THermalization in Axion Inflation

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In collaboration with Alessio Notari [arXiv:1706.00373]

Axions in inflation

- Appealing way of realizing large field inflation, their mass is protected by the (discrete) shift symmetry. E.g.: Natural Inflation [Freese, Frieman and Olinto '90]

$$\mathcal{L}_\phi = K(\phi) + \Lambda^4 (1 + \cos(\phi/f))$$

- Axions (ϕ) are expected to couple to gauge fields through an axial coupling

$$\frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}, \qquad \tilde{F}^{\mu\nu} \equiv \frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}} F_{\alpha\beta}$$

where ϕ is the inflaton and f is the axion decay constant.

- When ϕ develops a VEV, parity is broken and the eom for the massless gauge field ($A_\pm$) during inflation becomes [Anber and Sorbo 06']

$$A''_\pm(\eta, k) + \left(k^2 \pm \frac{2k\xi}{\eta} \right) A_\pm(\eta, k) = 0, \qquad \xi = \frac{\dot{\phi}}{2fH}$$

c.f. talks by P. Adshead, R. Caldwell, A. Maleknejad, M. Peloso, E. Sfakianakis

- Instability band: $(8\xi)^{-1} < -k\eta < 2\xi$. If $\xi \simeq$ constant: [Anber and Sorbo 06']

$$A_k(\eta) \simeq \frac{1}{\sqrt{2k}} e^{-ik\eta}, \qquad \text{subhorizon}$$
$$A_k(\eta) \simeq \frac{1}{2\sqrt{\pi k \xi}} e^{\pi\xi}, \qquad \text{superhorizon}$$

- Phenomenology:
    - Large loop corrections to ζ induced through the coupling $\xi \zeta F\tilde{F}$

$$\text{2-point function :} \quad P^{\zeta}_{\text{1-loop}} = \mathcal{O}(10^{-4}) P^2_{\text{obs}} e^{4\pi\xi}$$
$$\text{non-Gaussianity:} \quad f^{\text{equi}}_{NL}|_{\text{1-loop}} = \mathcal{O}(10^{-7}) P_{\text{obs}} e^{6\pi\xi}$$

    - Large tensor modes, backreaction, preheating, ...
    - Observations constraint $\xi \lesssim 2.5$ $(\xi < 2.2)$ which imposes a lower bound on f.

[Anber&Sorbo 09', Sorbo 11', Barnaby&Peloso 11', Linde et al. 13', Bartolo et al. 14', RZF&Sloth 14', Adshead et al. 15', Planck 15', RZF et. al 15', ...]

Particle production and thermalization [RZF&Notari 1706.00373]

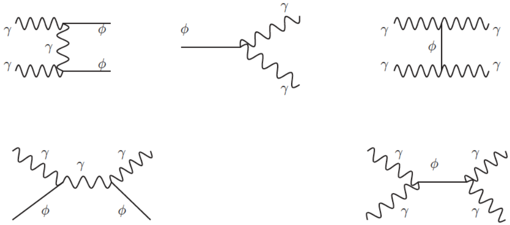
Instability $\Rightarrow$ particle production of modes

- Instability band covers subhorizon modes where particle interpretation is meaningfull.
- Gauge field effective particle number (N_γ) per mode k:

$$1/2 + N_\gamma(k) = \frac{\rho_\gamma(k)}{\omega(k)} = \sum_{\text{pol}} \frac{A_k'^2 + k^2 A_k^2}{2\,\omega(k)} \quad \Rightarrow \quad \begin{cases} N_\gamma(k) \simeq 0, & k/a \gg H \\ N_\gamma(k) \simeq \frac{e^{2\pi\xi}}{8\pi\xi}, & k/a \ll H \end{cases}$$

What happens when there are many particles around...?

