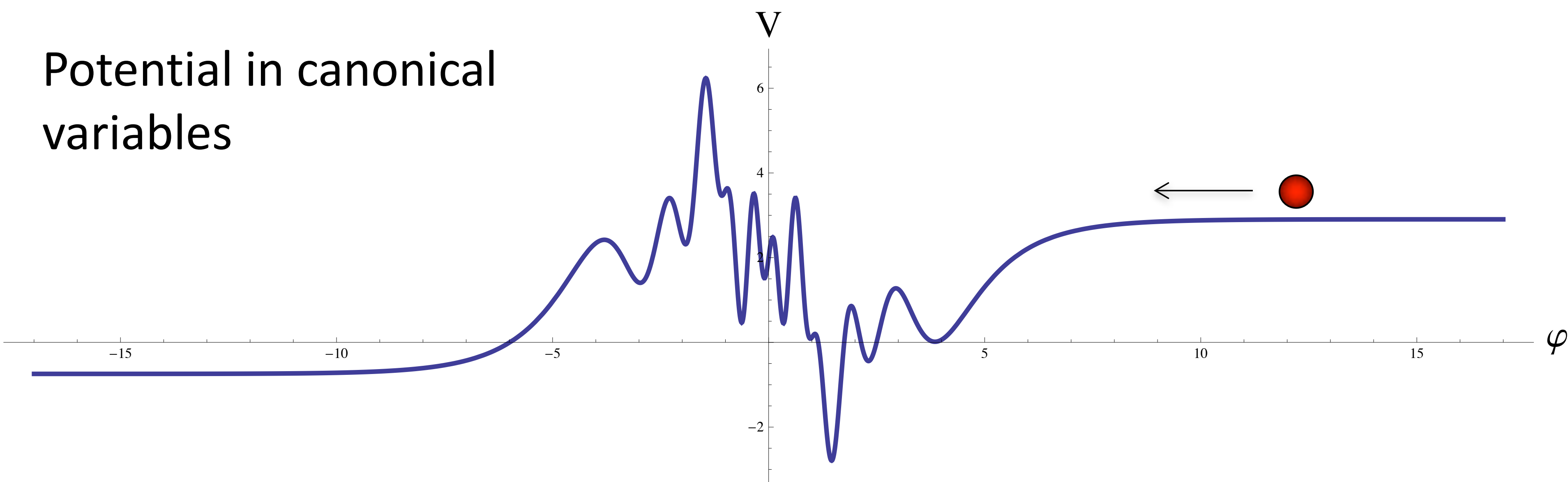
$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha\frac{12}{N^2}$$

Stretching and flattening of the potential is similar to stretching of inhomogeneities during inflation

Potential in the original variables of the conformal theory



Potential in canonical variables



Inflation **in** the landscape is facilitated by inflation **of** the landscape

The scale of inflation in α-attractors

These are models of **large field** inflation. However, **the scale M of inflation in these models can be small:**

$$\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - V_0(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + ...)$$

$$M = \sqrt{\frac{3\alpha}{2}} \lesssim 1 \quad \text{for} \quad \alpha \lesssim \frac{2}{3}$$

In the recent generation of inflationary models based on maximal supergravity, one can have M < 1 for several different scalar field, and then merge different inflaton fields to get

$$3\alpha = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$$

$$10^{-3} \lesssim r \lesssim 10^{-2}$$

see a discussion below and the talk by Kallosh

The essence of α-attractors

Galante, Kallosh, AL, Roest 1412.3797

THE BASIC RULE:

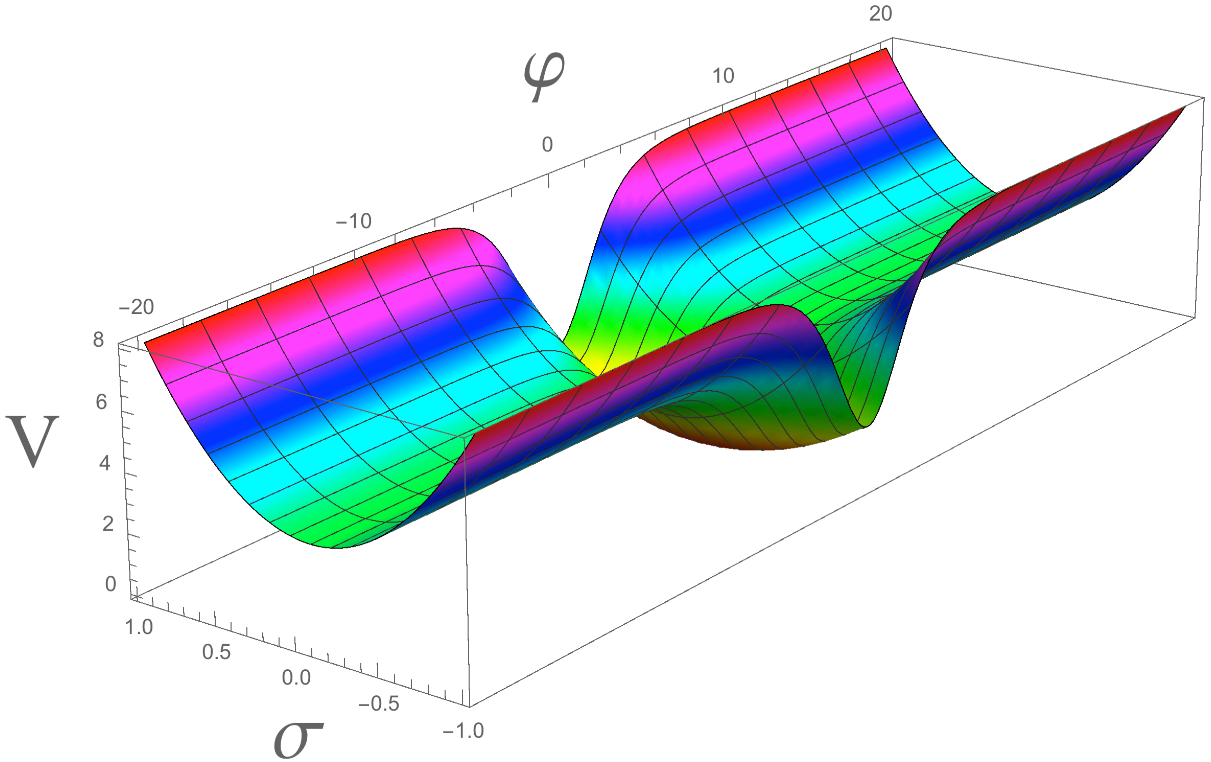
For a broad class of cosmological attractors, the spectral index **n_s** depends mostly on the order of the pole in the kinetic term, while the tensor-to-scalar ratio **r** depends on the residue. **Choice of the potential almost does not matter**, as long as it is non-singular at the pole of the kinetic term. Geometry of the moduli space, not the potential, determines much of the answer.

