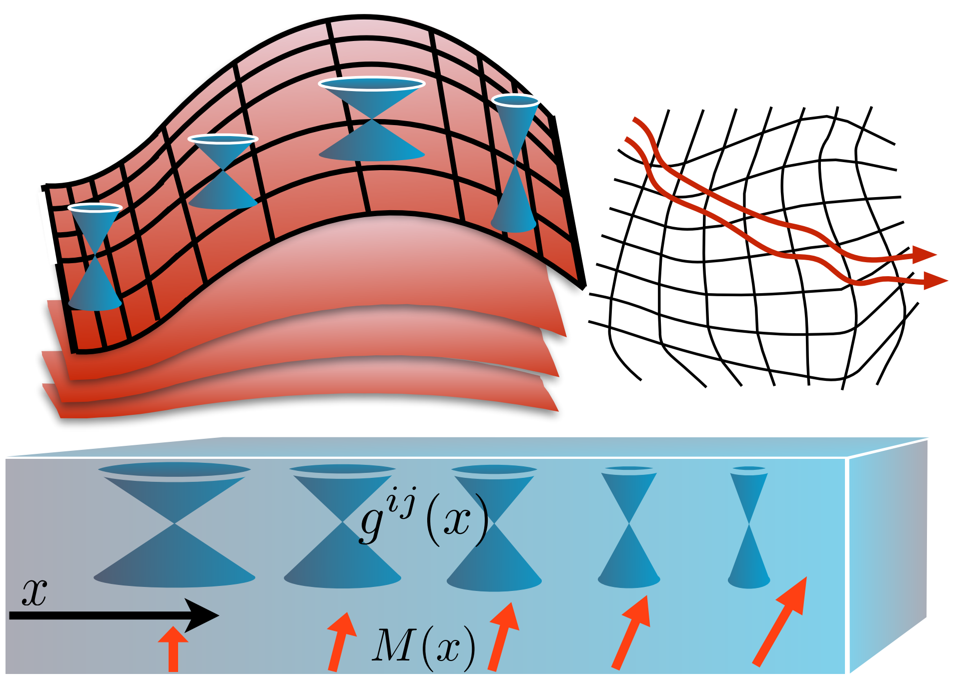
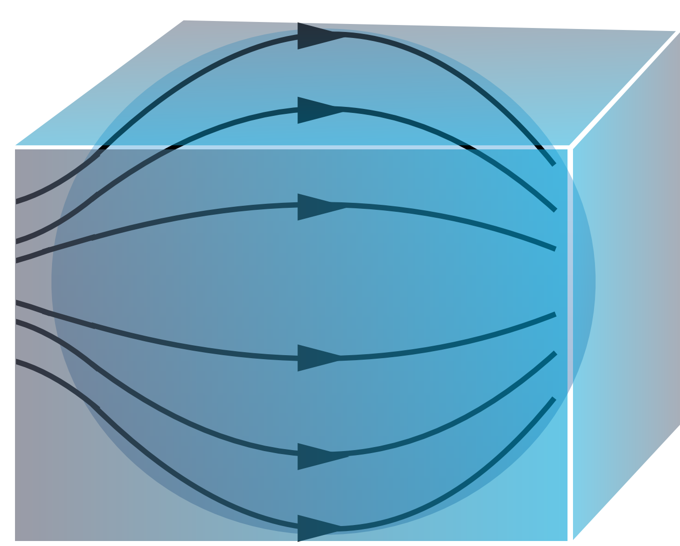
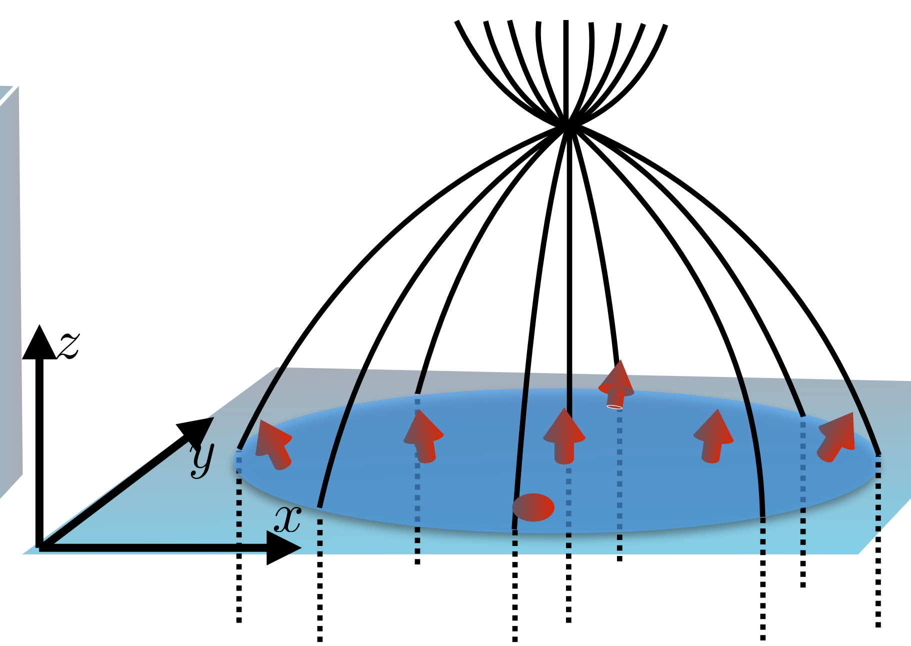
Designer curved-space geometry in Weyl metamaterials

