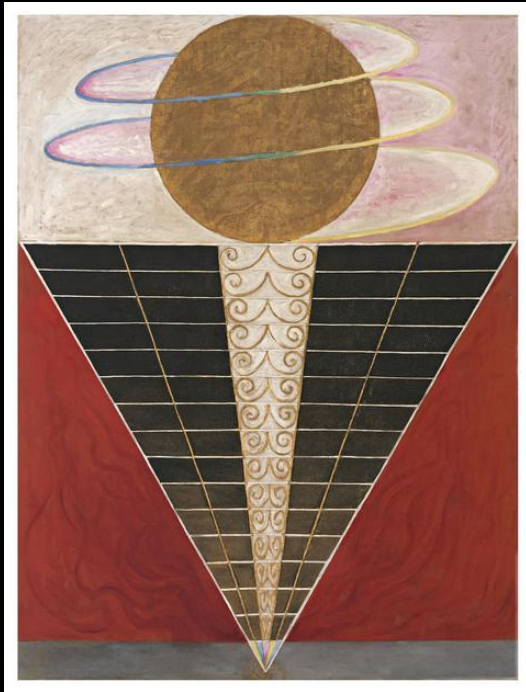




Hilma af Klint, Alterbilled No. 2, 1915
Moderna Museet, Stockholm

Anna Lisa Varri
University of Edinburgh

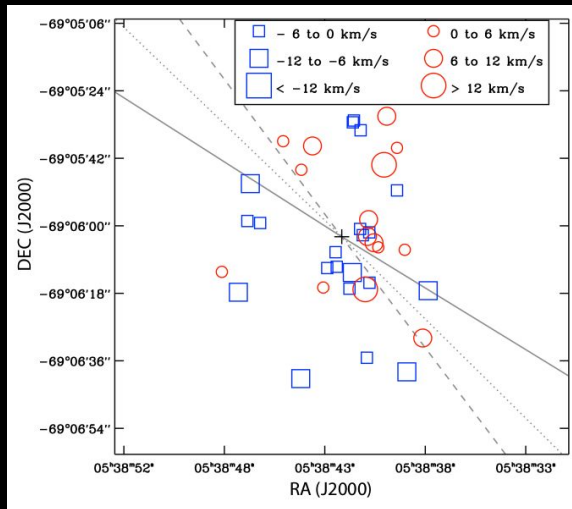
Marie Curie Fellow | Institute for Astronomy

Enrico Vesperini, **Maria Tiongco**, Catherine Gosmeyer (Indiana),
Steve McMillan (Drexel), Douglas Heggie (Edinburgh)

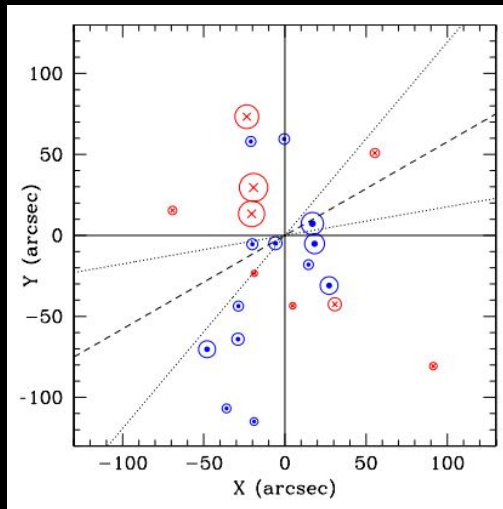


Dissipationless collapse with non-vanishing angular momentum

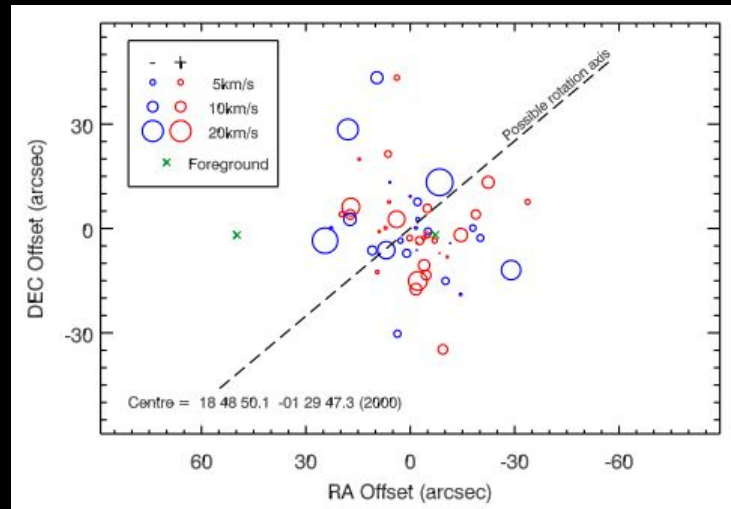
yet another possible Gaia Challenge playground



R136 (LMC; 2 Myr) | Hénault-Brunet et al. 2012



NGC 1846 (LMC; 1.5Gyr) | Mackey et al. 2013



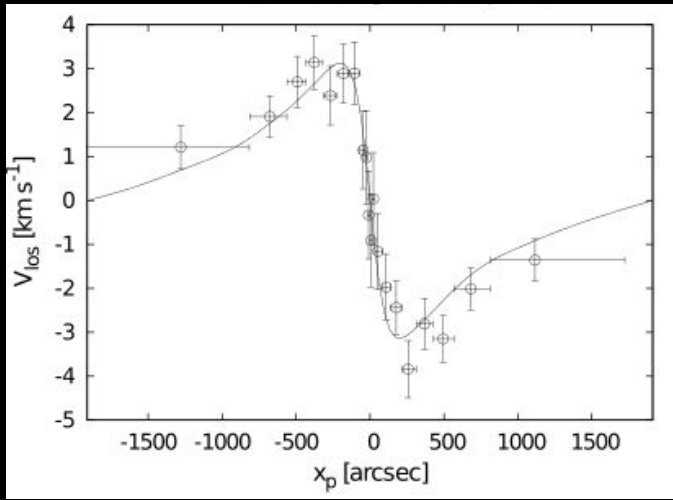
GLIMPSE-C01 (500 Myr) | Davies et al. 2011

Kinematical and morphological complexity in young and intermediate age star clusters

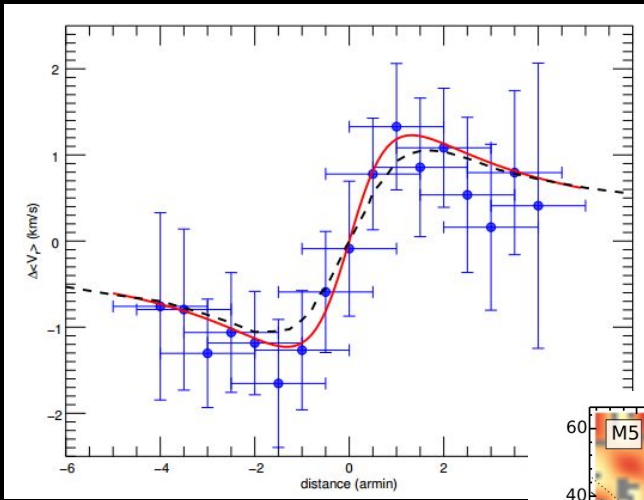
Challenging to measure, yet increasingly appreciated

Detailed dynamical study of
two other LMC clusters:
NGC 1866 (100 Myr)
NGC 1850 (50 Myr)

