BERKELEY CENTER *for*
COSMOLOGICAL PHYSICS

Forward Model Approach to LSS Analysis

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Low Redshift Universe, Stockholm, July 2017

Forward Model Approach to LSS Analysis

Statistical Bayesian analysis:

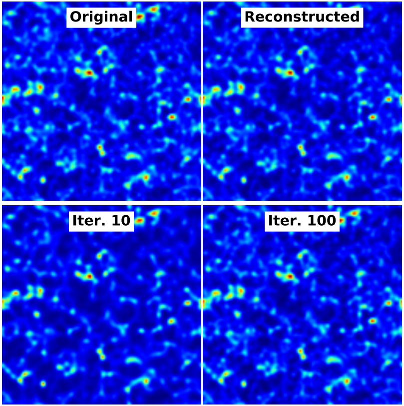
$$L(s|d) = (2\pi)^{\frac{-(M+N)}{2}} \det(SN)^{\frac{-1}{2}} \exp\left(\frac{-s^{\dagger}S^{-1}s - [d - f(s)]^{\dagger} N^{-1} [d - f(s)]}{2}\right)$$

$$\chi^2(s) = s^{\dagger}S^{-1}s + [d - f(s)]^{\dagger}N^{-1}[d - f(s)],$$

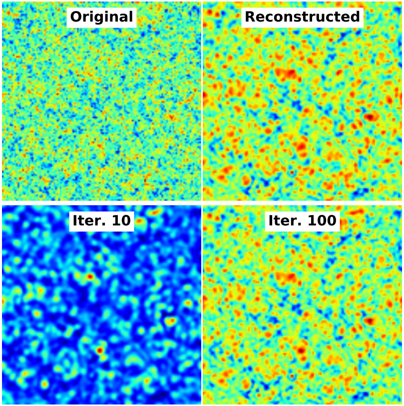
- **Prior**: Gaussian initial density Fourier modes 's' with linear power spectrum 'S'
- **Forward Model *f*(*s*)**: take the initial density modes 's' and evolve them to the final observable corresponding to the data 'd'
- **High dimensional optimization problem** - (L)BFGS, HMC, Conjugate gradient ⇒ Need gradients of forward model F(s)

MAP of Map

Final Conditions



Initial Conditions



Seljak et.al(arXiv:1706.06645)

Power Spectrum Reconstruction

Integrate out the modes around the minimum variance map

$$P(\mathbf{\Theta}|\mathbf{d}) \propto L(\mathbf{d}|\mathbf{\Theta}) = \int P(\mathbf{s},\mathbf{d}|\mathbf{\Theta}) d^M \delta \mathbf{s}$$

$$= (2\pi)^{\frac{-(M+N)}{2}} \det(\mathbf{SN})^{\frac{-1}{2}} \exp\left(-\frac{1}{2}\left[\hat{\mathbf{s}}^\dagger \mathbf{S}^{-1}\hat{\mathbf{s}} + (\mathbf{d}-\mathbf{f}(\hat{\mathbf{s}}))^\dagger \mathbf{N}^{-1}(\mathbf{d}-\mathbf{f}(\hat{\mathbf{s}}))\right]\right) \times$$

$$\int \exp\left\{-\frac{1}{2}\delta\mathbf{s}^\dagger \mathbf{D}\delta\mathbf{s}\right\} d^M \mathbf{s}$$