Scatterings and decays involving $\phi\gamma$ interactions are enhanced by powers of N_γ



For example, the scatering rate of $\gamma\gamma \to \gamma\gamma$

$$S_{\gamma\gamma\to\gamma\gamma} = \frac{1}{E_1}\int \prod_{i=2}^{4}\left(\frac{d^3p_i}{(2\pi)^3(2E_i)}\right)|M_n|^2\,(2\pi)^4\delta^{(4)}\left(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu\right) B_{\gamma\gamma\to\gamma\gamma}(k,p_2,p_3,p_4)$$

where $B_{\gamma\gamma\to\gamma\gamma}(p_1,p_2,p_3,p_4)$ contains the phase space factors given by

$$B_{\gamma\gamma\to\gamma\gamma}(p_1,p_2,p_3,p_4) = N_\gamma(p_1)N_\gamma(p_2)\left[1+N_\gamma(p_3)\right]\left[1+N_\gamma(p_4)\right] - (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4)$$

and is Bose enhanced by N_γ^3.

- Scatterings and decays rates are $\propto N_\gamma^3, N_\gamma^2$.
  Therefore, when N_γ reaches a given threshold then

$$t_{\text{scatterings, decays}} \ll H^{-1} \quad \Rightarrow \quad \text{thermalization}$$

- To estimate the conditions for thermalization we derive, from the eom, Boltzmann-like eqs. for $N_{\gamma_+}(k)$, $N_{\gamma_-}(k)$ and $N_\phi(k)$:

$$\begin{aligned} N'_{\gamma_+}(k,\eta) &= -\frac{4k\xi}{\eta}\frac{\mathrm{Re}\left[g(k,\eta)\right]}{|g(k,\eta)|^2+k^2}\left(N_{\gamma_+}(k,\eta)+1/2\right) \\ N'_{\gamma_-}(k,\eta) &\simeq N'_\phi(k,\eta) \simeq 0 \end{aligned}$$

  where $g(k,\eta) = A'(k,\eta)/A(k,\eta)$.

- Then, add the scatterings and decays.

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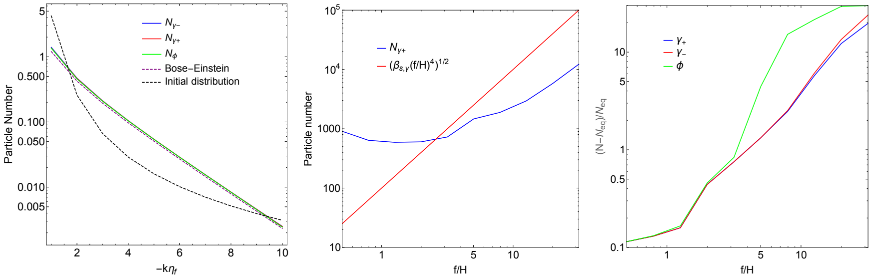
- Solve the system numerically and verify if the distribution approaches a Bose-Einstein distribution.

Numerical results

- Box with $\mathcal{O}(10)$ modes of comoving momentum: $k \in [1, \mathcal{O}(10)]H$.
  Duration of simulation: $\simeq 1$ e-fold, $\{\eta_0 = -2, \eta_f = -1\}$

- Checking thermalization by looking at the average difference to a BE distribution

$$\frac{\Delta N}{N} \equiv \frac{1}{N_{\text{tot}}} \sum_k \frac{N^{\text{norm}}(k) - N^{\text{eq}}(k,T)}{N^{\text{eq}}(k,T)},$$



Left: Change in the particle numbers after thermalization for $f = 0.1H$, $\xi = 2$. Center: Final particle number vs f/H for $\xi = 3.9$ Right: Average difference to Bose-Einstein distribution vs f/H for $\xi = 3.9$.

Numerical results:

- Distribution of particles approaches a BE distribution
- Numerically the system thermalizes when

$$\xi \gtrsim 0.44 \log\left(\frac{f}{H}\right) + 3.4\,,$$

- Observations impose $f\xi/H \gtrsim 10^3$. This means $\xi \gtrsim 5.8 \Rightarrow$ backreacting and non-perturbative regime $\Rightarrow$ unclear. [RZF et al. 15', Peloso et al. 16']

