Inflationary Paradigm in Modified Gravity

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Jul. 21th, Nordita, Stockholm/Sweden

Theoretical modelling of the Universe

General Relativity

$$S = \int \frac{M_{\rm Pl}^2}{2}\sqrt{-g}R + \mathcal{L}_{\rm matter}$$

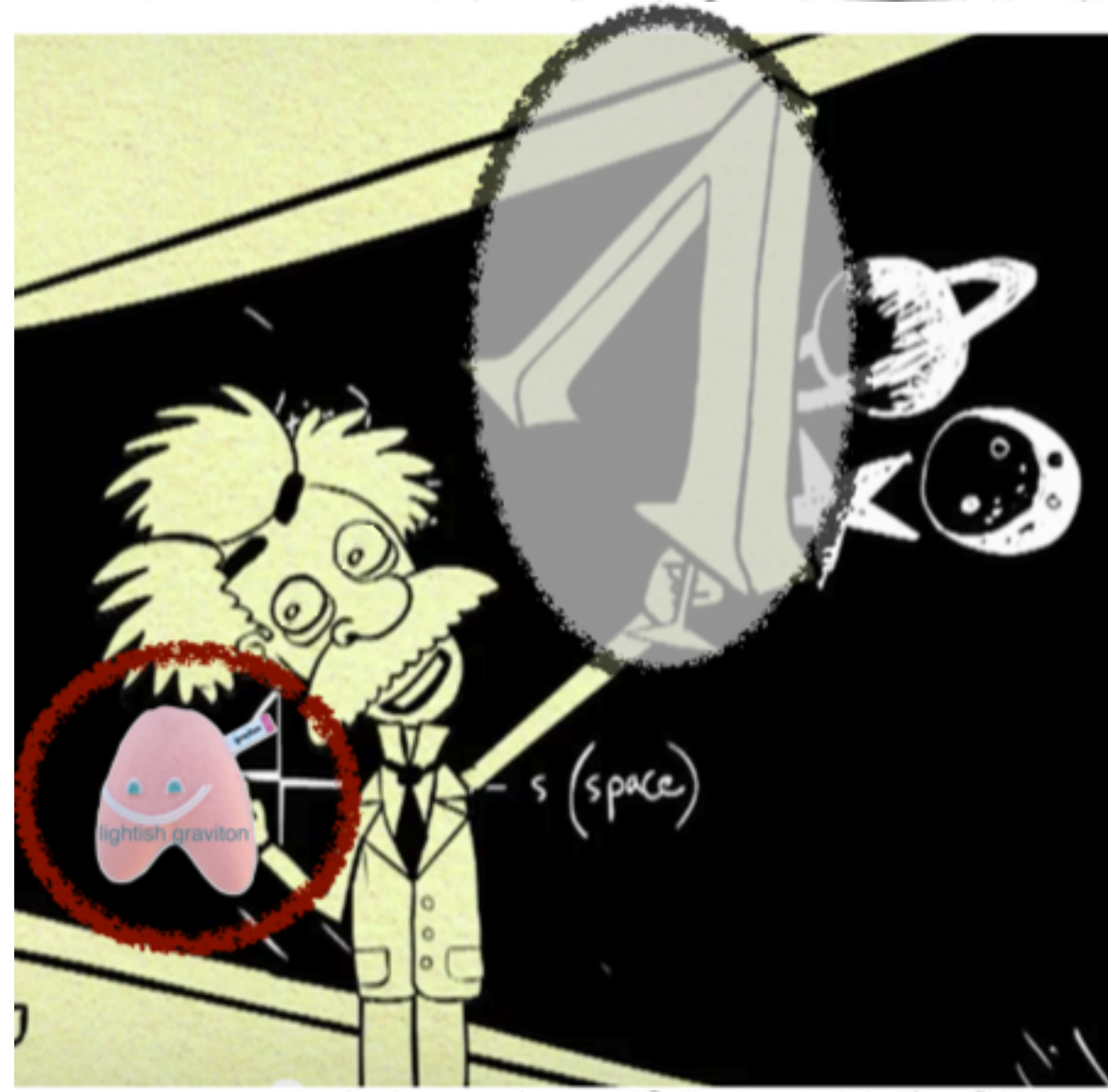
varying the action

- **geometry = matter**

Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

$\partial^2 g$ $\qquad (\rho, p)$



Homogeneity & Isotropy

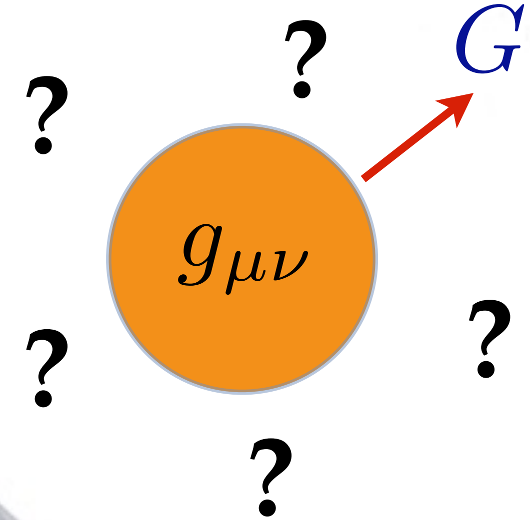
Friedmann Lemaitre Robertson Walker (FLRW) solutions