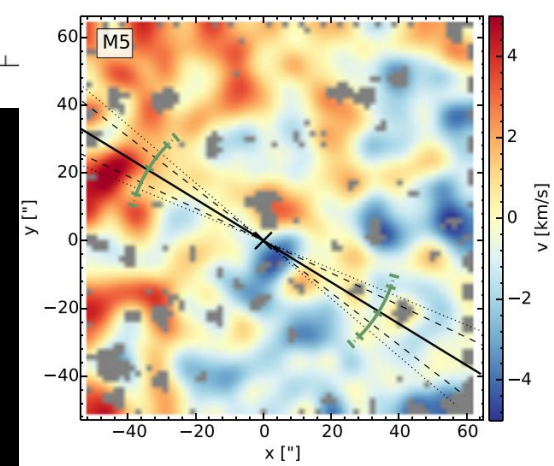
Fisher et al. 1992, 1993



47 Tuc | Bianchini, Varri, Bertin, Zocchi 2013



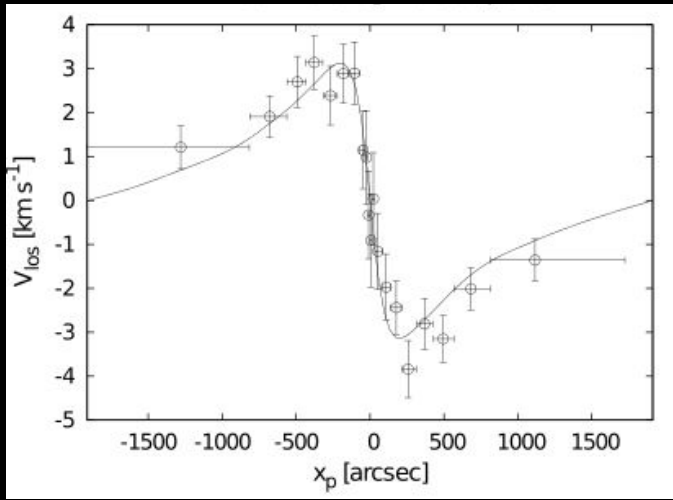
NGC 4372 | Kacharov, Bianchini et al. 2014



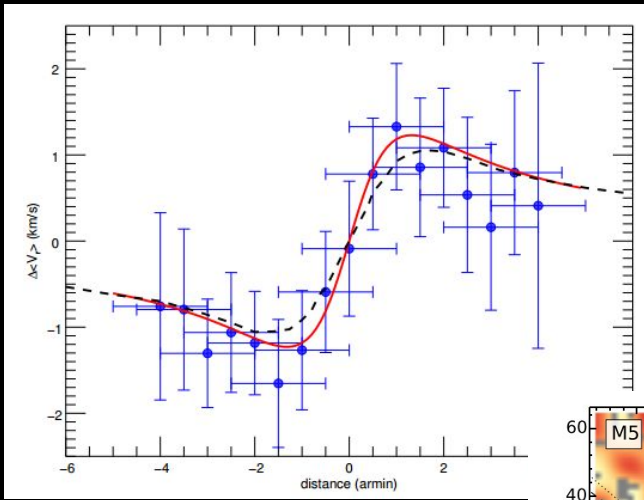
M5 | Fabricius et al. 2014

Internal rotation in old star clusters
should be interpreted as a lower limit

Transport and loss of angular momentum during dynamical evolution (Spurzem et al '90s)



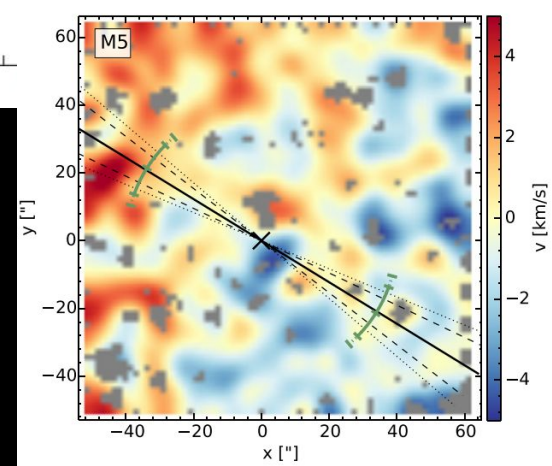
47 Tuc | Bianchini, Varri, Bertin, Zocchi 2013



NGC 4372 | Kacharov, Bianchini et al. 2014

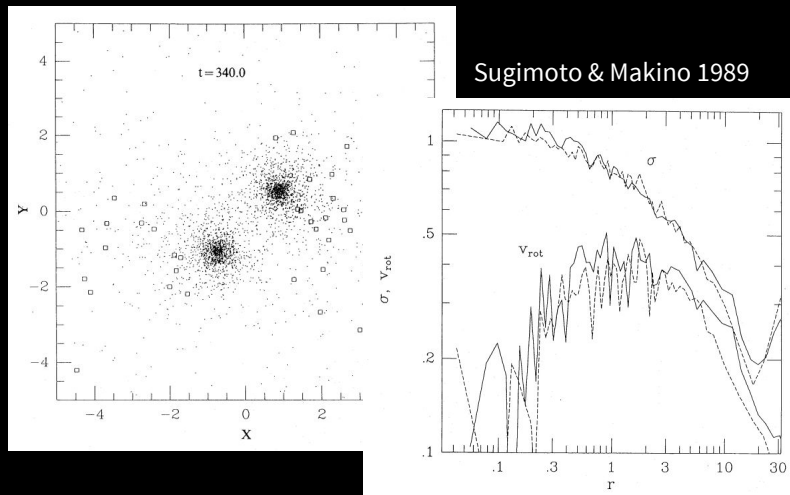
Synergy between ground-based spectroscopic surveys (Gaia-ESO) and HST + Gaia proper motions will be key

Internal rotation in old star clusters should be interpreted as a lower limit



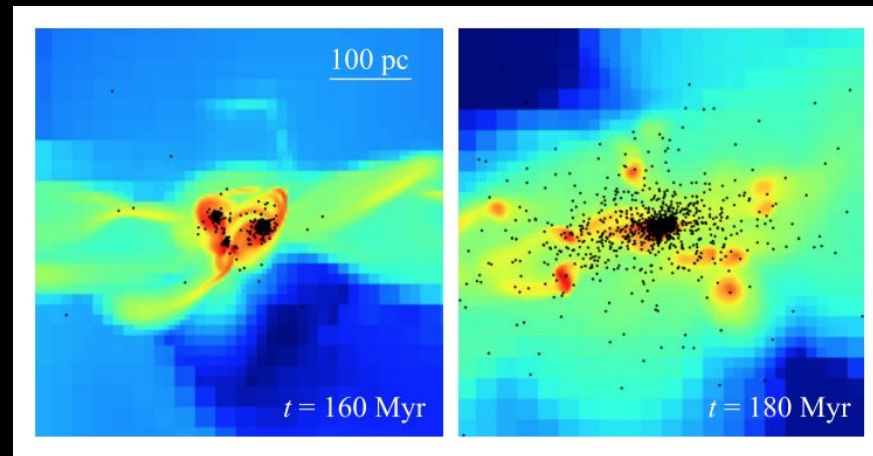
M5 | Fabricius et al. 2014

Transport and loss of angular momentum during dynamical evolution (Spurzem et al '90s)



Collision and merger of
small clouds or proto-clusters

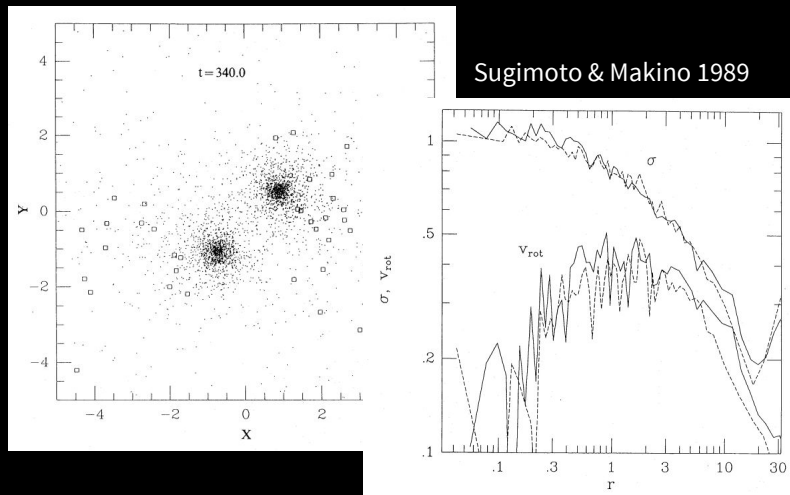
Akiyama & Sugimoto 1990, Makino, Okumura, Ebisuzaki & Makino 1991,
Fujimoto & Kumai 1997, Baumgardt et al. 2003 [G1], Gieles et al. in prep.



