Explorations in non-linear cosmostatistics

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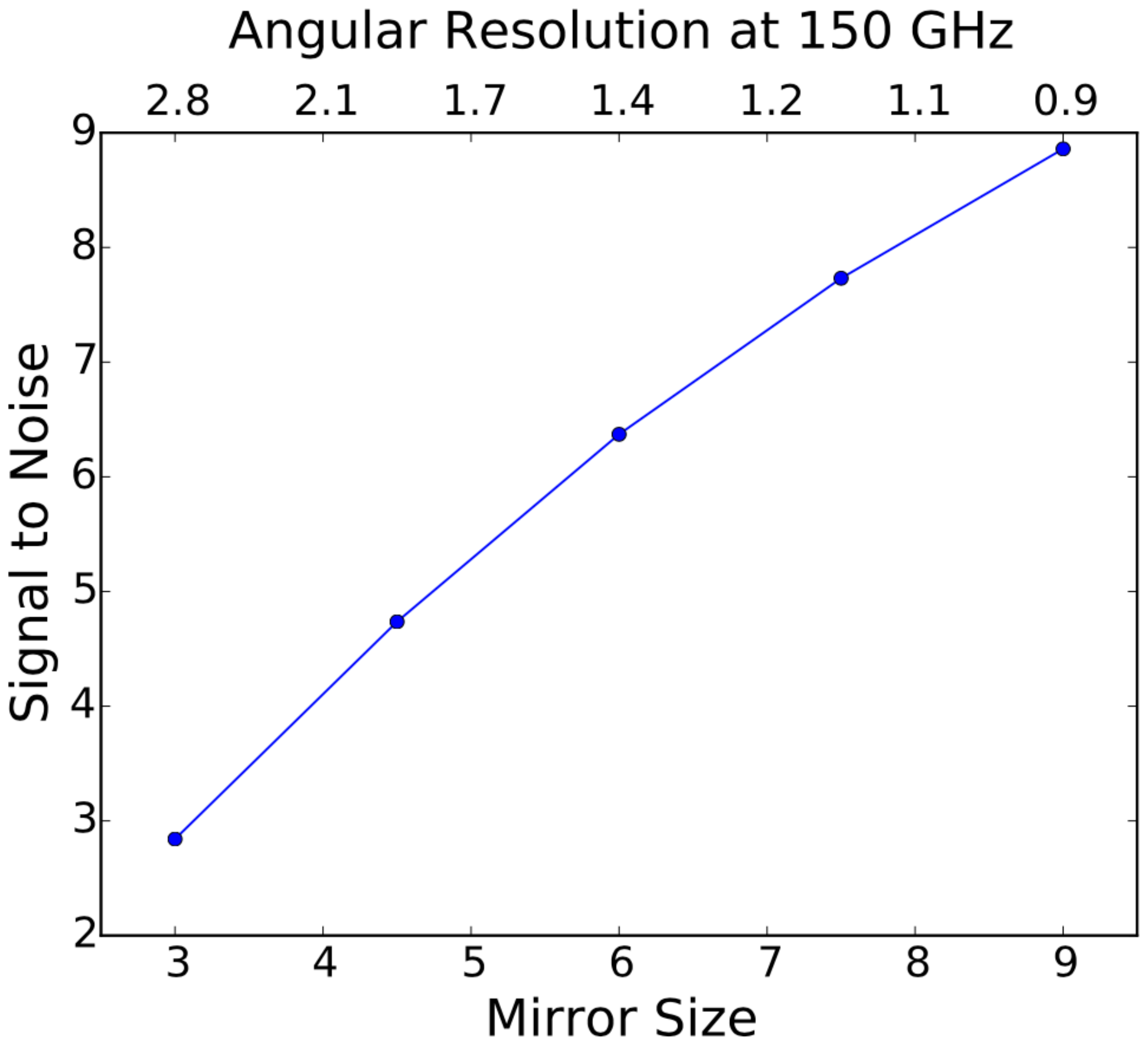
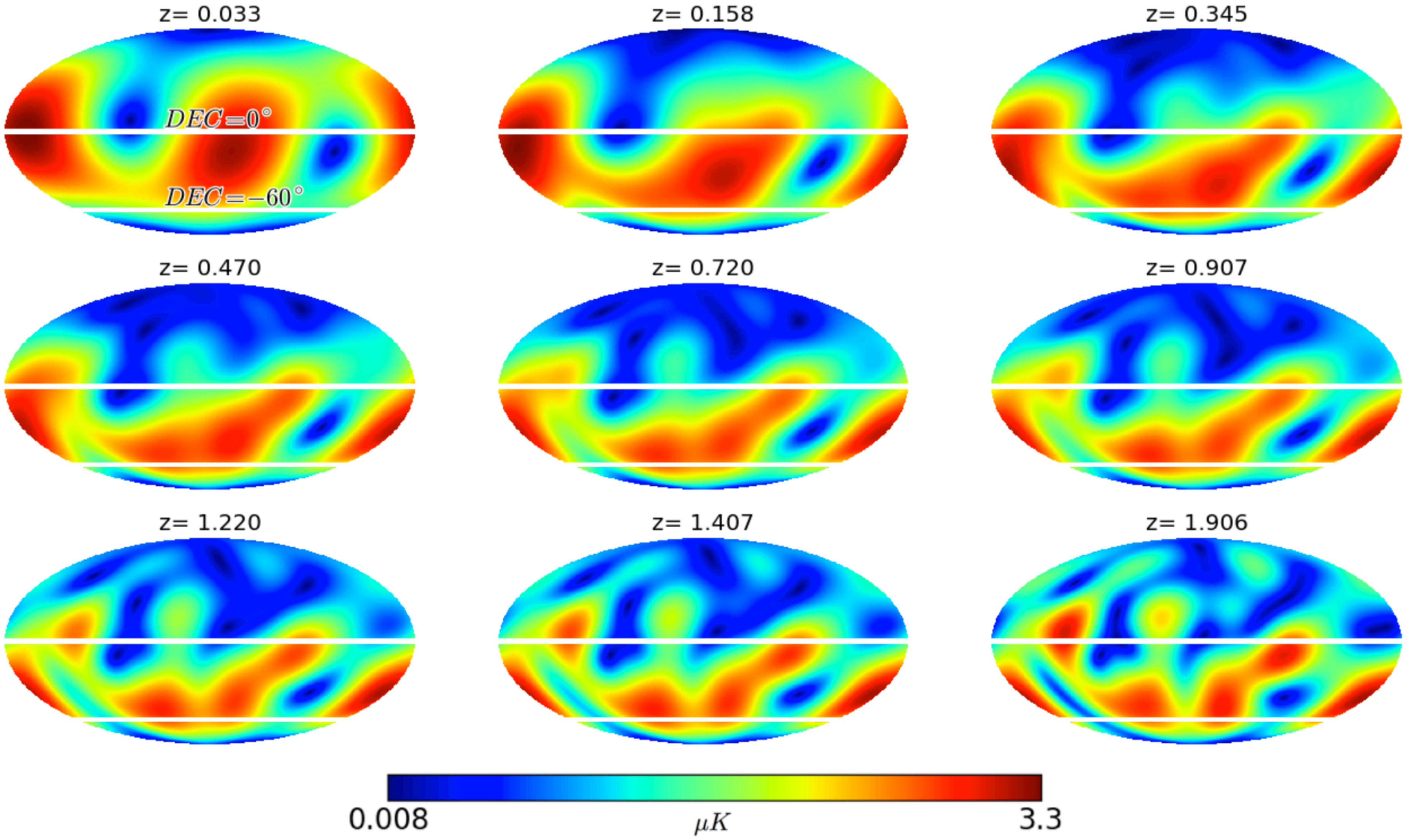
What I am not talking about:



- "**Measuring Polarized Emission in Clusters in the CMB S4 Era**," Thibaut Louis, Bunn, Wandelt & Silk (arXiv: 1707.04102)
- Use polarized SZ effect to calibrate cluster τ-M relation.

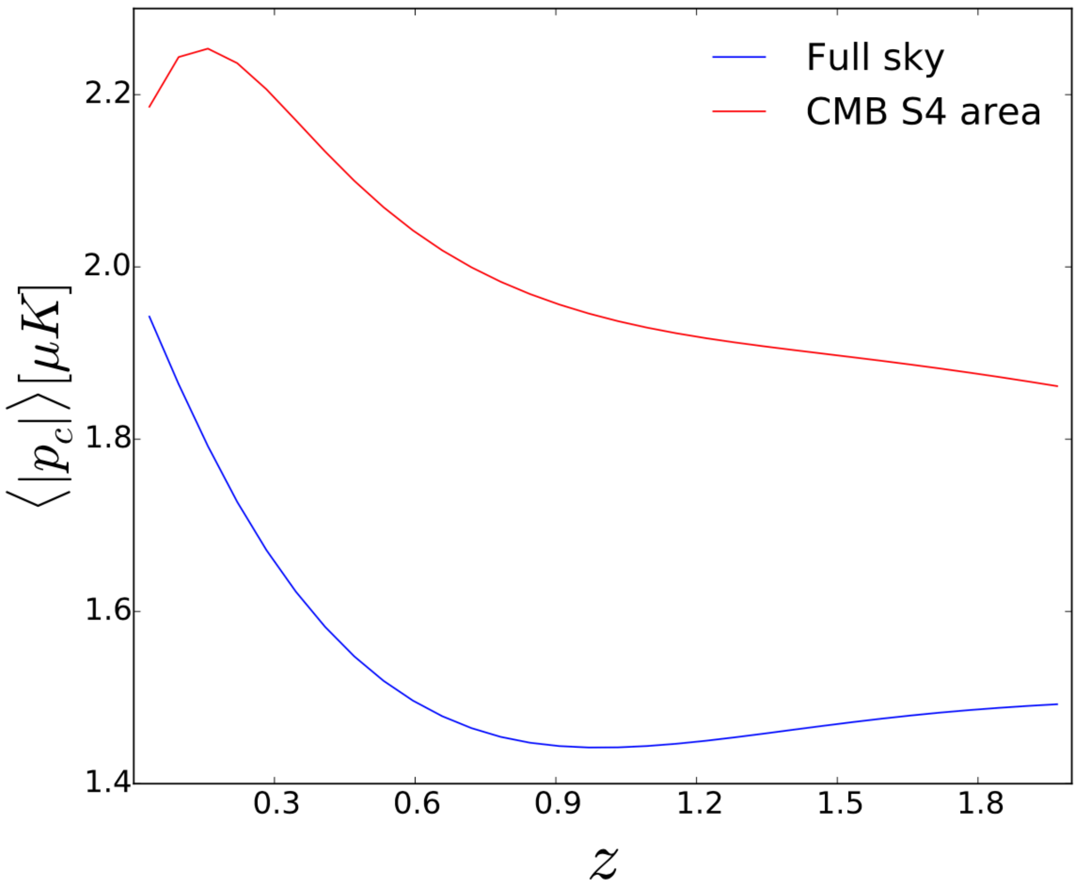
Polarized SZ on the sky



Benjamin Wandelt

Louis, et al. arXiv: 1707.04102

Detecting pSZ on clusters with S4



T. Louis, et al. arXiv: 1707.04102

For measurements of the neutrino mass and ultimate constraints on primordial gravitational waves need to do go beyond quadratic CMB lensing analysis

Anderes, Lavaux, Wandelt

arXiv:1412.4079

Millea, Anderes, Wandelt

in prep.

