Pushing the (Astrophysical) Boundaries of Kinetic Plasma Simulations

[The Art of (Exa?)Rescaling]

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Recent PIC successes:

collisionless shocks
reconnection
laboratory plasma

Astrophysical phenomena that can be studied with kinetic simulations:

Persistent sources:

accreting black holes
neutron star magnetosphere (pulsars, magnetars)

Explosions:

magnetar flares (cf. solar flares)
cosmological gamma-ray bursts (GRBs), supernovae

Energy transformation in compact objects

gravity -> kinetic energy -> magnetic fields, heat -> radiation

ways of particle acceleration

reconnection, shocks, electric gaps

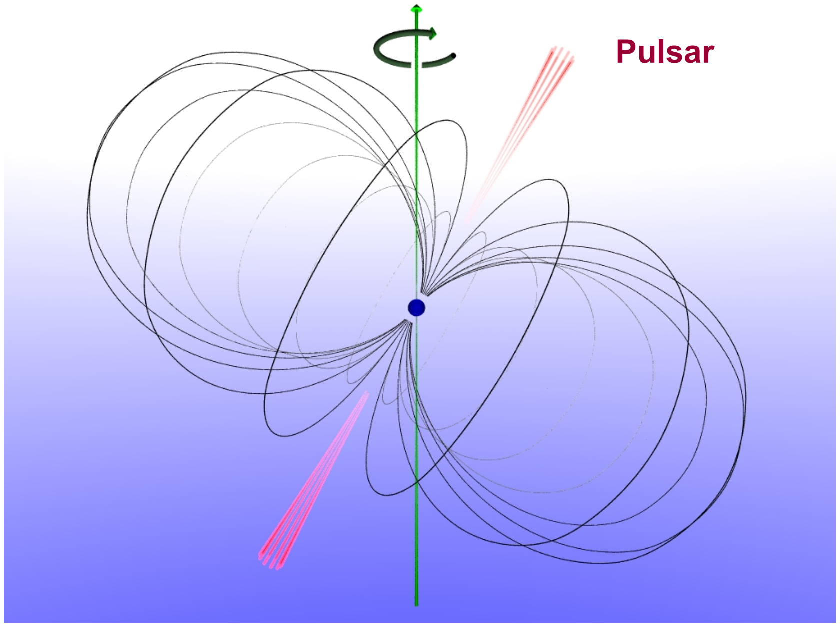
new processes for kinetic plasma simulations:

radiative cooling, e+- pair creation, coherent emission

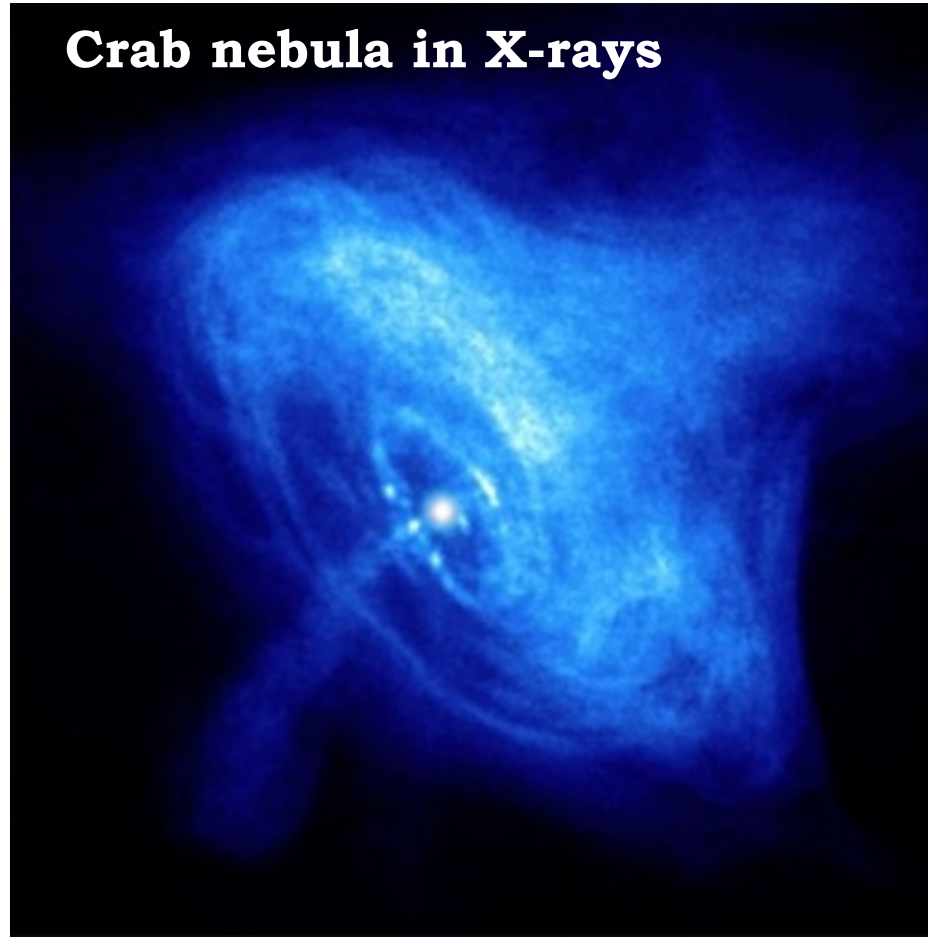
in strong (often dominant) magnetic fields