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$$

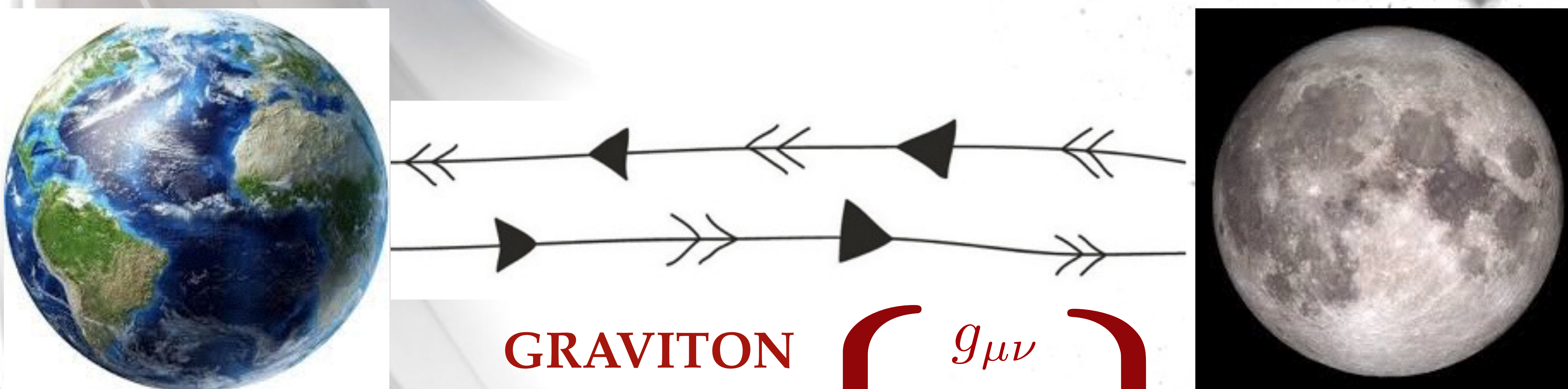
Challenges

- Cosmological and black hole singularities → failure of GR for gravitational phenomena at UV?
- Quantum theory of gravity → UV completion?
- Cosmological Constant Problem
- The necessity of Inflation?

Modified Gravity



$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



GRAVITON $\left\{ \begin{matrix} g_{\mu\nu} \\ f_{\mu\nu} \\ A_\mu \\ \phi \end{matrix} \right\}$

Modified Gravity

MG

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

add new additional fields to the gravity sector

- **scalar-tensor theories (Horndeski interactions)**
- **vector-tensor theories (generalized Proca interactions)**
- **tensor-tensor theories (massive gravity, bigravity)**

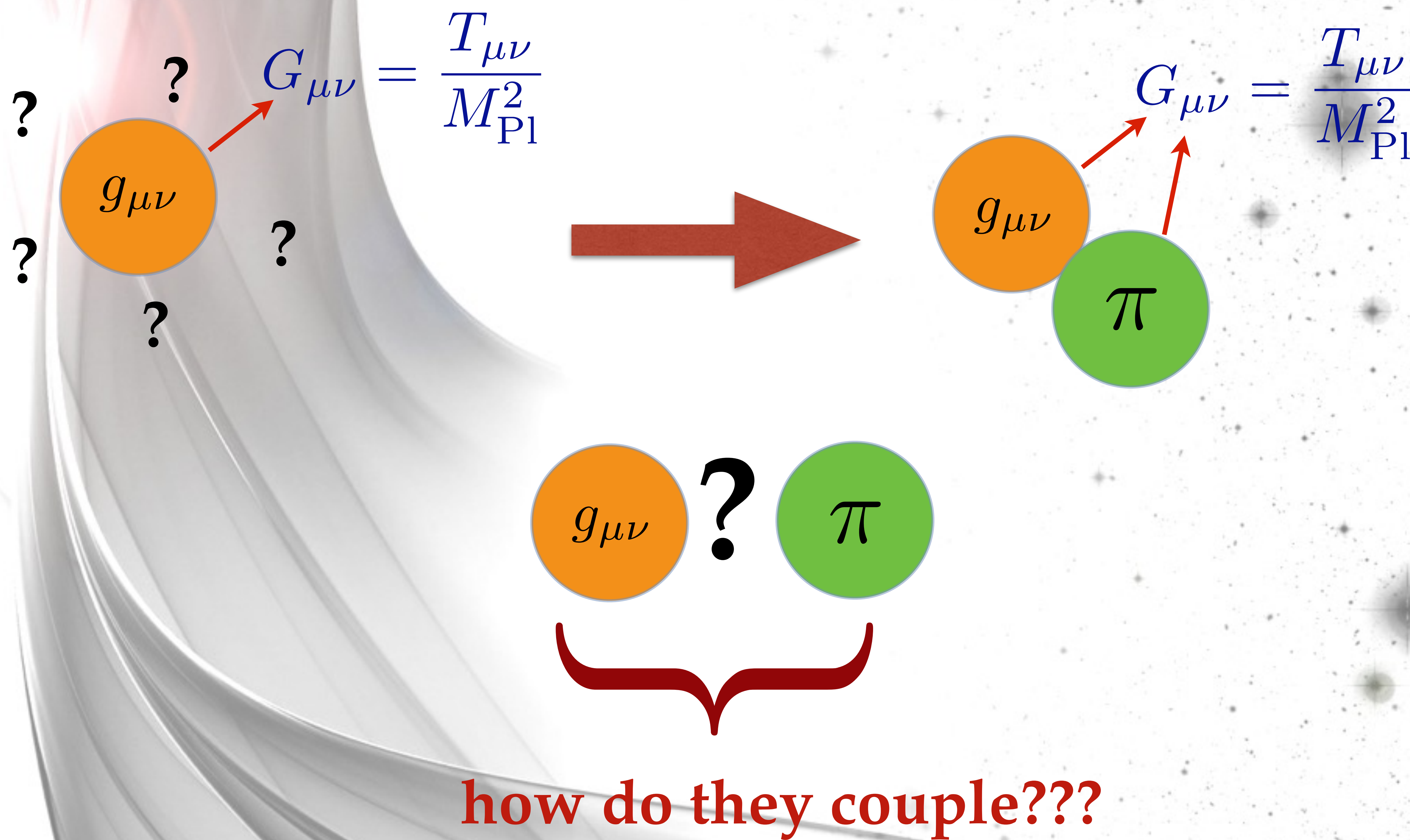
Modified Gravity

MG

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

- **Maybe not modifying that much!**

Horndeski theory (scalar-tensor theory)