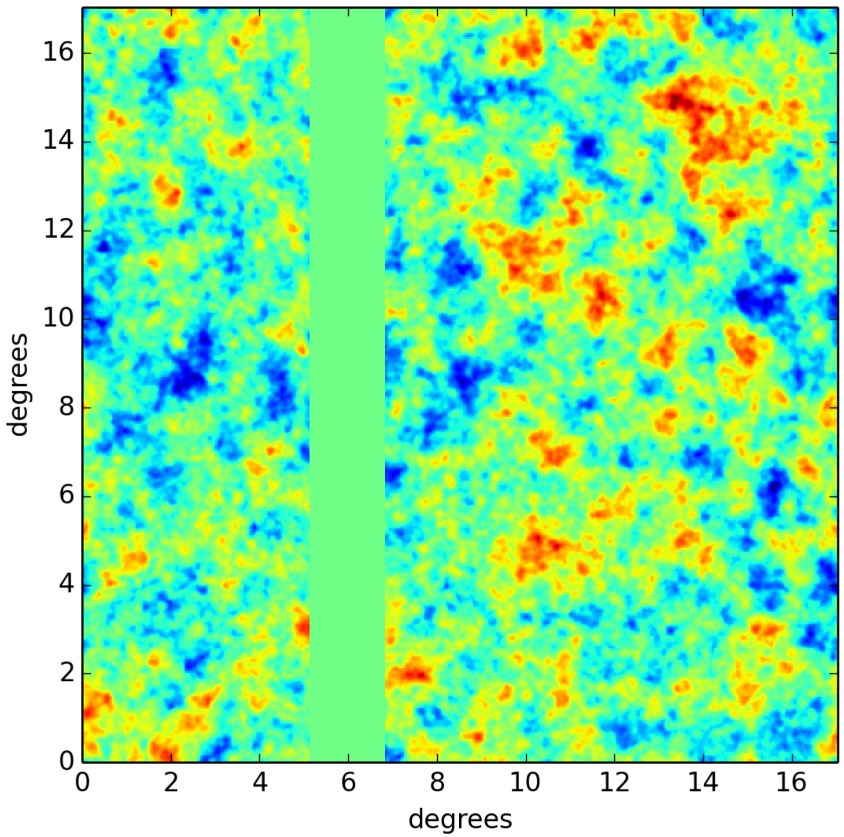
Bayesian Lensing potential reconstruction

- Standard quadratic estimator approach known to be **suboptimal** for upcoming high S/N experiments
- A Bayesian approach would be **optimal**. But studying the posterior pdf for the lensing potential given the data is an "intractable" problem.
- All standard approaches fail due to the large number of parameters or due to hugely inefficient sampling of the parameter space

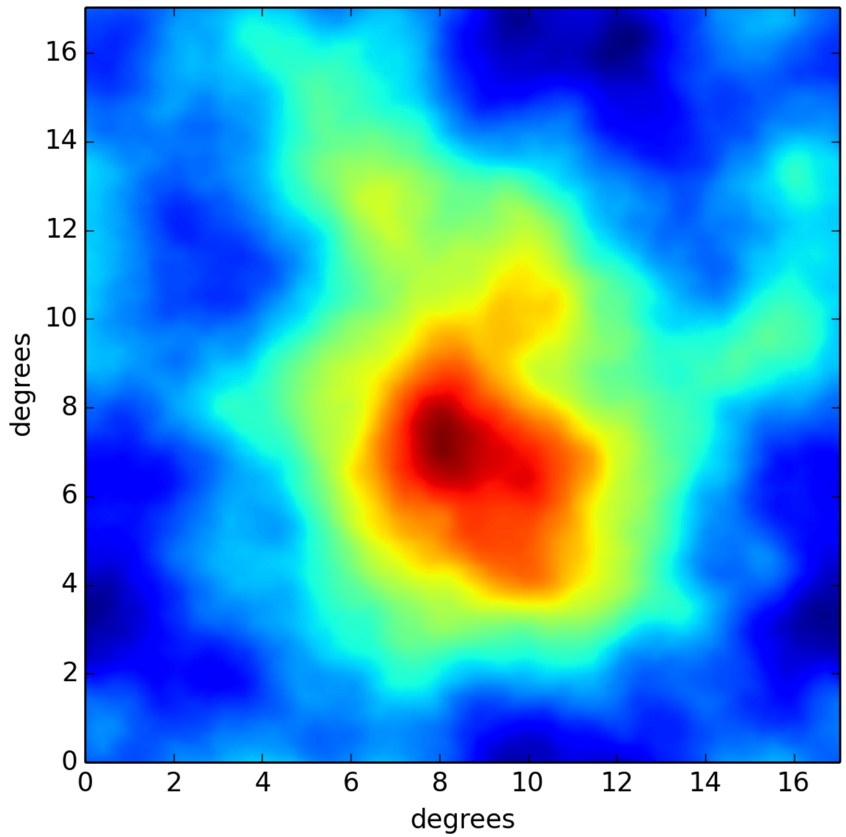
Anderes, Lavaux, BDW (arXiv:1412.4079) demonstrated fully Bayesian delensing and ϕ reconstruction MCMC sampler for temperature data. Carron and Lewis (arXiv:1704.08230) show optimal reconstruction of ϕ including polarization.

Example

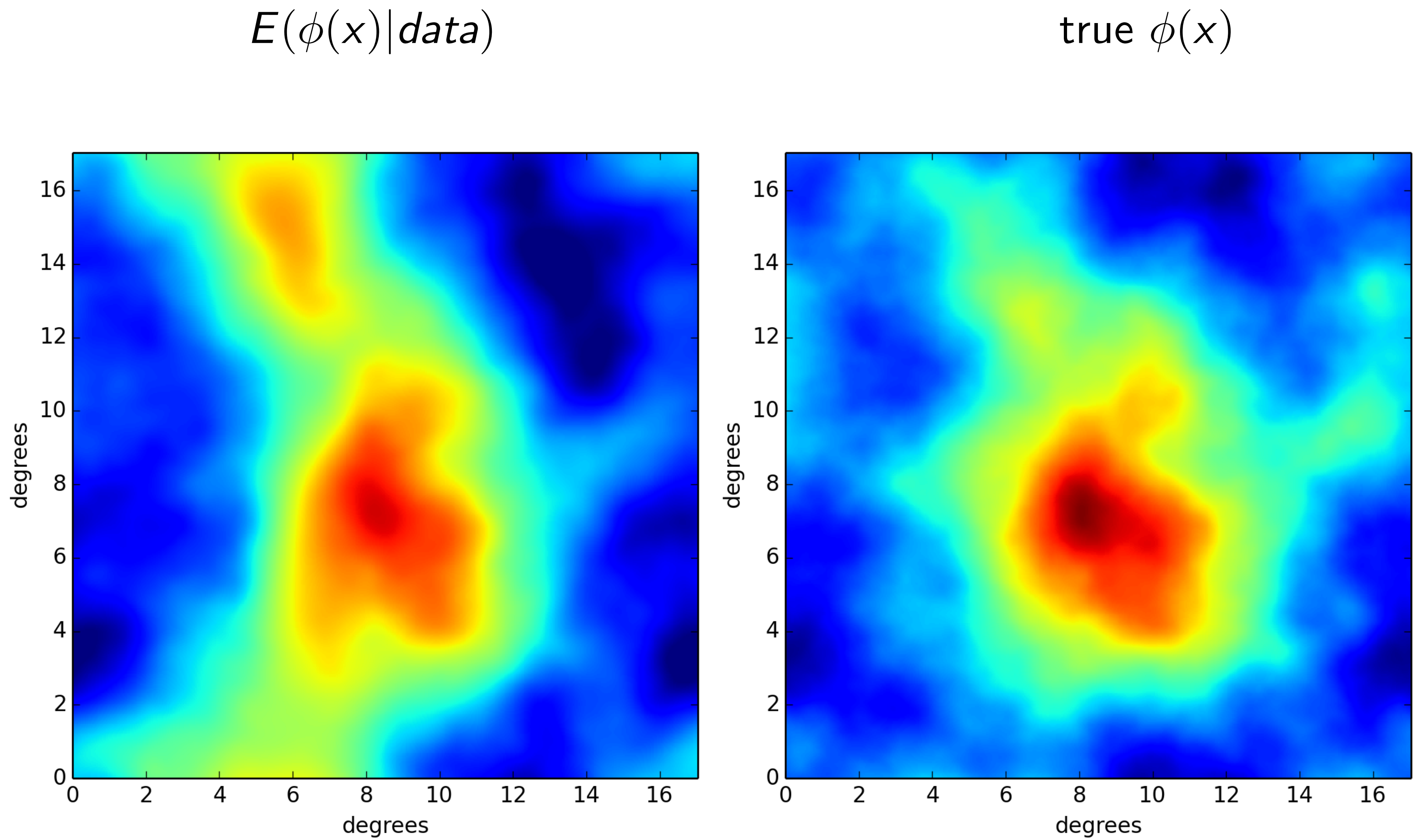
Data



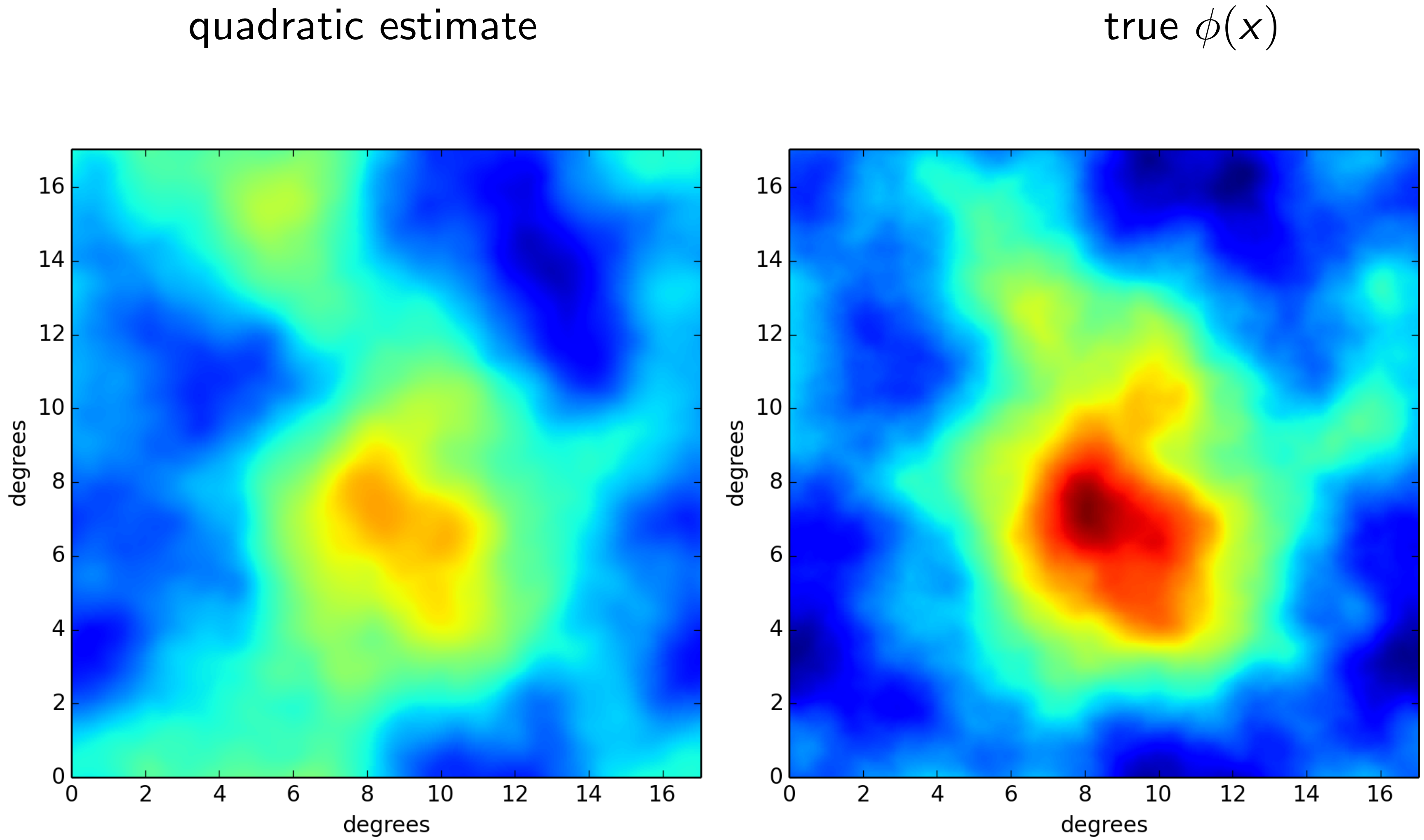
true $\phi(x)$



Posterior mean lensing potential



Posterior mean vs the quadratic estimator (based on 4-point correlation measurement)



Quadratic estimator computed *without the mask.*

Lensing potential

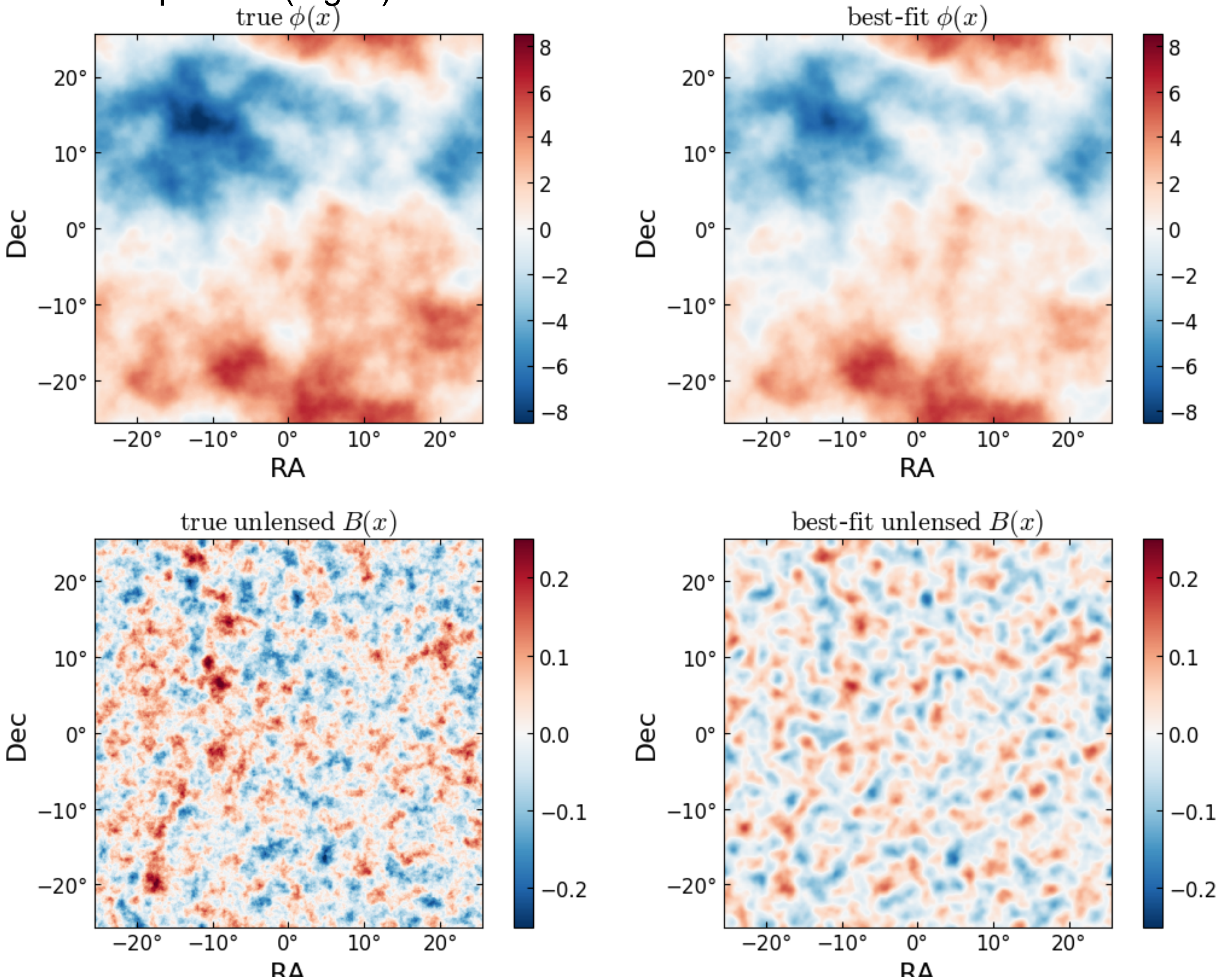
Unlensed CMB maps

$$\mathcal{P}(T, E, B, \phi, r \,|\, \mathrm{data})$$

Cosmo params (e.g. r)

See Marius' talk Friday @ 9:30

about Millea, Anderes & Wandelt (in prep).



