SU(2) Gauge Fields and Physics of Inflation

Azadeh Maleknejad

IPM

A. M. and M. M. Sheikh-Jabbari, Phys. Rev. D (2011)

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B (2013)

A. M, M. M. Sheikh-Jabbari and Jiro Soda Phys. Rept. 528 (2013)

A. M. JHEP 1607 (2016) 104

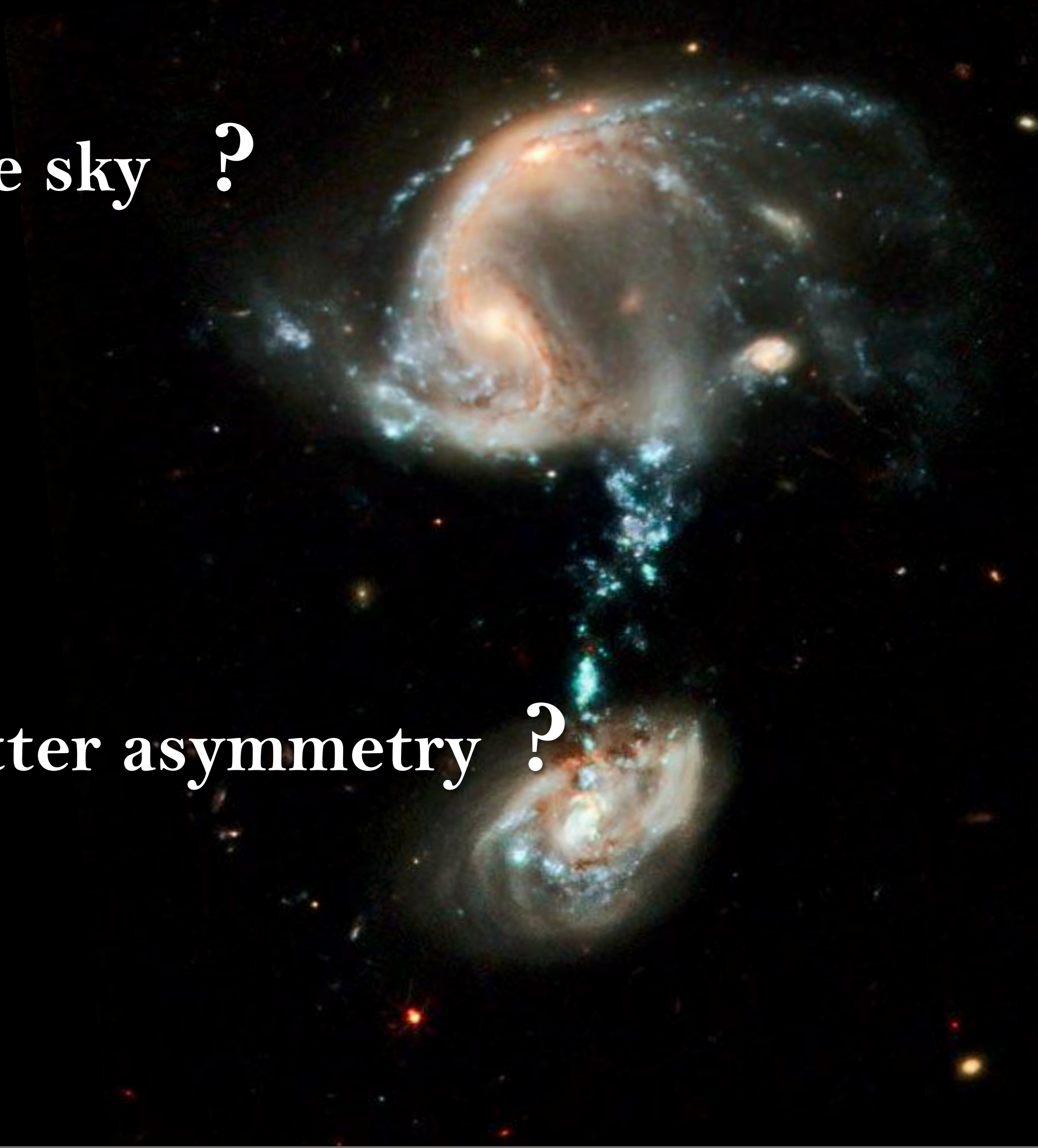
B. A. M. JCAP 1612 (2016) no.12, 027

NORDITA Inflation and the Cosmic Microwave Background

July 2017

- Non-Abelian gauge fields in the sky ?

- Gravitational origin for the matter asymmetry ?



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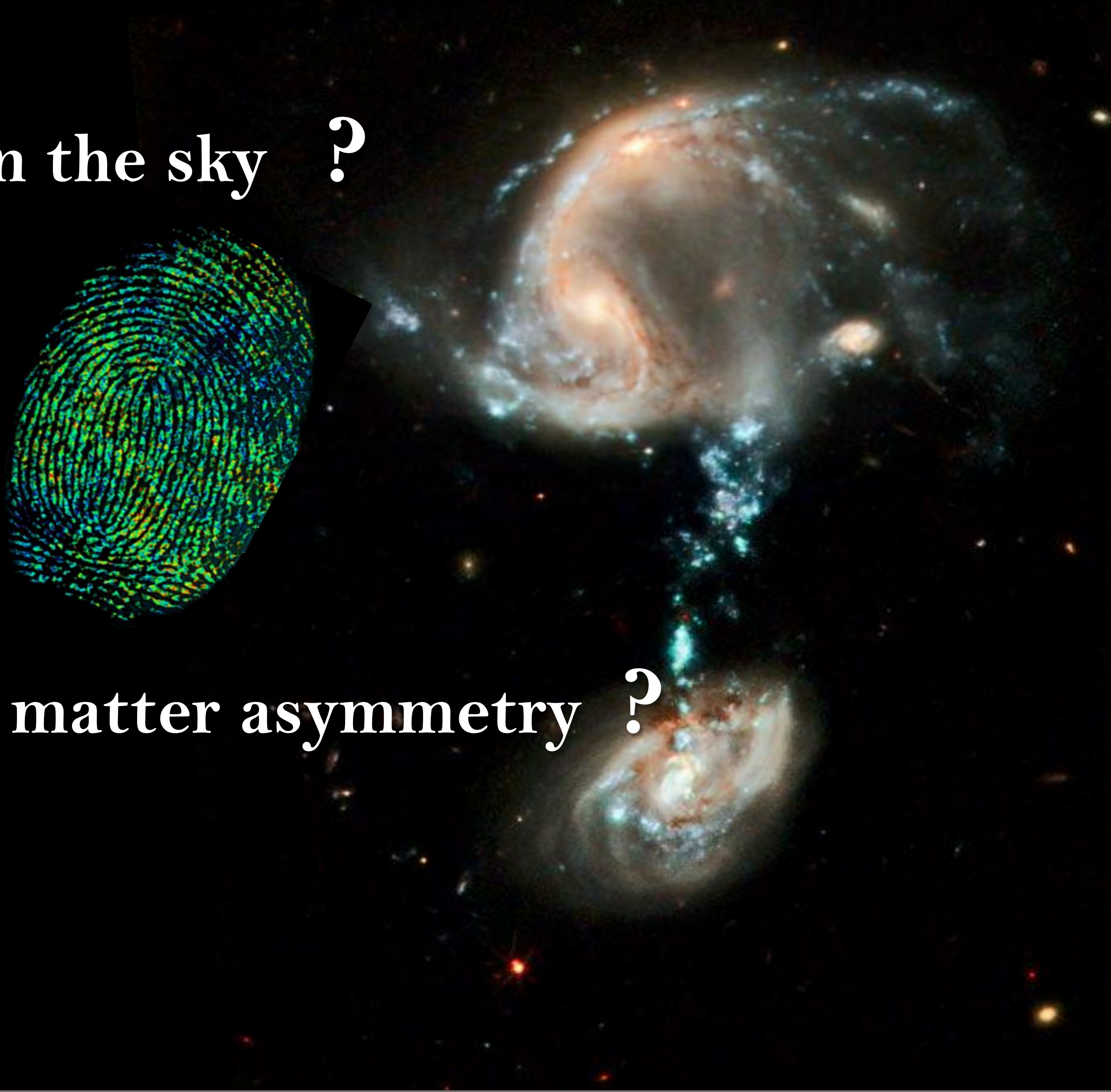

