Lynden-Bell, phase space parity, and rotating stellar systems

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Motivation

- Role of angular momentum in the early and long-term evolution of collisional stellar systems still relatively unexplored (Spurzem et al., '90) e.g., dynamical instabilities, acceleration of core collapse
- Crucial to decouple the effects of structural and kinematical properties.
- Why would you care?
    - Simple and cost-effective method to construct DF-based rotating models (for alternative approach, see Alice's talk)

Observational evidence

- Nuclear Clusters
  - NSCs usually elongated along the plane of the galaxy disk (Seth+ 2006 ApJ)
  - NGC 4244 shows clear rotation of 30 km/s within the central 10 pc (0.5”) of the cluster. (Seth+ 2008 ApJ)
- Globular Clusters
  - Young and intermediate-age massive clusters may show significant rotation (R136: Hénault-Brunet+ 2012 A&A ; NGC 1846: Mackey+ 2013 MNRAS)
  - Evidence of rotation in the central regions of 11 GGCs. (Fabricius+ ApJL 2014)
  - Phase space of several GGC soon accessible for the first time (Gaia + HST+ radial velocities) -> kinematic complexity unveiled

Lyden-Bell "Trick"

- The "trick" is a method of creating a rotating model from a non-rotating one (Lyden-Bell 1962)
- Start with a non rotating model axisymmetric model $f(E,J_z)$
  - find an increment function $\Delta f(E, J_z)$ with is antisymmetric in J_z
  - it follows that $\int \Delta f(E, J_z)\, dv^3 = 0$
  - $f_0 = f(E, J_z) + \Delta f(E, J_z)$ is a new distribution function with the same mass density distribution (as long as f0 > 0 everywhere)
- We are exploiting a symmetry in velocity space at each position, to reduce the amount of particles with $-J_z$ and increasing amount with J_z
- $F(E,J_z)=F_p(E)\{(1+x)H(J_z)+(1-x)H(-J_z)\}$
  - Application to three families of non-rotating models

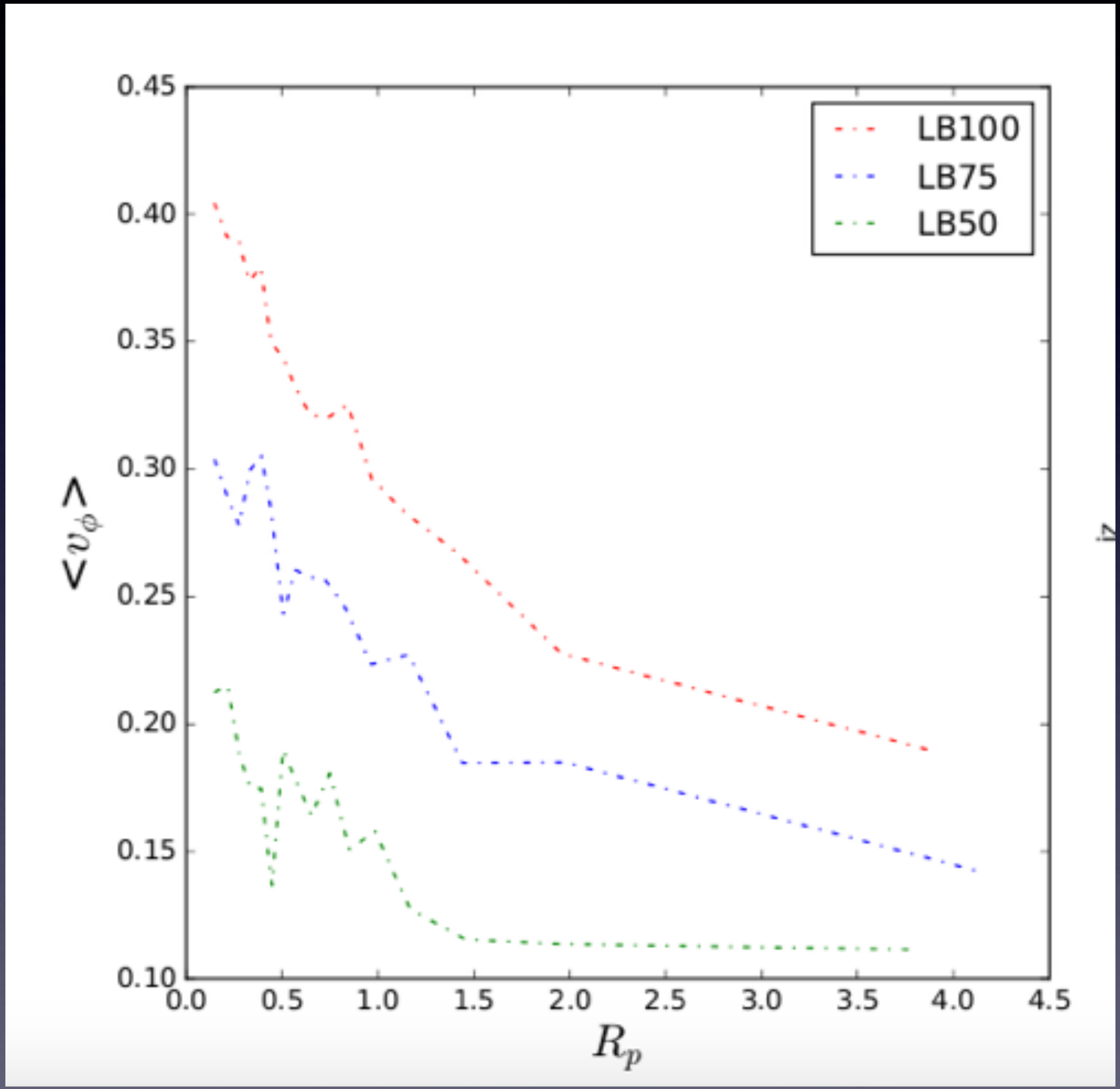
Plummer model

- Well-studied, spherical, non-rotating model (Plummer 1911)

$$F(E) = \frac{3}{7\pi^3}(2E)^{7/2},$$

- We can create a family of Plummer rotating models with varying degree of rotation
  - $F(E, J_z) = F_p(E)\{(1+x)H(J_z) + (1-x)H(-J_z)\}$
  - where x is the percentage of particles (with $-J_z$ convert to $+J_z$)

Rotation Profiles



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Anisotropic Plummer Model

- The maximum about of rotation is limited by the tangential velocity profile
- We can increase the amount of rotation by considering anisotropic models
- One parameter analytical family of anisotropic Plummer models (Dejonghe 1987)