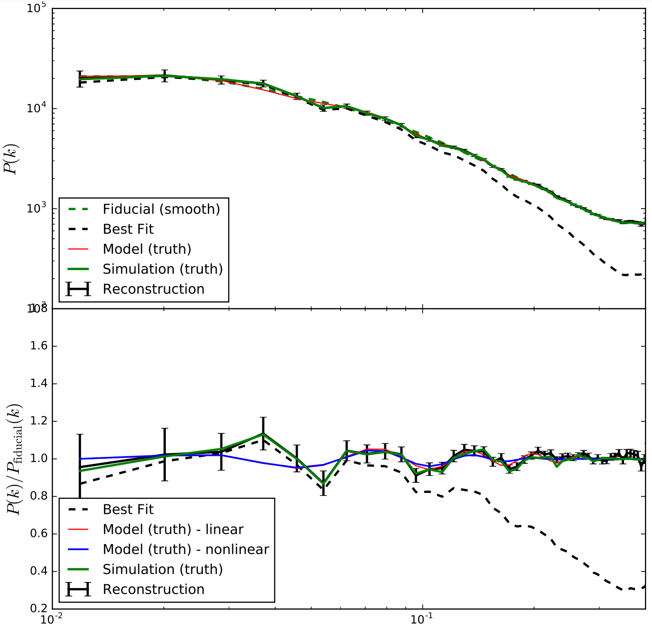
$$= (2\pi)^{-N/2} \det(\mathbf{SN}\tilde{\mathbf{D}})^{-1/2} \exp\left(-\frac{1}{2}\left[\hat{\mathbf{s}}^\dagger \mathbf{S}^{-1}\hat{\mathbf{s}} + (\mathbf{d}-\mathbf{f}(\hat{\mathbf{s}}))^\dagger \mathbf{N}^{-1}(\mathbf{d}-\mathbf{f}(\hat{\mathbf{s}}))\right]\right)$$

Use Newton's Method to maximize the posterior

$$\ln L(\mathbf{\Theta}_{\text{fid}} + \Delta\mathbf{\Theta}) = \ln L(\mathbf{\Theta}_{\text{fid}}) + \sum_l \left[\frac{\partial \ln L(\mathbf{\Theta})}{\partial \Theta_l}\right]_{\mathbf{\Theta}_{\text{fid}}} \Delta\Theta_l + \frac{1}{2}\sum_{ll'}\left[\frac{\partial^2 \ln L(\mathbf{\Theta})}{\partial\Theta_l \partial\Theta_{l'}}\right]_{\mathbf{\Theta}_{\text{fid}}} \Delta\Theta_l \Delta\Theta_{l'}$$

$$(\mathbf{F}\Delta\hat{\mathbf{\Theta}})_l = \frac{1}{2}\hat{\mathbf{s}}^\dagger \mathbf{S}_{\text{fid}}^{-1} \mathbf{\Pi}_l \mathbf{S}_{\text{fid}}^{-1}\hat{\mathbf{s}} - b_l$$

Power Spectrum Reconstruction



Beyond Matter

- Redshift surveys observe galaxies
- Traditionally modeled as residing in halos
- Halo finders are complicated and not differentiable, need an analytic model
- Poisson sampling of halos

Bias Prescription

$$\delta_h(x) = b_1\delta_m(x) + b_2\delta_m^2(x)$$

Pros:

- Simple
- Well studied
- Performs quite well to model power spectra

Cons:

- Which biasing scheme?
- Mass information is marginalized over
- Negative b_2 leads to negative over-densities
- Model a discrete field with continuous field

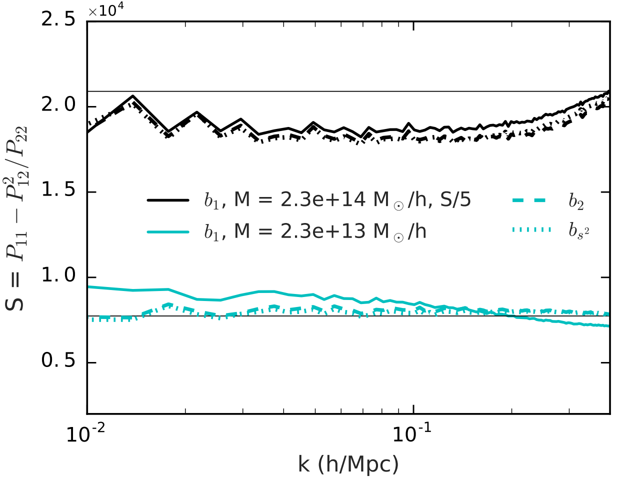
Forward model approach requires comparing the two fields at point level.

