# Shot noise and the halo model

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*Advances in theoretical cosmology in light of data*
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*with Dimitry Ginzburg and Kwan Chuen Chan*

# Halo shot noise

- *Perturbative bias expansion:*

$$\delta_h(\mathbf{x}) = b_1\delta(\mathbf{x}) + b_{\nabla^2\delta}\nabla^2\delta(\mathbf{x}) + \epsilon_0(\mathbf{x})$$
$$+ \frac{1}{2}b_2\delta^2(\mathbf{x}) + b_{K^2}\left(K_{ij}^2\right)(\mathbf{x}) + \epsilon_\delta(\mathbf{x})\delta(\mathbf{x}) + \ldots$$

$$\langle\epsilon_0\delta\rangle = \langle\epsilon_\delta\delta\rangle = 0$$

[Desjacques+ 16 & many references therein]

- *Our definition of the halo shot noise field:*

$$\epsilon(\mathbf{x}) \equiv \delta_h(\mathbf{x}) - b_1\delta(\mathbf{x})$$
$$= \epsilon_0(\mathbf{x}) + b_{\nabla^2\delta}\nabla^2\delta(\mathbf{x}) + \ldots$$

[Following Hamaus+ 10]

# Shot noise power spectrum

- *Split halo population into different mass bins:*

$$\epsilon_i(\mathbf{x}) = \delta_i(\mathbf{x}) - b_i\delta(\mathbf{x})$$
$$= \epsilon_{0i}(\mathbf{x}) + \ \ldots \ + \epsilon_{\delta i}(\mathbf{x})\delta(\mathbf{x}) + \ \ldots$$

- *Compute the halo noise power spectrum or covariance*

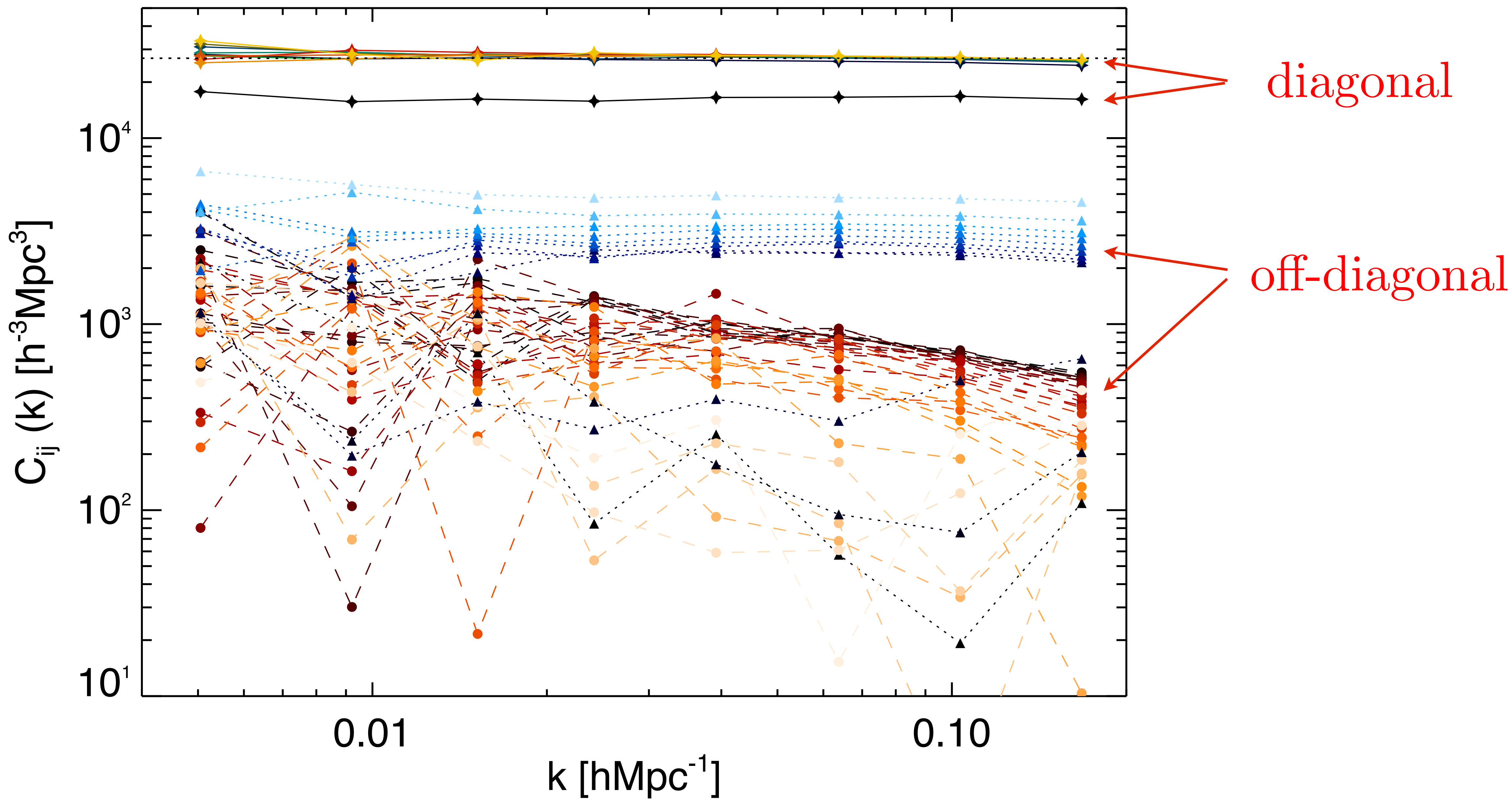
$$P_{\epsilon_i\epsilon_j}(k) = P_{\epsilon_{0i}\epsilon_{0j}}(k) + \ \ldots$$

- *For a Poisson sampling:*

$$P_{\epsilon_i\epsilon_j}(k\to 0) = P_{\epsilon_{0i}\epsilon_{0j}}(k\to 0) \equiv P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}}$$
$$= \frac{1}{\bar{n}_i}\delta^K_{ij}$$

[Hamaus+ 10]

*Halo bins with equal number density:*




[Hamaus+ 10]

# Halo model

- *Matter:* $$\frac{M}{\bar{\rho}_m} u(k|M)$$
- *Halos:* $$\frac{1}{\bar{n}_i} \Theta(M, M_i)$$
- *Galaxies:* $$\frac{N_g(M)}{\bar{n}_g} u_g(k|M)$$

[Seljak 00; Peacock & Smith 00; Scoccimarro+ 01; Cooray & Sheth 02]

# Halo model

*From the definition of the halo noise field:*

$$
\begin{aligned}
P_{\epsilon_i\epsilon_j}(k) &= \langle\delta_i\delta_j\rangle(k) - b_i\langle\delta_j\delta\rangle(k) \\
&\quad - b_j\langle\delta_i\delta\rangle(k) + b_ib_j\langle\delta\delta\rangle(k) \\
&= P_{ij}(k) - b_iP_{j\delta}(k) - b_jP_{i\delta}(k) + b_ib_jP_{\delta\delta}(k)\ .
\end{aligned}
$$