- Joint posterior contains all possible* information on these quantities
- Develop new computational techniques that enable maximizing and sampling propagating non-G uncertainties) from $\mathcal{P}(T, E, B, \phi, r \,|\, \mathrm{data})$

*assuming $TEB\phi$ Gaussianity

Key ingredient: optimal filtering

$$d = s + n$$

Not sparse in any easily accessible basis

$$\boxed{(\mathbf{S}^{-1} + \mathbf{N}^{-1})}\, s_{\mathrm{WF}} = \mathbf{N}^{-1} d$$

Sparse in Fourier space

Sparse in pixel space

Usual solution strategies

- Iterative conjugate gradients (optimal for SPD matrices)
- Preconditioner (diagonal in Fourier space, multi-grid) (Smith et al.; Eriksen, Wehus, et al.)
- Problems:
  - Even for Planck: ***extremely ill-conditioned***, condition number $>10^9$
  - Preconditioner not universal
  - Stability issues (Jacobi smoother in multi-grid)

The Messenger method: Wiener Filtering *without* preconditioner

- Introduce auxiliary field (*messenger* field) t with covariance $\mathbf{T}$. Then

$$\left(\bar{\mathbf{N}}^{-1} + (\;\;\mathbf{T})^{-1}\right) t = \bar{\mathbf{N}}^{-1} d + (\;\;\mathbf{T})^{-1} s$$

$$\left(\mathbf{S}^{-1} + (\;\;\mathbf{T})^{-1}\right) s = (\;\;\mathbf{T})^{-1} t\,,$$

$$\bar{\mathbf{N}} \equiv \mathbf{N} - \mathbf{T}$$

is solved by the Wiener filter.

Can solve each of these equations *exactly* algebraically.

Iterate. Easy to show that this converges and is unconditionally stable.

Elsner & BDW, arXiv:1210.4931
Kodi-Ramanah, Lavaux & BDW, arXiv:1702.08852

The Messenger method: Wiener Filtering *without* preconditioner

- Introduce auxiliary field (*messenger* field) t with covariance $\mathbf{T}$ and a parameter λ≥1. Then

$$\left(\bar{\mathbf{N}}^{-1} + (\boxed{\lambda}\mathbf{T})^{-1}\right)\, t = \bar{\mathbf{N}}^{-1}\, d + (\boxed{\lambda}\mathbf{T})^{-1}\, s$$

$$\left(\mathbf{S}^{-1} + (\boxed{\lambda}\mathbf{T})^{-1}\right)\, s = (\boxed{\lambda}\mathbf{T})^{-1}\, t\,,$$

$$\bar{\mathbf{N}} \equiv \mathbf{N} - \mathbf{T}$$

is solved by the Wiener filter **for λ=1**.

Can solve each of these equations *exactly* algebraically.

Iterate. Easy to show that this converges and is unconditionally stable.

Elsner & BDW, arXiv:1210.4931
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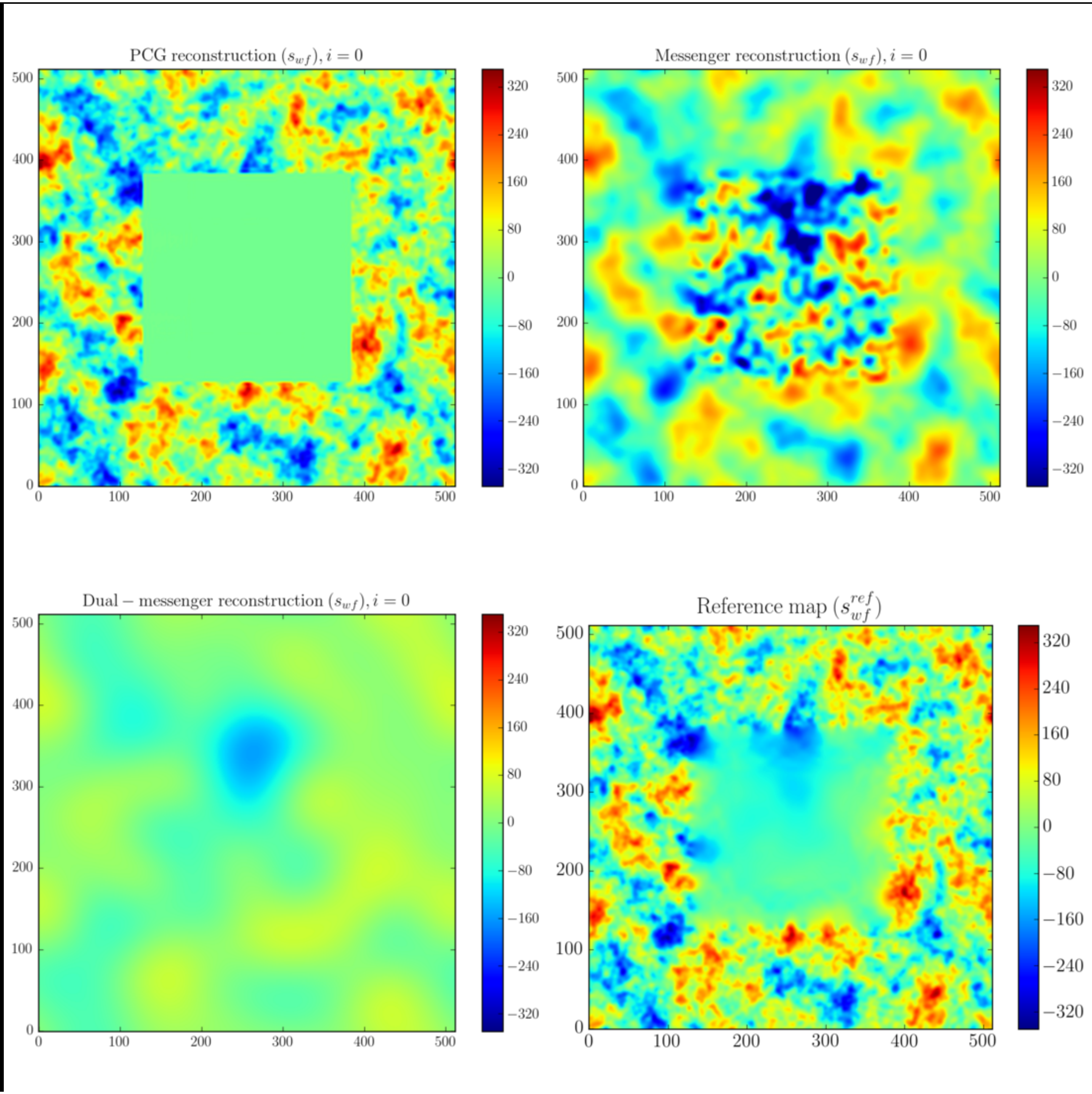
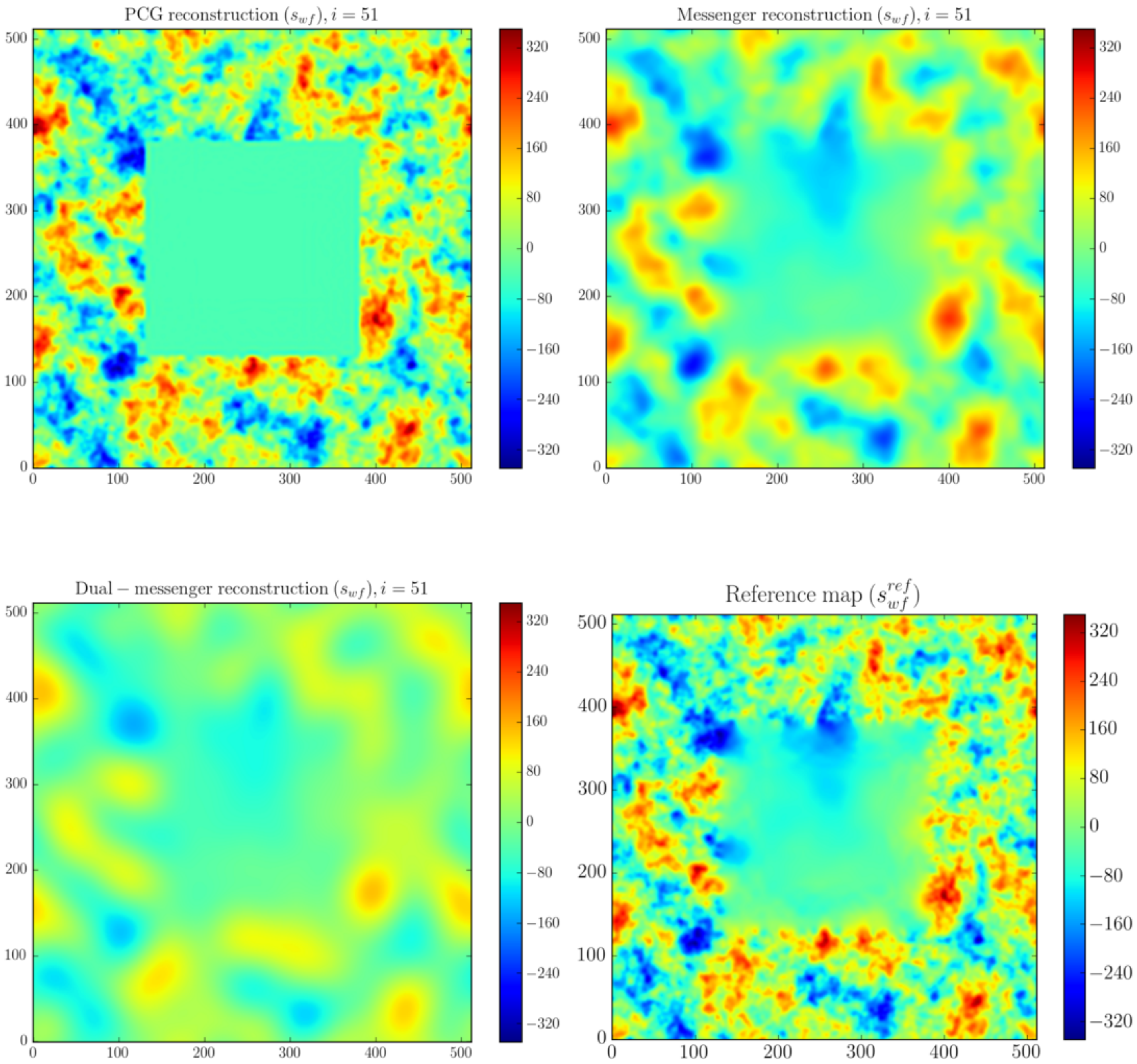
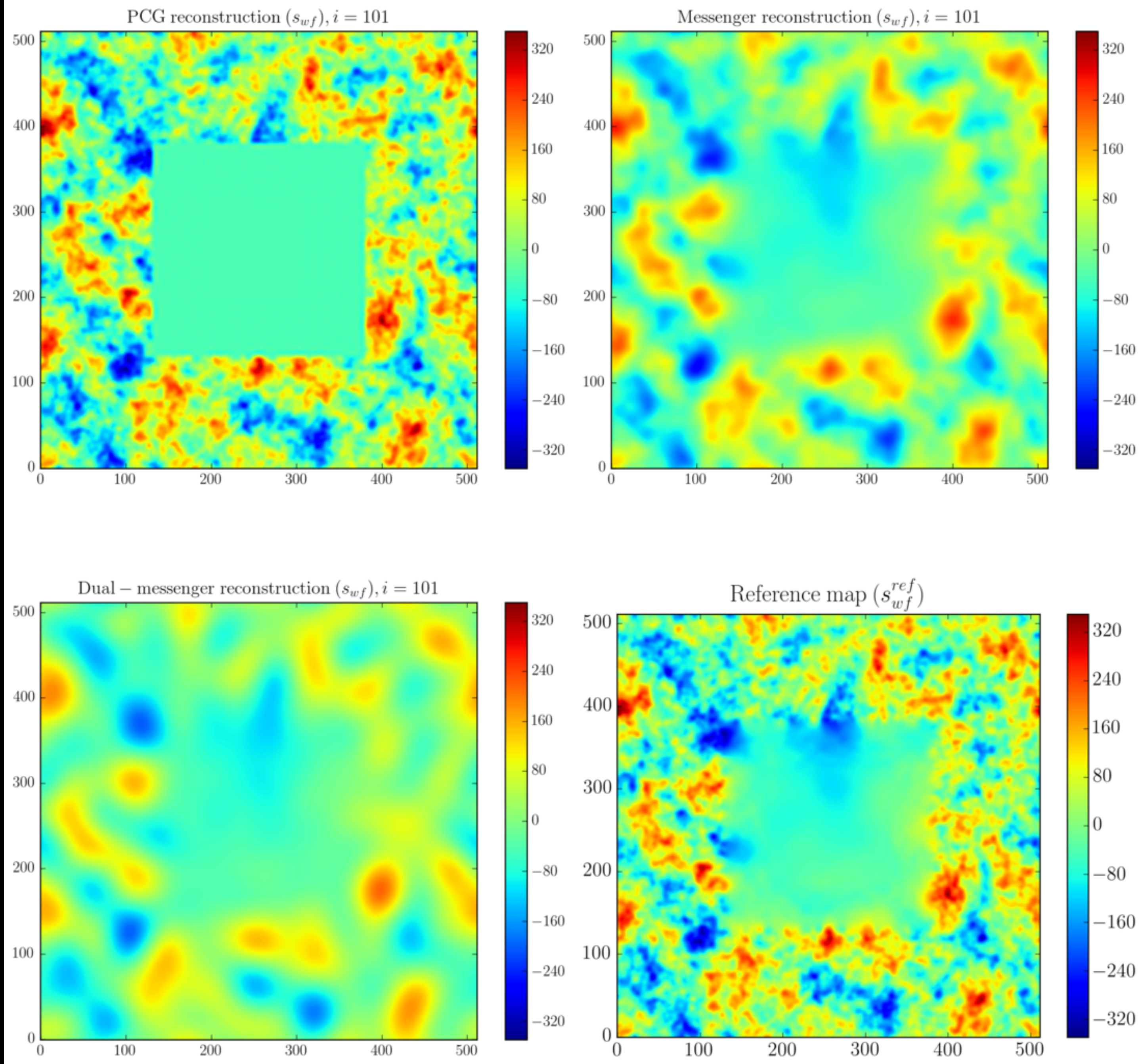