An often discussed concern about higher order corrections to the potential for large field inflation does not apply to these models.

What happens if we add other fields?

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}M^2\sigma^2 - \frac{g^2}{2}\phi^2\sigma^2$$

Potential in canonical variables has a plateau at large values of the inflaton field, and it is quadratic with respect to σ.



Asymptotic freedom of the inflaton

Kallosh, AL, 1604.00444

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}(\partial\sigma)^2 - V(\phi,\sigma)$$

Couplings of the canonically normalized fields are determined by derivatives such as

$$\lambda_{\varphi,\sigma,\sigma} = \partial_\varphi\partial_\sigma^2 V(\phi,\sigma) = 2\sqrt{\frac{2}{3\alpha}}\,\underline{e^{-\sqrt{\frac{2}{3\alpha}}\varphi}}\,\partial_\phi\partial_\sigma^2 V(\phi,\sigma)|_{\phi\to\sqrt{6\alpha}}$$

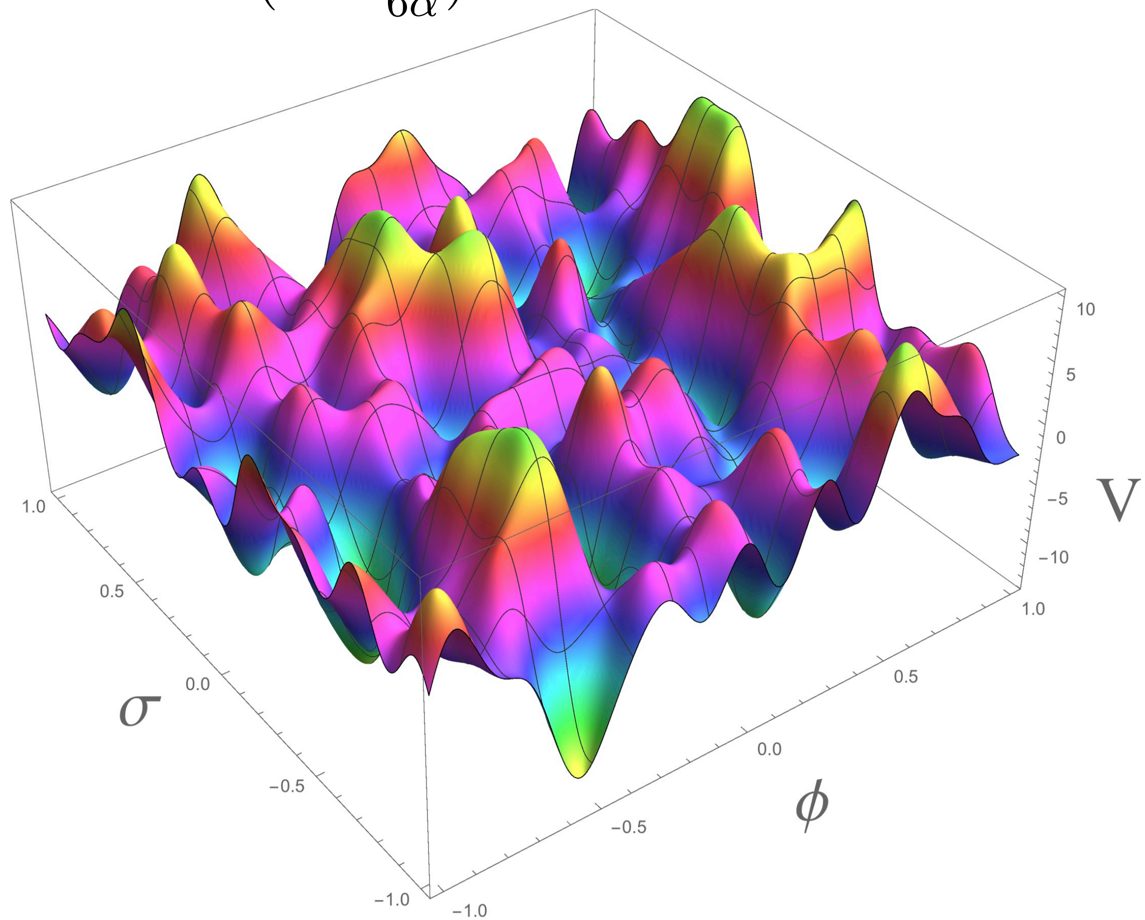
As a result, **couplings of the inflaton field to all other fields are exponentially suppressed during inflation**. The asymptotic shape of the plateau potential of the inflaton is **not** affected by quantum corrections.

Inflation in Random Potentials and Cosmological Attractors

AL 1612.04505

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - \frac{(\partial_\mu \sigma)^2}{2} - V(\phi,\sigma)$$

