A Cosmological Open Quantum System

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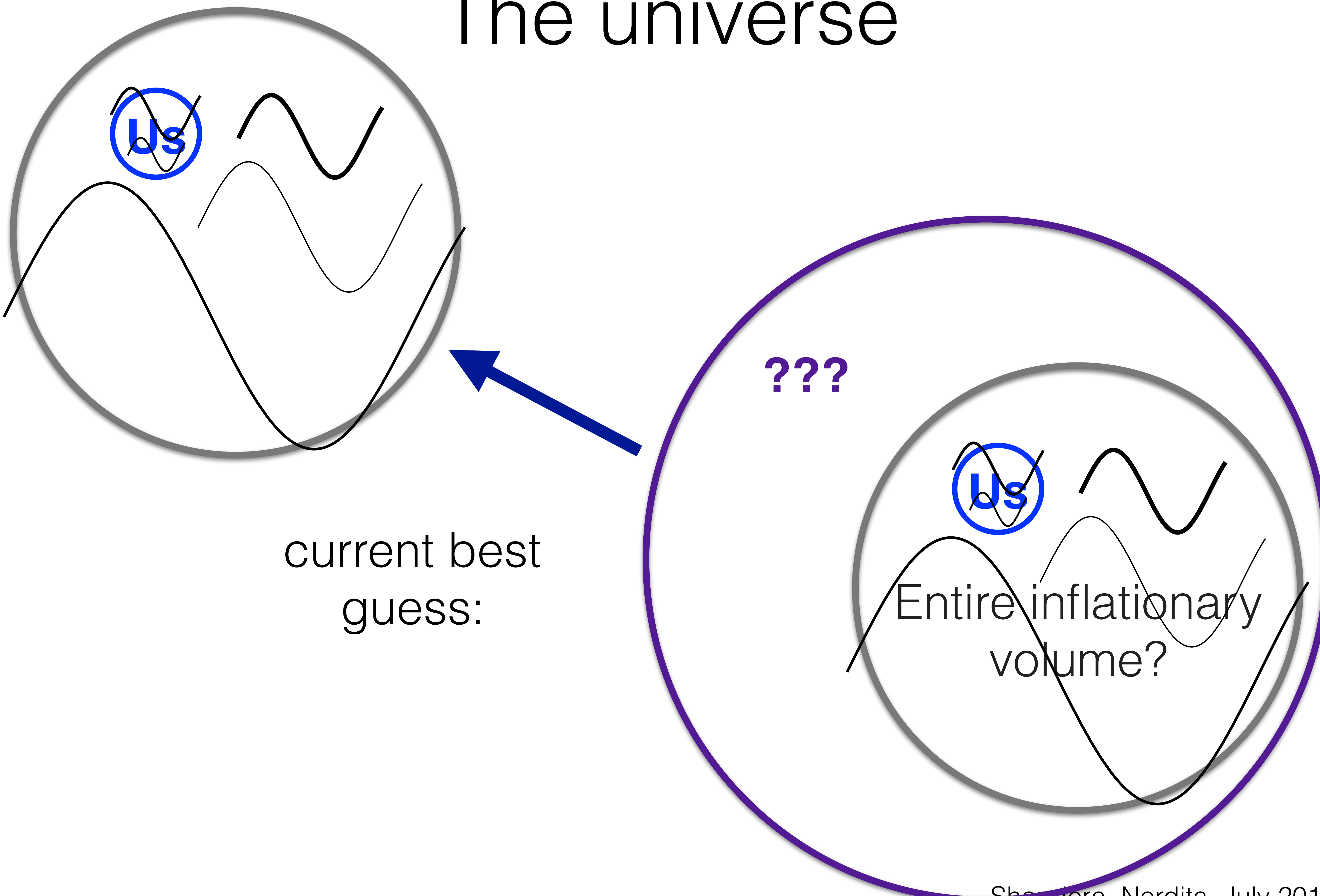
Inflation and the Cosmic Microwave Background
Nordita, 18 July 2017

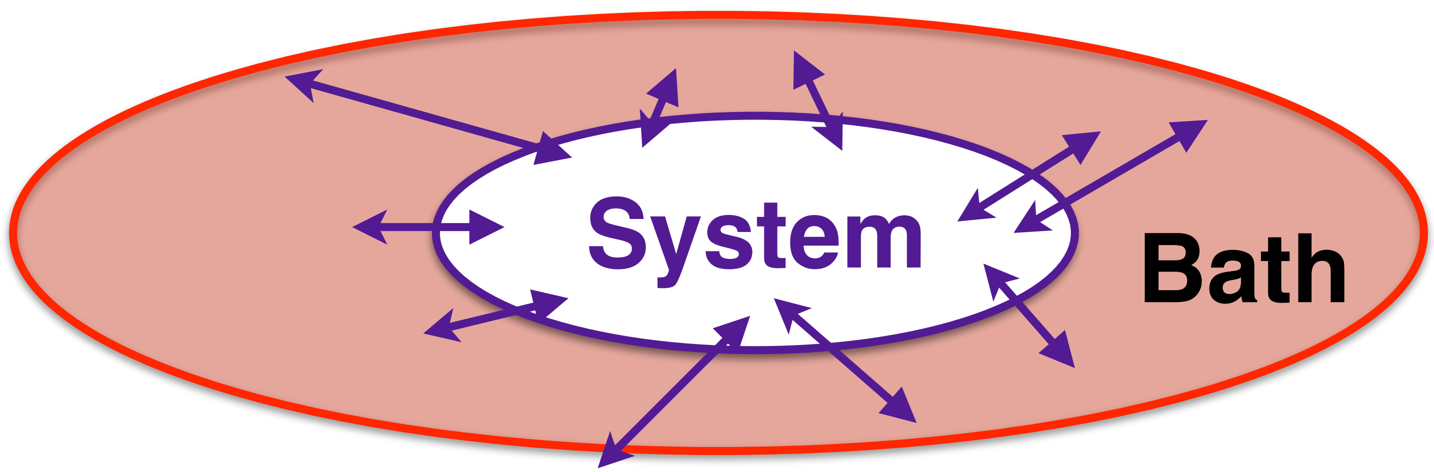
The universe



current best guess:

What are the dynamics of *only* the observable modes?