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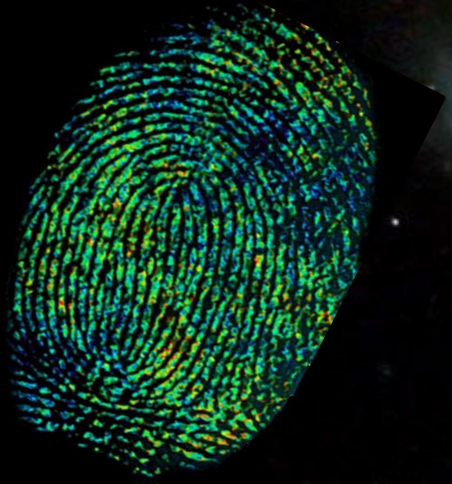

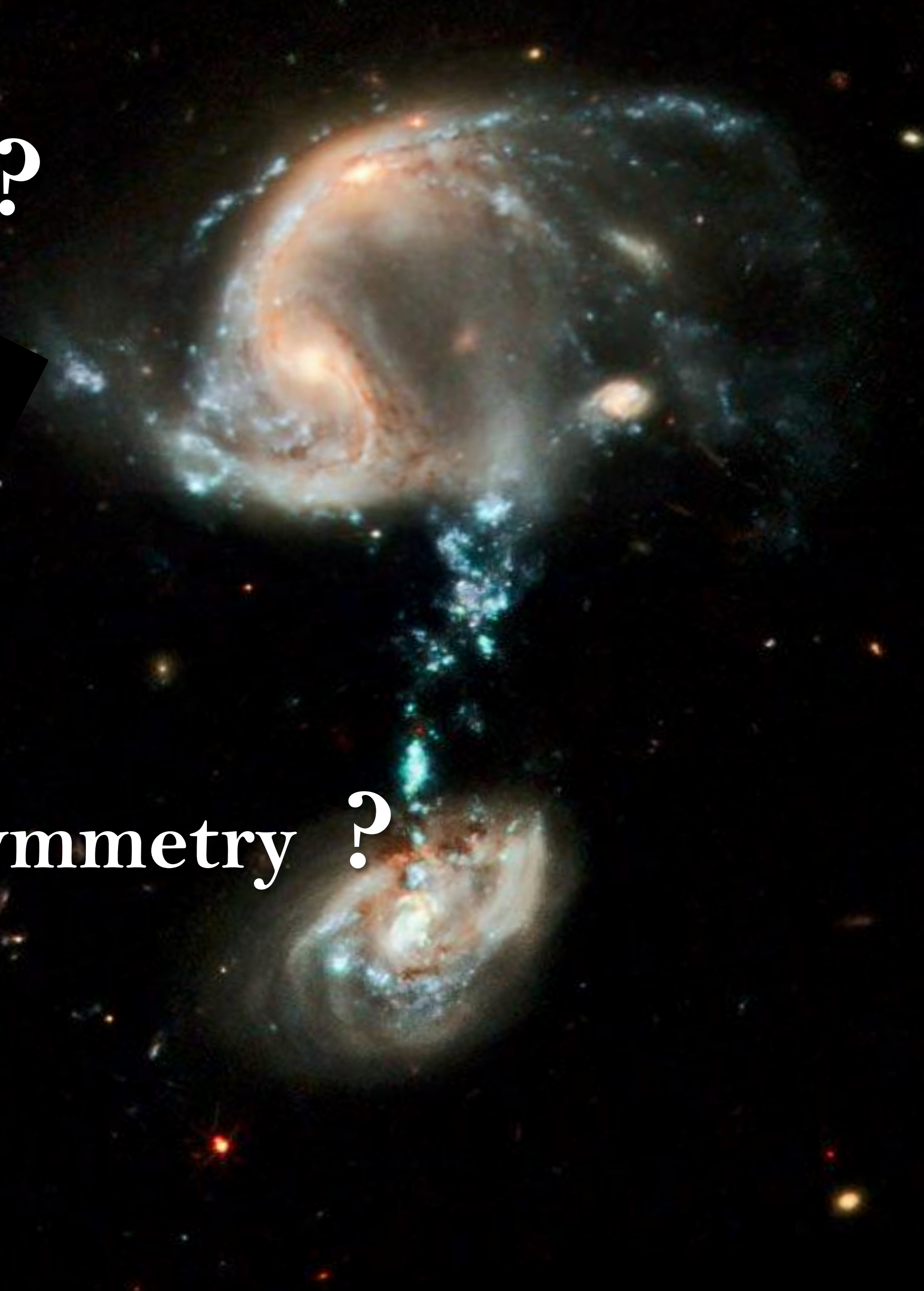

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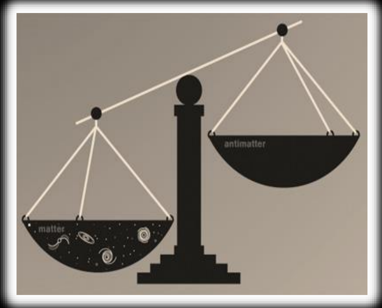
Gravity?



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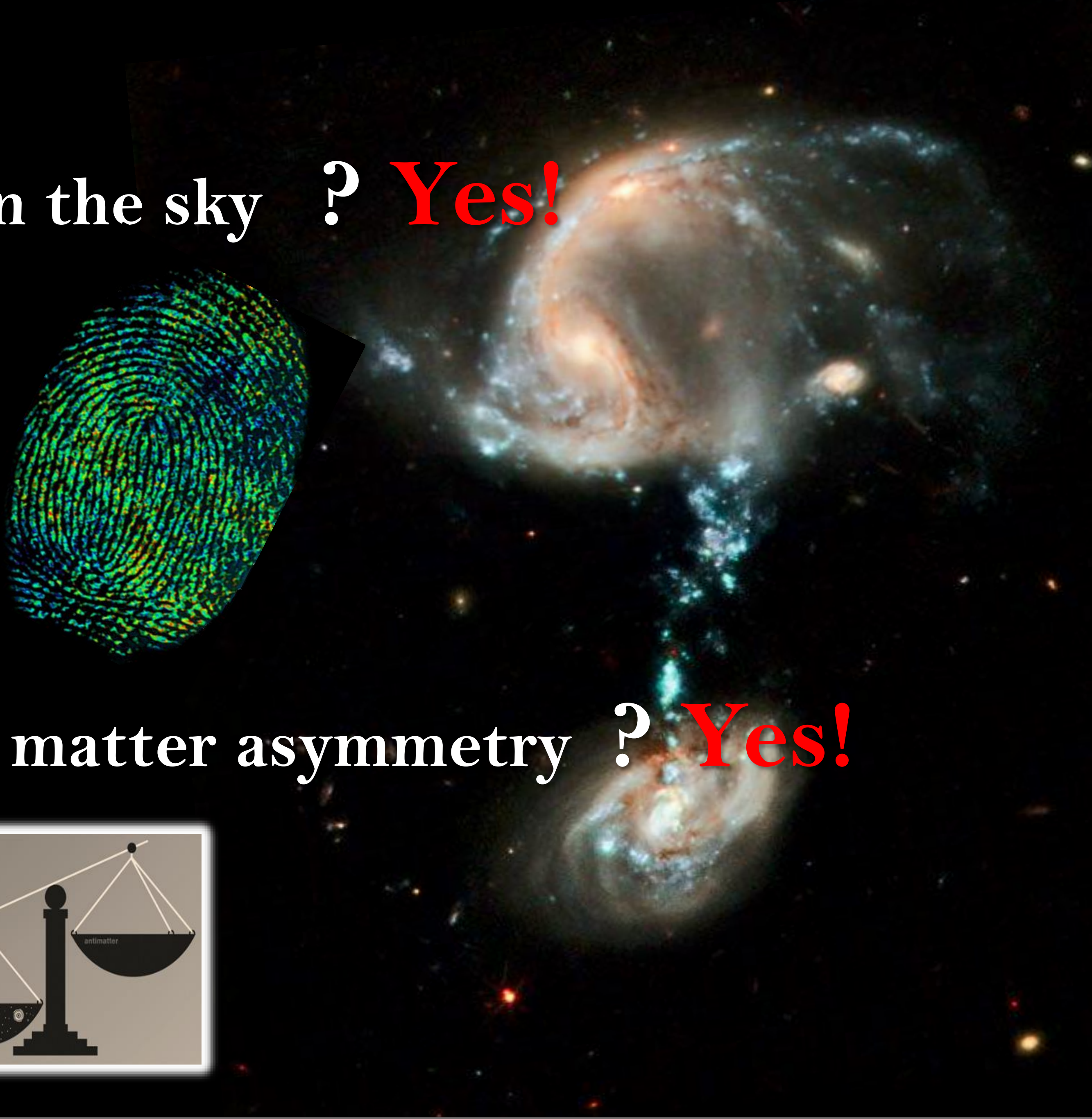