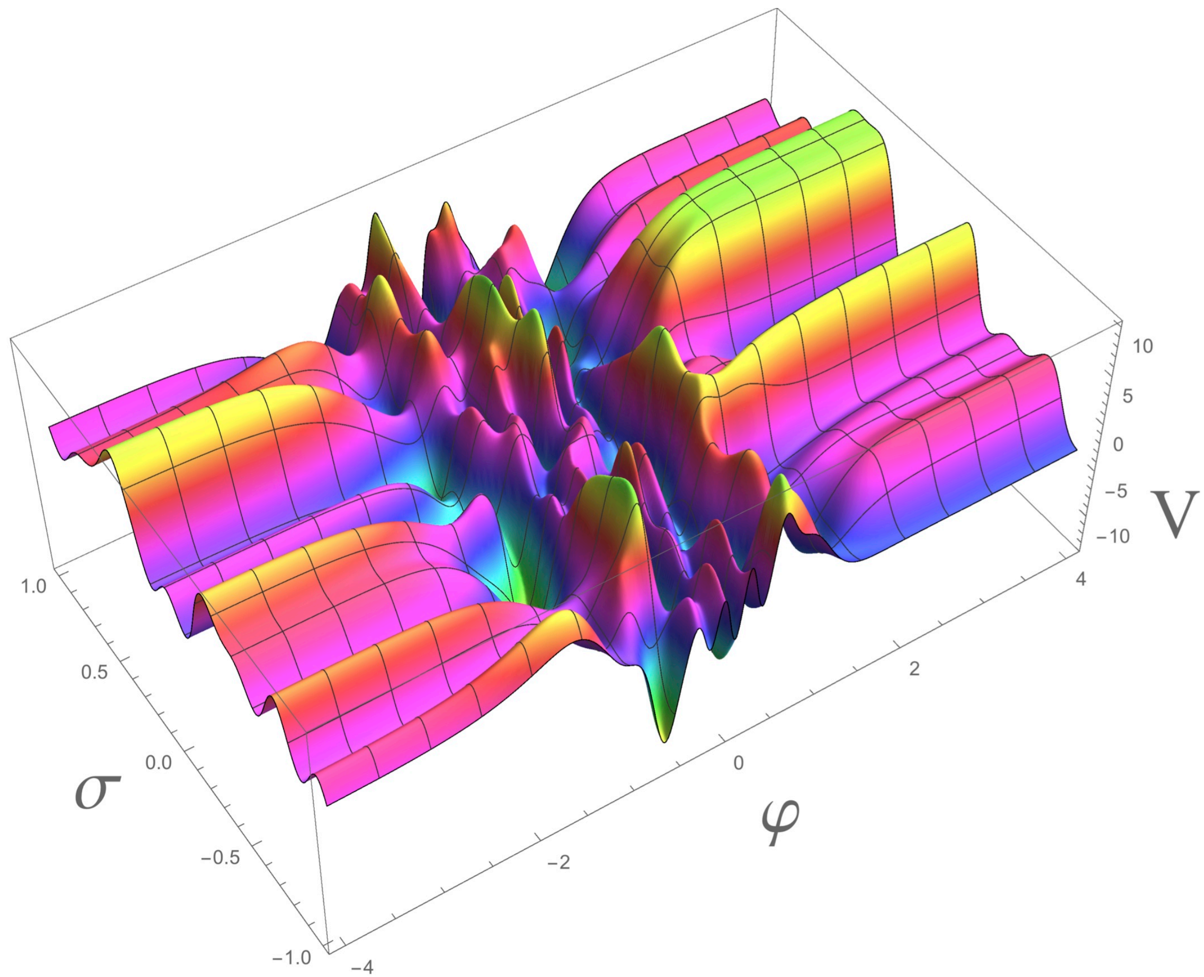
Can we have inflation in such potentials?



In terms of canonical fields φ with the kinetic term $\frac{(\partial_\mu \varphi)^2}{2}$, the potential is

$$V(\varphi, \sigma) = V(\sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}, \sigma)$$

Many inflationary valleys representing alpha-attractors



Double Attractors

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2(1-\frac{\phi^2}{6\alpha})^2} - \frac{(\partial_\mu \sigma)^2}{2(1-\frac{\sigma^2}{6\beta})^2} - V(\phi,\sigma)$$

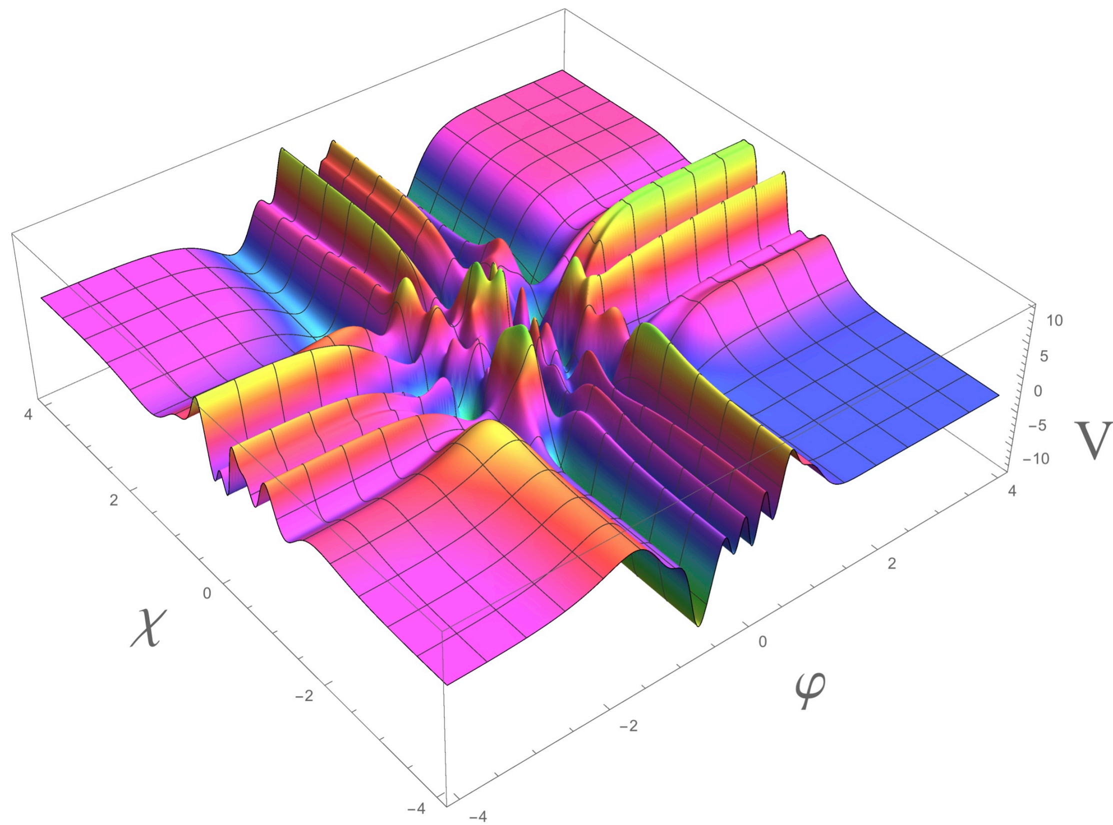
In terms of canonical fields $V(\varphi,\chi) = V(\sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}, \sqrt{6\beta}\,\tanh\frac{\chi}{\sqrt{6\beta}})$

Two families of attractors, related to the valleys along the two different inflaton directions:

$$1-n_s \approx \frac{2}{N}\,, \qquad r \approx \frac{12\alpha}{N^2}\,.$$

or

$$1-n_s \approx \frac{2}{N}\,, \qquad r \approx \frac{12\beta}{N^2}\,.$$



Up to now, we discussed bosonic models of cosmological attractors, but most of them have supergravity versions.

Construction of models of SUGRA inflation is especially simple now, using the new methods described in the talk by Kallosh. These methods allow to provide SUGRA versions of any bosonic inflationary potential, and describe arbitrary values of the cosmological constant and the gravitino mass.

What if interaction between attractors is very strong?