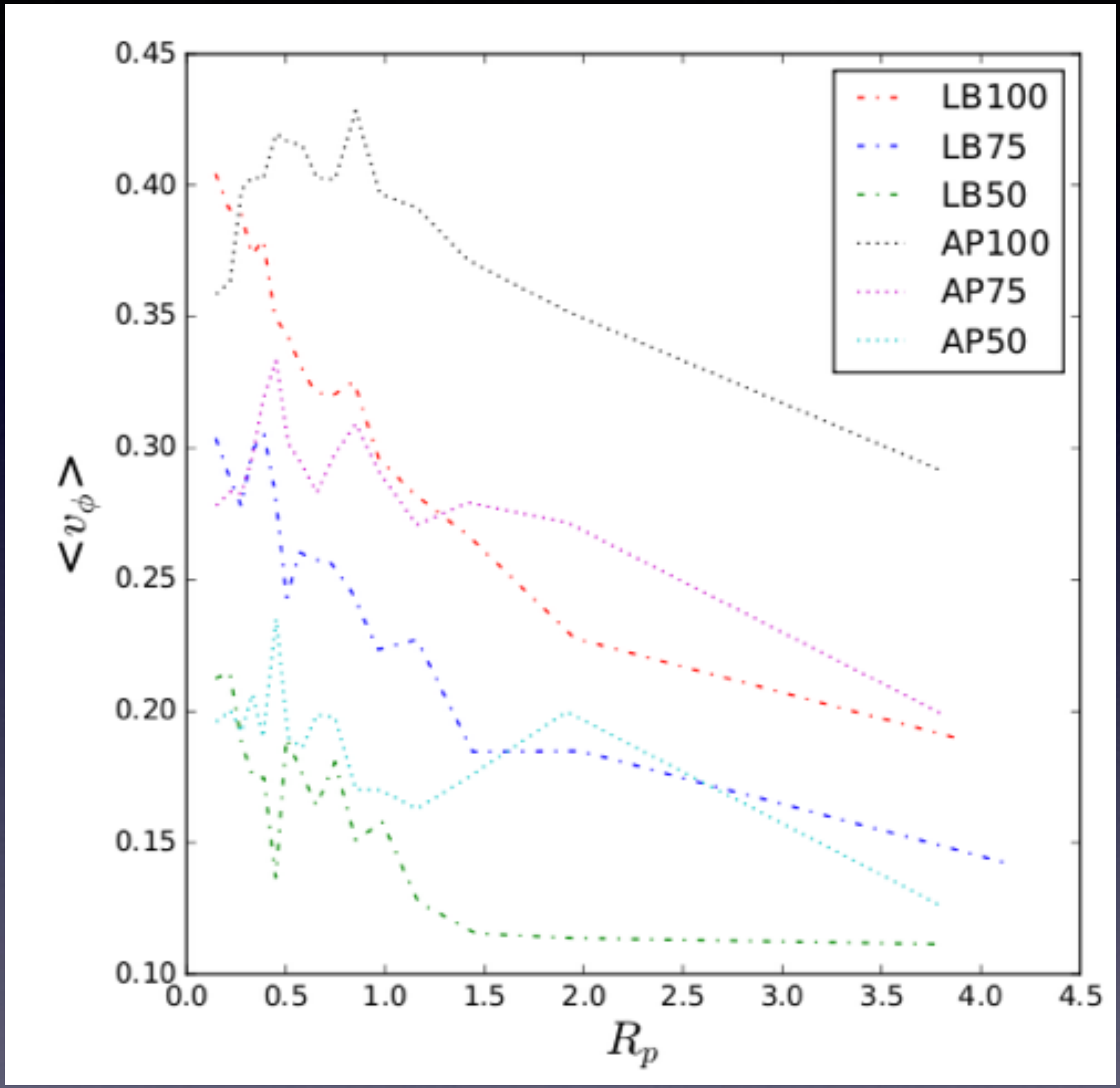
Anisotropic Plummer Model

$$F_q(E,L) = \frac{3\Gamma(6-q)}{2(2\pi)^{5/2}\Gamma(\tfrac{1}{2}q)} E^{7/2-q} \mathbb{H}\left(0,\ \tfrac{1}{2}q,\ \tfrac{9}{2}-q,\ 1;\ \frac{L^2}{2E}\right),$$

$$\mathbb{H}(a,\ b,\ c,\ d;\ x) = \begin{cases} \dfrac{\Gamma(a+b)}{\Gamma(c-a)\Gamma(a+d)} x^a {}_2F_1(a+b,\ 1+a-c;\ a+d;\ x), & x \le 1; \\[2ex] \dfrac{\Gamma(a+b)}{\Gamma(d-b)\Gamma(b+c)} \left(\dfrac{1}{x}\right)^b {}_2F_1\left(a+b,\ 1+b-d;\ b+c;\ \dfrac{1}{x}\right), & x \ge 1. \end{cases}$$

