An optimal Bayesian solution to the CMB delensing problem

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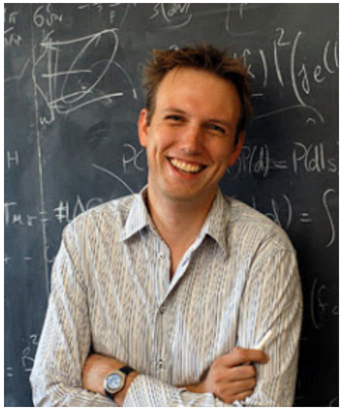
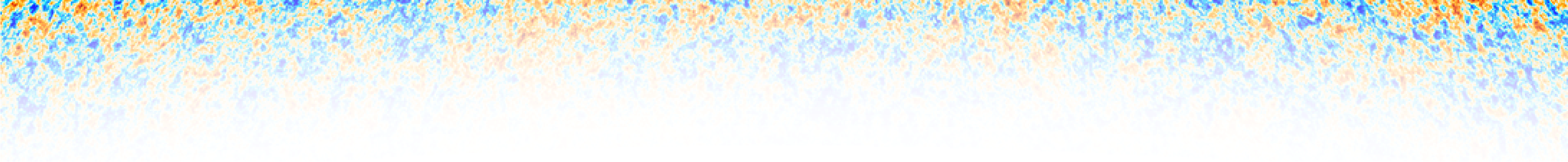
with

Ethan Anderes



Ben Wandelt

Nordita July 21, 2017



How do we optimally delense future CMB data to obtain the best possible estimates of r?

CMB “fields”

$f \equiv (T, Q, U)$

Lensing potential

Cosmo params

Data

$$\boxed{\mathcal{P}(f, \phi, r \,|\, d)}$$

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All current analyses are based on this
Currently near-optimal but will be sub-optimal for next-gen noise levels

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$$= \int df \mathcal{P}(f, \phi \,|\, r, d)$$

Carron & Lewis (2017), Hirata & Seljak (2003) give algorithm to *maximize* this

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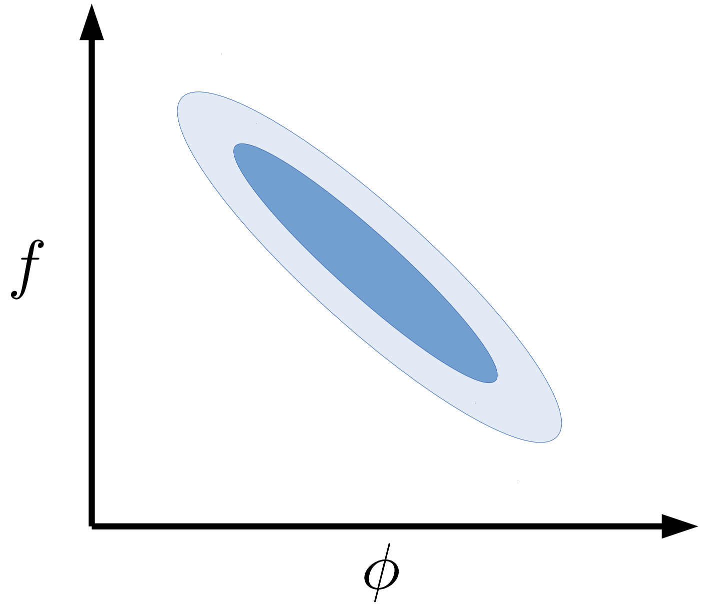

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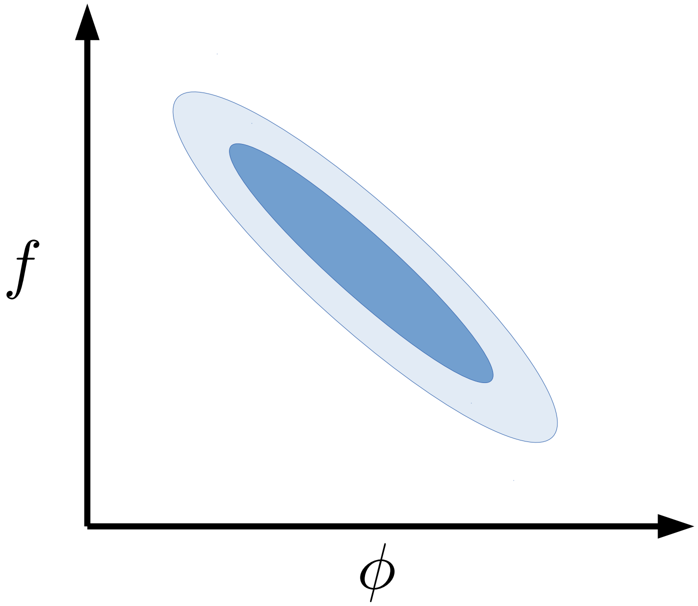

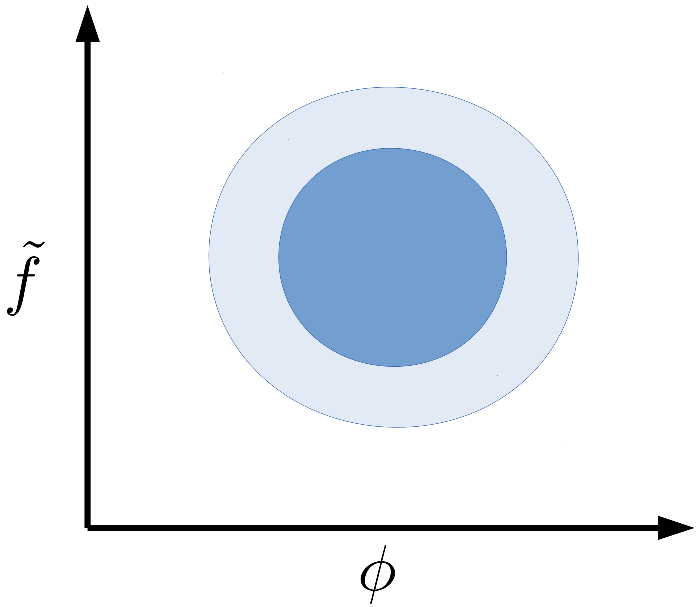

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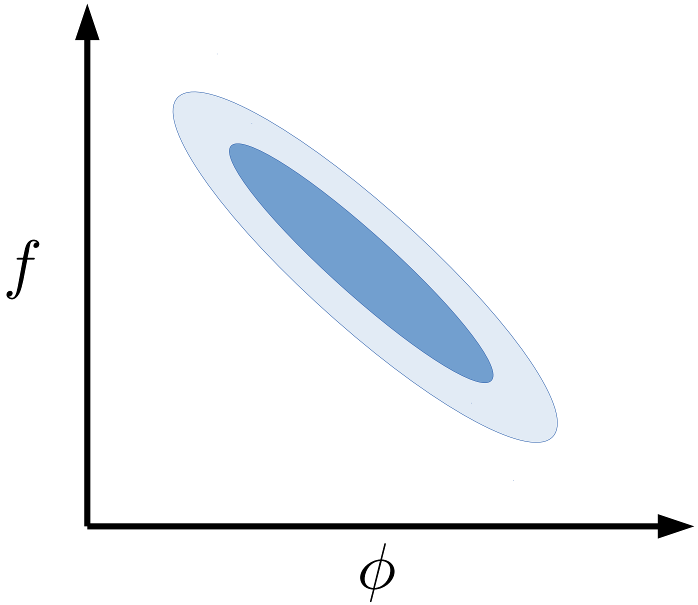

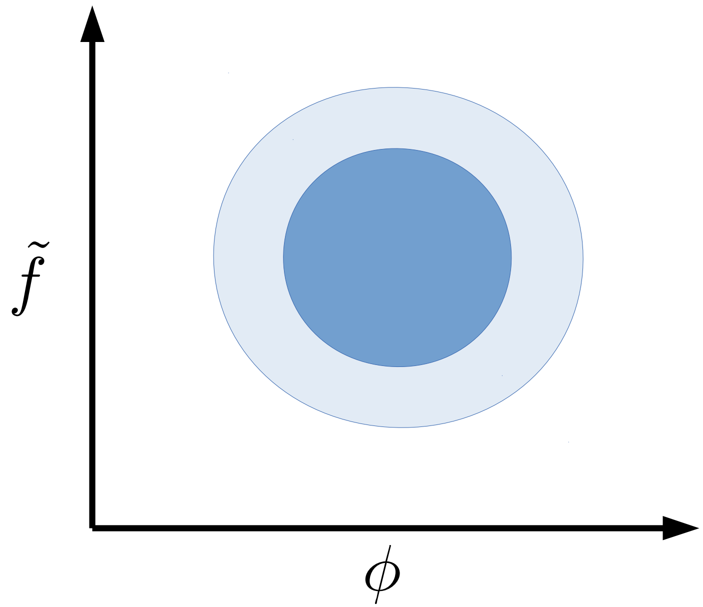

So, as pointed out by Anderes et al. 2015, its *very* beneficial to reparametrize,

$$\mathcal{P}(\tilde{f}, \phi \,|\, d) = \mathcal{P}(f(\tilde{f}), \phi \,|\, d) \left| \frac{df}{d\tilde{f}} \right|$$

But now we introduce this determinant...

where $\quad \tilde{f} = \mathcal{L}(\phi) f \implies \left| \frac{df}{d\tilde{f}} \right| = 1/\left|\mathcal{L}(\phi)\right|$

