高等研究院 名古屋大学

Career Development Project for Researchers of Allied Universities

Large gauge transformation, Soft theorem, and Infrared divergence in inflationary universe

Yuko Urakawa

(Nagoya university, Institute for Advanced Research)

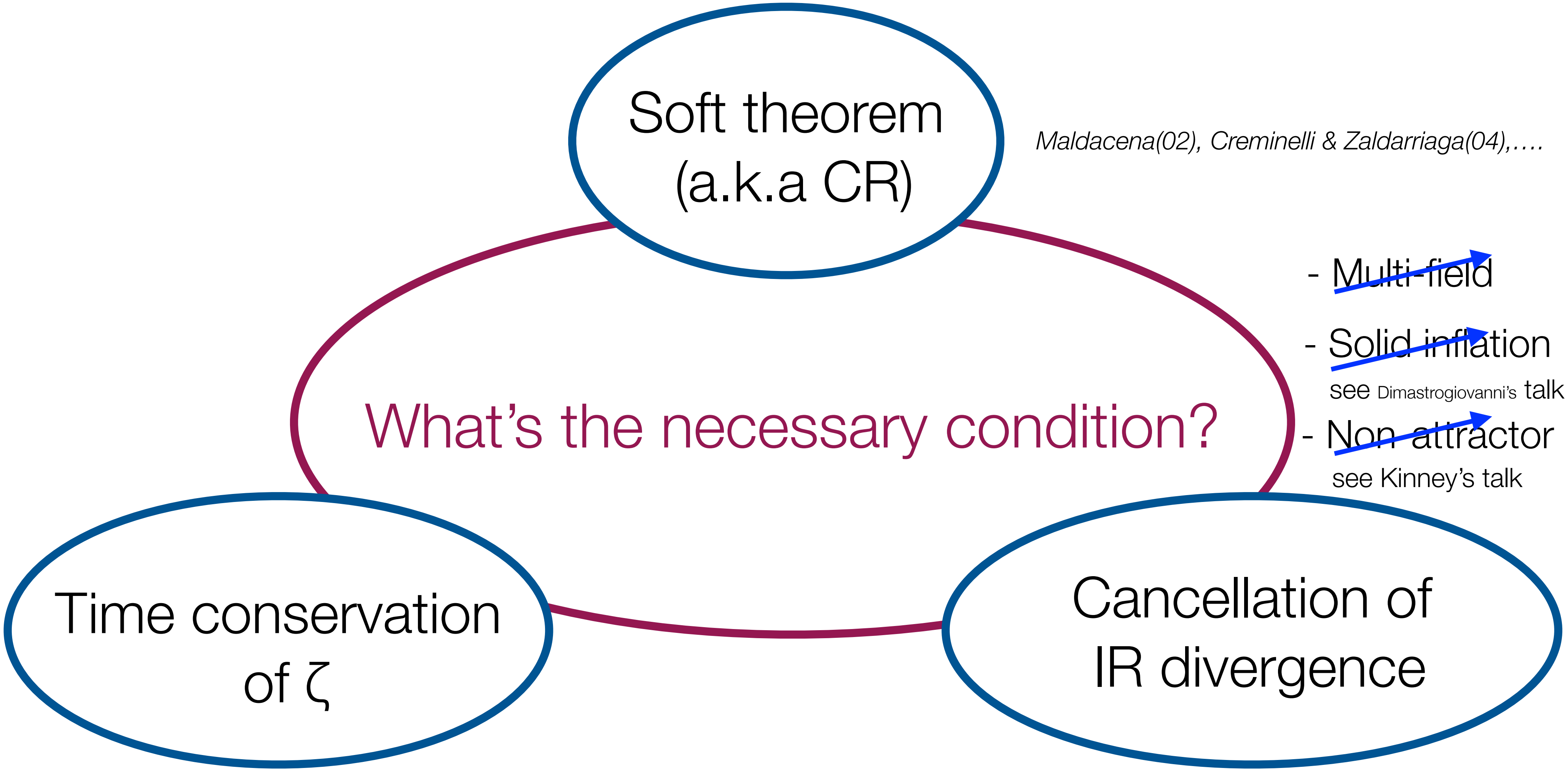
w/ Takahiro Tanaka (Kyoto university)

Based on JCAP 1606 (16) 020

arXiv:1707.05485

Universal properties in infrared (IR)

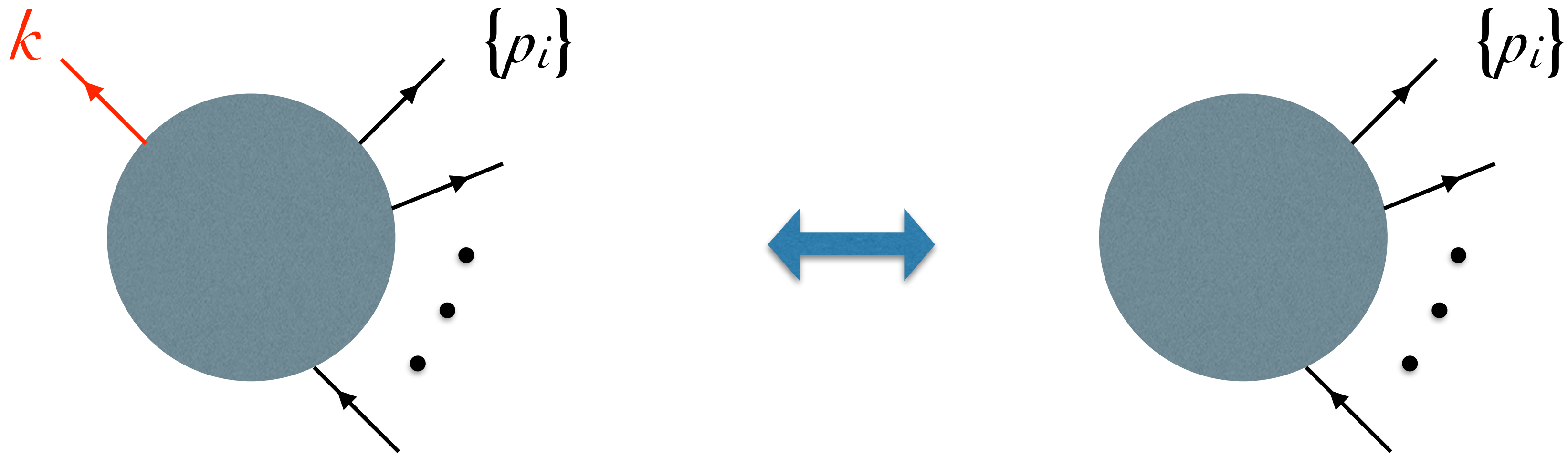
Going beyond case studies for IR (*k/aH* <<1)



Wands et al (00),Weinberg (03),….