- Dejonghe 1987

Rotation Profiles II



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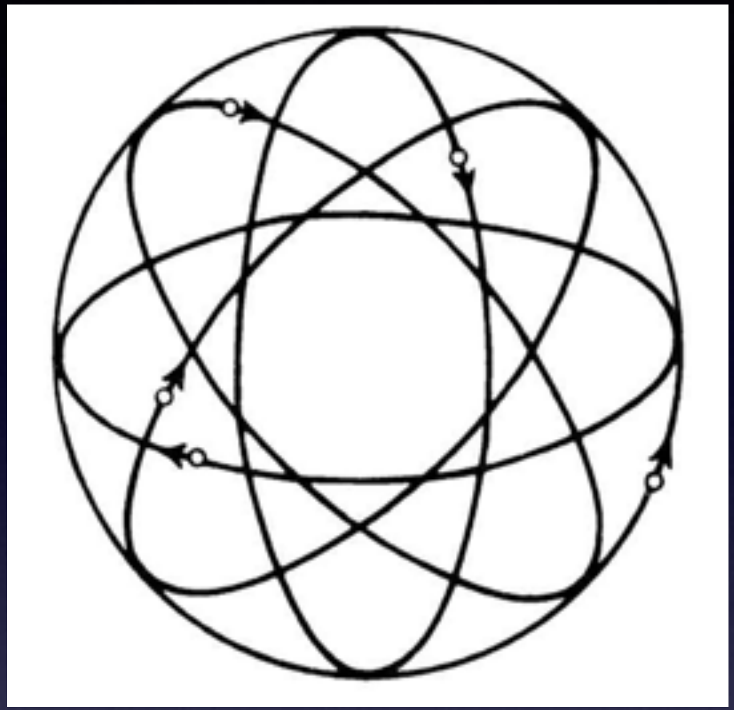
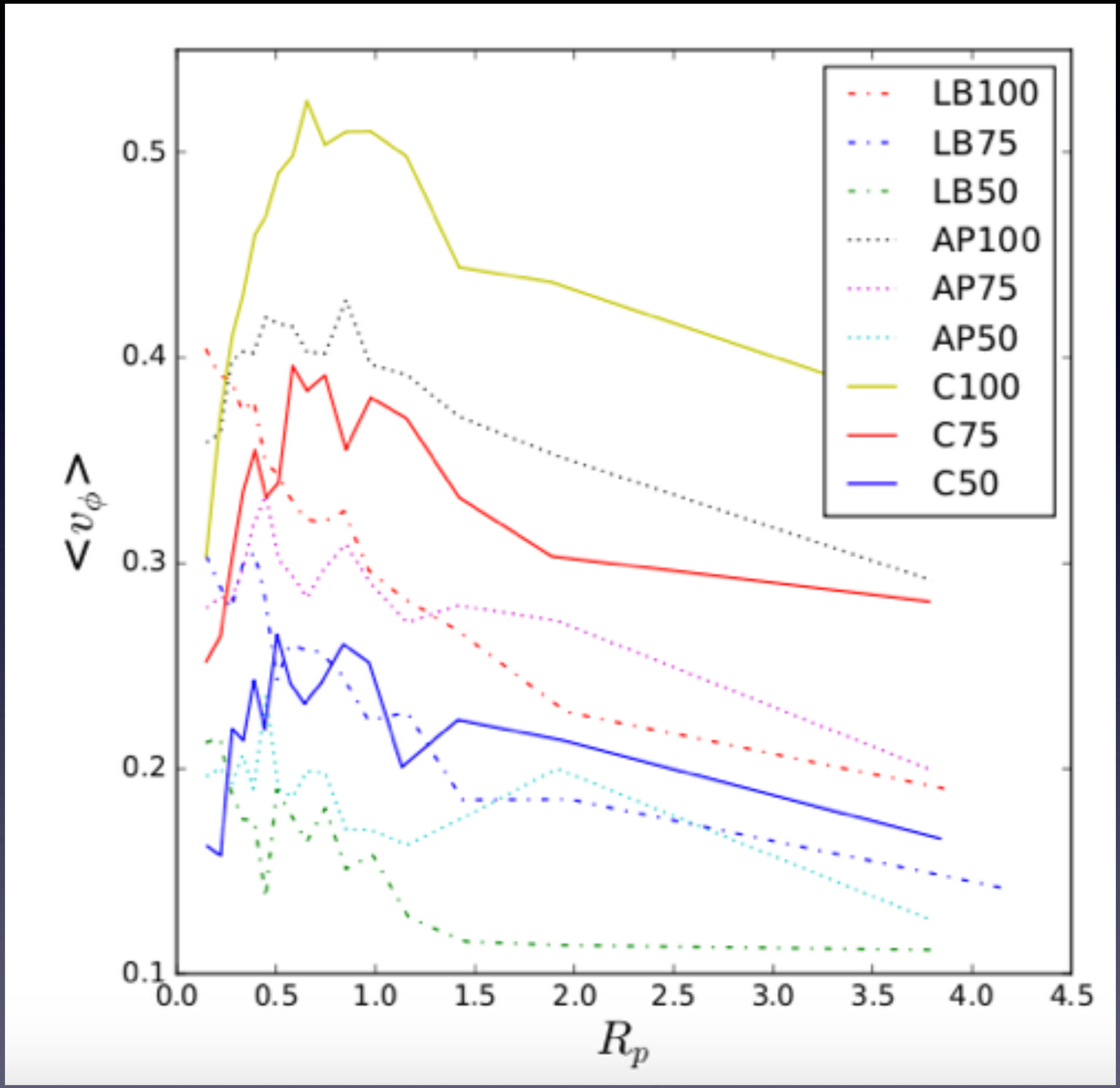
Einstein Sphere



- Model consisting of entirely circular orbits (Einstein 1939)
- Limiting case of anisotropic Plummer model
- Maximum net angular momentum and very easy to create realisation

Rotation Profiles III



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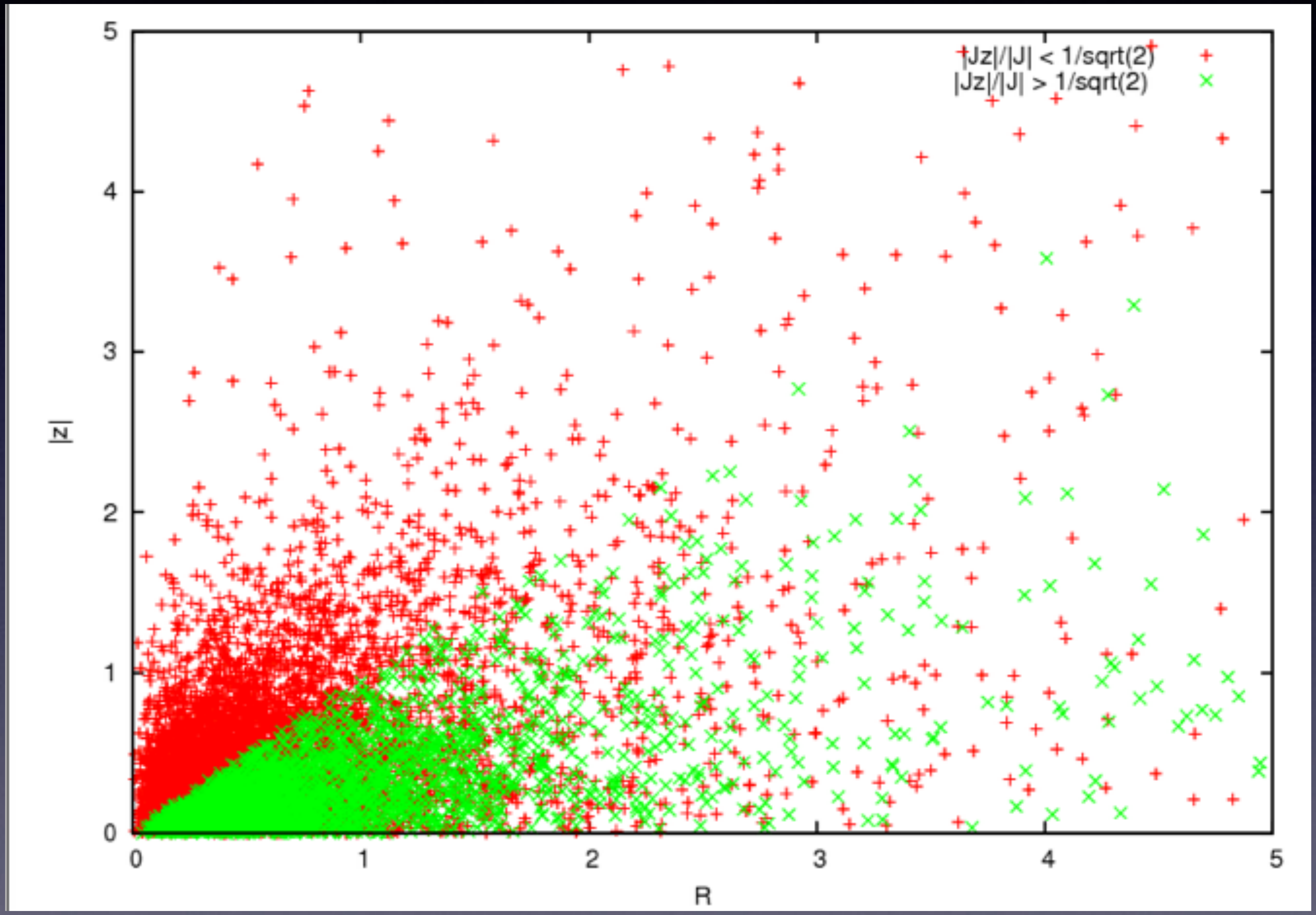
Other Models