Collapse of a single cloud with
complex kinematics and
morphology, either in isolated or
merging galaxies

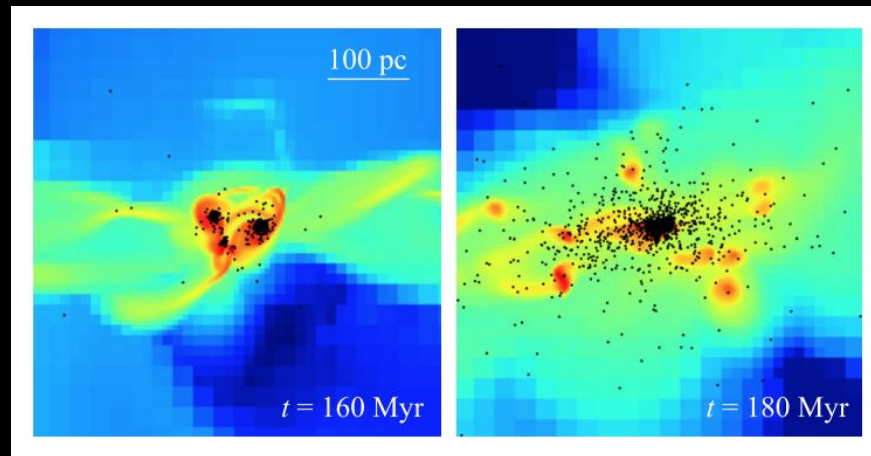
Bonnell et al. 2003, Fall & Rees 1985,
and many, many others

Formation* of rotating star clusters?



Collision and merger of
small clouds or proto-clusters

Akiyama & Sugimoto 1990, Makino, Okumura, Ebisuzaki & Makino 1991,
Fujimoto & Kumai 1997, Baumgardt et al. 2003 [G1], Gieles et al. in prep.

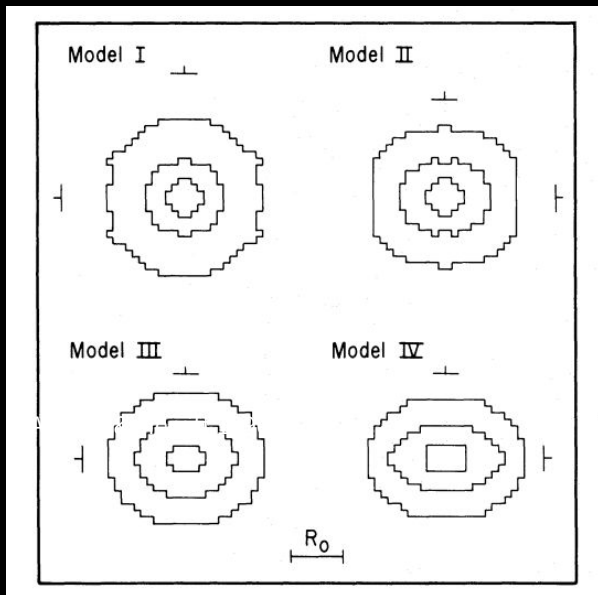


Collapse of a single cloud with
complex kinematics and
morphology, either in isolated or
merging galaxies

Bonnell et al. 2003, Fall & Rees 1985,
and many, many others

Formation* of rotating star clusters?

* Should actually read as “early dynamical evolution”; this talk is gas-free, sorry!



Gott 1973

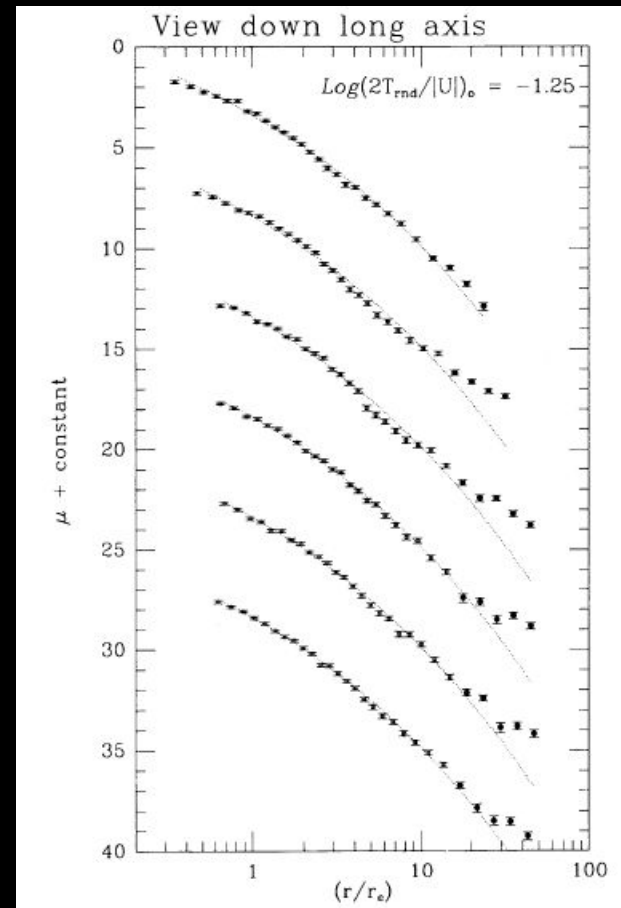
Explored as formation scenario for Ellipticals

Gott 1973, Hohl & Zang 1979,
Akiyama & Sugimoto 1989,
Aguilar & Merritt 1990

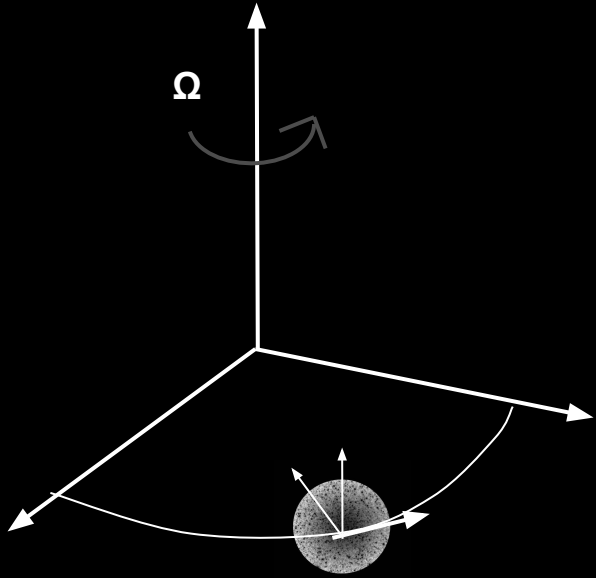
... what about
star clusters?

**Violent relaxation* with non-vanishing
total angular momentum**

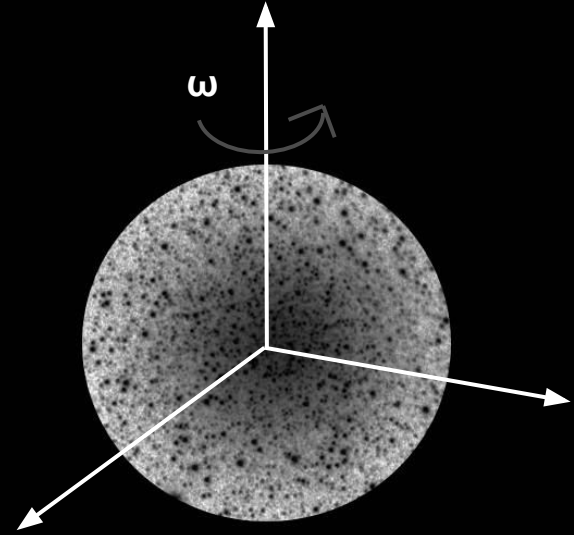
* i.e., “dissipationless collapse” - à la Lynden Bell 1967, van Albada 1982 ...