Stochasticity

$$S_{XY} = P_{XX} - \frac{P_{XY}^2}{P_{YY}}$$

X = halo field (δ_h)
Y = model field
$(b_1\delta_m + \frac{b_2}{2}\delta_m^2 + b_{s^2}s^2)$

Modi et.al (arXiv: 1612.01621)

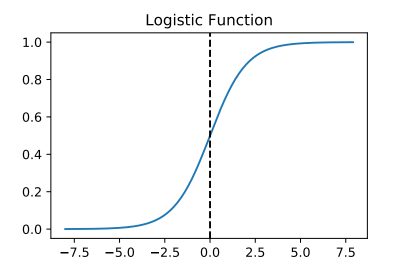


Analytic Transformations of Density Field- Sigmoid

Halos form at positions where the over-density crosses a threshold (δ_T)

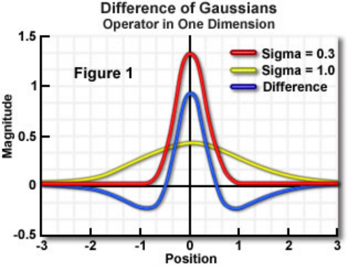
$$g(\delta) = \frac{1}{1 + e^{-z}}$$
$$z = \alpha(\delta - \delta_T)$$

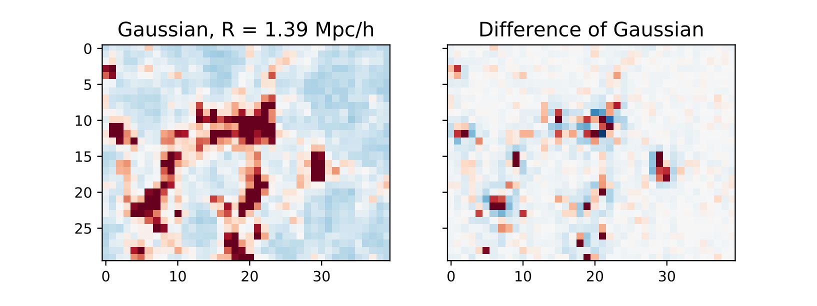
In a spherical collapse universe: $\delta_T = 1.686\,(178)$



Analytic Transformations - Difference of Gaussians

- Halos are peaks
- Blob detection in image processing - Difference of Gaussian
- Excursion set peaks in scales



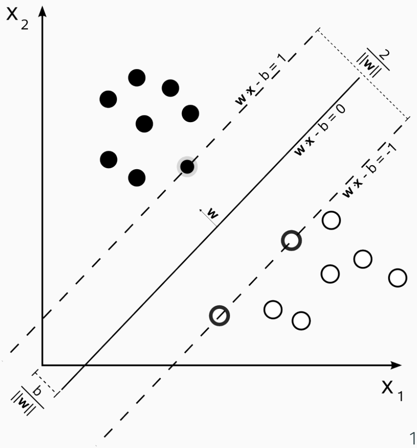


Analytic Transformations - Support Vector Machines

Given labeled training data, the support vector machines outputs an optimal hyperplane which categorizes new examples

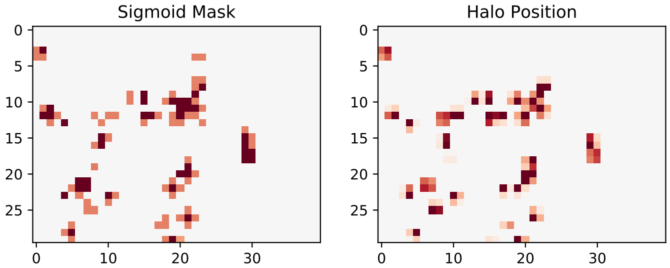
- Allows us to go beyond 2 features - shear, velocity field
- A semi-rigorous way to find the thresholds

$$\sum_i w_i \times f_i + z = 0$$



Mask of positions

How do positions look like?