- We can generalise this trick by considering the depend on E, J (total angular model)

- By allowing condition to depend on J_z and J, we can confine rotation to the centre of the system

    - $F(E,J_z) = F_p(E)\{ (1+x)H(G(E,J_z,J)) + (1-x)H(-G(E,J_z,J))\}$

Example

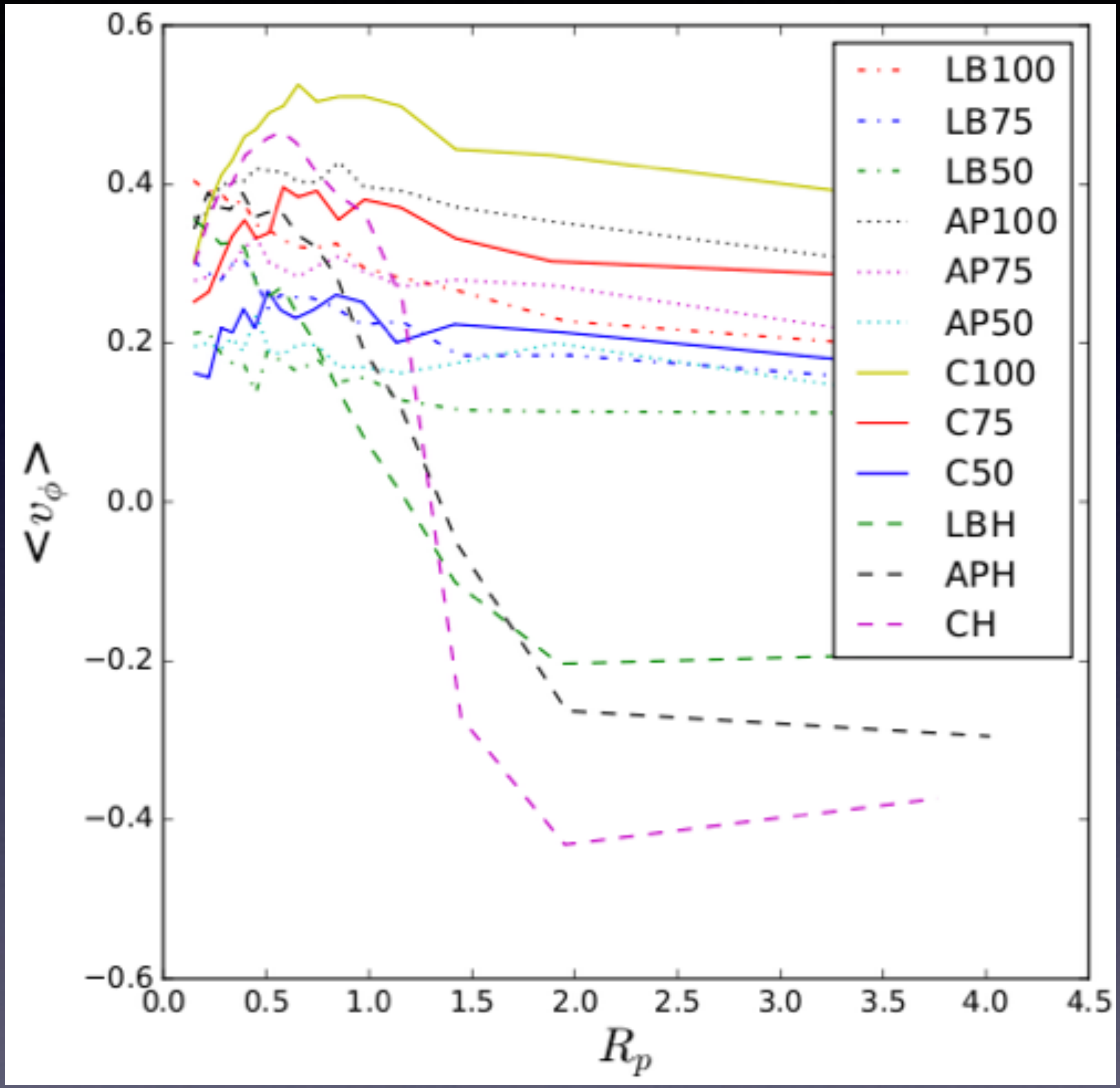


Condition $J_z/J > 1/\sqrt{2}$

Other Models II

- In a spherical symmetric potential J_x, J_y are also integrals of motion
- By using an appropriate condition we can change the axis of rotation for the most bounds stars (M15, Gebhardt+ 2000)
- In fact we reserve the direction of rotation of rotation for the most bound stars creating a model with no net angular moment