Aguilar & Merritt 1990



1. System in an external tidal field,
without initial internal rotation

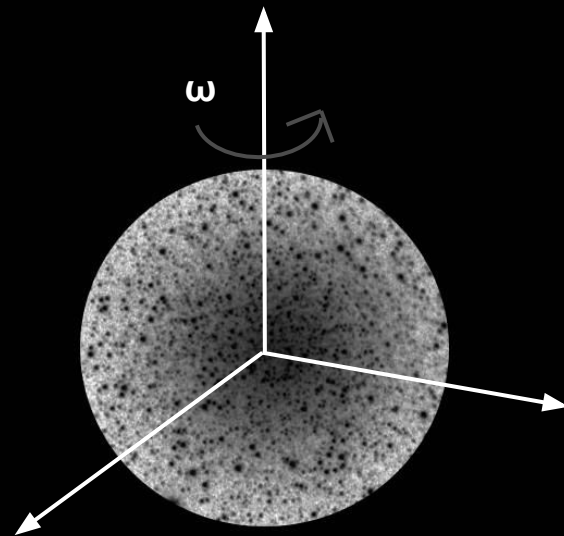


2. Isolated system, with
non-vanishing initial total
angular momentum

Generalization in two flavors

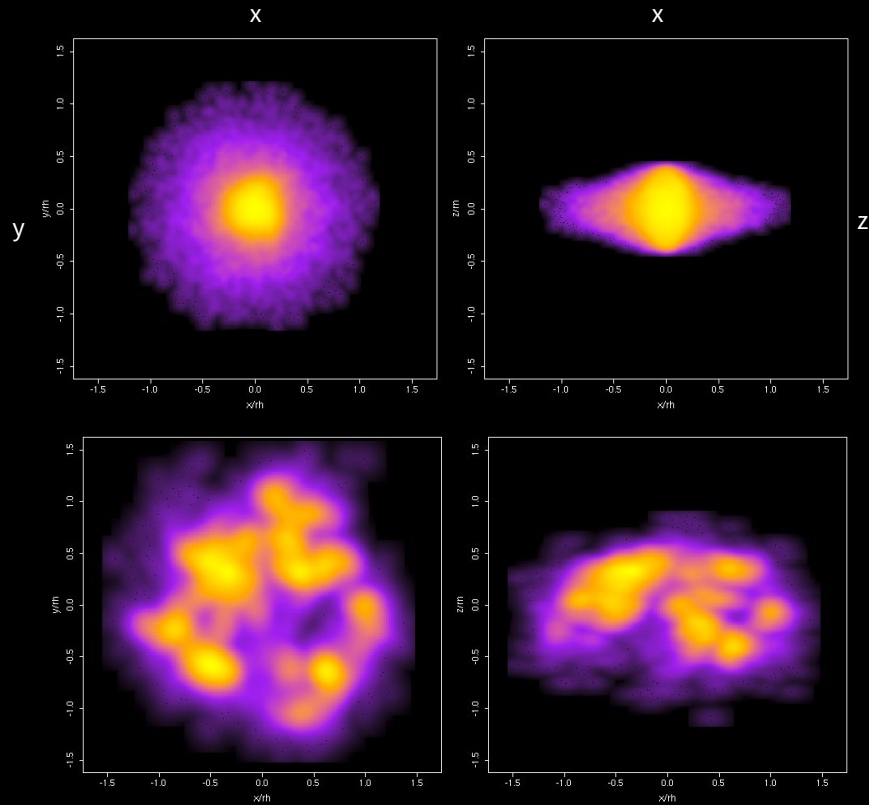


1. System in an external tidal field,
without initial internal rotation



2. Isolated system, with
non-vanishing initial total
angular momentum

Generalization in two flavors



Reference models

Homogeneous and fractal spheres
($D=3.0, 2.8, 2.4$)

Initial solid body rotation

$Q_{\text{ran}} = 2K_{\text{ran}}/|W| = 0.1, 0.25, 0.5, 0.75$

$Q_{\text{rot}} = 2K_{\text{rot}}/|W| = [0.0], 0.16, 0.33, 0.50$

$N=60000$, equal-mass particles
Starlab, survey of 32 models

Varri, Vesperini, Tionco et al., in preparation

**Violent relaxation of isolated systems
with non-vanishing total angular momentum**

Time [Hénon units]