In some cases: an open quantum system



Open system: exchanges information with the bath

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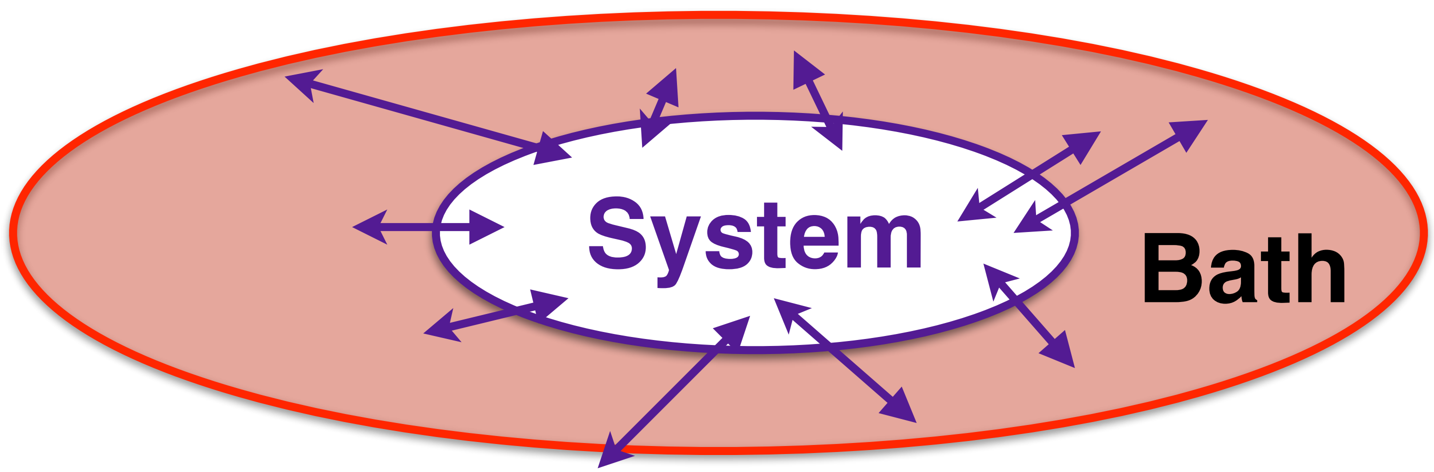
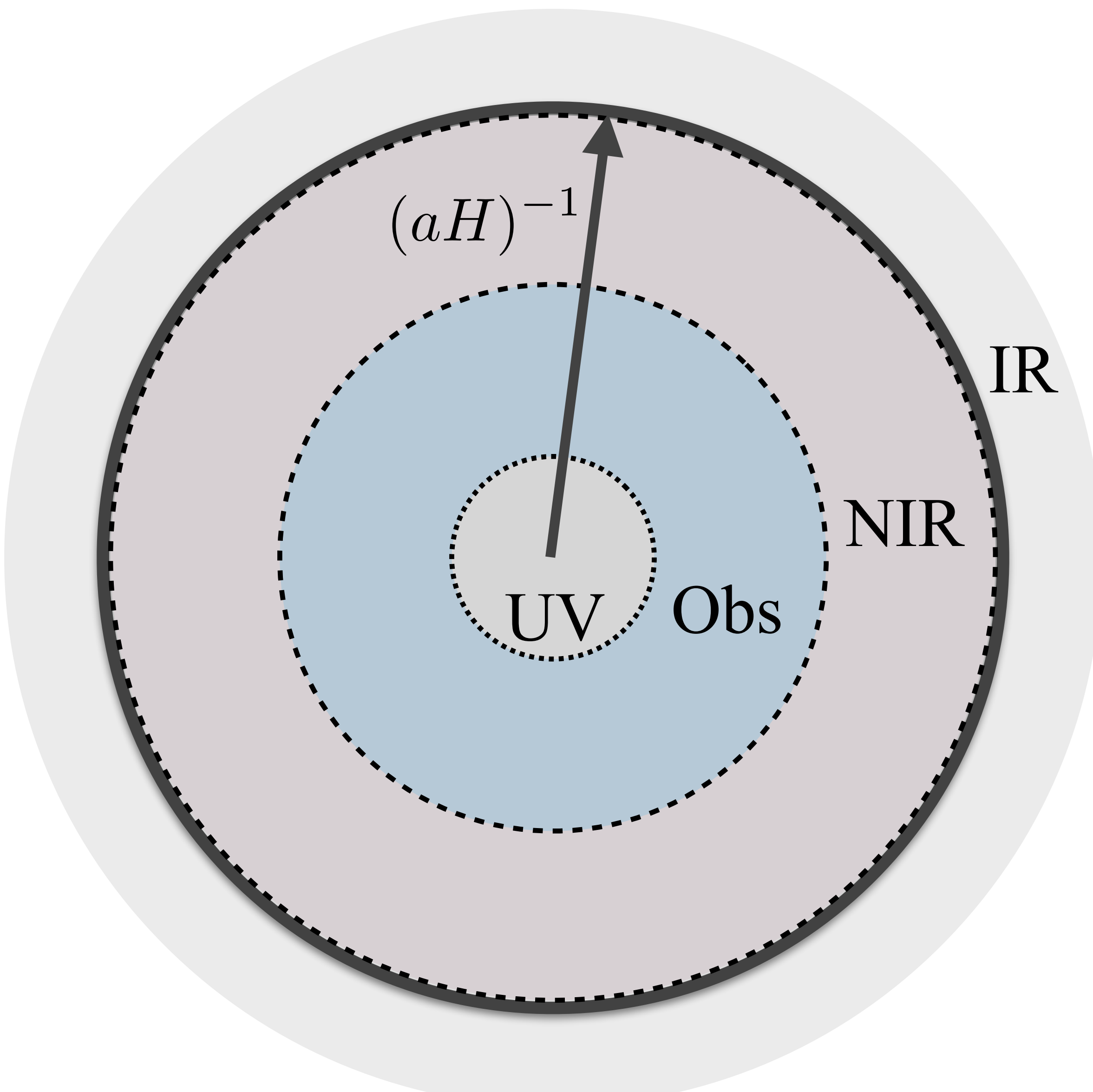


