UNIVERSITY OF AMSTERDAM

Many-body strategies for multi-qubit gates

Kareljan Schoutens

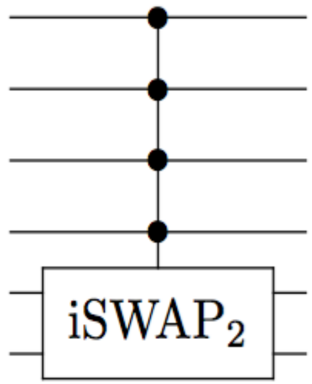
NORDITA workshop, July 31 2017

QuSoft

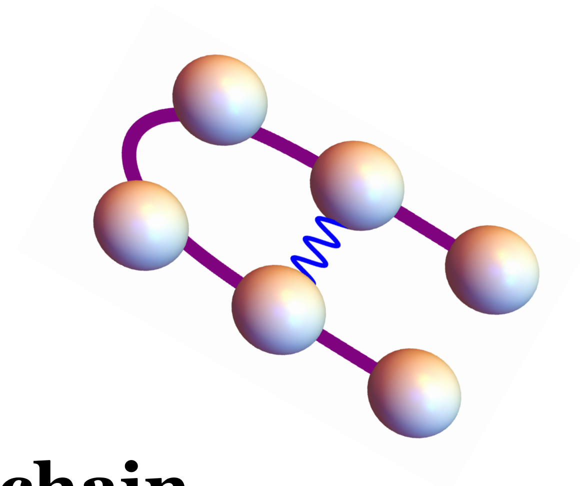
Delta Institute for Theoretical Physics

1-page summary

quantum circuits for quantum algorithms typically need **multi-qubit gates**: unitaries acting on more than 2 qubits

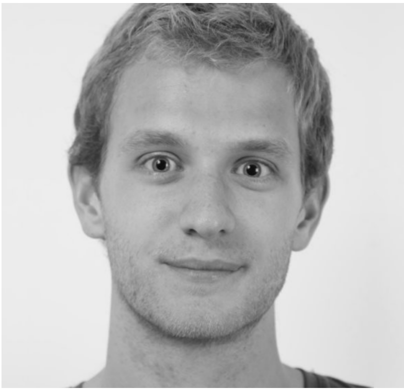


multi-qubit gates can be built from 1-qubit and 2-qubit gates, but such constructions can be cumbersome



we realize *N*-qubit gates via driven dynamics of ***N* coupled qubits**

main mechanism is resonant coupling of eigenstates of **Krawtchouk qubit chain**



Many-body strategies for multi-qubit gates - quantum control through Krawtchouk chain dynamics

Koen Groenland[1,2,3] and Kareljan Schoutens[1,2]

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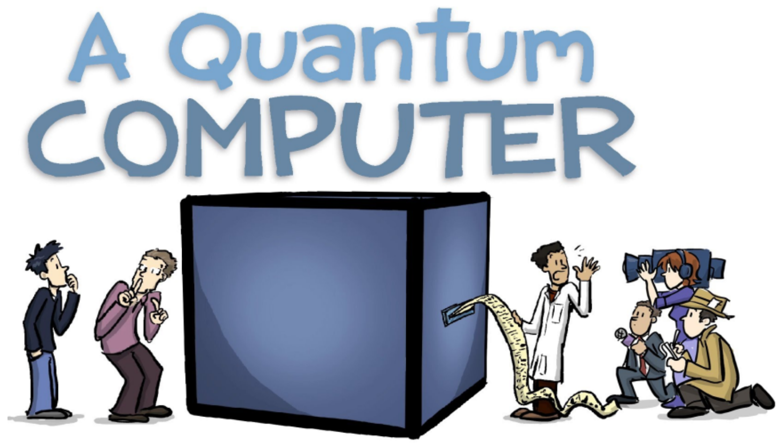
[3] *CWI, Science Park 123, 1098 XG Amsterdam, the Netherlands*

(Dated: 17 July 2017)

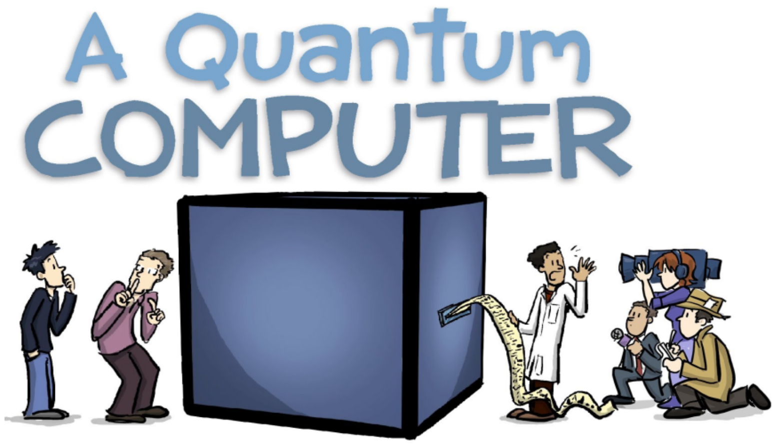
We propose a strategy for engineering multi-qubit quantum gates. As a first step, it employs an *eigengate* to map states in the computational basis to eigenstates of a suitable many-body Hamiltonian. The second step employs resonant driving to enforce a transition between a single pair of eigenstates, leaving all others unchanged. The procedure is completed by mapping back to the computational basis. We demonstrate the strategy for the case of a linear array with an even number N of qubits, with specific $XX + YY$ couplings between nearest neighbors. For this so-called Krawtchouk chain, a 2-body driving term leads to the iSWAP_N gate, which can be reworked to an iSWAP_2 gate with $N - 2$ controls or, using a single auxiliary qubit, to an $(N - 1)$-Toffoli gate.

arXiv:1707.05144v1 [quant-ph] 17 Jul 2017



outline

- background and motivation
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk chain



outline

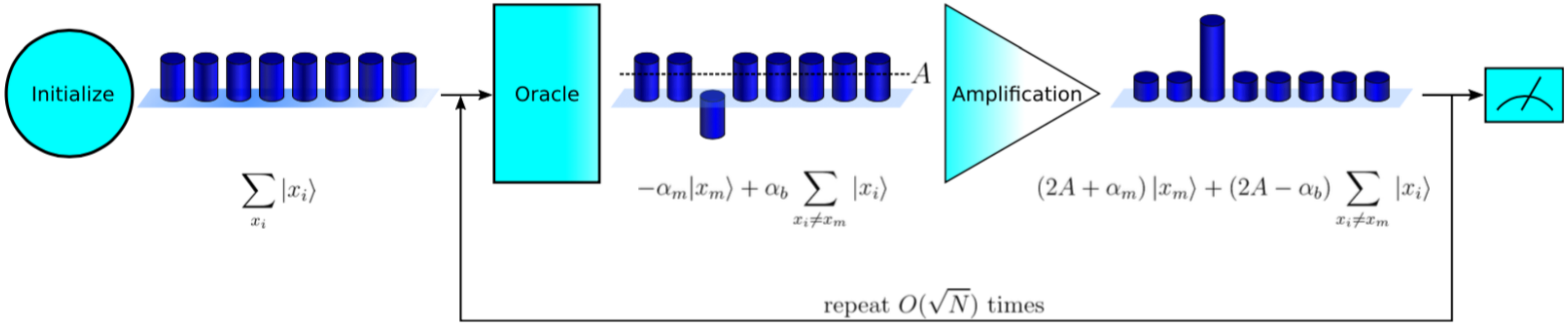
- background and motivation
- many-body strategies for multi-qubit gates
- quantum control on the Krawtchouk chain

quantum algorithms

For specific problems quantum algorithms can be made to outperform classical computers by cunningly combining quantum parallelism with interference.

Grover search algorithm:

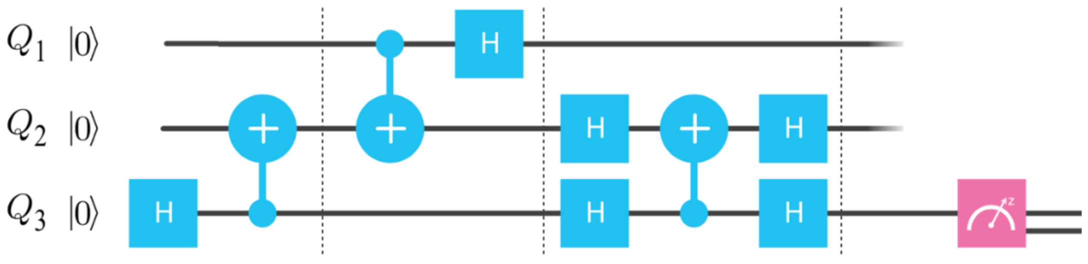
finding tagged element in size-N database in $O(\sqrt{N})$ steps