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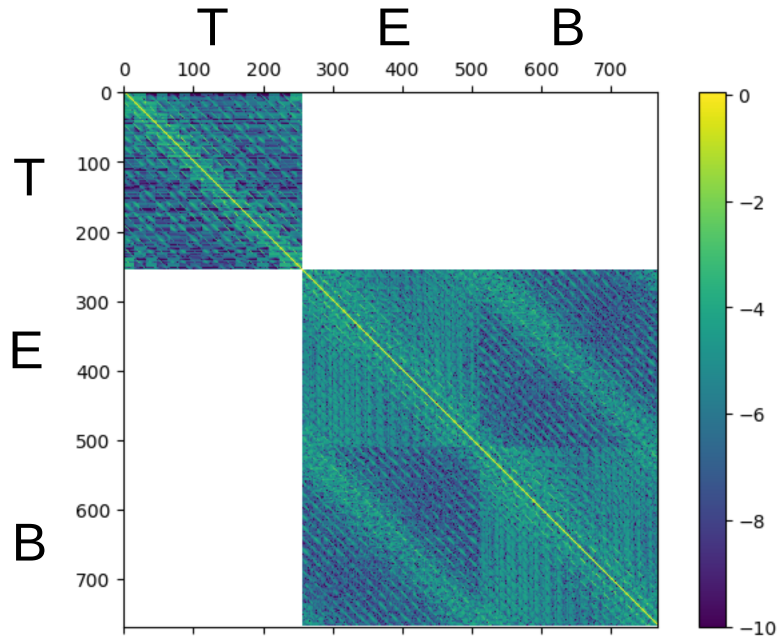

$\log(\mathrm{abs}(\mathcal{L}(\phi)_{ij}))$

Matrix representation of $\mathcal{L}(\phi)$ for 16x16 1’ pixel TEB maps for 7th order Taylor series approximation

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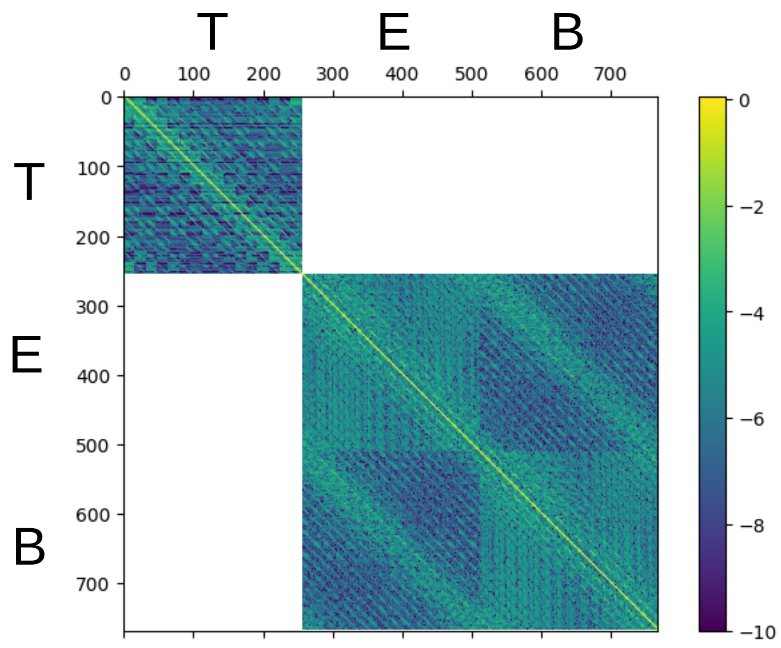

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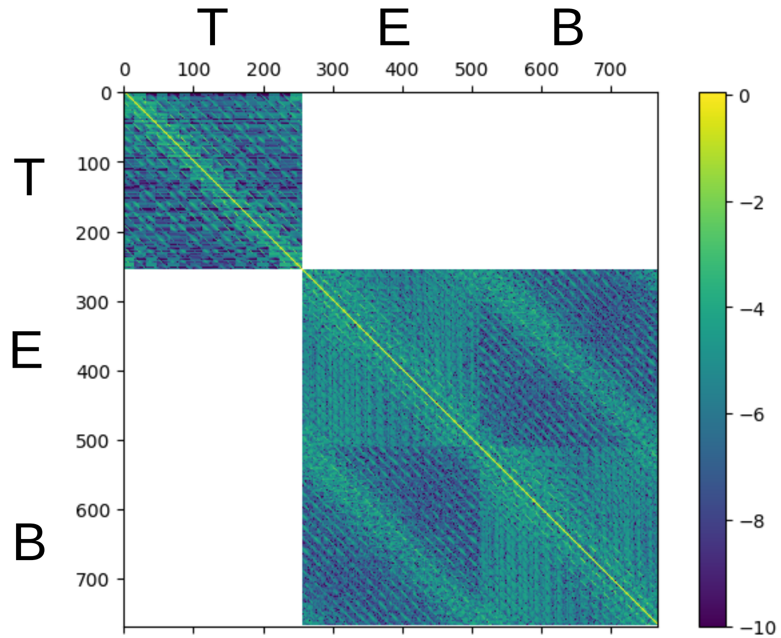

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Additionally, the variation of the determinant with ϕ is significant.

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One can show f_t obeys an ODE “flow” equation

$$\frac{df_t(x)}{dt} = \nabla\phi(x) \cdot \left[\mathbb{1} + t\nabla\nabla\phi(x)\right]^{-1} \cdot \nabla f_t(x)$$

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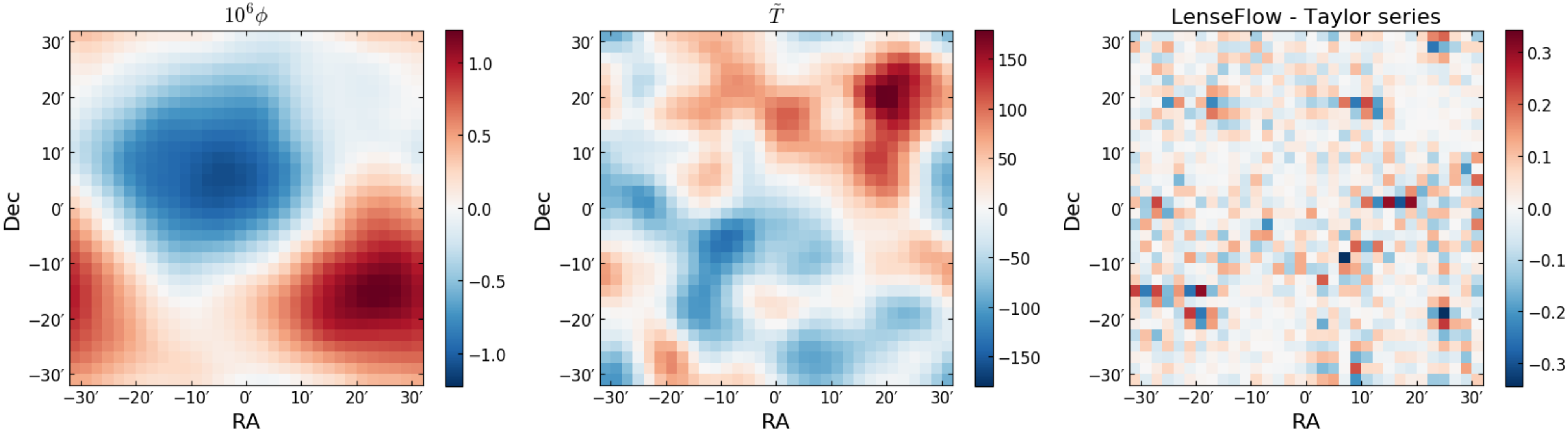
(In practice we use 4$^{\text{th}}$ order Runge-Kutta with 7 time-steps.)

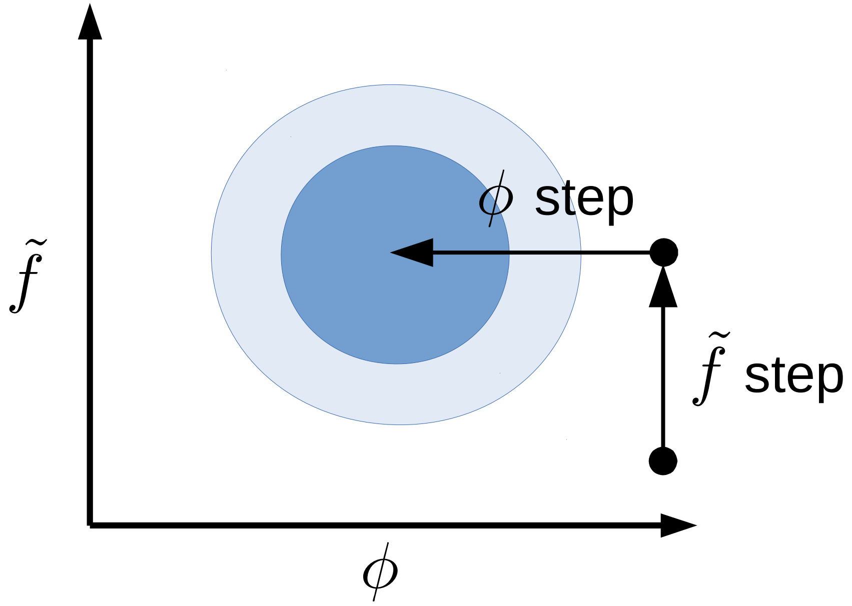
LenseFlow vs. Taylor series



Differences between the two which lead to different determinants

Ok, let's maximize & sample!