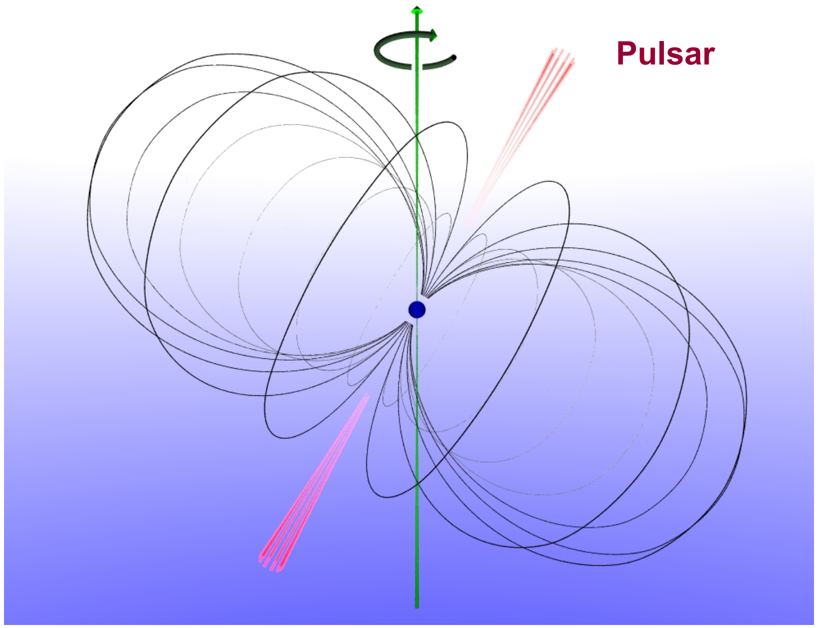
— unfamiliar PIC territory: what problems are doable?