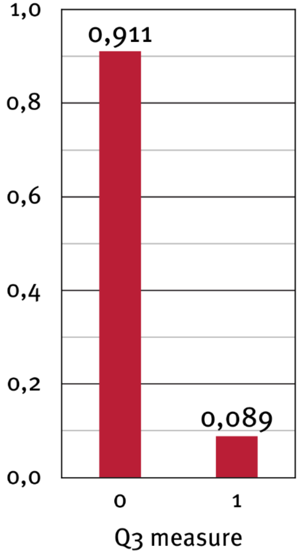


quantum circuit

3-step implementation of quantum algorithm on
N-qubit quantum register

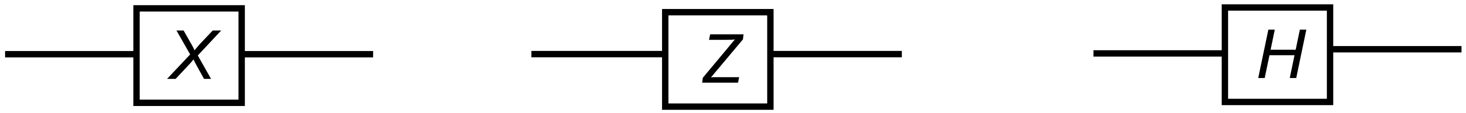
- **initialization**
- **unitary evolution** via quantum gates
- read-out through **measurement**





quantum gates

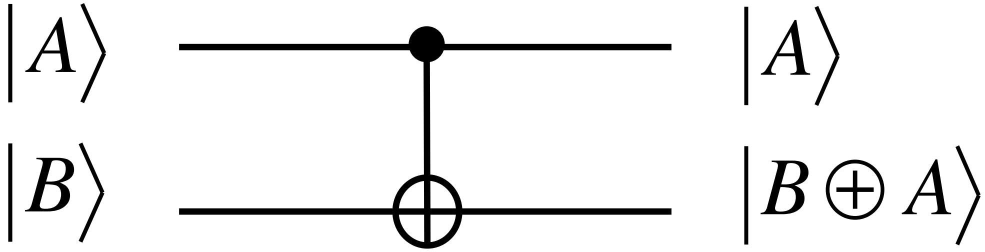
- **1-qubit gates:** *X, Z, H, …*



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- **2-qubit gates:** CNOT, *XX(θ)*, SWAP, …

CNOT:



universal gate sets

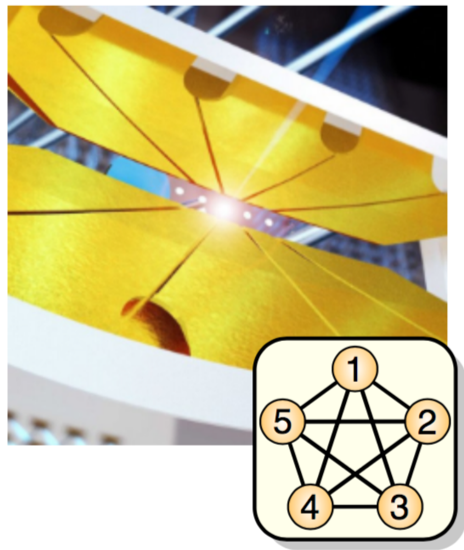
- **strong universality**
  all N-qubit unitaries can be built from CNOTs
  plus sufficiently many 1-qubit gates

- **weak universality**
  all N-qubit unitaries can be approximated to arbitrary
  precision using CNOTs plus a suitable (finite) set
  of 1-qubit gates

native gates and quantum compiling

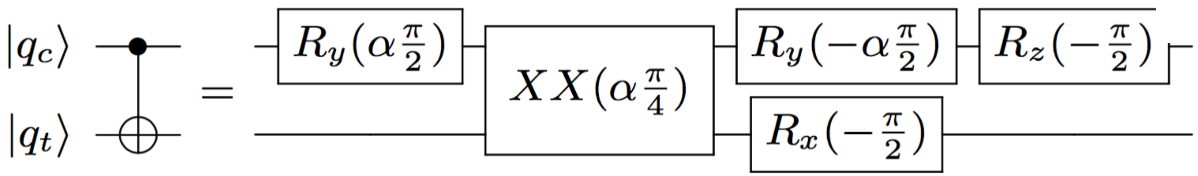
- **native gate libraries**
  the 1-qubit and 2-qubit interactions that are natural for a given qubit platform lead to a `native gate library'.

- **quantum compiling**
  expressing universal gates in native gates



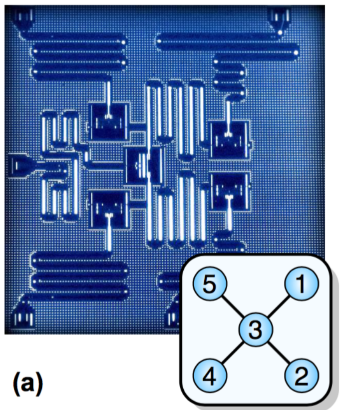
example: native gate library for trapped ions

- all 1-qubit rotations $R_\alpha(\theta)$
- 2-qubit gates $X_i X_j\,(\theta)$

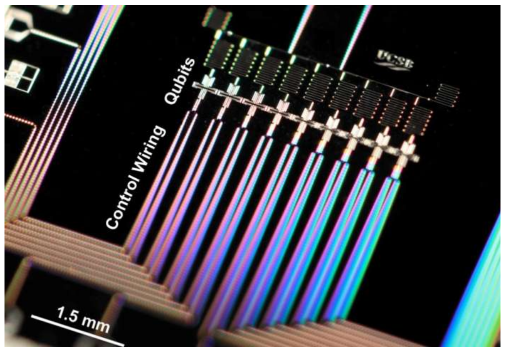


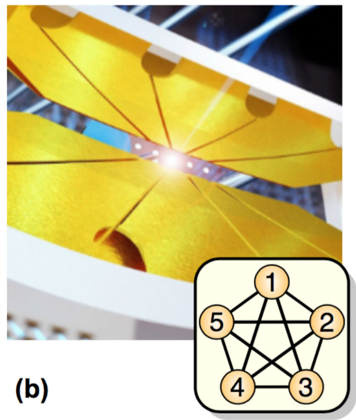
state of the art

quantum hardware has progressed to the point that programmable qubit platforms with up to some 20 qubits are available → real-world testing of few-qubit quantum algorithms!



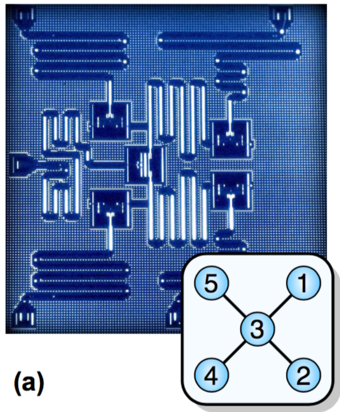
(a)





(b)

IBM Q
`Quantum Experience’



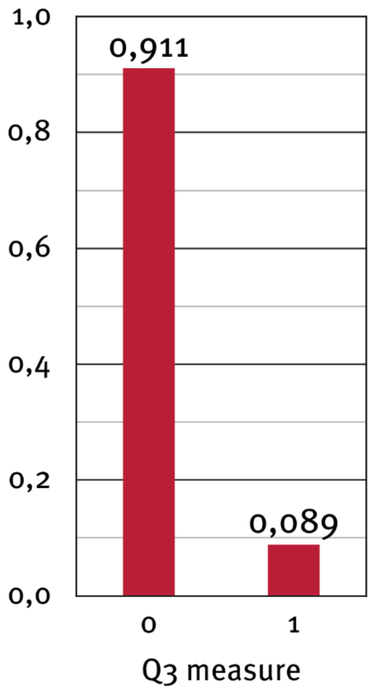
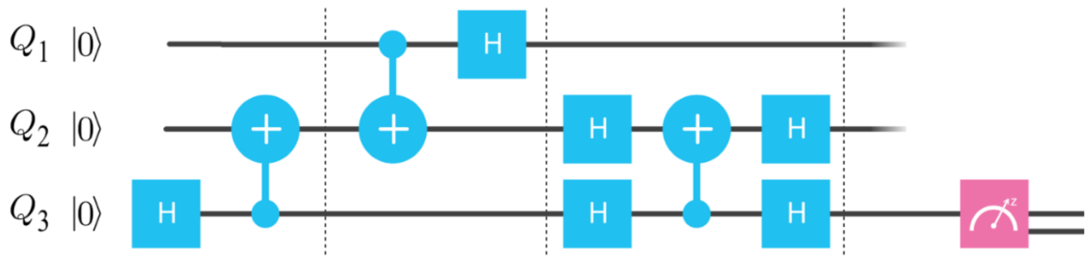


FIGURE C.3: The results of 8192 runs of the quantum circuit teleporting the state $|0\rangle$ shown in figure C.1.

Quantum teleportation: transfering qubit Q1 to Q3 at distant location



Bachelor thesis Jorran de Wit (2016)

Complete 3-Qubit Grover Search on a Programmable Quantum Computer

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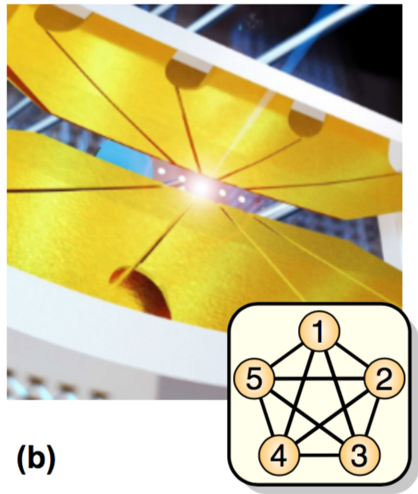
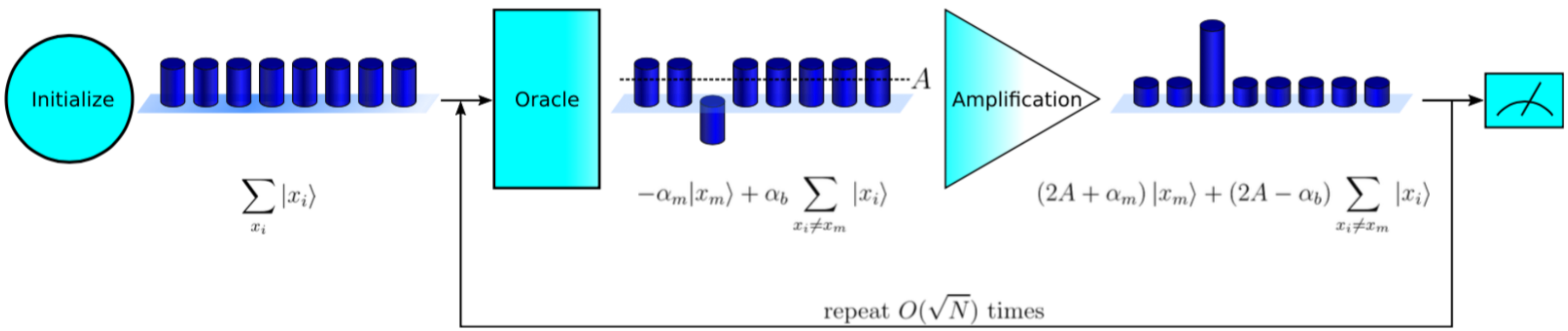
[1] *Joint Quantum Institute, Department of Physics, and Joint Center for Quantum Information and Computer Science, University of Maryland, College Park, MD 20742, USA*
[2] *National Science Foundation, Arlington, VA 22230, USA*
[3] *IonQ Inc., College Park, MD 20742, USA*
(Dated: March 31, 2017)