The algorithm we devise is a *coordinate descent*



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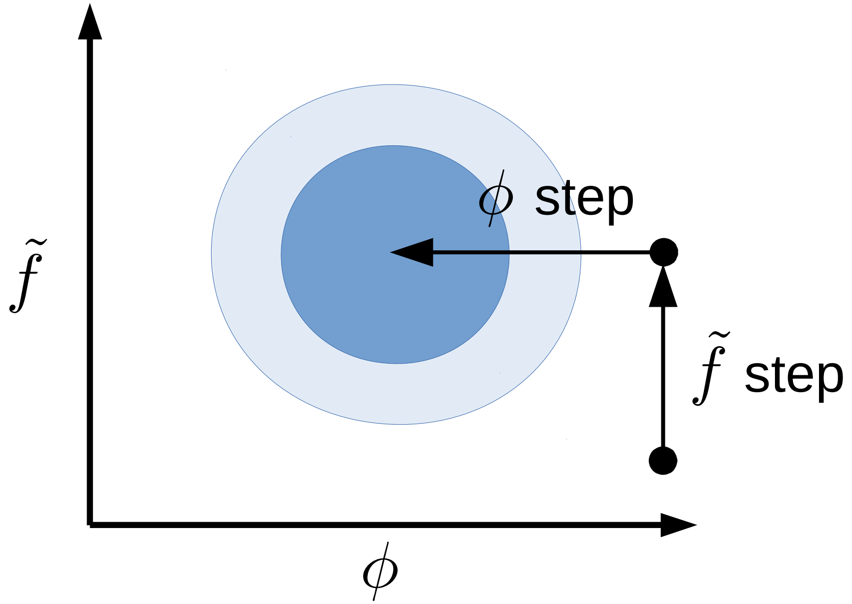

$$-2\ln\,\mathcal{P}(\tilde{f},\phi\,|\,d) =$$

$$= \underbrace{(d-\tilde{f})^{\dagger}\mathcal{C}_n^{-1}(d-\tilde{f})}_{\text{likelihood}} + \underbrace{\tilde{f}^{\dagger}\mathcal{L}(\phi)^{-\dagger}\mathcal{C}_f^{-1}\mathcal{L}(\phi)^{-1}\tilde{f}}_{\text{prior on } f} + \underbrace{\phi^{\dagger}\mathcal{C}_{\phi}{}^{-1}\phi}_{\text{prior on } \phi}$$

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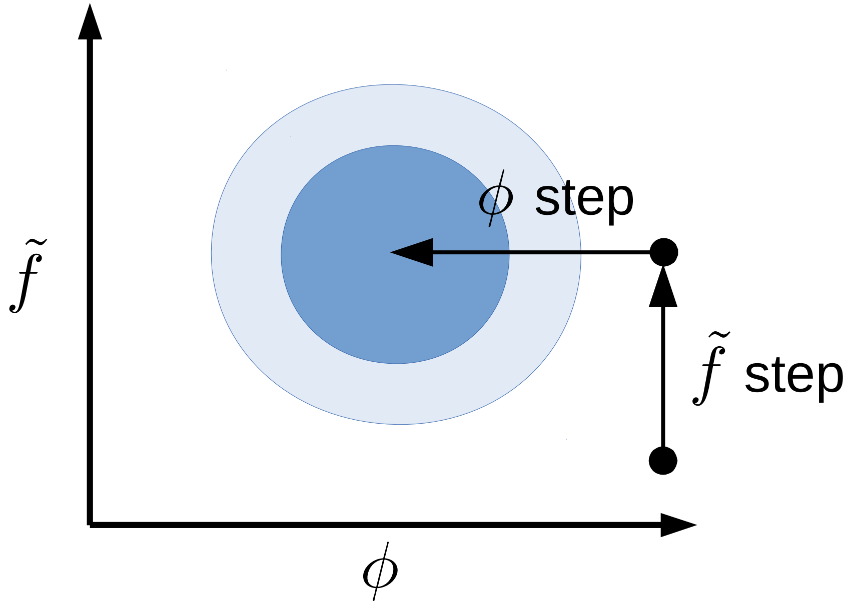

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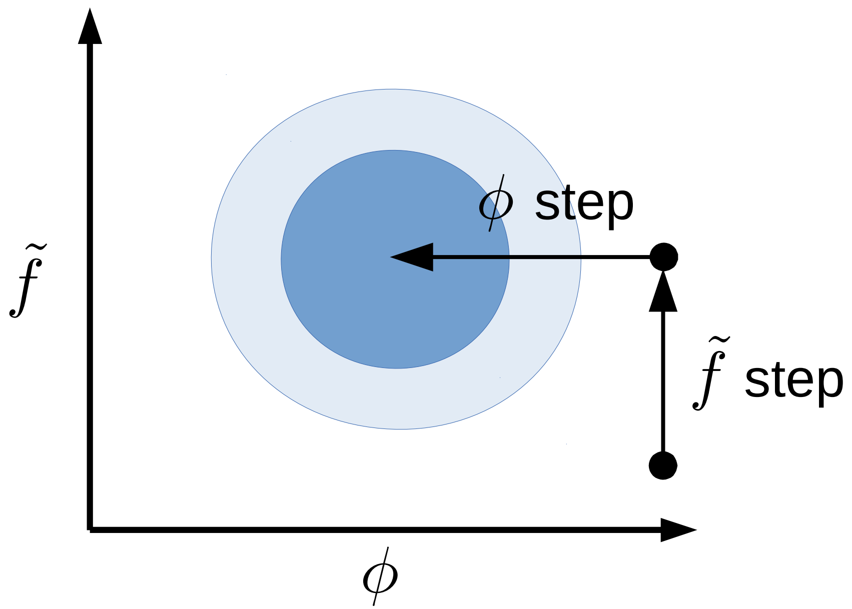

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ϕ step

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Simulated data with: 1uK-arcmin noise, r=0.05

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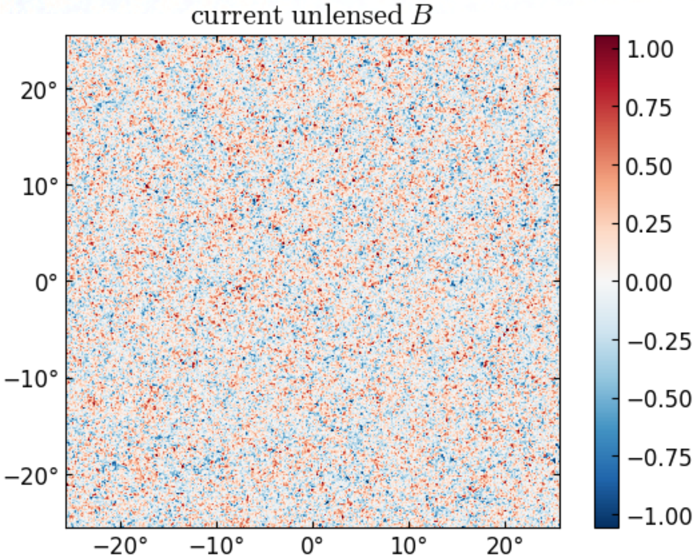

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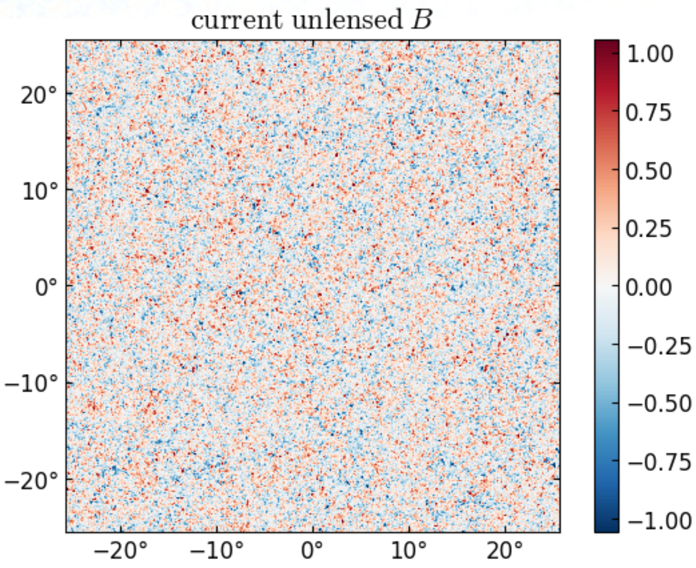

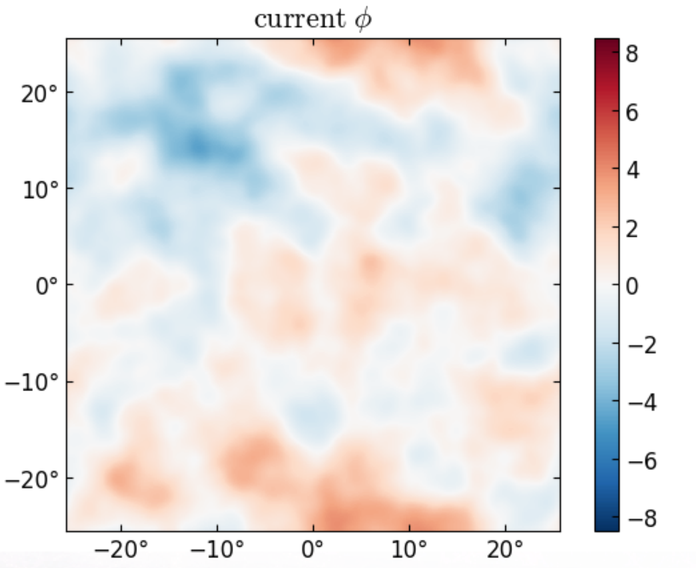

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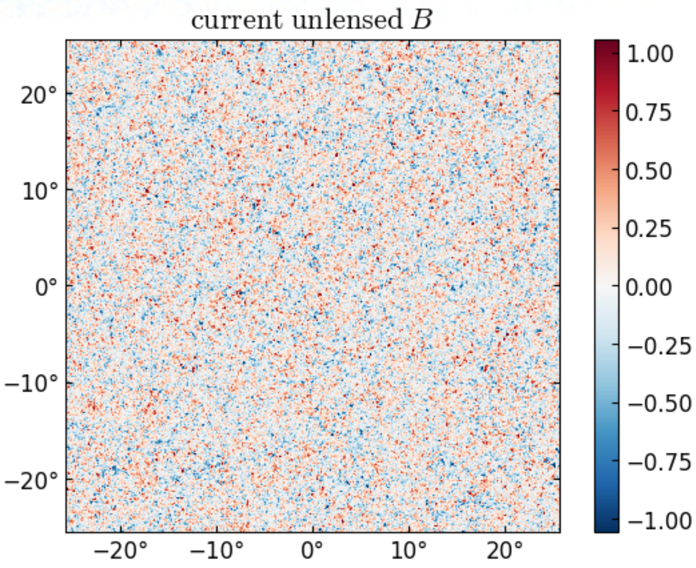