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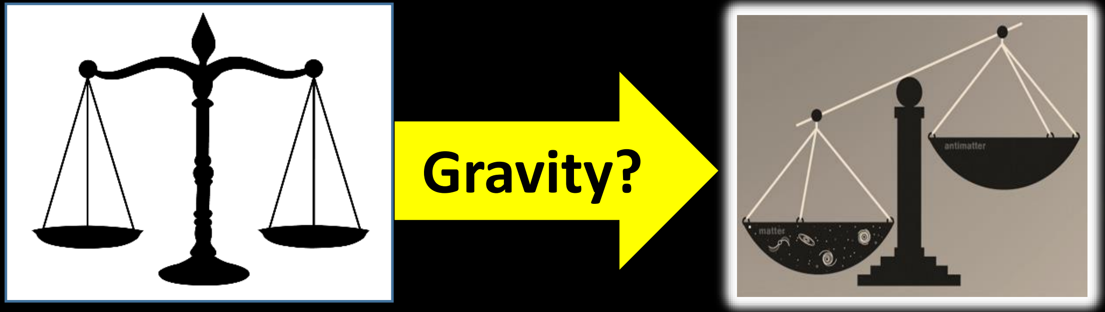

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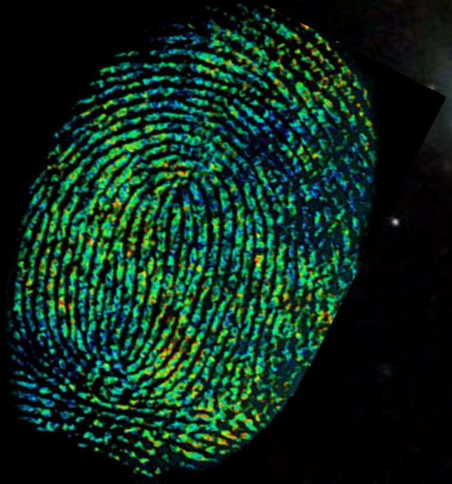

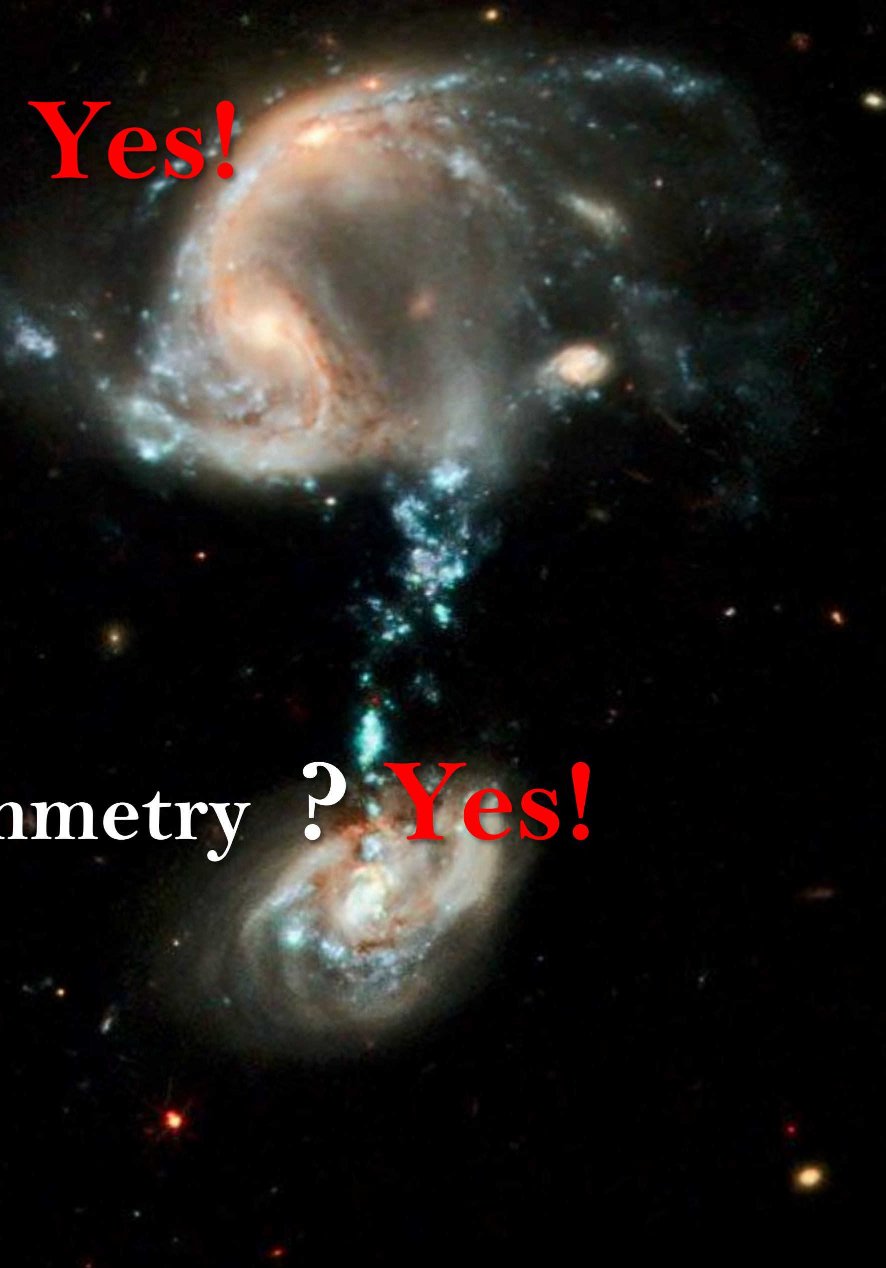

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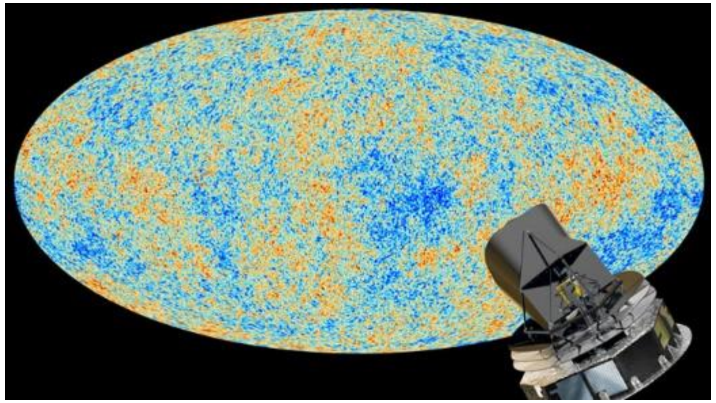
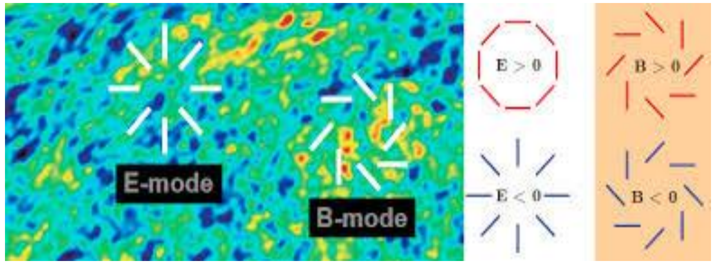
It is an Exciting time for doing Cosmology!





- Current measurements are in great agreement with the concept of inflation.

- While many key predictions of inflation have been verified, still
- the primordial gravitational waves (PGW) are missing!

- Joint Planck & BICEP2/Keck array set an upper bound $r < 0.07$ at 95% CL.
- Next generation of CMB experiments, such as the **LiteBIRD, CORE, CMB-S4** and … will measure $r \lesssim 10^{-3}$.

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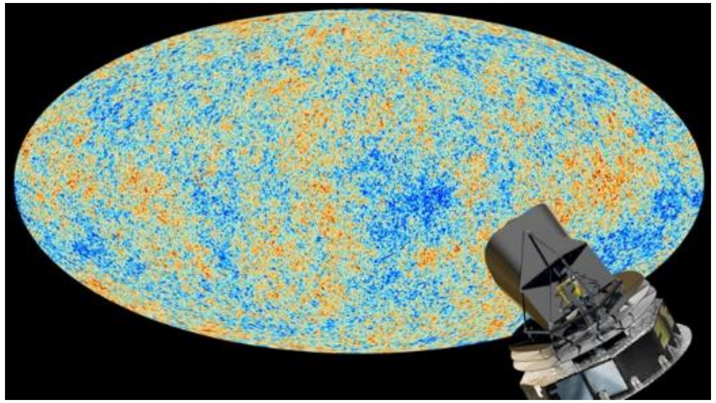

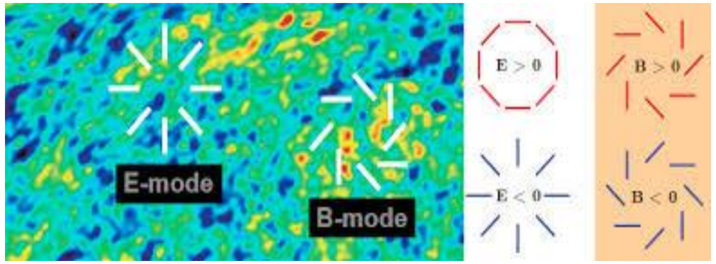

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Better understanding of this exciting data requires better theoretical understanding of the possible mechanisms of generation of PGWs!