Horndeski theory (scalar-tensor theory)

$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4,X}([\Pi]^2 - [\Pi^2])$$

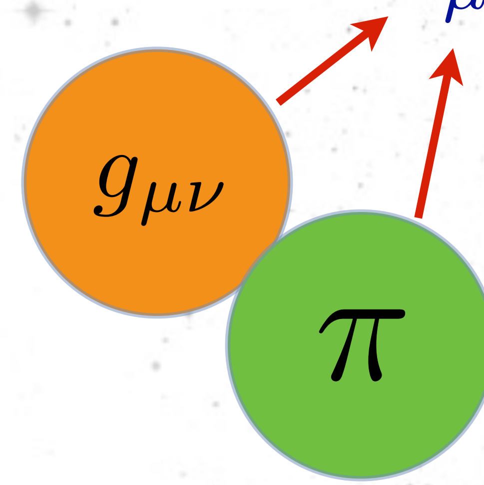
$$\mathcal{L}_5 = G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu} - \frac{G_{5,X}}{6}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^2])$$

G.W.Horndeski Int. J. Theo.Phys. 10, 363-384 (1974)

C.Deffayet, Esposito-Farese,Vikman Phys. Rev.D (79), 084003 (2009)

Horndeski theory (scalar-tensor theory)

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

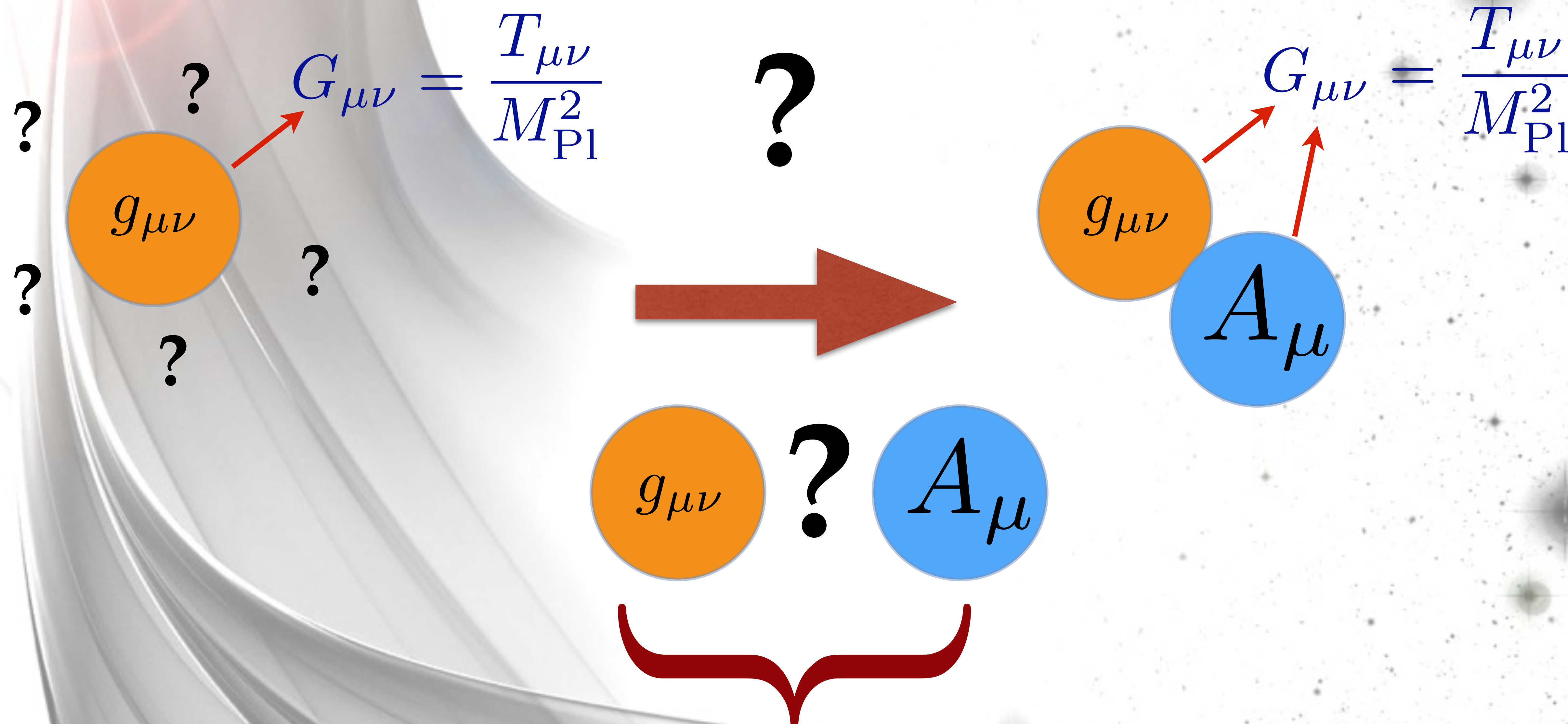
homogeneous &
isotropic solutions

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$\pi = \pi(t)$$

- (quasi-)de Sitter solutions
- bouncing solutions

? Vektor-Tensor Theories ?



how do they couple???

