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Constraints and symmetries in the EFT of single-field inflation

in collaboration with

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Introduction and outline

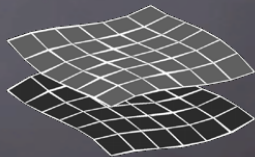
- Inflation: some primordial data, many models.
- Idea: understand the role of symmetries (exact or approximate) without relying on specific UV models and the consequences/constraints for observables.
- A simple case: shift-symmetry in single-clock inflation. Assuming an exact shift-symmetry provides specific relations among correlation functions.
Can we understand whether it is exact or broken?
- Adding higher-derivative operators: the case of theories with weakly-broken galileon symmetry in inflation.

EFT for single-field cosmology

[Creminelli, Luty, Nicolis, Senatore '06], [Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '08]

- Irrespective of what is really driving the evolution at microscopic level, one can capture the low-energy physics just in terms of **symmetry breaking patterns**.
- The system can be thought of as being equipped with a “clock” $\Phi(t)$ scanning the status of the Universe, defining a privileged time-slicing.

Exact symmetries: $x^i \rightarrow x^i + \xi^i(t, \vec{x})$.
Broken symmetries: $t \rightarrow t + \xi^0(t, \vec{x})$.



EFT for single-field cosmology

[Creminelli, Luty, Nicolis, Senatore '06], [Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '08]

- The residual symmetries (spatial diffs) enforce the following action

$$S = \int d^4x \sqrt{-g} \mathcal{L}(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu; t).$$

- Because of the breaking of time diffs, it exhibits **3 d.o.f.**: **2** graviton helicities + **1** scalar mode (use a **Stückelberg transformation** to make it explicit).
- The action for perturbations can be found expanding around FRW

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2.$$

- At this level, no input either from inflation (this is really the **EFT of single-clock cosmology!**) or from additional symmetries.

EFT for single-field inflation

[Creminelli, Luty, Nicolis, Senatore '06], [Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '08]

- Neglecting for the moment higher-derivative operators,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + \sum_{n=0}^{\infty} \frac{M_n(t)}{n!} (g^{00} + 1)^n \right].$$

- $M_0 = -M_{\text{Pl}}^2(3H^2 + 2\dot{H})$ and $M_1 = M_{\text{Pl}}^2\dot{H}$ are fixed by the background.
- Yet there is infinite freedom because the coefficients $M_{n \geq 2}(t)$ can be in principle *arbitrary* functions of time.
(E.g. in $P(X)$, with $X = -g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$, M_n are *not* necessarily constant.)

Shift-symmetry in the EFT of inflation

[Finelli, Goon, Pajer, L.S. *in progress*]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + \sum_{n=0}^{\infty} \frac{M_n(t)}{n!} (g^{00} + 1)^n \right].$$

- Imposing a **shift-symmetry** for the *clock*, $\Phi \rightarrow \Phi + c$, fixes the time-dependence of *all* free coefficients.

Shifting the clock is equivalent to performing a specific *t*-shift:

$$t \rightarrow t + \Delta t, \quad \Delta t \equiv \frac{c}{\dot{\Phi}(t)} + \dots$$

As a result

$$\chi \dot{M}_n - n \dot{\chi} M_n + \dot{\chi} M_{n+1} = 0.$$

- We are simply imposing that, after the time re-parametrization

$$t \rightarrow \Phi(t(\phi)) = \Phi(\phi) = \phi,$$

there is no explicit ϕ -dependence in the action

$$S = \int d\phi d^3\bar{x} \sqrt{-g} \mathcal{L}(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_{\mu} \cancel{\chi})$$

Shift-symmetry in the EFT of inflation

[Finelli, Goon, Pajer, L.S. *in progress*]

- The condition

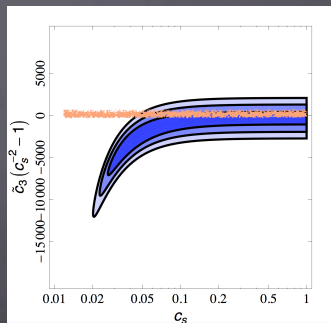
$$X\dot{M}_n - n\dot{X}M_n + \dot{X}M_{n+1} = 0$$

is simply telling us that all M_n are specific functions of the slow-roll parameters.

- As a result:

$$\check{c}_3(c_s^{-2} - 1) = \frac{1}{2} (3c_s^2 - 4 + c_s^{-2}) + (1 + c_s^{-2}) \frac{s}{2\varepsilon - \eta}.$$

- Relations among observables are more easily derived using Ward identities, without relying on perturbation theory.