Kallosh, AL, Wrase, Yamada 1704.04829, Kallosh, AL, Roest, Yamada 1705.09247

We will study it in SUGRA, by methods described in the talk by Kallosh

$$\mathcal{G} = \log W_0^2 - \frac{1}{2}\sum_{i=1}^{2}\log\frac{(1-Z_i\overline{Z}_i)^2}{(1-Z_i^2)(1-\overline{Z}_i^2)} + S + \overline{S} + g_{S\overline{S}}S\overline{S}$$

where

$$g^{S\overline{S}} = W_0^{-2}\,V + 3$$

and the scalar potential is

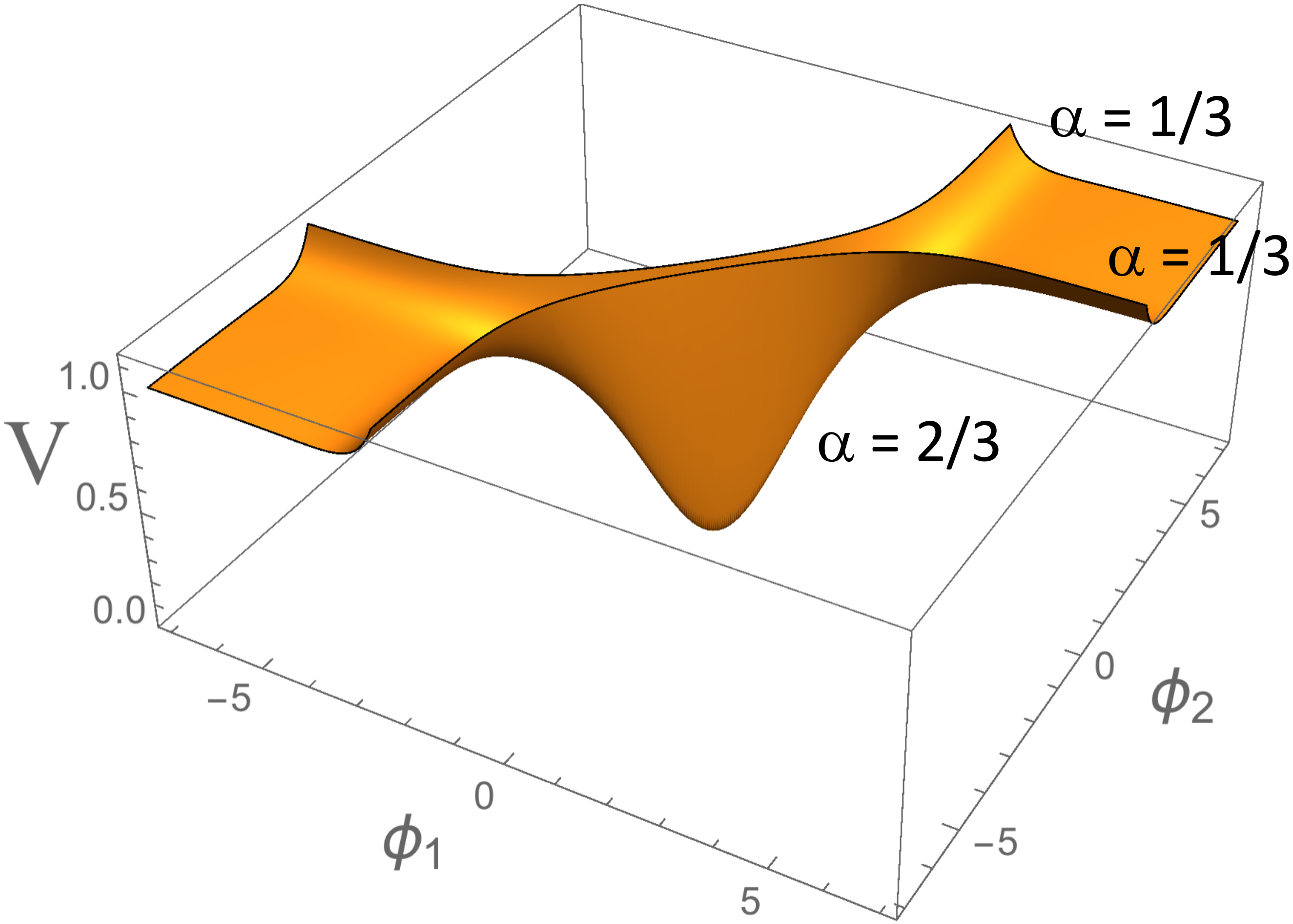
$$\mathbf{V} = \Lambda + \frac{m^2}{2}(|Z_1|^2 + |Z_2|^2) + \frac{M^2}{4}\big((Z_1+\overline{Z}_1) - (Z_2+\overline{Z}_2)\big)^2$$

$$Z_i = \tanh\frac{\phi_i + \mathrm{i}\theta_i}{\sqrt{2}}$$

For M >> m, the last term in the potential forces the two inflaton fields to coincide,