Generalized Proca action

Interactions on curved space-time requires the presence of non-minimal couplings to gravity

L.H & J.Beltran, Phys.Lett.B757 (2016) 405-411, arXiv:1602.03410

L. H., JCAP 1405, 015 (2014), arXiv:1402.7026

G.Tasinato JHEP 1404 (2014)067 arXiv:1402.6450

Allys, Peter, Rodriguez, JCAP 1602 (2016) 02, 004

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = G_3(Y)\nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(Y)R + G_{4,Y}\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]$$

$$\mathcal{L}_5 = G_5(Y)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,Y}\Big[(\nabla\cdot A)^3 + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3(\nabla\cdot A)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\Big] - \tilde{G}_5(Y)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta$$

$$\mathcal{L}_6 = G_6(Y)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu$$

Cosmology with Vector Fields

Flat Friedmann-Lemaitre-Robertson-Walker background

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$$

Homogeneity & Isotropy

$$A_\mu = (\phi(t), 0, 0, 0)$$

$$A_i^0 = 0$$
$$A_i^a = A(t)\delta_i^a$$

A_μ

A_μ^a

Cosmology in Multi-Proca

Broken SU(2) symmetry A^a_μ

L.H & J.Beltran,
arXiv:1610.08960

$$\mathcal{L}_2 = G_2(A^2, F^a_{\mu\nu}) \,,$$

$$\mathcal{L}_4 = G_4 R + G_4' \delta_{ab} \frac{S^a_{\mu\nu} S^{b\mu\nu} - S^{a\mu}_{\mu} S^{b\nu}_{\nu}}{4} \,,$$

$$\mathcal{L}_6 = G_6 L^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \frac{G_6'}{2} \tilde{F}^{a\alpha\beta} \tilde{F}^{a\mu\nu} S^b_{\alpha\mu} S^b_{\beta\nu}$$

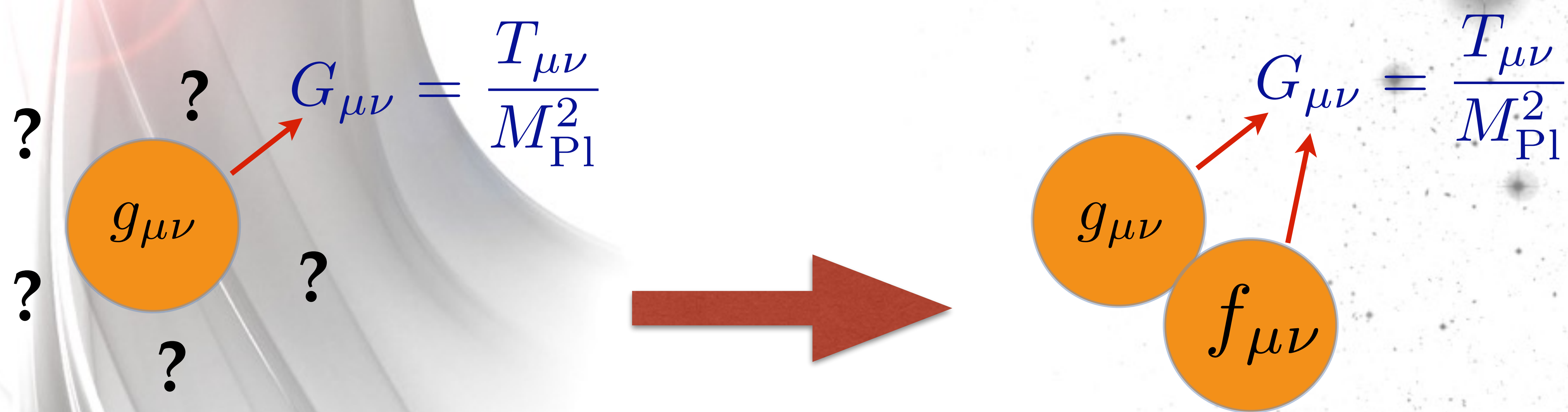
Homogeneity & Isotropy →

(New field configuration!)

(Not so far considered in the literature!)

$$A^0_i = \phi_i$$
$$A^a_i = A(t)\delta^a_i$$

Massive Gravity (tensor-tensor theory)



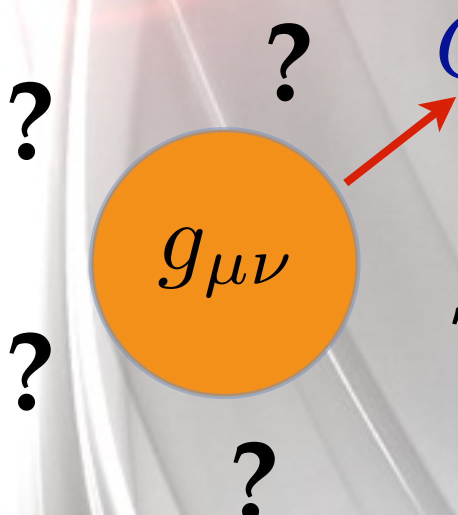
$$m^2\sqrt{g^{-1}f}$$

(for theoretical consistency)

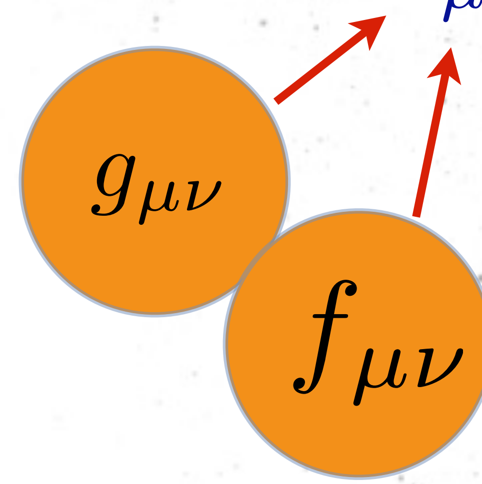
C. de Rham, G. Gabadaze, A.J.Tolley, PRL106 (2011)

S.F. Hassan, R.A. Rosen JHEP1107 (2011)

Massive Gravity (tensor-tensor theory)



