Lattice Simulations in Natural Inflation

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Advances in theoretical cosmology in the light of data

with P. Adshead (UIUC), J. T. Giblin (Kenyon), T. R. Scully (UIUC)

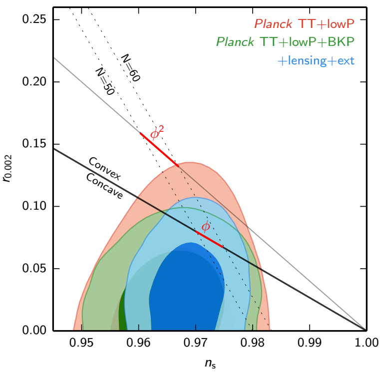
- P. Adshead, J. T. Giblin, T. R. Scully & EIS, JCAP **1610**, 039 (2016) [arXiv:1606.08474 [astro-ph.CO]]
- P. Adshead, J. T. Giblin, T. R. Scully & EIS, JCAP **1512**, no. 12, 034 (2015) [arXiv:1502.06506 [astro-ph.CO]]

Probing inflation

- Tensor modes $\sim H^2$
- Scalar modes $\sim H^4/\dot{\phi}^2$

$$r \equiv \frac{\text{Tensor}}{\text{Scalar}} \sim \frac{\dot{\phi}^2}{m_{\rm Pl}^2 H^2}$$

BICEP suggests $r \lesssim 0.1 \quad \Rightarrow$
Large r values lead to
super-Planckian field excursions



Formally, Lyth Bound:

$$N = \int \frac{da}{a} = \int \frac{da}{dt}\frac{dt}{a} = \int \frac{H m_{\rm Pl}}{\dot{\phi}}\frac{d\phi}{m_{\rm Pl}} = \frac{8}{\sqrt{r}}\frac{\Delta\phi}{m_{\rm Pl}}$$

Models with super-Planckian field excursions $\Rightarrow$ UV sensitivity!

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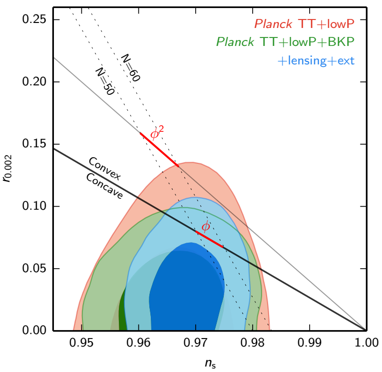
Large r values ~~lead to~~ suggest **super-Planckian** field excursions (with interesting exceptions)



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Axions as inflatons

Shift symmetry $\phi \to \phi + c$ protects inflation from UV physics.

Shift symmetry makes (ending) inflation impossible, since the potential, e.g. ϕ^n, does not respect the symmetry.

$\Rightarrow$ It has to be broken (softly).

Examples include

- Quadratic inflation: $V(\phi) = \frac{1}{2} m^2 \phi^2$
- Original natural inflation: $V(\phi) = \mu^4 \left(1 - \cos(\phi/f)\right)$
- Axion monodromy: $V(\phi) = \mu^3 \left(\sqrt{\phi^2 + \phi_c^2} - \phi_c\right)$

Allowed couplings

A field with a shift symmetry can only couple **derivatively** to other degrees of freedom

$$\mathcal{L}_{\rm Int} \supset \boxed{\frac{\alpha}{8f}\phi\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}} + \frac{C}{f}\partial_\mu\phi\bar{\psi}\gamma_5\gamma^\mu\psi$$

$$\updownarrow$$

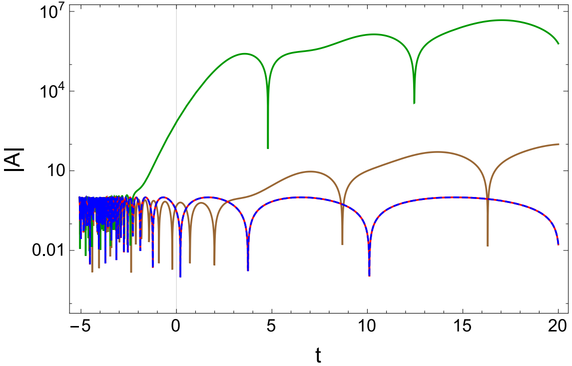
$$\boxed{-\frac{\alpha}{f}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\phi A_\nu\partial_\alpha A_\beta}$$

From a EFT perspective, these interactions must be present.