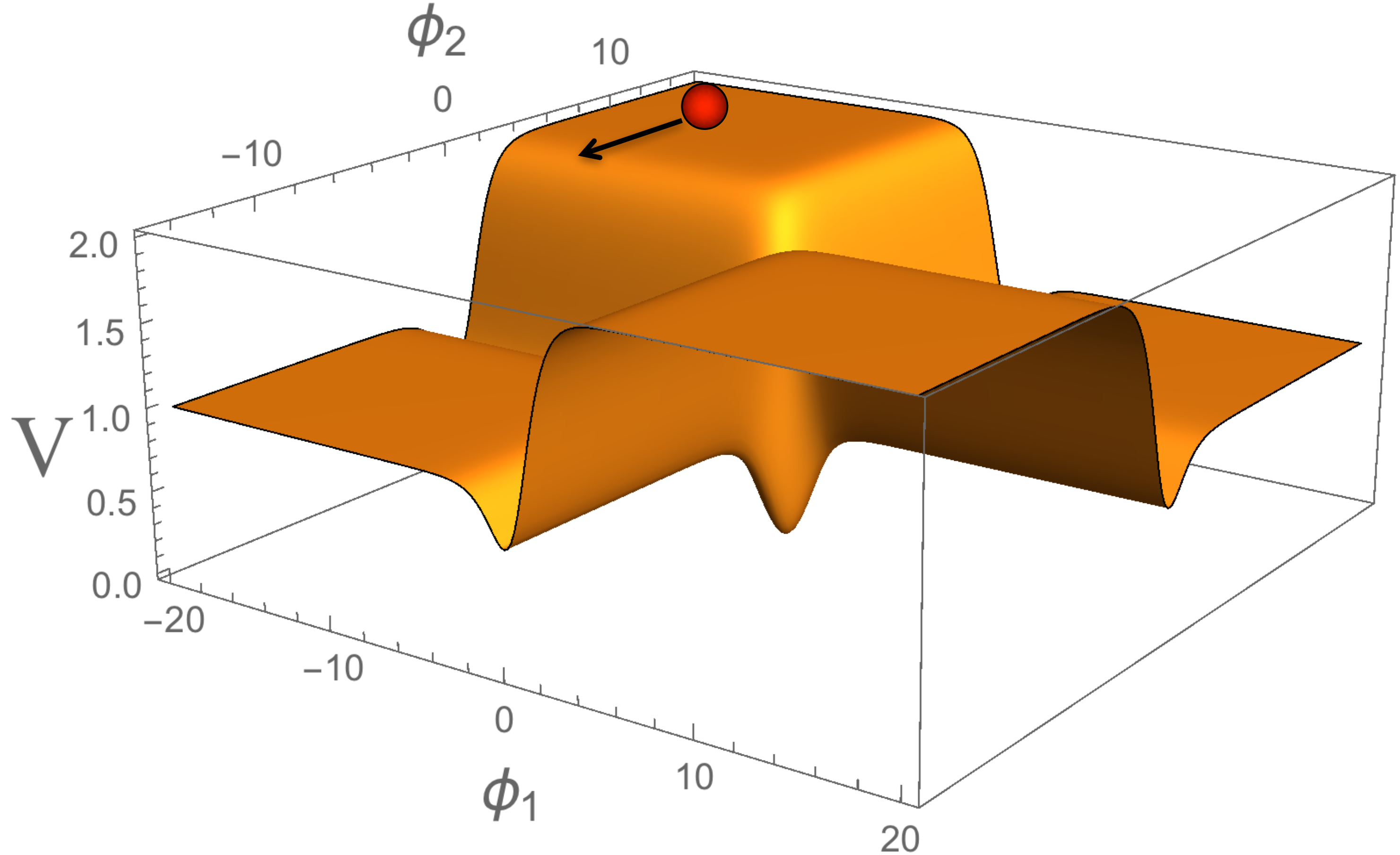
$$\phi_1 = \phi_2$$

Two strongly interacting attractors with $\alpha = 1/3$ merge into one attractor with $\alpha = 2/3$.



This figure shows only the lower part of the potential, cutting the upper part. Now look at the full potential

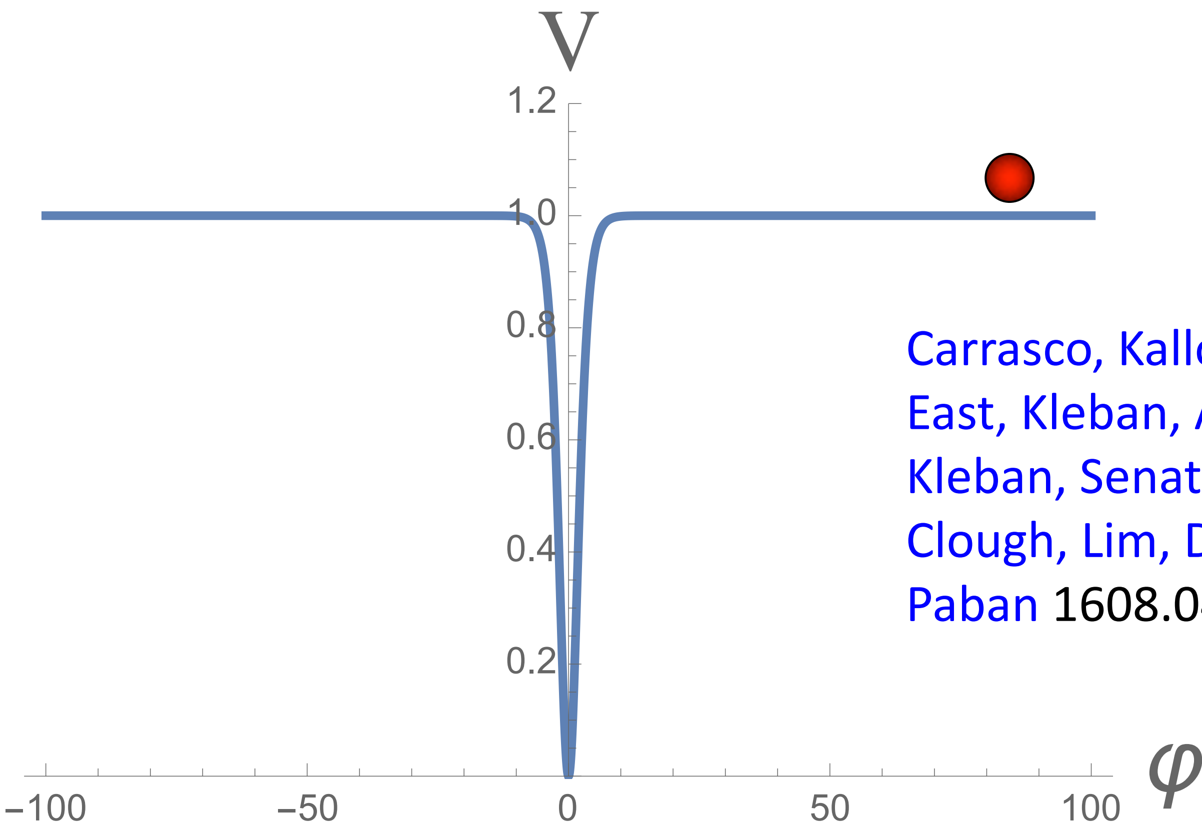
Cascade Inflation



The minimum corresponds to the attractor merger shown at the previous slide. This is where inflation ends. But it begins at the infinitely long upper plateau of height $O(M^2)$.