Grover search: finding tagged element in size-*N* database in $O(\sqrt{N})$ steps

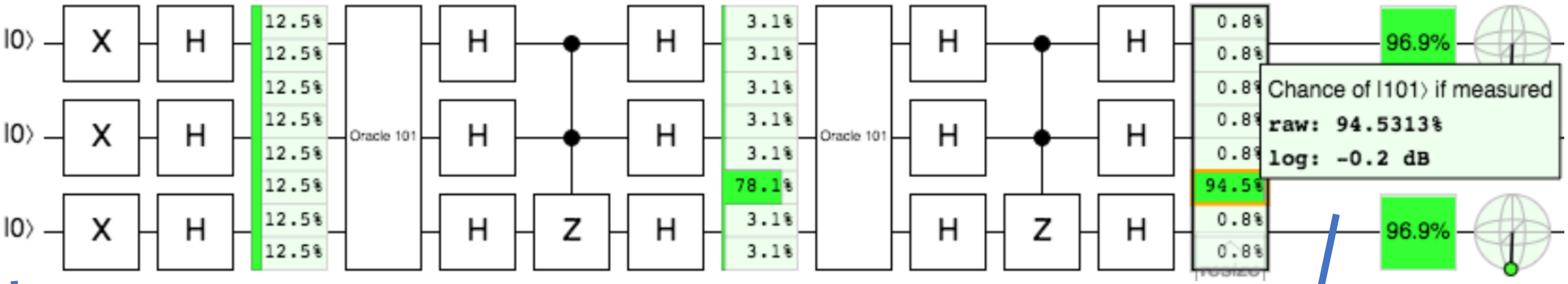


(b)



3-qubit Grover search on Quirk:

finds 1 out of 8 elements in two steps



Oracle tagging the element |101>

Initializing the qubits to |0>

read-out gives
tagged element |101>
with 94.5% chance

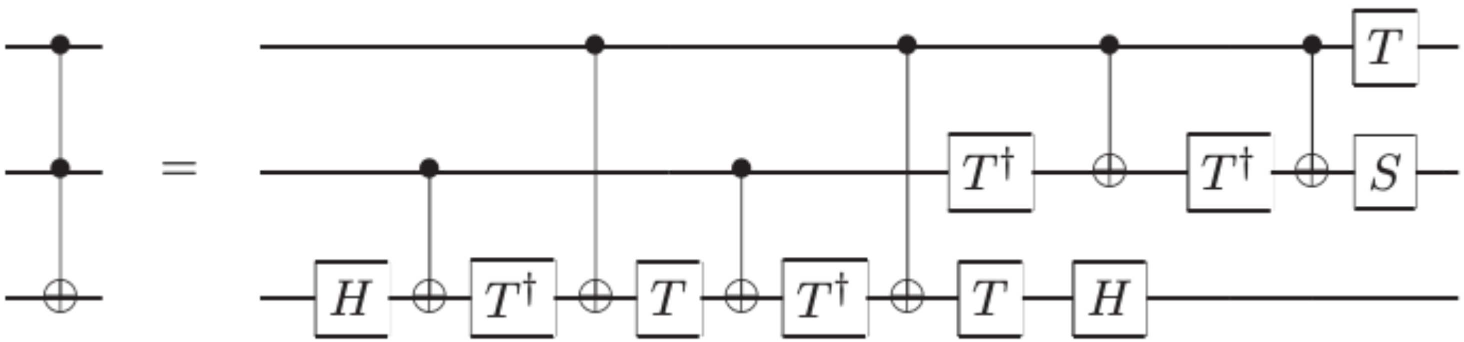
multi-qubit gates

- quantum algorithms such as Grover search use gates like

$$\text{CCNOT (Toffoli), CCZ, ... , C}^{N\text{-}1}\text{NOT, C}^{N\text{-}1}\text{Z, etc}$$

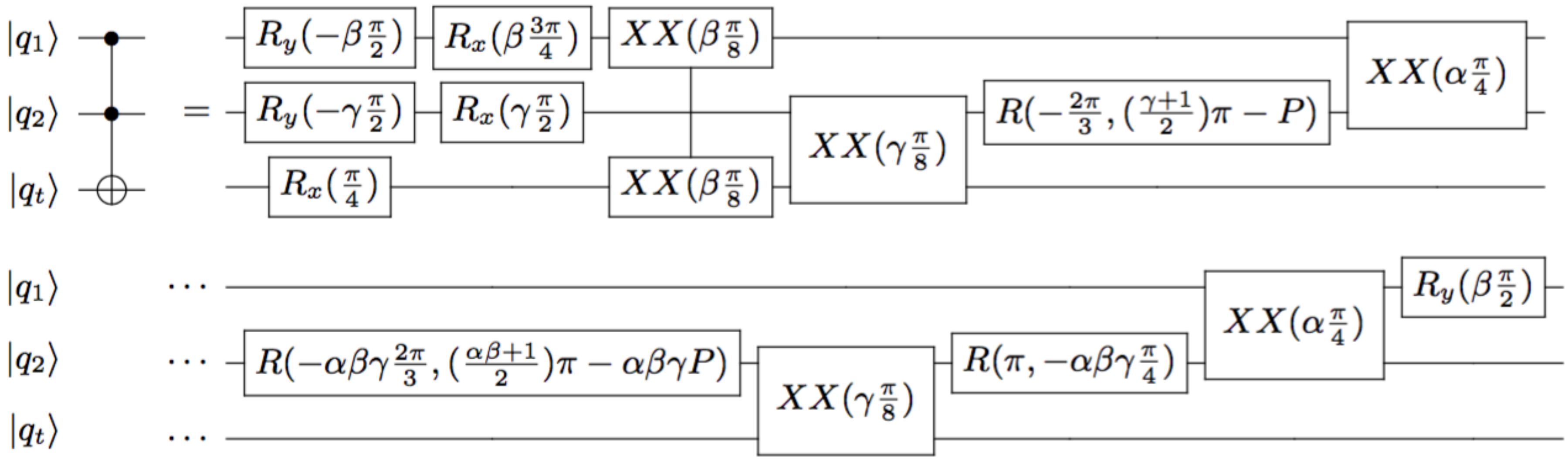
- building these from 1-qubit and 2-qubit gates requires lengthy circuits