Tanaka & Y.U. (09, 10,….), …..

Soft theorem



- Weinberg's soft theorem

Weinberg (65)

Graviton scattering in (asym.) flat spacetime

$$\mathcal{M}_n(q_1, \cdots, q_{n-1}, k) = \lim_{k \to 0} \sum_{a=1}^{n-1} \frac{\epsilon_{\mu\nu} q_a^\mu q_a^\nu}{k \cdot q_a} \mathcal{M}_{n-1}(q_1, \cdots, q_{n-1})$$

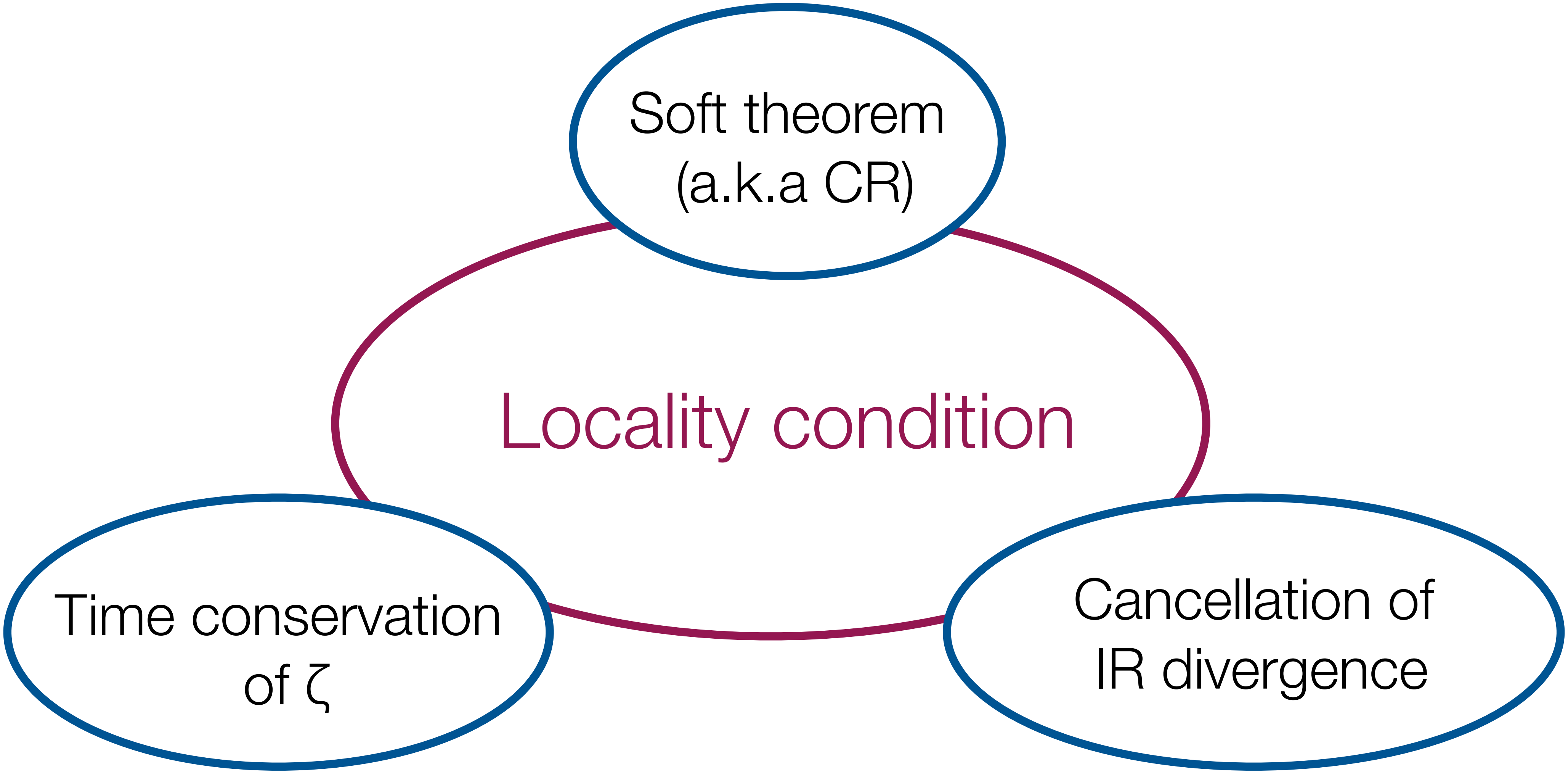
- Consistency relation

Maldacena(02), Creminelli & Zaldarriaga(04),….

$$\lim_{k_n \to 0} \frac{\mathcal{C}^{(n)}(\{\boldsymbol{k}_i\}_n)}{P(k_n)} = -\left(\sum_{i=2}^{n-1} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + 3(n-2)\right) \mathcal{C}^{(n-1)}(\{\boldsymbol{k}_i\}_{n-1})$$