I. Rotation-powered pulsars



- beamed coherent radio emission
- X/γ-ray emission
- e+-loaded wind





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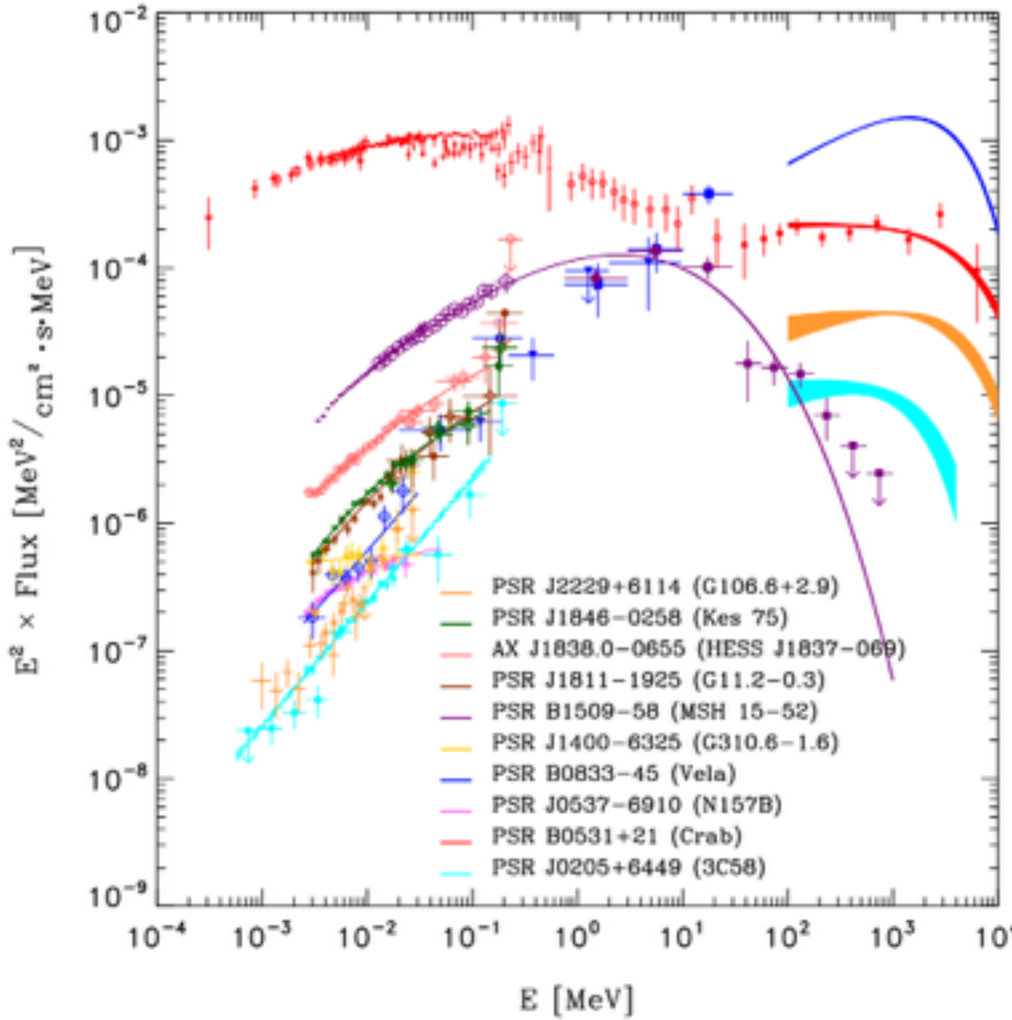
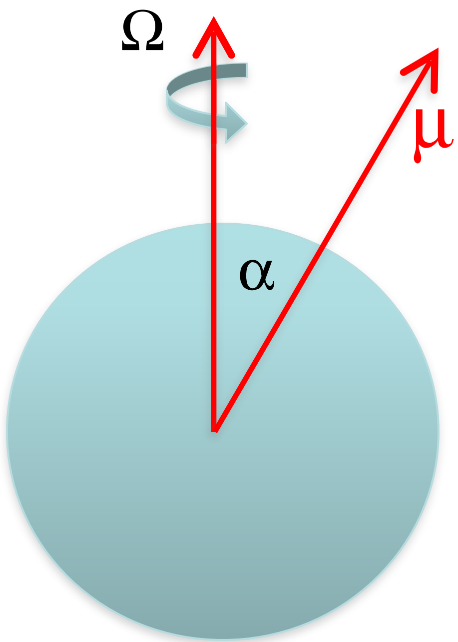
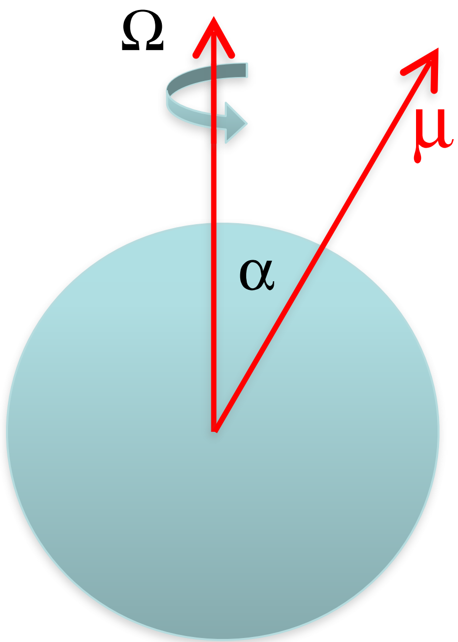