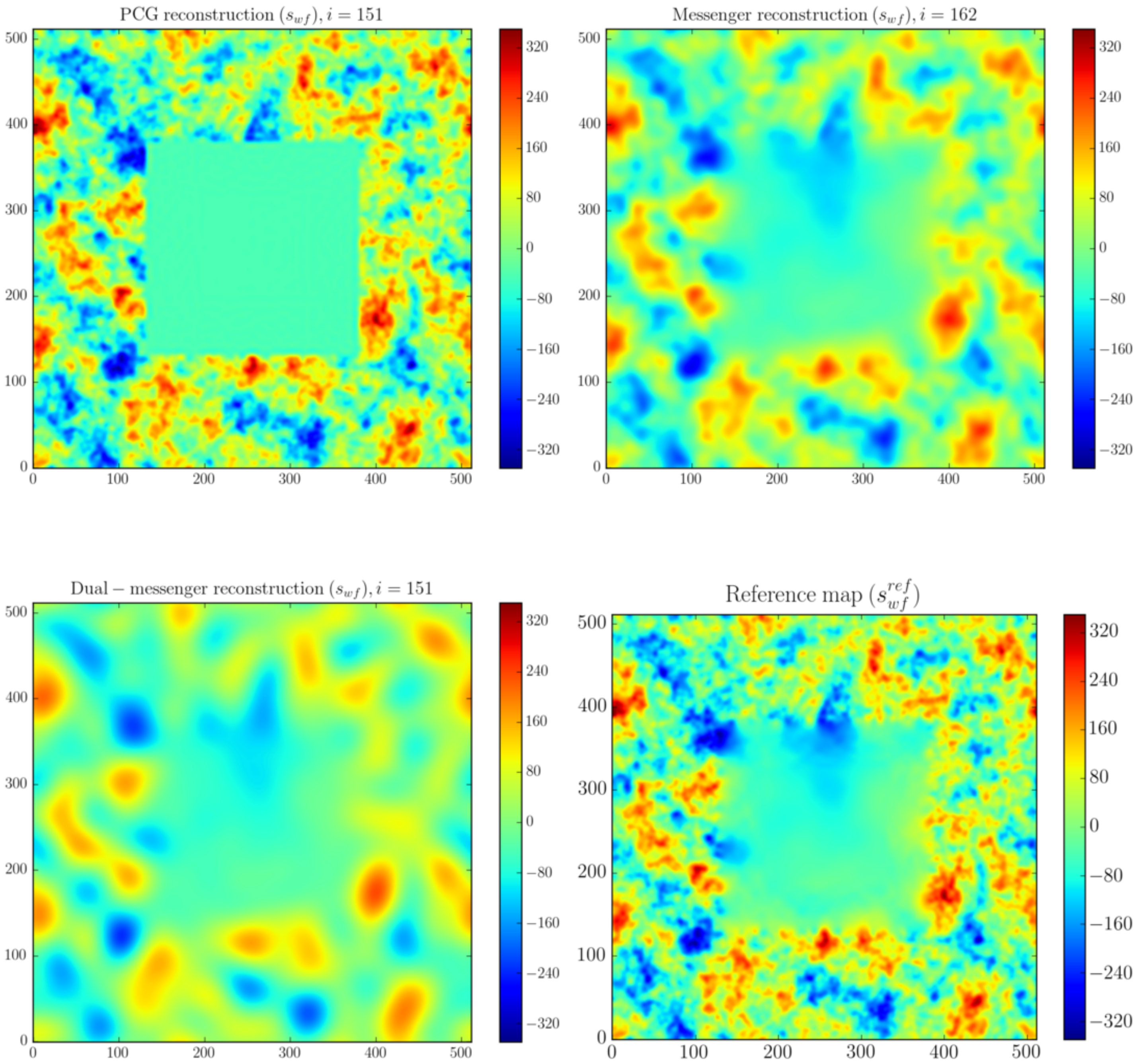
Kodi-Ramanah, Lavaux & BDW, arXiv:1702.08852



Kodi-Ramanah, Lavaux & BDW, arXiv:1702.08852



Kodi-Ramanah, Lavaux & BDW, arXiv:1702.08852



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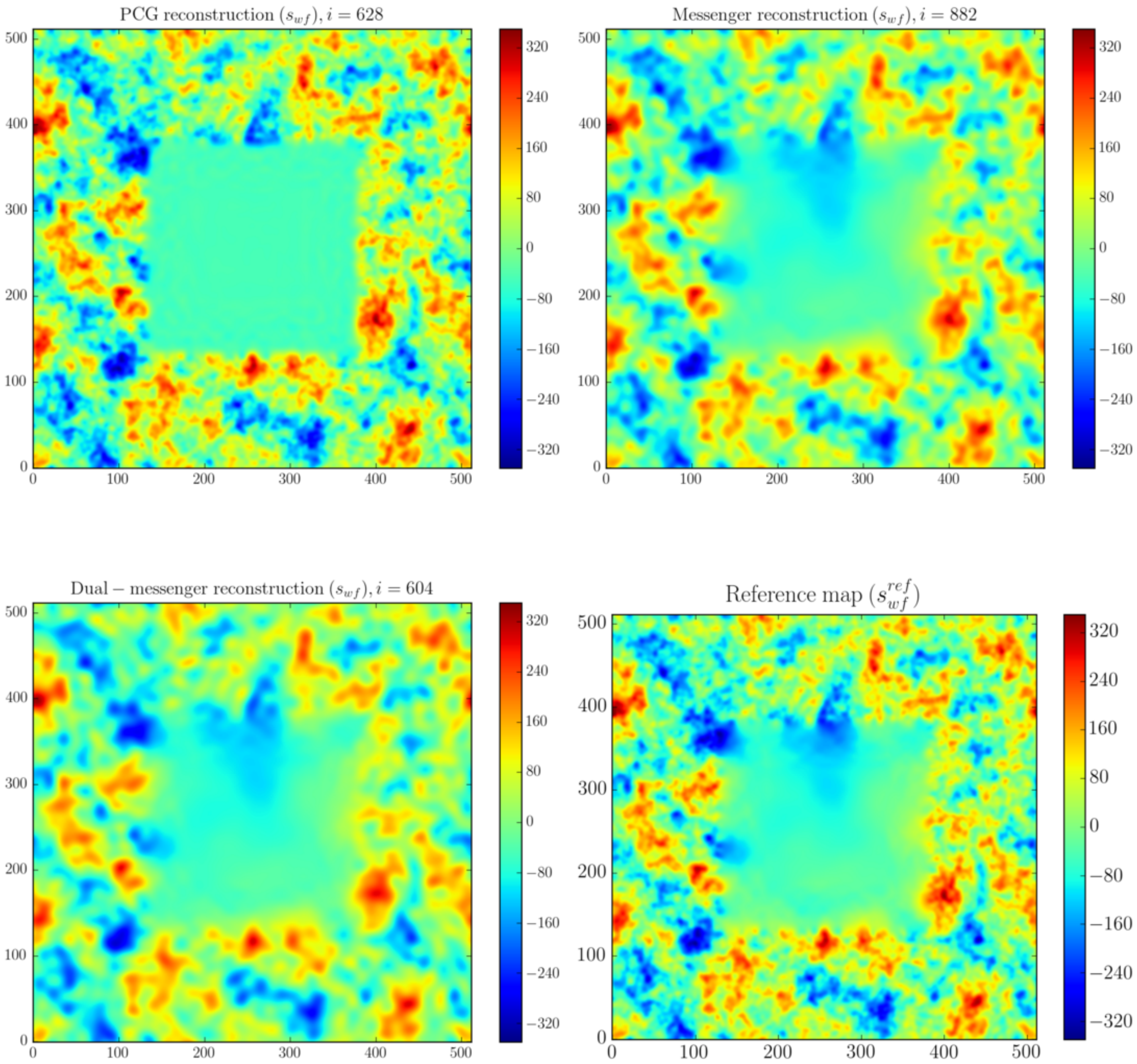