Open system: exchanges information with the bath

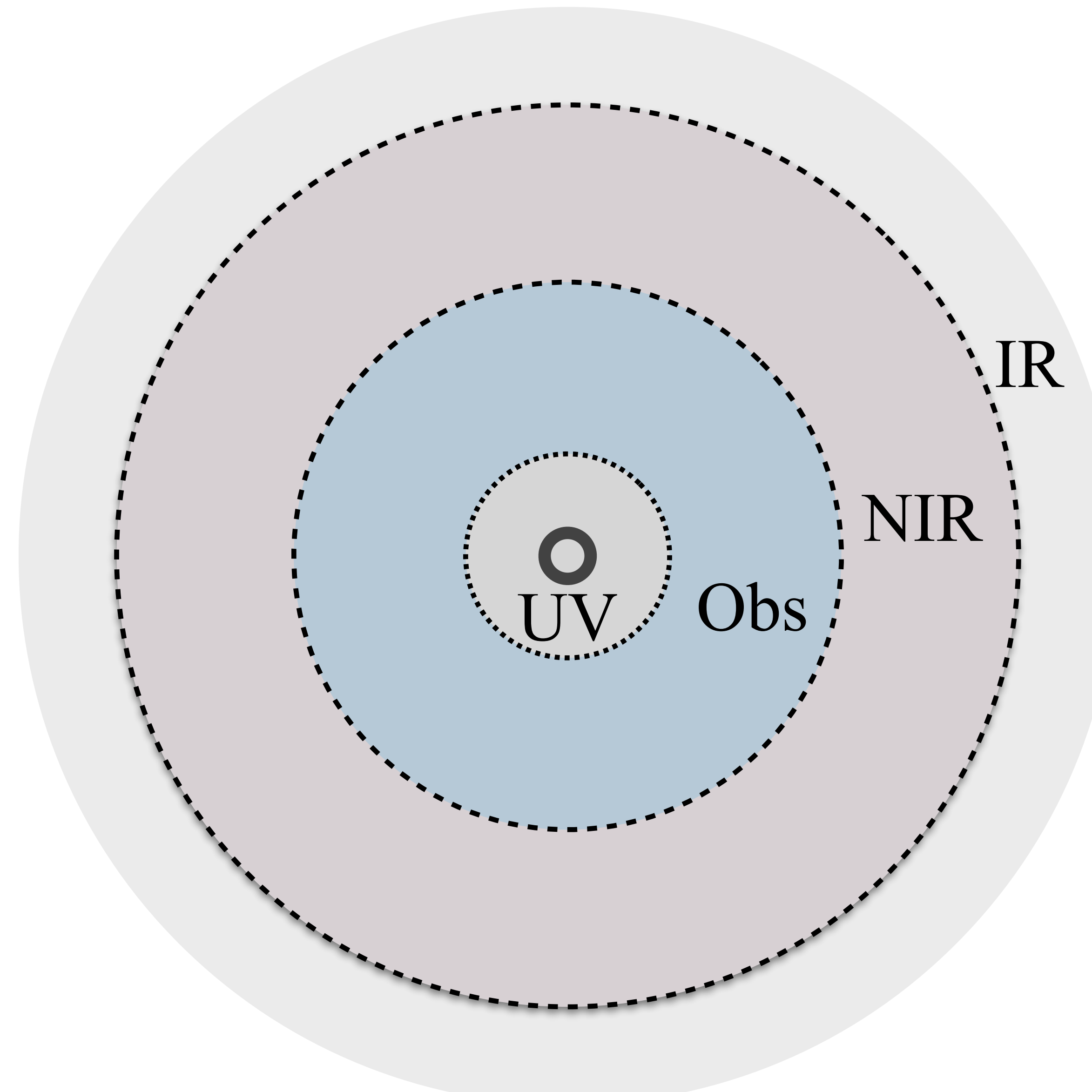
(not just lost to the the bath)

Consider bands of comoving momenta



η_0

$(aH)^{-1} = k_{\rm IR}^{-1}$



$\eta \gg \eta_0$

$(aH)^{-1} \ll k_{\rm UV}^{-1}$

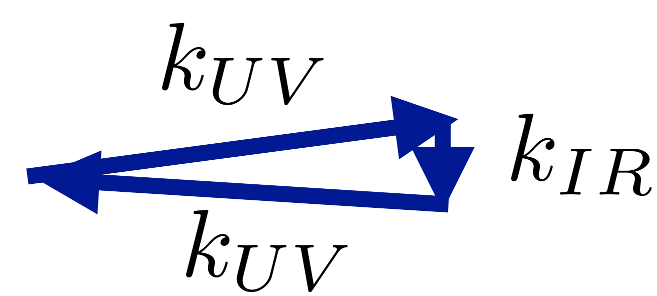
Are the observable modes an open system?