multi-qubit gates



Toffoli-3 using standard Clifford + T gate library

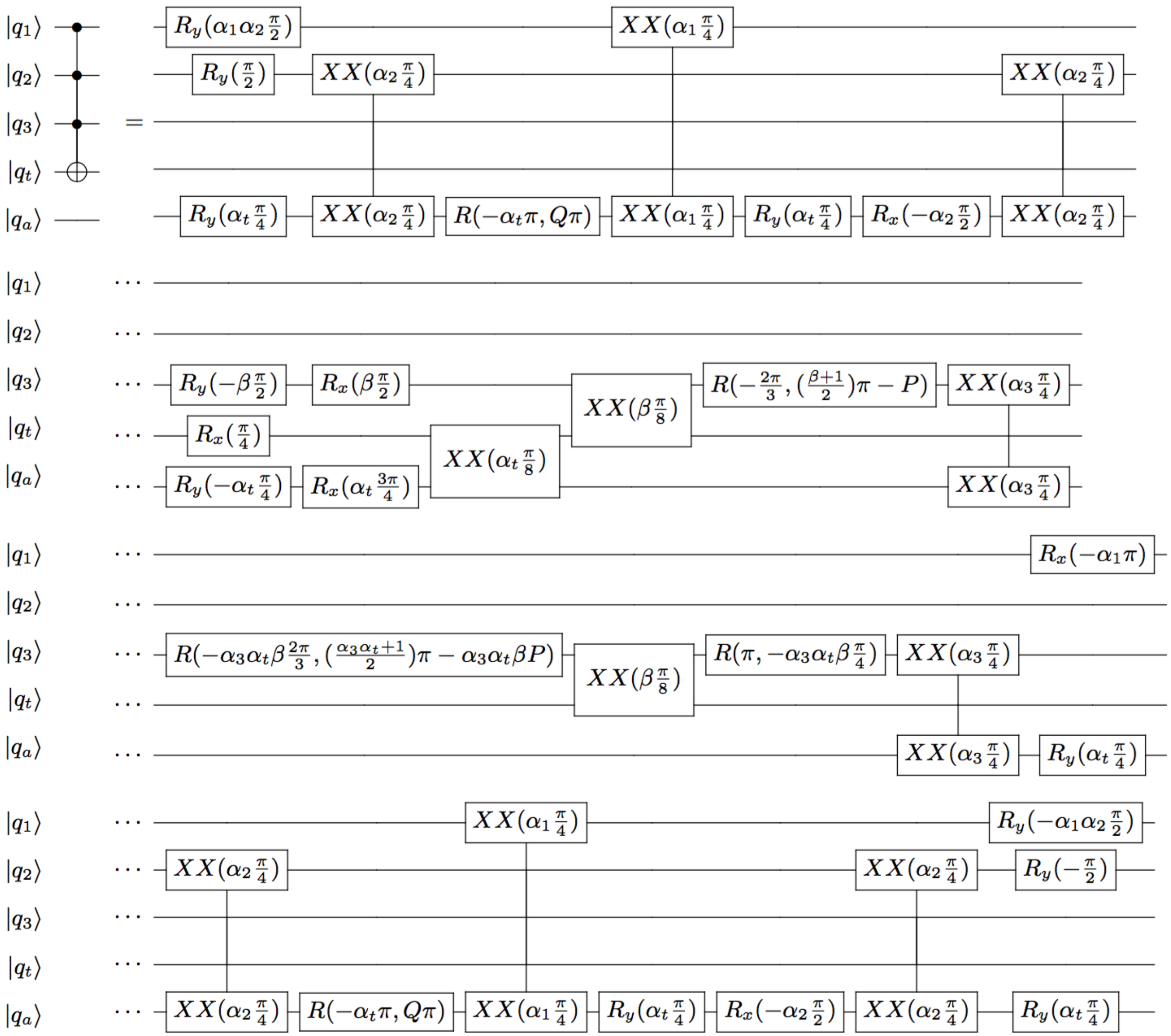
multi-qubit gates

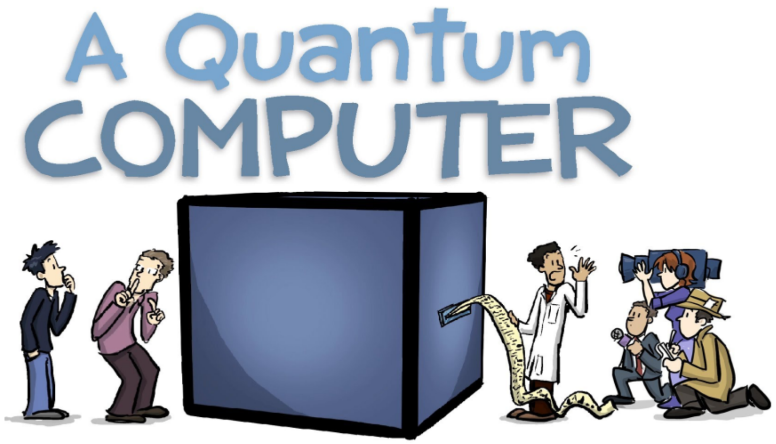


Toffoli-3 using XX/R gate library

multi-qubit gates

Toffoli-4 using XX/R gate library





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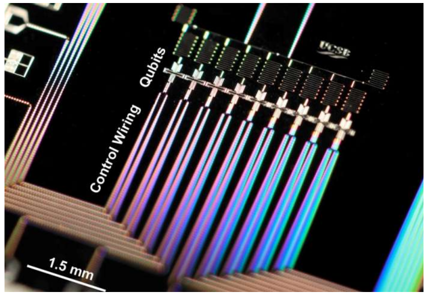
- quantum control on the Krawtchouk chain

many-body strategy



idea

couple N qubits, leading to a many-body spectrum

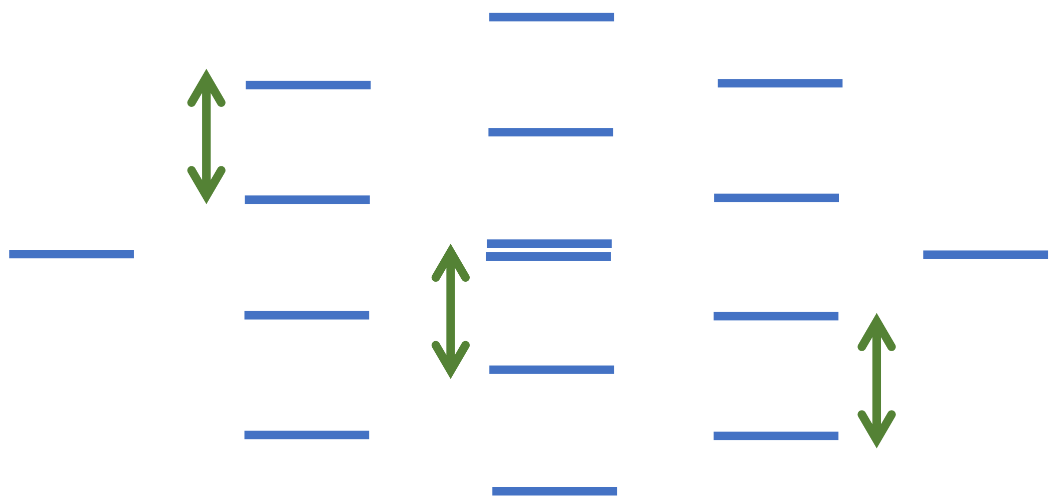
proposed protocol

- apply quantum circuit for *eigengate* to produce eigenstates from states in computational basis
- use resonant driving to selectively couple and interchange 2 out of 2^N eigenstates
- apply eigengate to return to computational basis

many-body strategy…

protocol requires

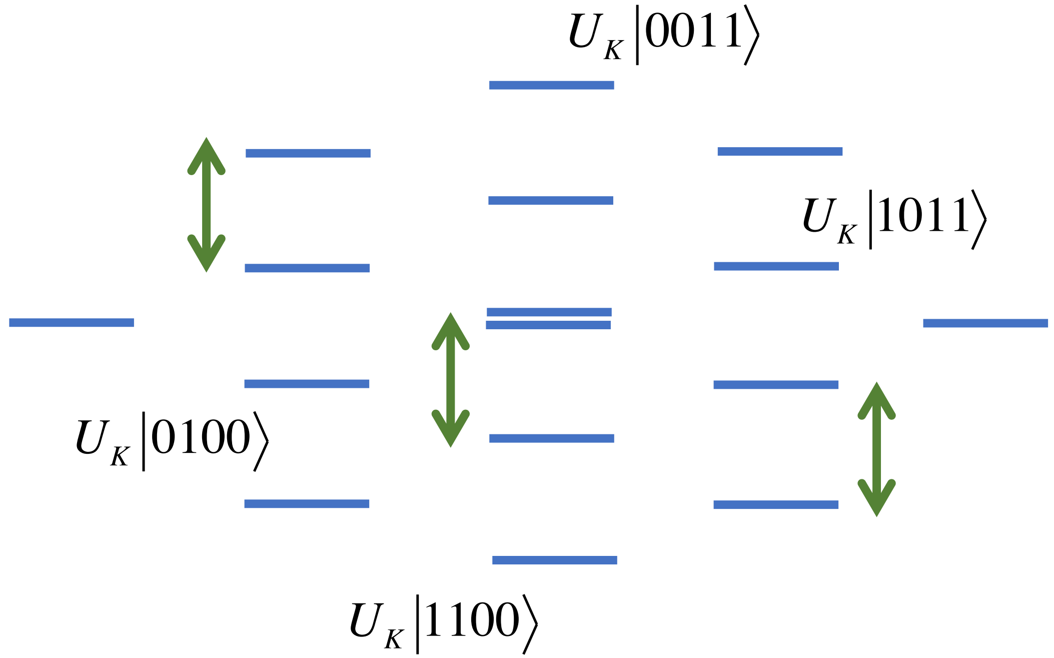
1. commensurate many-body spectrum



many-body strategy…

protocol requires

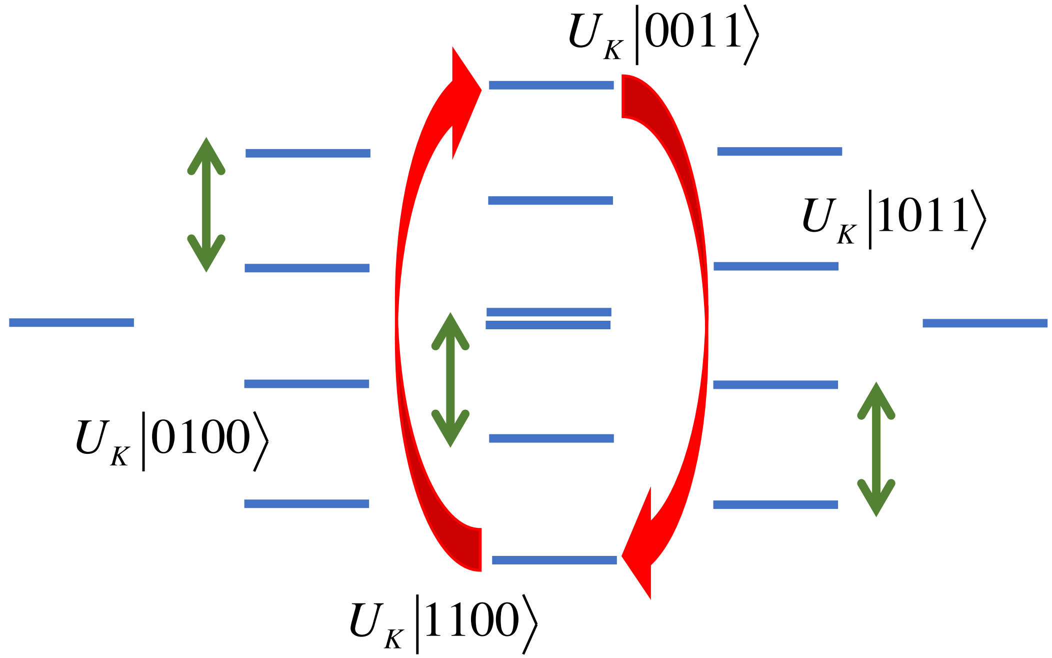
2. eigengate U_K producing many-body eigenstates