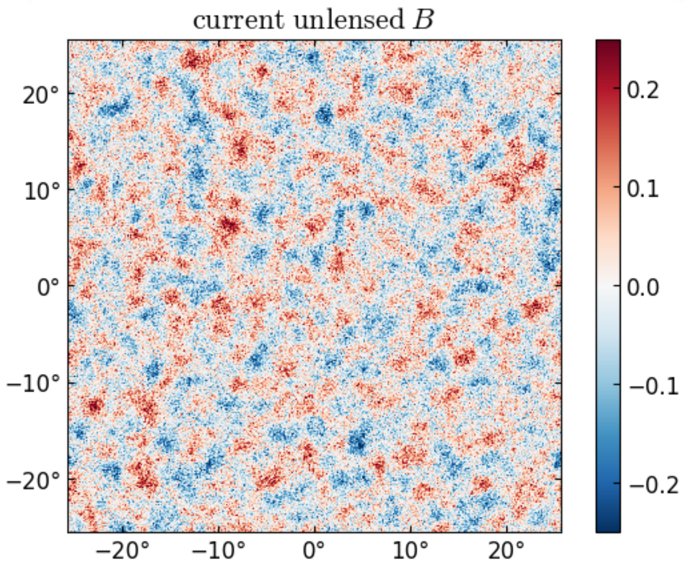

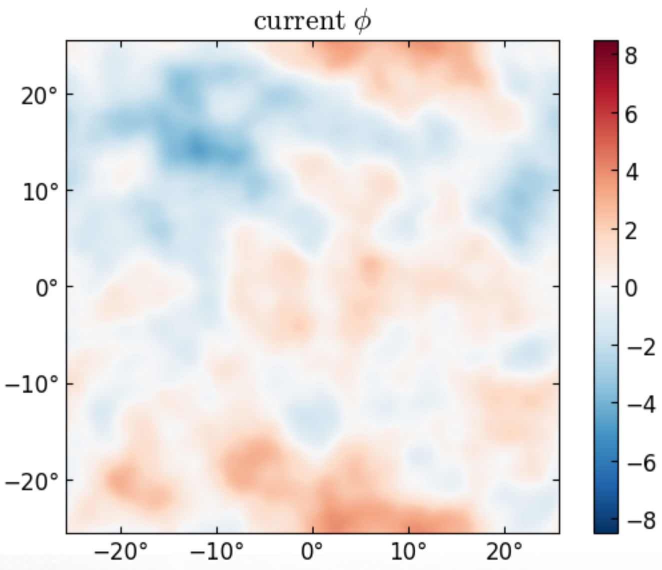

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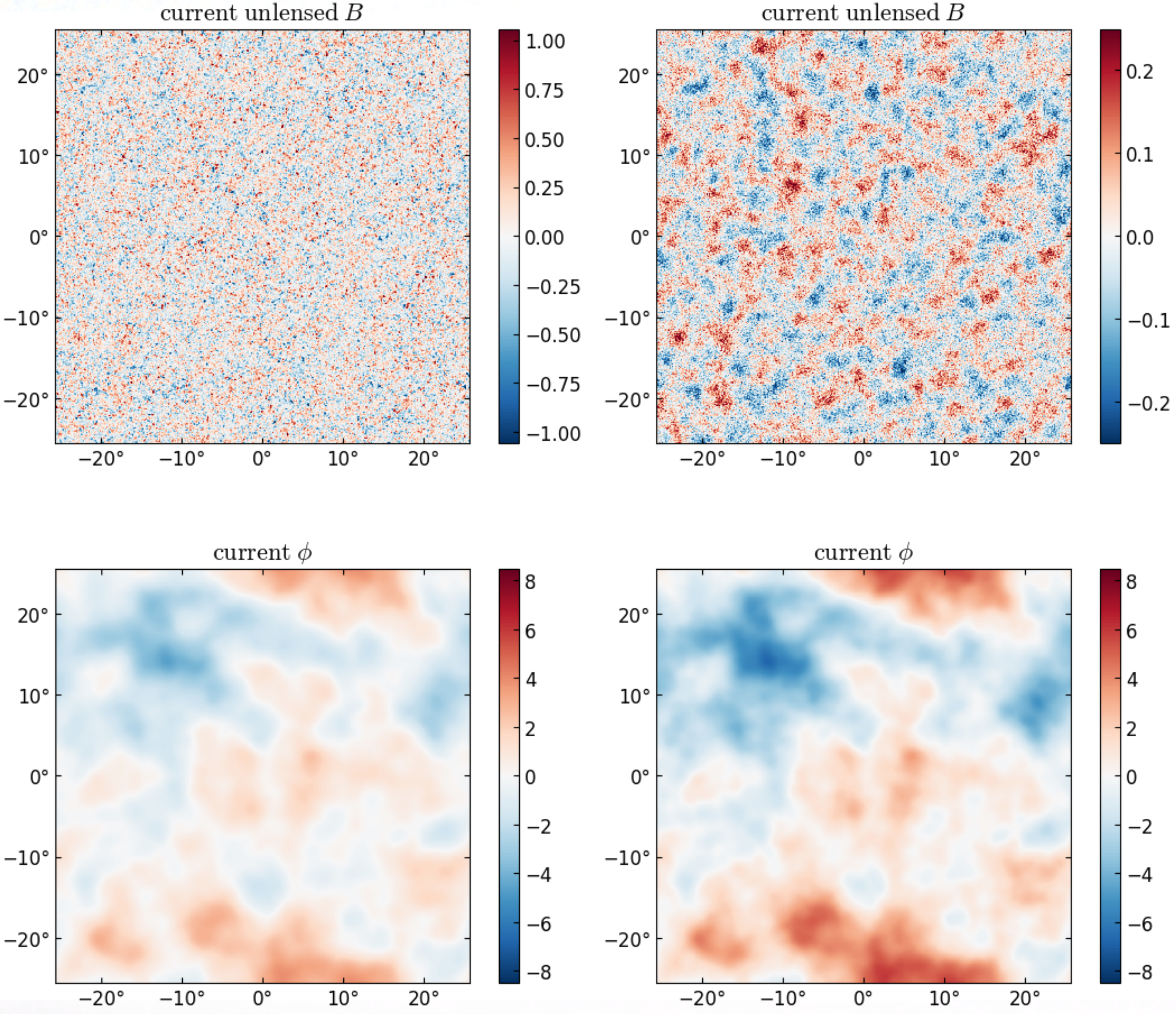

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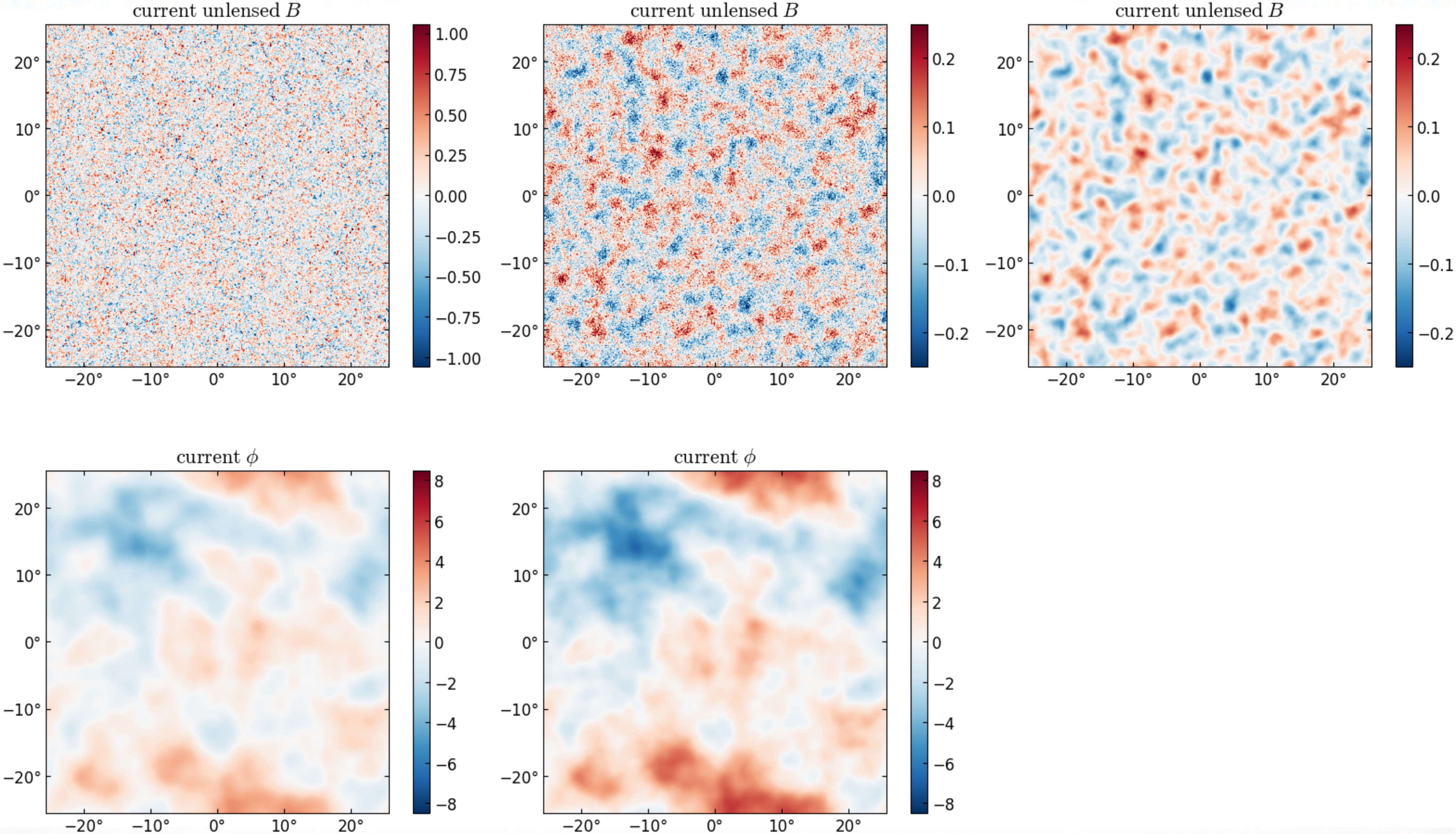