Two classes of primordial models

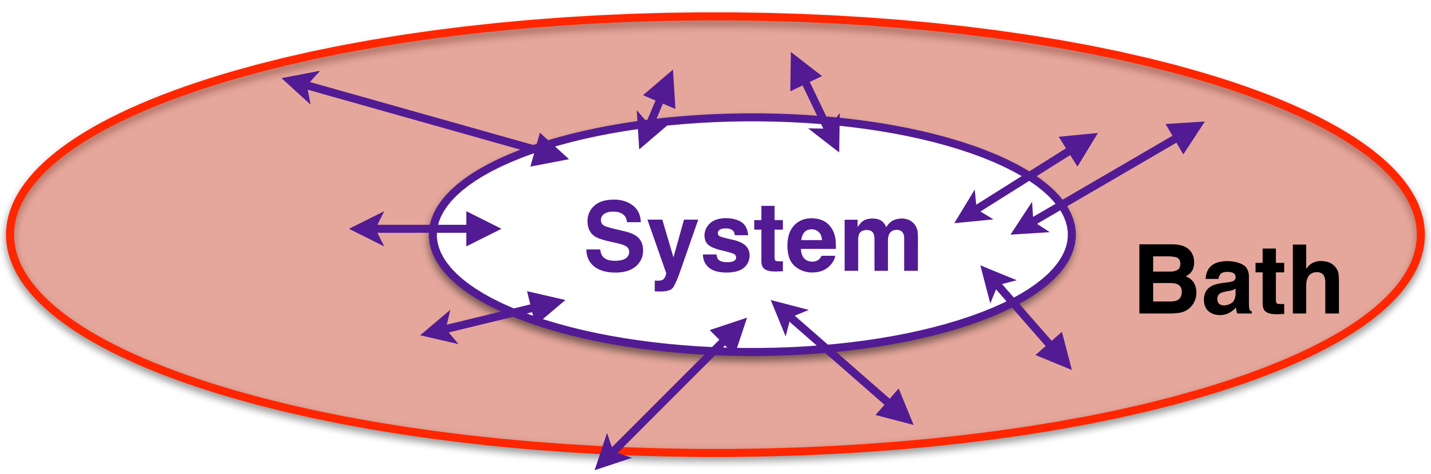
(1) No significant coupling between modes of different wavelengths: “single-clock inflation”

(2) Long-short mode-coupling possible: additional light scalars (quasi-single field, curvaton), any dynamics where ζ evolves outside the horizon

Bispectrum: squeezed triangles



An inherently gravitational open quantum system



$$k_{\rm FIR} < (aH)|_{\eta=\eta_0} < k_{\rm NIR} < k_{\rm obs} < k_{\rm UV}$$

“bath” “system”

Previous work, and usual Effective Field Theory, puts UV physics as the ‘unknown’

Boyanovsky
Burgess et al;

The Plan

Look at a single interaction term:

$$S_3 = \int d^3x\, d\eta\, a^4 \frac{3\epsilon(c_s^2 - 1)}{c_s^2} \zeta \dot{\zeta}^2$$

But two different dynamics:

slow-roll vs "non-attractor"

Kinney 2005

Write down the equation for the evolution of the system

$$\frac{d\rho}{d\eta} = \frac{1}{i\hbar}[\hat{H}, \rho] + \hat{L}_1^\dagger(\eta)\hat{L}_2(\eta)\rho + \ldots$$

Usual story at quadratic order:

$$\chi = z\zeta = \frac{\sqrt{2}\epsilon a}{c_s}\zeta$$

$$\hat{H} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[c_s k\left(\hat{c}_{\vec{k}}\hat{c}^\dagger_{\vec{k}} + \hat{c}_{-\vec{k}}\hat{c}^\dagger_{-\vec{k}}\right) - i\frac{z'}{z}\left(\hat{c}_{\vec{k}}\hat{c}_{-\vec{k}} - \hat{c}^\dagger_{\vec{k}}\hat{c}^\dagger_{-\vec{k}}\right)\right]$$

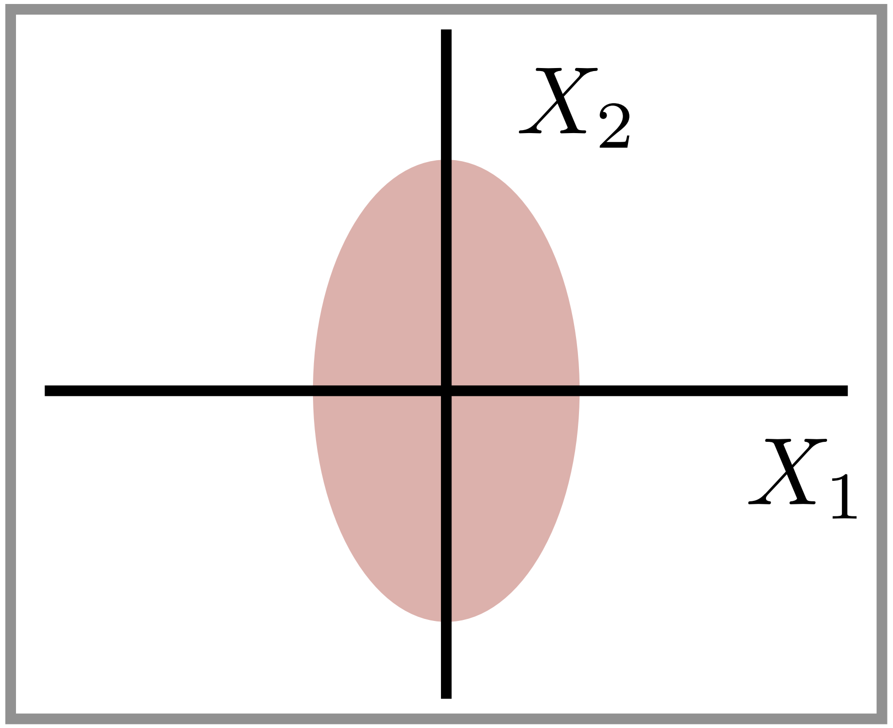
"Two-mode squeezing"

Grishchuk et al 1992;
Albrecht et al 1994;