Each of them can lead to new connections to **data** through magnetogenesis and leptogenesis.

We work with an abelian $U(1)$ gauge field & decompose in two polarizations $(+, -)$.

$$\ddot{A}_k^{\pm} + H\dot{A}_k^{\pm} + \left[\left(\frac{k}{a}\right)^2 \mp \frac{\alpha}{f}\frac{k}{a}\dot{\phi}\right] A^{\pm} = 0$$



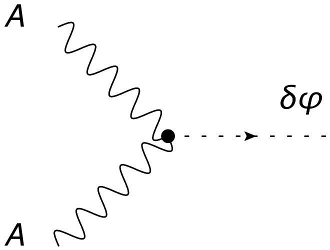
For non-zero coupling each polarization $(+, -)$ exhibit different **exponential enhancement**.

Backreaction

Gauge fields source **density fluctuations** by back-reacting on the inflaton through the usual **axion-photon interaction**

$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} + V_{\phi\phi}\right)\right]\delta\phi = \frac{\alpha}{f}\frac{1}{a^2}\left(\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right)$$

$$\boxed{|A| = e^{\pi\frac{\alpha}{f}\frac{|\dot{\phi}|}{H}}}$$



Constraints on the coupling through:

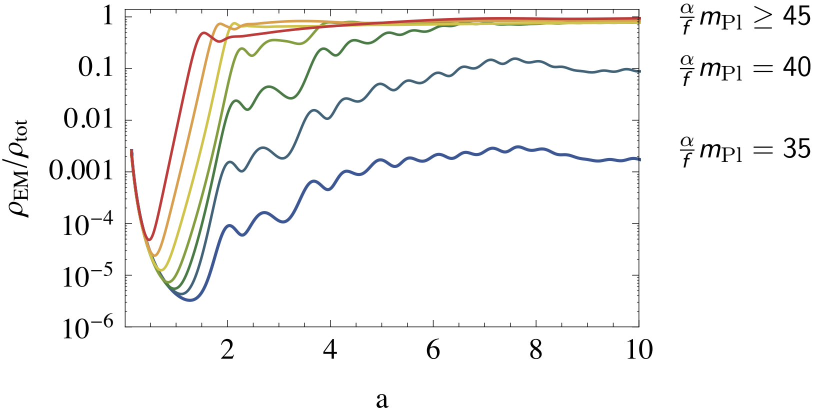
- non-Gaussianity at the CMB
- Primordial Black Hole production

$$\Rightarrow \quad \frac{\alpha}{f} \lesssim 110 m_{\mathrm{Pl}}^{-1}$$

$\Longrightarrow$ **Lattice simulations** are needed to compute strong back-reaction effects for large coupling

Reheating Efficiency

Coupling the axion to gauge fields can lead to explosive transfer of energy from the inflaton.

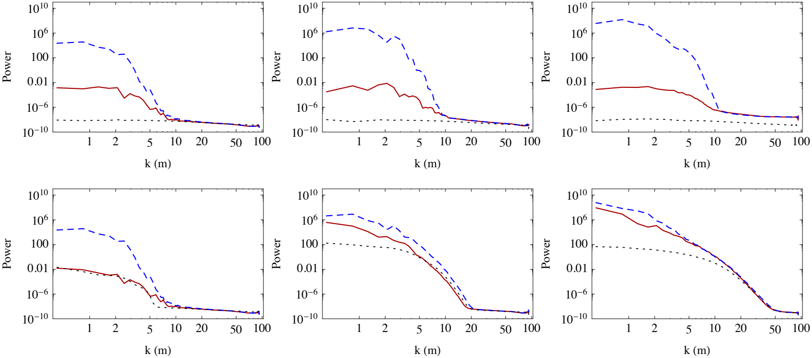


Reheating occurs after a **single axion oscillation** for $\frac{\alpha}{f} m_{\rm Pl} > 45$.

$$\frac{\alpha}{f} m_{\rm Pl} = 35 \qquad \frac{\alpha}{f} m_{\rm Pl} = 45 \qquad \frac{\alpha}{f} m_{\rm Pl} = 60$$

NO back-reaction

WITH back-reaction

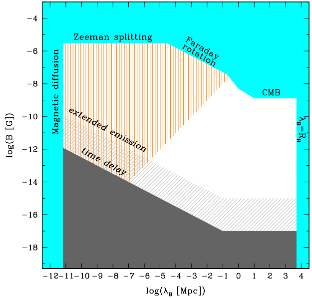


Strong re-scattering **suppresses polarization** on sub-horizon scales for large couplings.