Shift-symmetry in the EFT of inflation

[Finelli, Goon, Pajer, L.S. *in progress*]

- In particular, using the background conditions for M_0 and M_1 , the equations of motion can be written as

$$\varepsilon \left[\eta - 2\varepsilon + 3(1 + c_s^2) \right] = 0,$$

where $\varepsilon \equiv -\dot{H}/H^2$ and $\eta \equiv \dot{\varepsilon}/(H\varepsilon)$.

- No “slow-roll” in pure $P(X)$ -theories!
- Enforcing the shift symmetry requires higher-derivative operators...

EFT for single-field inflation

- One can add derivative operators in the EFT: $(N \equiv 1/\sqrt{-g^{00}})$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{M_{\text{Pl}}^2 \dot{H}}{N^2} - M_{\text{Pl}}^2 (3H^2 + \dot{H}) \right. \\ \left. + M_1^4 \delta N^2 + M_2^4 \delta N^3 + \dots \right. \\ \left. + \hat{M}_1^3 \delta K \delta N + \hat{M}_2^3 \delta K \delta N^2 + \dots \right].$$

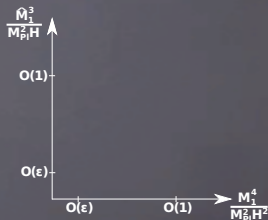
- For a classification of the operators in the unitary gauge, see [Langlois, Mancarella, Noui, Vernizzi '17].

EFT for single-field inflation

At the leading order in derivatives:

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 + \hat{M}_1^3 \delta K \delta N + \dots \right].$$

- One expects that physical observables are predominantly determined by the operators with the least number of derivatives.
- Within the regime of validity of the effective theory, $\delta K \delta N$ is always suppressed with respect to δN^2 . Unless there is some symmetry...



Message. In theories with (weakly broken) galileon symmetry, one can have $M_1^4 \sim \hat{M}_1^3 H \lesssim M_{\text{pl}}^2 H^2$, still within the regime of validity of the EFT.

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Flat space galileons [Nicolis, Rattazzi, Trincherini '09]

Invariant operators under $\phi \rightarrow \phi + c + b_\mu x^\mu$ (at the lowest order in derivatives):

$$\mathcal{L}_2 = (\partial\phi)^2$$

$$\mathcal{L}_3 = (\partial\phi)^2 [\Phi]$$

$$\mathcal{L}_4 = (\partial\phi)^2 ([\Phi]^2 - [\Phi^2])$$

$$\mathcal{L}_5 = (\partial\phi)^2 ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

where $[\Phi] = \square\phi$, $[\Phi^2] = \partial^\mu \partial_\nu \phi \partial^\nu \partial_\mu \phi$, ...

The main properties of the theory

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \sum_{n=3}^5 \frac{c_n}{\Lambda^{3(n-2)}} \mathcal{L}_n$$

are:

- the coefficients c_n are not renormalized (**non-renormalization theorem**) [Luty, Porrati, Rattazzi '03];
- second order equations of motion.

Coupling to gravity: non-minimal coupling

Gravity breaks the symmetry. For a certain class of theories not badly...

$$\mathcal{L}_3^{\text{nm}} = \sqrt{-g}(\partial\phi)^2[\Phi],$$

$$\mathcal{L}_4^{\text{nm}} = \sqrt{-g}[(\partial\phi)^4 R - 4(\partial\phi)^2([\Phi]^2 - [\Phi^2])],$$

$$\mathcal{L}_5^{\text{nm}} = \sqrt{-g} \left[(\partial\phi)^4 G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{2}{3}(\partial\phi)^2([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]) \right].$$

One can prove that only operators of the form

$$\frac{(\partial\phi)^{2n}}{M_{\text{Pl}}^n \Lambda^{3n-4}}$$

are generated. [Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Result. *The smallest scale by which the operators $(\partial\phi)^{2n}$ are suppressed is $\Lambda_2 \equiv (M_{\text{Pl}} \Lambda^3)^{1/4} \Rightarrow$ WBG symmetry.*

Remark. The theory has 2nd order e.o.m. (covariant galileon).

[Deffayet, Esposito-Farese, Vikman '09]

Higher derivative operators

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Because of the non-renormalization properties, one can still focus on a finite set of operators in the case

$$\frac{(\partial\phi_0)^2}{\Lambda_2^4} \sim \frac{\partial^2\phi_0}{\Lambda^3}.$$