Two-mode squeezing

$$\langle(\Delta\hat{X}_1)^2\rangle_{\text{squeezed}} = \frac{1}{4}e^{-2\xi}$$

$$\langle(\Delta\hat{X}_2)^2\rangle_{\text{squeezed}} = \frac{1}{4}e^{2\xi}$$



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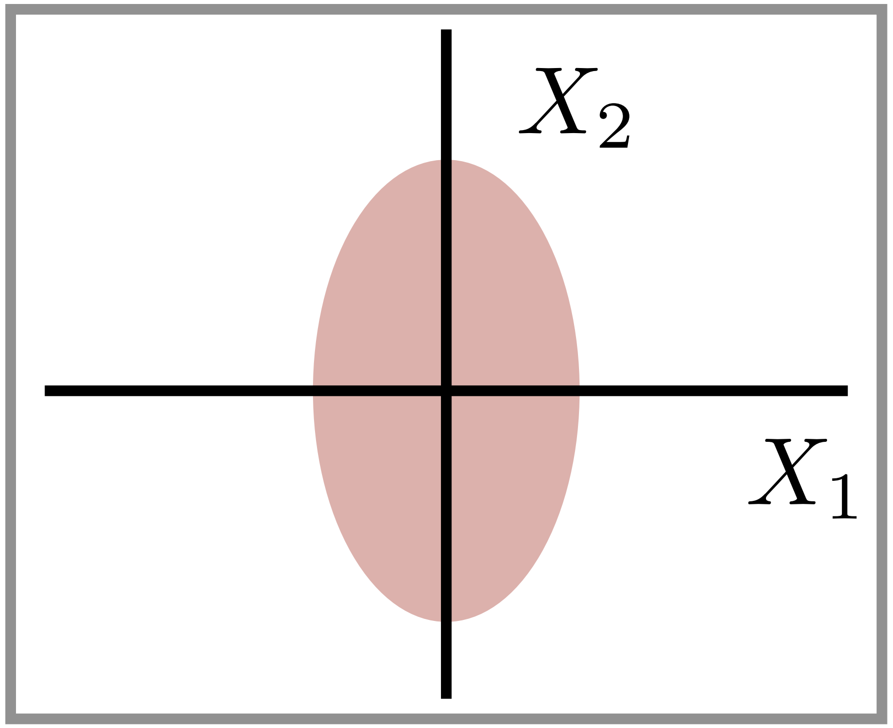

Squeezing is in quadratures that depend on both modes:

(for example)

$$\hat{X}_1 = \frac{1}{2^{3/2}}\left(\hat{a}_{\vec{k}} + \hat{a}^\dagger_{\vec{k}} + \hat{a}_{-\vec{k}} + \hat{a}^\dagger_{-\vec{k}}\right)$$

$$\hat{X}_2 = \frac{1}{2^{3/2}i}\left(\hat{a}_{\vec{k}} - \hat{a}^\dagger_{\vec{k}} + \hat{a}_{-\vec{k}} - \hat{a}^\dagger_{-\vec{k}}\right)$$

Slow-roll vs non-attractor

$$\hat{H}^{\rm sq.} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[-i\left(\frac{z'}{z}\right)\left(\hat{c}_{\vec{k}}\hat{c}_{-\vec{k}} - \hat{c}^{\dagger}_{\vec{k}}\hat{c}^{\dagger}_{-\vec{k}}\right)\right]$$

slow-roll:

$$\epsilon \approx \text{constant},$$

$$\frac{z'}{z} \approx \frac{a'}{a}$$

non-attractor:

$$\dot{\phi} + 3H\phi = \text{constant}$$

$$\epsilon \propto a^{-6}$$

$$\frac{z'}{z} \approx -2\frac{a'}{a}$$

- Gravitational field is the zero momentum pump
- Squeezing parameters have a different time dependence in two cases

The interaction term

$$\lambda(\eta)\hat{H}_I = \frac{3(c_s^2-1)}{8M_p c_s^2 a\sqrt{\epsilon}} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)$$
$$\times\left[\sqrt{\frac{k_2k_3}{k_1}}\left(\hat{c}^\dagger_{-\vec{k}_1}\hat{c}^\dagger_{-\vec{k}_2}\hat{c}^\dagger_{-\vec{k}_3}+\hat{c}_{\vec{k}_1}\hat{c}^\dagger_{-\vec{k}_2}\hat{c}^\dagger_{-\vec{k}_3}+\ldots\right)+\mathrm{symm}\right]$$

Namjoo et al 2012
Chen et al 2013;

$$\frac{3}{5}f_{\rm NL}^{\rm local} = \frac{3}{4c_s^2}(1+c_s^2)$$

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Time-dependence differs in slow-roll vs non-attractor

Namjoo et al 2012
Chen et al 2013;

$$\frac{3}{5}f_{\mathrm{NL}}^{\mathrm{local}} = \frac{3}{4c_s^2}(1+c_s^2)$$

The evolution equation

Recall the usual unitary evolution from quantum mechanics

$$\hat{H}|\psi\rangle = i\hbar \frac{d}{dt}|\psi\rangle$$

Now consider the matrix $\rho = |\psi\rangle\langle\psi|$

It evolves as

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[\hat{H}, \rho]$$

The density matrix

If $|\psi\rangle$ is the state of the system,
Then $\rho = |\psi\rangle\langle\psi|$ is the “density matrix”.