Homogeneous, cold,
non-rotating

H1a: $D=3.0$, $Q_{\text{ran}}=0.1$, $Q_{\text{rot}}=0.0$

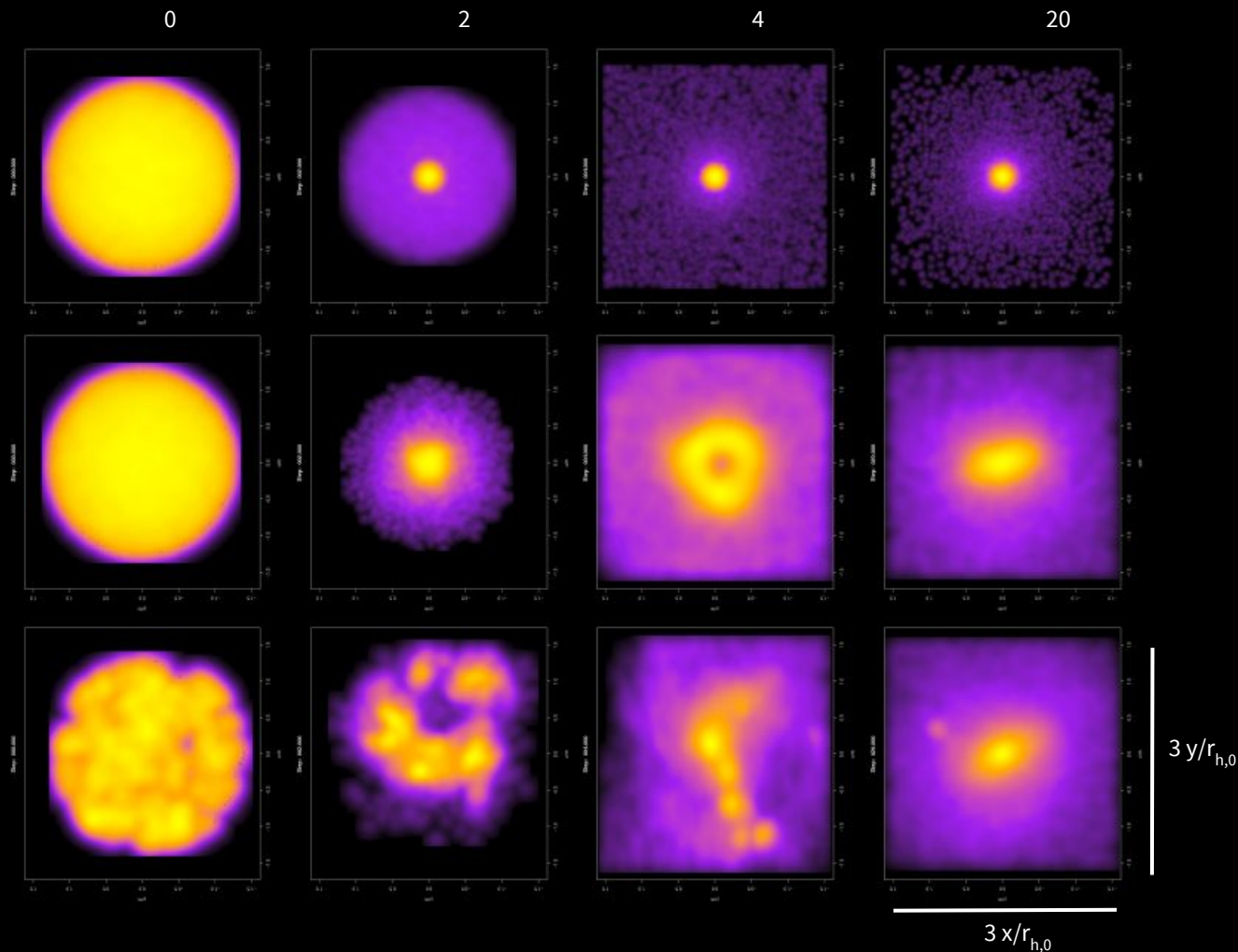
Homogeneous, cold,
rotating

H3a: $D=3.0$, $Q_{\text{ran}}=0.1$, $Q_{\text{rot}}=0.33$

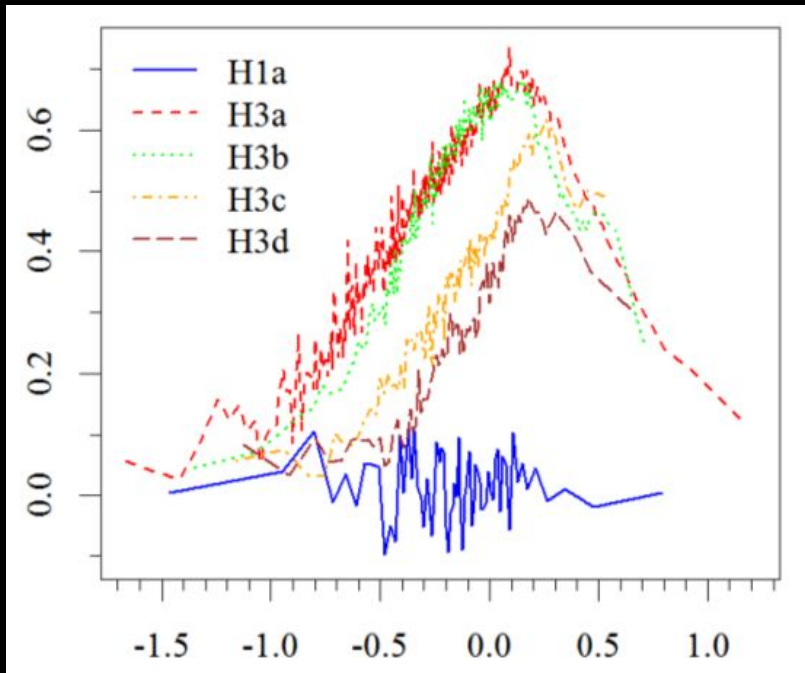
Clumpy, cold,
rotating

F24_3a : $D=2.4$, $Q_{\text{ran}}=0.1$, $Q_{\text{rot}}=0.33$

Varri, Vesperini, Tiongco et al., in
preparation

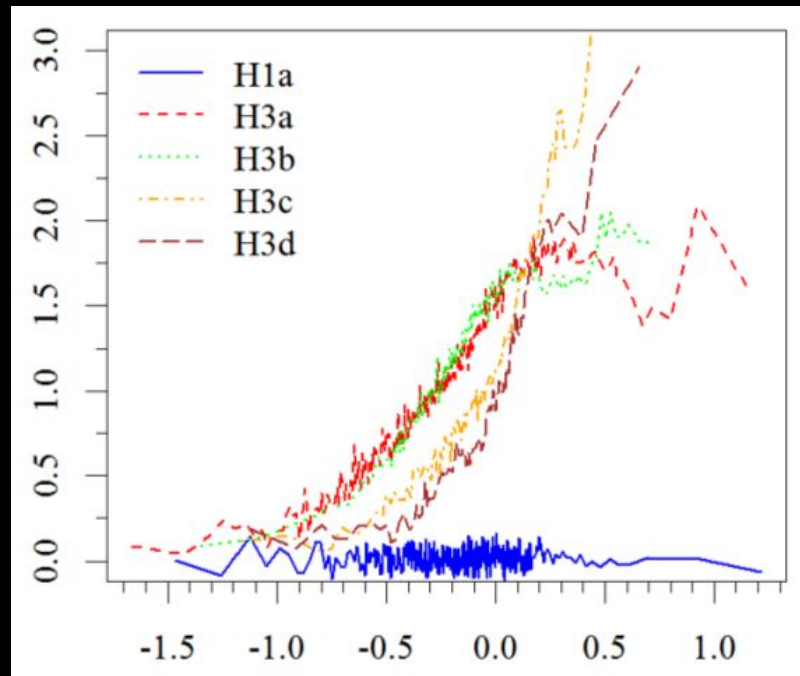


$\langle v_\phi \rangle$



$\log(r/r_h)$

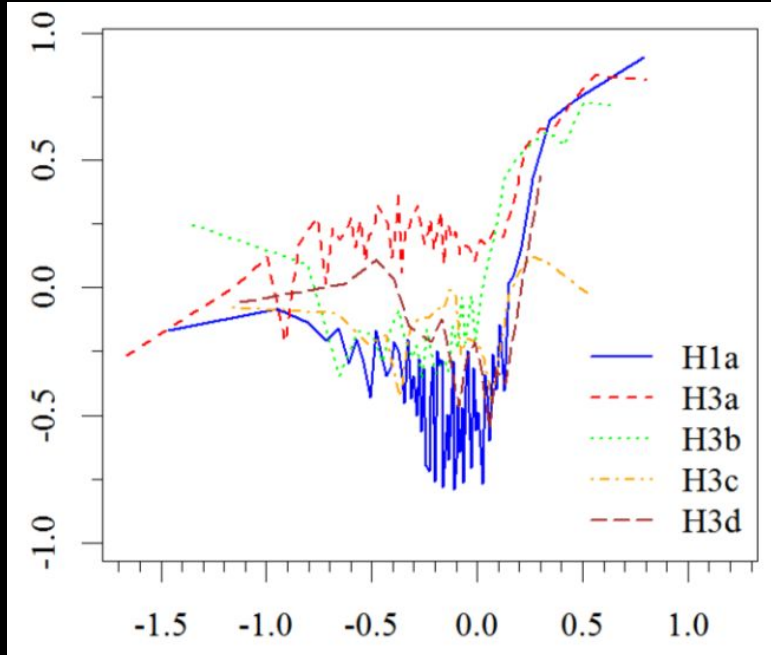
$\langle v_\phi \rangle / \sigma$



$\log(r/r_h)$

Rotation curves and velocity dispersion profiles

β



$\log(r/r_h)$

$$\beta = 1 - \frac{\sigma_{\theta}^2 + \sigma_{\phi}^2}{2\sigma_r^2}$$

Tangential anisotropy, for the cold homogeneous non-rotating case, noted - but not interpreted - also by Trenti, Bertin, van Albada 2005