Inflation Paradigm

In most of the inflationary models, inflation is driven by the coupling of one or more scalar fields to gravity:

$$\frac{1}{2}(\partial_\mu\varphi)^2 \ll V(\varphi)\ , \quad (\frac{V_\varphi}{V})^2 \ll 1 \quad \text{and} \quad \frac{V_{\varphi\varphi}}{V} \ll 1$$

Axions fields are abundant in string theory and very well-motivated candidates for inflaton field.

Enjoying shift symmetry, their effective potential is **protected** from dangerous quantum corrections which guaranteed flatness of the potential.

Axion inflation models are attractive phenomenologically due to their ability to generate observable gravitational waves (either as large field models or their coupling with the gauge fields).

Theoretical Setup

- A single field axion model minimally coupled to Einstein gravity

$$\mathcal{L}_{\rm inf} = \frac{R}{2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) + \mathcal{L}_A(A^a_\mu, g_{\mu\nu}, \varphi)$$

A. M. JHEP 1607 (2016) 104 arXiv:1604.03327
A. M. JCAP 1612 (2016) no.12, 027 arXiv:1604.06520

- $V(\varphi)$ is an arbitrary potential able to support slow-roll inflation.

- In addition, a non-Abelian SU(2) gauge field ${A^a}_\mu$

$${F^a}_{\mu\nu} = \partial_\mu {A^a}_\nu - \partial_\nu {A^a}_\mu - g{\epsilon^a}_{bc}{A^a}_\mu {A^b}_\nu$$

coupled to the axion with a Chern-Simons interaction

$$\mathcal{L}_A(A^a_\mu, g_{\mu\nu}, \varphi) = -\frac{1}{4}\left(F^a_{\mu\nu}F^{\mu\nu}_a + \frac{\lambda}{f}\varphi\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\right)$$

This action has been inspired by the Chromo-natural inflation model by P. Adshead & M. Wyman PhysRevLett.108.261302

Why SU(2) gauge fields and not U(1)?

- Rotational Symmetry
- a U(1) gauge field breaks spatial isometry

On the other hand, an SU(2) gauge field can have spatially isotropic and homogenous solution in FRW background.

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B (2013)
A. M. and M. M. Sheikh-Jabbari, Phys. Rev. D (2011)

- Independent modification of scalar and tensor perturbations
- At the *nonlinear level* U(1) gauge field coupled to graviton & inflaton

$$\delta A + \delta A \to \delta g \qquad \delta A + \delta A \to \delta\varphi$$

- So the resulting sourced gravity wave signal is correlated to a large scale non-Gaussianity.

(For a model that can fix it: R. Namba, M. Peloso, M. Shiraishi, L. Sorbo and C. Unal, JCAP **1601**, no. 01, 041 (2016))

However, mixing between SU(2) gauge field and perturbations in scalar & tensor sectors are at the *linear order* and coming from different fluctuations.

How to get an isotropic model?

In the isotropic and homogenous *FRW* background, with a Lagrangian of the form

$$\mathrm{L} = -\frac{1}{2}R + \mathrm{L_m}(\phi_I, \mathrm{F}_{\mu\nu})$$

gauge fields are determined up to gauge and rotation translations

$$A_0^a = 0 \qquad \mathrm{A}_i^{\mathrm{a}} \overset{SO(3)}{\to} R_i^j\, \mathrm{A}_j^{\mathrm{a}} \quad \& \quad \mathrm{A}_i^{\mathrm{a}} \overset{SU(2)}{\to} \Lambda_b^a\, \mathrm{A}_j^{\mathrm{b}}$$

The non-Abelian gauge field in the background

$$\mathrm{A}_\mu^{\mathrm{a}} = \begin{cases} 0 & \mu = 0 \\ a(t)\psi(t)\delta_i^a & \mu = i \end{cases} \quad \text{and} \quad \phi_I = \phi_I(t)$$

A. M. and M. M. Sheikh-Jabbari, Phys.Lett. B (2013)
A. M. and M. M. Sheikh-Jabbari, Phys. Rev. D (2011)

we have a homogenous and isotropic configuration

Theoretical Setup

- how to break the conformal invariance?

A. M. JHEP 1607 (2016) 104 arXiv:1604.03327
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$$\dot{\rho}_\varphi + 3H(\rho_\varphi + P_\varphi) = -\frac{\lambda}{f}\dot{\varphi}\vec{E}^a.\vec{B}_a,$$

$$\dot{\rho}_{\rm YM} + 4H\rho_{\rm YM} = \frac{\lambda}{f}\dot{\varphi}\vec{E}^a.\vec{B}_a.$$

Chern-Simons interaction

breaks the conformal symmetry.

The gauge field is very small on the background $\frac{\rho_{YM}}{\rho} \lesssim \epsilon^2$

And the coupling is $\frac{\lambda}{f} \sim \mathcal{O}(10)$

We have one dim-less parameter $\xi_\psi \simeq \frac{g\psi}{H}$

$$\sqrt{2} < \xi_\psi < 3, \qquad \psi \sim \epsilon.$$

Cosmic perturbations

Perturbing the metric and the fields around the background, we have

- $\delta\varphi = \delta\varphi_S$ $\quad + \quad$ $\delta g_{\mu\nu} = \delta g^S{}_{\mu\nu} + \delta g^V{}_{\mu\nu} + \delta g^T{}_{\mu\nu}$

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- $\delta\varphi = \delta\varphi_S$ + $\delta g = \delta g_S + \delta g_V + \delta g_T$

Although gauge field has no contribution on the BG evolution, its fluctuations have important effects to the cosmic perturbations:

- + $\delta A^a = \delta A^a_S + \delta A^a_V + \delta A^a_T$

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- $$+ \qquad \delta A^a = \delta A^a_S + \delta A^a_V + \delta A^a_T$$

- SU(2) gauge field has scalar and tensor fluctuations which sources the curvature fluctuations and gravitational waves:

$$R = R_{vacuum} + R_{Sourced} \qquad h = h_{vacuum} + h_{Sourced}$$

Cosmic perturbations - scalar

The gauge field sourced the scalar fluctuations and we have

- $\delta\varphi = \delta\varphi_{vacuum} + \delta\varphi_{Sourced}$ where $\delta\varphi_{Sourced} = \alpha(k,\tau)\delta\varphi_{vacuum}$

Only between mode functions, operators are incoherent.

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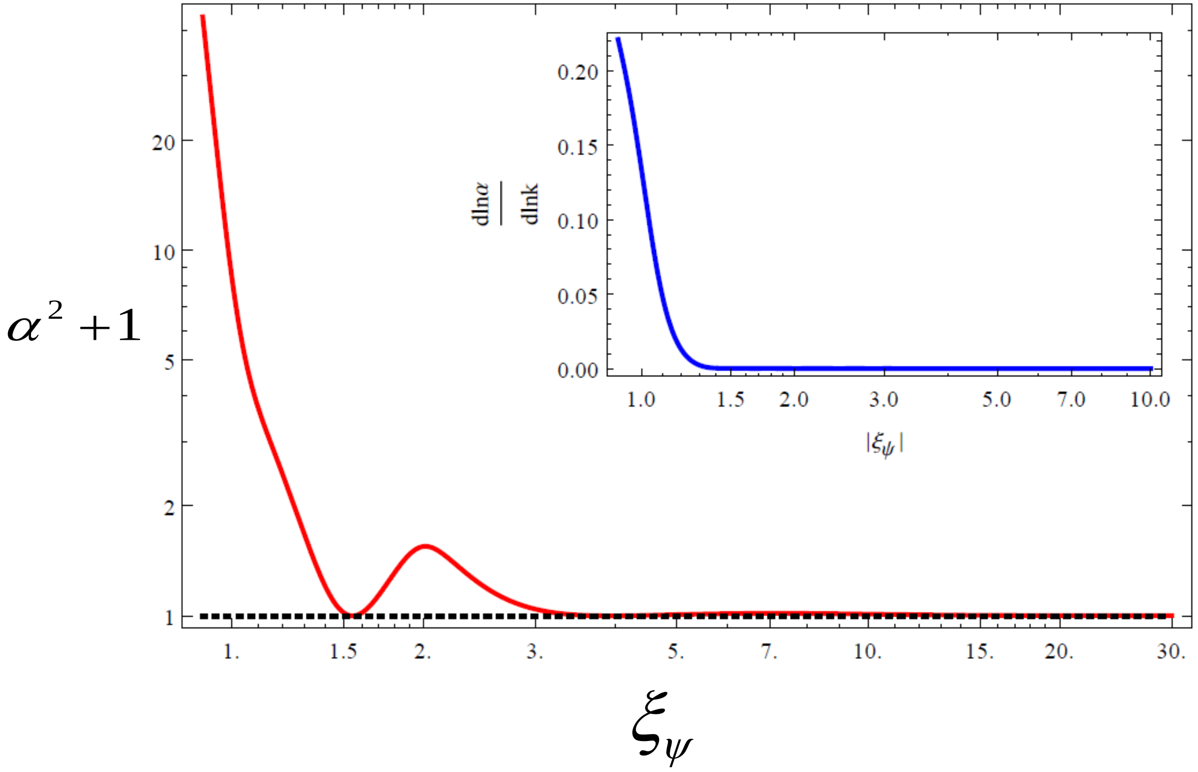