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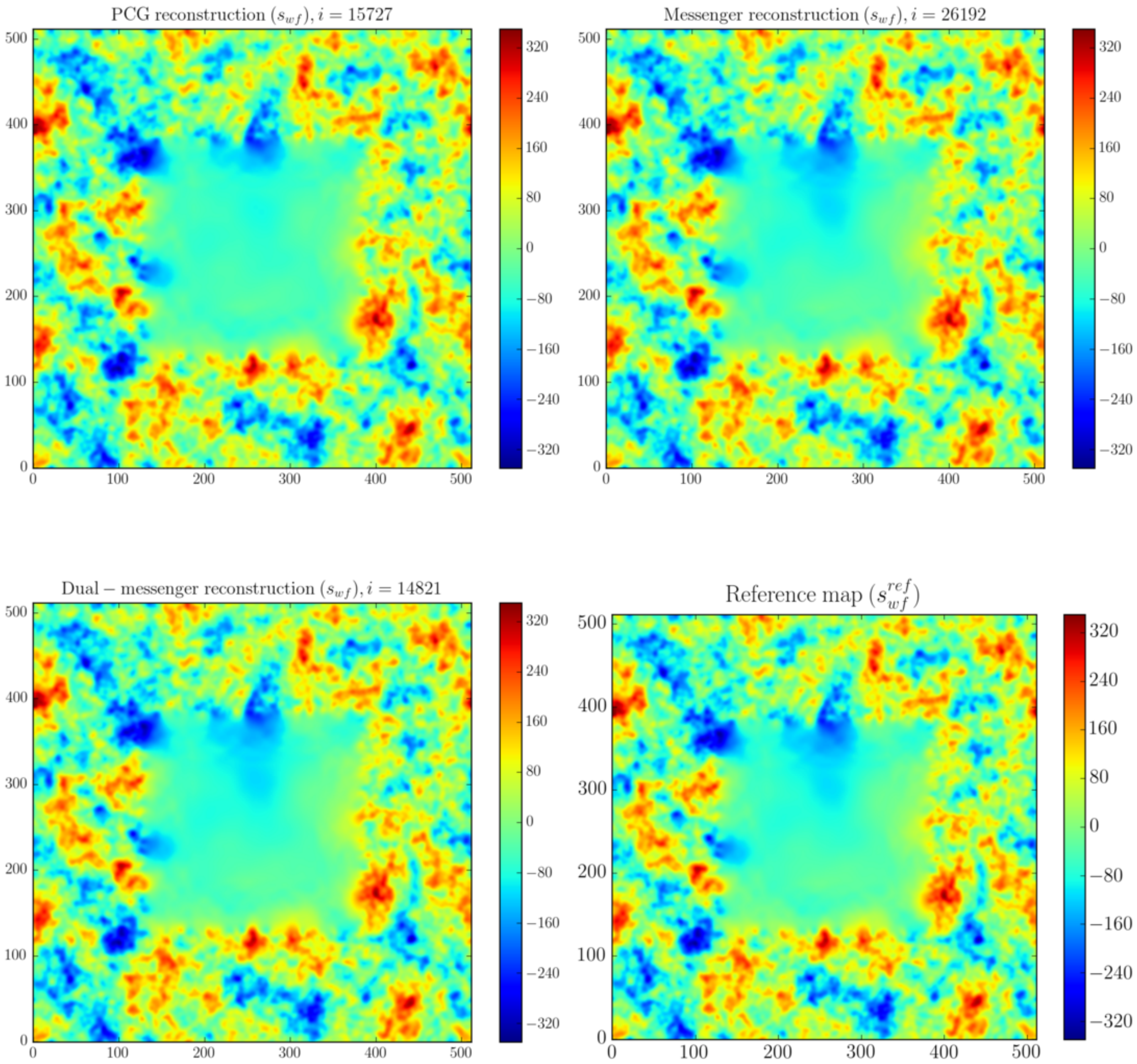
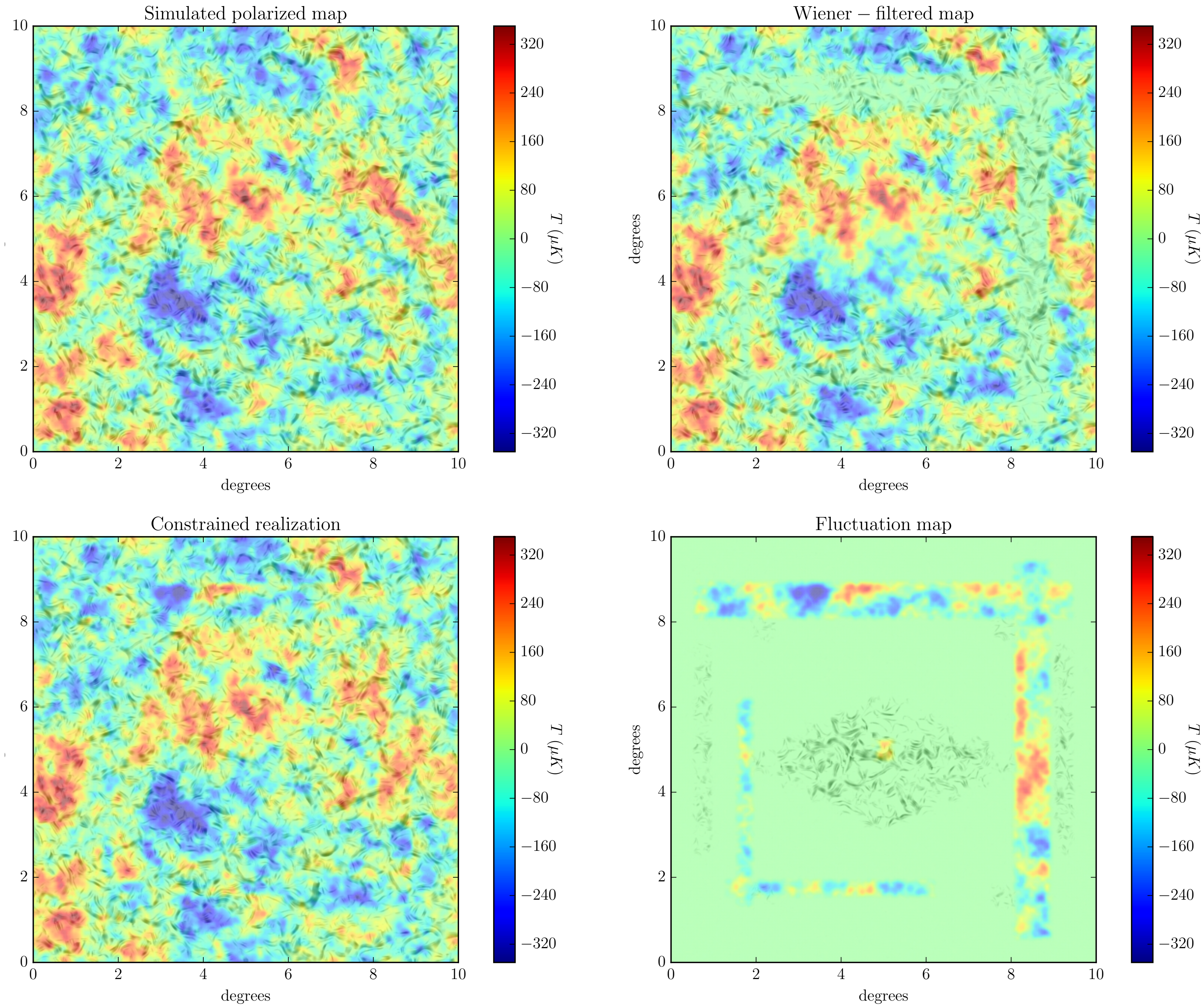




Now fully polarized: Kodi Ramanah, Lavaux, Wandelt, in prep.

Parameter maps

(S. Mukherjee & BDW, in prep.)

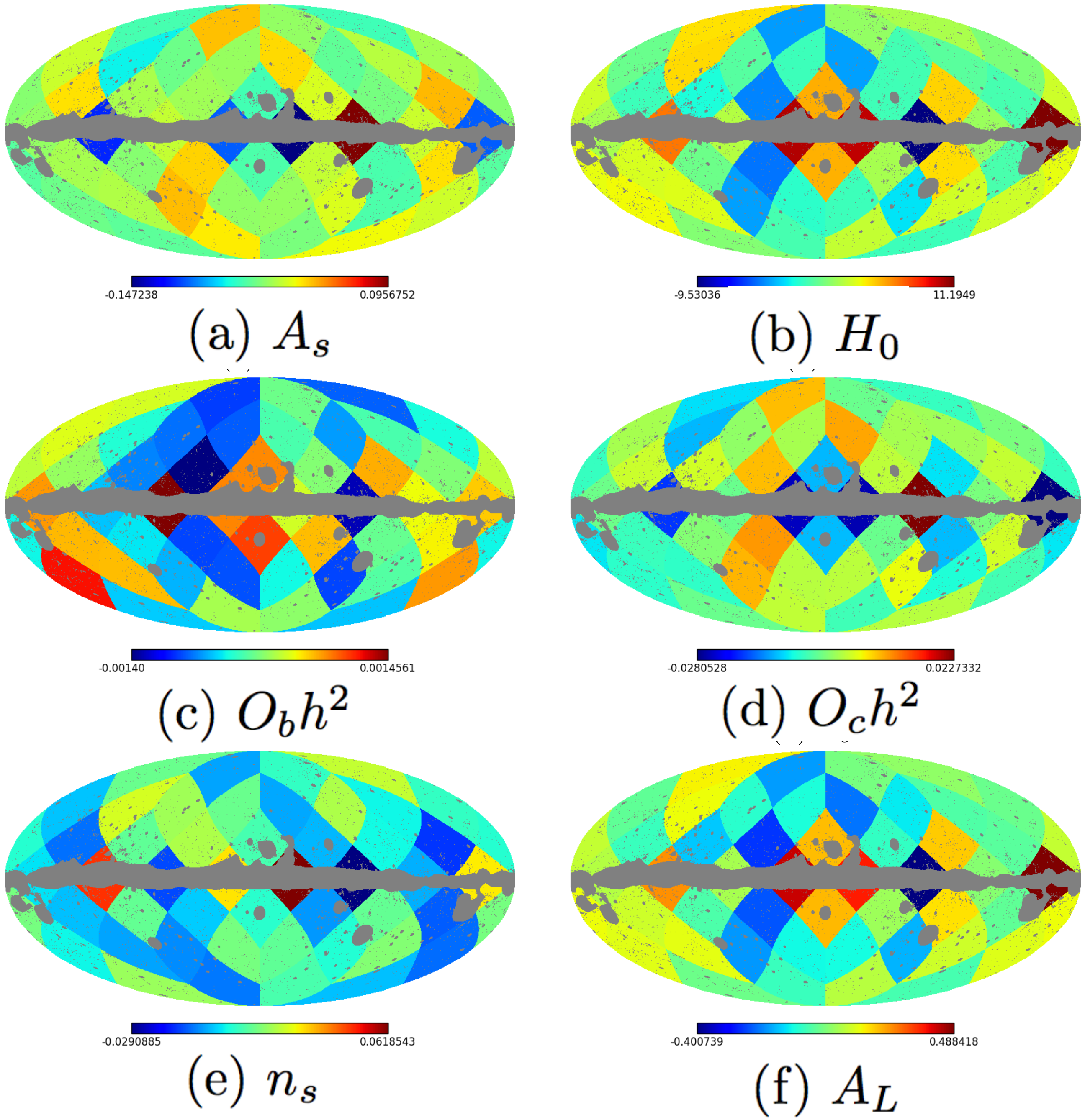
Why would we want parameter maps?

- Physics?
  - Test for isotropy breaking.
  - Large angle anomalies?
  - Spatially varying constants, abundances,…?
- Systematics?
  - Diagnostic tool: check how systematics (foregrounds, instrument) project on the physical parameters.

Superfast parameter estimation

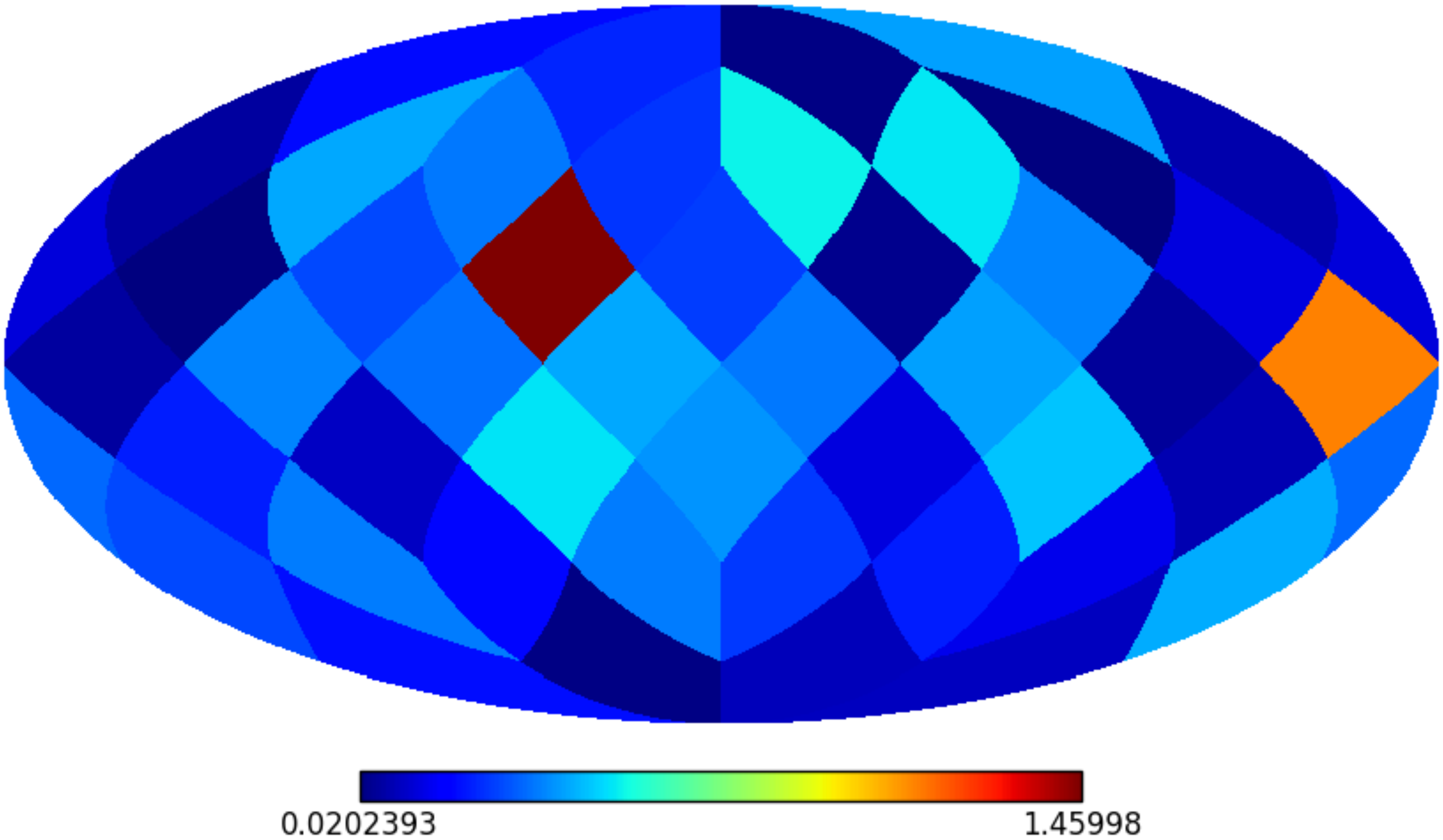
- Optimal quadratic estimator for each parameter is simple in round patches on the sky − can make *maps* using convolution!
- Avoids doing MCMC in each patch − get estimates and covariance matrix for parameters
- Requires only $4n+1$ high-res spherical harmonic transforms for n parameter maps.