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Equal superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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Can also describe mixed states: $\rho = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$

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50% of particles in $|\psi\rangle = |0\rangle$

50% of particles in $|\psi\rangle = |1\rangle$

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Inflationary quadratic evolution

$$\hat{U}_0(\eta,\eta_0)|0_{\vec{k}},0_{-\vec{k}}\rangle = e^{-i\int_{\eta_0}^{\eta}\hat{H}_0(k,\eta_1)d\eta_1}|0_{\vec{k}},0_{-\vec{k}}\rangle$$

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$$= \hat{S}_k(\eta)\hat{R}_k(\eta)|0_{\vec{k}},0_{-\vec{k}}\rangle$$

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$$= \hat{S}_k(\eta)\hat{R}_k(\eta)|0_{\vec{k}},0_{-\vec{k}}\rangle$$

$$= \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{-2in\phi_k}\tanh^n r_k|n_{\vec{k}},n_{-\vec{k}}\rangle$$

$$\equiv |SQ(k,\eta)\rangle = \sum_n c_n^{(sq)}(\eta)|n_{\vec{k}},n_{-\vec{k}}\rangle$$

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$r_k(\eta),\ \phi_k(\eta)$ differ in slow-roll vs non-attractor

Describing only the observable modes

The full density matrix:

$$\sigma(\eta_0) = |\overline{NIR}\rangle |\psi_{\rm obs.}(\eta_0)\rangle\langle\psi_{\rm obs.}(\eta_0)| \langle\overline{NIR}|$$

The reduced density matrix:

$$\rho(\eta) = Tr_{NIR}\, \sigma(\eta)$$

$$= \sum_N \langle N|\hat{U}(\eta,\eta_0)|\overline{NIR}\rangle|\psi_{\rm obs.}(\eta_0)\rangle\langle\psi_{\rm obs.}(\eta_0)|\langle\overline{NIR}|\hat{U}^\dagger(\eta,\eta_0)|N\rangle$$

The evolution:

Evolution of the full system:

$$\sigma(\eta) = \hat{U}(\eta,\eta_0)\sigma(\eta_0)\hat{U}^\dagger(\eta,\eta_0)$$

$$\hat{U}(\eta,\eta_0) = e^{-i\int_{\eta_0}^{\eta}\hat{H}_0(\eta_1)d\eta_1} T e^{-i\int_{\eta_0}^{\eta}\hat{H}_{\mathrm{I,i}}(\eta_1)d\eta_1}$$

After tracing out over infra-red modes, interesting part is second order in interaction strength…

The evolution:

For just the observable modes, interesting part is second order in interaction strength:

$$\partial_\eta \rho^{(2)}(\eta) = -i\left[\hat{H}_0^{\rm obs}, \rho^{(2)}(\eta)\right] - i\left[\hat{H}^{(2)}_{eff}, \rho^{(0)}(\eta)\right] + \{\hat{A}(\eta), \rho^{(0)}(\eta)\} + \sum_N \left[\hat{L}_{N1}\rho^{(0)}(\eta)\hat{L}^\dagger_{N2} + \hat{L}_{N2}\rho^{(0)}(\eta)\hat{L}^\dagger_{N1}\right]$$

$$\hat{H}^{(2)}_{eff} = -\frac{i}{2}\sum_N(\hat{L}^\dagger_{N1}\hat{L}_{N2} - \hat{L}^\dagger_{N2}\hat{L}_{N1})$$

$$\hat{A}(\eta) = -\frac{1}{2}\sum_N\left(\hat{L}^\dagger_{N1}\hat{L}_{N2} + \hat{L}^\dagger_{N2}\hat{L}_{N1}\right)$$

(H.P. Breuer)

The “Lindblad”-like operators

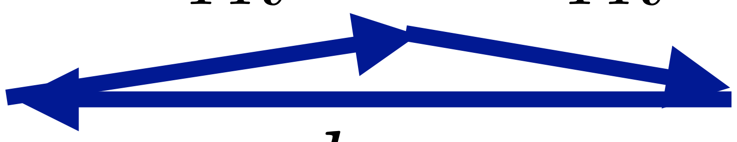
$$\hat{L}_{N1}(\eta) = \lambda(\eta)\langle N|\hat{H}_I(\eta_0)|SQ(\eta)\rangle$$

$$\hat{L}_{N2}(\eta) = \int_{\eta_0}^{\eta} d\eta_1 \lambda(\eta_1)\langle N|\hat{H}_{I,int}(\eta_1-\eta)|SQ(\eta)\rangle$$

These will take a different form depending on how many of the interaction modes are in the system (observable) vs the bath (near-infrared)

(compare: Agon et al 1412.3148)

Folded triangles



Two bath (IR) modes and one system mode:

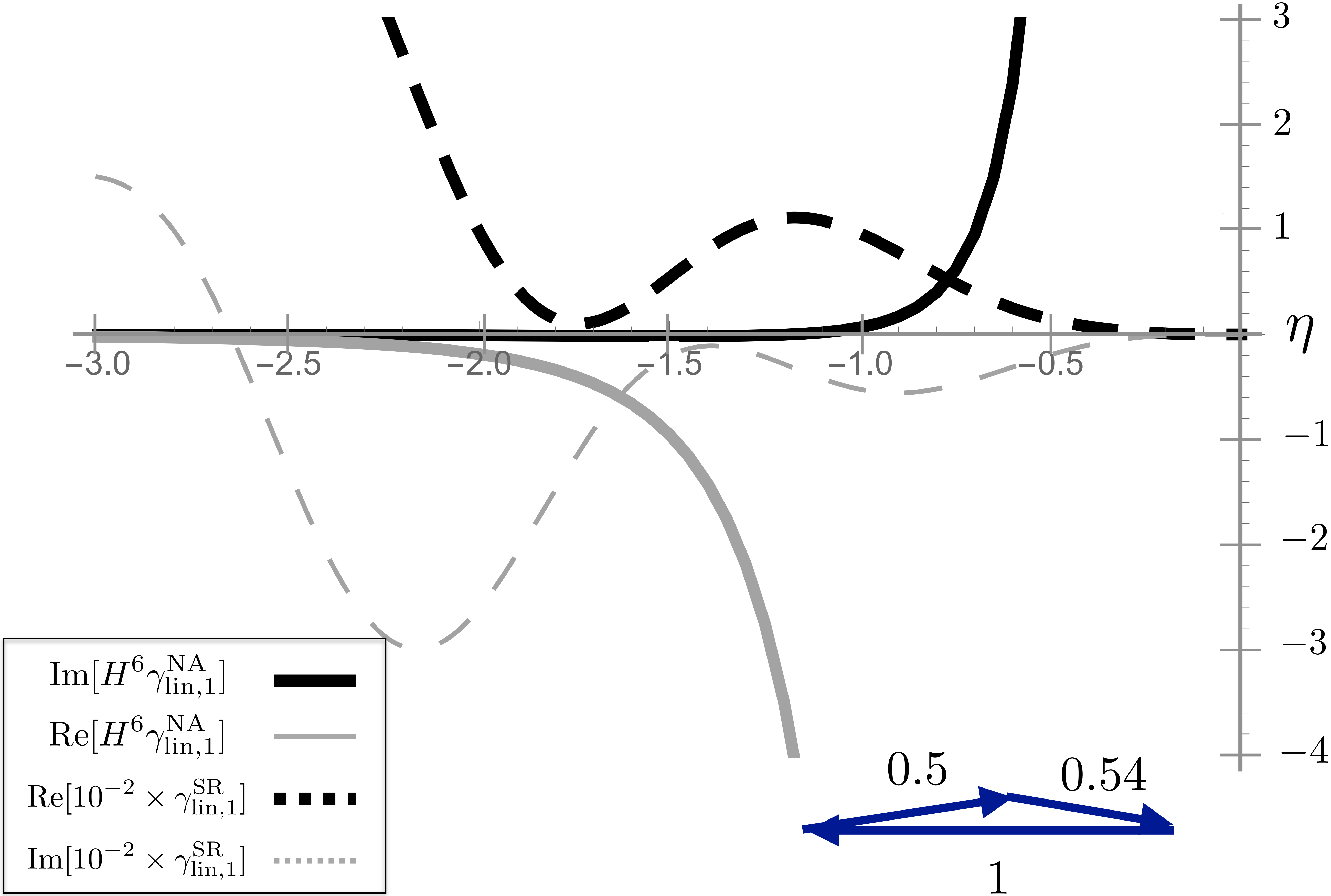
Only possible for modes near the system/bath boundary

Then $L_{N1}\rho^{(0)}(\eta)L^{\dagger}_{N2}$ contains:

$$\int \frac{d^3k_s}{(2\pi)^3}\hat{c}_{\vec{k}_o}(\eta_0)\rho^{(0)}(\eta)\hat{S}_{k_s}(\eta)\hat{R}_{k_s}(\eta)\left[\hat{c}^{\dagger}_{\vec{k}_o}(\eta_0)f_1(m_i,k_i\eta)\right.$$
$$\left.+\hat{c}_{-\vec{k}_o}(\eta_0)f_2(m_2,m_3,k_i\eta)\right]\hat{R}^{\dagger}_{k_s}(\eta)\hat{S}^{\dagger}_{k_s}(\eta)+\ldots$$

"Linear dissipation terms"

Time-dependence of linear terms in $L_{N1}(\eta)L^\dagger_{N2}(\eta)$



Squeezed triangles

One bath (IR) modes and two system modes

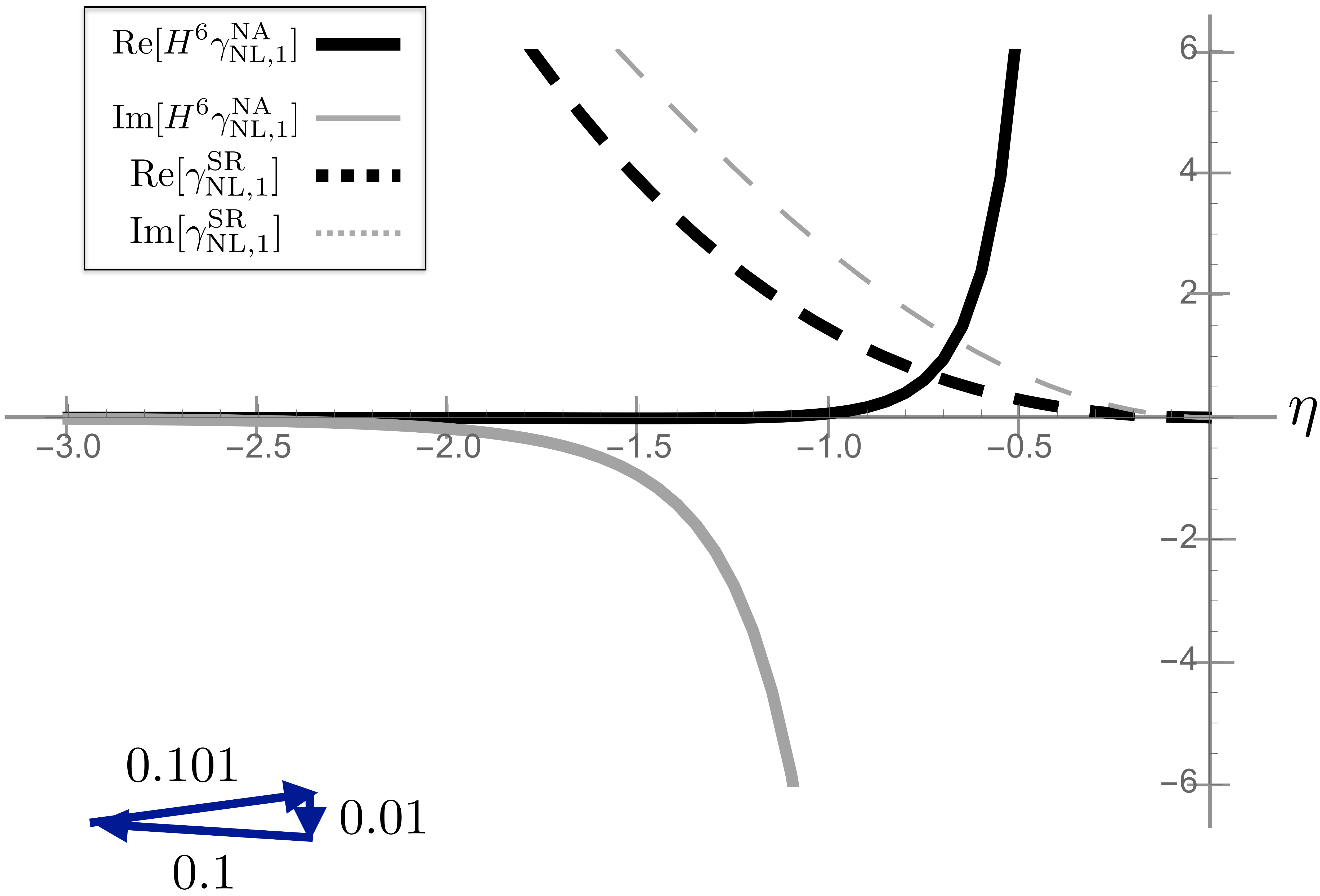
All system modes can receive this contribution