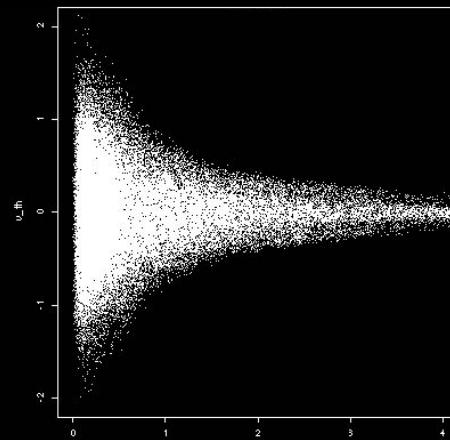
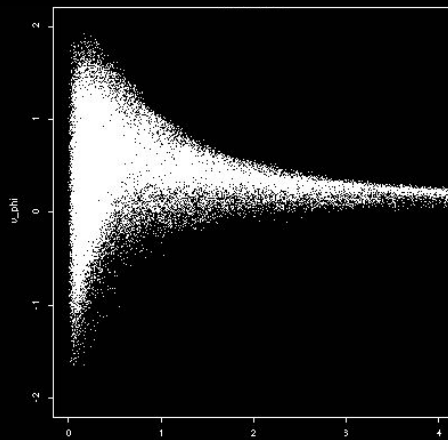
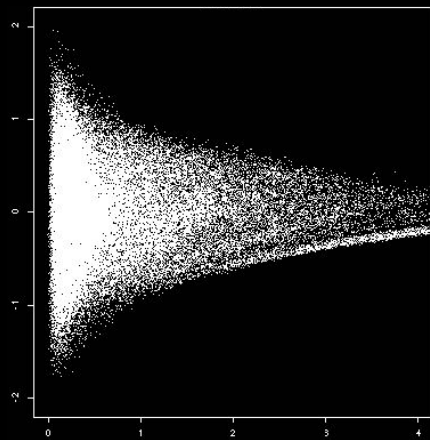
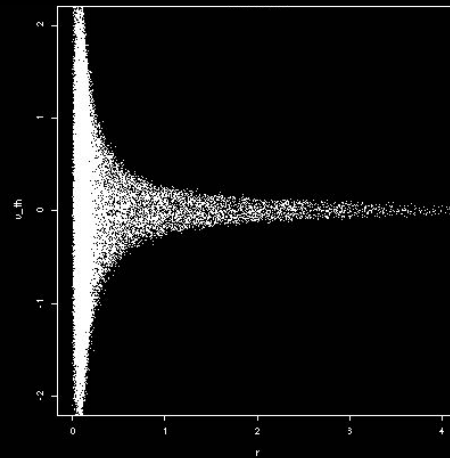
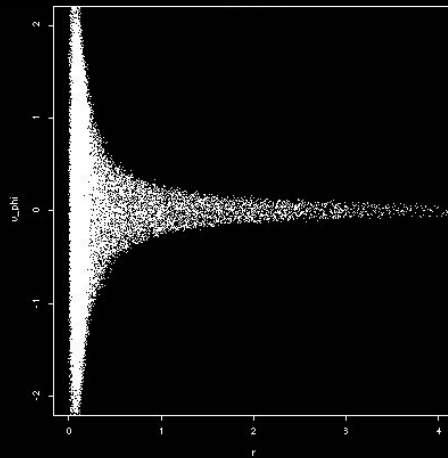
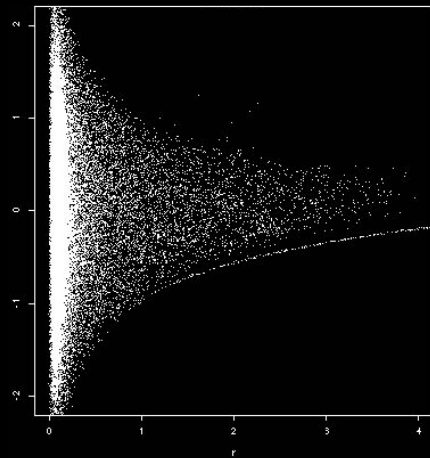
Anisotropy in the velocity space

Homogeneous, cold,
non-rotating

A glimpse of the phase space

Homogeneous, cold,
rotating

Varri, Vesperini, et al., in preparation



(r, v_r)

(r, v_ϕ)

(r, v_θ)

Consistency criteria
for spherical,
anisotropic systems

A CUSP SLOPE–CENTRAL ANISOTROPY THEOREM

JIN H. AN^{1,2} AND N. WYN EVANS¹

Received 2005 September 29; accepted 2006 January 6

ABSTRACT

For a wide class of self-gravitating systems, we show that if the density is cusped like $r^{-\gamma}$ near the center, then the limiting value of the anisotropy parameter $\beta = 1 - \langle v_r^2 \rangle / (2 \langle v_t^2 \rangle)$ at the center cannot be greater than $\gamma/2$. Here $\langle v_r^2 \rangle$ and $\langle v_t^2 \rangle$ are the radial and tangential velocity second moments. This follows from the nonnegativity of the phase-space density. We compare this theorem to other proposed relations between the cusp slope and the central anisotropy to clarify their applicabilities and underlying assumptions. The extension of this theorem to tracer populations in an externally imposed potential is also derived. In particular, for stars moving in the vicinity of a central black hole, this reduces to $\gamma \geq \beta + \frac{1}{2}$, indicating that an isotropic system in Keplerian potential should be cusped at least as steep as $r^{-1/2}$. Similar limits have been noticed before for specific forms of the distribution function, but here we establish this as a general result.

Subject headings: galaxies: kinematics and dynamics — methods: analytical — stellar dynamics

Consistency criteria for spherical, anisotropic systems

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Mon. Not. R. Astron. Soc. **408**, 1070–1074 (2010)

doi:10.1111/j.1365-2966.2010.17184.x

How general is the global density slope–anisotropy inequality?

Luca Ciotti[★] and Lucia Morganti[†]

Astronomy Department, University of Bologna, via Ranzani 1, 40127 Bologna, Italy

Accepted 2010 June 11; Received 2010 June 9; in original form 2010 April 23

ABSTRACT

Following the seminal result of An & Evans, known as the central density slope–anisotropy theorem, successive investigations unexpectedly revealed that the density slope–anisotropy inequality holds not only at the centre, but at all radii in a very large class of spherical systems whenever the phase-space distribution function is positive. In this paper we derive a criterion that holds for all spherical systems in which the augmented density is a separable function of radius and potential: this new finding allows us to unify all the previous results in a very elegant way, and opens the way for more general investigations. As a first application, we prove that the global density slope–anisotropy inequality is also satisfied by all the explored additional families of multicomponent stellar systems. The present results, and the absence of known counterexamples, lead us to conjecture that the global density slope–anisotropy inequality could actually be a universal property of spherical systems with positive distribution function.

Key words: celestial mechanics – galaxies: elliptical and lenticular, cD – galaxies: kinematics and dynamics.

Consistency criteria
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doi:10.1088/0004-637X/726/2/80

ON THE UNIVERSALITY OF THE GLOBAL DENSITY SLOPE–ANISOTROPY INEQUALITY

EMMANUEL VAN HESE, MAARTEN BAES, AND HERWIG DEJONGHE
Sterrenkundig Observatorium, Universiteit Gent, Krijgslaan 281 S9, B-9000 Gent, Belgium; emmanuel.vanhese@gmail.com,
maarten.baes@ugent.be, herwig.dejonghe@ugent.be

Received 2010 October 4; accepted 2010 October 20; published 2010 December 17

ABSTRACT

Recently, some intriguing results have led to speculations whether the central density slope–velocity dispersion anisotropy inequality (An & Evans) actually holds at all radii for spherical dynamical systems. We extend these studies by providing a complete analysis of the global slope–anisotropy inequality for all spherical systems in which the augmented density is a separable function of radius and potential. We prove that these systems indeed satisfy the global inequality if their central anisotropy is $\beta_0 \leq 1/2$. Furthermore, we present several systems with $\beta_0 > 1/2$ for which the inequality does not hold, thus demonstrating that the global density slope–anisotropy inequality is not a universal property. This analysis is a significant step toward an understanding of the relation for general spherical systems.