Inflation in the light of “other” data

Magnetic fields are observed at all scales. We focus on large scales

- Galactic magnetic fields at *kpc* scales of $10^{-6}G$
- Intergalactic magnetic fields with correlation length of λ



$$B \gtrsim 10^{-17}G \;\left(or\ 10^{-15}G\right) \quad \text{for } \lambda \geq 1\text{Mpc}$$

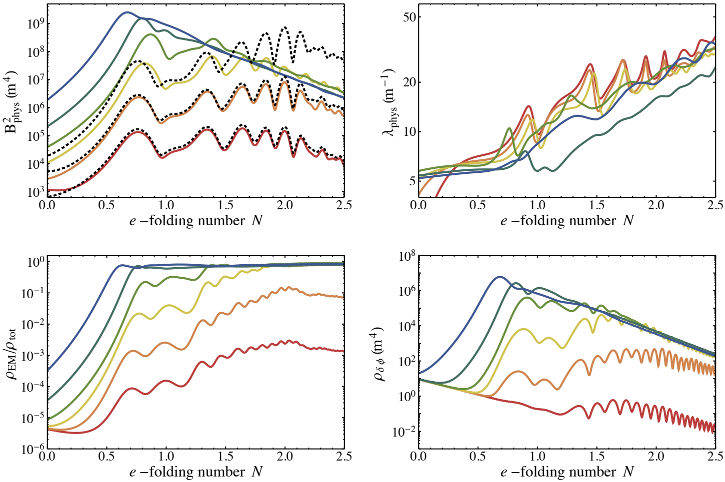
$$B \gtrsim \sqrt{\frac{1Mpc}{\lambda}}10^{-17}G \qquad \text{for } \lambda < 1\text{Mpc}$$

define $B_{\rm eff} \equiv B\sqrt{\lambda/1Mpc} > 10^{-17}G$

EGMF Constraints from Simultaneous GeV-TeV Observations of Blazars

A. M. Taylor[1], I. Vovk[1] and A. Neronov[1]

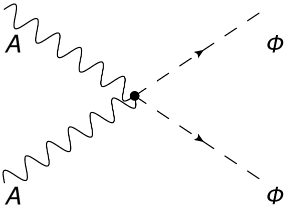
Lattice Results



Photons $\rightarrow$ Charged Plasma

Instantaneous preheating efficiently generates gauge fields, but we are not made of gauge fields...

$\implies$ The "missing" link are Standard Model interactions

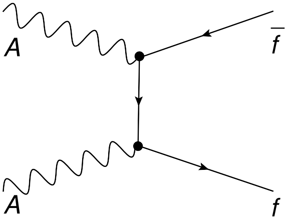


$$\sigma_{AA\to\Phi\Phi} \sim \frac{\alpha_Y^2}{s}$$

$$\frac{\Gamma}{H} = \frac{n\sigma v}{H} \sim \alpha_Y^2 \left(\frac{m_{\rm Pl}}{m}\right)^2 \gg 1$$

Fast interactions lead to

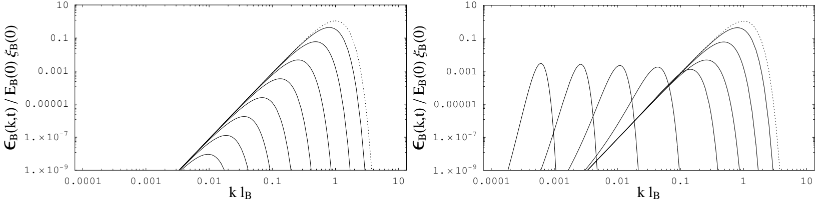
$$T_{\rm reh} \sim \sqrt{m \times m_{\rm Pl}} \sim 10^{-3}\, m_{\rm Pl}$$



Evolution of Helical Fields

In a turbulent plasma $B-$fields undergo **inverse cascade** :

- **helicity conservation**
- energy transfer from smaller to larger scales.



Campanelli, arXiv:0705.2308

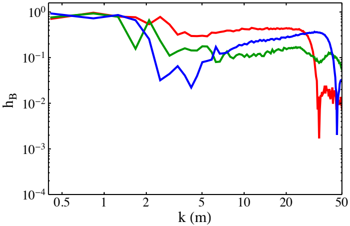
This protects magnetic fields from fast decay

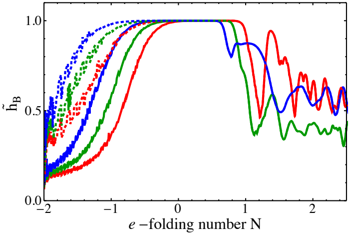
$\Longrightarrow$ **stronger magnetic fields today**.