*the halo model predicts:*

$$
\begin{aligned}
P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} &= P^{1\mathrm{H}}_{ij}(0) - b_iP^{1\mathrm{H}}_{j\delta}(0) - b_jP^{1\mathrm{H}}_{i\delta}(0) + b_ib_jP^{1\mathrm{H}}_{\delta\delta}(0) \\
&= \frac{1}{\bar{n}_i}\delta^K_{ij} - b_i\frac{\overline{M_j}}{\bar{\rho}_m} - b_j\frac{\overline{M_i}}{\bar{\rho}_m} + b_ib_j\frac{\langle nM^2\rangle}{\bar{\rho}_m^2}
\end{aligned}
$$

[Hamaus+ 10]

# Halo model

*From the definition of the halo noise field:*

$$P_{\epsilon_i\epsilon_j}(k) = \langle\delta_i\delta_j\rangle(k) - b_i\langle\delta_j\delta\rangle(k) - b_j\langle\delta_i\delta\rangle(k) + b_ib_j\langle\delta\delta\rangle(k) = P_{ij}(k) - b_iP_{j\delta}(k) - b_jP_{i\delta}(k) + b_ib_jP_{\delta\delta}(k)\ .$$

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$$P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} = P^{1\mathrm{H}}_{ij}(0) - b_iP^{1\mathrm{H}}_{j\delta}(0) - b_jP^{1\mathrm{H}}_{i\delta}(0) + b_ib_jP^{1\mathrm{H}}_{\delta\delta}(0) = \frac{1}{\bar{n}_i}\delta^K_{ij} - b_i\frac{\overline{M_j}}{\bar{\rho}_m} - b_j\frac{\overline{M_i}}{\bar{\rho}_m} + b_ib_j\frac{\langle nM^2\rangle}{\bar{\rho}_m^2}$$

average mass in bin $j$

integrated over all mass range

[Hamaus+ 10]

# Halo model

*From the definition of the halo noise field:*

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\begin{aligned}
P_{\epsilon_i\epsilon_j}(k) &= \langle\delta_i\delta_j\rangle(k) - b_i\langle\delta_j\delta\rangle(k) \\
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&= P_{ij}(k) - b_i P_{j\delta}(k) - b_j P_{i\delta}(k) + b_i b_j P_{\delta\delta}(k)\ .
\end{aligned}
$$

*the halo model predicts:*

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P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} &= P^{1\mathrm{H}}_{ij}(0) - b_i P^{1\mathrm{H}}_{j\delta}(0) - b_j P^{1\mathrm{H}}_{i\delta}(0) + b_i b_j P^{1\mathrm{H}}_{\delta\delta}(0) \\
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\end{aligned}
$$

[Hamaus+ 10]

# Halo model

*From the definition of the halo noise field:*

$$P_{\epsilon_i \epsilon_j}(k) = \langle \delta_i \delta_j \rangle(k) - b_i \langle \delta_j \delta \rangle(k) - b_j \langle \delta_i \delta \rangle(k) + b_i b_j \langle \delta \delta \rangle(k) = P_{ij}(k) - b_i P_{j\delta}(k) - b_j P_{i\delta}(k) + b_i b_j P_{\delta\delta}(k) \; .$$

*the halo model predicts:*

$$P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} = P^{1\mathrm{H}}_{ij}(0) - b_i P^{1\mathrm{H}}_{j\delta}(0) - b_j P^{1\mathrm{H}}_{i\delta}(0) + b_i b_j P^{1\mathrm{H}}_{\delta\delta}(0) = \frac{1}{\bar{n}_i}\delta^K_{ij} - b_i \frac{\overline{M_j}}{\bar{\rho}_m} - b_j \frac{\overline{M_i}}{\bar{\rho}_m} + b_i b_j \frac{\langle n M^2 \rangle}{\bar{\rho}_m^2}$$

[Hamaus+ 10]

# Halo exclusion

- *Halo fluctuation field:* $$\delta_i(\mathbf{x}) = \frac{n_i(\mathbf{x})}{\bar{n}_i} - 1$$

- *2-point correlation:* $$\left\langle \delta_i(\mathbf{x}_1)\delta_j(\mathbf{x}_2)\right\rangle = \frac{1}{\bar{n}_i}\delta^K_{ij} + \xi_{ij}(\mathbf{r}_{12})$$

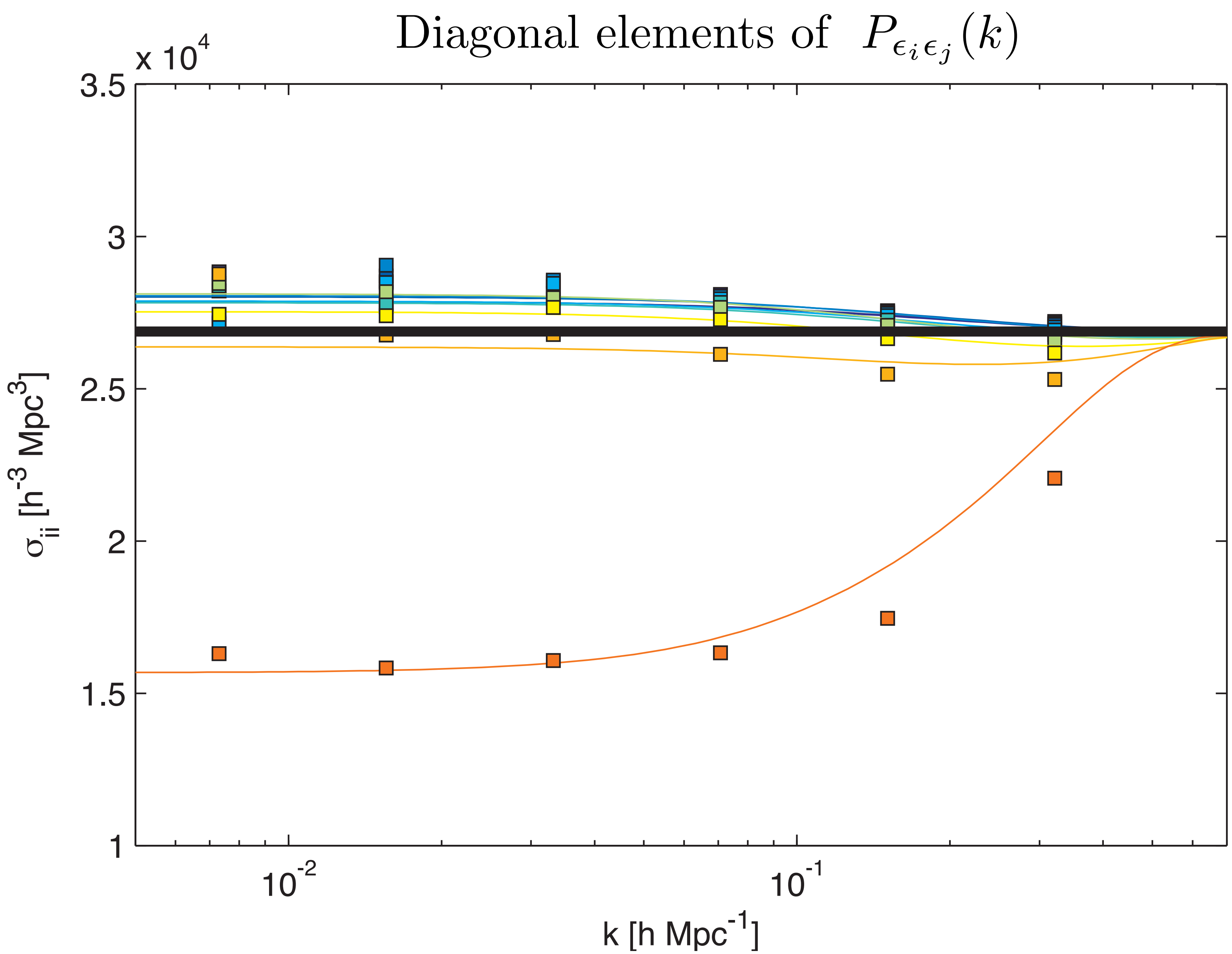
- *Power spectrum:* $$P_{ij}(k \to 0) = \left(\frac{1}{\bar{n}_i}\delta^K_{ij} + \Xi_{ij}\right) + b_i b_j P_{\rm lin}(k) + \ \ldots$$