many-body strategy…

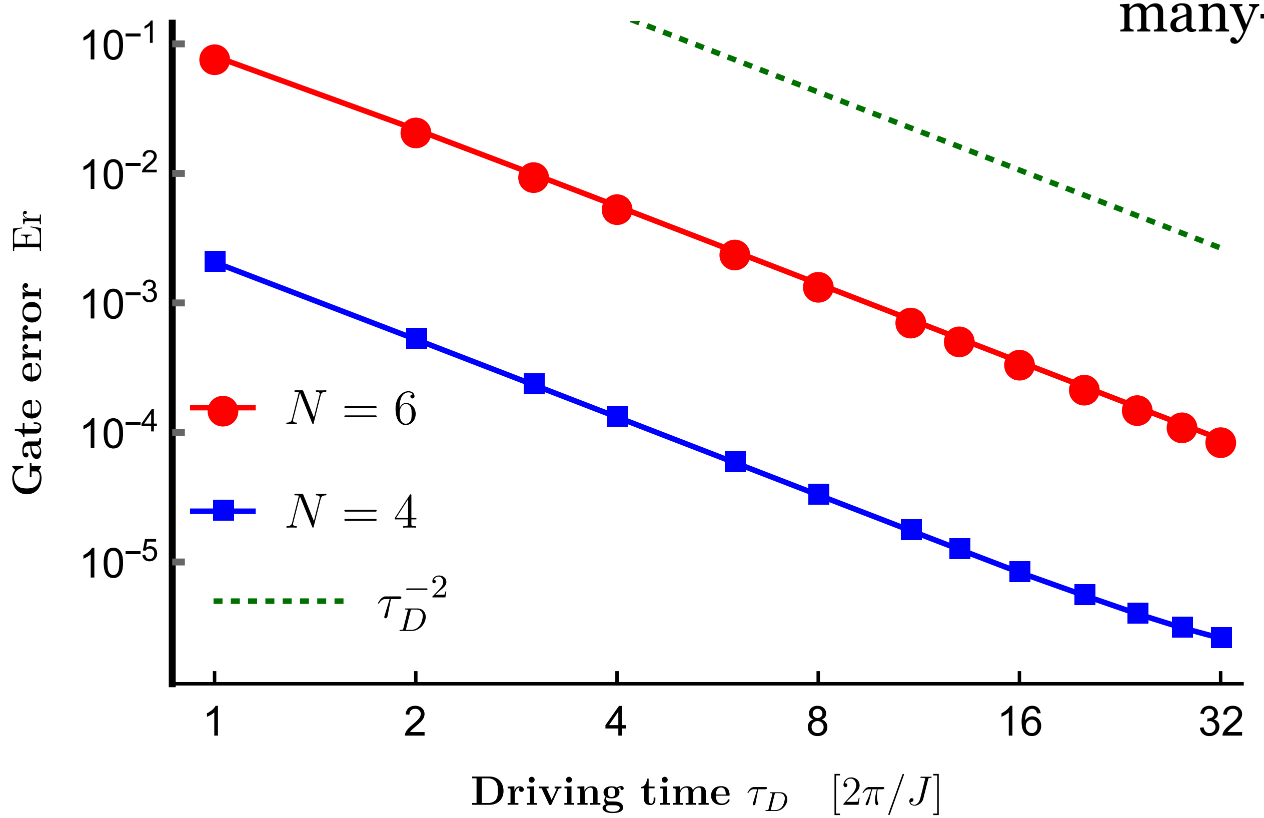
protocol requires

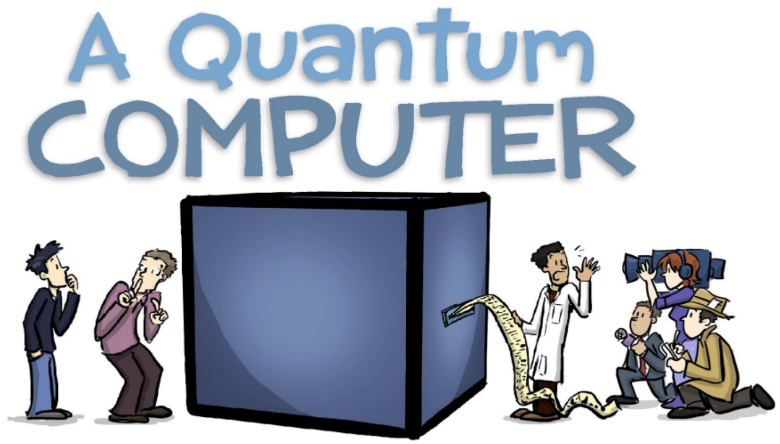
3. driving operator H_D



… for multi-qubit gates

iSWAP$_4$ and iSWAP$_6$ gates realized through many-body protocol





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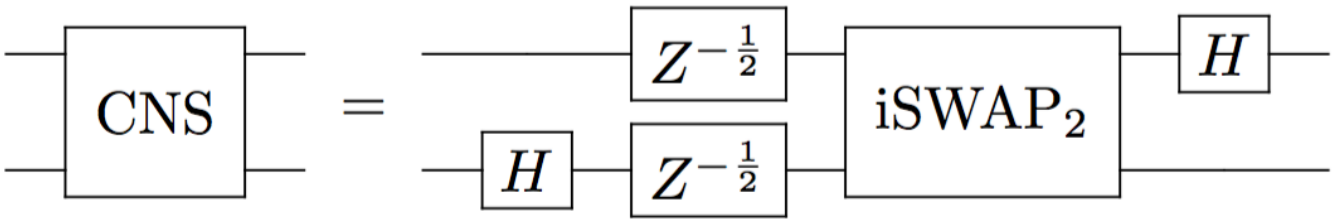
2-qubit *XX+YY* coupling

$$H^{(2)} = -\frac{J}{2}(X_1X_2 + Y_1Y_2)$$

- $t=\pi/J$ pulse of $H^{(2)}$ gives gate iSWAP$_2$,

  $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow i\,|10\rangle$, $|10\rangle \rightarrow i\,|01\rangle$, $|11\rangle \rightarrow |11\rangle$

- combining iSWAP$_2$ with 1-qubit gates gives gate CNS, which is CNOT followed by SWAP



Krawtchouk chain (*N*=*n*+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

- 1-body spectrum

$$\lambda_k = J(k - \frac{N-1}{2}), \qquad k = 0, 1, .., n$$

- eigenstates

$$\left|\{k\}\right\rangle_{H^K} = \sum_{x=0}^{n} \phi_{k,x}^{(n)} \left|\{x\}\right\rangle \qquad \phi_{k,x}^{(n)} = K_{k,x}^{(n)} \sqrt{\frac{\binom{n}{x}}{\binom{n}{k} 2^n}}$$

with $K^{(n)}$ the **Krawtchouk polynomials**

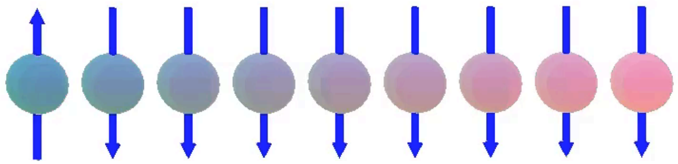
$$K_{k,x}^{(n)} = \sum_{j=0}^{k} (-1)^j \binom{x}{j}\binom{n-x}{k-j}$$

Krawtchouk chain

dynamics for Krawtchouk couplings known to be special

Christandl-Datta-Ekert-Landahl 2004

time evolution over time $t=\pi/J$ gives Perfect State Transfer (PST) for state with single `particle' or `spin-flip'



animation:
van der Jeugt

$t=\pi/J$ pulse on product state $|+\rangle^{\otimes N}$ leads to *graph states* (or GHZ states)

Clark-Moura Alves -Jaksch 2014

Realizations of $XX+YY$ qubit chains

$XX+YY$ chains can be realized with trapped ions, superconducting qubits (transmons), or cold atoms in an optical lattice.

cold atoms

A 2-species *1D* Bose-Hubbard model in the limit $U>>T$ can be tuned to form a Krawtchouk qubit chain

Clark-Moura Alves-Jaksch 2014

Krawtchouk chain from Bose-Hubbard model

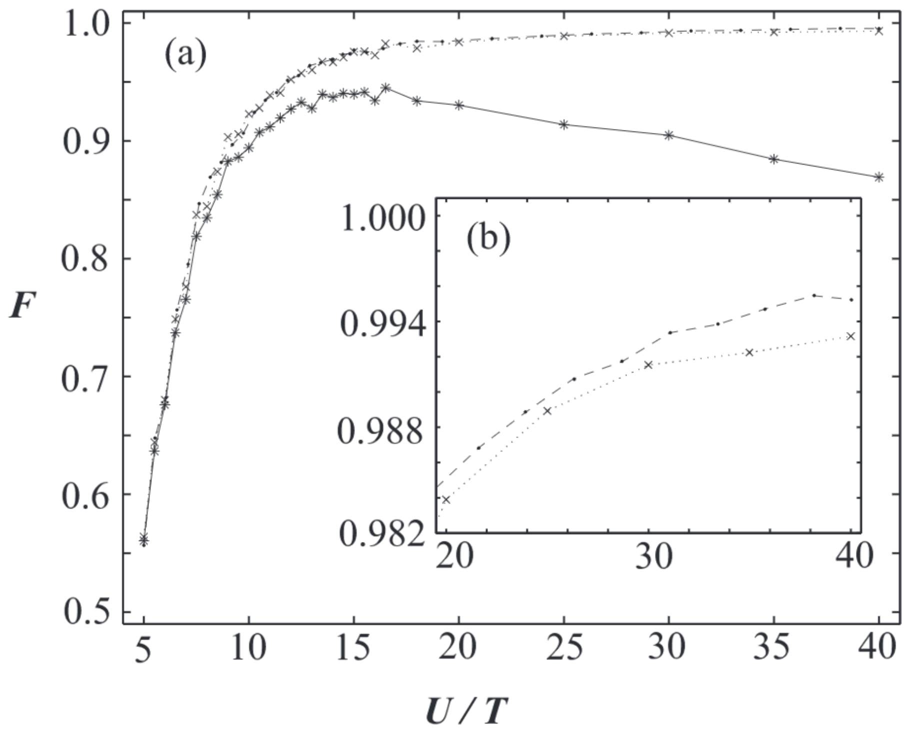


FIG. 3: (a) The fidelity F of the effective XY spin-chain implemented by the 2-species BHM with the ratio U/T, for no noise $\Delta = 0$ $(\cdot)$, $\Delta = 1\%$ $(\times)$ and $\Delta = 5\%$ $(*)$. (b) A close-up of (a). The solid, dashed and dotted lines are drawn to guide the eye.