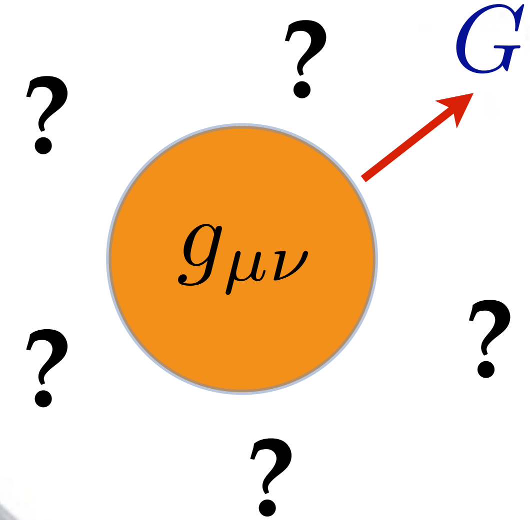
$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



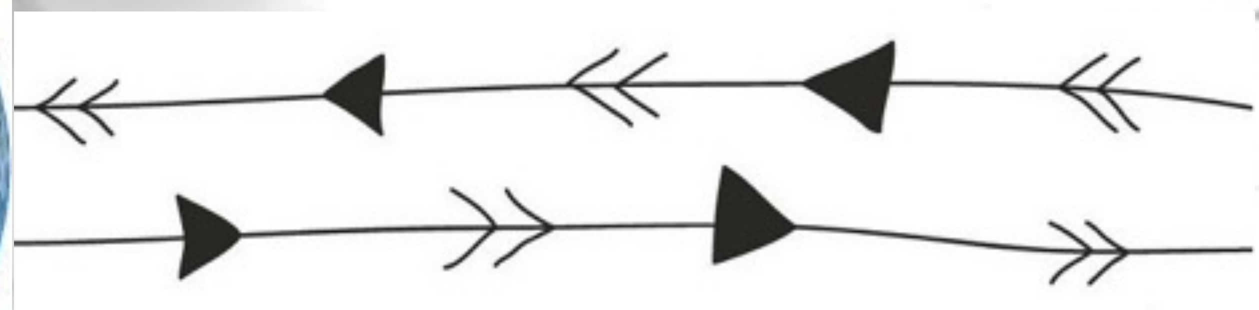
$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

$$ds_g^2 = -N_g^2 dt^2 + a_g^2 \delta_{ij} dx^i dx^j$$
$$ds_f^2 = -N_f^2 dt^2 + a_f^2 \delta_{ij} dx^i dx^j$$

Modified Gravity



$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



GRAVITON $\left\{ \; g_{\mu\nu} \; \right\}$

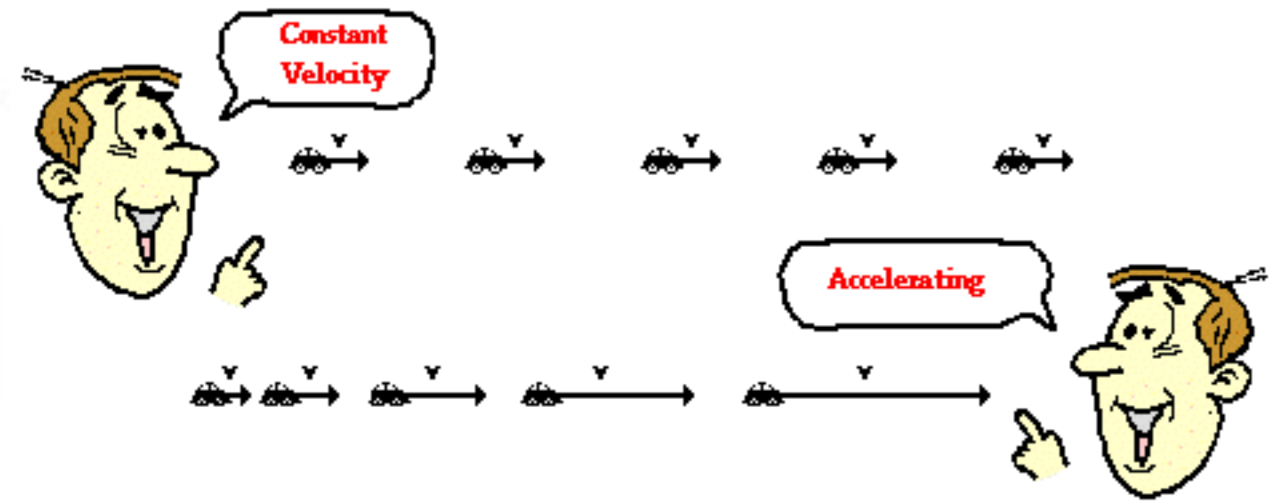
without adding additional degrees

beyond Riemanian geometry

Relativistic Particle

A free non-relativistic particle

$$\mathcal{S}_{\rm NR} = \int \frac{1}{2} m v^2 dt \qquad \vec{v} = \frac{d\vec{x}}{dt}$$



- The free particle moves with constant velocity
- This action allows the particle to move with any constant velocity

Principle of finiteness

A free relativistic particle

$$\mathcal{S}_{\rm R} = \int mc^2 dt \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

- The constraint of maximal velocity is implemented.
- In the limit of small velocities $|\vec{v}| << c$ we recover the non-rel. limit

Born-Infeld electromagnetism

$$\mathcal{S}_{\mathrm{Maxwell}} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$$

Principle of finiteness

$$\frac{1}{2}m^2\int \mathrm{d}t\, v^2 \to mc^2\int \mathrm{d}t\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)$$

$$\mathcal{S}_{\mathrm{BIE}} = -\lambda^4\int \mathrm{d}^4x\left[\sqrt{-\det\left(\eta_{\mu\nu}+\lambda^{-2}F_{\mu\nu}\right)}-1\right]$$

M. Born and L. Infeld. Proc.Roy.Soc.Lond. A144 .(1934)

For small electromagnetic fields it recovers Maxwell's theory:

$$\mathcal{S}_{\mathrm{BIE}}(F_{\mu\nu}\ll\lambda^2)\simeq\mathcal{S}_{\mathrm{Maxwell}}$$

For large electromagnetic fields it prevents the unlimited growth of electric and magnetic fields!