$$\Xi_{ij} \equiv \int d^3r\, \xi_{ij}(\mathbf{r})$$

[Baldauf+ 13]

# Halo exclusion

*Use, e.g., BBKS peaks to get a prediction for $\xi_{ij}(r)$ at all r*

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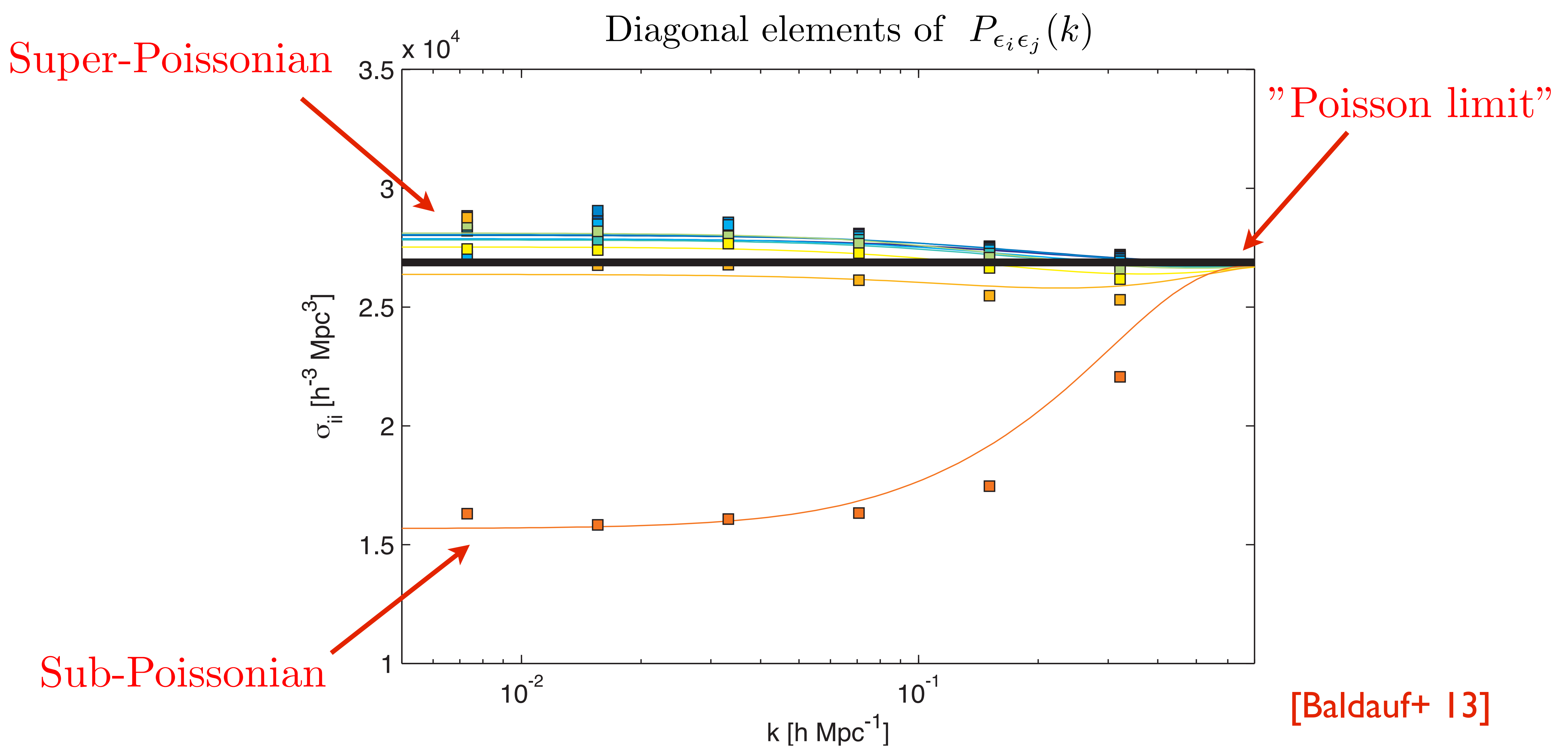



[Baldauf+ 13]

# Question

*Can we retain the simplicity of the halo model and, at the same time, get meaningful predictions for the halo shot noise ?*

# Another look at the halo model

- *On inspecting 1-halo, 2-halo etc. terms, you notice that*

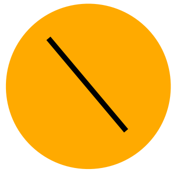
$$\delta_i(\mathbf{x}) = \big(b_i + \tilde{\epsilon}_{\delta i}(\mathbf{x})\big)\delta(\mathbf{x}) + \tilde{\epsilon}_{0i}(\mathbf{x}) + \ \ldots$$

$$\delta_m(\mathbf{x}) = \big(1 + \tilde{\epsilon}_{\delta m}(\mathbf{x})\big)\delta(\mathbf{x}) + \tilde{\epsilon}_{0m}(\mathbf{x}) + \ldots \ ,$$

*reproduces the original halo model predictions*

# Another look at the halo model

- *From 1-halo power spectra:*



$$P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0j}} = \frac{\delta^K_{ij}}{\bar{n}_i} \,, \quad P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0m}} = \frac{\overline{M_i}}{\bar{\rho}_m} \,,$$

$$P^{\{0\}}_{\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}} = \frac{\langle \bar{n} M^2 \rangle}{\bar{\rho}_m^2}$$

# Another look at the halo model

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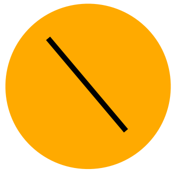

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$$P^{\{0\}}_{\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}} = \frac{\langle \bar{n} M^2 \rangle}{\bar{\rho}_m^2}$$

- *From 1-halo bispectra:*

$$B^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0j}\tilde{\epsilon}_{0k}} = \frac{1}{\bar{n}_i^2}\delta^K_{ijk}\,, \quad B^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0j}\tilde{\epsilon}_{0m}} = \frac{\overline{M_i}}{\bar{n}_i\bar{\rho}_m}\delta^K_{ij}$$

$$B^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}} = \frac{\overline{M_i^2}}{\bar{\rho}_m^2}\,, \quad B^{\{0\}}_{\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}} = \frac{\langle n M^3 \rangle}{\bar{\rho}_m^3}$$