Necessary condition for universal IR

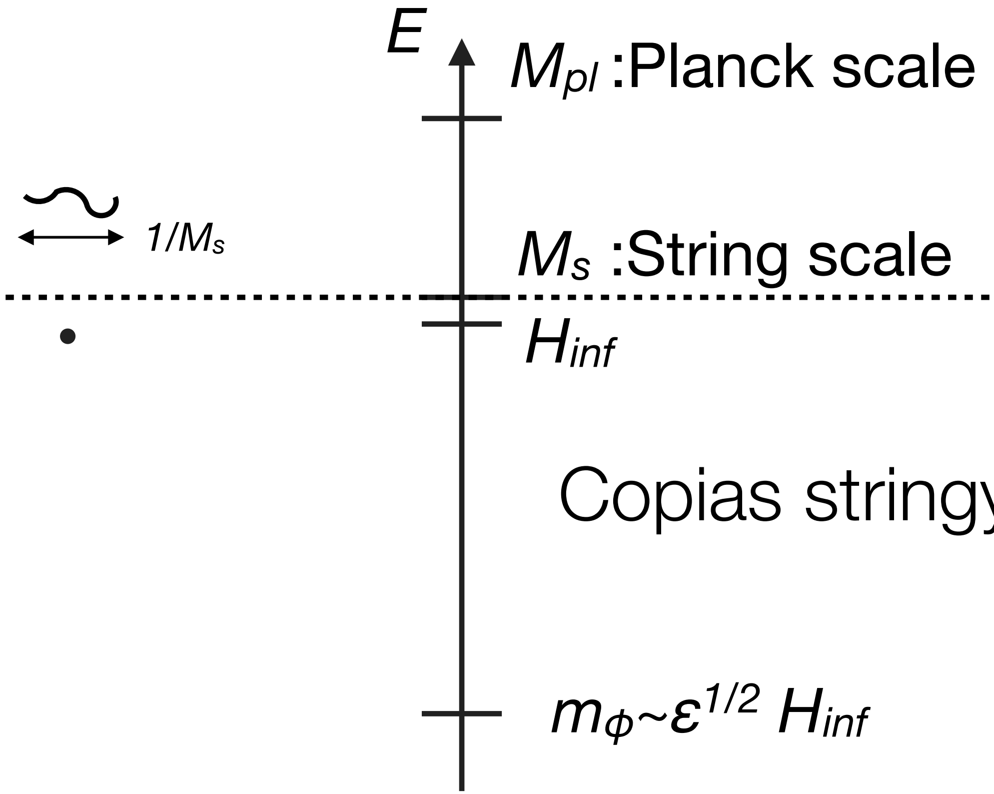
$k/aH \ll 1$



Formulation in model independent language!!

See also Berezhiani & Khoury (13)

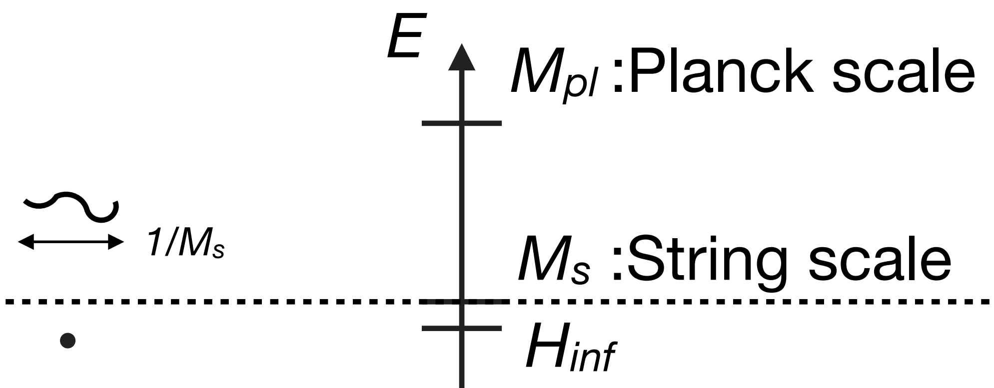
Application: Probe of string theory



Copias stringy corrections during inflation

N.B. No successful model building yet.

Application: Probe of string theory



Copias stringy corrections during inflation

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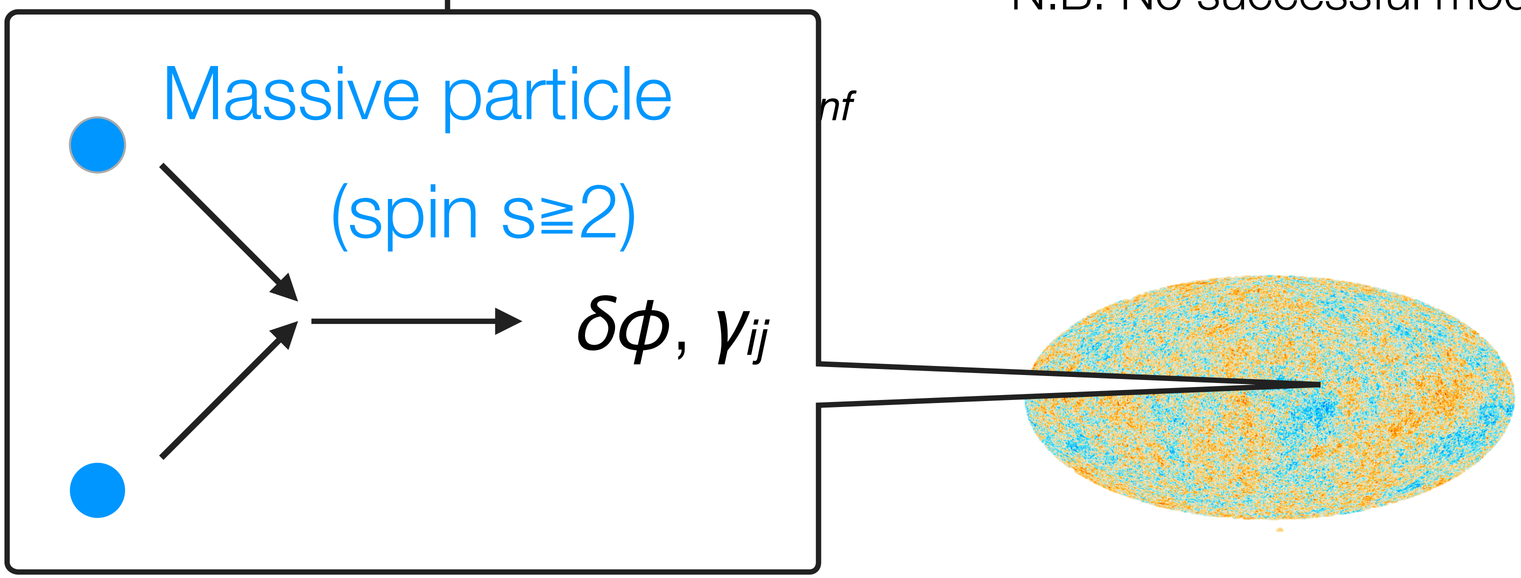
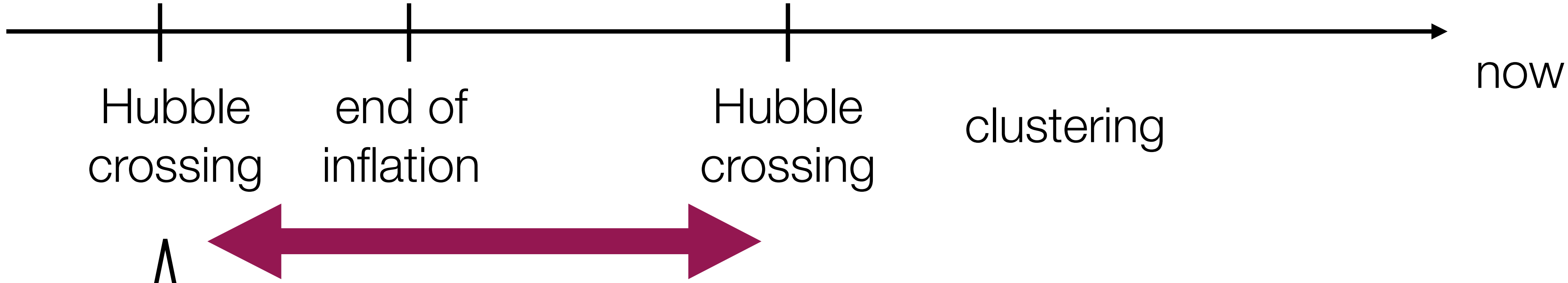