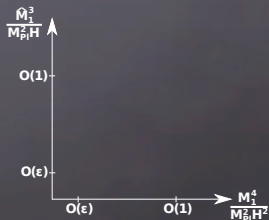
In the unitary gauge language,

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 + \hat{M}_1^3 \delta K \delta N + \dots \right],$$

it means

$$M_1^4 \sim \hat{M}_1^3 H.$$

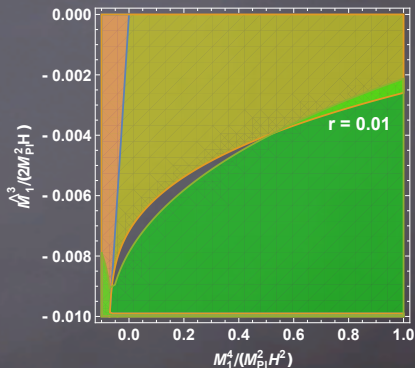
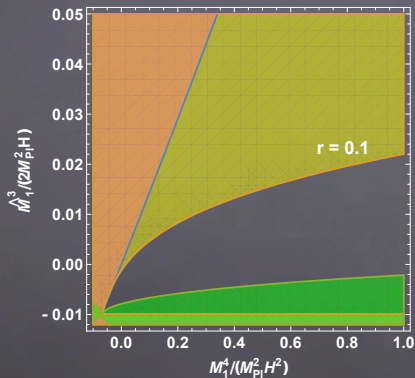
The “enhancement” of the operator $\delta K \delta N$ in theories with **WBG symmetry** may result in large, possibly measurable non-Gaussianity, $f_{\text{NL}} \gtrsim 1$.



Constraints on single-field inflation

[Pirtskhalava, L.S., Trncherini '15]

$$\varepsilon = 10^{-2}$$



Backup slides

Part I

Introduction to WBG theories

Effective Field Theories

General recipe. The main ingredients in the construction of an EFT

$$\mathcal{L}_{\text{EFT}} = -\frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{\Lambda^4} + \frac{(\partial\phi)^2\partial^2\phi}{\Lambda^3} + \mathcal{O}\left(\frac{\partial^m(\partial^2\phi)^n}{\Lambda^{m+3n-4}}\right) + \dots$$

are

- the **low-energy degrees of freedom** (not necessarily the *fundamental* ones);
- the **symmetries** (establishing the allowed operators).

Power counting. Then, the *natural* size of the couplings is fixed by the size of **quantum corrections**.

Higher derivative operators

General idea. Can higher derivative operators become important in an EFT?

$$\mathcal{L}_{\text{EFT}} = -\frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^4}{\Lambda^4} + \frac{(\partial\phi)^2\partial^2\phi}{\Lambda^3} + \mathcal{O}\left(\frac{\partial^m(\partial^2\phi)^n}{\Lambda^{m+3n-4}}\right) + \dots$$

Typically, higher derivative operators are not easy to deal with. Indeed, once expanded around the background, either

- they are **irrelevant** (at low energies, they are generically expected to be negligible with respect to lower derivative operators)

or

- lead to **instabilities**,

unless some **symmetry** protects the theory.

This is the case of the **galileon symmetry**:

$$\phi \rightarrow \phi + c + b_\mu x^\mu.$$

Galileon transformations

Purpose. We want to study the properties of an **approximate** galileon symmetry for some scalar field ϕ :

$$\phi \rightarrow \phi + c + b_\mu x^\mu.$$

Why approximate? It is unavoidable in cosmology (the galileon symmetry can only be defined in flat space-times, being explicitly broken by **gravity**).

Result. We will look for theories which retain as much as possible the galileon symmetry \Rightarrow ***Weakly Broken Galileon invariance***.

The non-renormalization theorem

It simply follows from integrations by parts. In D -dimensional flat space-time ($D \geq n$)

$$\mathcal{L}_{n+1} = \varepsilon^{i_1 \dots i_n k_1 \dots k_{D-n}} \varepsilon^{j_1 \dots j_n}_{k_1 \dots k_{D-n}} \phi \partial_{i_1} \partial_{j_1} \phi \dots \partial_{i_n} \partial_{j_n} \phi.$$

Statement. In any loop diagram involving \mathcal{L}_n , the external legs are all differentiated at least twice.

$$\begin{aligned} \varepsilon^{i_1 \dots i_n k_1 \dots k_{D-n}} \varepsilon^{j_1 \dots j_n}_{k_1 \dots k_{D-n}} \phi_{\text{ext}} \partial_{i_1} \partial_{j_1} \phi_{\text{int}} \dots \\ = \varepsilon^{i_1 \dots i_n k_1 \dots k_{D-n}} \varepsilon^{j_1 \dots j_n}_{k_1 \dots k_{D-n}} \partial_{i_1} \partial_{j_1} \phi_{\text{ext}} \phi_{\text{int}} \dots \end{aligned}$$

\Rightarrow One finds loop corrections of the form $(\partial^2 \phi)^n$.

\Rightarrow Galileon interactions are not renormalized.

Weakly Broken Galileon symmetry: a simple example in flat space-time

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi$$

- Exactly invariant under $\phi \rightarrow \phi + c + b_\mu x^\mu$.
- Λ_3 is the cutoff.