Born-Infeld inspired gravity

$$\mathcal{S}_{\rm DG} = \int \mathrm{d}^4x \sqrt{-\det\left(a g_{\mu\nu} + b R_{\mu\nu} + c X_{\mu\nu}\right)}$$

S. Deser and G. Gibbons, CQG15 (1998)

- $X_{\mu\nu} =$ Higher order curvature terms to be tuned to avoid ghosts
- $X_{\mu\nu}$ contains terms of quadratic and higher orders in $R_{\mu\nu}$
- There is a large freedom in the choice of $X_{\mu\nu}$ and no clear immediate criterion.

Born-Infeld inspired gravity

$$\mathcal{S}_{\rm Ed} = \lambda^4 \int d^4x \sqrt{\det R_{(\mu\nu)}(\Gamma)}$$

Determinantal actions for gravity were considered by Eddington (1924) as a purely affine theory.

Couplings to matter: The metric enters as an auxiliary field that can then be integrated out.

M. Ferraris and J. Kijowski,
Letters in Mathematical
Physics, 5 127-135, (1981)

Levi-Civita connection

$$\Gamma^\alpha_{\mu\nu} = \gamma^\alpha_{\mu\nu} + L^\alpha_{\mu\nu}(Q) + K^\alpha_{\mu\nu}(T)$$

$$\nabla_\mu g_{\alpha\beta} = Q_{\mu\alpha\beta} \qquad T^\alpha_{\mu\nu} = \Gamma^\alpha_{[\mu\nu]}$$

Non-metricity   Torsion

For Einstein-Hilbert   metric ⟺ Palatini

In general there are two independent traces of the Riemann tensor:

$$R^\alpha{}_{\alpha\mu\nu} \qquad R^\alpha{}_{\mu\alpha\nu}$$

also independent from $g^{\alpha\beta} R^\mu{}_{\alpha\beta\nu}$

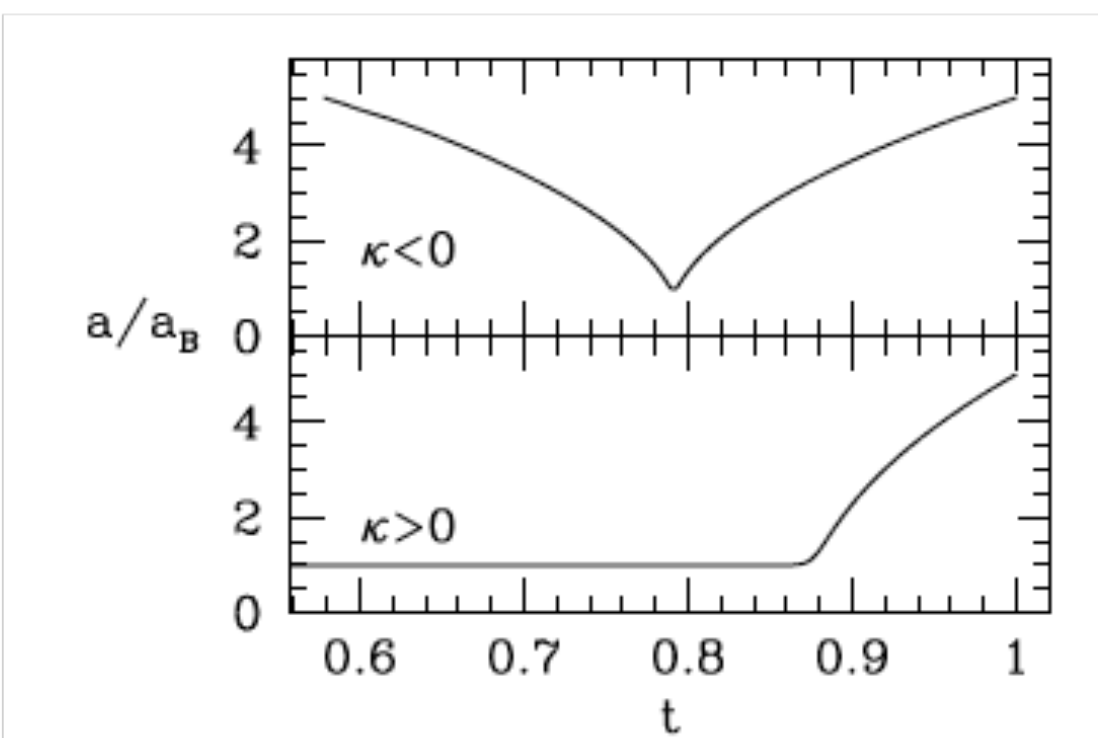
Born-Infeld inspired gravity

$$\mathcal{S}_{\mathrm{BIP}} = \lambda^4 \int \mathrm{d}^4x \left[\sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}(\Gamma)\right)} - \sqrt{-\det(g_{\mu\nu})} \right]$$

D. N. Vollick, PRD 69 (2004) 064030.

In the Palatini formulation the ghost can be avoided without further corrections

Existence of bouncing solutions...