Rotation Profiles IV



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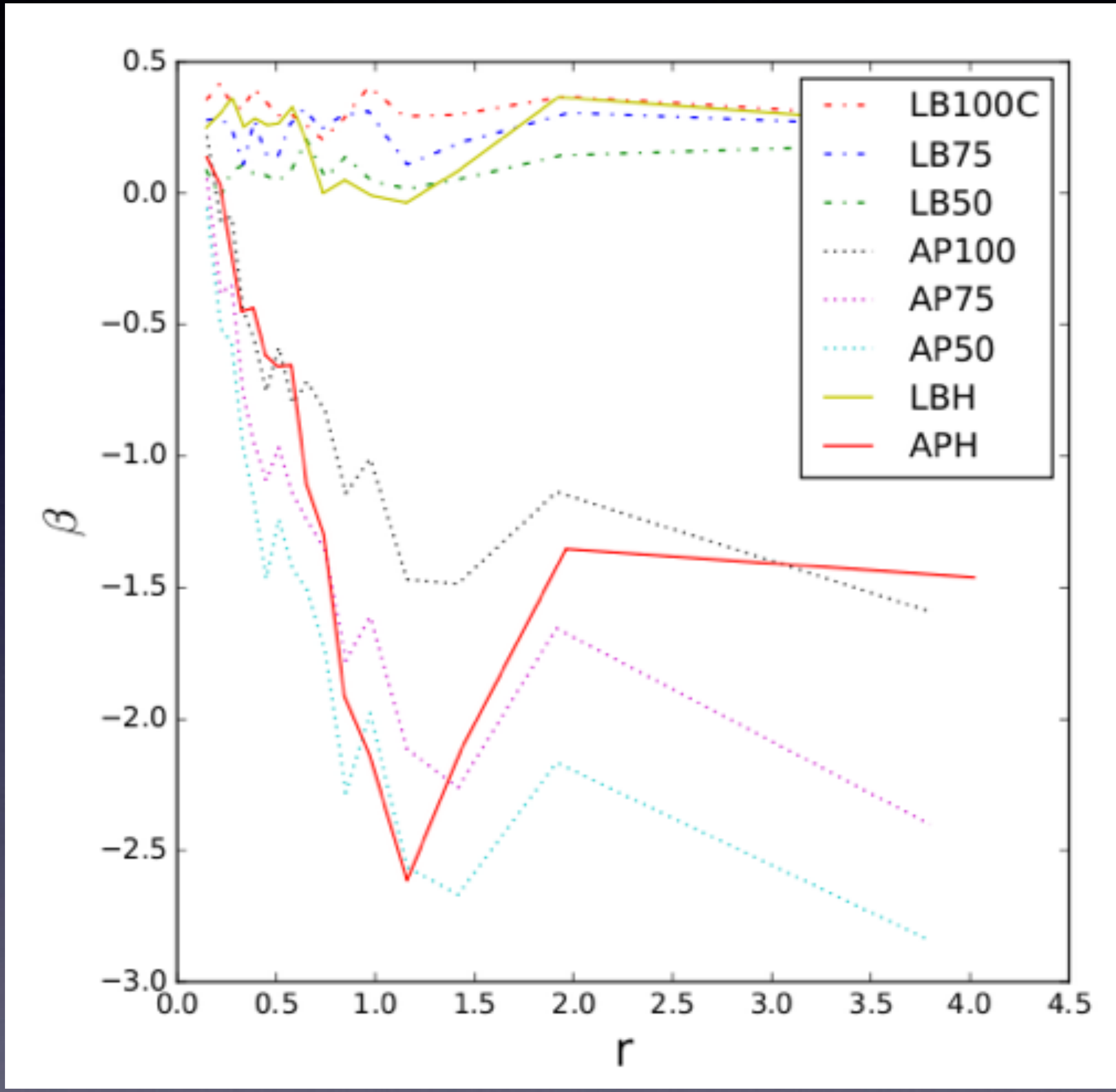
Research Goals

- Numerical and analytical analysis of stability properties of these families of rotating models
  - Attention to degree of differentiality
  - Fundamental understanding of the nature of dynamical instabilities in spheroidal stellar systems still lacking
- Investigation of role of angular momentum in the long-term dynamical evolution of collisional systems
  - Ideal framework to decouple structural and kinematical effects
  - Attention to collisional /less effects
- Exploration of the interplay with many other physical ingredients (IM/SMBH, tides, mass segregation)

Anisotropy Profiles



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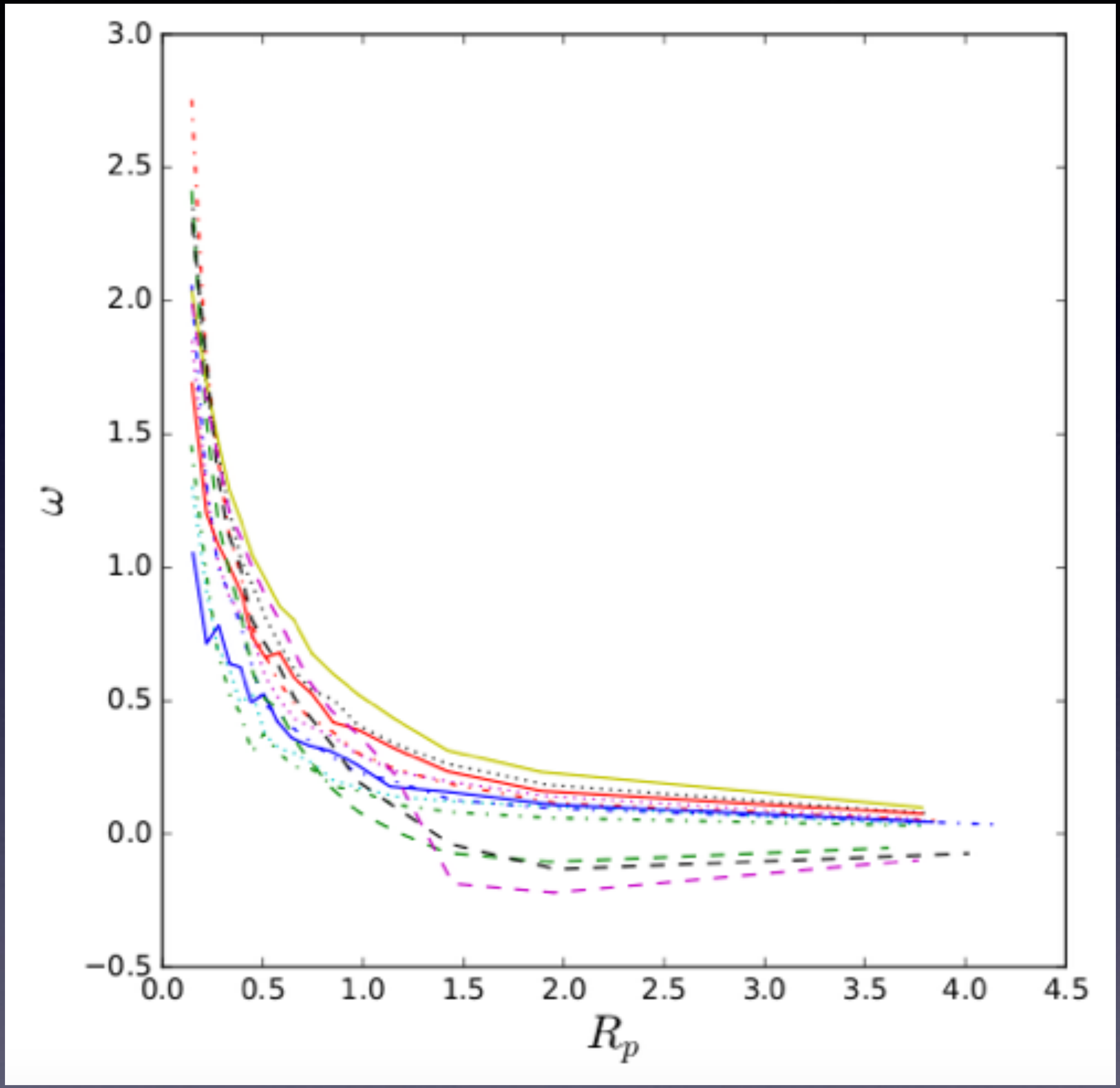
ω Profiles