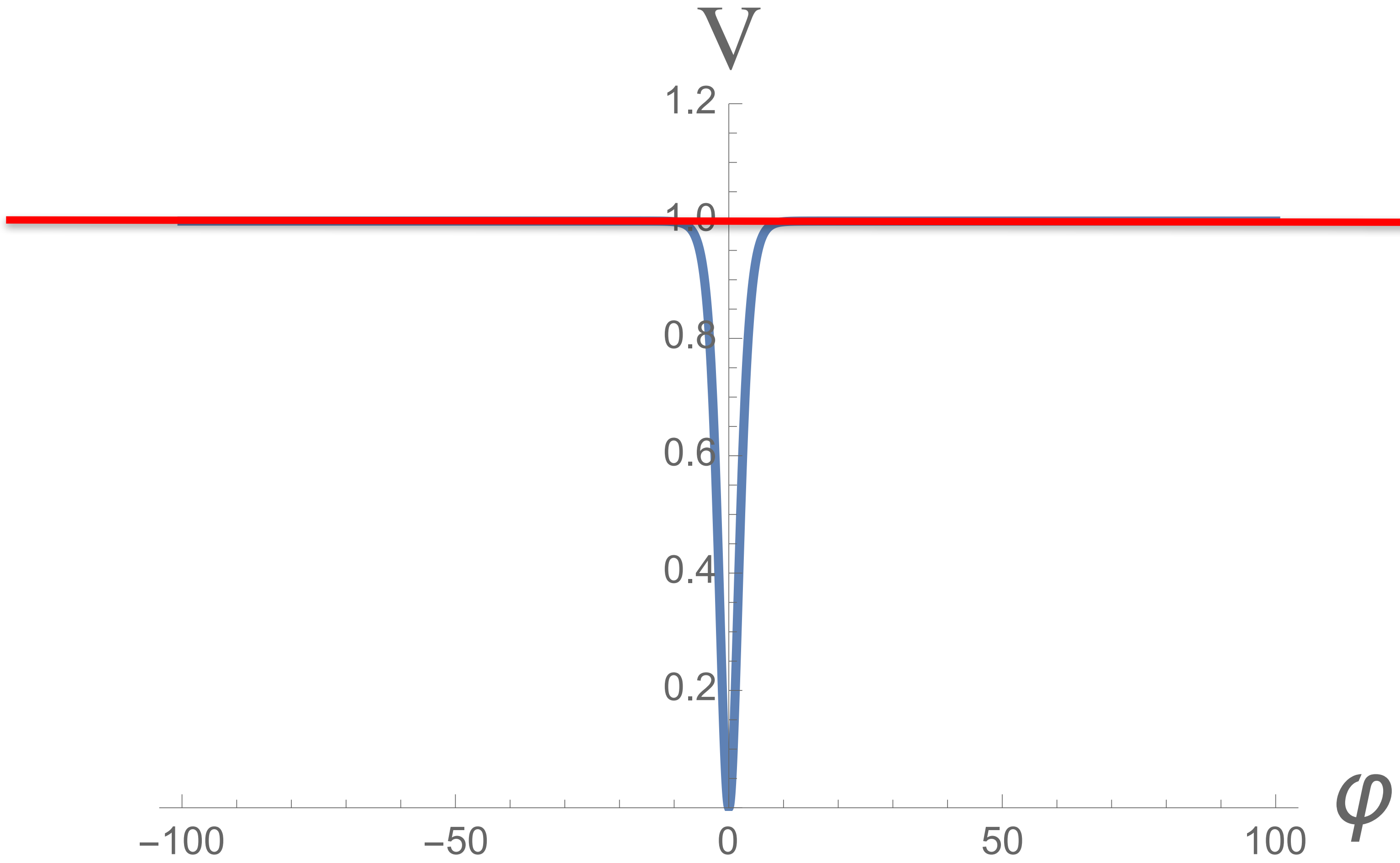
α-attractors: Initial conditions for inflation

At large fields, the α-attractor potential remains 10 orders of magnitude below Planck density. Can we have inflation with natural initial conditions here? The same question applies for the Starobinsky model and Higgs inflation.



Carrasco, Kallosh, AL 1506.00936
East, Kleban, AL, Senatore 1511.05143
Kleban, Senatore 1602.03520
Clough, Lim, DiNunno, Fischler, Flauger, Paban 1608.04408

To explain the main idea, note that this potential coincides with the cosmological constant almost everywhere.



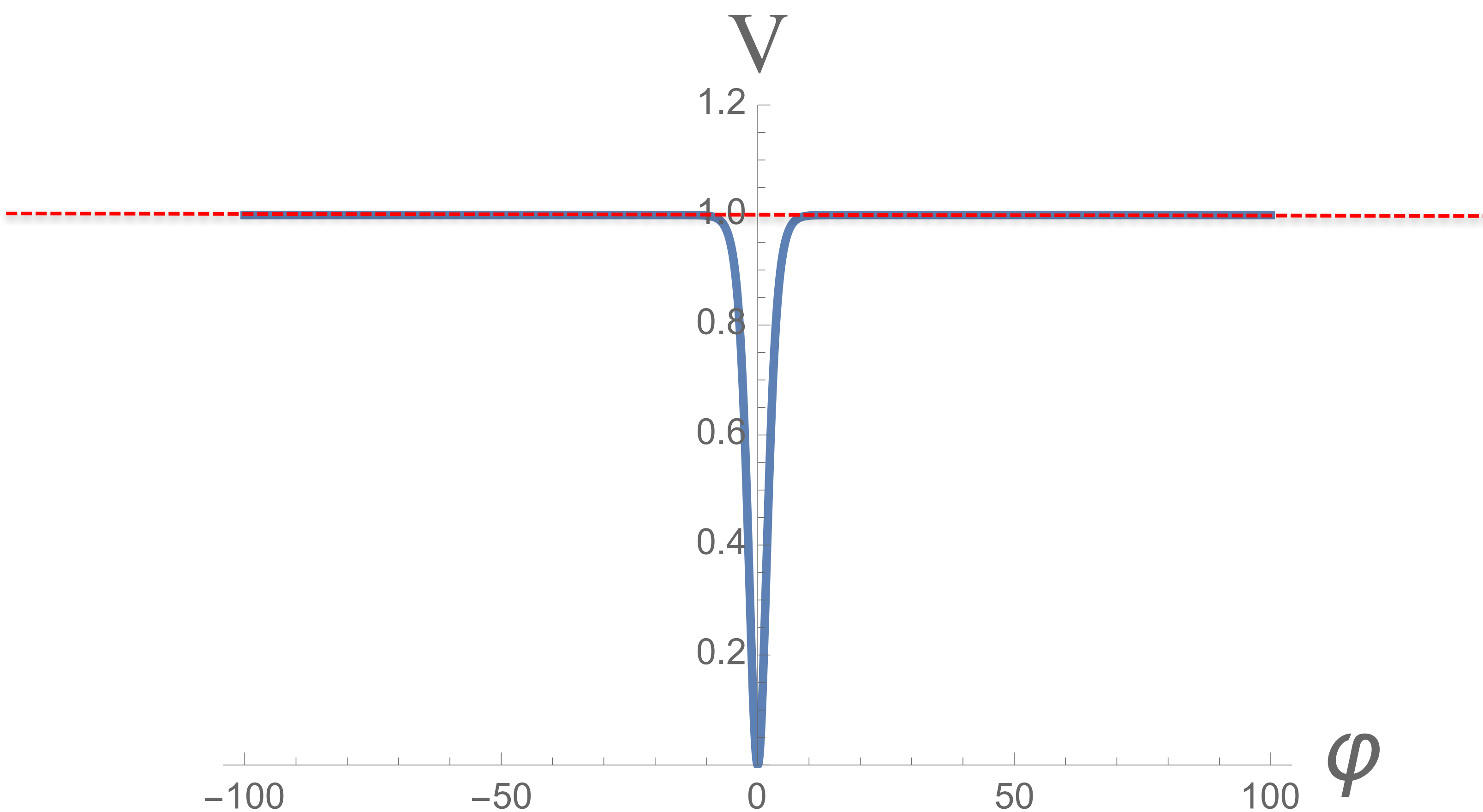
For the universe with a cosmological constant, the problem of initial conditions is nearly trivial.