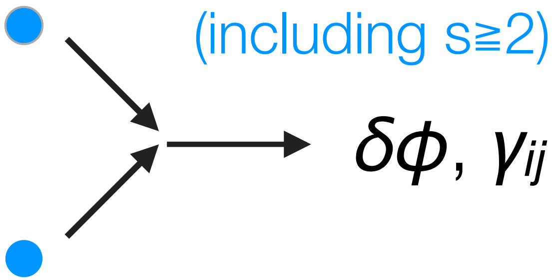
Application: Cosmological collider



ζ, γ_{ij} is conserved, if L.C. is valid.

Massive particles

(including s≧2)



$$B_\phi(\mathbf{k}_s, \mathbf{k}_L) = \sum_{l=0,2,\cdots} A_l \mathcal{P}_l(\hat{\mathbf{k}}_L \cdot \hat{\mathbf{k}}_S)...$$

Arkani-Hamed&Maldacena (15)

see also Lee et al.(16)

→ Galaxy (intrinsic) alignment

Schmidt et al. (15, 16)

Large gauge transformation

Gauge transformation g: g $\not\to$ 1 in $|x| \to \infty$

<u>Large Gauge transformations</u>

Classical action is invariant under

- Dilatation transformation $x^i \to e^s x^i$

Constant shift in homogeneous mode of ζ

$$\zeta(t,\,\boldsymbol{x}) \to \zeta_s(t,\,\boldsymbol{x}) = \zeta(t,\,e^{-s}\boldsymbol{x}) - s$$

$$\Delta_s \zeta(t,\,\boldsymbol{x}) = -s(1 + \boldsymbol{x}\cdot\partial_{\boldsymbol{x}}\zeta(t,\,\boldsymbol{x})) + \mathcal{O}(s^2)$$

- Shear transformation $x^i \to \left[e^{\frac{S}{2}}\right]^i{}_j\, x^j$ S_{ij}: Symmetric traceless constant tensor

Constant shift in homogeneous mode of γ_{ij}

Noether charge

Noether charge for dilatation

Hinterbichler et al. (14)

$$Q_\zeta \equiv \frac{1}{2}\int d^3\boldsymbol{x}\,[\Delta_s\zeta(t,\,\boldsymbol{x})\pi_\zeta(t,\,\boldsymbol{x}) + \pi_\zeta(t,\,\boldsymbol{x})\Delta_s\zeta(t,\,\boldsymbol{x})]$$

Generator of dilatation $\quad [Q_\zeta,\,\zeta(x)] = -i\Delta_s\zeta(x)$

Decomposition

$$|\Psi\rangle = \int d\bar{\zeta}^c\,|\psi(\bar{\zeta}^c)|\,|\,\bar{\zeta}^c\,\rangle|\,\Psi\rangle_{\bar{\zeta}^c} \qquad \langle\bar{\zeta}^c\,|\Psi\rangle = |\psi(\bar{\zeta}^c)|\,|\,\Psi\rangle_{\bar{\zeta}^c}$$

$|\,\bar{\zeta}^c\,\rangle$: Eigenstate of k=0 mode

$|\,\Psi\rangle_{\bar{\zeta}^c}$: Projected k ≠ 0 mode onto $|\,\bar{\zeta}^c\,\rangle$

WT identity for Dilatation invariance

Tanaka & Y.U. (17)

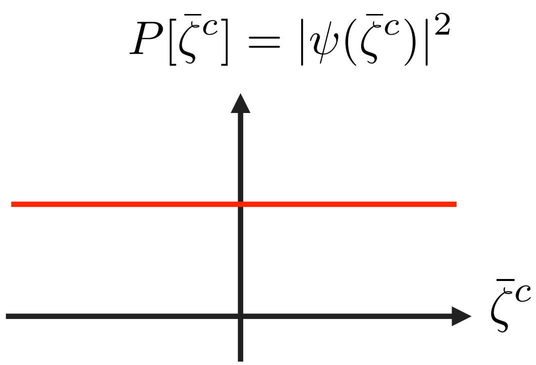
Ward-Takahashi identity $$\langle \bar{\zeta}^{c'} | Q_\zeta | \Psi \rangle = 0$$

$$|\Psi\rangle = \int d\bar{\zeta}^c \, |\psi(\bar{\zeta}^c)| \, |\, \bar{\zeta}^c \rangle |\, \Psi\rangle_{\bar{\zeta}^c}$$

Q_ζ shifts k=0 mode $$iQ_\zeta |\, \bar{\zeta}^c\rangle = \frac{\partial}{\partial \bar{\zeta}^c} |\, \bar{\zeta}^c\rangle$$

1) Condition on k=0

Re[WT]=0 → $$\frac{\partial}{\partial \bar{\zeta}^c} |\psi(\bar{\zeta}^c)| = 0$$



N.B. Scale-invariant Gaussian

$$\langle |\zeta_k|^2 \rangle \propto 1/k^3 \to \infty \;\; (k \to 0)$$

WT identity for Dilatation invariance 2

Tanaka & Y.U. (17)

Ward-Takahashi identity $\quad \langle \bar{\zeta}^{c'} | Q_\zeta | \Psi \rangle = 0$

$$|\Psi\rangle = \int d\bar{\zeta}^c \, |\psi(\bar{\zeta}^c)| \, | \, \bar{\zeta}^c \rangle | \, \Psi \rangle_{\bar{\zeta}^c}$$