# Another look at the halo model

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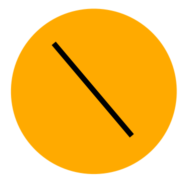

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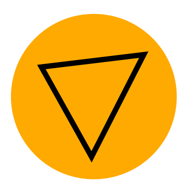
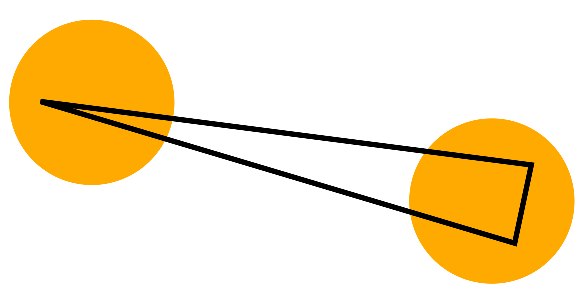
- *From 1-halo bispectra:*



$$B^{\{0\}}_{\tilde\epsilon_{0i}\tilde\epsilon_{0j}\tilde\epsilon_{0k}} = \frac{1}{\bar n_i^2}\delta^K_{ijk}\,, \qquad B^{\{0\}}_{\tilde\epsilon_{0i}\tilde\epsilon_{0j}\tilde\epsilon_{0m}} = \frac{\overline{M_i}}{\bar n_i\bar\rho_m}\delta^K_{ij}$$

$$B^{\{0\}}_{\tilde\epsilon_{0i}\tilde\epsilon_{0m}\tilde\epsilon_{0m}} = \frac{\overline{M_i^2}}{\bar\rho_m^2}\,, \qquad B^{\{0\}}_{\tilde\epsilon_{0m}\tilde\epsilon_{0m}\tilde\epsilon_{0m}} = \frac{\langle nM^3\rangle}{\bar\rho_m^3}$$

- *From 2-halo bispectra:*



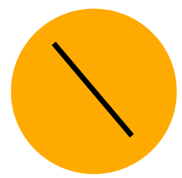
$$P^{\{0\}}_{\tilde\epsilon_{\delta m}\tilde\epsilon_{0m}} = \frac{\langle nM^2 b_1\rangle}{2\bar\rho_m^2}\,, \qquad P^{\{0\}}_{\tilde\epsilon_{\delta i}\tilde\epsilon_{0m}} + P^{\{0\}}_{\tilde\epsilon_{0i}\tilde\epsilon_{\delta m}} = b_i\frac{\overline{M_i}}{\bar\rho_m}\,,$$

$$P^{\{0\}}_{\tilde\epsilon_{\delta i}\tilde\epsilon_{0j}} + P^{\{0\}}_{\tilde\epsilon_{0i}\tilde\epsilon_{\delta j}} = \frac{b_i}{\bar n_i}\delta^K_{ij}$$

*etc.*

# Another look at the halo model

- *From 1-halo power spectra:*



$$P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0j}} = \frac{\delta^K_{ij}}{\bar{n}_i}\,, \quad P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{0m}} = \frac{\overline{M_i}}{\bar{\rho}_m}\,,$$

$$P^{\{0\}}_{\tilde{\epsilon}_{0m}\tilde{\epsilon}_{0m}} = \frac{\langle \bar{n} M^2 \rangle}{\bar{\rho}_m^2}$$

Valid at all $k$ !

- *From 1-halo bispectra:*

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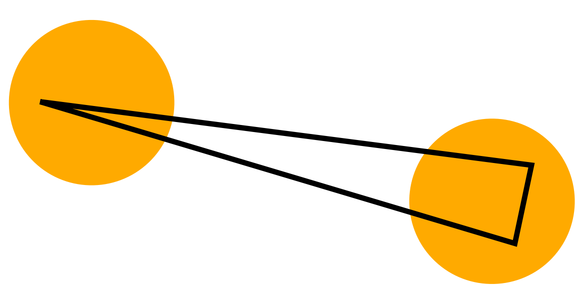
- *From 2-halo bispectra:*



$$P^{\{0\}}_{\tilde{\epsilon}_{\delta m}\tilde{\epsilon}_{0m}} = \frac{\langle n M^2 b_1 \rangle}{2\bar{\rho}_m^2}\,, \quad P^{\{0\}}_{\tilde{\epsilon}_{\delta i}\tilde{\epsilon}_{0m}} + P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{\delta m}} = b_i\frac{\overline{M_i}}{\bar{\rho}_m}\,,$$

$$P^{\{0\}}_{\tilde{\epsilon}_{\delta i}\tilde{\epsilon}_{0j}} + P^{\{0\}}_{\tilde{\epsilon}_{0i}\tilde{\epsilon}_{\delta j}} = \frac{b_i}{\bar{n}_i}\delta^K_{ij}$$

*etc.*

# Possible solution

- *On inspecting 1-halo, 2-halo etc. terms, you notice that*

$$\delta_i(\mathbf{x}) = \big(b_i + \tilde{\epsilon}_{\delta i}(\mathbf{x})\big)\delta(\mathbf{x}) + \tilde{\epsilon}_{0i}(\mathbf{x}) + \ \ldots$$

$$\delta_m(\mathbf{x}) = \big(1 + \tilde{\epsilon}_{\delta m}(\mathbf{x})\big)\delta(\mathbf{x}) + \tilde{\epsilon}_{0m}(\mathbf{x}) + \ldots \ ,$$

*reproduces the original halo model predictions*

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*reproduces the original halo model predictions*

- *"Halo model" perturbative expansions reorganized such that:*

$$\delta_i(\mathbf{x}) = \big(b_i + \tilde{\epsilon}_{\delta i}(\mathbf{x}) - b_i\tilde{\epsilon}_{\delta m}(\mathbf{x})\big)\delta(\mathbf{x}) + \tilde{\epsilon}_{0i}(\mathbf{x}) - b_i\tilde{\epsilon}_{0m}(\mathbf{x}) + \ \dots$$

$$\delta_m(\mathbf{x}) = \delta(\mathbf{x})$$

# Possible solution

- *On inspecting 1-halo, 2-halo etc. terms, you notice that*

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*reproduces the original halo model predictions*

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$$\delta_i(\mathbf{x}) = \big(b_i + \underbrace{\tilde{\epsilon}_{\delta i}(\mathbf{x}) - b_i\tilde{\epsilon}_{\delta m}(\mathbf{x})}_{\equiv\, \epsilon_{\delta i}(\mathbf{x})}\big)\delta(\mathbf{x}) + \underbrace{\tilde{\epsilon}_{0i}(\mathbf{x}) - b_i\tilde{\epsilon}_{0m}(\mathbf{x})}_{\equiv\, \epsilon_{0i}(\mathbf{x})} + \ \dots$$