Start at the Planck density, in an expanding universe dominated by inhomogeneities. The energy density of matter is diluted by the cosmological expansion as $1/t^2$. **What could prevent the exponential expansion of the universe which becomes dominated by the cosmological constant Λ after the time $t = \Lambda^{-1/2}$?**

Inflation does NOT happen in the universe with the cosmological constant $\Lambda = 10^{-10}$ only if the whole universe collapses within 10^{-28} seconds after its birth.

In other words, only instant global collapse could allow the universe to avoid exponential expansion dominated by the cosmological constant. If the universe does not instantly collapse, it inflates.

This optimistic conclusion related to the cosmological constant applies to α-attractors as well, because their potential coincides with the cosmological constant almost everywhere.



These arguments are valid for general large field inflationary models as well. Recently they have been confirmed by the same methods of numerical GR as the ones used in simulations of BH evolution and merger. The simulations show how BHs are produced from large super-horizon initial inhomogeneities, while the rest of the universe enters the stage of inflation.

East, Kleban, AL, Senatore 1511.05143

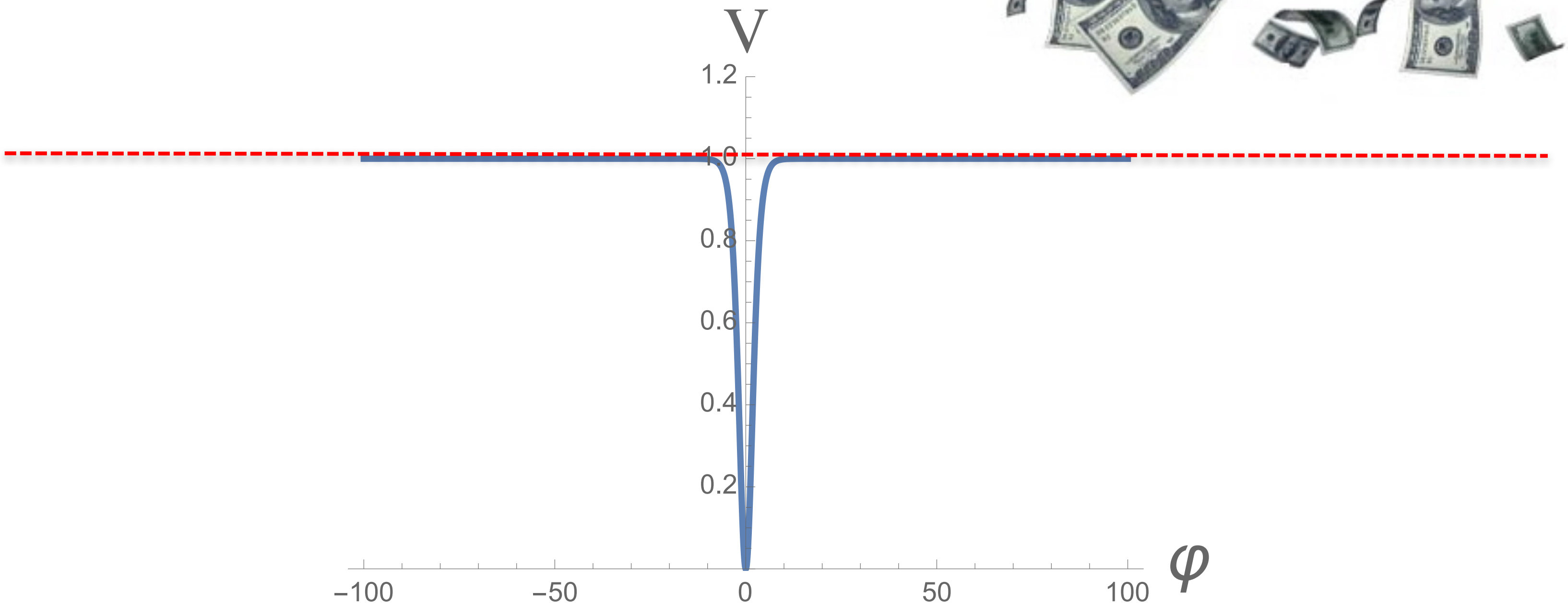
These results obtained by sophisticated calculations have a very simple interpretation in terms of inflation in economy.

It is well known that dropping money from a helicopter may lead to inflation, unless all money miss the target

A simple interpretation of our results

suggested by Starobinsky

Money dropped from a helicopter have no choice but lend on an infinitely long plateau. This inevitably leads to inflation



Adding other fields simplifies it even further

$$\frac{1}{\sqrt{-g}}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\frac{(\partial\phi)^2}{(1-\frac{\phi^2}{6\alpha})^2} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}M^2\sigma^2 - \frac{g^2}{2}\phi^2\sigma^2$$