Teemu Ojanen
Aalto University, Finland



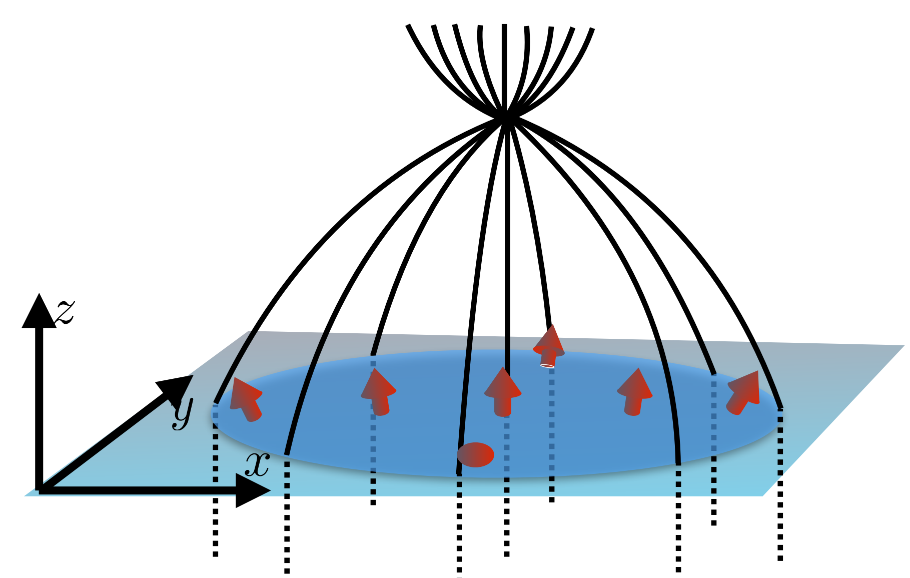




Alex Weström and T. O., arXiv:1703.10408

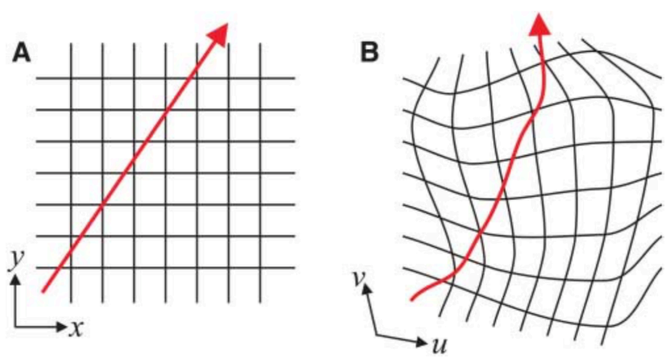
Outline

- Electromagnetic metamaterials: engineering geometry through local control
- Designer curved-space geometry for electrons: why Weyl should be good
- Engineering geometry (frame fields, metric etc) and gauge fields through general TR and I breaking
- Semiclassical dynamics in Weyl metamaterials: geodesics plus Berry forces
- Application: 3D Weyl electron lens

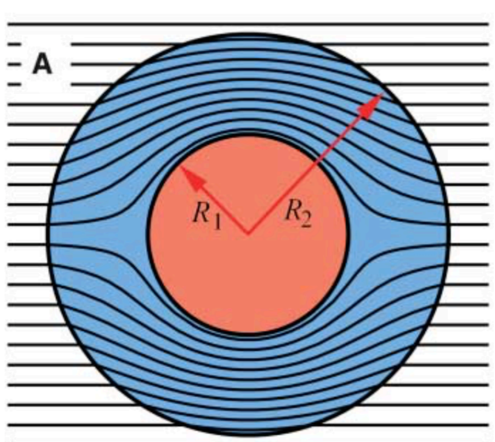


EM metamaterials

- By controlling permittivity and permeability locally, one can tailor EM propagation
- Wide tuneability of constitutive coefficients possible, even negative values (Veselago lens 1968)