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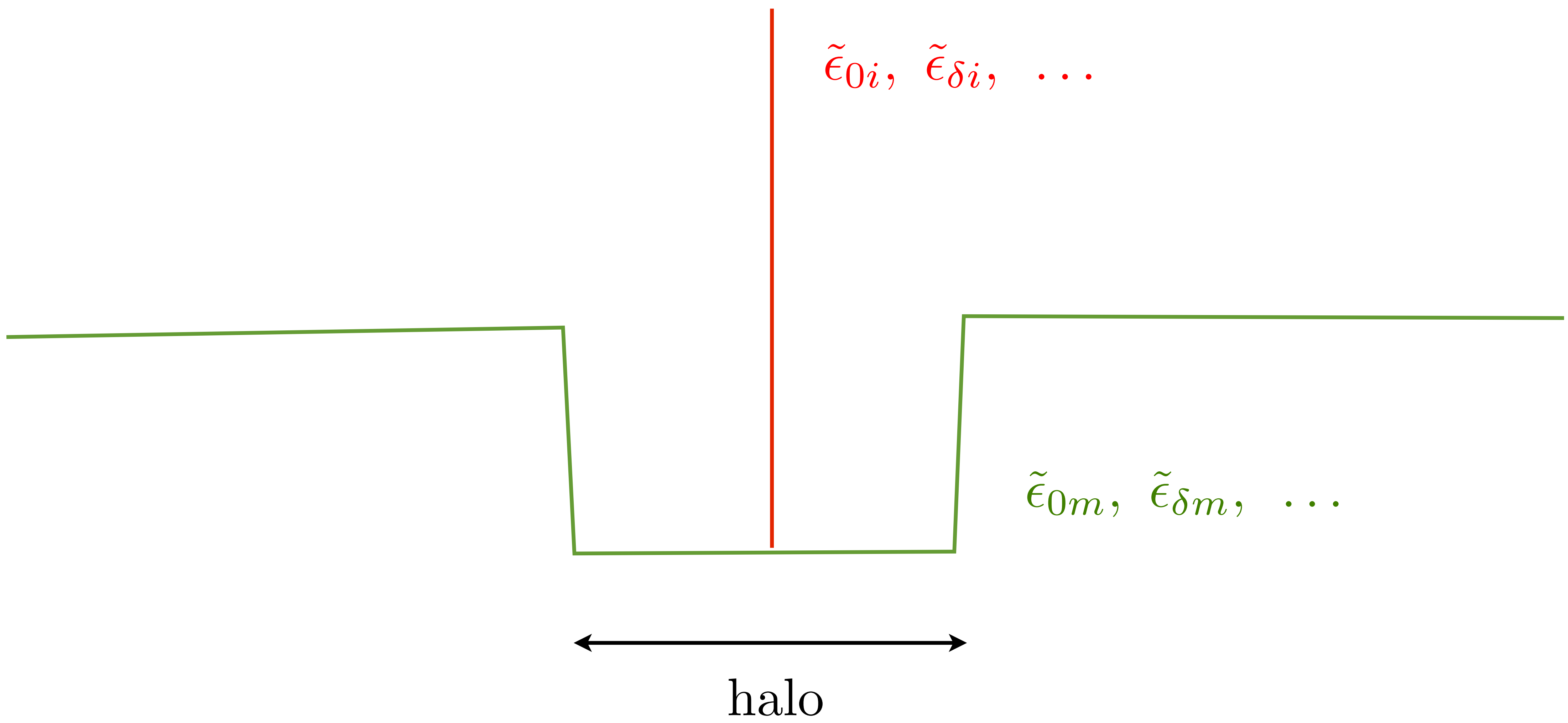
$\tilde{\epsilon}_{0i},\ \tilde{\epsilon}_{\delta i},\ \ldots$
$\tilde{\epsilon}_{0m},\ \tilde{\epsilon}_{\delta m},\ \ldots$
halo

# Power spectra

*Halo power spectrum:*

$$P_{ij}(k) \approx P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} + b_i b_j P_{\rm lin}(k) + \quad \ldots$$

*Low-k white noise:*

$$P^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}} = \frac{1}{\bar{n}_i}\delta^K_{ij} + \Xi_{ij}$$

*Exclusion:*

$$\Xi_{ij} = -b_i \frac{\overline{M_j}}{\bar{\rho}_m} - b_j \frac{\overline{M_i}}{\bar{\rho}_m} + b_i b_j \frac{\langle n M^2 \rangle}{\bar{\rho}_m^2}$$

*and, consistently,*

$$P_{i\delta}(k \to 0) \equiv 0$$

$$P_{\delta\delta}(k \to 0) \equiv 0$$

# Bispectra

$$B_{hhh} \approx B^{\{0\}}_{\epsilon_0\epsilon_0\epsilon_0} + b_1^3 B_{\delta\delta\delta} + \left\{ b_1^2 \left[ b_2 + 2b_{K^2} \left( \mu_{23}^2 - \frac{1}{3} \right) \right] P_{\rm lin}(k_2) P_{\rm lin}(k_3) + (2 \text{ cyc.}) \right\}$$

$$B_{hh\delta} \approx 2P^{\{0\}}_{\epsilon_0\epsilon_\delta} P_{\rm lin}(k_3) + b_1^2 B_{\delta\delta\delta} + \left\{ b_1 \left[ b_2 + 2b_{K^2} \left( \mu_{23}^2 - \frac{1}{3} \right) \right] \times P_{\rm lin}(k_2) P_{\rm lin}(k_3) + (2 \longleftrightarrow 3) \right\}$$

# Bispectra

$\epsilon_0$

$$B_{hhh} \approx B^{\{0\}}_{\epsilon_0\epsilon_0\epsilon_0} + b_1^3 B_{\delta\delta\delta} + \left\{ b_1^2 \left[ b_2 + 2b_{K^2}\left(\mu_{23}^2 - \frac{1}{3}\right)\right] P_{\rm lin}(k_2) P_{\rm lin}(k_3) + (2 \text{ cyc.}) \right\}$$

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$\eta_0$

$$\left. \times P_{\rm lin}(k_2) P_{\rm lin}(k_3) + (2 \longleftrightarrow 3) \right\}$$

# Bispectra

$$B_{hhh} \approx B^{\{0\}}_{\epsilon_0\epsilon_0\epsilon_0} + b_1^3 B_{\delta\delta\delta} + \left\{ b_1^2 \left[ b_2 + 2b_{K^2} \left( \mu_{23}^2 - \frac{1}{3} \right) \right] P_{\rm lin}(k_2) P_{\rm lin}(k_3) + (2 \text{ cyc.}) \right\}$$

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- *What we already know:*

$$B_{\epsilon_0\epsilon_0\epsilon_0}(k \to \infty) = \frac{1}{\bar{n}^2}$$

$$P_{\epsilon_0\epsilon_\delta}(k \to \infty) = \frac{b_1}{2\bar{n}}$$

[Peebles 80]

# Bispectra

$$B_{hhh} \approx B^{\{0\}}_{\epsilon_0\epsilon_0\epsilon_0} + b_1^3 B_{\delta\delta\delta} + \left\{ b_1^2 \left[ b_2 + 2b_{K^2}\left(\mu_{23}^2 - \frac{1}{3}\right)\right] P_{\rm lin}(k_2)P_{\rm lin}(k_3) + (2\ \text{cyc.}) \right\}$$

$$B_{hh\delta} \approx 2P^{\{0\}}_{\epsilon_0\epsilon_\delta} P_{\rm lin}(k_3) + b_1^2 B_{\delta\delta\delta} + \left\{ b_1 \left[ b_2 + 2b_{K^2}\left(\mu_{23}^2 - \frac{1}{3}\right)\right] \right.$$
$$\left. \times P_{\rm lin}(k_2)P_{\rm lin}(k_3) + (2 \longleftrightarrow 3) \right\}$$