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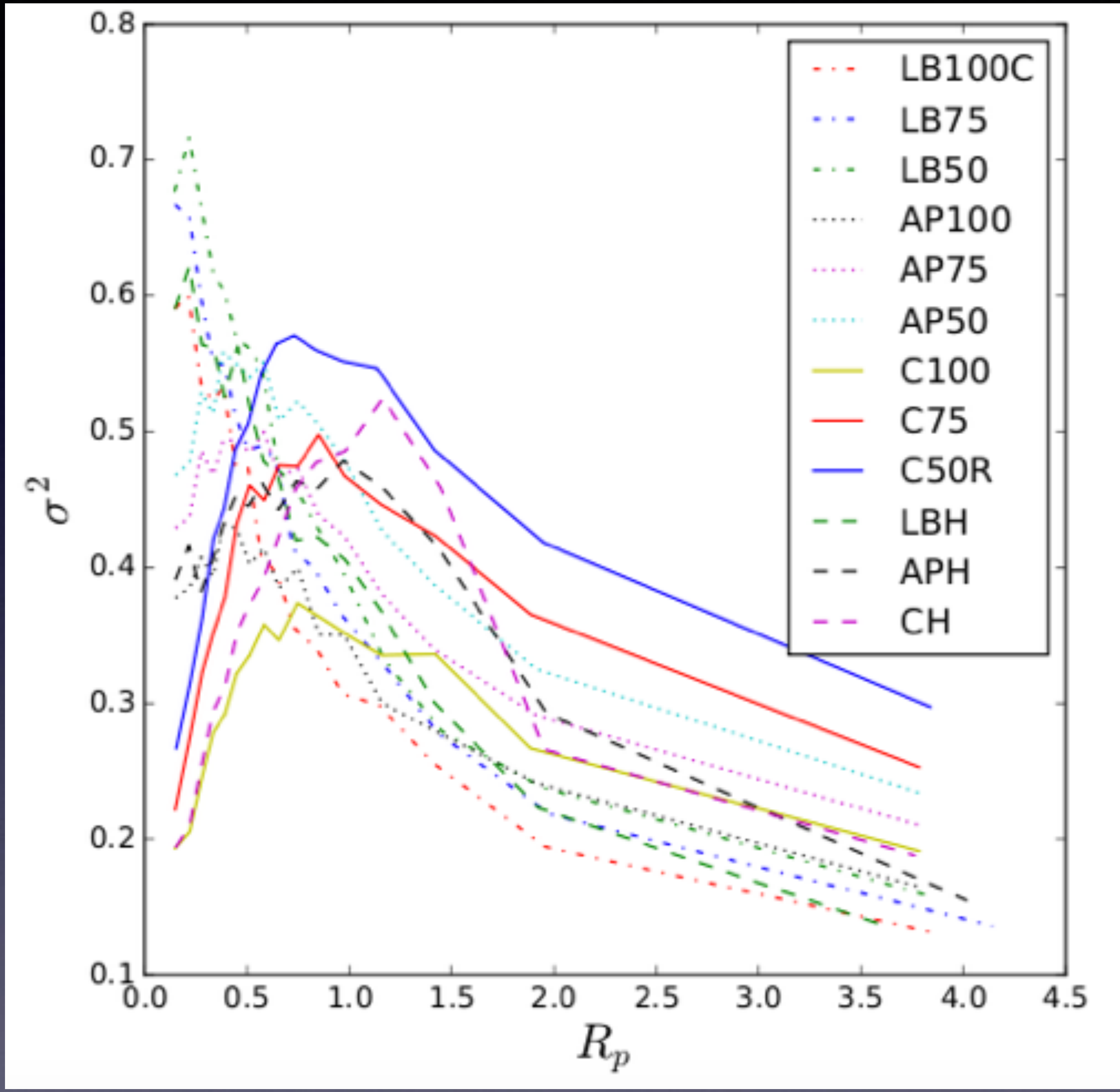
Velocity Dispersion Profiles



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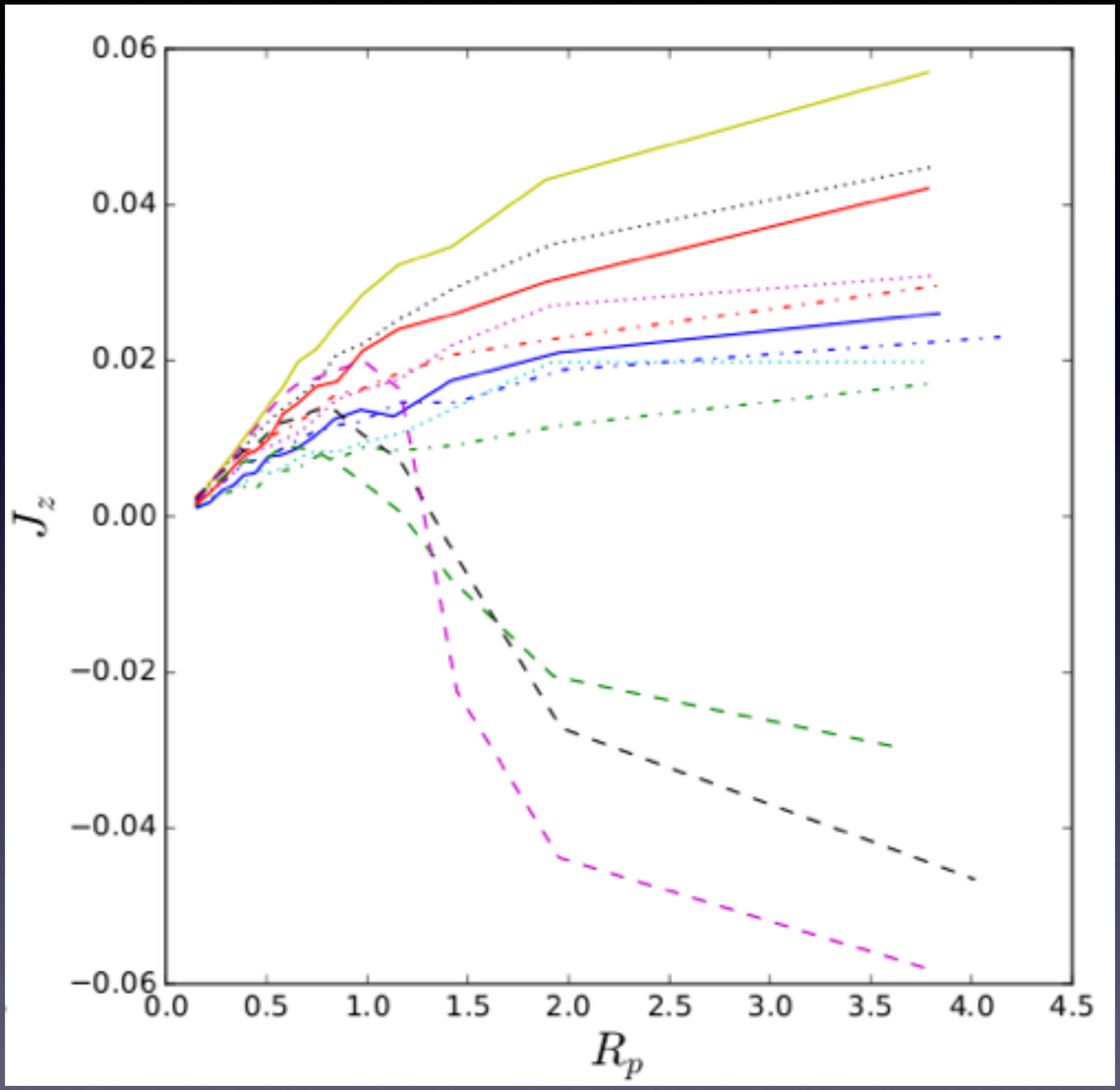
J_z Profiles