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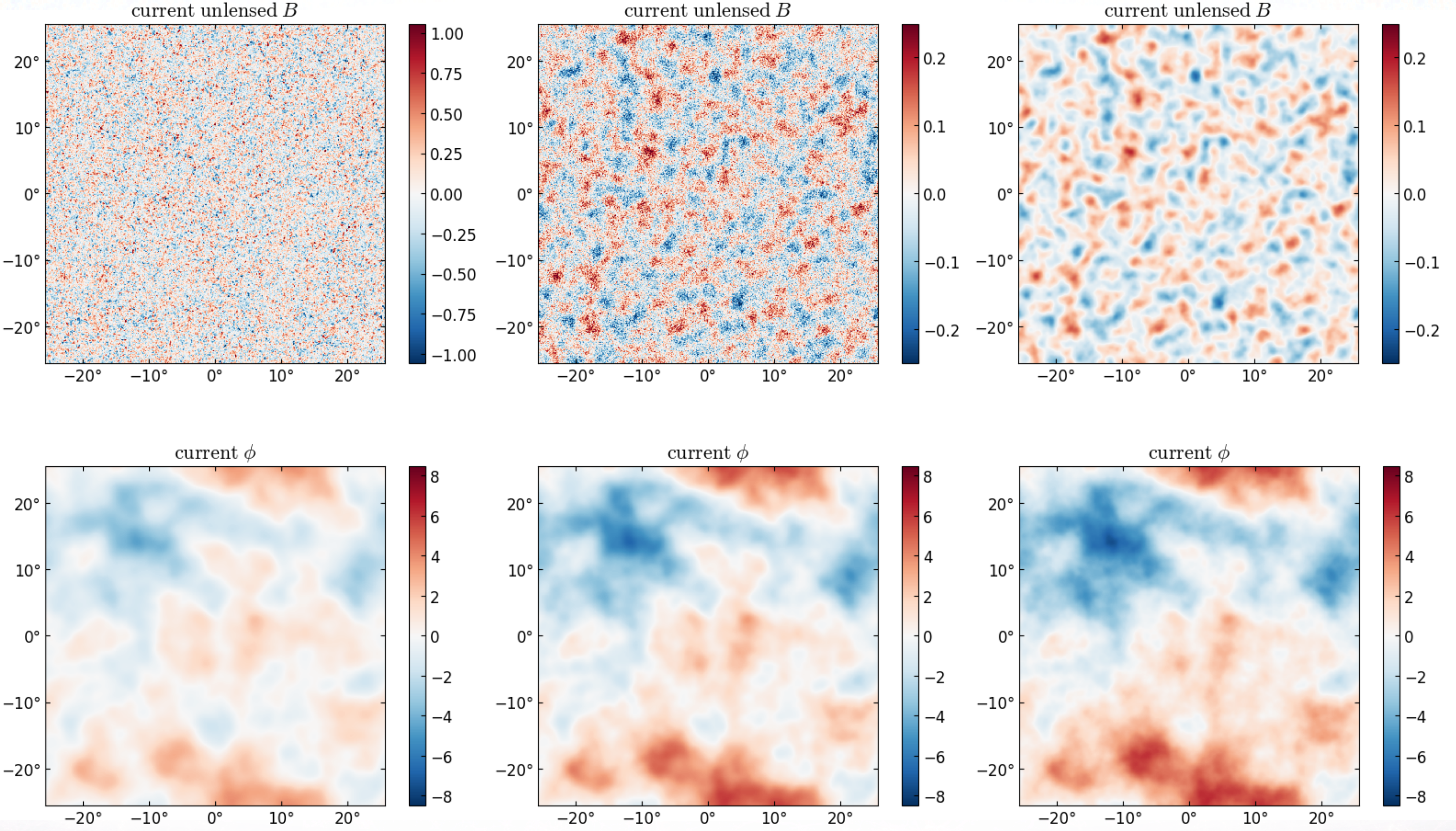

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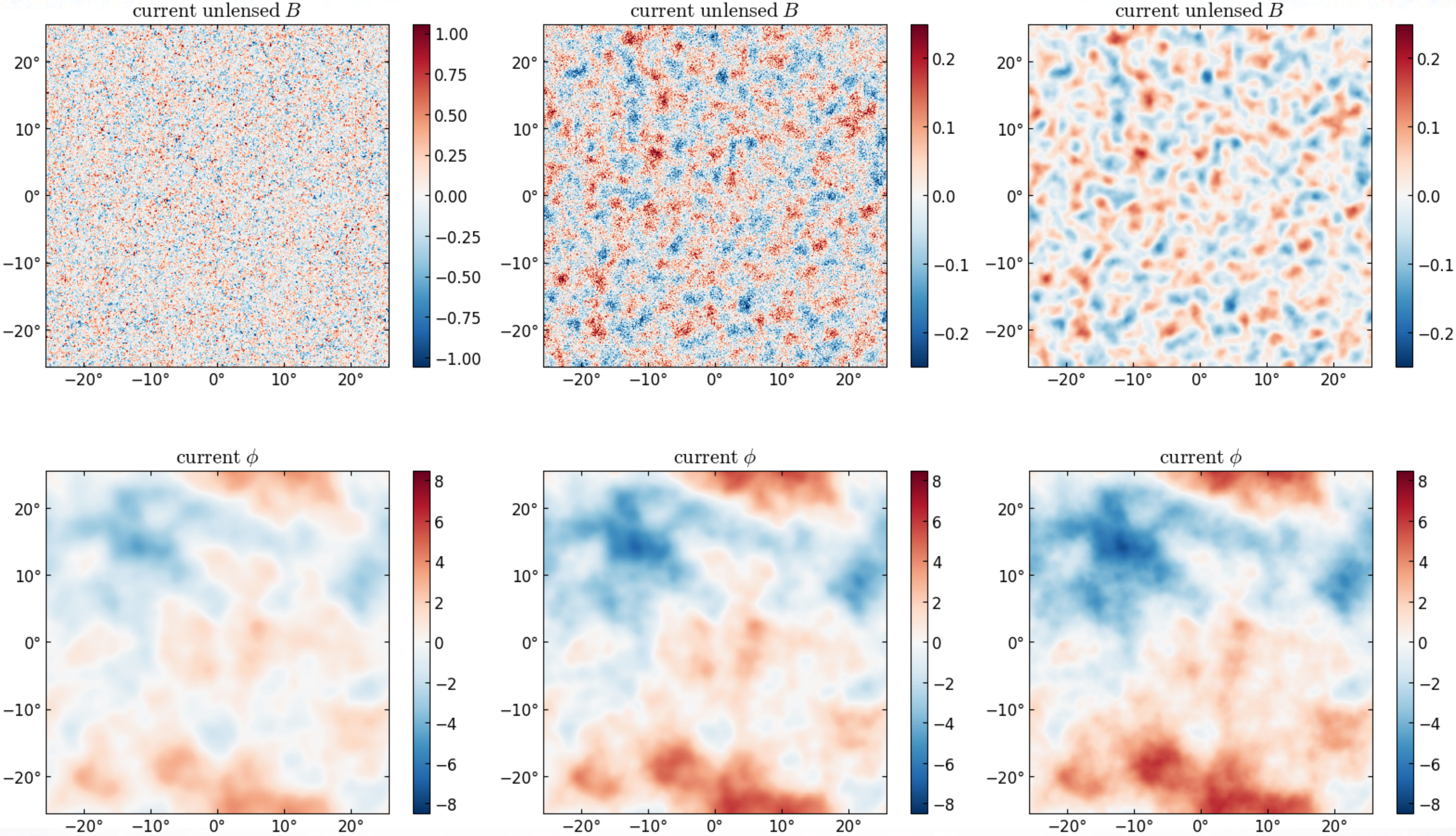
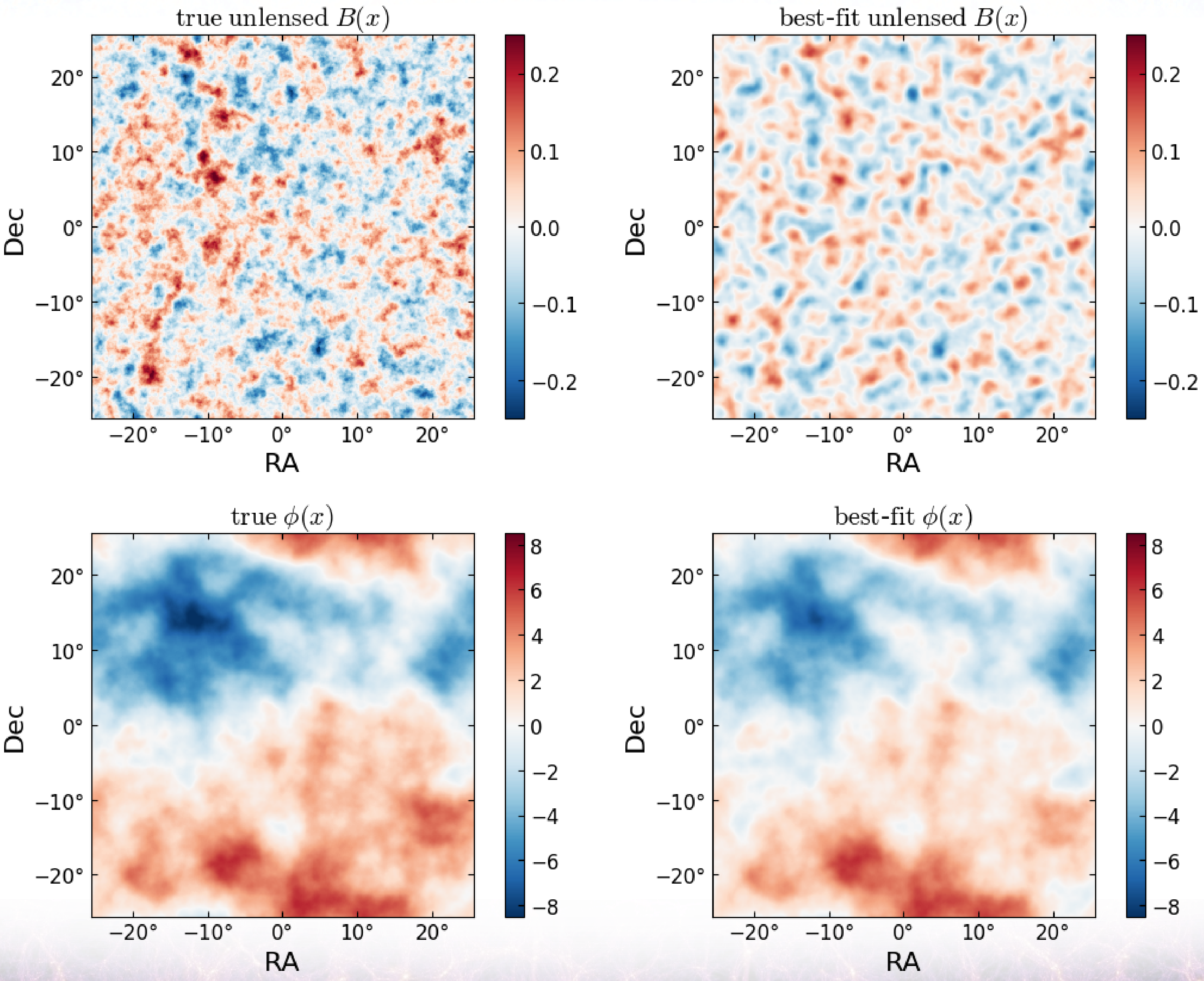