- Key insight: effective metric tensor is given by permeability and permittivity tensors. (Leonhardt 2006, Pendry et al. 2006)



- Fermat principle: light takes shortest optical path -> Tune the geometry, obtain desired geodesics, control the propagation (General relativity kinematics)
- Enable exotic devices such as perfect lenses and cloaking devices



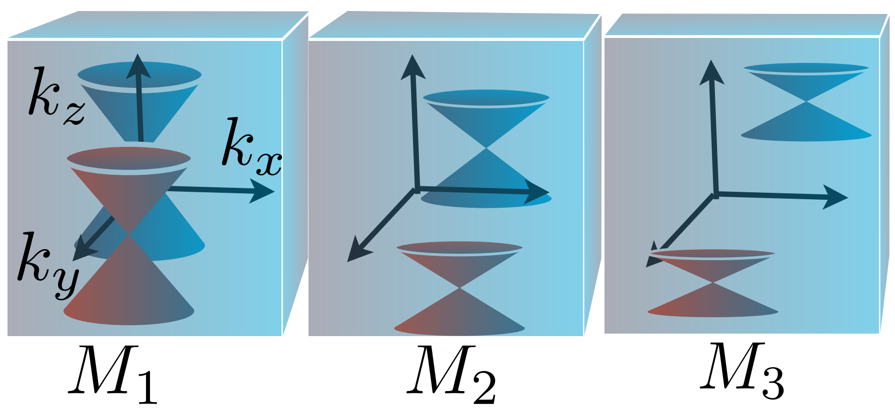
Geometry-engineering in electronic systems

- Geometry and gauge field engineering has been studied extensively in context of 2d materials
- For example, strain-induced gauge fields and topological states
- Strain-induced chiral anomaly and related effects recently studied also in Weyl and Dirac semimetals
- Why bother with geometry-engineering? Tailor-made electronic transport properties, novel exotic electronic devices, test lab for curved-space quantum physics

What is Weyl good for?