Cosmological Parameters in 2015 Planck SMICA map



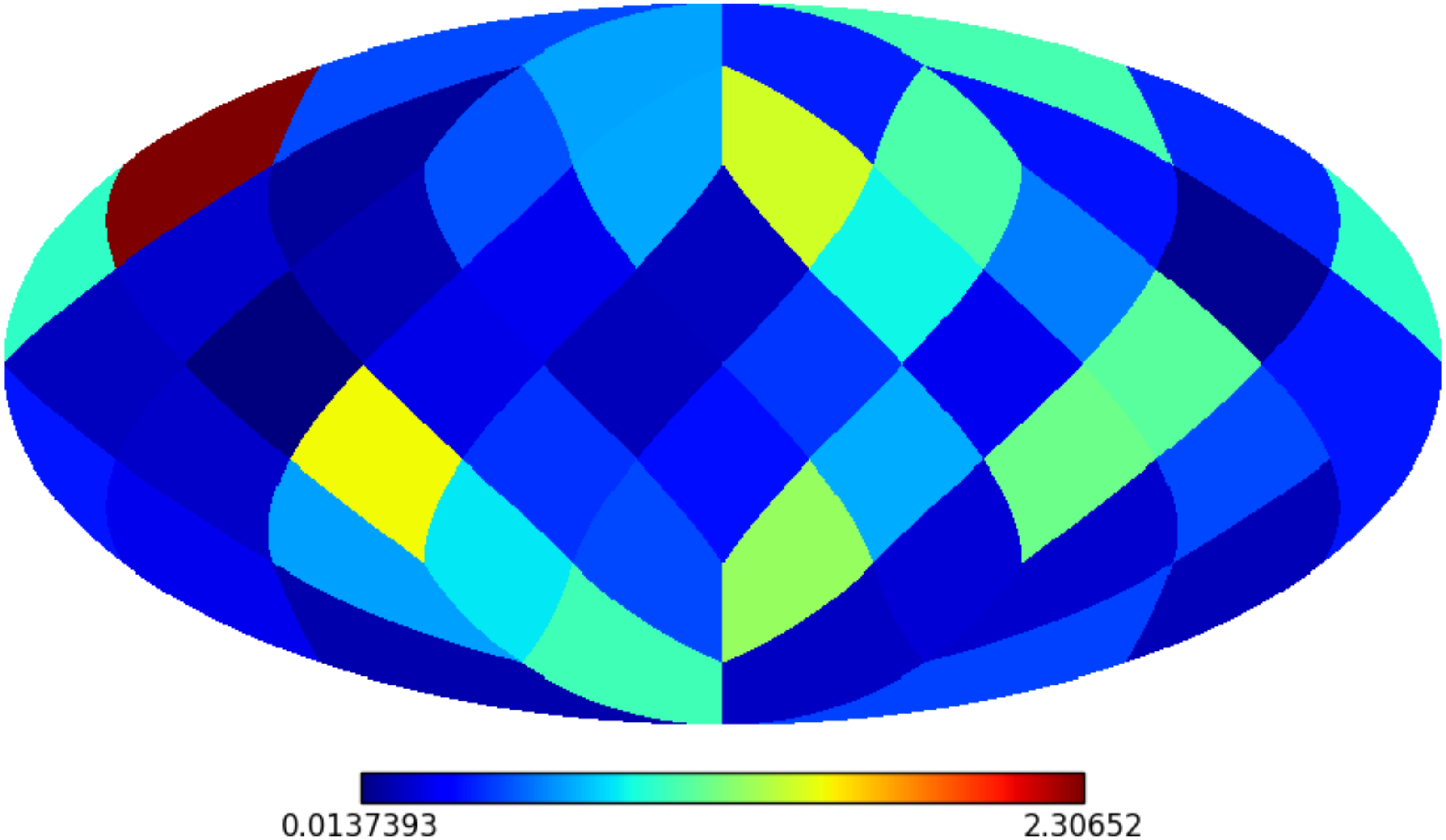
S. Mukherjee & BDW, in prep.

χ^2-map of parameter deviations from Planck best fit



S. Mukherjee & BDW, in prep.

χ^2-map of parameter deviations (simulation)



S. Mukherjee & BDW, in prep.

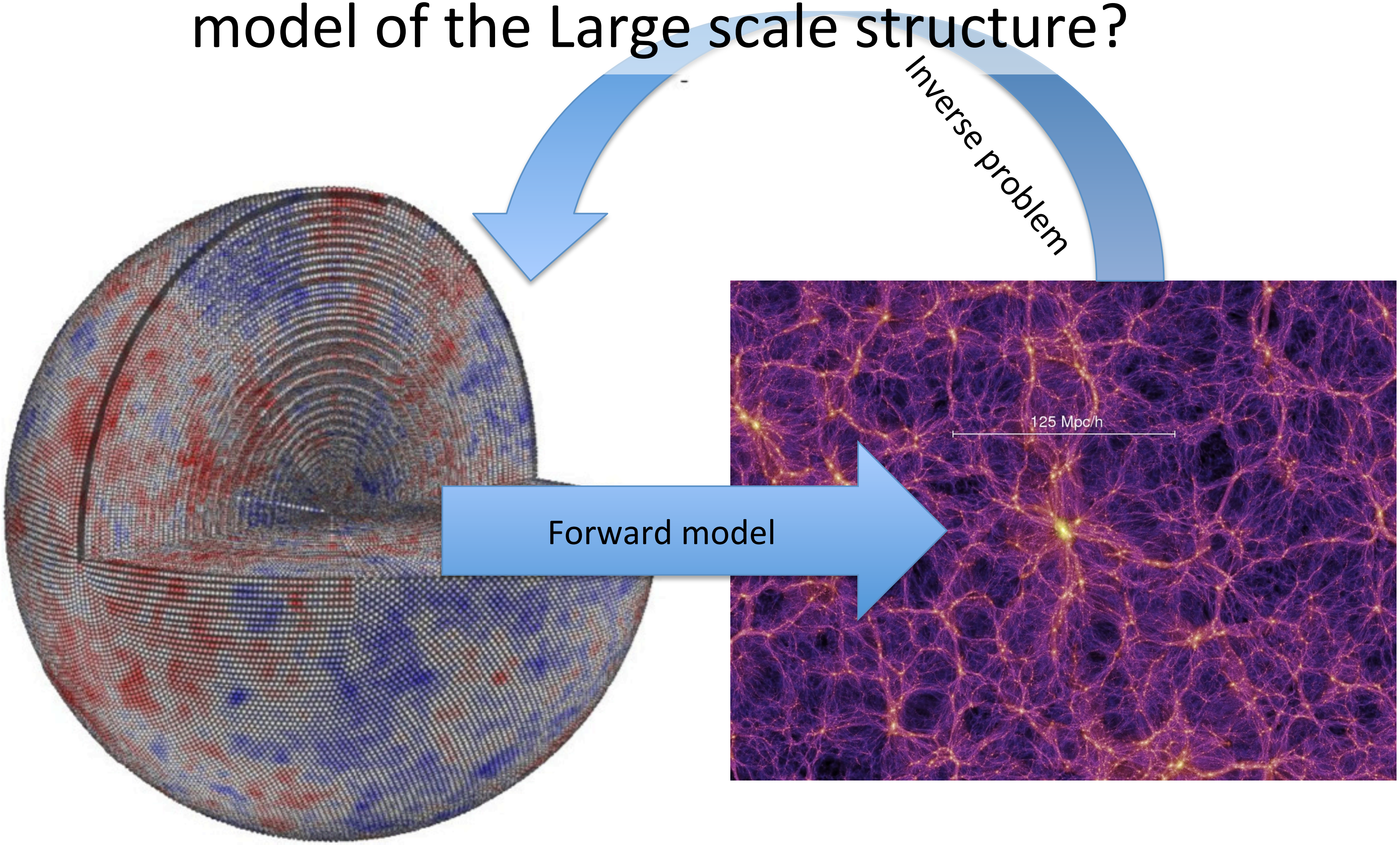
Beyond the CMB

The large scale structure challenge

- ***Problem: to access the small scales need to deal with non-linearity and "bias"***

- Possible approaches:
  - **Avoid**: Observe at high redshift before density contrast became non-linear
  - **Adapt**: Focus on less complicated parts of the Universe, e.g. those that retain more memory of the initial conditions: cosmic voids
  - **Attack**: Physical & statistical forward model of the survey, bias, etc. (perturbative or non-linear)

"Attack" mode: can we invert a full forward model of the Large scale structure?



Primordial curvature perturbation

Non-linear structure

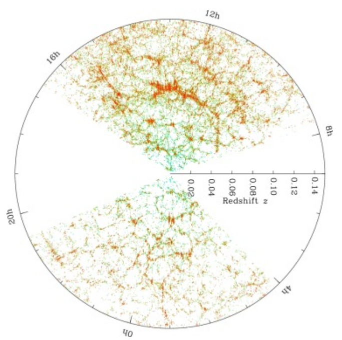
"It is impossible to predict anything with perfect determinism because it deals with probabilities from the outset."

Arthur Eddington

A probabilistic model of Galaxy surveys

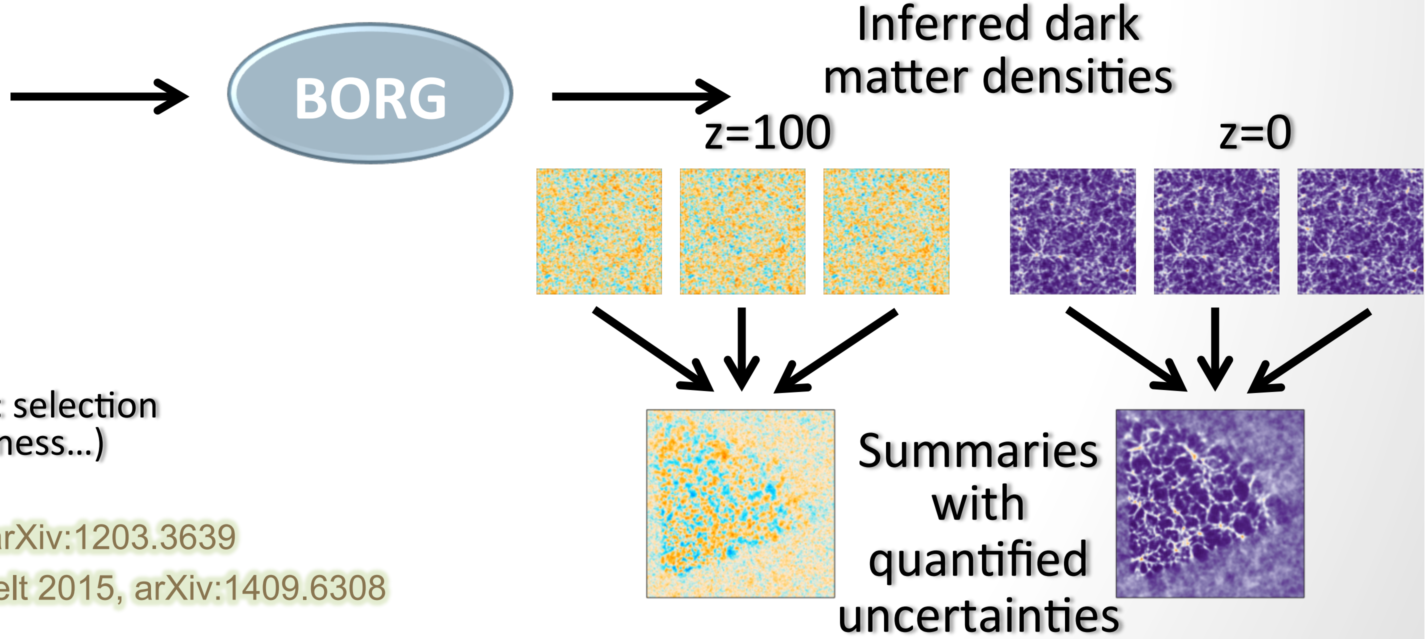
BORG: *Bayesian Origin Reconstruction from Galaxies*

- **Data model**: Gaussian prior – Second-order Lagrangian perturbation theory (2LPT) + likelihood for galaxies
  (also treats: bias model, automatic noise level calibration, selection function, mask, …)
- **Sampler**: Hamiltonian Markov Chain Monte Carlo method