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$$P_{\epsilon_0\epsilon_\delta}(k \to \infty) = \frac{b_1}{2\bar{n}}$$

[Peebles 80]

- *What we predict:*

$$B^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}\epsilon_{0k}} = -b_i b_j b_k \frac{\langle nM^3\rangle}{\bar{\rho}_m^3} + \left[ b_i b_j \frac{\overline{M_k^2}}{\bar{\rho}_m^2} + (2\ \text{cyc.})\right]$$
$$- \left[ b_i \left( \frac{\overline{M_j}}{\bar{n}_j \bar{\rho}_m}\right) \delta^K_{jk} + (2\ \text{cyc.})\right] + \frac{\delta^K_{ijk}}{\bar{n}_i^2}$$

$$P^{\{0\}}_{\epsilon_{0i}\epsilon_{\delta j}} + P^{\{0\}}_{\epsilon_{0j}\epsilon_{\delta i}} = \frac{b_i}{\bar{n}_i}\delta^K_{ij} - b_i \frac{\overline{M_j b_j}}{\bar{\rho}_m} - b_j \frac{\overline{M_i b_i}}{\bar{\rho}_m} + b_i b_j \frac{\langle nM^2 b_1\rangle}{\bar{\rho}_m^2}$$

# Test with numerical simulations

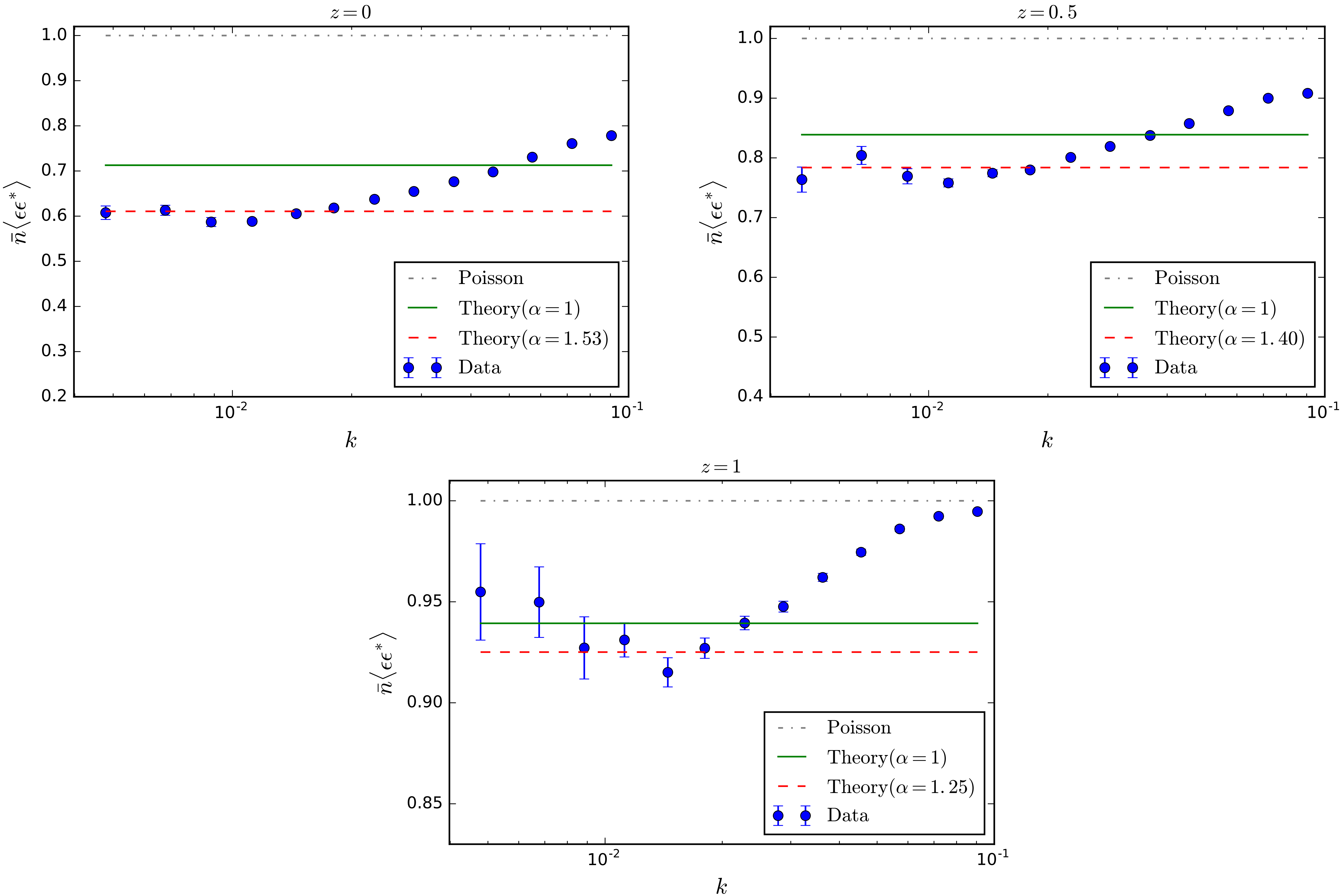
- *We use a series of 512 N-body simulations from the DEUS-FUR project*
- *Halos with mass > $10^{14}$ $M_\odot/h$ resolved*
- *Compute cumulative bispectra:*

$$B_{\epsilon\epsilon\epsilon}(< k_{\rm max}) = \frac{V^2}{N_t} \sum \epsilon(\mathbf{k}_1)\epsilon(\mathbf{k}_2)\epsilon^*(\mathbf{k}_1 + \mathbf{k}_2) \qquad \rightarrow B_{\epsilon_0\epsilon_0\epsilon_0}$$