Weakly Broken Galileon symmetry: a simple example in flat space-time

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- Exactly invariant under $\phi \rightarrow \phi + c + b_\mu x^\mu$.
- Λ_3 is the cutoff.

Let us introduce a *small* breaking ($\Lambda_2 \gg \Lambda_3$):

$$\mathcal{L}' = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi + \frac{1}{\Lambda_2^4}(\partial\phi)^4$$

- Symmetry breaking operators of the form $(\partial\phi)^{2n}$ are generated by quantum corrections.
- They are generated at a scale which is parametrically higher than Λ_2 .
 \Rightarrow *The operator $(\partial\phi)^4$ gets only small corrections through loop effects.*



Coupling to gravity [Pirtskhalava, L.S., Trincherini, Vernizzi '15]

- What happens if we introduce **gravity**? Galileon invariance is unavoidably broken...
- How can we couple the theory to gravity? There are many ways... In analogy with the simple example in flat space

$$\mathcal{L}' = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi + \frac{1}{\Lambda_2^4}(\partial\phi)^4,$$

we would like to preserve the **non-renormalization properties** of the galileons as much as possible.

Coupling to gravity [Pirtskhalava, L.S., Trincherini, Vernizzi '15]

- The scale at which symmetry breaking operators are generated depends on the **coupling to gravity**.

Let us play with the possible couplings to gravity!

We concentrate on quantum-mechanically generated operators of the form

$$(\partial\phi)^{2n}.$$

Coupling to gravity: minimal coupling

The **minimally coupled** galileon theory is obtained by replacing

$$\partial_\mu \rightarrow \nabla_\mu.$$

Operators of the form

$$\frac{(\partial\phi)^{2n}}{M_{\text{Pl}}^k \Lambda_3^{4n-k-4}}, \quad k, n > 0,$$

are generated by graviton loops.

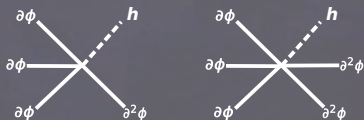
One can show that $k = 2n/3$ and the *smallest scale by which such operators are suppressed is $(M_{\text{Pl}} \Lambda_3^5)^{1/6}$.*

Can one do better? Can one enhance such a scale?

Coupling to gravity: non-minimal coupling

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Proof. One can show that all vertices of the form



cancel at *tree level*.

At most, one is left with diagrams involving $2 \times (\partial\phi)$ and 1 graviton, leading to the estimation

$$\frac{(\partial\phi)^{2n}}{M_{\text{Pl}}^n \Lambda_3^{3n-4}}.$$

Theories with WBG symmetry in curved space

Then we can add other symmetry breaking terms that do not spoil the properties (and that turn out to be in the Horndeski class):

$$\mathcal{L}_2^{\text{WBG}} = \Lambda_2^4 G_2(X)$$

$$\mathcal{L}_3^{\text{WBG}} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X) [\Phi]$$

$$\mathcal{L}_4^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X) R + 2 \frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X) ([\Phi]^2 - [\Phi^2])$$

$$\mathcal{L}_5^{\text{WBG}} = \frac{\Lambda_2^8}{\Lambda_3^9} G_5(X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi - \frac{\Lambda_2^4}{3\Lambda_3^9} G_{5X} ([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3])$$

where $X \equiv -\frac{(\partial\phi)^2}{\Lambda_2^4}$ and $[\Phi] = \square\phi$, $[\Phi^2] = \nabla^\mu \nabla_\nu \phi \nabla^\nu \nabla_\mu \phi$, ...

Remark. The coefficients of the polynomials $G_i(X)$ are not renormalized up to some powers of $\frac{\Lambda_3}{M_{\text{Pl}}}$ (WBG symmetry).

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

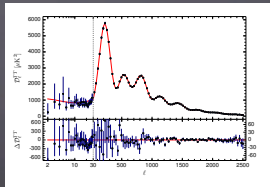
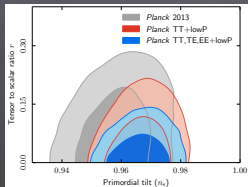
Part II

WBG theories in Cosmology

The standard Big Bang model and inflation

The Big Bang model. In the last decades, experiments have provided the following picture of the Universe:

- it is expanding and accelerating;
- it is highly homogeneous and isotropic;
- it is extremely flat;
- the spectrum of primordial scalar perturbations, imprinted on the CMB ($\delta T/T \sim 10^{-5}$), is *almost*
 - scale-invariant ($n_s = 0.968 \pm 0.006$),
 - adiabatic ($|\alpha_{\text{non-adi}}| \lesssim 10^{-2}$),
 - Gaussian ($f_{\text{NL}}^{\text{equil}} = -4 \pm 43$);
- no tensor modes ($r < 0.07$, 95% CL).