THE TURBULENT CHIRAL-MAGNETIC CASCADE IN THE EARLY UNIVERSE

Axel Brandenburg[1,2,3,4], Jennifer Schober[3], Igor Rogachevskii[5,1,3], Tina Kahniashvili[6,7], Alexey Boyarsky[8], Jürg Fröhlich[9], Oleg Ruchayskiy[10], and Nathan Kleeorin[5,3]

Late Universe Magnetic Field

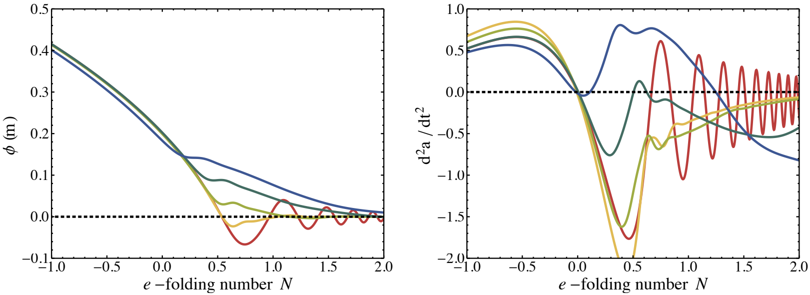




- Conversion of gauge fields to charged particles $\mathcal{O}(1)$
- Conversion of hypercharge to EM $\cos\theta_W \sim 0.9$
- Inverse cascade starts shortly after inflation

$$\boxed{B_{\text{eff}} \gtrsim 10^{-16}\, G} \quad \Leftrightarrow \quad B_{\text{phys}} \sim 10^{-13} G \quad \& \quad \lambda_{\text{phys}} \sim 10\, pc$$

Who ordered that?



- Strong back-reaction from the gauge-field traps the inflaton.
- Inflation ends momentarily.
- Once the gauge fields red-shift enough, inflation re-starts.

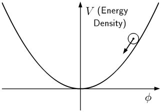
Take the case of a single scalar field. If the field has quantum fluctuations $\delta\phi(\vec{x}, t)$ on top of a classical trajectory $\phi_0(t)$, then one can write

$$\begin{aligned} \phi(\vec{x}, t) &= \phi_{\rm cl}(t) + \delta\phi(\vec{x}, t) = \phi_{\rm cl}(t) - \delta\tau(\vec{x})\dot{\phi}_{\rm cl}(t) \\ &\Rightarrow \boxed{\phi(\vec{x}, t) = \phi_{\rm cl}(t - \delta\tau(\vec{x}))} \end{aligned}$$

Intuitively inflation ends on different times at different places.

The time delay field $\delta\tau(\vec{x})$ is given by

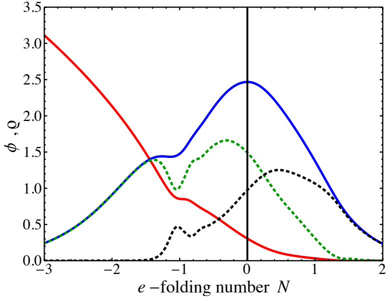
$$\boxed{\delta\tau(\vec{x}) = \frac{\delta\phi(\vec{x}, t)}{\dot{\phi}_{\rm cl}(t)}}$$

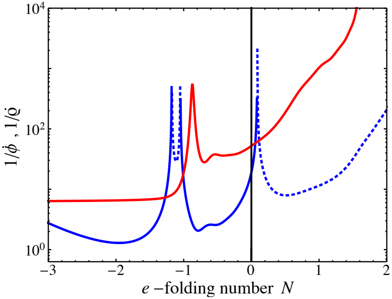


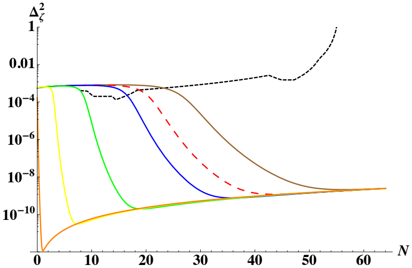
and is related to the density perturbations or temperature fluctuations

$$\frac{\delta T(\vec{x})}{T} = \frac{\delta\rho(\vec{x})}{\rho} \propto \delta\tau(\vec{x})$$

Inflaton trapping







Linde, Mooij & Pajer, arXiv:1212.1693

- An example of "trapped inflation"
- Black Hole production is altered

$\Longrightarrow$ Re-computing bounds on α/f

$\Longrightarrow$ Possible PBH scenario?