Step 1 Step 3 Step 30

30min on 1 single multi-core CPU for these 2500deg^2

1024x1024, 3 arcmin pixels



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(Only affects the Wiener filter step which needs more conjugate gradient steps => 4 hours)

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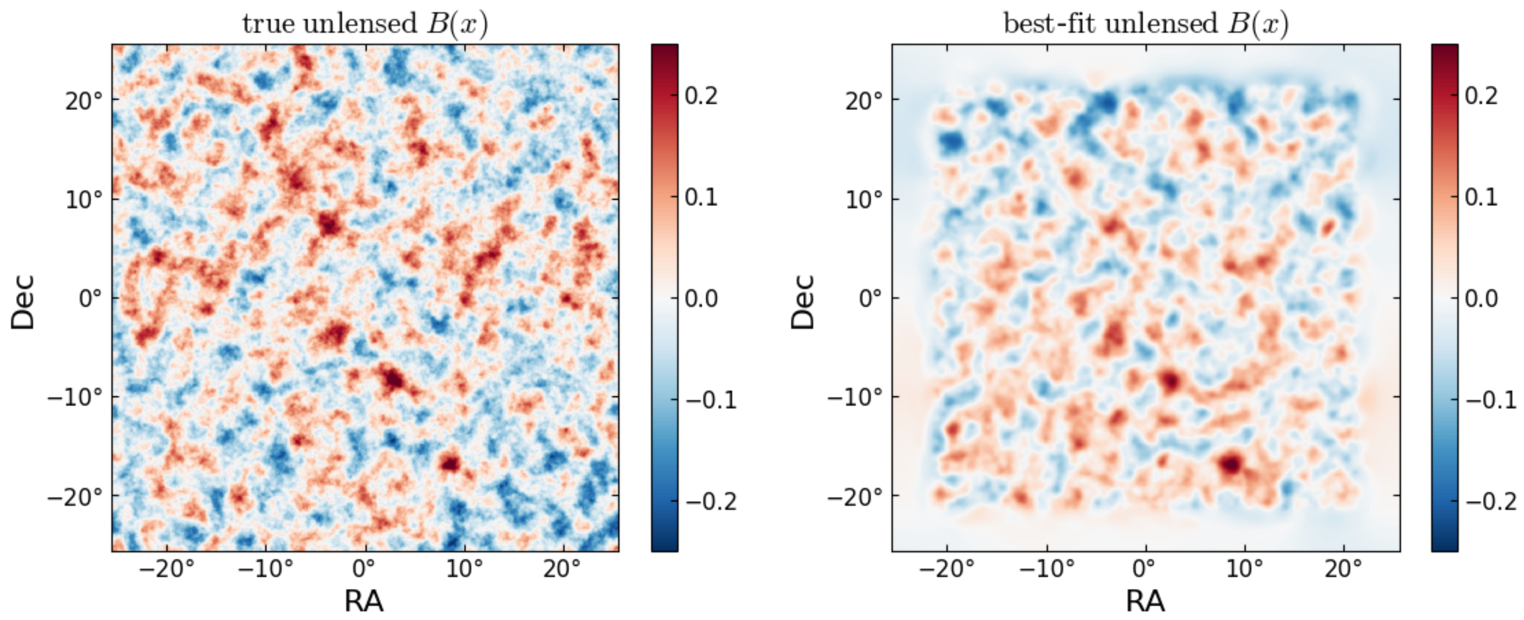

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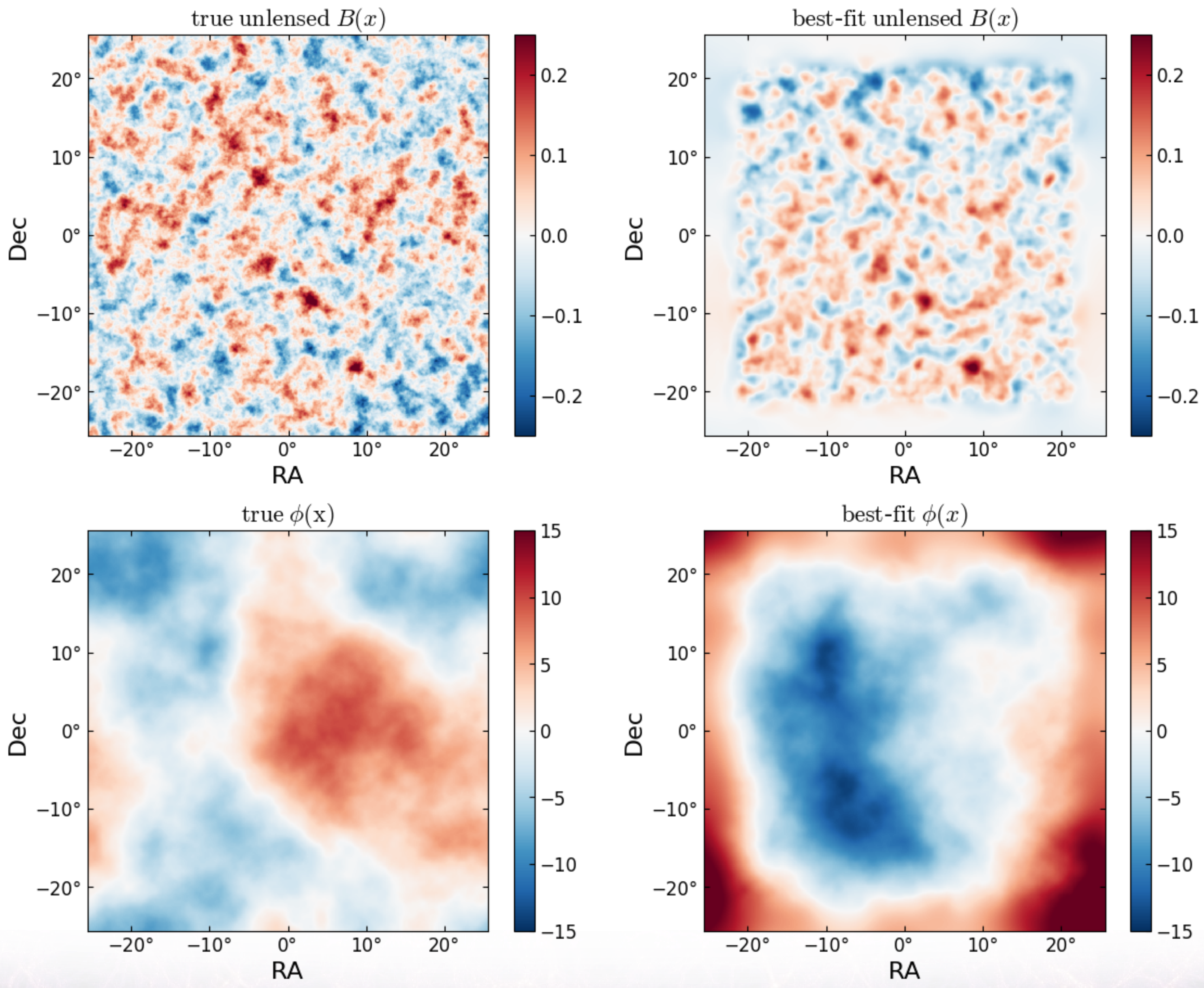