Contact with observations: scalar modes

The inflaton fluctuations $\delta\phi$ are responsible for the CMB anisotropies. The distribution function of the **scalar fluctuations** is parametrized in terms of

- *power spectrum*:

$$\langle \delta\phi_{\vec{k}} \delta\phi_{\vec{k}'} \rangle = (2\pi)^3 P_s(k) \delta(\vec{k} + \vec{k}'), \quad P_s(k) k^3 \equiv \mathcal{A}_s k^{n_s-1};$$

- *bispectrum*:

$$\langle \delta\phi_{\vec{k}_1} \delta\phi_{\vec{k}_2} \delta\phi_{\vec{k}_3} \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$$

The bispectrum is related to the field's self-interactions. A possible detection could discriminate among different models. A useful parametrization is

$$f_{\text{NL}} \propto \frac{B(k, k, k)}{P_s^2(k)}.$$

f_{NL} is highly suppressed in *slow-roll* models: the extreme flatness of the potential gives rise to small non-Gaussianity. What about derivative interactions?

Observable non-Gaussianity

In a generic **low-energy EFT**

$$\mathcal{L} = \sqrt{-g} \left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{(\partial\phi)^4}{\Lambda^4} + \dots \right],$$

what is the impact of derivative contributions at the level of non-Gaussianity? One finds [Creminelli '03]

$$f_{\text{NL}} \sim \frac{\dot{\phi}_0^2}{\Lambda^4}.$$

Any amount of $f_{\text{NL}} \gtrsim 1$ would be out of the regime of validity of the effective theory.

It **can not be trusted**, unless for instance the infinite series of derivative operators can be re-summed because of some **symmetry**, e.g. in DBI inflation.

DBI inflation

[Silverstein, Tong '03], [Alishahiha, Silverstein, Tong '04]

In DBI inflation, the effective expansion

$$\mathcal{L} \sim \sqrt{-g} \left[\Lambda^4 \sum_n c_n \frac{(\partial\phi)^{2n}}{\Lambda^{4n}} - V(\phi) \right]$$

can be re-summed as

$$\mathcal{L}_{\text{DBI}} = \sqrt{-g} \left[-\Lambda^4 \sqrt{1 + \frac{(\partial\phi)^2}{\Lambda^4}} - V(\phi) \right].$$

It predicts

$$f_{\text{NL}}^{\text{equil}} = \frac{5}{27} - \frac{35}{108} \frac{1}{c_s^2} + \dots, \quad c_s^2 \ll 1.$$

WBG theories in Cosmology: de Sitter solutions

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

General idea. Let us see how this can happen in a theory with (approximate) galileon symmetry for the cosmological scalar fields:

$$\phi \rightarrow \phi + c + b_\mu x^\mu.$$

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An inflationary theory with WBG symmetry can be written as

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{Pl}}^2 R - V(\phi) + \sum_{i=2}^5 \mathcal{L}_i^{\text{WBG}} \right]$$

We distinguish two cases:

- (i) $V \equiv 0$: we need at least $G_3(X) \square \phi$ (no slow-roll in pure $P(X)$ -theories!)

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We distinguish two cases:

- (i) $V \equiv 0$: we need at least $G_3(X) \square \phi$ (no slow-roll in pure $P(X)$ -theories!)
- (ii) $V \neq 0$: the evolution is of slow-roll type.

Remark. In both cases *quantum corrections are fully under control*.

The EFT of inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Background quantum stability: $X \equiv \frac{\dot{\phi}_0^2}{\Lambda_2^4} \lesssim 1$, $Z \equiv \frac{H\dot{\phi}_0}{\Lambda_3^3} \lesssim 1$.

$$M_1^4 = \Lambda_2^4 [2X^2 G_{2XX} - 3XZ (G_{3X} + 2XG_{3XX}) + \dots]$$

$$\hat{M}_1^3 H = \Lambda_2^4 [-2XZ G_{3X} + \dots]$$

- As anticipated, a *radiatively stable* hierarchy follows:

$$M_1^4 \sim \hat{M}_1^3 H.$$

- Depending on the values of X and Z , two phenomenologically distinct regimes:

- kinetically driven phase** ($X \sim Z \sim 1$):

$$M_1^4 \sim M_{\text{Pl}}^2 H^2, \quad \hat{M}_1^3 \sim M_{\text{Pl}}^2 H, \quad \dots$$

- potentially driven phase** ($X \sim \sqrt{\epsilon}$, $Z \sim 1$):

$$M_1^4 \sim \epsilon M_{\text{Pl}}^2 H^2, \quad \hat{M}_1^3 \sim \epsilon M_{\text{Pl}}^2 H, \quad \dots$$

The EFT of inflation with WBG symmetry

Coming back to our previous question...

$$S = \int d^4x \sqrt{\gamma} N [\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \dots].$$

- One can show that in theories with **WBG symmetry**, operators with a different number of derivatives produce comparable effects:

$$|M_1^4| \sim |\hat{M}_1^3 H| \lesssim M_{\text{Pl}}^2 H^2.$$

- $\delta K \delta N$ relevant for non-Gaussianity.