Kuiper, Hermsen 2015


Ω
μ
α

Vacuum model



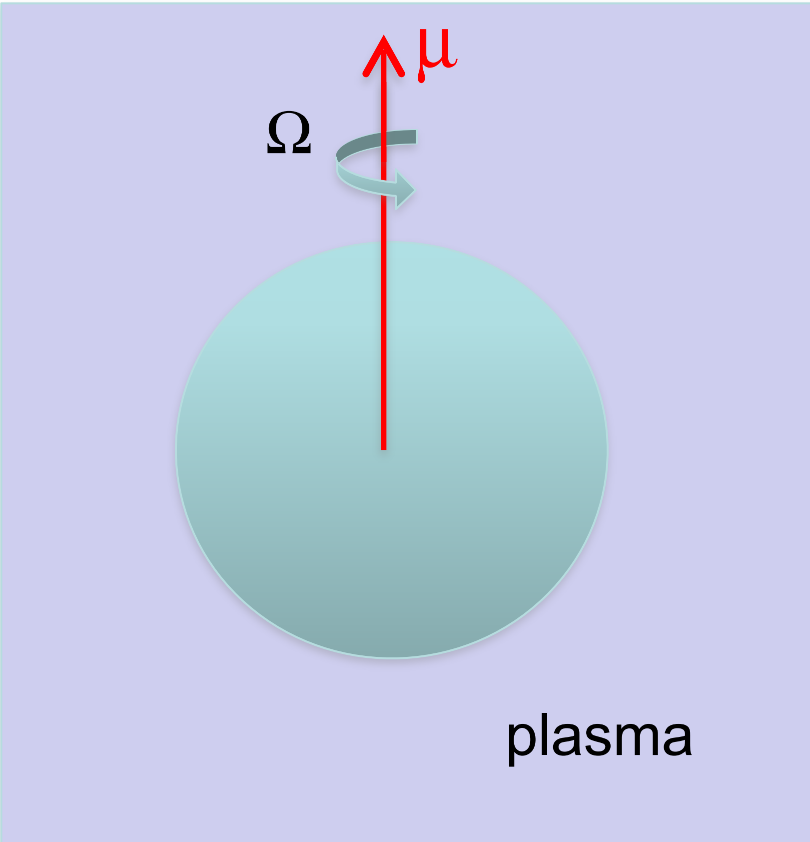
Vacuum magneto-dipole radiation

$$\frac{d}{dt}\frac{I\Omega^2}{2} = -\frac{2\ddot{\mu}^2}{3c^3}$$

Induced electric field; Poynting flux

Gould 1968
Pacini 1968
Gunn, Ostriker 1969

Force-free model

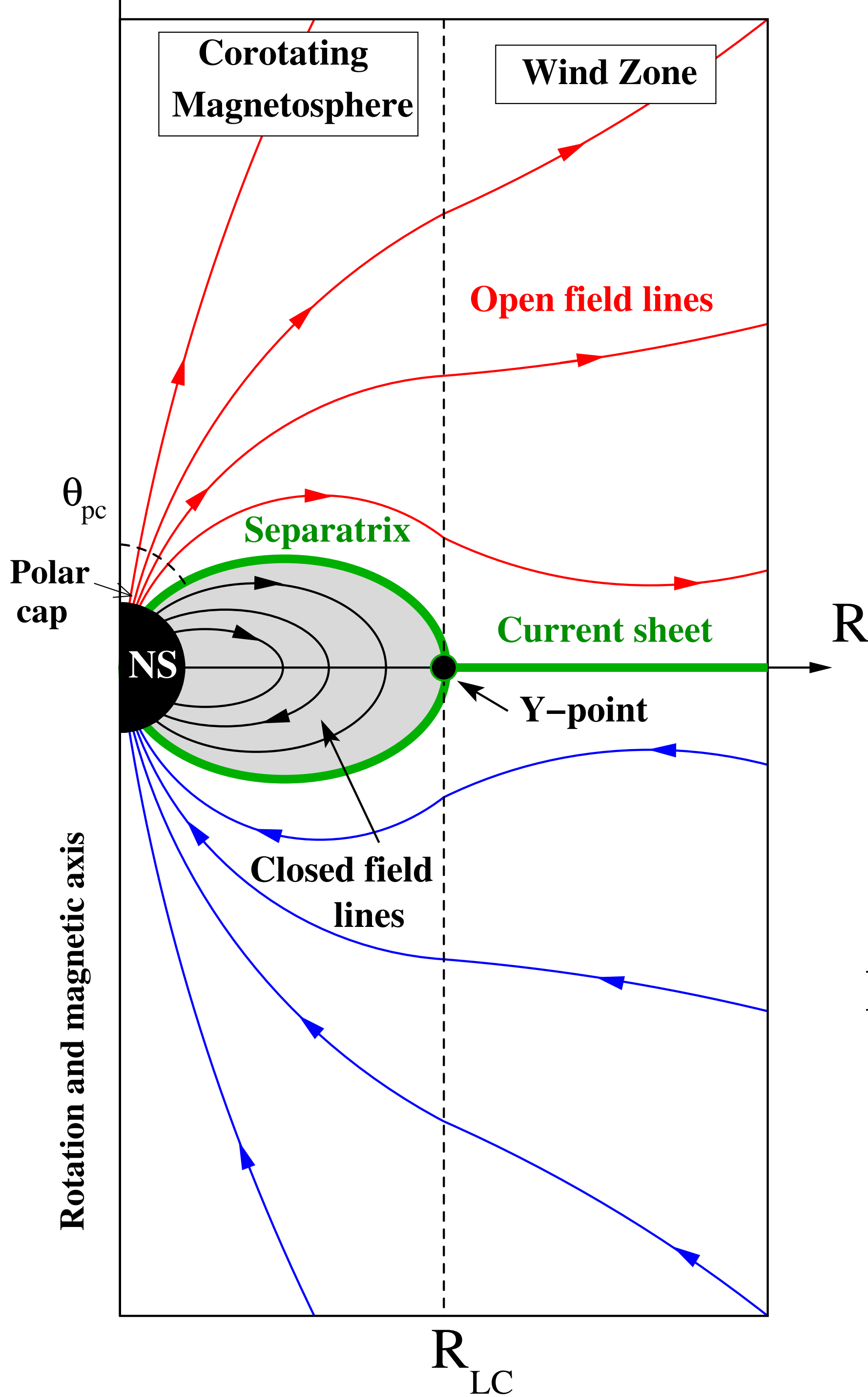


(aligned rotator: $\Omega \parallel \mu$)

Co-rotating plasma: $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B}/c = 0$$

Goldreich, Julian (1969)



$$I \approx \Omega^2 \mu / c$$

$$L_{\rm sd} \approx \Omega^4 \mu^2 / c^3$$

Force-free numerical solution:

Contopoulos et al. 1999
Timokhin 2006
Spitkovsky 2006
Parfrey et al. 2012

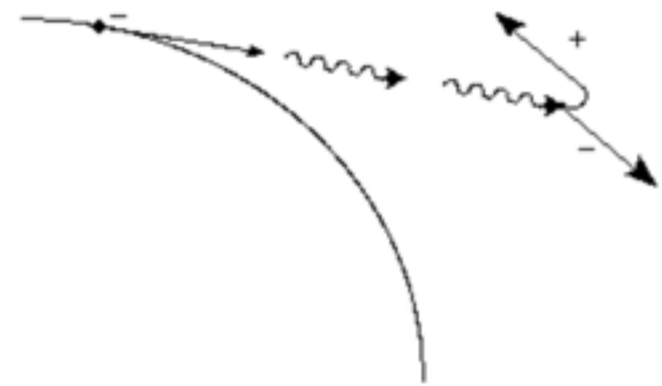
Both vacuum and force-free models address the question of how rotational energy is extracted from the star but not how it is **dissipated**