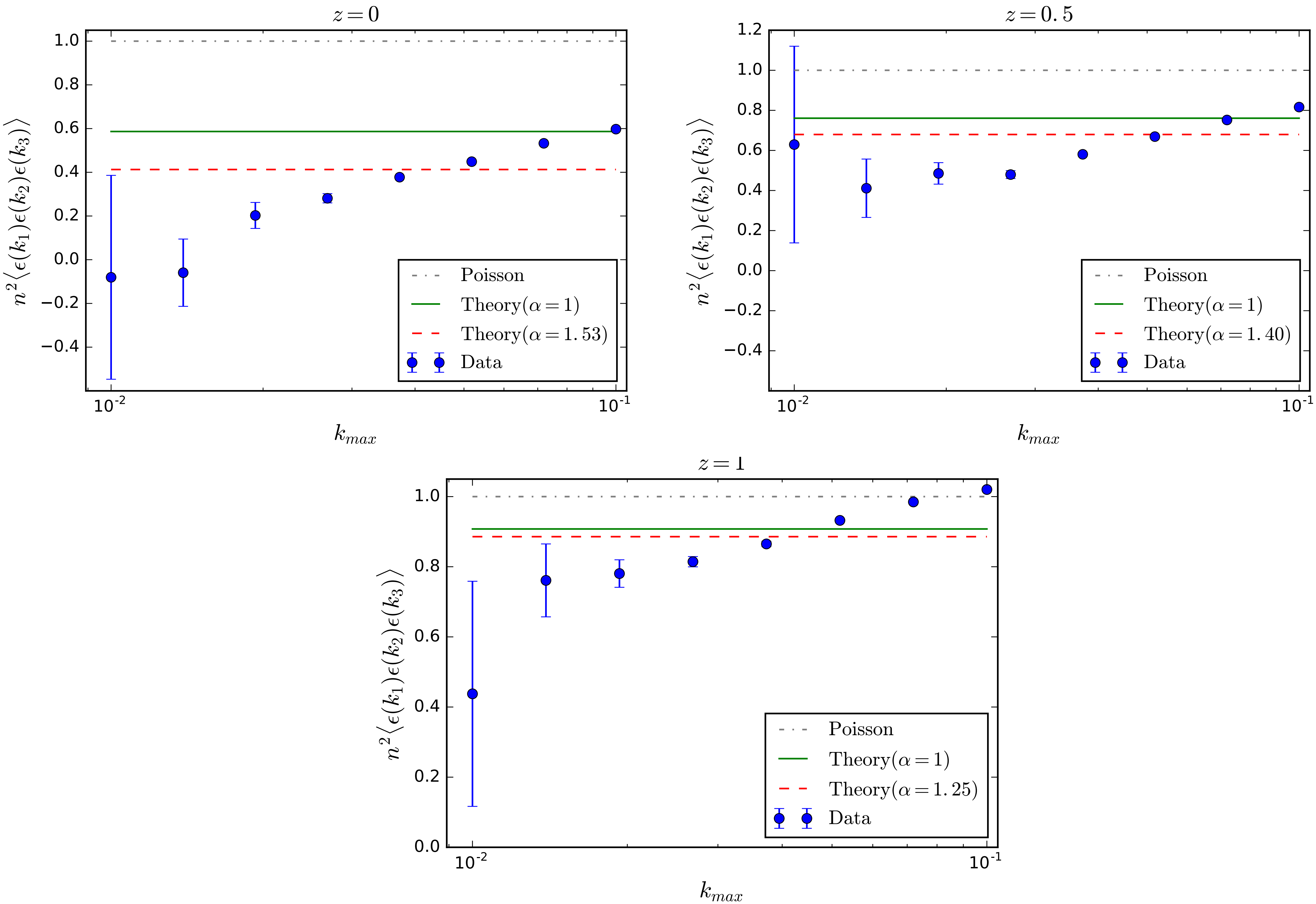
$$B_{\epsilon\epsilon\delta}(< k_{\rm max}) = \frac{V^2}{N_t} \sum \epsilon(\mathbf{k}_1)\epsilon(\mathbf{k}_2)\delta^*(\mathbf{k}_1 + \mathbf{k}_2) \qquad \rightarrow 2P_{\epsilon_0\epsilon_\delta}P_{\rm lin}$$

- *Add one fudge factor α to the model = adjust exclusion volume*

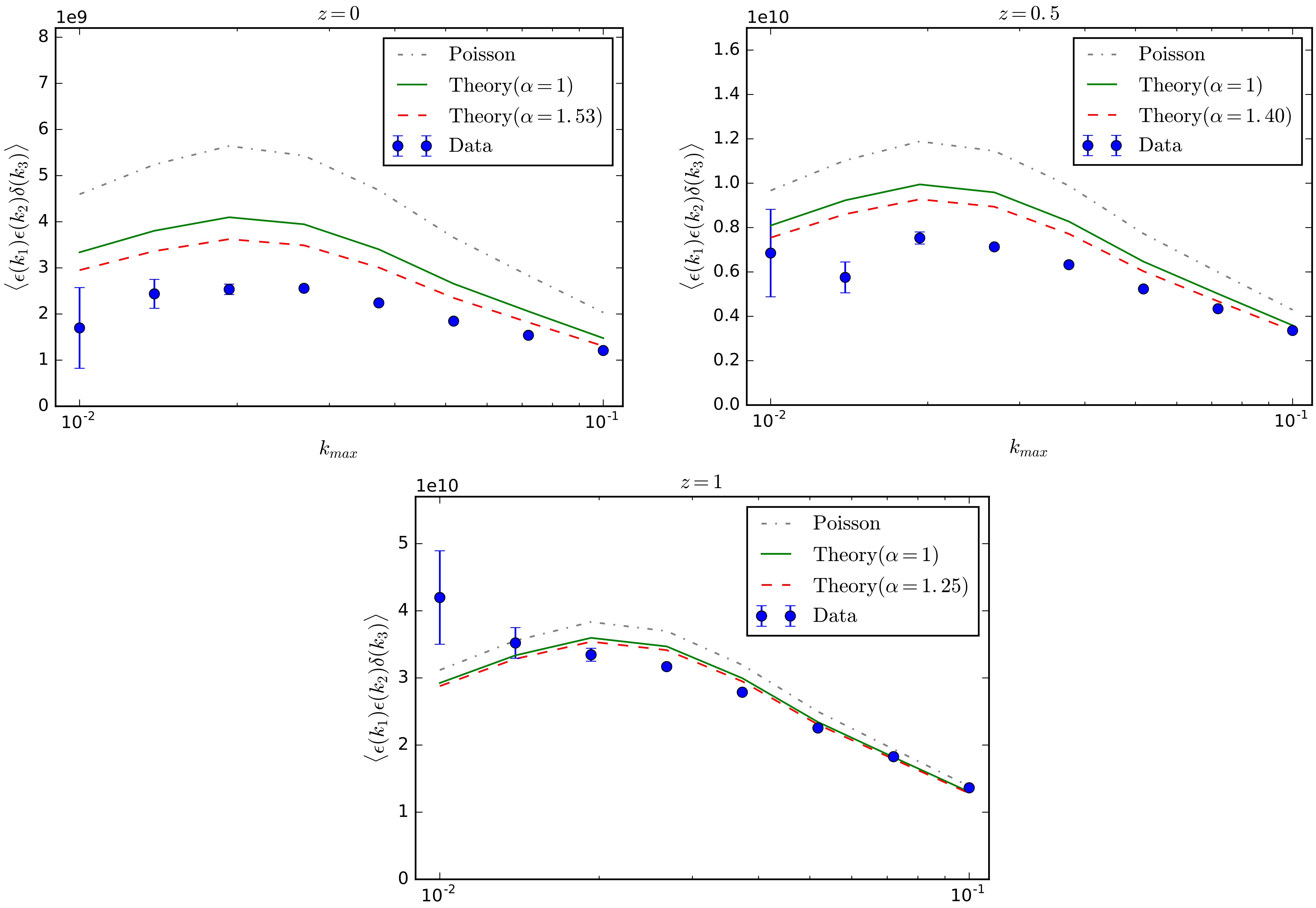
# $P_{\epsilon\epsilon}(k)$ :