Q_ζ shifts k=0 mode $\quad iQ_\zeta | \, \bar{\zeta}^c \rangle = \frac{\partial}{\partial \bar{\zeta}^c} | \, \bar{\zeta}^c \rangle$

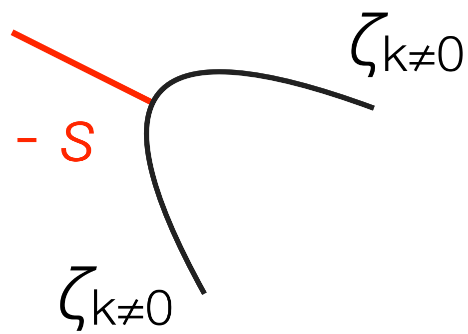
2) Condition on k≠0 which interacts w/ k=0

Im[WT]=0 $\longrightarrow$ $iQ_\zeta | \, \Psi \rangle_{\bar{\zeta}^c} = \frac{\partial}{\partial \bar{\zeta}^c} | \, \Psi \rangle_{\bar{\zeta}^c}$

$x^i \to e^s x^i$

s: constant

$\Longleftrightarrow$ $\zeta_{k=0} \to \zeta_{k=0} - s$



Locality condition

Tanaka & Y.U. (17)

Extension of WT identity to soft mode $k_L \to 0$, but $\neq 0$

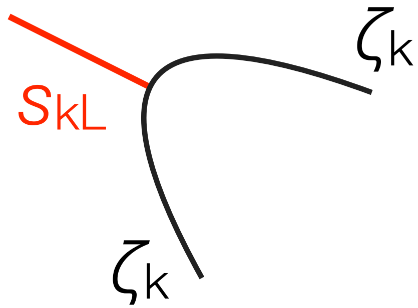
$$\text{LC} \qquad iQ^W_\zeta(\boldsymbol{k}_L)|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}} = \frac{\partial}{\partial \zeta^c_{-\boldsymbol{k}_L}}|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}}$$

Generator of inhomogeneous dilatation $\quad Q^W_\zeta(\boldsymbol{k}_L)$

$$x^i \to e^{s_{\boldsymbol{k}_L}} x^i$$

s_{kL}: constant

$\zeta_{kL} \to \zeta_{kL} - s_{kL}$



LC → Quantum version of Weinberg's adiabatic mode

Weinberg (03)

Why Locality condition?

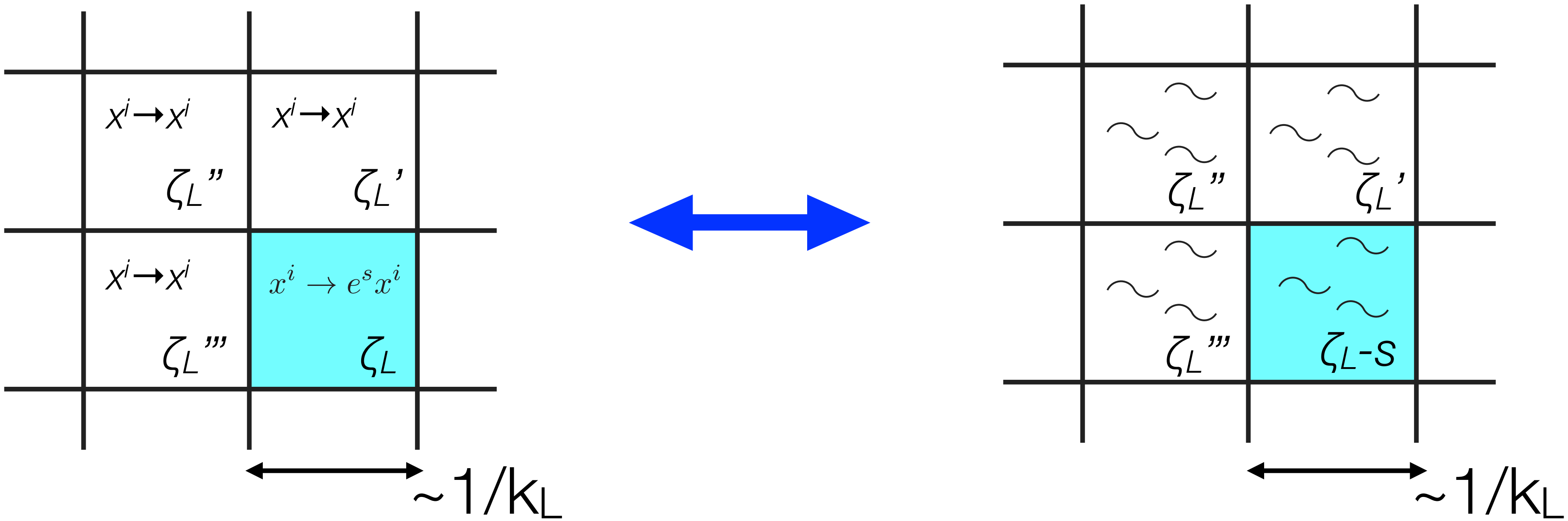
Tanaka & Y.U. (17)

Extension of WT identity to soft mode $k_L \to 0$, but $\neq 0$

LC $$iQ^W_\zeta(\boldsymbol{k}_L)|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}} = \frac{\partial}{\partial \zeta^c_{-\boldsymbol{k}_L}}|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}}$$

Generator of inhomogeneous dilatation $Q^W_\zeta(\boldsymbol{k}_L)$

<u>In position space</u>



Soft theorem (C.R.)