simulation of N=6 Krawtchouk chain using 2-species Bose-Hubbard model, as function of U/T and noise Δ

Clark-Moura Alves -Jaksch 2014

Experimental Perfect State Transfer

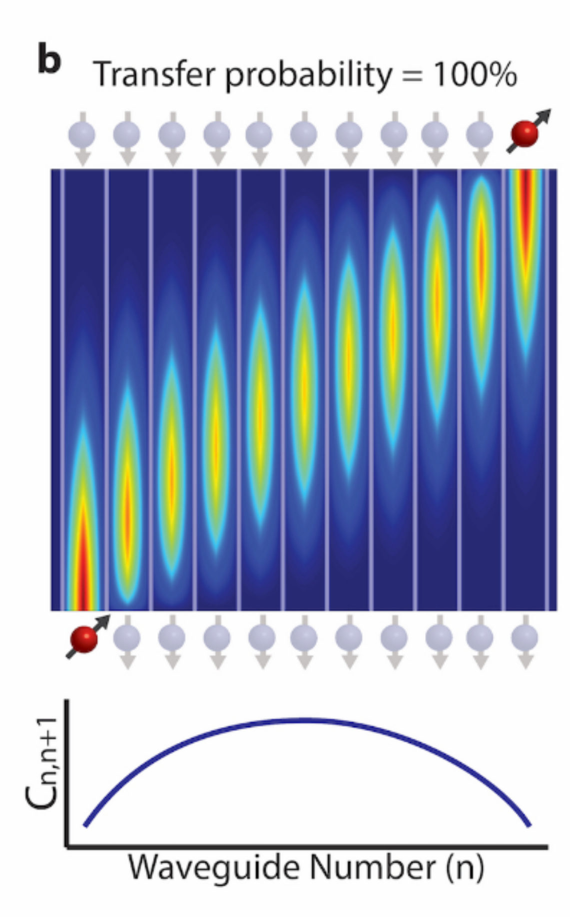
Optical waveguides

Krawtchouk couplings have been engineered, and Perfect State Transfer experimentally realized, in an array of 11 coupled optical waveguides (polarization encoded photonic qubit)

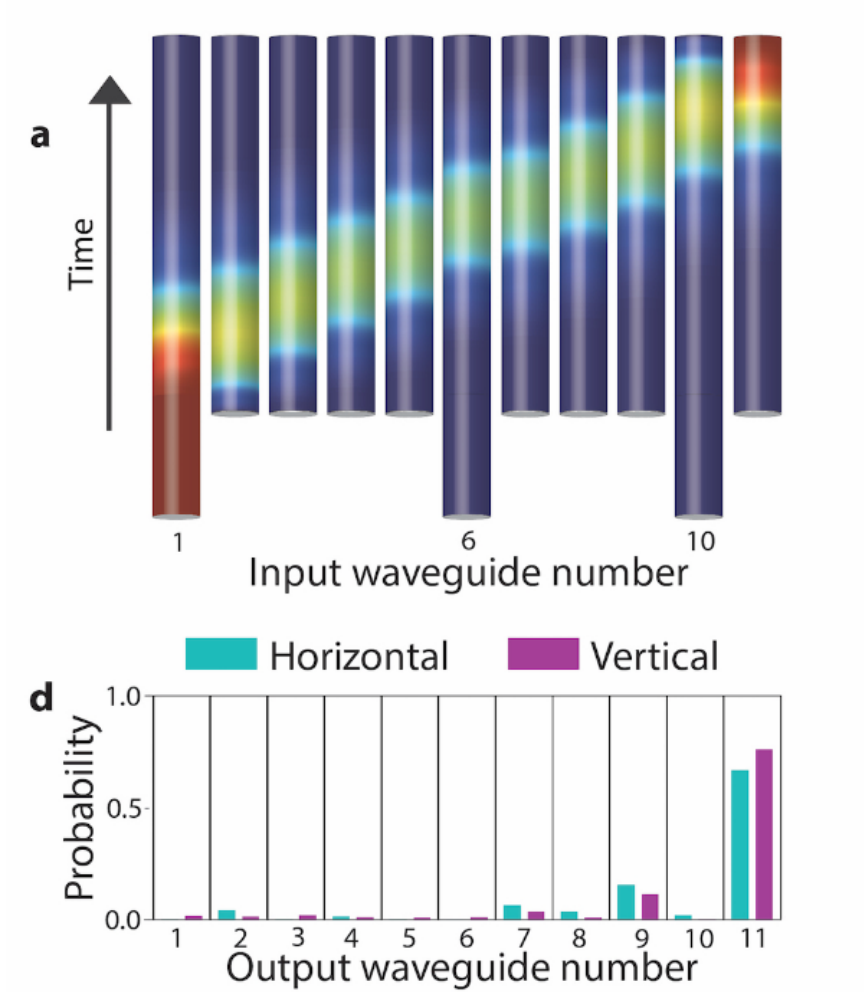
Chapman et al. 2016

Optical waveguides with Krawtchouk couplings

simulation



experimental realizaton



Chapman et al. 2016

Krawtchouk chain (*N*=*n*+1)

$$H^K = -\frac{J}{2}\sum_{x=0}^{n}\sqrt{(x+1)(n-x)}\left[X_x X_{x+1} + Y_x Y_{x+1}\right]$$

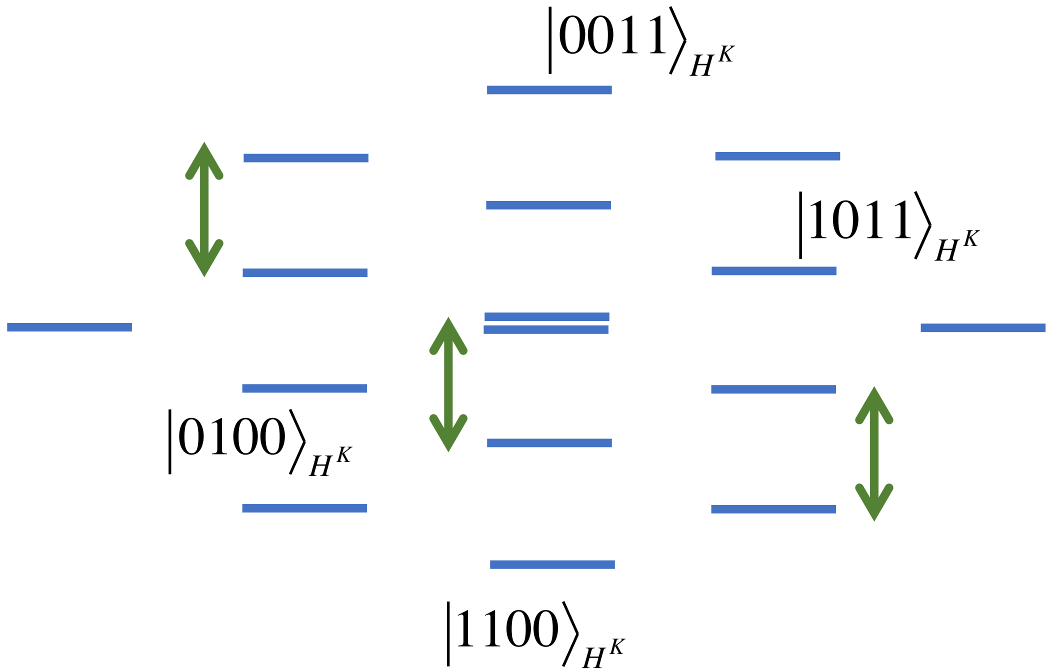
- **important clue:** mapping to free fermions through
  Jordan-Wigner transformation

$$\frac{1}{2}\left(X_j + iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j \qquad \frac{1}{2}\left(X_j - iY_j\right) = \prod_{i=0}^{j-1}(1-2n_i)f_j^+$$

- many-body eigenstates built from fermionic eigenmodes

$$c_k^+ = \sum_{j=0}^{n}\phi_{k,j}^{(n)} f_j^+$$

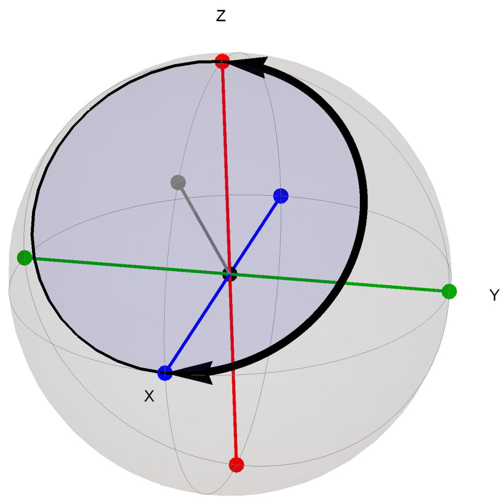
Krawtchouk chain (*N*=*4*)