M. Bañados, P. G. Ferreira, PRL 105, 011101 (2010)

Extended Born-Infeld gravity

We can rewrite the action as

$$\mathcal{S} = \lambda^4 \int d^4x \sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}\right)} = \lambda^4 \int d^4x \sqrt{-g}\, \det \sqrt{\delta^\mu{}_\nu + \lambda^{-2} g^{\mu\alpha} R_{\alpha\nu}}$$

$$\Rightarrow \quad \mathcal{S} = \lambda^4 \int d^4x \sqrt{-g}\, \det \sqrt{\hat{g}^{-1}\hat{q}}$$

$$q_{\alpha\nu} \equiv g_{\alpha\nu} + \lambda^{-2} R_{\alpha\nu}(\Gamma)$$

This reminds of the massive gravity potential:

$$\mathcal{S}_{MG} = \int d^4x \sqrt{-g} \sum_{n=0}^{4} \frac{\beta_n}{n!(4-n)!} e_n\left(\sqrt{g^{-1} f}\right)$$

C. de Rham, G. Gabadaze, A.J.Tolley, PRL106 (2011)

S.F. Hassan, R.A. Rosen JHEP1107 (2011)

elementary symmetric polynomials

Extended Born-Infeld gravity

...and so, a natural generalization of BI inspired gravity is

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JCAP 11 (2014) 004

$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2} \hat{g}^{-1} \hat{R}(\Gamma)}$$

$$\begin{aligned}
e_0(\hat{M}) &= 1, \\
e_1(\hat{M}) &= [\hat{M}], \\
e_2(\hat{M}) &= \frac{1}{2!}\Big([\hat{M}]^2 - [\hat{M}^2]\Big), \\
e_3(\hat{M}) &= \frac{1}{3!}\Big([\hat{M}]^3 - 3[\hat{M}][\hat{M}^2] + 2[\hat{M}^3]\Big), \\
e_4(\hat{M}) &= \frac{1}{4!}\Big([\hat{M}]^4 - 6[\hat{M}]^2[\hat{M}^2] + 8[\hat{M}][\hat{M}^3] + 3[\hat{M}^2]^2 - 6[\hat{M}^4]\Big).
\end{aligned}$$

with matter minimally coupled.

Low curvature limit

$$\mathcal{S} \simeq \int \mathrm{d}^4x \sqrt{-g} \left[\tilde{\lambda}^4 (\beta_0 + 4\beta_1 + 6\beta_2 + 4\beta_3 + \beta_4) + \frac{\tilde{\lambda}^4}{2\lambda^2} (\beta_1 + 3\beta_2 + 3\beta_3 + \beta_4) g^{\mu\nu} R_{\mu\nu}(\Gamma) \right]$$

Cosmological constant

Newton's constant

Extended Born-Infeld gravity

...and so, a natural generalization of BI inspired gravity is

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$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4 x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2} \hat{g}^{-1} \hat{R}(\Gamma)}$$

Field equations

$$\frac{\tilde{\lambda}^4 \lambda^{-2}}{2} \left(R_{\alpha\lambda} W^{\lambda}{}_{\beta} + R_{\beta\lambda} W^{\lambda}{}_{\alpha} \right) - \mathcal{L}_G g_{\alpha\beta} = T_{\alpha\beta}$$

$$\nabla_\lambda \left(\sqrt{-g} W^{\beta\nu} \right) - \delta^\nu_\lambda \nabla_\rho \left(\sqrt{-g} W^{\beta\rho} \right) + 2\sqrt{-g} \left(\mathcal{T}^\kappa_{\lambda\kappa} W^{\beta\nu} - \delta^\nu_\lambda \mathcal{T}^\kappa_{\rho\kappa} W^{\beta\rho} + \mathcal{T}^\nu_{\lambda\rho} W^{\beta\rho} \right) = 0$$

$$\hat{W} = f_1 \hat{M}^{-1} + f_2 \mathbb{1} + f_3 \hat{M} + f_4 \hat{M}^2$$

$$\begin{aligned} f_1 &= \beta_1 e_0 + \beta_2 e_1 + \beta_3 e_2 + \beta_4 e_3 \\ f_2 &= -(\beta_2 e_0 + \beta_3 e_1 + \beta_4 e_2) \\ f_3 &= \beta_3 e_0 + \beta_4 e_1 \\ f_4 &= -\beta_4 e_0 \end{aligned}$$

The equations have the same structure for all the terms.

Minimal Born-Infeld extension

$$\mathcal{S}_{\rm min} = \lambda^2 M_{\rm Pl}^2 \int {\rm d}^4x \sqrt{-g}\,{\rm Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Metric field equations

$$(M^{-1})^{\alpha}{}_{(\mu}R_{\nu)\alpha} - {\rm Tr}(\hat{M} - \mathbb{1})\lambda^2 g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}$$

$$\Downarrow \quad \hat{R} = \lambda^2 \hat{g}(\hat{M}^2 - \mathbb{1})$$

$$\frac{1}{2}\left[\hat{g}(\hat{M} - \hat{M}^{-1}) + (\hat{M} - \hat{M}^{-1})^T\hat{g}\right] - {\rm Tr}(\hat{M} - \mathbb{1})\hat{g} = \frac{1}{\lambda^2 M_{\rm Pl}^2}\hat{T}$$

This equation allows to express $M^{\alpha}{}_{\beta}$ as a function of the matter content and the metric tensor.

Minimal Born-Infeld extension

$$\mathcal{S}_{\rm min} = \lambda^2 M_{\rm Pl}^2 \int {\rm d}^4x \sqrt{-g}\,{\rm Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Connection field equations

$$\nabla_\lambda\left(\sqrt{-g}W^{\beta\nu}\right) - \delta^\nu_\lambda\nabla_\rho\left(\sqrt{-g}W^{\beta\rho}\right) + 2\sqrt{-g}\left(\mathcal{T}^\kappa_{\lambda\kappa}W^{\beta\nu} - \delta^\nu_\lambda\mathcal{T}^\kappa_{\rho\kappa}W^{\beta\rho} + \mathcal{T}^\nu_{\lambda\rho}W^{\beta\rho}\right) = 0$$

$$\hat{W} = \hat{M}^{-1}$$

We will consider solutions without torsion $\mathcal{T}^\alpha{}_{\mu\nu} = 0$

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho(\nu}W^{\beta)}{}_\rho\right) = 0 \quad\Longrightarrow\quad \Gamma = \Gamma(\tilde{g}) \qquad \tilde{g}^{\mu\nu} = \sqrt{\det\hat{M}}\,g^{\alpha\mu}(\hat{M}^{-1})^\nu{}_\alpha$$

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho[\nu}W^{\beta]}{}_\rho\right) = 0 \quad\Longrightarrow$$

We set torsion to zero a posteriori. This is a consistency equation for this Ansatz.