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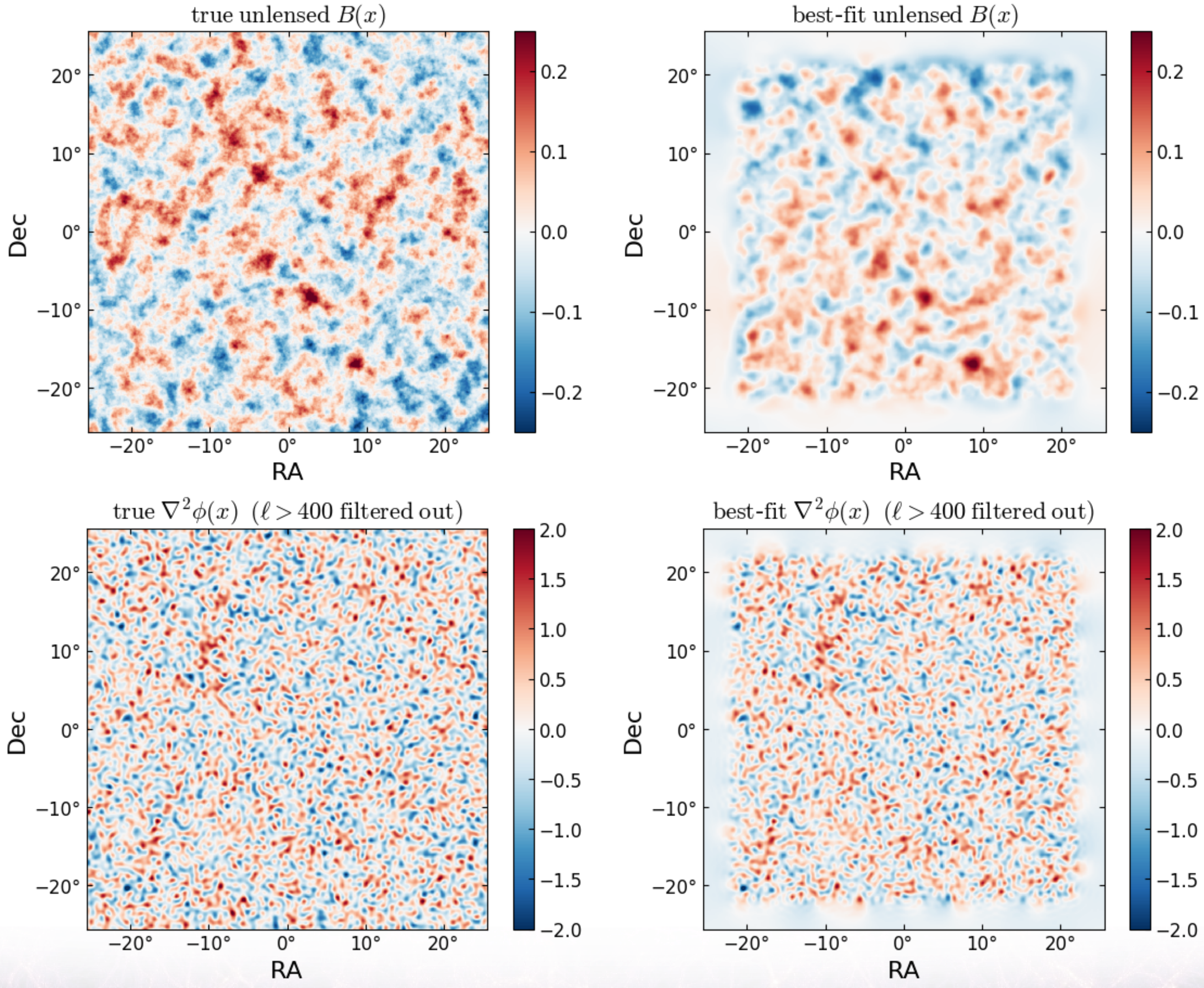
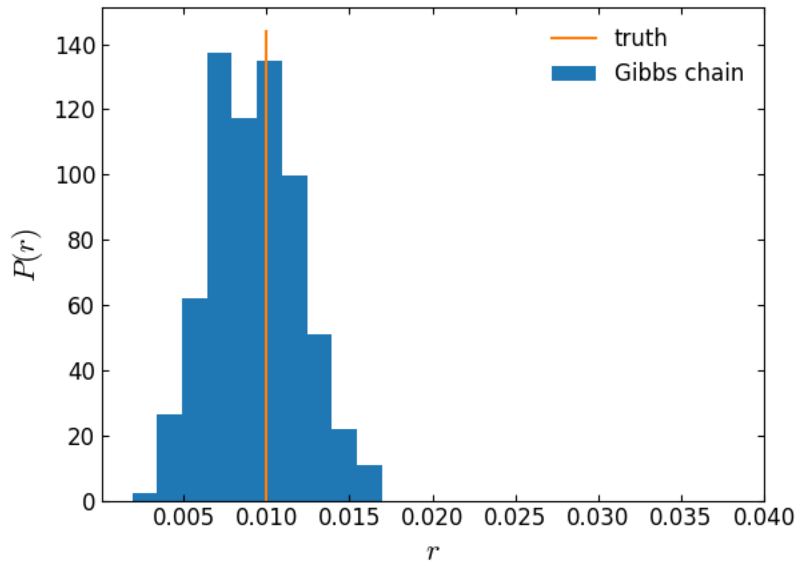
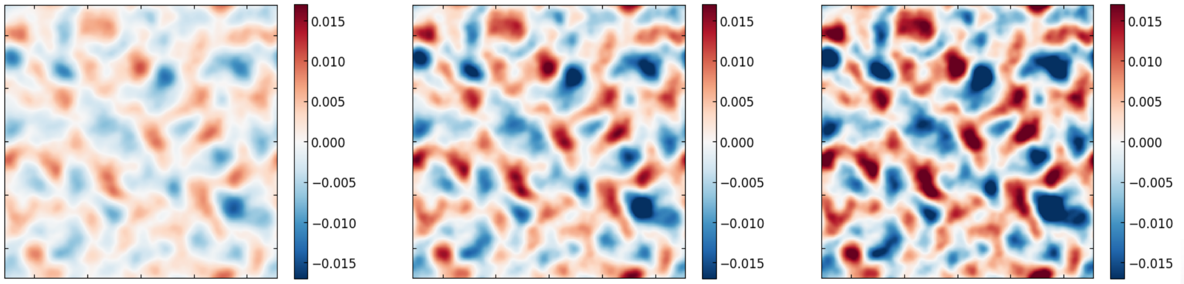


What about r?

For now, a slightly simplified preview: $\mathcal{P}(f, \hat{\phi}, r \,|\, d)$



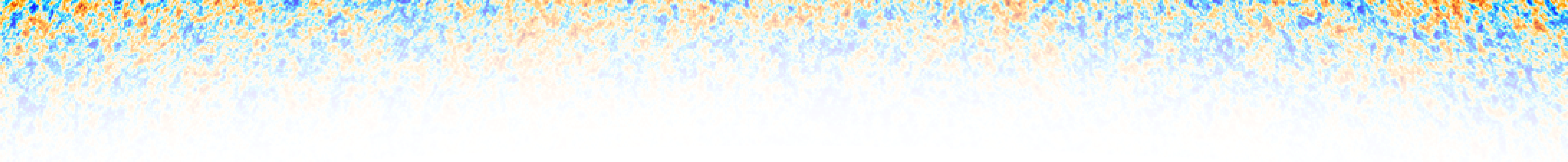
Samples of :

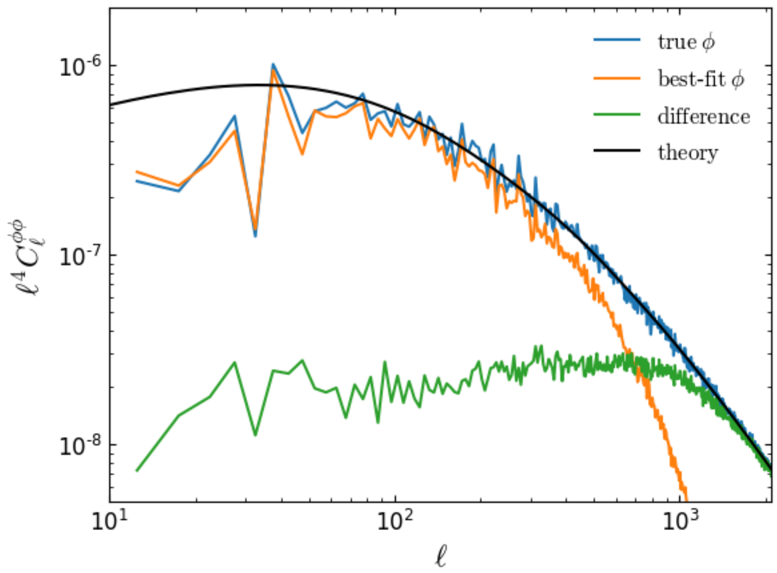


...