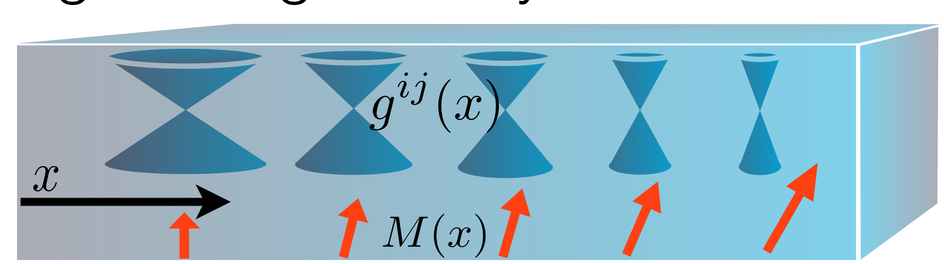
- Crystals with I and TR intact have two-fold degenerate energy bands-> Weyl nodes require breaking of either (or both)

- Accidental degeneracy: location of the Weyl nodes and Weyl cones depend sensitively on nature of TR and I breaking-> offers lot of tuneability

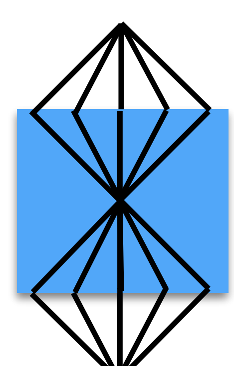


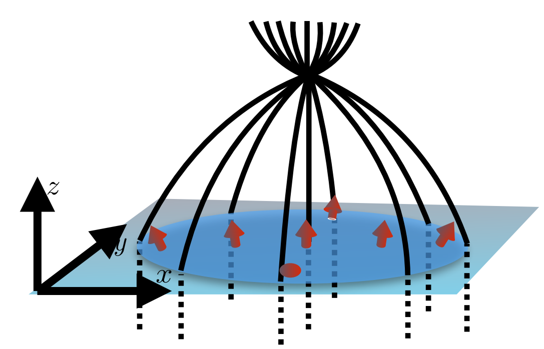
- Local deformation of local light cones translates into curved geometry

- This suggests a strategy for geometry-engineering: identify TR and I breaking couplings, allow them vary locally



….and 3d vs. 2d





Geometry engineering in a nutshell

- Write down general four-band model with TR and I intact

$$H_0 = n(\mathbf{k})\mathbb{I} + \boldsymbol{\kappa}(\mathbf{k})\cdot\boldsymbol{\gamma} + m(\mathbf{k})\gamma_4 \qquad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$$

$$m(\mathbf{k}) = m(-\mathbf{k}) \qquad \boldsymbol{\kappa}(\mathbf{k}) = -\boldsymbol{\kappa}(-\mathbf{k})$$

- Identify all possible TR and I breaking terms (and promote them to smooth fields)

$$H = H_0 + \mathbf{u}\cdot\mathbf{b} + \mathbf{w}\cdot\mathbf{p} + \mathbf{u}'\cdot\mathbf{b}' + f\varepsilon$$

$$\gamma_{ij} \equiv -i[\gamma_i, \gamma_j]/2$$

	TR	I
$\mathbf{b} = (\gamma_{23}\ \gamma_{31}\ \gamma_{12})$	-1	$+1$
$\mathbf{p} = (\gamma_{14}\ \gamma_{24}\ \gamma_{34})$	$+1$	-1
$\mathbf{b}' = (\gamma_{15}\ \gamma_{25}\ \gamma_{35})$	-1	$+1$
$\varepsilon = \gamma_{45}$	$+1$	-1

- Find *U* that block diagonalises *H* to obtain two-band Weyl Hamiltonian (can be done when $\boldsymbol{u}' = \varepsilon = 0$)

$$H' = \begin{pmatrix} D_0(\mathbf{k})\sigma^0 + \mathbf{D}(\mathbf{k})\cdot\boldsymbol{\sigma} & 0 \\ 0 & d_0(\mathbf{k})\sigma^0 + \mathbf{d}(\mathbf{k})\cdot\boldsymbol{\sigma}, \end{pmatrix}$$