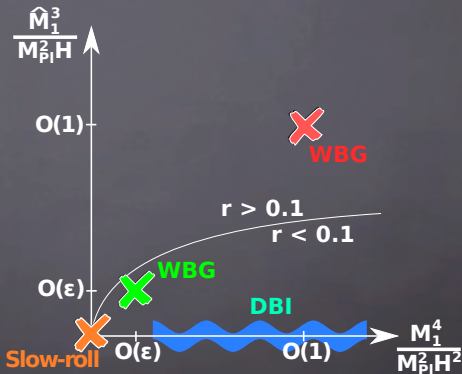
Experimental constraints on single-field inflation

[Pirtskhalava, L.S., Trincherini '15]

A qualitative sketch of the experimental constraints on the parameter space.

$$S = \int d^4x \sqrt{\gamma} N \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \dots \right].$$

- The bound on r alone puts strong and robust constraints on the parameter space of the effective theory.
- Further constraints are imposed by f_{NL} .



Constraints on single-field inflation

[Pirtskhalava, L.S., Trincherini '15]

Let us define the dimensionless parameters

$$\alpha \equiv \frac{\hat{M}_1^3}{M_{\text{Pl}}^2 H}, \quad \beta \equiv \frac{M_1^4}{M_{\text{Pl}}^2 H^2}, \quad \gamma \equiv \frac{\hat{M}_2^3}{M_{\text{Pl}}^2 H}, \quad \delta \equiv \frac{M_2^4}{M_{\text{Pl}}^2 H^2}.$$

Models of single field inflation

Inflationary models	Parameter hierarchy	c_s^2	Non-Gaussianity amplitude
Canonical slow-roll	$\epsilon \ll 1; \alpha, \beta, \gamma, \delta = 0$	1	$f_{\text{NL}} \sim \epsilon$
DBI-like theories	$\alpha, \gamma = 0; \epsilon \ll \beta \ll \delta$	$\frac{\epsilon}{\beta}$	$f_{\text{NL}} \sim \frac{1}{c_s^2}$
Galileon inflation	$\epsilon \ll \alpha, \beta, \gamma, \delta$	$\frac{\alpha(1-\alpha)}{\beta-3\alpha(2-\alpha)}$	$f_{\text{NL}} \sim \frac{1}{c_s^2}$ (or $\sim \frac{1}{c_s^4}$)
Kinetically driven WBG	$\epsilon \ll \alpha, \beta, \gamma, \delta \sim 1$	$\frac{\alpha(1-\alpha)}{\beta-3\alpha(2-\alpha)}$	$f_{\text{NL}} \sim \frac{1}{c_s^6}^\dagger$
Potentially driven WBG	$\epsilon \sim \alpha, \beta, \gamma, \delta$	$\frac{\epsilon+\alpha}{\epsilon+\beta-6\alpha}$	$f_{\text{NL}} \sim \frac{1}{c_s^4}^\ddagger$

† Such a steep growth is ruled out by experimental constraints.

‡ Slight deformation of slow-roll inflation, but with possible large non-Gaussianity.

WBG symmetry in presence of a scalar potential

Let's introduce a *flat* ($\epsilon_V, \eta_V \ll 1$) potential $V(\phi)$:

$$\int d^4x \sqrt{-g} [\mathcal{L}^{\text{WBG}} - V(\phi)], \quad V(\phi) \sim \phi^m.$$

Warning. Do quantum corrections to the vertices $V(\phi) \frac{\hbar^n}{M_{\text{Pl}}^n}$ spoil the notion of WBG symmetry of the theory? No. Indeed, for instance,

- only internal graviton lines: $\sim \left(\frac{\Lambda_3}{M_{\text{Pl}}}\right)^n$;
- one internal scalar field: $\sim \frac{V' \Lambda_3^2}{VM_{\text{Pl}}} \sim \sqrt{\epsilon_V} \left(\frac{\Lambda_3}{M_{\text{Pl}}}\right)^2$;
- ...

Result. These loop corrections are consistent with the notion of WBG symmetry. [Pirtskhalava, L.S., Trincherini, Vernizzi '15]

de Sitter space and symmetries

Inflation takes place in (an approximate) de Sitter space,

$$ds^2 = \frac{1}{H^2 \eta^2} (-d\eta^2 + d\vec{x}^2) = -dt^2 + e^{2Ht} d\vec{x}^2,$$

whose isometry group is $SO(4,1)$. Spatial **translations** and **rotations** are exact symmetries because of homogeneity and isotropy of the background evolution. The other de Sitter isometries are

- **dilation**: $\eta \rightarrow \lambda\eta$, $\vec{x} \rightarrow \lambda\vec{x}$,
- $\eta \rightarrow \eta - 2\eta(\vec{b} \cdot \vec{x})$, $x^i \rightarrow x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b} \cdot \vec{x})$.