Then $L_{N1}\rho^{(0)}(\eta)L^{\dagger}_{N2}$ contains:

$$\int \frac{d^3k_{s1}}{(2\pi)^3}\int \frac{d^3k_{s2}}{(2\pi)^3}\left[\hat{c}_{\vec{k}_{s1}}(\eta_0)\hat{c}_{\vec{k}_{s2}}(\eta_0)\rho^{(0)}(\eta)\hat{\mathcal{S}}(k_{s1},k_{s2},\eta)\right.$$
$$\left.\times\hat{c}^{\dagger}_{\vec{k}_{s1}}(\eta_0)\hat{c}^{\dagger}_{\vec{k}_{s2}}(\eta_0)q_1(m,k_B\eta)\ldots\hat{\mathcal{S}}(k_{s1},k_{s2},\eta)^{\dagger}+\ldots\right]$$

“Non-linear dissipation terms”

Time-dependence of non-linear terms in $L_{N1}(\eta)L^{\dagger}_{N2}(\eta)$



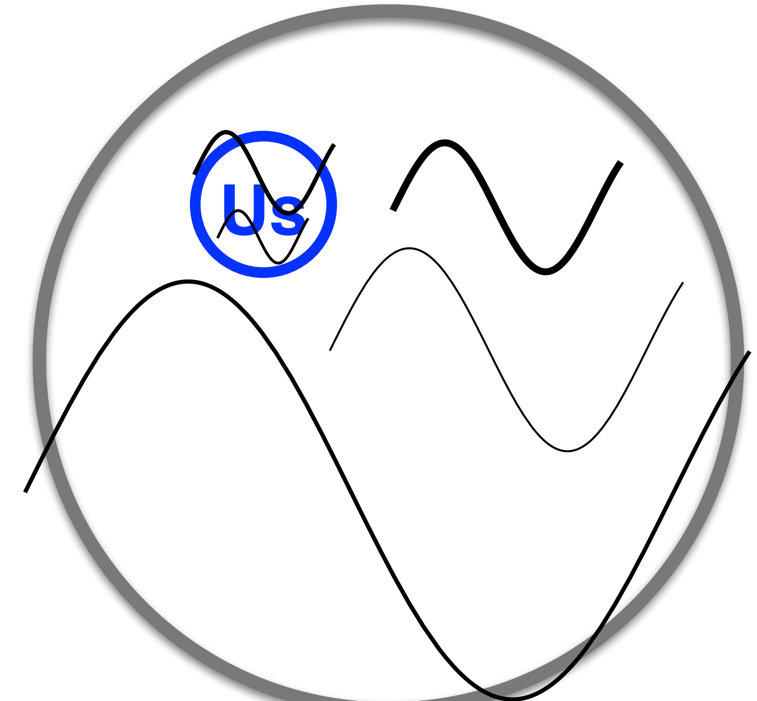
Summary

Cosmology suggests an open quantum system where gravity is key:

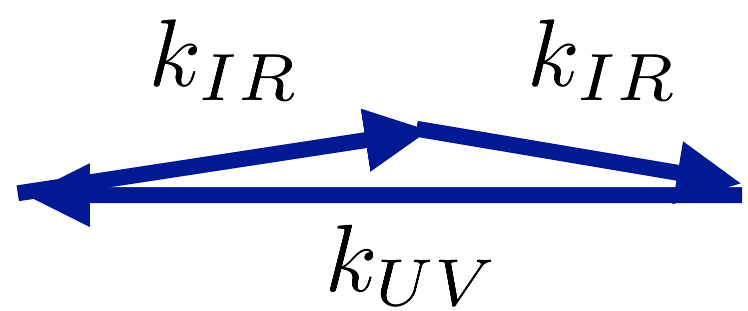
(1) in defining system and bath

(2) in sourcing correlated particle pairs

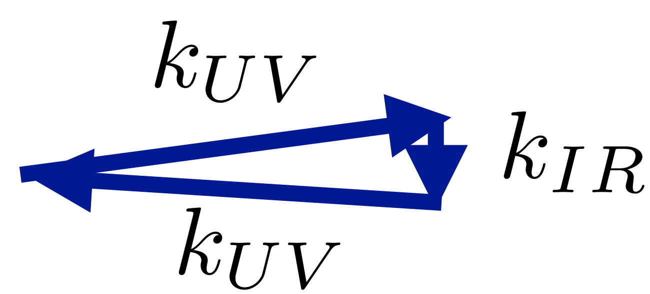
(3) in providing a nonlinear interaction term



Linear dissipation from



Non-linear dissipation from



The effective theory? Evolution of observable modes is generically non-Hamiltonian, non-Markovian