- Example: $H = \boldsymbol{\kappa}(\mathbf{k})\cdot\boldsymbol{\gamma} + m\gamma_4 + \mathbf{u}\cdot\mathbf{b}$

$$\boldsymbol{\kappa}(\mathbf{k}^W) = \pm\sqrt{u^2 - m^2}\,\hat{\mathbf{u}}$$

$$d_1 = -\frac{(\hat{\mathbf{u}}\times\boldsymbol{\kappa}(\mathbf{k}))\cdot\hat{\mathbf{z}}}{\sqrt{1-\hat{u}_3^2}}$$

$$d_2 = -\frac{[\hat{\mathbf{u}}\times(\boldsymbol{\kappa}(\mathbf{k})\times\hat{\mathbf{u}})]\cdot\hat{\mathbf{z}}}{\sqrt{1-\hat{u}_3^2}}$$

$$d_3 = u - \sqrt{(\hat{\mathbf{u}}\cdot\boldsymbol{\kappa}(\mathbf{k}))^2 + m^2}$$

- Linearise at low energy $H'_W = d_0(\mathbf{k}_W)\sigma^0 + e^j{}_\nu(\mathbf{r})\sigma^\nu(k_j - k_j^W)$ $\qquad e^i{}_\nu(\mathbf{r}) = \left.\dfrac{\partial d_\nu}{\partial k_i}\right|_{\mathbf{k}=\mathbf{k}W}$

- Linearize at low energy $H'_W = d_0(\mathbf{k}_W)\sigma^0 + e^j{}_\nu(\mathbf{r})\sigma^\nu(k_j - k_j^W)$ $\quad e^i{}_\nu(\mathbf{r}) = \left.\dfrac{\partial d_\nu}{\partial k_i}\right|_{\mathbf{k}=\mathbf{k}W}$

$$H'_W\Psi = \mathcal{E}\Psi \qquad \mathcal{E}^2 = g^{ij}p_ip_j \qquad g^{\mu\nu} = \eta^{ab}e^\mu{}_a e^\nu{}_b \qquad p_i = k_i - k_i^W$$

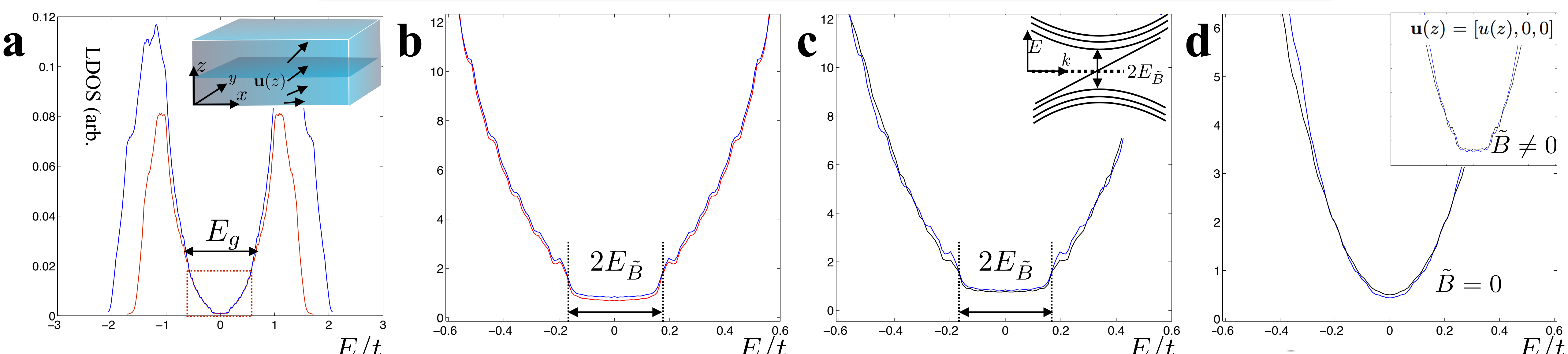
- Postulate effective quantum (curved-space Weyl) Hamiltonian at low energy

$$H_W = \frac{1}{2}\{e^j{}_\nu\sigma^\nu, -i\partial_j - a_j\} =$$

$$= e^j{}_\nu\sigma^\nu\left(-i\partial_j - a_j - \frac{i}{2}\tilde{e}_j{}^\mu\partial_n e^n{}_\mu\right) \qquad \mathbf{a} = \mathbf{k}^W(\mathbf{r}) \qquad \tilde{\mathbf{B}} = \nabla\times\mathbf{k}^W$$