Key words: dark matter – Galaxy: kinematics and dynamics – methods: analytical

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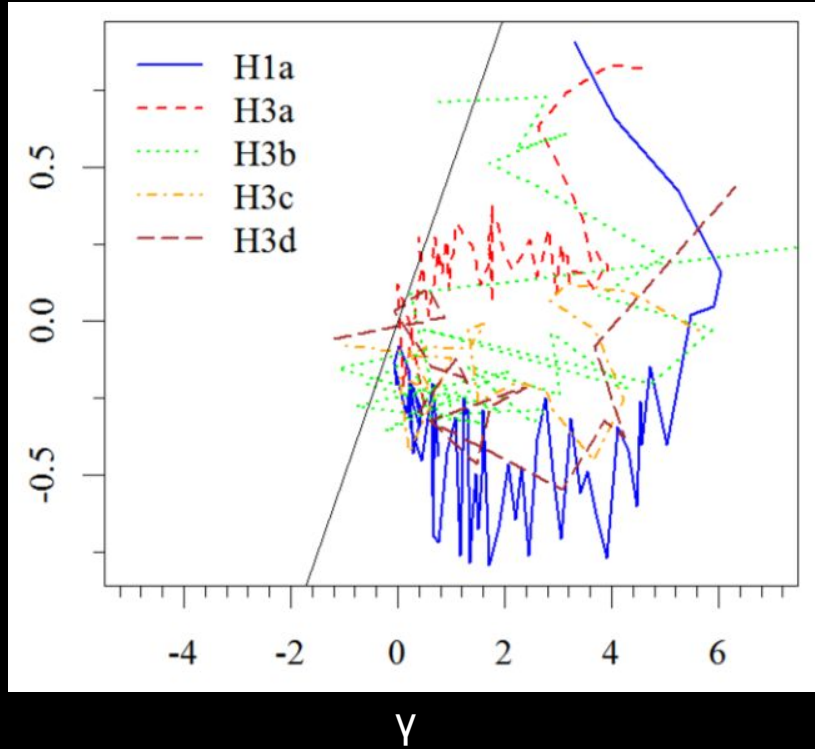
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... what about axisymmetric,
rotating systems?

β



$$\gamma(r) = -\frac{d \ln \rho}{d \ln r}(r)$$

$$\beta = 1 - \frac{\sigma_{\theta}^2 + \sigma_{\phi}^2}{2\sigma_r^2}$$

$$\gamma(r) \geq 2\beta(r)$$

Density slope - anisotropy plane

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Consistency criteria for spherical, anisotropic systems

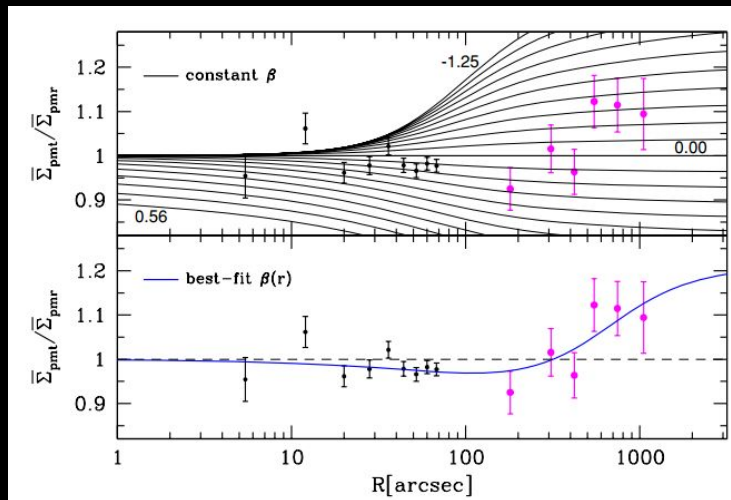
... what about axisymmetric,
rotating systems?

“...well, this smells like a Theorem
that someone should
really try to prove”

Douglas Hoggie
Blackford Hill, Edinburgh

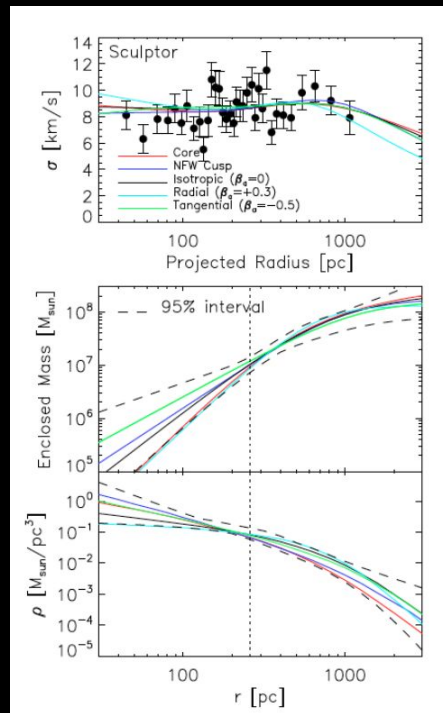


New chapter of our
fundamental understanding
of stellar dynamics



ω Cen | Van der Marel et al. 2010

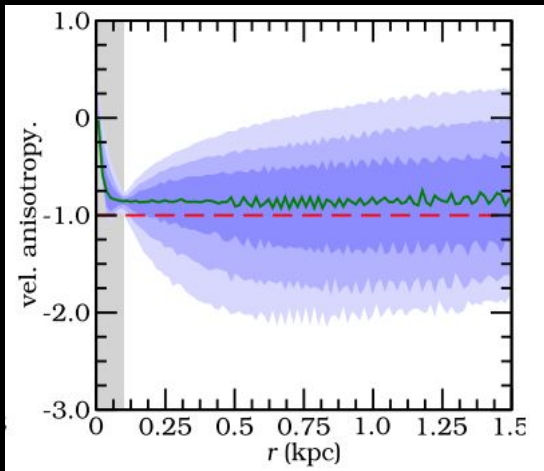
Star Clusters:
IMBH or Anisotropy?



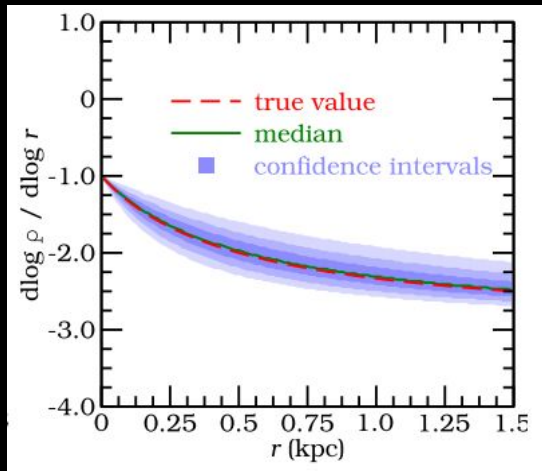
Sculptor (1 pop) | Walker 2012

Dwarf galaxies:
Core or Cusp?

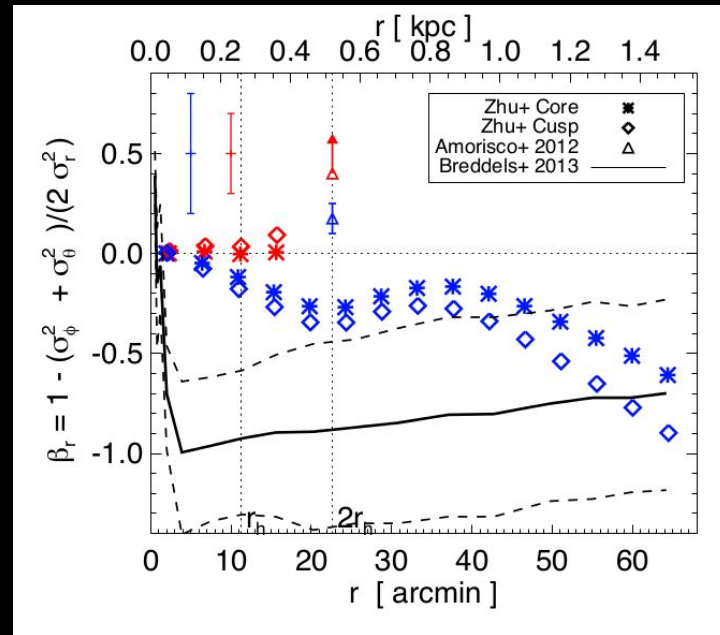
Why would it be important?



flat tangentially anisotropic ellipsoid should hold clues to the formation and dynamical evolution of Sculptor but it is as yet unclear whether a model exists that can reproduce this trend.