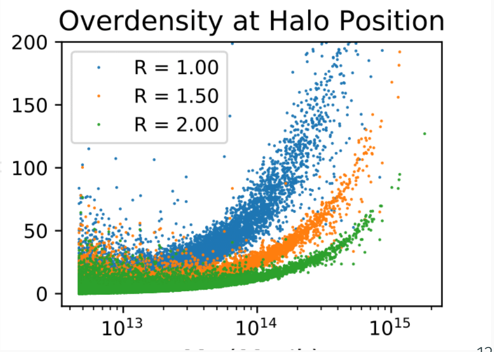


Mass Scaling

- Can use information about mass in addition to position from the data (through luminosity, number density etc.)
- From simulations, use gridded smoothed density field at multiple smoothing scales to get an estimate of mass
- Develop analytic formulation or simply use ML

$$M_h = 10^y \times (a\delta^2 + b\delta + c)$$

a, b, c, y : Free parameters that can be fit for



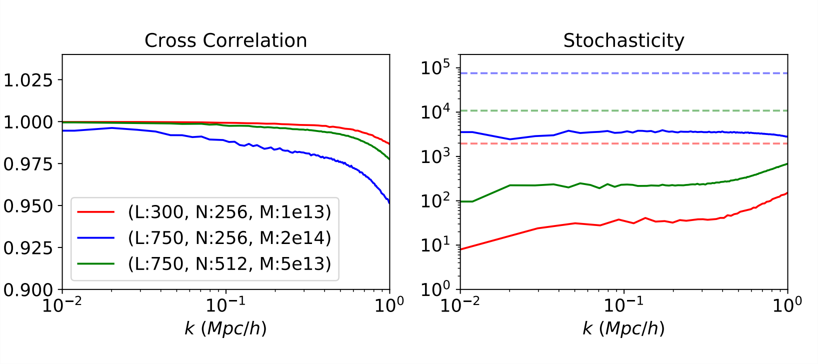
Performance: 2 Point Statistics

Cross correlation coefficient

$$C = \frac{P_{XY}}{\sqrt{P_{XX} P_{YY}}}$$

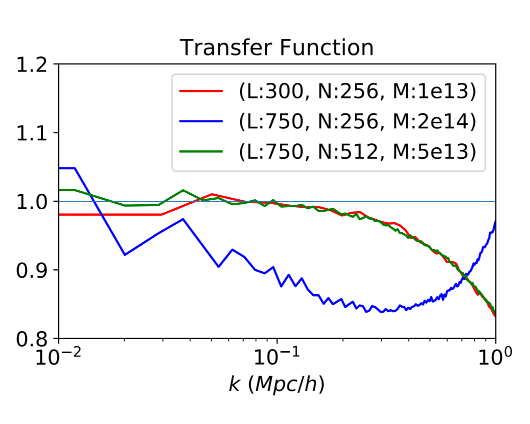
Stochasticity

$$S = P_{XX} - \frac{P_{XY}^2}{P_{YY}}$$

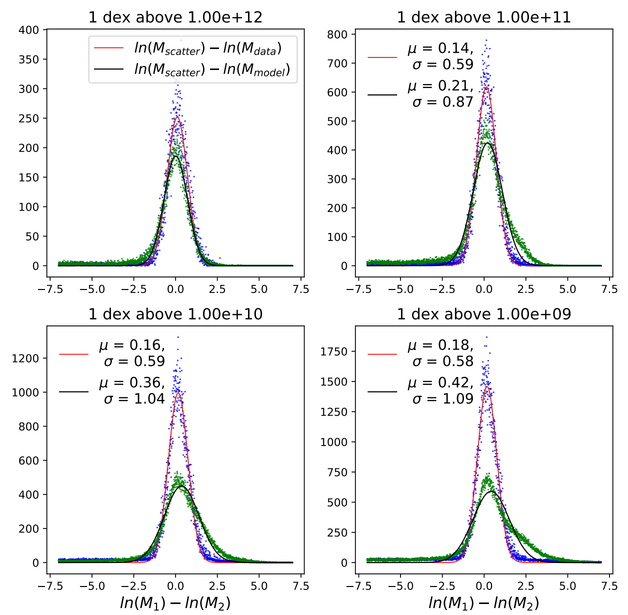


Performance: 2 Point Statistics

- Order of magnitude reduction in stochasticity
- Cross-correlation coefficient > 0.95
- Transfer function ~ 1
- Fit parameters independent of initial seed