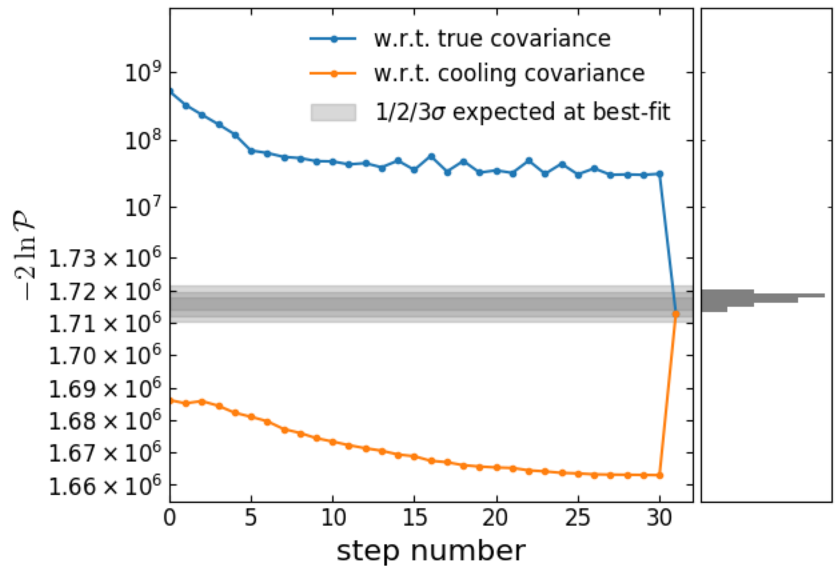
Conclusions

- We can maximize $\mathcal{P}(f, \phi, r \,|\, d)$
- Sampling is coming up and I've given you a preview of it
- Looking forward to more improvement, application to data, and feedback from the community (see our paper soon!)





true ϕ
best-fit ϕ
difference
theory
$\ell^4 C_\ell^{\phi\phi}$
ℓ
10^{-6}
10^{-7}
10^{-8}
10^1
10^2
10^3



w.r.t. true covariance
w.r.t. cooling covariance
1/2/3σ expected at best-fit
$-2\ln\mathcal{P}$
step number
10^9
10^8
10^7
1.73×10^6
1.72×10^6
1.71×10^6
1.70×10^6
1.69×10^6
1.68×10^6
1.67×10^6
1.66×10^6
0
5
10
15
20
25
30

LenseFlow determinant

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$$\operatorname{logdet}[\mathbb{1} + \varepsilon \; p_t \cdot \nabla] = \varepsilon \operatorname{Tr}[p_t \cdot \nabla] + \mathcal{O}(\epsilon^2)$$

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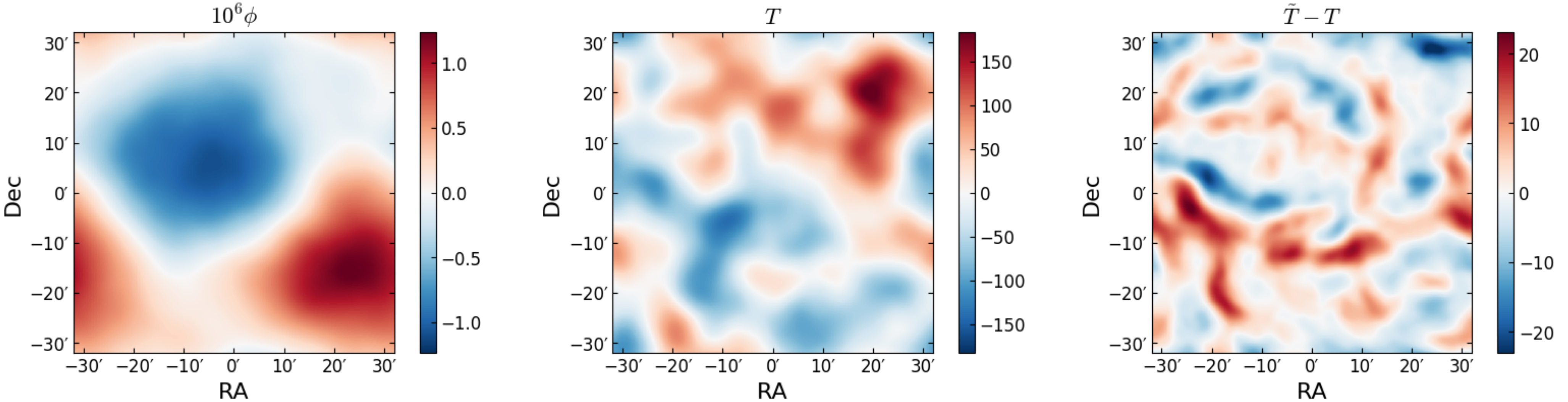
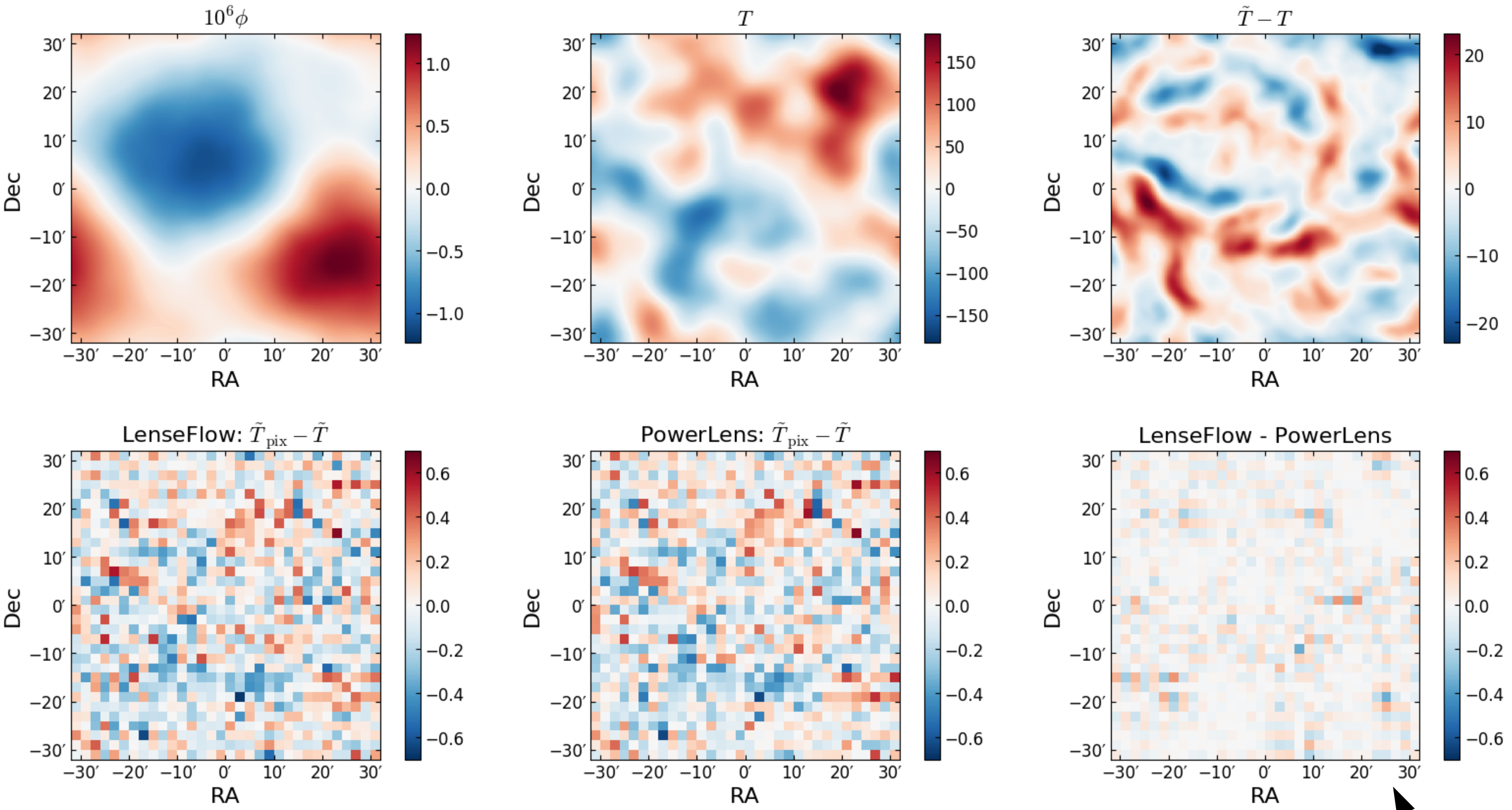
So for LenseFlow $\det|\mathcal{L}(\phi)| = 1$ so we can ignore it!



$10^6\phi$
T
$\tilde{T} - T$
Dec
RA
−30′ −20′ −10′ 0′ 10′ 20′ 30′
1.0 0.5 0.0 −0.5 −1.0
150 100 50 0 −50 −100 −150
20 10 0 −10 −20



LenseFlow

Taylor series

Differences between two which lead to different determinants