Krawtchouk eigengate

- exact *eigengate* for Krawtchouk chain eigenstates

$$U_K = \exp\left(-i\frac{\pi}{J}\frac{(H^K + H^Z)}{\sqrt{2}}\right)$$



with

$$H^Z = \frac{J}{2}\sum_{x=0}^{n}(x - \frac{n}{2})(I - Z)_x$$

- important clue: Krawtchouk operators $L_X = H^K$ and $L_Z = H^Z$ satisfy angular momentum commutation relations
- use this to prove that

$$U_K H^Z = H^K U_K \quad \Rightarrow \quad U_K |s\rangle = |s\rangle_{H^K}$$

Krawtchouk eigengate, II

- equivalent expression

$$U_K = \exp\left(-i\frac{\pi}{2J}H^Z\right)\exp\left(-i\frac{\pi}{2J}H^K\right)\exp\left(-i\frac{\pi}{2J}H^Z\right)$$

- action on 1-particle states implies

$$\sum_{k=0}^{n}(-i)^k K_{x,k}^{(n)}K_{k,y}^{(n)} = i^{x+y-n/2}2^{n/2}K_{x,y}^{(n)}$$

(agrees with Meixner's expansion formula)

Multi-qubit gate: iSWAP$_N$

- idea: for N even, driving term $H_D(t)$ that resonantly couples the

  highest energy state $U_K|00\ldots01\ldots11\rangle$

  to the

  lowest energy state $U_K|11\ldots10\ldots00\rangle$

- need to annihilate the $N/2$ fermionic modes with $\lambda_k>0$ and create the $N/2$ modes with $\lambda_k<0$

- can be done by the following 2-qubit operator

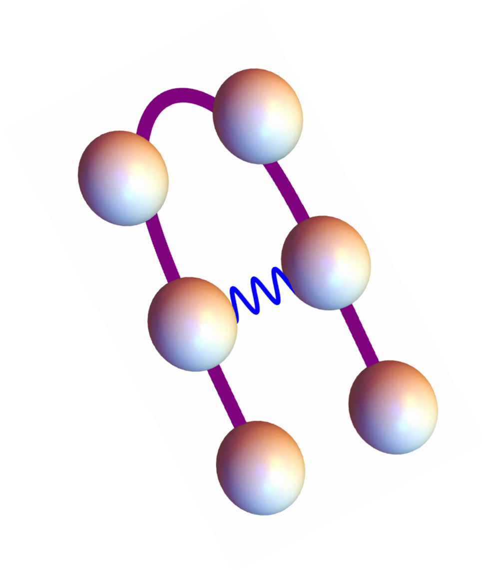
$$\sigma_j^- \sigma_{j+N/2}^+ = f_j^+ [1-2 f_{j+1}^+ f_{j+1}] \ldots [1-2 f_{j+N/2-1}^+ f_{j+N/2-1}] f_{j+N/2}$$

Multi-qubit gate: iSWAP$_N$

- for N=6: matrix element

$$\langle 111000 | U_K (\sigma_1^+ \sigma_4^- - \sigma_4^+ \sigma_1^-) U_K | 000111 \rangle = \frac{5}{32}$$

- resonant driving term



$$H_D^{(1,-)}(t) = i\, J_D \cos[9Jt]\, [\sigma_1^+ \sigma_4^- - \sigma_4^+ \sigma_1^-]$$

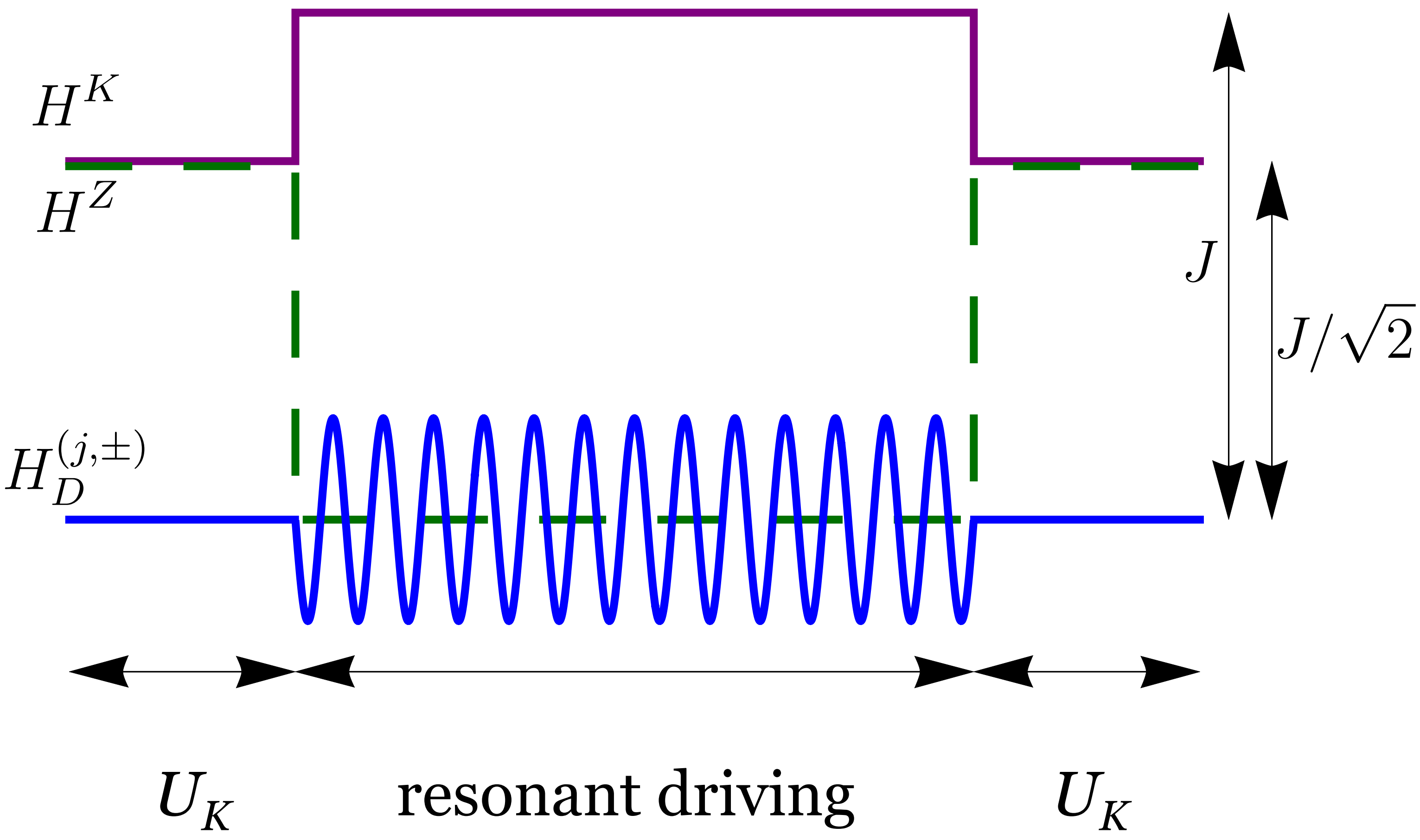
- conditions on driving time τ_D

$$\tau_D (5 J_D / 64) = \pi / 2 \qquad \tau_D = M(2\pi / J)$$

so that (in leading order) |*000111*> and |*111000*> are interchanged and all dynamical phases return to 1

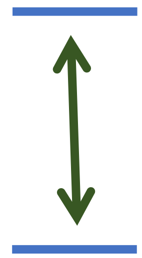
many-body protocol for iSWAP$_6$

$$|000111\rangle \rightarrow i|111000\rangle, \qquad |111000\rangle \rightarrow i|000111\rangle$$



resonant driving – fidelity

$$H_D(t) = \begin{pmatrix} E_1 & d\,\mathrm{e}^{i\omega t} \\ d\,\mathrm{e}^{-i\omega t} & E_2 \end{pmatrix} \qquad \Delta = \omega - (E_2 - E_1)$$

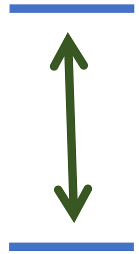


on resonance: $\Delta = 0$

$$v_1(t) = e^{-iE_1 t}\cos(d\,t), \quad v_2(t) = -ie^{-iE_2 t}\sin(d\,t)$$

resonant driving – fidelity

$$H_D(t) = \begin{pmatrix} E_1 & d\,\mathrm{e}^{i\omega t} \\ d\,\mathrm{e}^{-i\omega t} & E_2 \end{pmatrix} \qquad \Delta = \omega - (E_2 - E_1)$$