for the parameter regime $\xi_\psi \gtrsim \sqrt{2}$

power spectrum $P_{\mathcal{R}} = \frac{(1+\alpha^2(\xi_\psi))}{2\epsilon}\left(\frac{H}{2\pi}\right)^2$

spectral index $n_{\mathcal{R}} - 1 \simeq -2(3\epsilon - \eta)$

$at(-k\tau = 0.01)$

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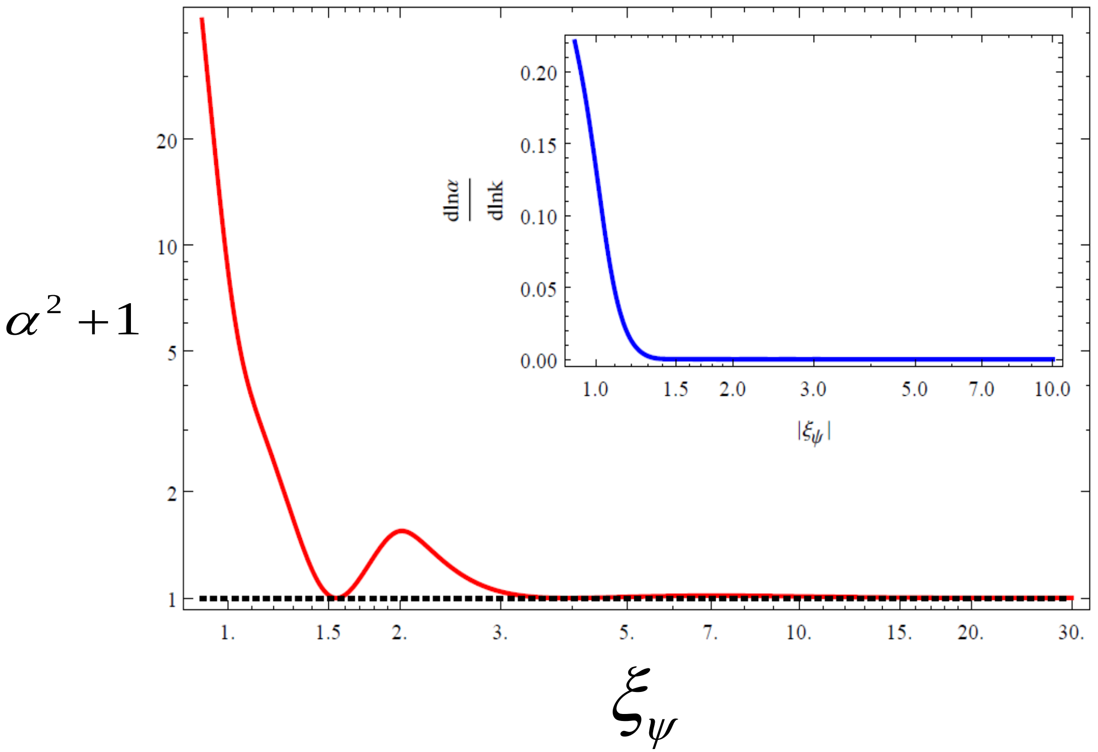

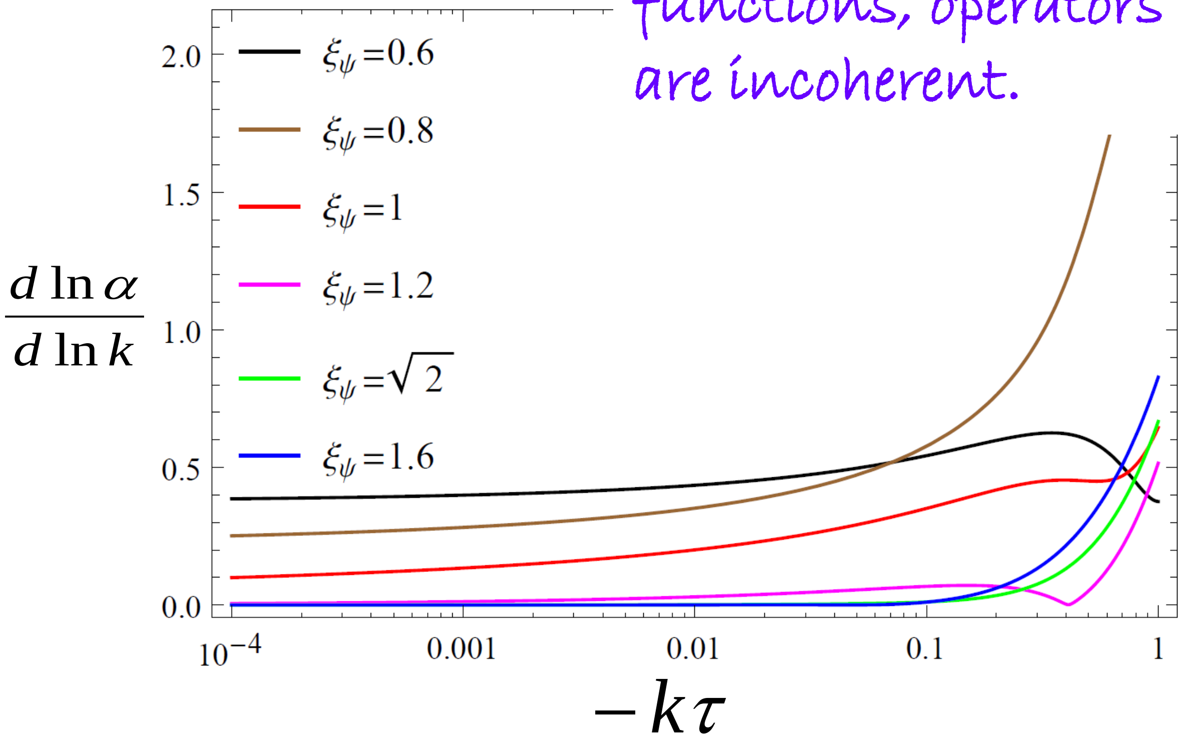

Cosmic perturbations – gravitational waves

The gauge field has a spin-2 fluctuation

$$\delta_T A^a_\mu = \begin{cases} 0 & \mu = 0 \\ \delta^{aj}\tilde{h}_{ij} & \mu = i \end{cases}$$

Which sources the gravitational waves

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij},$$

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Which sources the gravitational waves, $\tilde{h}_{ij}$