Example: $\boxed{H = \boldsymbol{\kappa}(\mathbf{k})\cdot\boldsymbol{\gamma} + m\gamma_4 + \mathbf{u}\cdot\mathbf{b}}$ $\qquad \boldsymbol{\kappa}(\mathbf{k}^W) = \pm\sqrt{u^2 - m^2}\,\hat{\mathbf{u}}$

$$\mathfrak{g} = \begin{pmatrix} -1 & 0 \\ 0 & \boldsymbol{g} \end{pmatrix} \qquad \boxed{g^{ij} = (\partial_{k_i}\boldsymbol{\kappa})\cdot(\partial_{k_j}\boldsymbol{\kappa}) - \frac{m^2}{u^2}(\partial_{k_i}\boldsymbol{\kappa}\cdot\hat{\mathbf{u}})(\partial_{k_j}\boldsymbol{\kappa}\cdot\hat{\mathbf{u}})\Big|_{\mathbf{k}=\mathbf{k}^W}}$$



$$H = \boldsymbol{\kappa}(\mathbf{k})\cdot\boldsymbol{\gamma} + m\gamma_4 + \mathbf{u}\cdot\mathbf{b} \qquad \kappa_i(\mathbf{k}) = t\sin k_i$$

$$\mathbf{u}(z) = u_0(\cos k_0 z, \sin(k_0 z), 0)$$

$$\mathbf{u}(z) = [0, 0, u(z)]$$

$$u(z) = u_1 + (u_2 - u_1)z/N_z$$

Examples:

Burkov-Balents TI-F sandwich structure

$$
\begin{aligned}
H =& v_F k_x \gamma_1 + v_F k_y \gamma_2 + \Delta_D \sin k_z d \gamma_3 \\
&+ (\Delta_S + \Delta_D \cos k_z d)\gamma_4 + \mathbf{m} \cdot \mathbf{b},
\end{aligned}
$$

TR breaking fields

Vazifeh-Frantz Weyl semimetal model

$$H = 2\lambda(\gamma_2 \sin k_y + \gamma_1 \sin k_x) + 2\lambda_z \gamma_3 \sin k_z + \gamma_4 M_{\mathbf{k}} + u_0 \varepsilon + \mathbf{u} \cdot \mathbf{b}$$

Dirac Semimetal (Cd_3As_2) with broken inversion:

I breaking fields

$$H = \varepsilon_0(\mathbf{k}) + A k_x \gamma_1 + A k_y \gamma_2 + M(\mathbf{k})\gamma_4 + \mathbf{w} \cdot \mathbf{p}$$

$$\varepsilon_0(\mathbf{k}) = C_0 + C_1 k_z^2 + C_2(k_x^2 + k_y^2)$$

$$M(\mathbf{k}) = M_0 + M_1 k_z^2 + M_2(k_x^2 + k_y^2)$$

Write in terms of gamma matrices-> plug&play

Semiclassical dynamics in Weyl metamaterials

- In optical metamaterials light moves along geodesics, electron motion in Weyl metamaterials is more complicated
- Semiclassical equations contain phase-space Berry curvature tensor (Xiao, Chang, Niu, Rev. Mod. Phys. 2010)

$$\dot{r}^i = \partial_{k_i}\mathcal{E} - \Omega_{k_i r^l}\dot{r}^l - \Omega_{k_i k_l}\dot{k}_l$$
$$\dot{k}_i = -\partial_{r^i}\mathcal{E} + \Omega_{r^i k_l}\dot{k}_l + \Omega_{r^i r^l}\dot{r}^l - e(E_i + \varepsilon_{ijk}\dot{r}^j B^k)$$