In general, these are not symmetries of the background. However, invariance under dilation, which guarantees a **scale-invariant** power spectrum and constraints the 2-point function to be

$$\langle \delta\phi_{\vec{k}} \delta\phi_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{F(k\eta)}{k^3},$$

can be recovered by an additional $\phi \rightarrow \phi + c$ invariance. [Creminelli '12]

Contact with observations: tensor modes

Fluctuations of the metric tensor, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \gamma_{\mu\nu}$, give rise to

$$\langle \gamma_{\vec{k}}^s \gamma_{\vec{k}'}^{s'} \rangle = (2\pi)^3 P_\gamma(k) \delta^{ss'} \delta(\vec{k} + \vec{k}'), \quad P_\gamma(k) = \frac{2H^2}{M_{\text{Pl}}^2 k^3 c_T}.$$

- The **tensor modes** are much more *model independent* and *robust*. Indeed, using

$$g_{\mu\nu} \mapsto g_{\mu\nu} + (1 - c_T^2) n_\mu n_\nu \quad \text{and} \quad g_{\mu\nu} \mapsto c_T^{-1} g_{\mu\nu},$$

one can always set $c_T = 1$. [Creminelli, Gleyzes, Noreña, Vernizzi '14]

Conversely, scalar modes can be adjusted in many ways (shape of the potential, many scalars, speed of sound c_s , ...).

- The amplitude fixes the energy scale of inflation.

The Effective Field Theory of inflation

Because of the breaking of time diffeomorphisms, the unitary gauge action describes **3 d.o.f.** : the **2** graviton helicities + **1** scalar mode.

(In analogy, the same happens for a spin-1 massive particle: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2$)

This mode can be made explicit by performing a broken gauge transformation, $t \rightarrow t + \pi(t, \vec{x})$ (**Stückelberg trick**).

(For a non-Abelian gauge group: $A_\mu \rightarrow UA_\mu U^\dagger - \frac{i}{g}U\partial_\mu U^\dagger$, $U \equiv e^{i\pi^a T^a}$)

In some cases the physics of the Goldstone decouples from the two graviton helicities at short distances (**decoupling limit**).

(In analogy with the *equivalence theorem* for the longitudinal components of the massive gauge boson: $m \ll E \ll 4\pi m/g$)

Inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

The Friedmann equations:

$$3M_{\text{Pl}}^2 H^2 = V + \Lambda_2^4 X \left[\frac{1}{2} - \frac{G_2}{X} + 2G_{2X} - 6ZG_{3X} + \dots \right],$$

$$2M_{\text{Pl}}^2 \dot{H} = - \frac{\Lambda_2^4 X [1 + 2G_{2X} - 3ZG_{3X} + \dots]}{1 + 2G_4 - 4XG_{4X} - 2ZXG_{5X}}.$$

Background quantum stability: $X = \frac{\dot{\phi}_0^2}{\Lambda_2^4} \lesssim 1, \quad Z \equiv \frac{H\dot{\phi}_0}{\Lambda_3^3} \lesssim 1.$

Depending on the values of X and Z , two phenomenologically distinct regimes:

- **kinetically driven phase** ($X \sim Z \sim 1$) \Rightarrow mixing with gravity is order-one important at **all** scales (*i.e.* decoupling limit does not apply);
- **potentially driven phase** ($X \sim \sqrt{\epsilon}, Z \sim 1$) with $M_{\text{Pl}}^2 H^2 \sim V \gg (\partial\phi)^2 \Rightarrow$ decoupling limit applies at the Hubble scale and possible large non-Gaussianity.

The EFT of inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

$\mathcal{L}_i^{\text{WBG}}$	$M_1^4 \Lambda_2^{-4}$	$\hat{M}_1^3 H \Lambda_2^{-4}$...
$i = 2$	$2X^2 G_{2XX}$	\times	...
$i = 3$	$-3XZ(G_{3X} + 2XG_{3XX})$	$-2XZG_{3X}$...
$i = 4$	$12XZ^2(3G_{4XX} + \dots)$	$8XZ^2\left(\frac{G_{4X}}{X} + \dots\right)$...
$i = 5$	$XZ^3\left(3\frac{G_{5X}}{X} + \dots\right)$	$2XZ^3\left(3\frac{G_{5X}}{X} + \dots\right)$...