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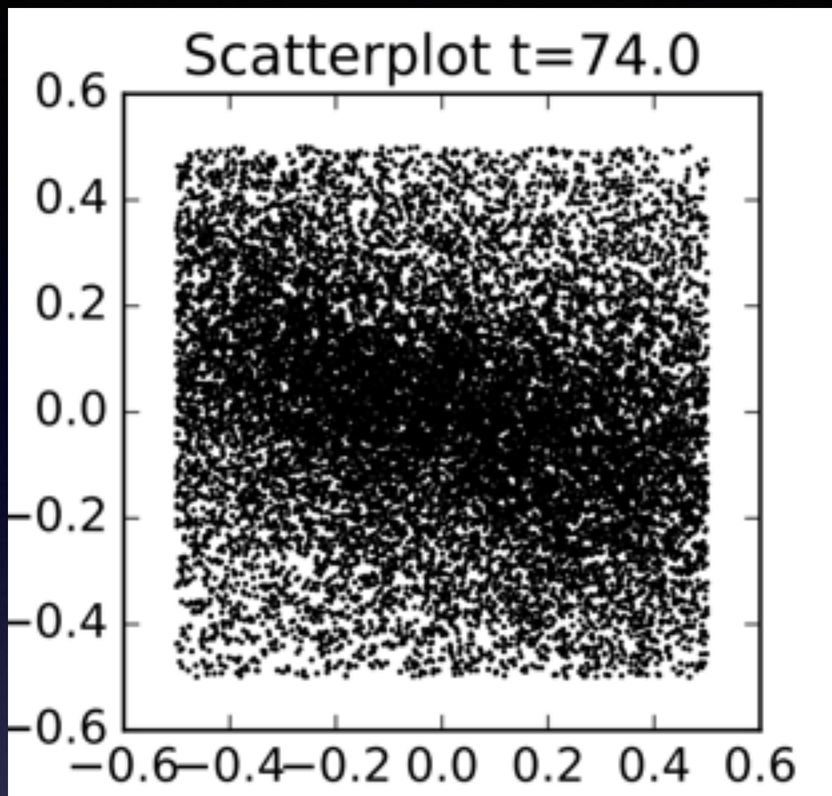
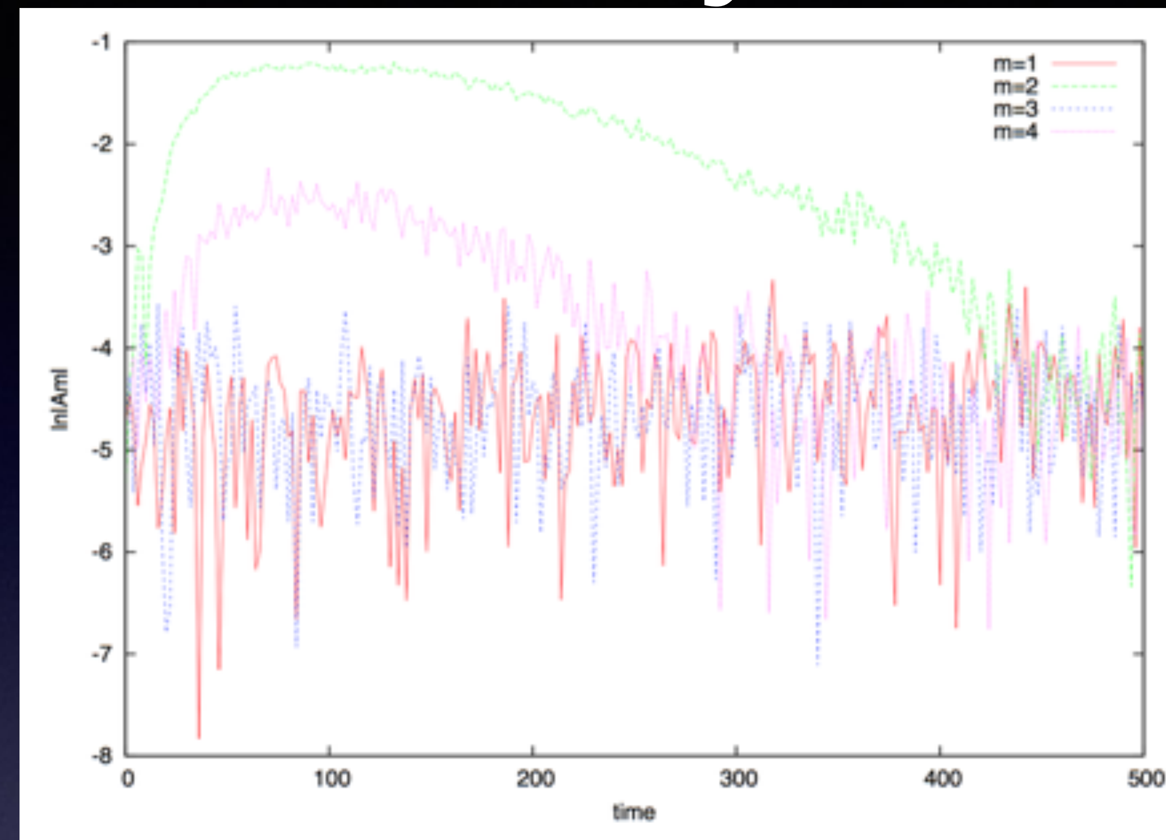
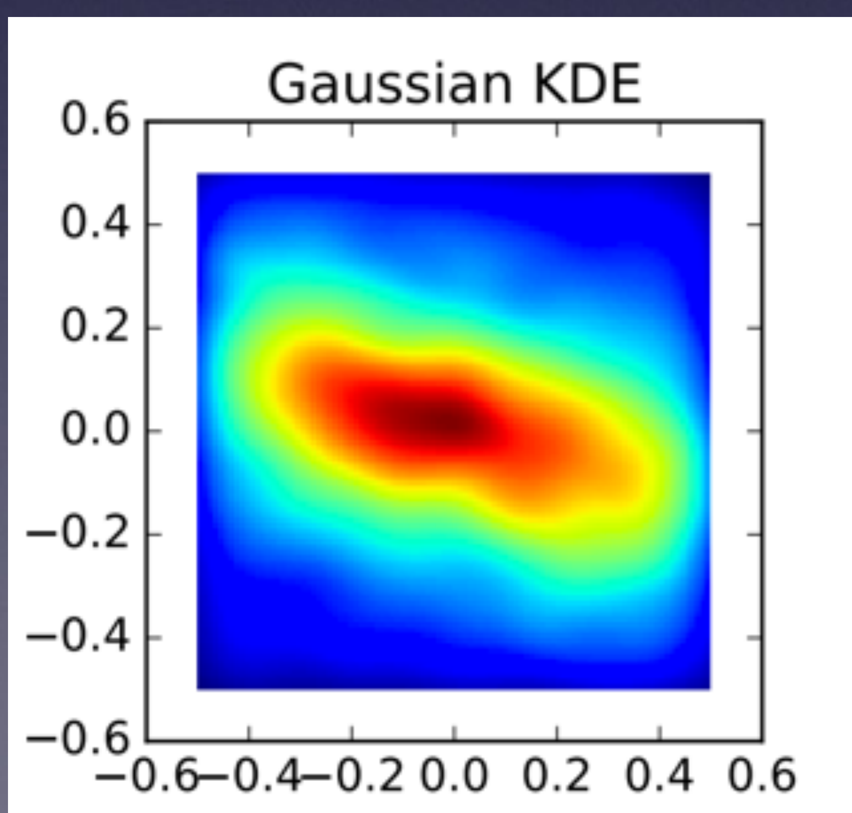
Bar instability