Still much to be done!

Summary

Axion inflation naturally has a Chern-Simons coupling to $U(1)$

$\Downarrow$

Lattice simulations needed for large coupling

$\Downarrow$

Instantaneous preheating & efficient scattering to the SM $\Longrightarrow$ **high reheat temperature**

$\Downarrow$

Largely **helical** magnetic fields & **inverse cascade**

$\Downarrow$

Possible origin of **intergalactic magnetic fields**

$\Downarrow$

Large backreaction effects $\Longrightarrow$ Inflaton **trapping** can mimic potential feature

$\Downarrow$

Possible enhanced **PBH** production

$\Downarrow$

Coupling constraints must be updated

Thank you!

Baryogenesis through magnetogenesis

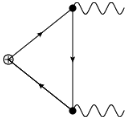
finite conductivity of the primordial plasma

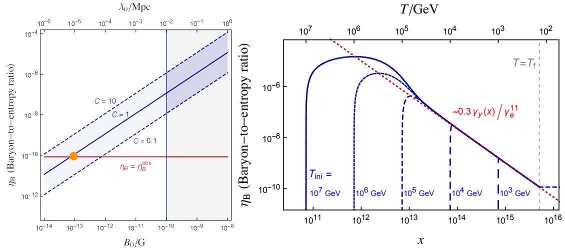
$\Downarrow$

helicity decay of the hypercharge fields

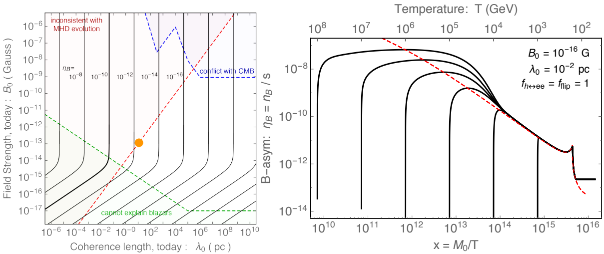
$\Downarrow$

baryon asymmetry through the SM chiral anomaly, without $B - L$ violation





Fujita & Kamada, arXiv:1602.02109



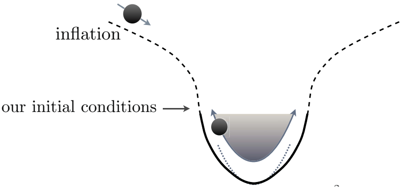
Kamada & Long, arXiv:1606.08891

Axion Monodromy Potential

The potential is quadratic near the origin and "flattens out" for larger values. This is a required condition for the formation of **oscillons**

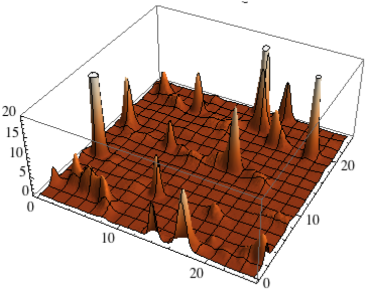
$$V(\phi) = \mu^3 \left(\sqrt{\phi^2 + \phi_c^2} - \phi_c \right) \approx \begin{cases} \mu^3 \phi & \text{for } \phi \gg \phi_c \\ \frac{\mu^3}{2|\phi_c|}\phi^2 & \text{for } \phi \ll \phi_c \end{cases}$$

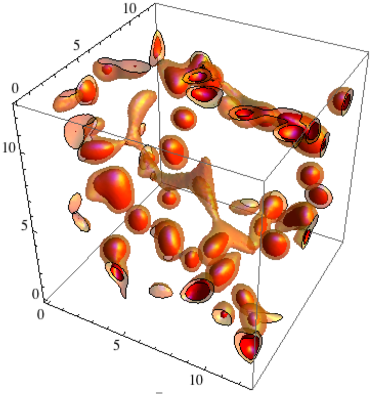
Oscillons are long-lived field configurations that are localized in space and oscillate in time.



Oscillon Emergence

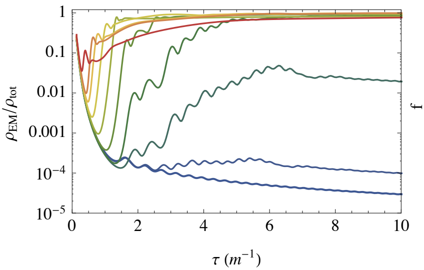
Oscillons have been numerically shown to emerge after inflation in axion monodromy models.

