

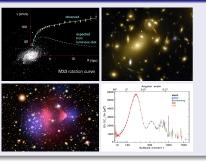
Advances in Theoretical Cosmology in Light of Data NORDITA, Stockholm 17-21 July 2017

Outline

- Dark Matter
- Primordial Black Holes
- Press-Schechter Formalism
- PBHs formation from
 - Gauge Production
 - Scalar Production
- Conclusion

Dark Matter

Evidences



Properties

- stable
- neutral
- weakly interacting
- right relic density

Candidates

- Axions
- Sterile neutrinos
- WIMPs
- Primordial Black Holes (PBHs)

Primordial Black Holes

Definition

A PBH is a type of black hole that is **not** formed by the gravitational collapse of a star, but by the extreme density of matter present during the Universe's early expansion.

PBHs properties

Mass:
$$M_{\mathrm{BH}} = 10^{15} \left(\frac{t}{10^{-23} \, \mathrm{s}} \right) \, \mathrm{g}$$

Temperature:
$$T_{\rm BH} \approx 10^{-7} \left(\frac{M}{M_{\odot}}\right)^{-1} {\rm K}$$

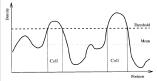
Lifetime:
$$au_{
m BH} pprox 10^{64} \left(rac{M}{M_{\odot}}
ight)^3 {
m y}$$

$$\textit{M}_{\odot} \simeq 2 \times 10^{33}\,\mathrm{g}$$

= = / . = 0	
$M_{_{ m BH}}$	$ au_{ ext{BH}}$
A man	$10^{-12}{ m s}$
A building	1 s
$10^{15}\mathrm{g}$	$10^{10} { m y}$
The Earth	$10^{49} { m y}$
The Sun	$10^{66} { m y}$
The Galaxy	$10^{99} { m y}$

Press-Schechter Formalism

The Press-Schechter formalism is a model for predicting the number density of bound objects of a certain mass.

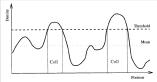


$$f(\geq M) = \gamma \int_{\zeta_{\rm th}}^{\infty} P(\zeta; M(R))$$

$$\zeta_{
m th}=0.4135 \qquad \qquad \gamma=w^{3/2}\simeq 0.2$$

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Gaussian PDF:
$$P_{\rm G}(\zeta; R) = \frac{1}{\sqrt{2\pi} \, \sigma_{\zeta}(R)} \exp\left(-\frac{\zeta^2}{2\sigma_{\zeta}^2(R)}\right)$$

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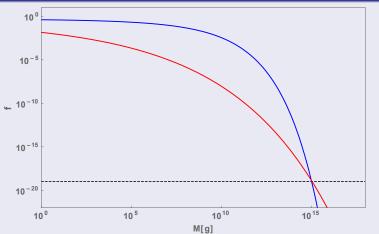
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$$\begin{aligned} &\text{non-Gaussian: } P_{\mathrm{NG}}(\zeta;\,R) = \frac{1}{\sqrt{2\pi\left(\zeta + \sigma_{g}^{2}(R)\right)}\,\sigma_{g}(R)} \exp\left(-\frac{\zeta + \sigma_{g}^{2}(R)}{2\sigma_{g}^{2}(R)}\right) \\ &f_{\mathrm{NG}} = \mathrm{erfc}\left(\sqrt{\zeta_{\mathrm{th}} + \sigma_{g}^{2}(R)}/\sqrt{2\sigma_{g}^{2}(R)}\right) \end{aligned}$$





Result

$$n_s(k_{\mathrm{PBH}}) \ge 1.418 \quad \Rightarrow \quad \mathcal{P}_\zeta \simeq 2 \times 10^{-2} \quad \text{ for Gaussian PDF}$$

 $n_s(k_{\mathrm{PBH}}) \ge 1.322 \quad \Rightarrow \quad \mathcal{P}_\zeta \simeq 4 \times 10^{-4} \quad \text{ for non-Gaussian PDF}$

Inflation

Inflation parameters

$$\begin{split} \mathcal{P}_{\zeta, \, \text{vac.}}(k) &= \mathcal{P}_{\zeta, \, \text{vac.}}(k_0) \left(\frac{k}{k_0}\right)^{n_{\text{s}}(k)-1} \\ n_{\text{s}}(k_0) - 1 &\equiv \frac{d \ln \mathcal{P}_{\zeta, \, \text{vac.}}(k)}{d \ln k} \\ r &= \frac{\mathcal{P}_{\text{t}}(k)}{\mathcal{P}_{\zeta}(k)}, \qquad \mathcal{P}_{\text{t}, \, \text{vac.}}(k) = \frac{2}{\pi^2} \left(\frac{H}{M_{\text{P}}}\right)^2 \left(\frac{k}{k_0}\right)^{n_{\text{t}}} \\ \mathcal{B}_{\zeta}(k_1, \, k_2, \, k_3) &= f_{\text{NL}} F(k_1, \, k_2, \, k_3) \end{split}$$