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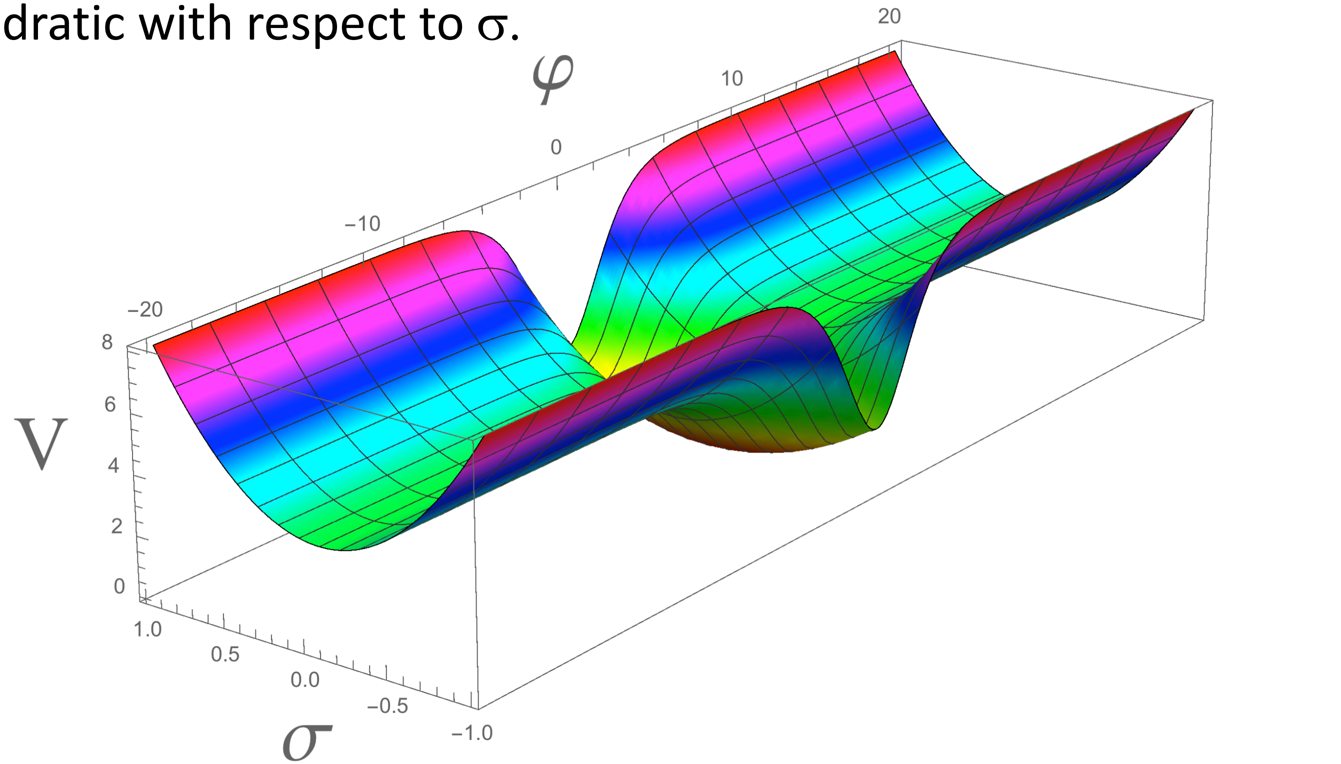
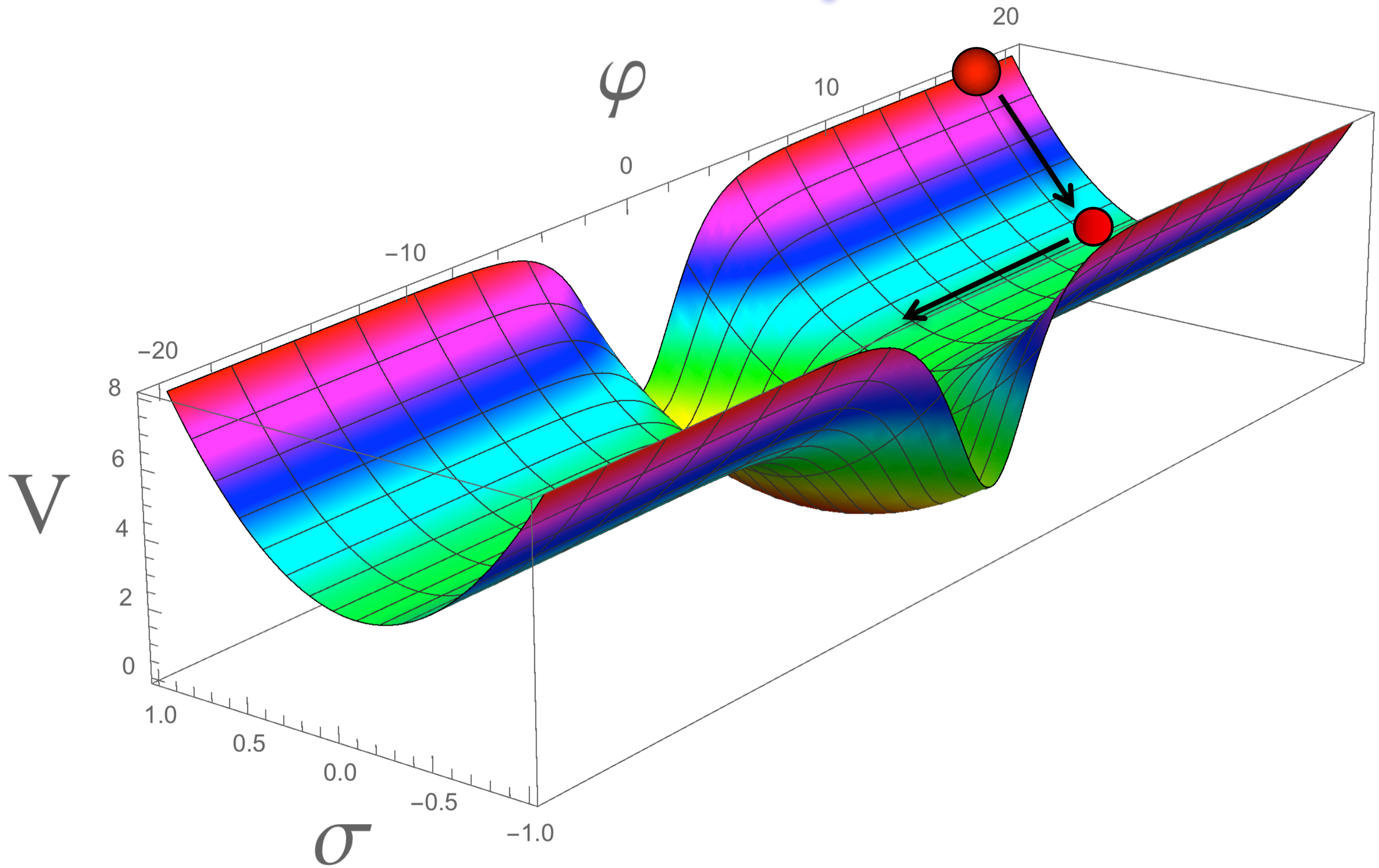


Initial conditions for plateau inflation



Chaotic inflation with a parabolic potential goes first, starting at nearly Planckian density. When the field down, the plateau inflation begins.

No problem with initial conditions

Conclusions:

Cosmological attractors allow to reconsider many usual assumptions with respect to the large field models, resolving some of their often discussed problems and offering new solutions to the problem of initial conditions in inflationary cosmology.