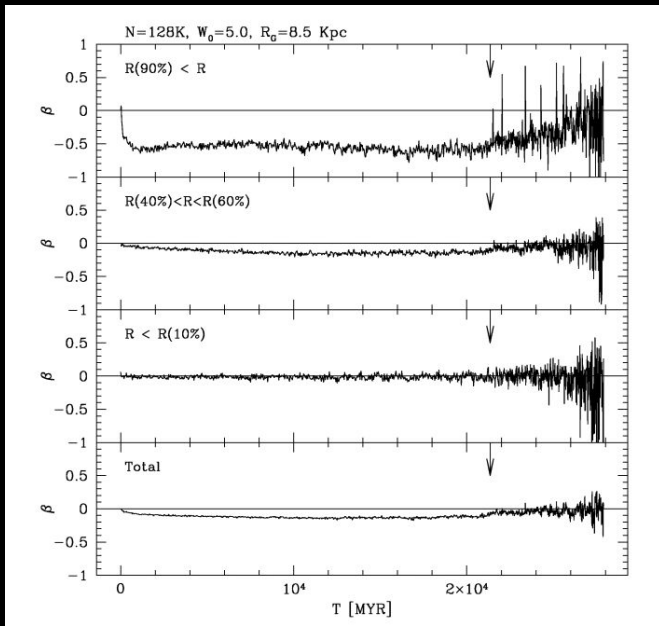
Sculptor (1 pop) | Breddels et al. 2013
Lokas et al. 2009, Battaglia et al. 2008,
Walker et al. 2009



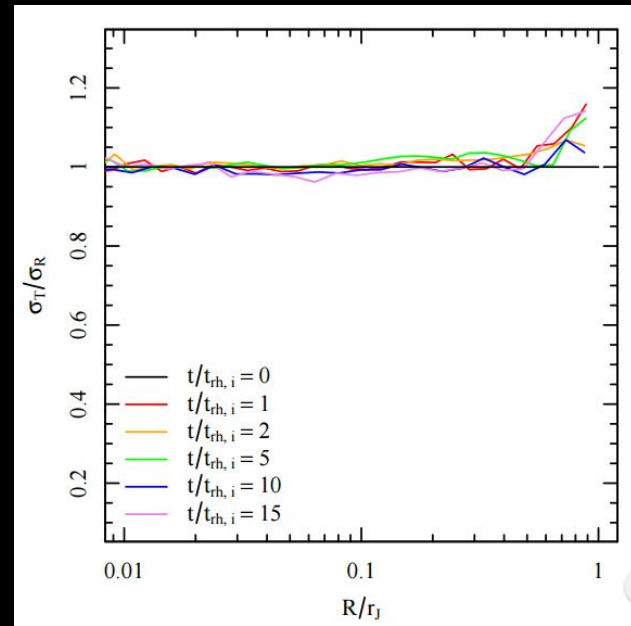
Sculptor (2 pop) | Zhu, van de Ven, Watkins, Posti 2016

Physical origin of tangential anisotropy?

1. Signature imprinted by formation process



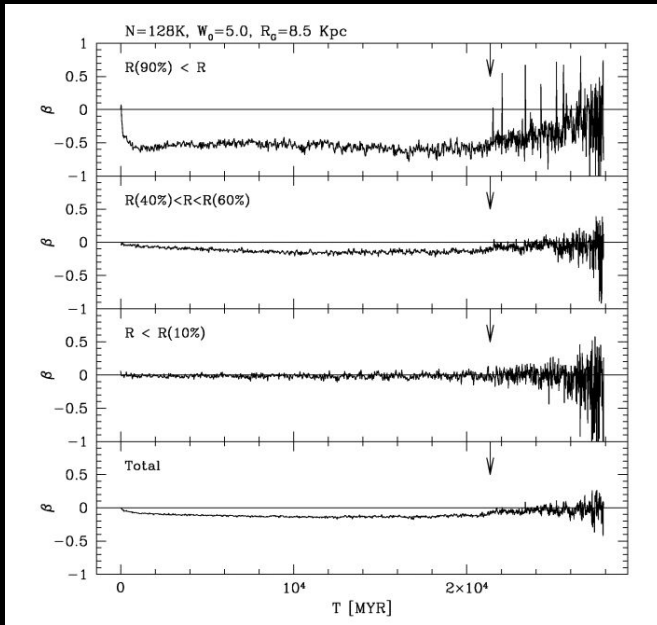
N-body model on circular orbit | Baumgardt & Makino 2003
Takahashi & Lee 2000, Hurley & Shara 2012



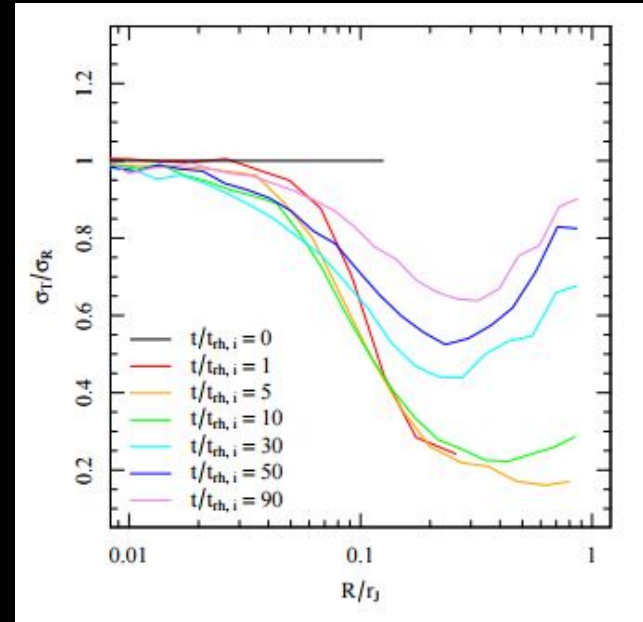
Strong dependence on filling factor | Tiongco, Vesperini, Varri 2016,
Sollima et al. 2015
Importance of potential escapers | Claydon, Gieles, Zocchi sub,
Daniel, Heggie, Varri sub.

Physical origin of tangential anisotropy?

2. Product of dynamical evolution in tidal field



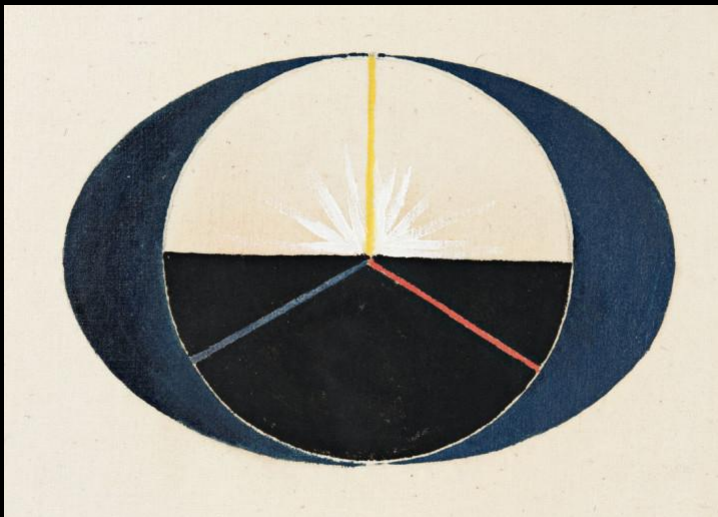
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Physical origin of tangential anisotropy?

2. Product of dynamical evolution in tidal field



Hilma af Klint, Series I No. 7, 1919, Moderna Museet, Stockholm

Parting thoughts

Some mocks available on the GC wiki

alvarri.com
annalisa.varri@gmail.com
@parallasseh

Ever-increasing observational evidence of kinematic complexity in star clusters, young and old.

Very much looking forward to the synergy Gaia + HST + ground-based spectroscopic surveys, for selected GGCs.

Non-vanishing total angular momentum in the violent relaxation scenario may leave unique fingerprints in phase space.

Varri, Vesperini et al., in preparation
Vesperini, Varri, McMillan, Zepf, MNRAS Letters, 2014

Star clusters are always my favourite excuse to study new chapters of stellar dynamics of rotating systems.

More on the construction of rotating equilibria, wait for Alice Zocchi and Phil Breen's talks later this week!