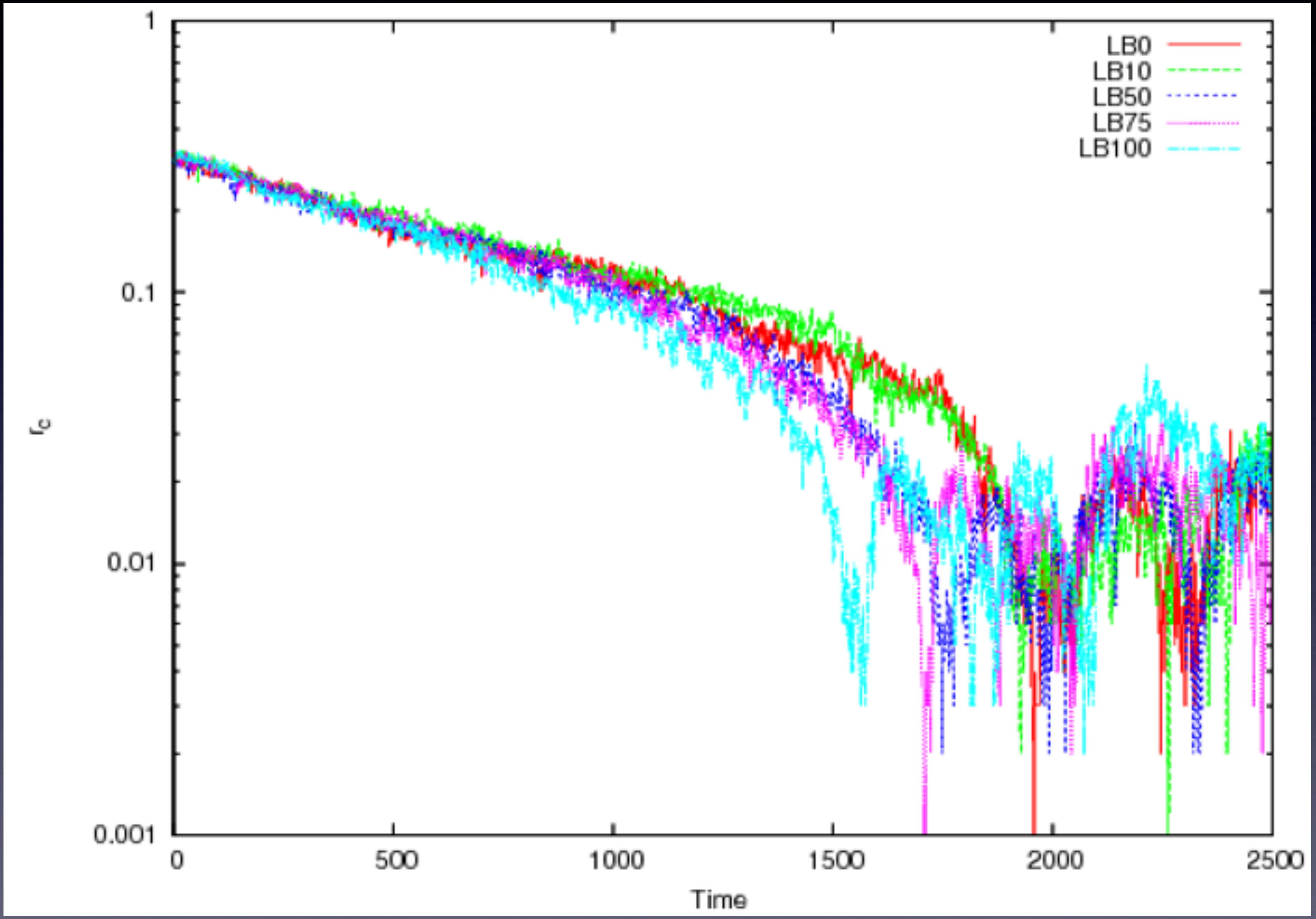






32k AP100 N-body realisation

Evolution of core radius



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