Tanaka & Y.U. (17)

LC $$iQ^W_\zeta(\boldsymbol{k}_L)|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}} = \frac{\partial}{\partial\zeta^c_{-\boldsymbol{k}_L}}|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}}$$

$$|\Psi\rangle = \int d\zeta^c_{\boldsymbol{p}_L}\,|\psi(\zeta^c_{\boldsymbol{p}_L})|\,|\,\zeta^c_{\boldsymbol{p}_L}\rangle|\,\Psi\rangle_{\zeta^c_{\boldsymbol{p}_L}}$$

<u>Assumption</u> $$|\psi(\zeta_{\mathbf{k}_L})| = \exp\left[-\frac{\zeta_{\mathbf{k}_L}\zeta_{-\mathbf{k}_L}}{4P_\zeta(k_L)}\right]$$

see also Hinterbichler et al. (14)

Compute $$\langle\Psi\,|\,\underbrace{[iQ^W_\zeta(\boldsymbol{k}_L),\,\zeta_{\boldsymbol{k}_{S1}}\cdots\zeta_{\boldsymbol{k}_{Sn}}]}_{(2)}\,|\,\Psi\rangle$$ (1)

$k_L/k_{Si} << 1$

(1) $$iQ^W_\zeta(\boldsymbol{k}_L)|\,\Psi\rangle = \int d\zeta^c_{-\boldsymbol{k}_L}\,|\psi(\zeta^c_{-\boldsymbol{k}_L})|\,\frac{\partial}{\partial\zeta^c_{-\boldsymbol{k}_L}}\left(|\,\zeta^c_{-\boldsymbol{k}_L}\rangle|\,\Psi\rangle_{\zeta^c_{-\boldsymbol{k}_L}}\right) \approx \frac{\zeta_{\boldsymbol{k}_L}}{2P_\zeta(k_L)}|\,\Psi\rangle$$

(2) $$\left[iQ^W_\zeta(\mathbf{k}_L),\,\zeta_{\mathbf{k}_S}\right] = \partial_{\mathbf{k}_S}\mathbf{k}_S\zeta_{\mathbf{k}_S}$$

Generalization of CR

Tanaka & Y.U. (17)

$$\langle \Psi \,|[iQ^W_\zeta(\boldsymbol{k}_L),\, \mathcal{O}_{\{i_{\alpha_1}\}\boldsymbol{k}_{S1}}(t_1)\cdots \mathcal{O}_{\{i_{\alpha_n}\}\boldsymbol{k}_{Sn}}(t_n)]|\,\Psi\rangle$$

$k_L/k_{Si} << 1$

Composite operator $\mathcal{O}^s_{\{i_\alpha\}}(t,\, e^s\mathbf{x}) = e^{-\Delta_\alpha s}\mathcal{O}_{\{i_\alpha\}}(t,\, \mathbf{x})$ $\quad x^i \to e^s x^i$

LC → CR

$$\lim_{k_L\to 0}\frac{\langle\Psi|\zeta_{\boldsymbol{k}_L}\mathcal{O}_{\{i_{\alpha_1}\}\boldsymbol{k}_{S1}}(t_1)\cdots\mathcal{O}_{\{i_{\alpha_n}\}\boldsymbol{k}_{S_n}}(t_n)|\Psi\rangle'}{P_\zeta(k_L)}$$

$$\overset{\text{s.l.}}{\approx} -\left(\sum_{i=2}^{n}\boldsymbol{k}_{Si}\cdot\frac{\partial}{\partial\boldsymbol{k}_{Si}} + d(n-1) - \Delta\right)\langle\Psi|\mathcal{O}_{\{i_{\alpha_1}\}\boldsymbol{k}_{S1}}(t_1)\cdots\mathcal{O}_{\{i_{\alpha_n}\}\boldsymbol{k}_{Sn}}(t_n)|\Psi\rangle'$$

$\Delta \equiv \sum_{i=1}^{n}\Delta_{\alpha_i}$

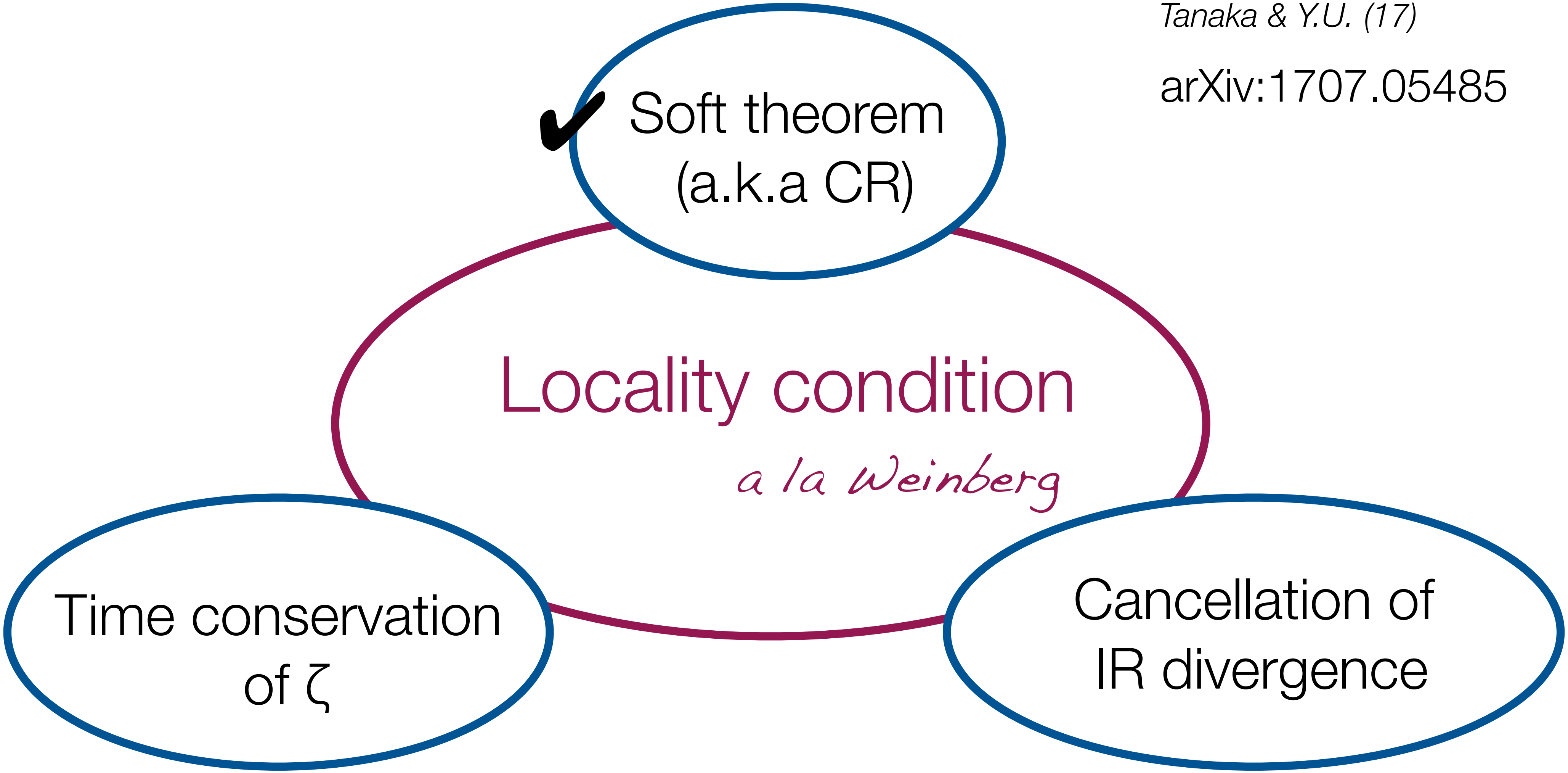
- General operator for short modes
- Non-perturbative contributions of short modes
- Short modes are not necessarily super Hubble
- Space-time dim d+1
- Time coordinates of short modes can be different

Universal properties in IR

k/aH <<1

Tanaka & Y.U. (17)

arXiv:1707.05485



Formulation in model independent language!!