$$\mathcal{E} = (g^{ij}k_i k_j)^{\frac{1}{2}} \qquad \Omega_{q_1 q_2} = -\frac{\epsilon^{ijk}}{2|d|^3} d_i \partial_{q_1} d_j \partial_{q_2} d_k \qquad d_j = e^i{}_j k_i$$

$$\Omega_{k_i k_l} \sim \frac{1}{|k|^2}, \quad \Omega_{r^i k_l} \sim \frac{1}{|k|}$$

Ballistic motion: $\ddot{r}^l + \Gamma^l{}_{ij}\dot{r}^i\dot{r}^j = 0 \qquad \Gamma^i{}_{jk} = \frac{1}{2}g^{il}(\partial_{r_j}g_{lk} + \partial_{r_k}g_{lj} - \partial_{r_l}g_{jk})$

+ higher order corrections

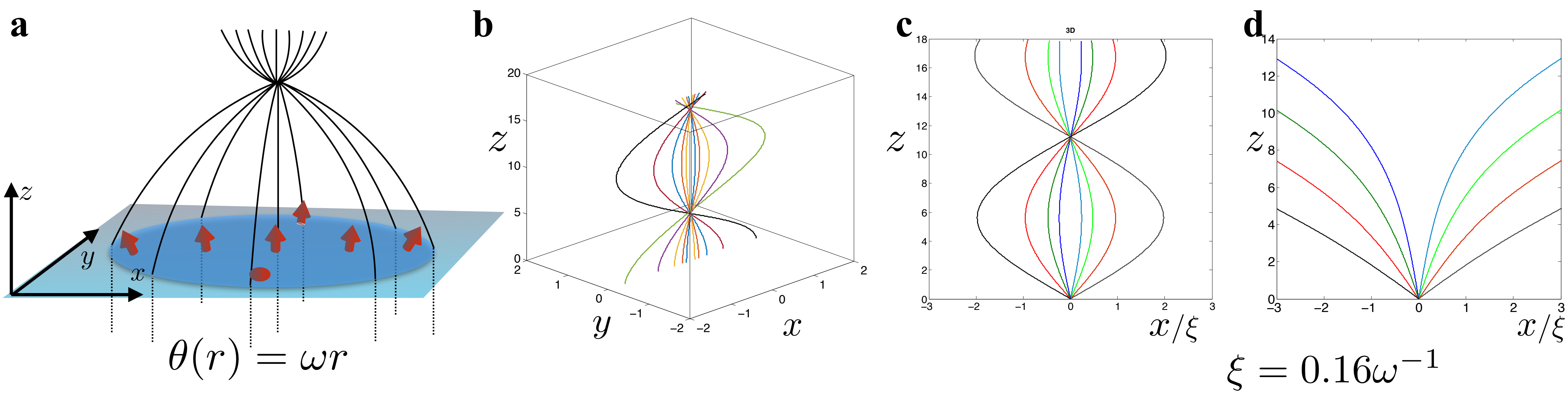
Application: 3d Weyl electron lens

- Surprisingly simple TR-breaking textures give rise to remarkable chirality-selective 3d lens effect

$$H = v_i k_i \gamma_i + m\gamma_4 + \mathbf{u}\cdot\mathbf{b}$$

$$\mathbf{u} = u\left[\cos\varphi\sin\theta(r), \sin\varphi\sin\theta(r), \cos\theta(r)\right]$$