Observations

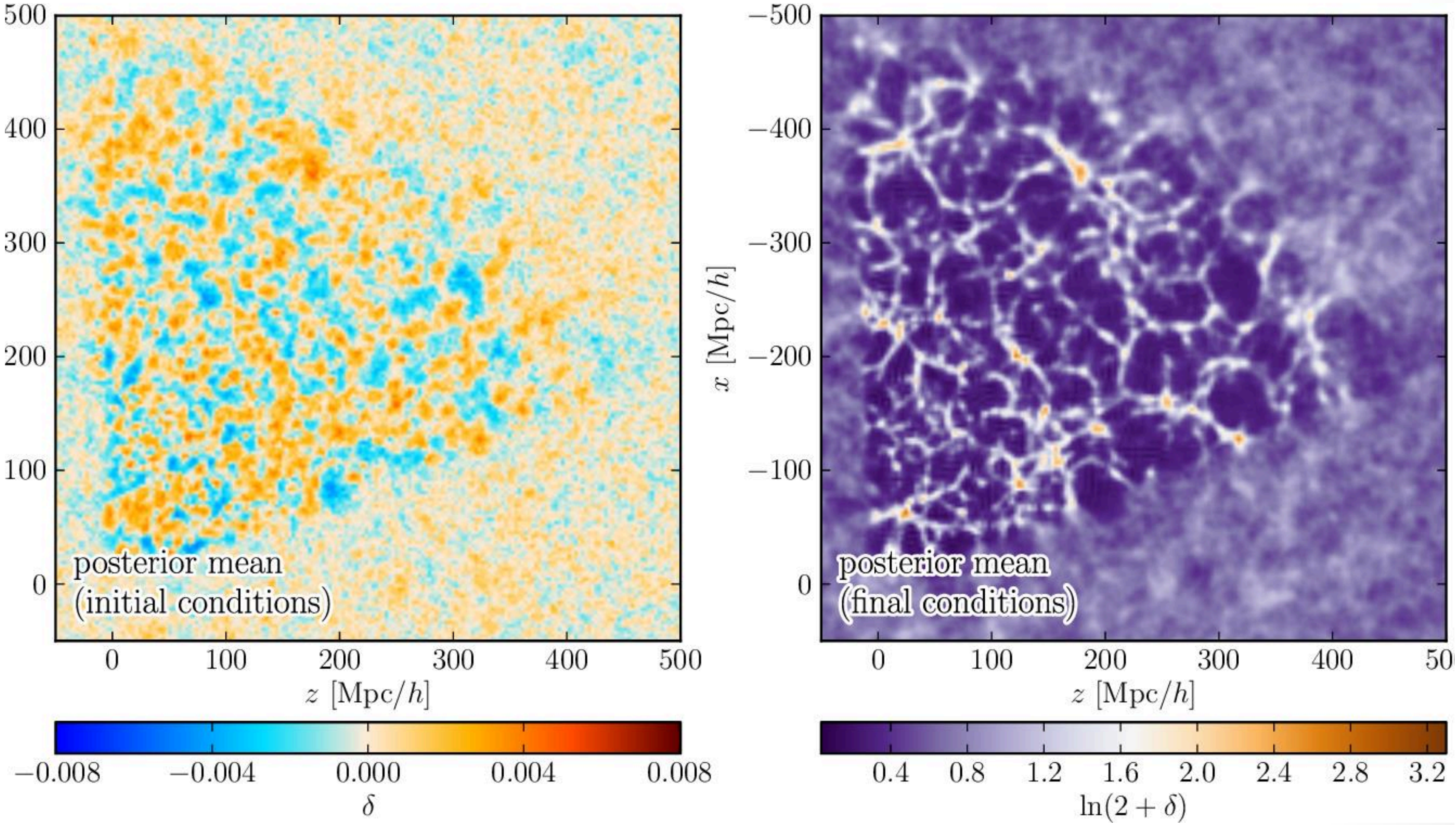
(galaxy catalog + meta-data: selection functions, completeness…)



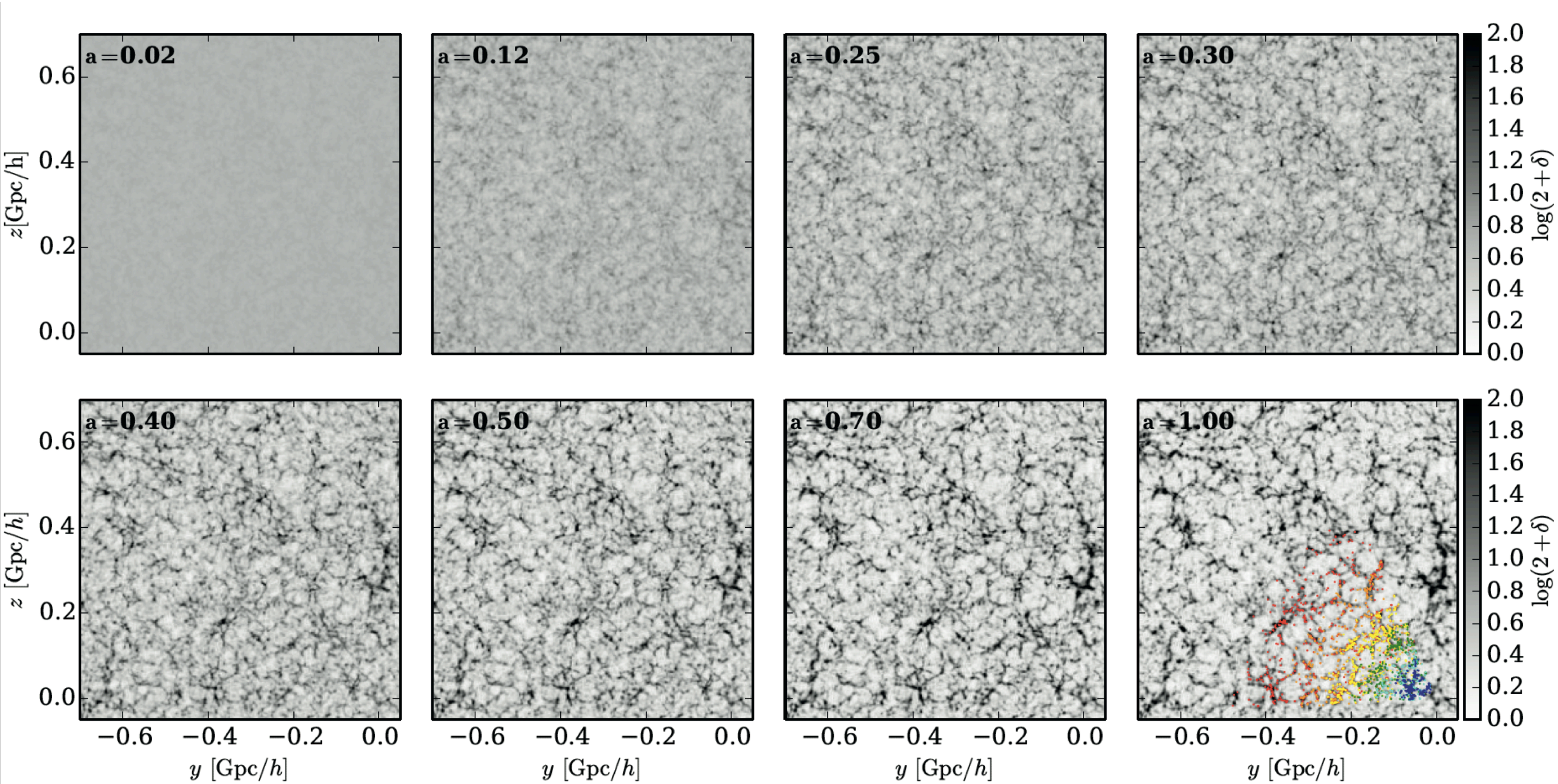
Jasche & Wandelt 2013, arXiv:1203.3639

Jasche, Leclercq & Wandelt 2015, arXiv:1409.6308

Bayesian chronocosmography from SDSS

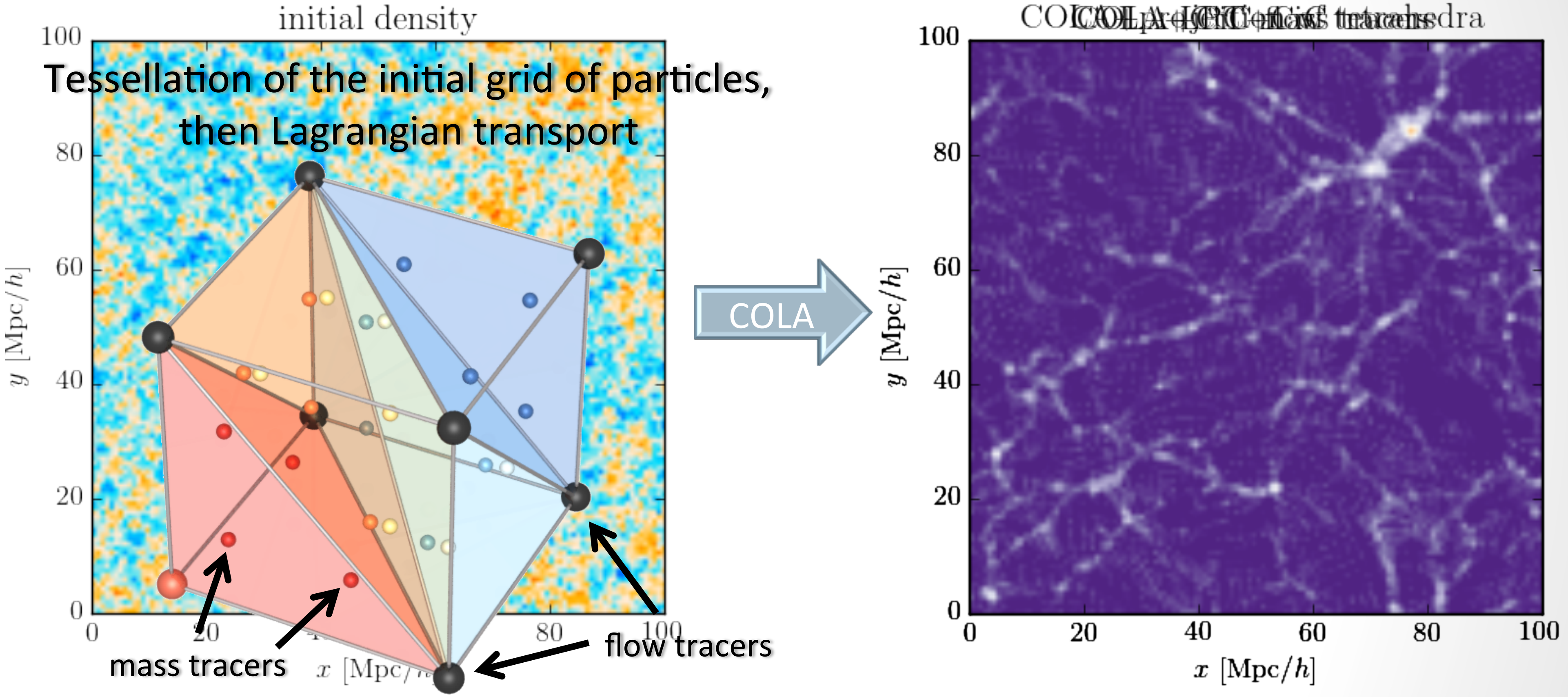


A reconstructed formation history of our Universe



Jasche, Leclercq & BDW 2014, arXiv:1409.6308

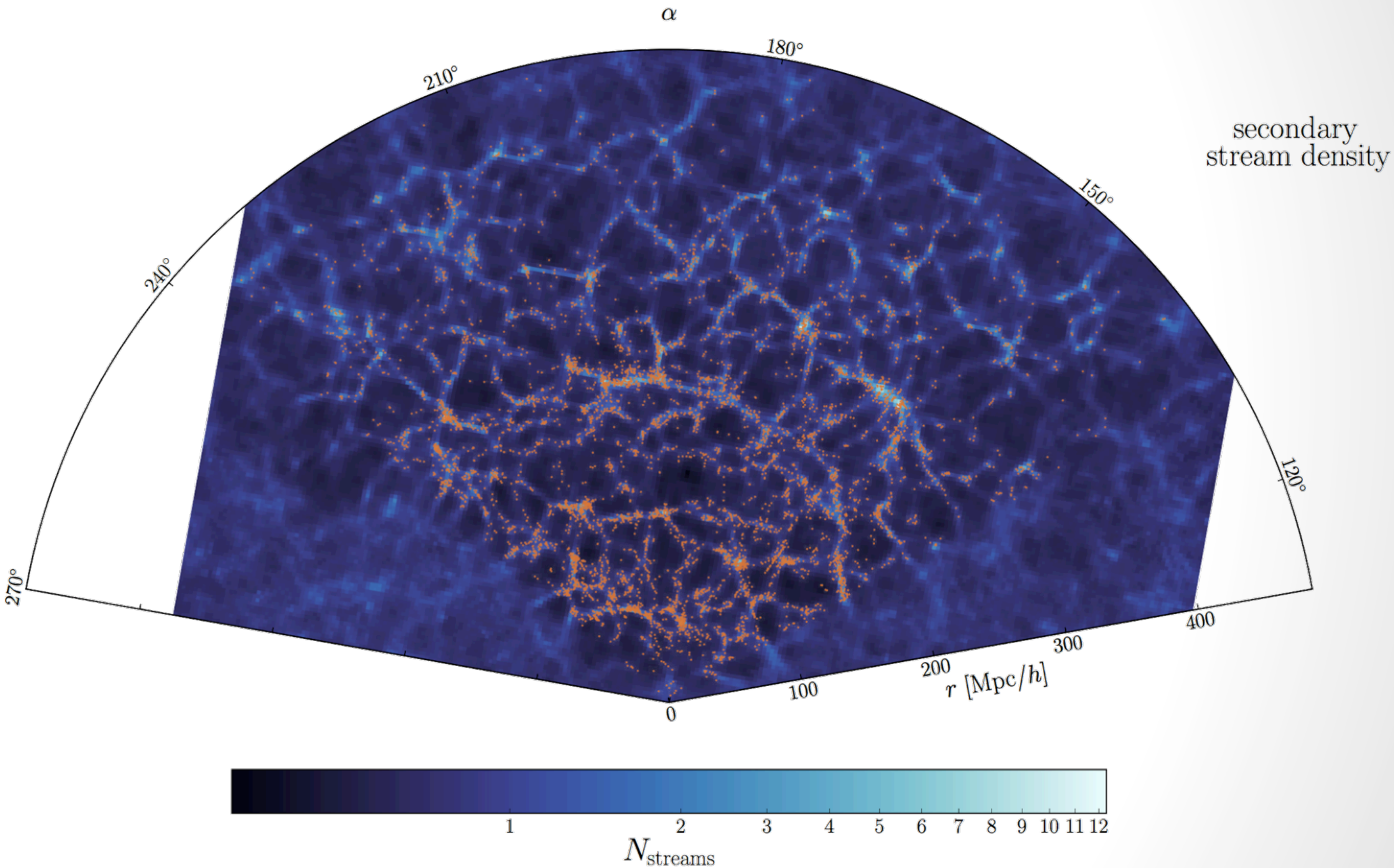
Methods that could only be used on simulations can now be used on data