Cosmological solutions

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$$\mathrm{d}\tilde{s}^2 = -N^2(M_0M_1^{-3})^{1/2}\mathrm{d}t^2 + \frac{a(t)^2}{\sqrt{M_0M_1}}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$$\lambda^{-2}H^2 = \frac{1 - M_0^2 + 3M_0M_1 - \frac{3M_0}{M_1}}{6\Big[1 - 3(\rho+p)\partial_\rho \ln[(M_0M_1)^{-1/4}]\Big]^2}$$

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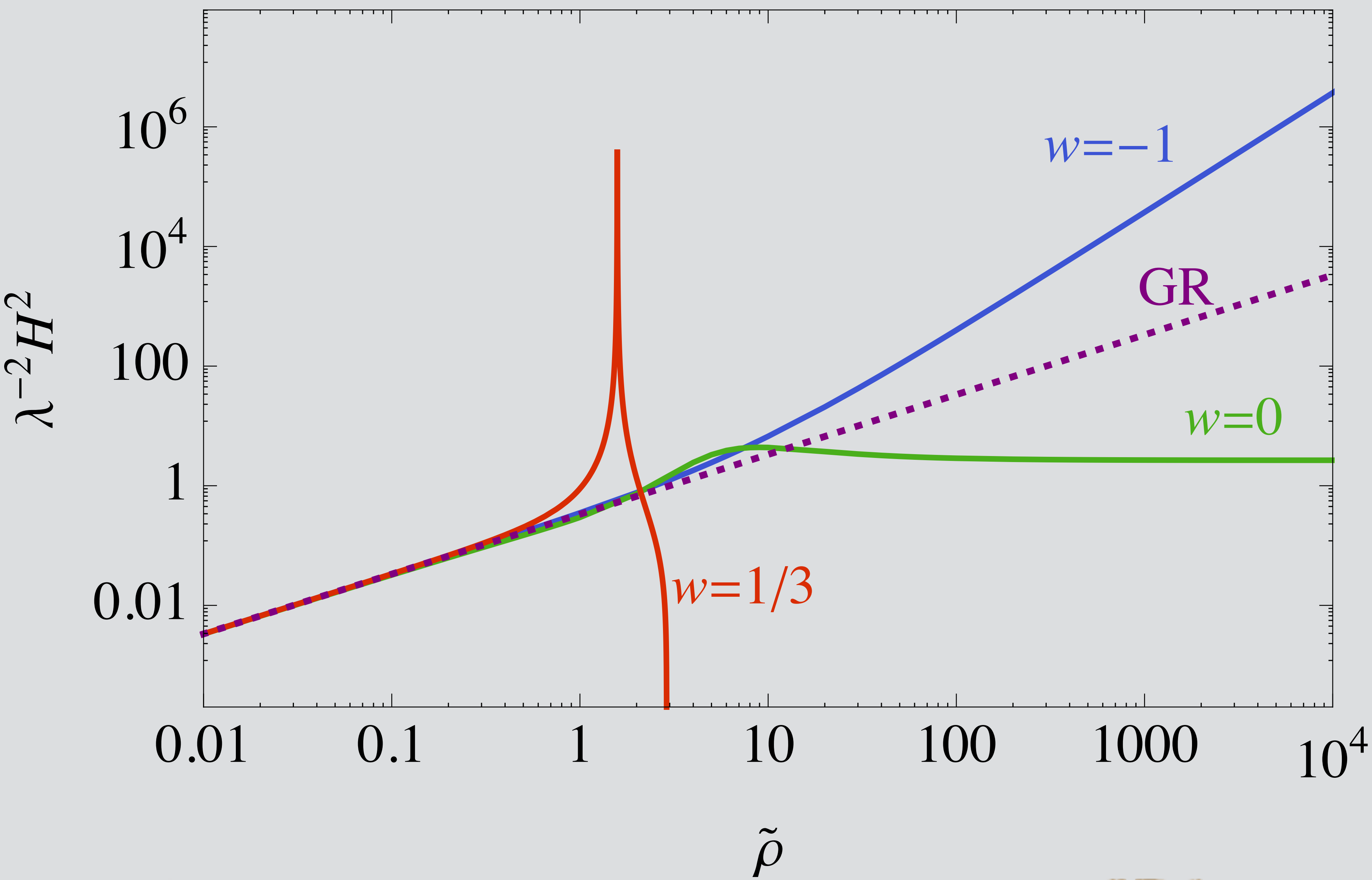

1704.03351

Born-Infeld inspired modifications of gravity

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Abstract

General Relativity has shown an outstanding observational success in the scales where it has been directly tested. However, modifications have been intensively explored in the regimes where GR seems either incomplete or signals its own limit of validity. In particular, the breakdown of unitarity at energies near the Planck scale strongly suggests that GR needs to be modified at high energies and quantum gravity effects are expected to be important. This is related to the existence of spacetime singularities when the solutions of General Relativity are extrapolated to regimes when the involved curvatures are large. In this sense, Born-Infeld inspired modifications of gravity have shown an extraordinary ability to regularise the gravitational dynamics, leading to non-singular cosmologies and regular black hole spacetimes in a very robust manner and without resorting to quantum gravity effects. This has boosted the interest in these theories in applications to stellar structure, compact objects, inflationary scenarios, cosmological singularities and black hole, and wormhole physics, among others. We review the motivations, various formulations, and main results achieved within this type of extensions beyond Einstein's gravity, including their observational viability, and provide an overview of current open problems and future research opportunities.

Keywords: Born-Infeld gravity, Astrophysics, Black Holes, Cosmology, Early universe, Compact objects, Singularities