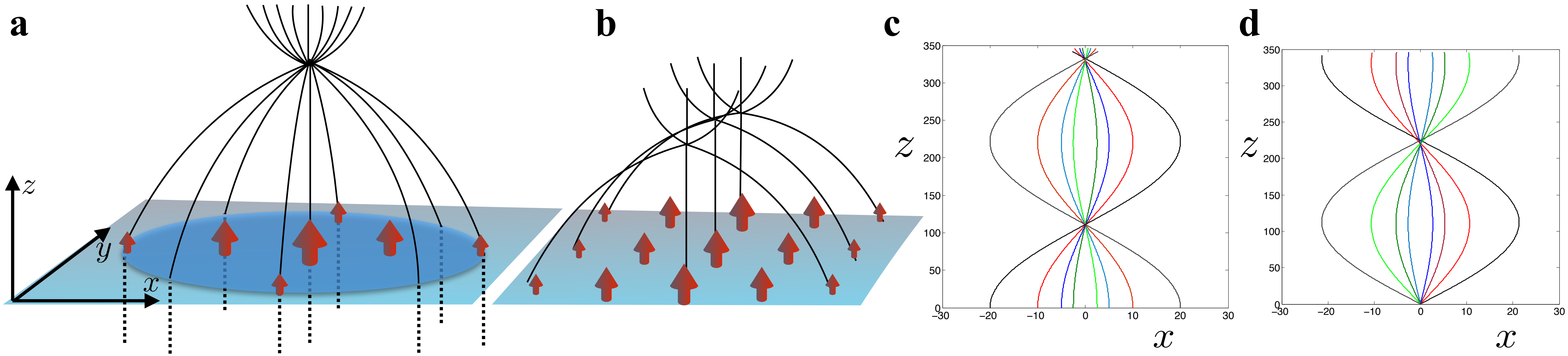
$$k^W = \pm\sqrt{u^2 - m^2}\,\hat{\mathbf{u}}$$



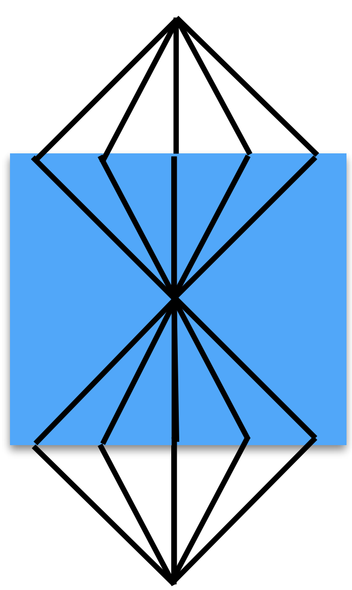
Combined effect of metric and effective gauge field

Distance between two focal points: $$\Delta z = 2\pi \frac{v_z}{v\omega}\sqrt{\frac{1-\frac{m^2}{u^2}}{\frac{m^2}{u^2}+\frac{u}{v_z k_z}\sqrt{1-\frac{m^2}{u^2}}}}$$

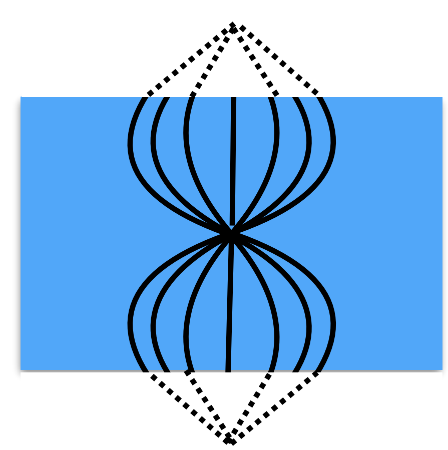
Alternative realizations



Comparison



Veselago



Weyl

Summary&outlook

- We proposed engineering 3d curved-space geometry and gauge fields for relativistic fermions by smooth TR and I breaking couplings
- Explicit solution for curved geometries as a function of TR and I breaking fields
- Semiclassical motion results from interplay classical and quantum geometric effects
- As a concrete application we discovered 3d chirality-selective Weyl electron lens
- Outlook: quantum effects with synthetic gauge fields and geometries, reverse engineering interesting geometries, exploring novel device concepts (cloaking devices?), effects of disorder…

Alex Weström and T. O., arXiv:1703.10408