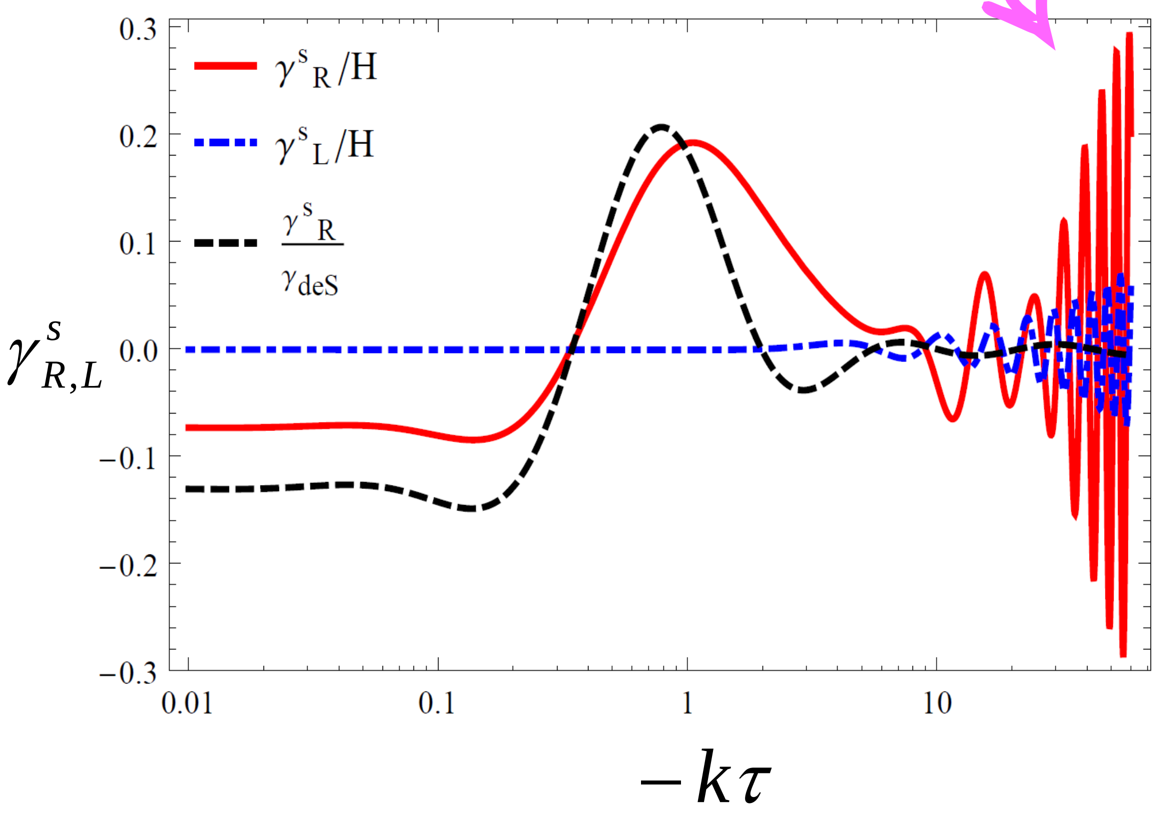
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 2a^2\pi^T_{ij},$$

$\tilde{h}_{ij}$ is polarized in a way that

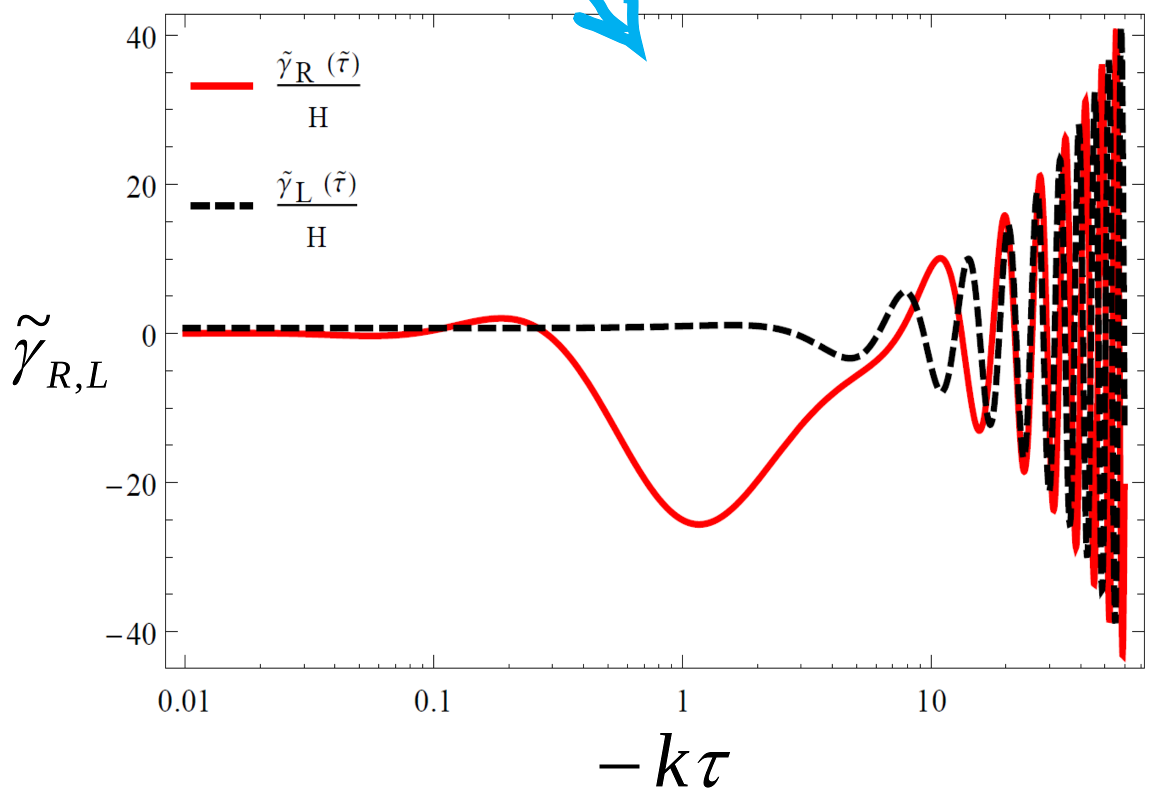
- $a^2\pi^T_\sigma(\tilde{h}_{ij}) = \begin{cases} S_k(\xi_\psi, \tau) & \sigma = + \\ 0 & \sigma = - \end{cases}$
- Therefore, the two polarization states of gravitational waves are
- $h_+ = h_+^{vacuum} + h_+^{Sourced}$ and $h_- = h_-^{vacuum}$

Cosmic perturbations – gravitational waves

$\gamma^s_{R,L}$ Gravitational Waves



$\tilde{\gamma}_{R,L}$ Tensor modes of Gauge field



Cosmic perturbations – gravitational waves

Gravitational power spectrum is $P_T \simeq \left(2 + \frac{\bar{\rho}_{\rm YM}}{\bar{\rho}}\mathcal{G}_+^2(\xi_\psi)\right)\left(\frac{H}{\pi M_{\rm pl}}\right)^2$

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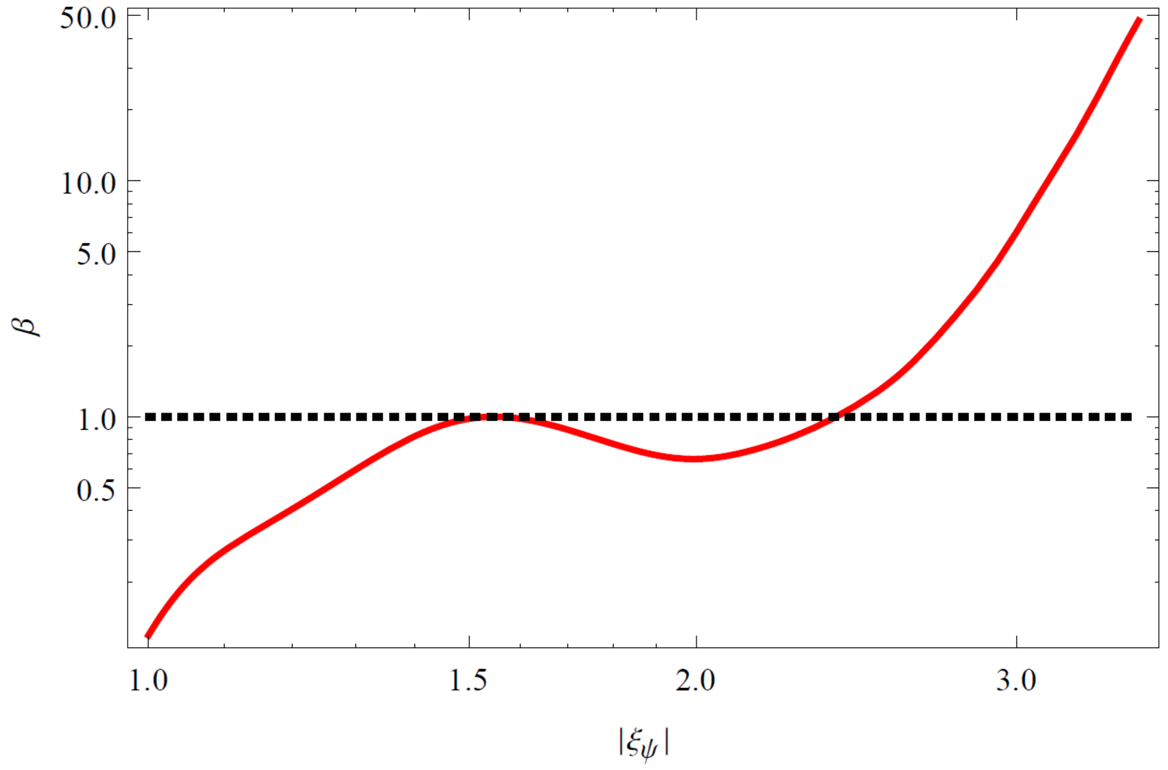
Tensor to scalar ratio is $r = 16\beta\varepsilon$