## $B_{\epsilon\epsilon\epsilon}(< k_{\max})$ :



## $B_{\epsilon\epsilon\delta}(< k_{\max})$ :



# Fourier vs. configuration space

- *In configuration space:*

$$\left\langle \delta_i(\mathbf{x}_1)\delta_j(\mathbf{x}_2)\delta_k(\mathbf{x}_3)\right\rangle = \xi_{ijk}(\mathbf{r}_{12},\mathbf{r}_{13}) + \left[\frac{\delta^K_{ij}}{\bar{n}_i}\delta^D(\mathbf{r}_{12})\xi_{ik}(\mathbf{r}_{13}) + (2\ \text{cyc.})\right] + \frac{\delta^K_{ijk}}{\bar{n}_i^2}\delta^D(\mathbf{r}_{12})\delta^D(\mathbf{r}_{13})$$

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$$B_{ijk}(k_i\to 0) = B^{\{0\}}_{\epsilon_{0i}\epsilon_{0j}\epsilon_{0k}} \equiv \Xi_{ijk} + \left[\frac{\delta^K_{ij}}{\bar{n}_i}\Xi_{jk} + (2\text{ cyc.})\right] + \frac{\delta^K_{ijk}}{\bar{n}_i^2}$$

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# Squeezed limit

- *Let us scrutinize $B_{ij\delta}$ in the limit* $k_1 = k_2 = 0\ , \quad 0 < k_3 \ll 1$

$$B_{ij\delta}(k_1, k_2, k_3) \to \int d^3r_{13} \int d^3r_{12}\, \xi_{ij\delta}(\mathbf{r}_{12}, \mathbf{r}_{13}) e^{-i\mathbf{k}_3\cdot\mathbf{r}_{13}} + \frac{\delta^K_{ij}}{\bar{n}_i} \int d^3r\, \xi_{i\delta}(\mathbf{r})\, e^{-i\mathbf{k}_3\cdot\mathbf{r}}$$

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$$\delta(\mathbf{k}_3) \equiv \delta_{\rm L} \qquad\qquad \approx \frac{b_i}{\bar{n}_i} P_{\rm lin}(k_3)\delta^K_{ij}$$

$$\xi_{ij\delta}(\mathbf{r}_{12},\mathbf{r}_{13}) \overset{k_3\ll 1}{=} \left\langle \xi_{ij}\left(\mathbf{r}_{12}|\delta_{\rm L}\right)\delta_{\rm L}\right\rangle$$

$$\approx \frac{\partial}{\partial\delta_{\rm L}} \xi_{ij}(\mathbf{r}_{12}|\delta_{\rm L})\Big|_{\delta_{\rm L}=0} \langle\delta_{\rm L}^2\rangle$$

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$$\approx \frac{\partial}{\partial \delta_\mathrm{L}} \xi_{ij}(\mathbf{r}_{12}|\delta_\mathrm{L})\Big|_{\delta_\mathrm{L}=0} \langle \delta^2_\mathrm{L}\rangle$$

$$\int d^3 r_{13} \left\{ \int d^3 r_{12}\, \xi_{ij\delta}(\mathbf{r}_{12}, \mathbf{r}_{13}) \right\} e^{-i\mathbf{k}_3\cdot\mathbf{r}_{13}}$$

$$\approx \frac{\partial}{\partial \delta_\mathrm{L}} \left\{ \int d^3 r_{12}\, \xi_{ij}(\mathbf{r}_{12}|\delta_\mathrm{L}) \right\}_{\delta_L=0} P_\mathrm{lin}(k_3)$$

$$= \bar{\rho}_m \frac{\partial \Xi_{ij}}{\partial \tilde{\rho}_m}\Big|_{\tilde{\rho}_m = \bar{\rho}_m} P_\mathrm{lin}(k_3)$$

# Shot noise consistency relations

- *A consistency (model-independent) relation:*

$$P^{\{0\}}_{\epsilon_{0i}\epsilon_{\delta j}} + P^{\{0\}}_{\epsilon_{0j}\epsilon_{\delta i}} = \frac{b_i}{\bar{n}_i}\delta^K_{ij} + \bar{\rho}_m \frac{\partial \Xi_{ij}}{\partial \bar{\rho}_m}$$

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- *Satisfied by our reformulation of the halo model ?*

$$\bar{\rho}_m \frac{\partial \Xi_{ij}}{\partial \bar{\rho}_m} = \bar{\rho}_m \frac{\partial}{\partial \bar{\rho}_m}\left\{ - b_i \frac{\overline{M_j}}{\bar{\rho}_m} - b_j \frac{\overline{M_i}}{\bar{\rho}_m} + b_i b_j \frac{\langle n M^2 \rangle}{\bar{\rho}_m^2} \right\} = \ ?$$

# Shot noise consistency relations

- *A consistency (model-in*

$$P^{\{0\}}_{\epsilon_{0i}\epsilon}$$

- *Satisfied by our reformu*

$$\bar{\rho}_m \frac{\partial \Xi_{ij}}{\partial \bar{\rho}_m} = \bar{\rho}_n$$

$$\begin{aligned}
\frac{\partial \overline{M_i}}{\partial \bar{\rho}_m} &= \frac{\partial}{\partial \bar{\rho}_m}\left(\frac{1}{\bar{n}_i}\int dM\, M n\, \Theta(M, M_i)\right) \\
&= \frac{1}{\bar{n}_i}\int dM\, M \left(\frac{\partial n}{\partial \bar{\rho}_m}\right)\Theta(M, M_i) \\
&= \frac{1}{\bar{n}_i \bar{\rho}_m}\int dM\, M n \left(\frac{n}{\bar{\rho}_m}\frac{\partial n}{\partial \bar{\rho}_m}\right)\Theta(M, M_i) \\
&= \frac{\overline{M_i b_i}}{\bar{\rho}_m}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \langle n M^2 \rangle}{\partial \bar{\rho}_m} &= \int dM\, M^2 \left(\frac{\partial n}{\partial \bar{\rho}_m}\right) \\
&= \frac{\langle n M^2 b_1 \rangle}{\bar{\rho}_m}
\end{aligned}$$

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- *Varying bar{M} and <n*M^2> only:*

$$\bar{\rho}_m \frac{\partial \Xi_{ij}}{\partial \bar{\rho}_m} = -b_i \frac{\overline{M_j b_j}}{\bar{\rho}_m} - b_j \frac{\overline{M_i b_i}}{\bar{\rho}_m} + b_i b_j \frac{\langle nM^2 b_1 \rangle}{\bar{\rho}_m^2}$$

$$\equiv P^{\{0\}}_{\epsilon_{0i}\epsilon_{\delta j}} + P^{\{0\}}_{\epsilon_{0j}\epsilon_{\delta i}} \quad !$$

# Conclusions

- *The original halo model predicts a halo shot noise power spectrum in good agreement with the data, but leads to unphysical noise in cross halo-matter statistics*
- *The halo model can be reformulated in such a way that it still qualitatively reproduces super-/sub-Poissonian noise in the low-k limit and, at the same time, is not plagued by unphysical noise*
- *Our prescription straightforwardly maps onto the perturbative bias expansion*
- *Non-Poissonian contributions are related to volume integrals over correlation functions and their response to long-wavelength density perturbations.*
- *The shot noise in the cross halo-matter bispectrum of cluster-size halos is strongly sub-Poissonian. The deviation from Poissonian noise is stronger than in the power spectrum*