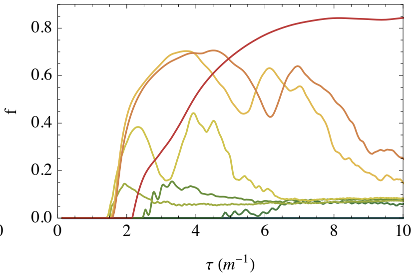




Amin, Easther, Finkel, Flauger, Hertzberg, arXiv: 1106.3335

Axion Monodromy inflation





Instantaneous preheating occurs for $\frac{\alpha}{f} m_{\rm Pl} \gtrsim 60$.

Explanation: $A_{\rm tach} \sim e^{\pi \frac{\alpha}{f} \frac{|\dot{\phi}|}{H}}$ during inflation

Indicator for oscillon emergence:

$$f = \frac{\int_{\rho_\phi > 4\langle \rho_\phi \rangle} \rho_\phi dV}{\int \rho_\phi dV}$$

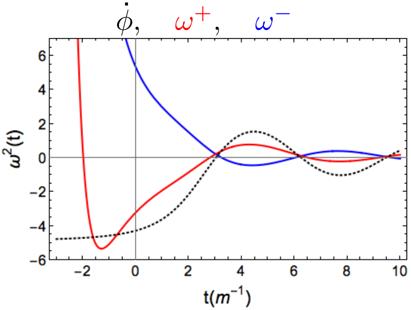
$$\ddot{A}_k^{\pm}+H\dot{A}_k^{\pm}+\left[\left(\frac{k}{a}\right)^2 \mp \frac{\alpha}{f}\frac{k}{a}\dot{\phi}\right]A^{\pm}=0$$

redefine as $\chi^{\pm}=a^{1/2}A^{\pm}$

$$\ddot{\chi}_k^{\pm}+\left[\omega_k^{\pm}(t)\right]^2\chi_k^{\pm}=0$$

with

$$\left[\omega^{\pm}(t)\right]^2=\frac{k^2}{a^2}\mp\frac{\alpha}{f}\frac{k}{a}\dot{\phi}+\frac{\dot{a}^2}{4a^2}-\frac{\ddot{a}}{2a}$$

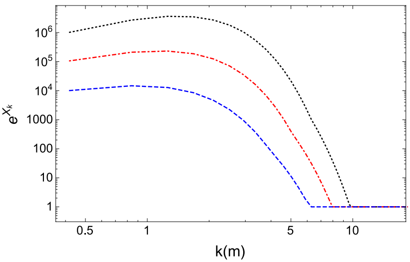


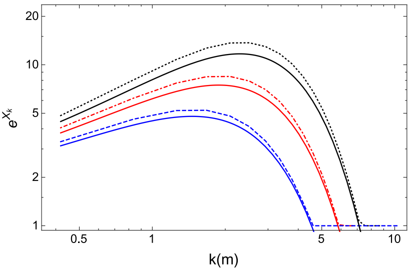
The amplification after the first tachyonic regime is (WKB):

$$\boxed{e^{X_k}=e^{\int\sqrt{-\omega^2(t)}dt}}$$

WKB result

We calculate gauge field amplification after the first tachyonic regime for both modes.





We expect the final state to be **strongly polarized**

Definitions

$$E_B(t) = \int_0^\infty dk\, \mathcal{E}_B, \qquad \mathcal{E}_B \equiv \frac{k^4}{(2\pi)^2} \left(|A_+|^2 + |A_-|^2\right)$$

$$H_B(t) = \int_0^\infty dk\, \mathcal{H}_B, \quad \mathcal{H}_B \equiv \frac{k^3}{2\pi^2} \left(|A_+|^2 - |A_-|^2\right)$$

Consistency relation $|\mathcal{H}_B| \leq 2k^{-2}\mathcal{E}_B$

We define the correlation length $\xi_B = \frac{1}{E_B} \int_o^\infty \frac{dk}{k} \mathcal{E}_B$

leading to the integral consistency relation $|H_B| \leq 2\xi_B E_B$

We distinguish the physical quantities

$$B^2_{\rm phys}(t) = \frac{2}{a^4(t)} E_B(t)\,, \quad \lambda_{\rm phys}(t) = a(t)\, 2\pi\, \xi_B(t)$$