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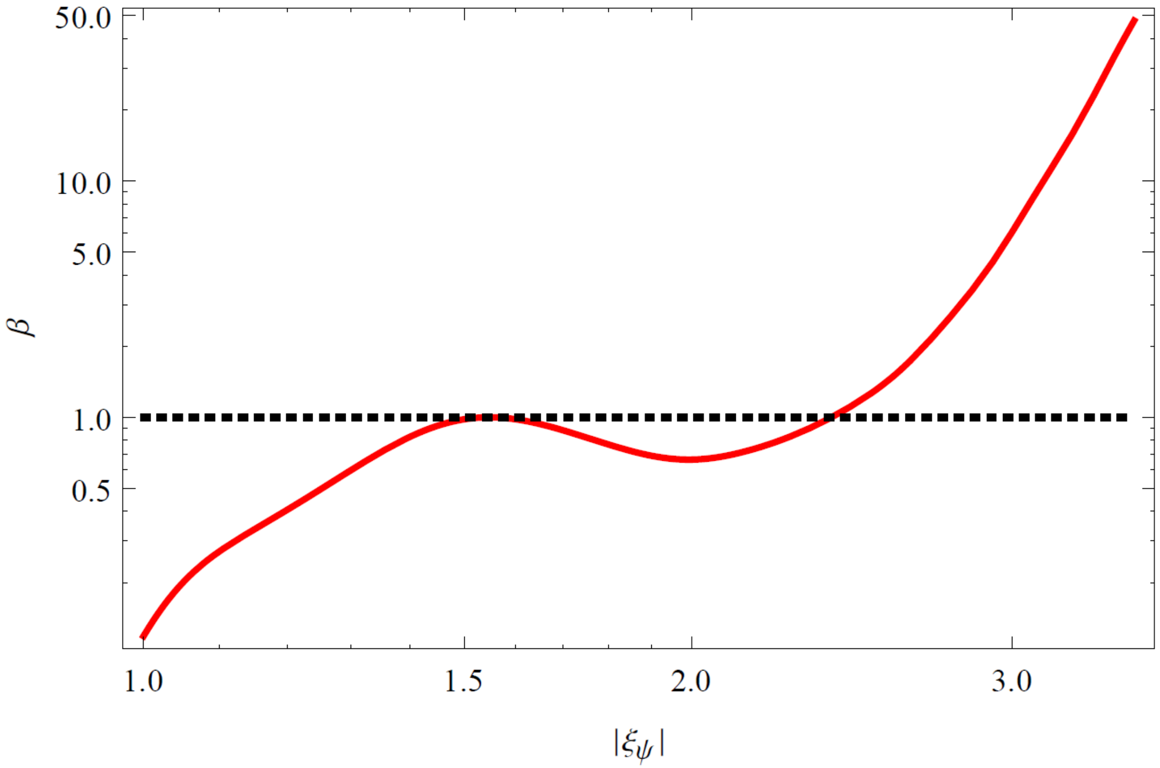
Tensor spectral tilt is

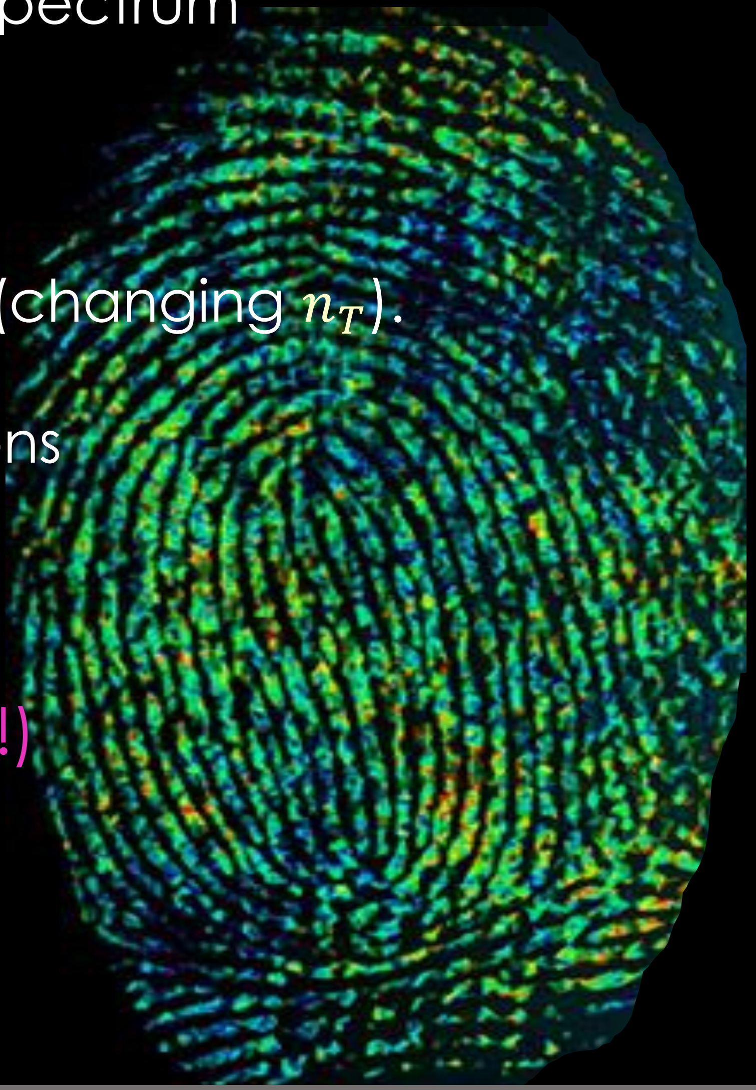
$$n_T = -2\epsilon - \frac{8/3}{(1+\xi_\psi^2)}\left(\frac{\bar{\rho}_{\rm YM}}{\bar{\rho}}\right)$$

The Lyth bound is modified to

$$\Delta\varphi \sim M_{\rm pl} N \sqrt{\frac{r}{8\beta}}$$



Presence of SU(2) gauge fields in the matter content of inflation



- Relaxing the direct relation between tensor power spectrum and the scale of inflation $P_T = (\frac{1+C_A}{\pi^2})\frac{H^2}{M_{Pl}^2}$

- Violation of the Lyth bound & consistency relation (changing n_T).

- Chiral gravitational waves and parity odd correlations $\langle TB \rangle$ and $\langle EB \rangle$

- Large Tensor Non-Gaussianity (Eiichiro will tell us!)

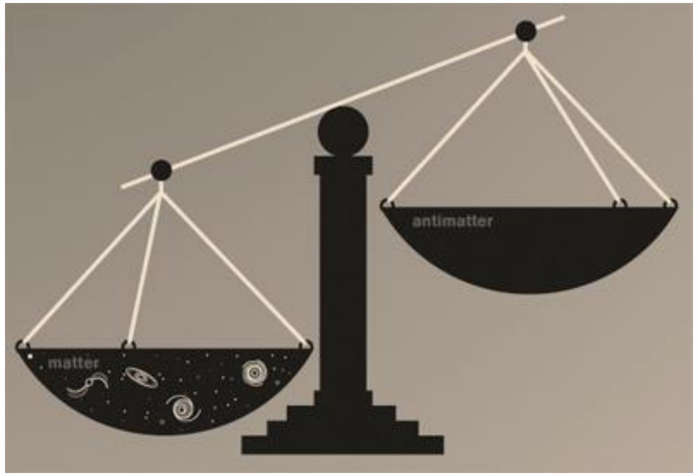
AM, M. M. Sheikh-Jabbari and Jiro Soda Phys. Rept. 528 (2013)
AM, Phys. Rev. D 90 (2014)
P. Adshead, E. Martinec, M. Wyman Phys. Rev. D88 021302 (2013)
R. Namba, M. Peloso, M. Shiraishi, L. Sorbo, C. Unal JCAP 1601 (2016) no.01, 041
B. Thorne, T. Fujita, M. Hazumi, N. Katayama, E. Komatsu, M. Shiraishi arXiv:1707.03240
A. Agrawal, T. Fujita, E. Komatsu arXiv:1707.03023

Origin of matter anti-matter asymmetry

Matter asymetry

- The observable Universe is highly matter-antimatter asymmetric.