Observation

Planck XX, arXiv: 1502.01592

$$ln(10^{10}\mathcal{P}_{\zeta,\,\mathrm{vac.}}(k_0)) = 3.094 \pm 0.034$$

$$k_0 = 0.05 \; \mathrm{Mpc^{-1}}$$

$$n_{\rm s} = 0.9645 \pm 0.0049$$

$$r_{0.002} < 0.10$$
 (95 % CL)

$$f_{\rm NL} = 22.7 \pm 25.5$$

PBHs formation from Particle Production

direct or gravitational coupling of the inflaton (ϕ) to another field (χ)

$$\mathcal{L}(\phi, \chi) = -\frac{1}{2}\partial_{\mu}\phi \,\partial^{\mu}\phi - V(\phi) - \frac{1}{2}\partial_{\mu}\chi \,\partial^{\mu}\chi - U(\chi) + \mathcal{L}_{\mathrm{int}}(\phi, \chi)$$

The equations of motion for the inflaton field:

$$H^{2} = \frac{1}{3M_{P}^{2}} \left(\frac{1}{2} \dot{\phi}^{2} + V(\phi) + \rho_{\chi} \right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{\partial \mathcal{L}_{\text{int}}}{\partial \phi}$$

The inflaton fluctuations satisfy

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2}{a^2}\delta\phi + V''(\phi)\delta\phi = \delta\left(\frac{\partial\mathcal{L}_{\text{int}}}{\partial\phi}\right)$$

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Result

$$\mathcal{P}_{\zeta}(k) = \mathcal{P}_{\zeta, \, \mathrm{vac.}}(k) + \mathcal{P}_{\zeta, \, \mathrm{src.}}(k)$$

$$\mathcal{P}_{t}(k) = \mathcal{P}_{t, \text{vac.}}(k) + \mathcal{P}_{t, \text{src.}}(k)$$

Gauge Production

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{lpha}{4f} \Phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Gauge Production

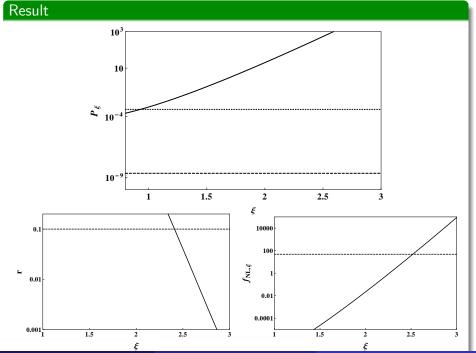
$$\mathcal{L}_{\rm int} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \Phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

direct coupling

$$\mathcal{P}_{\zeta} = \mathcal{P}_{\zeta, \, ext{vac.}} \left(1 + 7.5 \times 10^{-5} \, \epsilon^2 \, \mathcal{P}_{\zeta, \, ext{vac.}} X^2
ight)$$
 $r = 16\epsilon \, rac{1 + 2.2 \times 10^{-7} \, \mathcal{P}_{ ext{t, \, vac.}} X^2}{1 + 7.5 \times 10^{-5} \, \epsilon^2 \, \mathcal{P}_{\zeta, \, ext{vac.}} X^2}$
 $f_{ ext{NL.} \, \zeta}^{ ext{equil.}} pprox 4.4 \times 10^{10} \, \epsilon^3 \, \mathcal{P}_{\zeta, \, ext{vac.}}^3 X^3$

where

$$X \equiv \frac{e^{2\pi\xi}}{\xi^3} \qquad \xi \equiv \frac{\alpha}{2fH}\dot{\Phi}$$

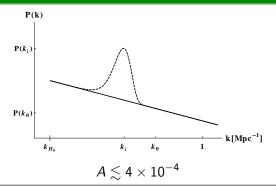


Scalar Production

$$\mathcal{L}_{\text{int}}(\phi, \chi) = -\frac{g^2}{2} (\phi - \phi_0)^2 \chi^2$$

$$\mathcal{P}_{\zeta,\,\mathrm{src.}}(k) \sim A \, k^3 e^{-rac{\pi}{2}\left(rac{k}{k_i}
ight)^2}$$

Result



Conclusions

- The fluctuation which arise at inflation are the most likely source of PBHs.
- The spectral index at scale of PBHs formation should be at least 1.418 (1.322) for Gaussian (non-Gaussian) PDF.
- The most stringent constraints on the gauge production parameter is derived from the non-production of DM PBHs at the end of inflation and the bounds from the bispectrum and the tensor-to-scalar ratio are weaker.
- In the scenario where the inflaton field coupled to a scalar field, the model is free of DM PBHs overproduction in the CMB observational range if the amplitude of the generated bump in the scalar power spectrum, A is less than 4×10^{-4} .