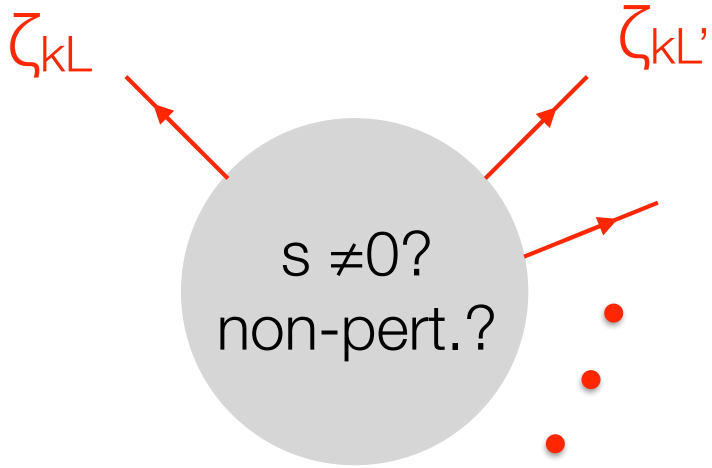
Conservation of ζ

Tanaka & Y.U. (16, 17)

<u>Claim</u> ζ_{k_L} is conserved in time in $k_L/aH \to 0$, if

1) Being perturbative about ζ_{k_L}

2) LC is valid



<u>Outline of argument</u>

1. Compute Influence functional $\Gamma[\zeta]$ by integrating out

   ~ non-local effective action

2. Using CR (←LC) for DOFs in

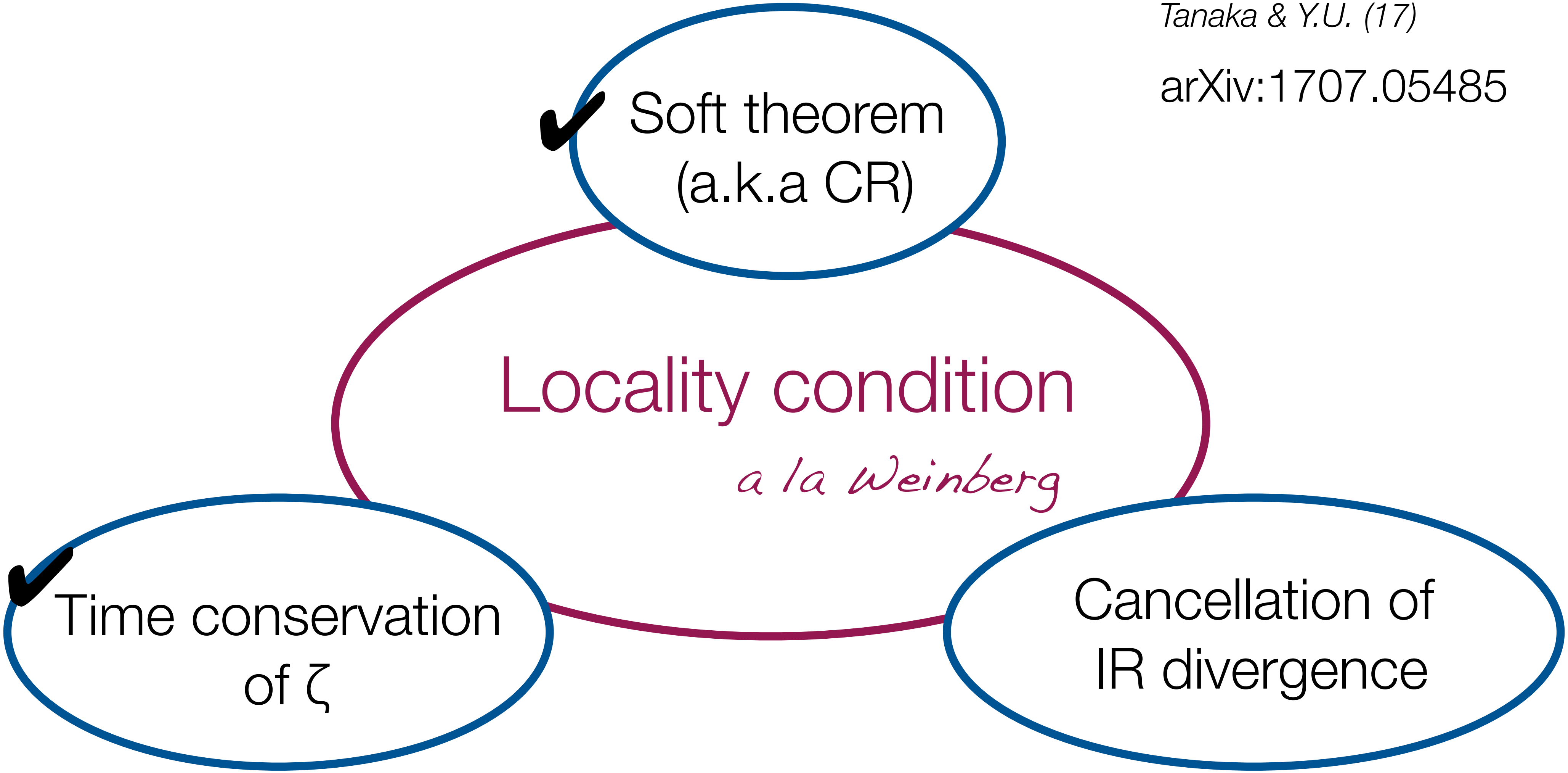
$\Gamma[\zeta_{k_L}]=\Gamma[\zeta_{k_L} - s]$ Presence of constant solution

Universal properties in IR

$k/aH << 1$

Tanaka & Y.U. (17)

arXiv:1707.05485



Formulation in model independent language!!