Problem solved if gauge fields belong to the standard model

- More, fixed and unsuppressed interactions (more predictive). More realistic, inflaton has to couple to SM.
- For $\gamma\psi$ scatterings or gluon self-interactions thermalization requires

$$\begin{cases} \left(\frac{\pi\alpha_{EM}}{2}\right)^2 \left(\frac{H}{k_*}\right)^2 H N^2_{\gamma\gamma\to e^-e^+} \gg N_{\gamma\gamma\to e^-e^+} H \quad \Rightarrow \quad \xi \gtrsim 2.9 \\ \left(\frac{9\pi\alpha_S}{32}\right)^2 \left(\frac{H}{k_*}\right)^2 H N^3_{gg\to gg} \gg N_{gg\to gg} H \quad \Rightarrow \quad \xi \gtrsim 2.9 \end{cases}$$

Under control

Generic properties of the thermal regime

- Thermalization shifts particles from horizon size to the UV. At horizon crossing the gauge field particle number is much smaller than in the non-thermal case.
- Effects on ζ are drastically modified!
- Loop corrections to ζ correlators are much smaller
  - $\langle \zeta^2 \rangle \propto \left(\frac{T}{H}\right)^4 \propto e^{2\pi\xi}$ instead of $e^{4\pi\xi}$ as in the non-thermal case;
  - $\langle \zeta^3 \rangle \propto \left\langle F\tilde{F} \right\rangle$ is suppressed because parity symmetry tends to be restored

$$F\tilde{F} \propto (n_{\gamma_+} - n_{\gamma_-}) \to \xi H^2 T^2$$

- Constraints on ξ become generically weaker.

Is the thermal regime stable?

- Moreover, after thermalization gauge field develops thermal mass $m_T \simeq \bar{g}T$

$$A''_{\pm} + \omega_T^2(k) A_{\pm} = 0, \qquad \omega_T^2(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2} \right).$$

- If $m_T > \xi H$ the instability disappears and thermal bath redshifts. However, the system should reach an equilibrium (or oscillate around it): if temperature is too small the thermal mass disappears and the instability opens again

- The system should reach an equilibrium temperature which balances the two terms:

$$\omega_T^2(k) \gtrsim 0 \qquad \Rightarrow \qquad T_{eq} \simeq \frac{\xi H}{\bar{g}}$$

At T_{eq}, ϕ is thermalized if $\Gamma(T_{eq}) \gg H$, i.e.:

$$\xi \gg \bar{g}\left(\frac{f}{H}\right)^{4/5}$$

Predictions for thermalized inflaton:

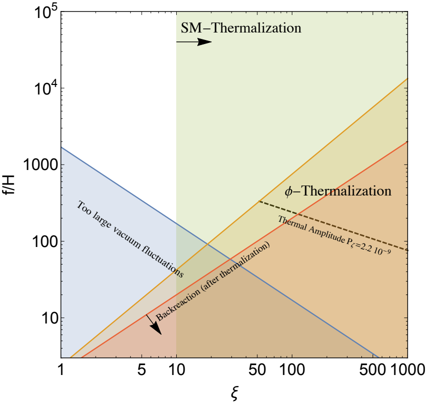
$$P_\zeta^{\text{thermal}} \simeq \frac{2T}{H} P_\zeta^{\text{vac}}$$

$$n_s - 1 = -6\epsilon_H + 2\eta + \frac{\dot{\xi}}{H\xi} = -4\epsilon_H + \eta$$

$$r = 16\epsilon\frac{H}{2T} = 8\epsilon\frac{\bar{g}}{\xi}$$

- $P_\zeta^{\text{thermal}}(T_{eq}) = 2.2 \times 10^{-9}$ gives a condition for $\xi(f)$.
- Non-Gaussianity estimated to be

$$f_{NL}^{\text{thermal}} \lesssim d\xi^4 P_\zeta^{\text{vac}} \mathcal{O}\left(\frac{T^5}{H^5}\right) \Rightarrow \xi < \mathcal{O}(20-100),$$



Conclusions & Future Work

Done:

- Controlled setup where a thermal bath can be sustained during inflation by the instability created due to the axial coupling.
- Couplings are shift symmetric so no thermal mass is generated for ϕ.
- Predictions are changed: constraints on ξ relaxed, spectral tilt fixed, r reduced (large field inflation (re)compatible with data!)

To be done:

- Confirm that the system reaches, or oscillates around, the equilibrium temperature.
- Improve non-Gaussianity calculations to derive more precise constraints on ξ.
- Is the backreacting regime possible?
- Many different features still to study.

Can inflation be ThAI?