off resonance: $\Delta \neq 0 \quad d << \Delta$

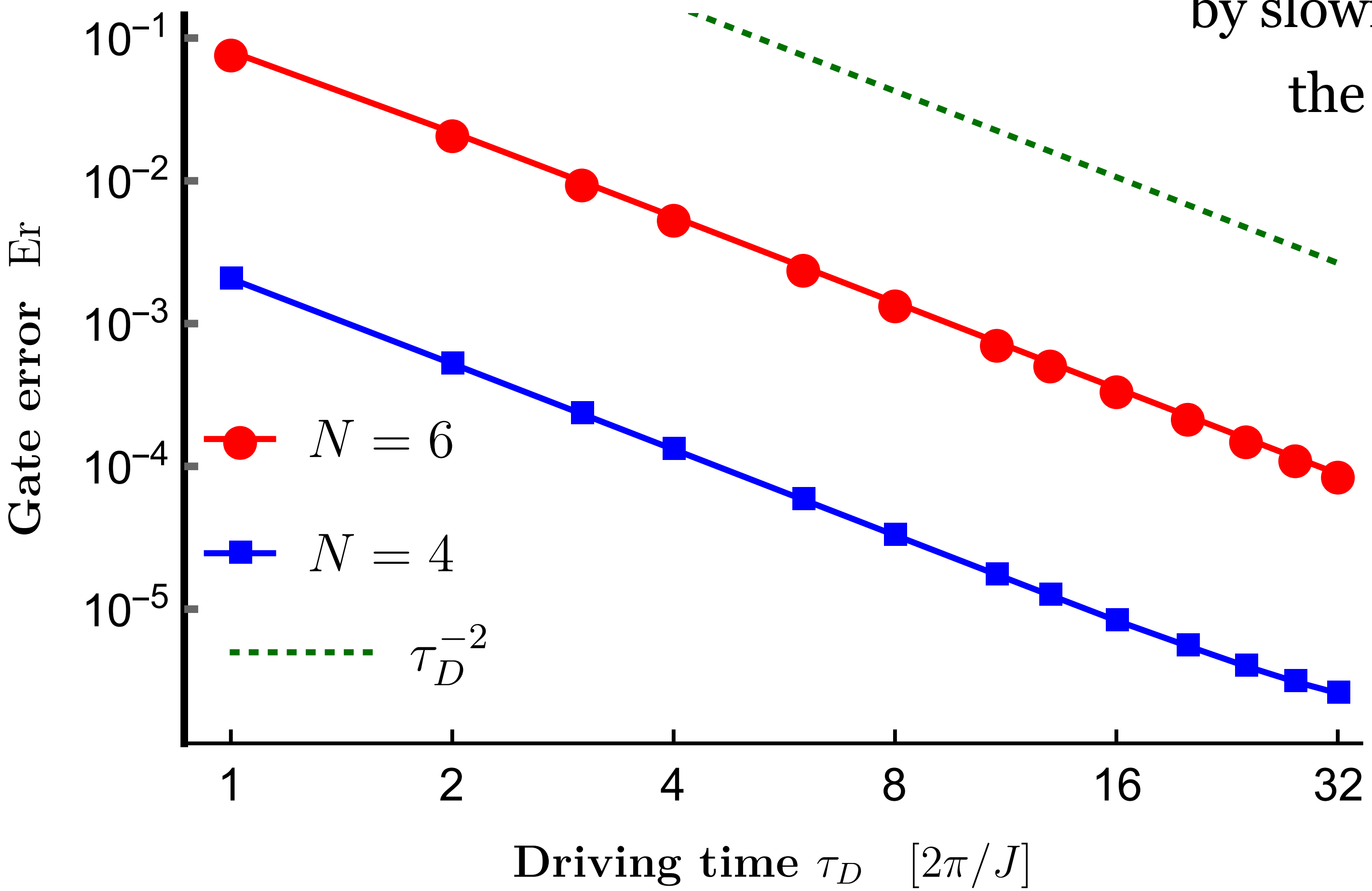
$$dt = \pi / 2 \qquad t = 2\pi M \qquad \Delta, (E_2 - E_1) \text{ integer}$$

$$U_D(t) = \begin{pmatrix} \exp(-\frac{\pi i}{2}\frac{d}{\Delta}) & -\pi i(\frac{d}{\Delta})^2 \\ -\pi i(\frac{d}{\Delta})^2 & \exp(\frac{\pi i}{2}\frac{d}{\Delta}) \end{pmatrix}$$

$$\mathrm{Er} = 1 - \frac{1}{2}\left|Tr\left[U_D\right]\right|$$

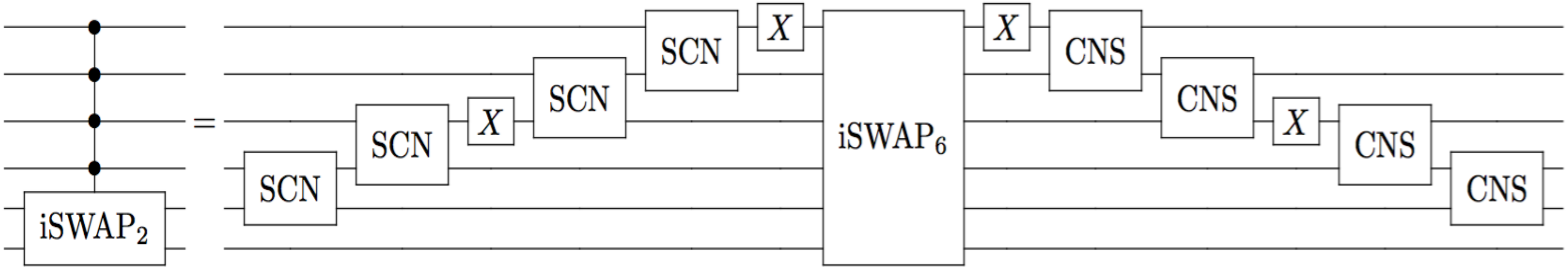
$$\approx \frac{1}{2}(\frac{\pi}{2})^2(\frac{d}{\Delta})^2 \propto \frac{1}{t^2}$$

gate fidelities for $iSWAP_4$ and $iSWAP_6$



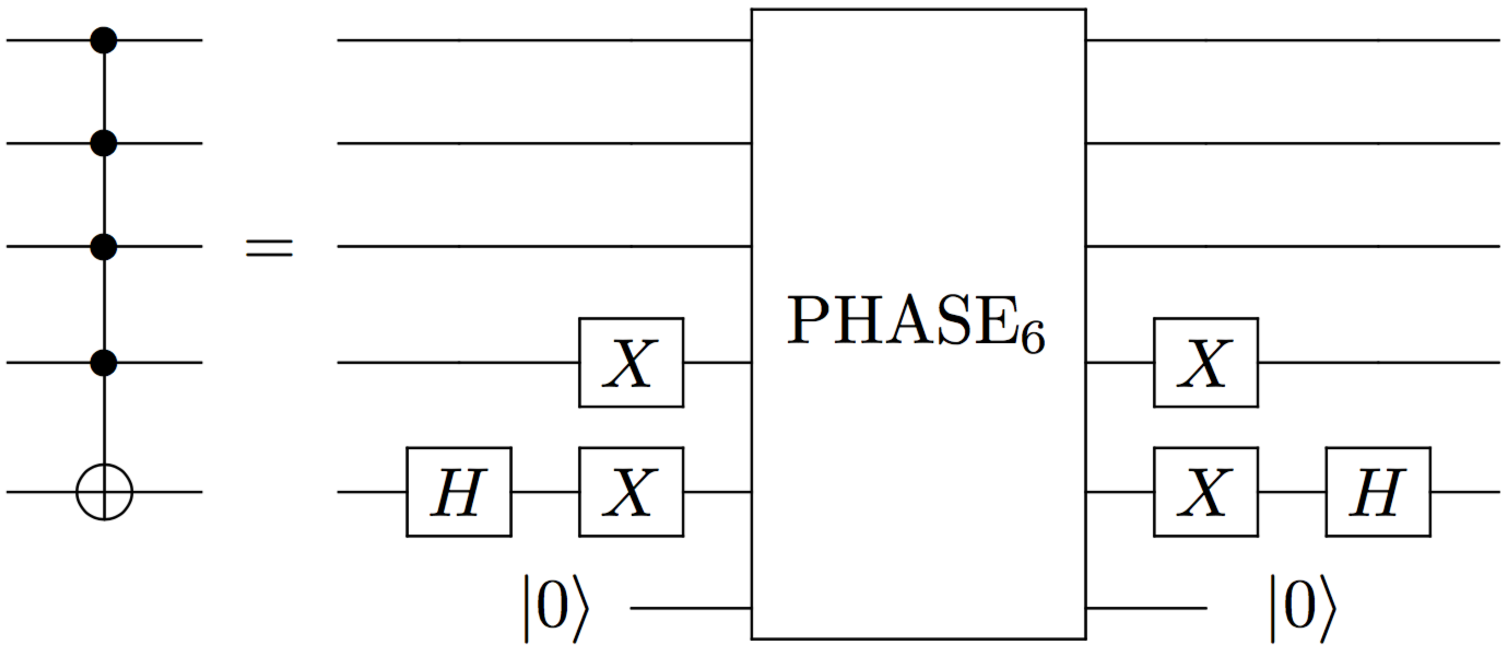
gate fidelity enhanced by slowing down the resonant driving

multi-qubit gates ...



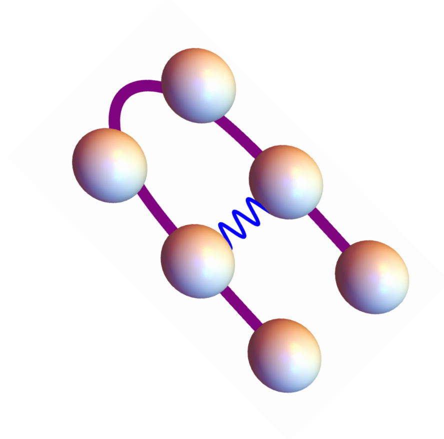
iSWAP$_2$ with 4 controls using iSWAP$_6$ gate

multi-qubit gates …

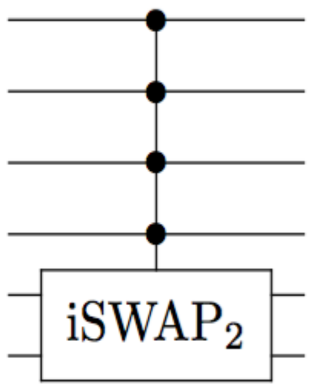


Toffoli-5 using double strength $iSWAP_6$ gate called $PHASE_6$

done & to be done



- ✓ exact eigengates giving fast quantum circuits for Krawtchouk eigenstates
- ✓ resonant driving targeting 2 out of 2^N states
- ✓ iSWAP_N reworked into multi-qubit gate with N-1 or N-2 controls.



- improve the resonant driving part (pulse shaping, correct for Lamb shifts)
- sensitivity to noise?
- window in N where protocol can be realistic?
- experimental implementation

- other incarnations of the many-body strategy