Cancellation of IR divergence

Properties of Infrared enhancements

- IR divergences of loop corrections in <ζk1...ζkn>
- Presence of IR divergence divergence free quantities ← LC

Tanaka & Y.U. (09, 10,....), ...

IR div. free variable $\quad {}^gR(x) = e^{iQ_\zeta^W(\boldsymbol{k}_L)}\, {}^gR(x) e^{-iQ_\zeta^W(\boldsymbol{k}_L)}$

$$\langle \Psi \,|{}^gR(x_1)\cdots {}^gR(x_n)|\, \Psi\rangle = \underbrace{\int d\zeta^c_{\boldsymbol{k}_L} |\psi(\zeta^c_{\boldsymbol{k}_L})|^2}_{\text{singular}} \; {}_{\zeta^c_{\boldsymbol{k}_L}}\langle \Psi \,|{}^gR(x_1)\cdots {}^gR(x_n)|\, \Psi\rangle_{\zeta^c_{\boldsymbol{k}_L}}$$

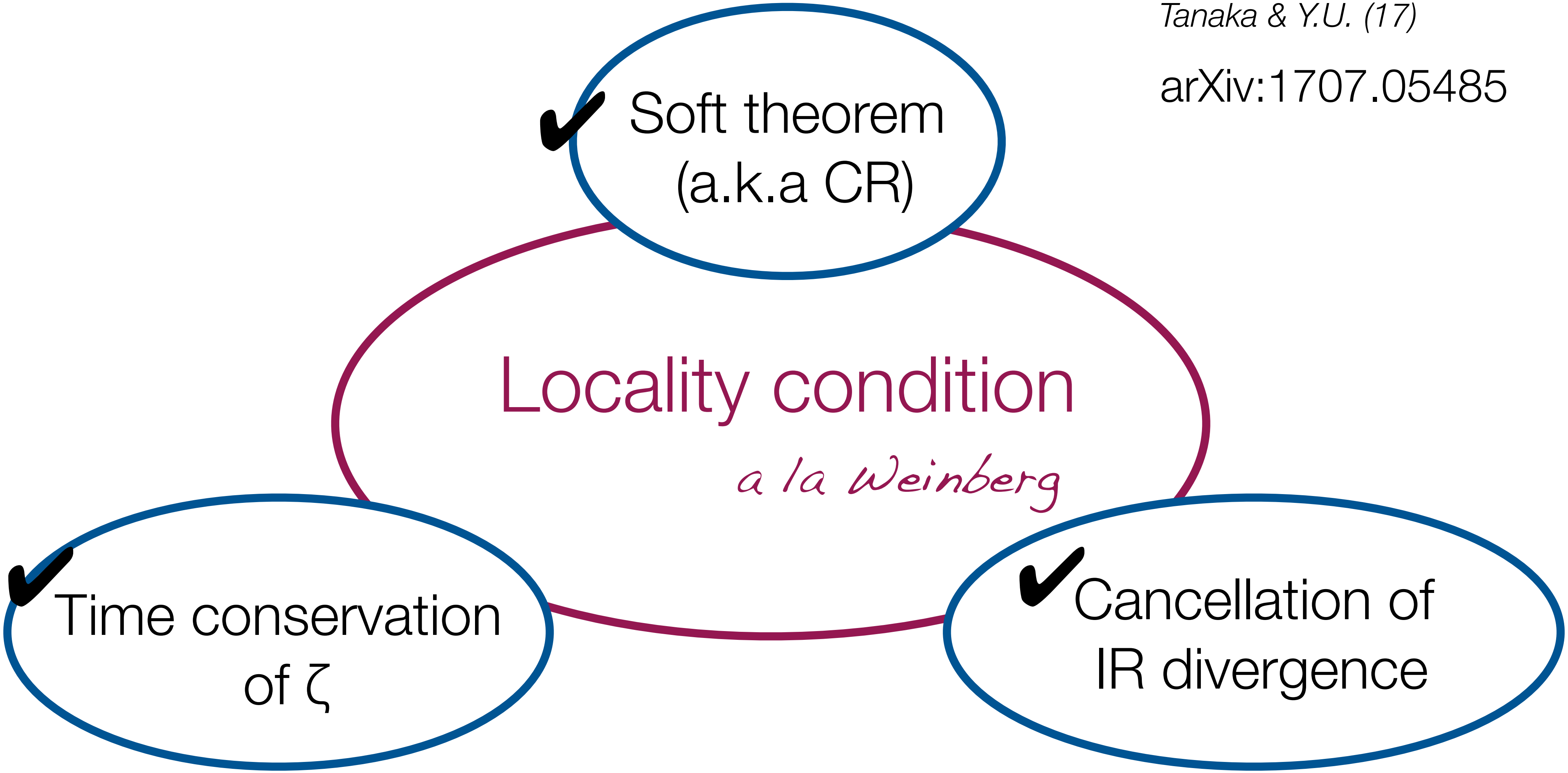
LC →

$$\begin{aligned} 0 &= {}_{\zeta^c_{\boldsymbol{k}_L}}\langle \Psi \,|\left[iQ_\zeta^W(\boldsymbol{k}_L),\, {}^gR(x_1)\cdots {}^gR(x_n)\right]|\, \Psi\rangle_{\zeta^c_{\boldsymbol{k}_L}} \\ &= \frac{\partial}{\partial \zeta^c_{\boldsymbol{k}_L}}\; {}_{\zeta^c_{\boldsymbol{k}_L}}\langle \Psi \,|{}^gR(x_1)\cdots {}^gR(x_n)|\, \Psi\rangle_{\zeta^c_{\boldsymbol{k}_L}} \;. \end{aligned}$$

Summery

Tanaka & Y.U. (17)

arXiv:1707.05485



Same story follows also for γ_{ij}

Thanks!

Example of ~~LC~~

Maldacena’s gauge can be non-local.

<u>Linear perturbation</u>

$$\partial_i N_i \supset \varepsilon \dot{\zeta}$$

In one field model (on attractor)

$$\text{in the limit } \boldsymbol{k} \to 0 \text{ as } \dot{\zeta}_{\boldsymbol{k}} = \mathcal{O}(k^p \zeta_{\boldsymbol{k}}) \text{ with } p \geq 1.$$

Otherwise, non-local