Hahn, Abel & Khaeler, arXiv:1210.6652

Neyrinck 2012, arXiv:1202.3364
Abel, Hahn & Kaehler 2012, arXiv:1111.3944
Shandarin, Habib & Heitmann 2012, arXiv:1111.2366

Counting Lagrangian streams in the SDSS volume

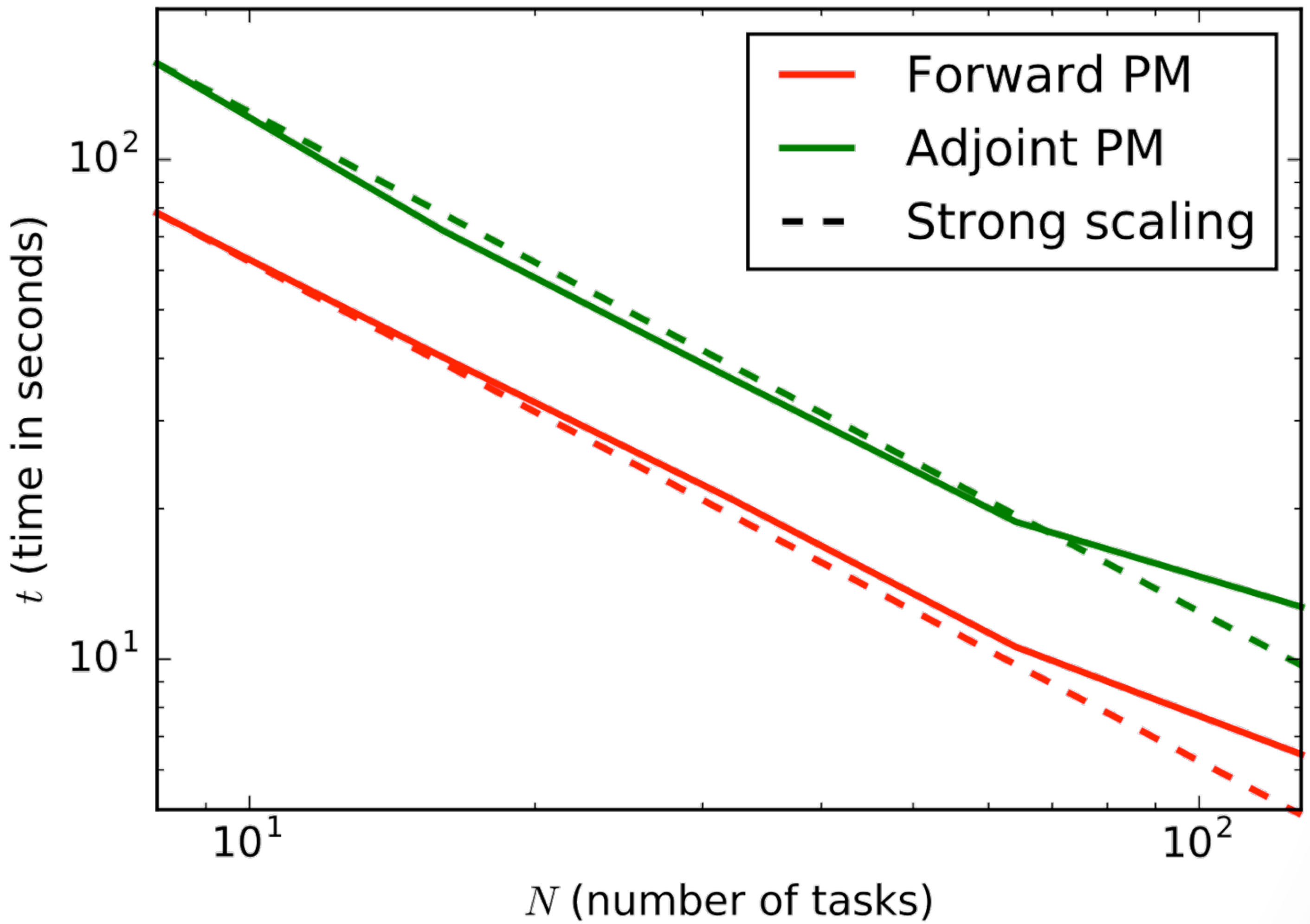


Leclercq et al. 2017

Recent news: Borg 3

- Lavaux and Jasche (in prep.)
- Full re-code of BORG
- Now includes
  - **fully non-linear PM** for gravitational dynamics
  - redshift space distortions
  - Multiplicative and additive foreground templates
  - sub-Poissonian likelihood
- Example:
  - 2M++ catalog (67,224 galaxies from the (based on 2MRS, 6dF and SDSS-DR7))
  - Comoving cubic box of 677.7 Mpc/h
  - 256^3 grid, resolution 2.64 Mpc/h => 17 million parameters
  - 3 bias parameters and mean galaxy density sampled, each per subcatalog
  - 4 luminosity bins

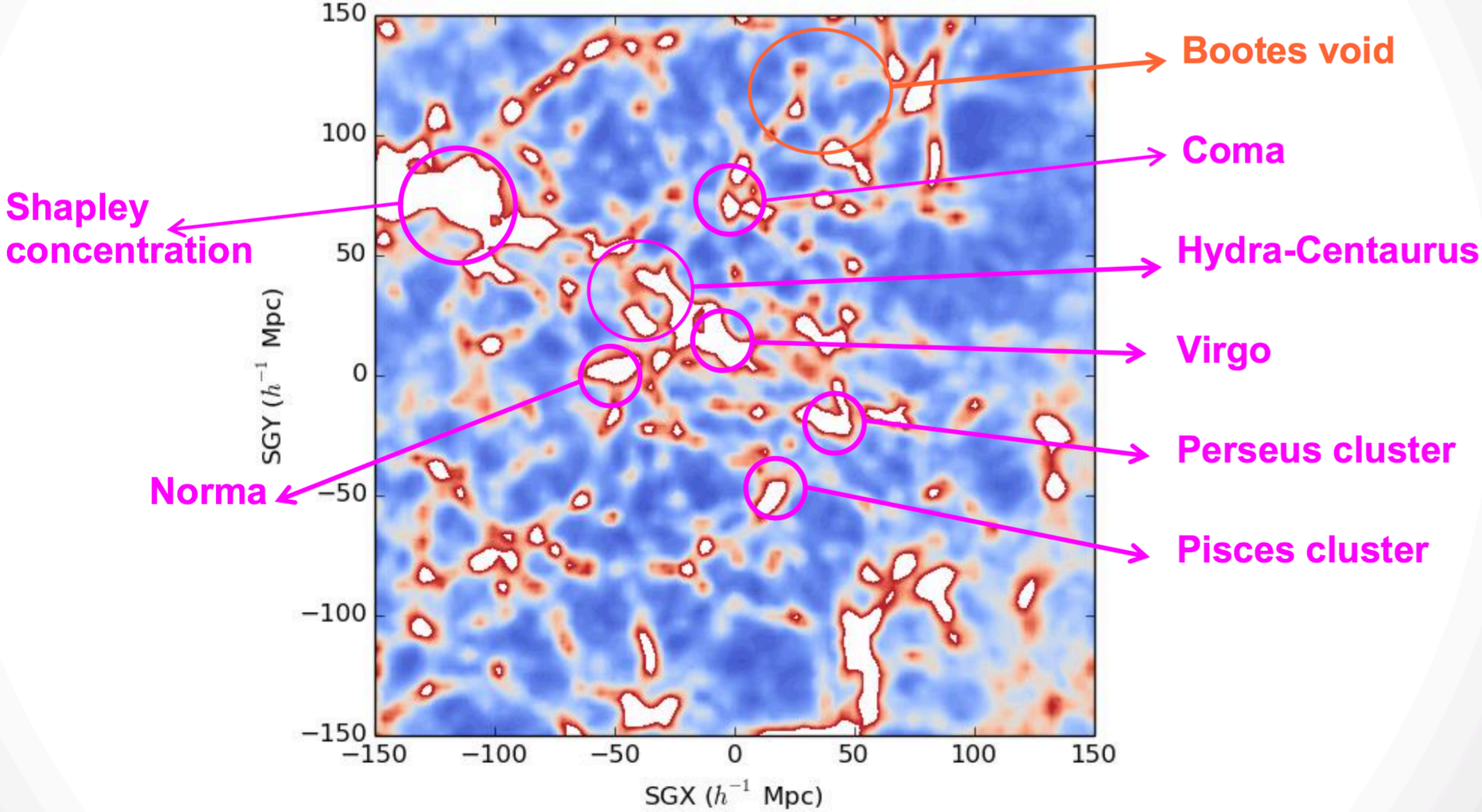
BORG3 scales to large problem sizes



512^3 particles and grid

BORG3 density field

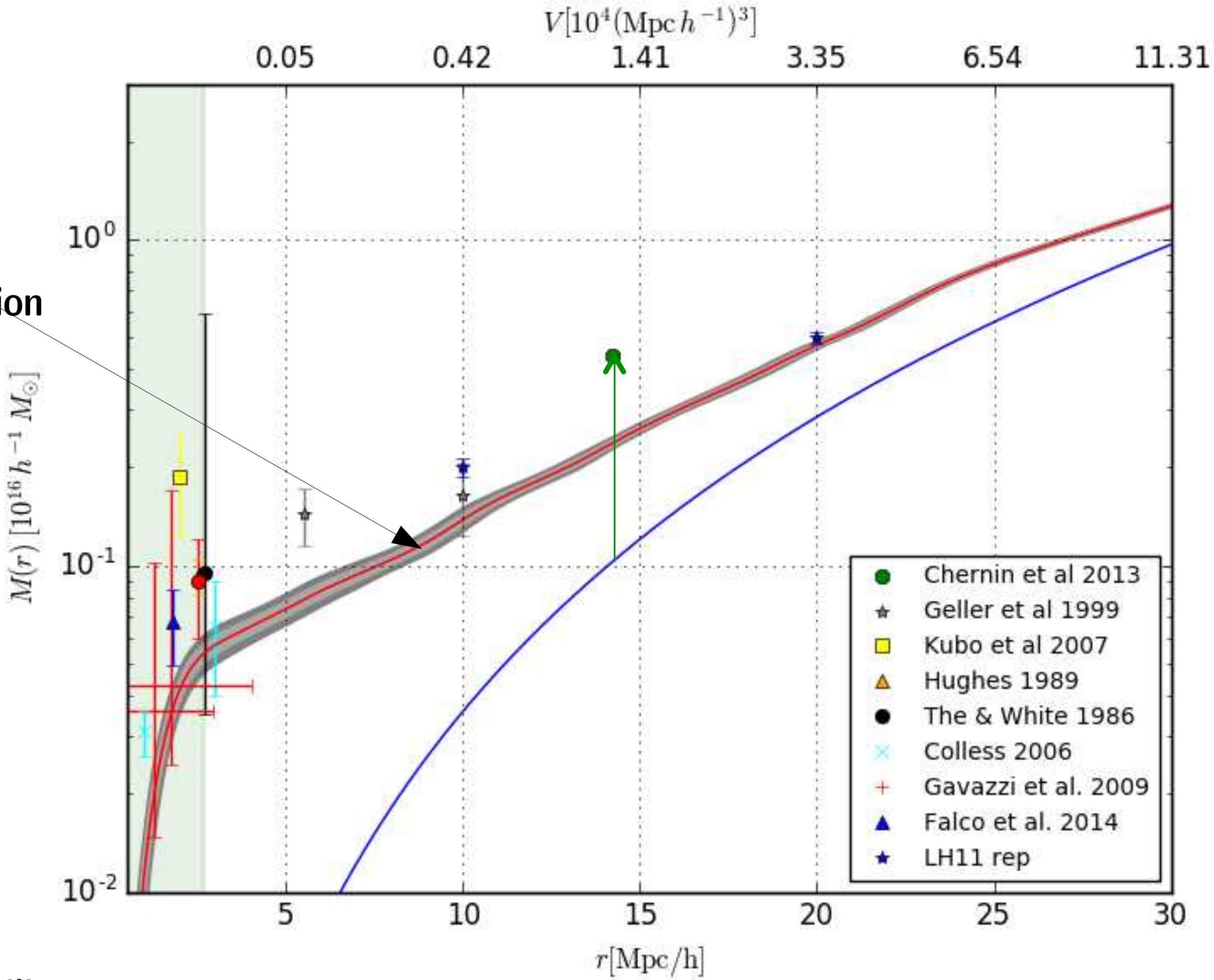
Supergalactic plane, no smoothing



Coma formation history - movie

Coma mass profile

Image of the posterior distribution



Mean

68% probability

95% probability

Lavaux & Jasche, 2017, in prep.

Summary

- We can constrain cluster optical depth with the correlated part of the polarized SZ effect
- The Messenger method is a powerful and simple solver for optimal filtering
- "Parameter map" diagnostic for cosmological surveys; feasible with fast parameter estimation using quadratic estimator

- Non-linear Bayesian analysis of CMB (lensing) and large scale structure data (full Bayesian model) is becoming feasible, opening up many new applications. Now have first validations with other probes.