This asymmetry can be quantified as



$$\eta = \frac{n_B - \bar{n}_B}{n_\gamma}\Big|_0 \approx \frac{n_B}{n_\gamma} = 6.19 \pm 0.15 \times 10^{-10}$$

- n_B = # density of barions
- $\bar{n}_B$ = # density of antibarions
- n_γ = # density of photons

P. A. R. Ade et al. Planck Collaboration, ``Planck 2013 results. XVI. Cosmological parameters, Astron. Astrophys. (2014)
`` Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations", Astrophys.J.Suppl.192:18,2011.

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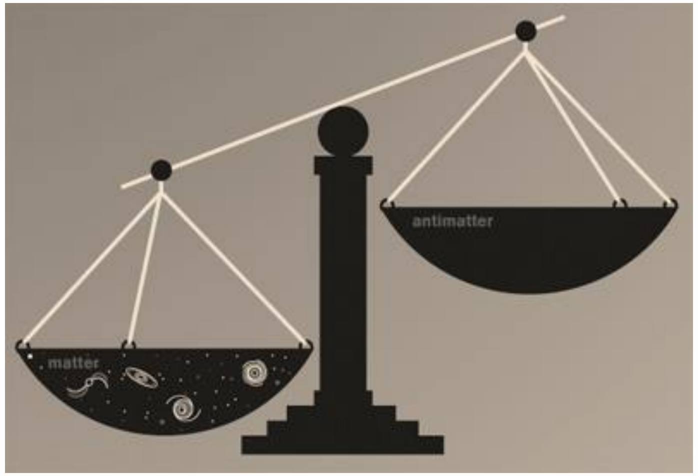
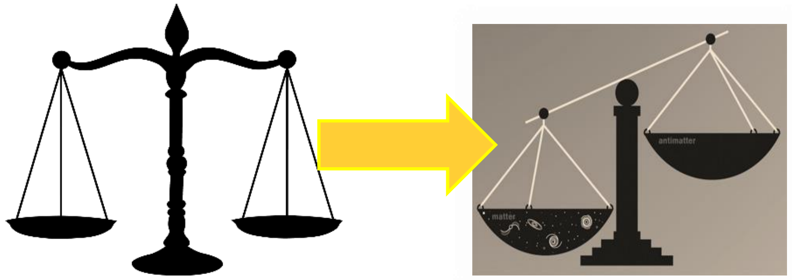


- *The big bang should have produced equal amount of matter and antimatter.*

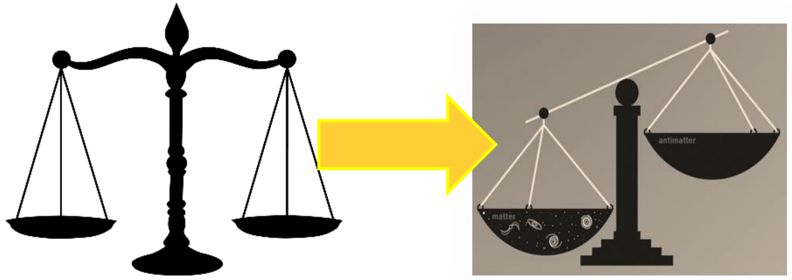
- The observed asymmetry must have been generated dynamically (baryogenesis)



- Sakharov conditions:
- the necessary & sufficient conditions to create a baryon-antibaryon asymmetry from symmetric initial conditions:
- *baryon number violating interactions,*
- *C and CP violation,*
- *Out of equilibrium state*

A. D. Sakharov, Pisma Zh.Eksp.Teor.Fiz. 5 (1967) 32-35 [JETP Lett. 5 (1967) 24-27].

- **Leptogenesis:** the cosmic baryon asymmetry is originated from an initial lepton number asymmetry in the early universe.



- In the standard approach of leptogenesis, the SM is extended by massive right handed neutrinos which (the source of CP violation in the model) decay and generate the initial lepton asymmetry.

- *the standard scenarios associate the matter-antimatter asymmetry to*

- the physics beyond the SM,

- after the inflationary era.

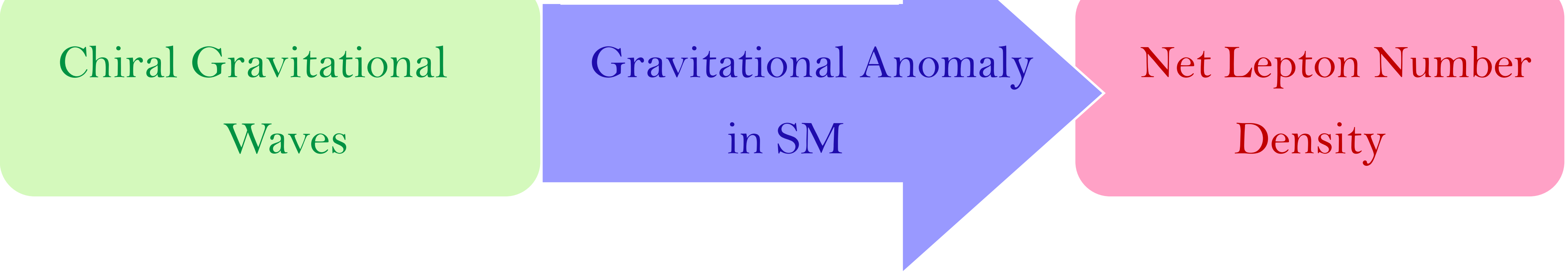
M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
C. S. Fong, E. Nardi and A. Riotto, Adv. High Energy Phys. 2012, 158303 (2012)

- **Inflato- Leptogenesis:** a leptogenesis model during inflation and within the SM of particle physics.

The chiral gravitational waves produced during inflation through the gravitational anomaly in the SM leads to a net lepton number density.

$$h_L \neq h_R$$

$$\nabla_\mu J_l^{\ \mu} = \frac{(N_L - N_R)}{16\pi^2} \tilde{R}R$$



S. Alexander, M. Peskin and M. M. Sheikh-Jabbari, Phys. Rev. Lett. 96, 081301 (2006)

Gravitational anomaly in the SM of particle physics

$$\nabla_\mu J_l{}^\mu = \frac{(N_L - N_R)}{16\pi^2}\tilde{R}R$$

- $J_l{}^\mu$ = lepton current,
- $N_{L,R}$ = #left/right-handed fermion degrees of freedom,
- $\tilde{R}R = \frac{1}{2}\varepsilon^{\lambda\mu\nu\xi}R_{\mu\nu\rho\sigma}R_{\nu\xi}{}^{\rho\sigma}$
- In the standard model of particle physics $N_L - N_R = 3$,
- In higher energy scales, eventually we reach to the mass scale of

right-handed neutrinos, Λ and $N_L - N_R = 0$.

- Lepton number generated during inflation

$$n_L \simeq \frac{6}{\pi^2}\Omega^{\rm GW}_{L-R}(\Lambda)H^3 \quad \text{where} \quad \Omega^{\rm GW}_{L-R} = \frac{\xi_\psi/72\pi^2}{(1+2\xi_\psi^2)}\left(\frac{\bar\rho_{\rm YM}}{M_{\rm pl}^4}\right)\left(\frac{\Lambda}{H}\right)^4$$

- Baryon to Photon ratio

$$\eta_{\rm B} \simeq 3\times 10^{-4}\frac{\xi_\psi/\sigma^{\frac{3}{4}}}{(2\xi_\psi^2+1)}\frac{\bar\rho_{YM}}{M_{\rm pl}^4}\left(\frac{\Lambda}{H}\right)^4\left(\frac{H}{M_{\rm pl}}\right)^{\frac{3}{2}}$$

- Reheating Temperature $T_{\rm reh} \lesssim 10^{10}$ GeV

reheating efficiency $\sigma \lesssim 10^{-13} - 10^{-18}$

- Mass of the Right-handed Neutrinos $M_{\rm R} \sim 10^{-5} M_{\rm pl}.$

A. M. JCAP 12 (2016) 027 arXiv:1604:06520

For another interesting similar scenario see R. R. Caldwell and C. Devulder arXiv:1706.03765

Conclusion

- Models of axion inflation with a small SU(2) gauge fields has a rich phenomenology and the following robust predictions:
  - Relaxing the direct relation between P_T and H
  - Chiral gravitational waves and parity odd correlations $\langle TB \rangle$ and $\langle EB \rangle$,
  - Violation of the Lyth bound,
  - Large Tensor Non-Gaussianity (Eiichiro will tell us!)

- Detection of B-mode polarization in the CMB from PGWs DOES NOT immediately imply discovery of vacuum fluctuations in the tensor metric perturbation!

- This class of models provides a natural setting for leptogenesis within SM and during inflation.

Thank You!