- As anticipated, a *radiatively stable* hierarchy follows:

$$M_1^4 \sim \hat{M}_1^3 H.$$

- kinetically driven phase** ($X \sim Z \sim 1$):

$$M_1^4 \sim M_{\text{Pl}}^2 H^2, \quad \hat{M}_1^3 \sim M_{\text{Pl}}^2 H, \quad \dots$$

- potentially driven phase** ($X \sim \sqrt{\epsilon}, Z \sim 1$):

$$M_1^4 \sim \epsilon M_{\text{Pl}}^2 H^2, \quad \hat{M}_1^3 \sim \epsilon M_{\text{Pl}}^2 H, \quad \dots$$

A more quantitative analysis

[Maldacena '05], [Chen, Huang, Kachru, Shiu '08], [Pirtskhalava, L.S., Trincherini '15]

Let's consider the effective theory

$$S = \int d^4x \sqrt{\gamma} N \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 + M_2^4 \delta N^3 - \hat{M}_1^3 \delta K \delta N + \hat{M}_2^3 \delta K \delta N^2 \right].$$

We want to explore configurations in which the decoupling limit does not apply and the full computation is required.

- Defining the gauge $\gamma_{ij} = a(t)^2 e^{2\zeta(t, \vec{x})} \delta_{ij}$,
- solving linearly the Hamiltonian constraints,
- expanding up to the 3rd order

yields the action for the scalar mode

$$S = \frac{1}{2} \int d^4x a^3 \left[\dot{\zeta}^2 - c_s^2 \frac{(\vec{\nabla} \zeta)^2}{a^2} + \mathcal{L}^{(3)} \right].$$

Constraints on single-field inflation

[Pirtskhalava, L.S., Trincherini '15]

In terms of the dimensionless parameters

$$\alpha \equiv \frac{\hat{M}_1^3}{M_{\text{Pl}}^2 H}, \quad \beta \equiv \frac{M_1^4}{M_{\text{Pl}}^2 H^2}, \quad \gamma \equiv \frac{\hat{M}_2^3}{M_{\text{Pl}}^2 H}, \quad \delta \equiv \frac{M_2^4}{M_{\text{Pl}}^2 H^2},$$

the speed of sound is

$$c_s^2 = \frac{(\varepsilon + \alpha)(1 - \alpha) + \frac{\dot{\alpha}}{H}}{\varepsilon + \beta - 3\alpha(2 - \alpha)}.$$

Different choices for the values of the parameters can yield a *small* speed of sound.

Inflationary models

In a single framework the EFT of inflation enables to assemble different models, which can be recovered by making particular choices for the effective coefficients.

Inflationary models	Parameter hierarchy	c_s^2	Non-Gaussianity amplitude
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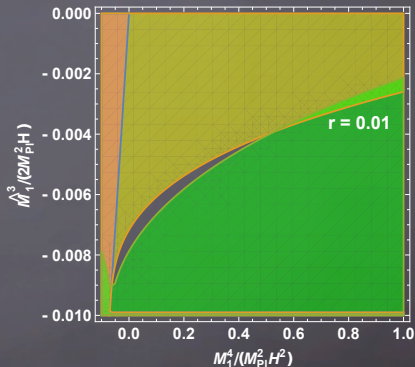
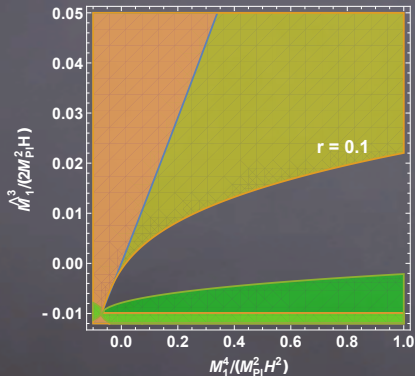
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Constraints on single-field inflation: summary

[Pirtskhalava, L.S., Trincherini '15]

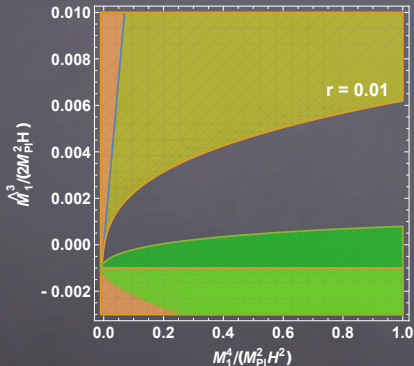
Parameters: $\varepsilon = 10^{-2}$, $\gamma \simeq \delta \simeq 1$



Constraints on single-field inflation: summary

[Pirtskhalava, L.S., Trincherini '15]

Parameters: $\varepsilon = 10^{-3}$, $\gamma \simeq \delta \simeq 1$



Remarks.

- DBI allowed.
- WBG inflation is allowed as well with the (stable) tuning $\alpha \lesssim 10^{-2}$, while $\beta, \gamma, \delta \lesssim 1$.
- In particular, slow-roll-WBG inflation is allowed.