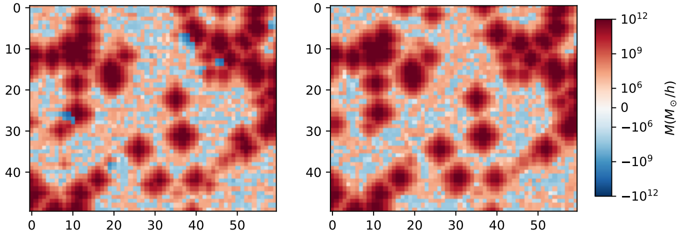
Noise Model



1 dex above 1.00e+12
$ln(M_{scatter}) - ln(M_{data})$
$ln(M_{scatter}) - ln(M_{model})$
1 dex above 1.00e+11
μ = 0.14,
σ = 0.59
μ = 0.21,
σ = 0.87
1 dex above 1.00e+10
μ = 0.16,
σ = 0.59
μ = 0.36,
σ = 1.04
$ln(M_1) - ln(M_2)$
1 dex above 1.00e+09
μ = 0.18,
σ = 0.58
μ = 0.42,
σ = 1.09
$ln(M_1) - ln(M_2)$

Picture Worth a Thousand Words?

One of these is the analytically created halo field and other shows FOF halos (both smoothed on some scale).

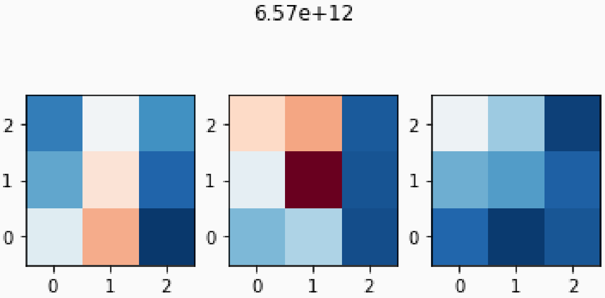


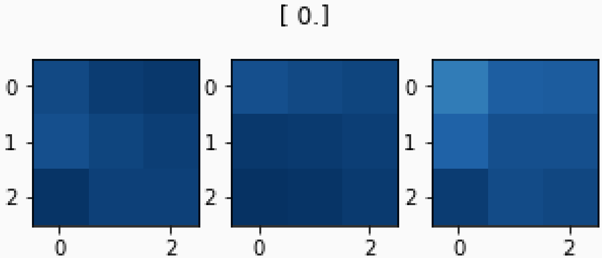
Summary

- Forward model approach to LSS in Bayesian framework
- Reconstruct initial density field and power spectrum starting from final matter field
- Need analytic methods to model galaxies and halos in order to use redshift survey data
- Bias model does not model halo field very well
- We propose modeling by using logistic function and difference of Gaussian to model halos as density threshold and peak
- This reduces stochasticity by about an order of magnitude

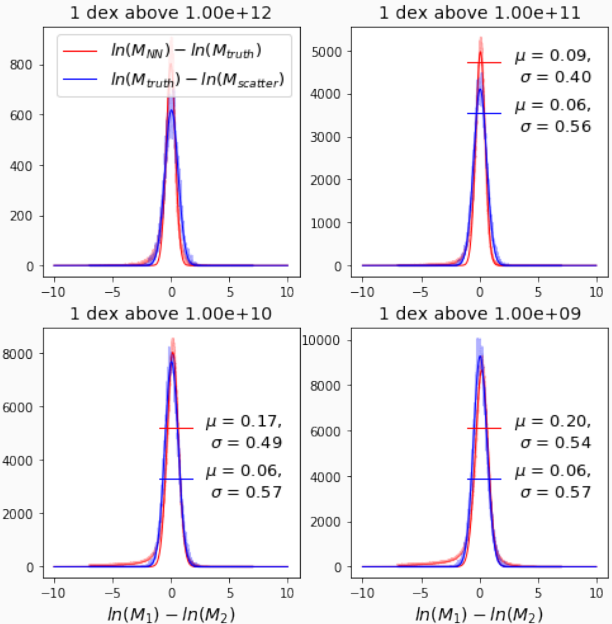
Thank You!

Neural Net Features





Point by point comparison - Neural Net



Noise Model - Neural Net