Neither vacuum nor force-free models can be correct, as they imply no energy dissipation => no emission

– there must be **gaps** in plasma-filled magnetosphere!

– continual electric **discharge** in the gap

Electric discharge



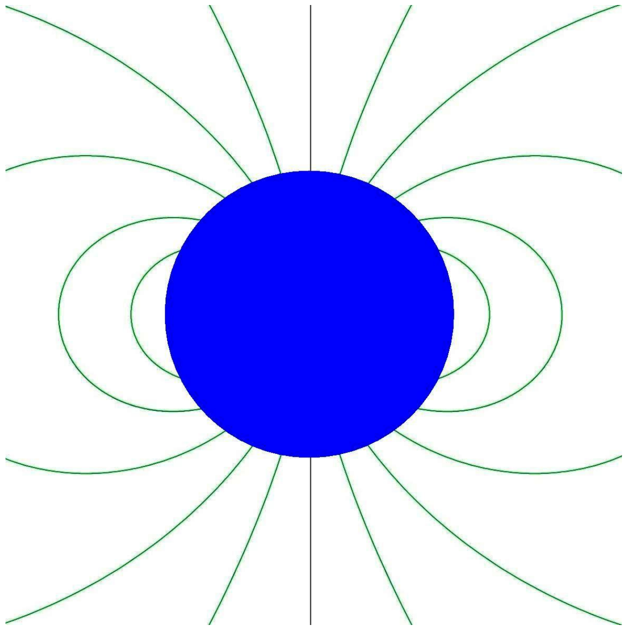
Unscreened E || B
⇒ particle acceleration
⇒ curvature radiation
⇒ photon conversion to e+-

Sturrock 1971;
Ruderman, Sutherland 1975
Arons, Scharlemann 1979
Cheng et al. 1986

New approach to the old puzzle: global PIC simulations

Chen, AB 2014; Philippov et al. 2014, 2015; Cerutti et al. 2015, 2016; Belyaev 2015

Numerical experiment



$$d\mathbf{p}/dt = e\,\mathbf{E} + e\,\mathbf{v} \times \mathbf{B}/c$$

$$\partial\,\mathbf{B}/\partial t = -c\nabla \times \mathbf{E}$$

$$\partial\,\mathbf{E}/\partial t = c\nabla \times \mathbf{B} - 4\pi\,\mathbf{J}$$

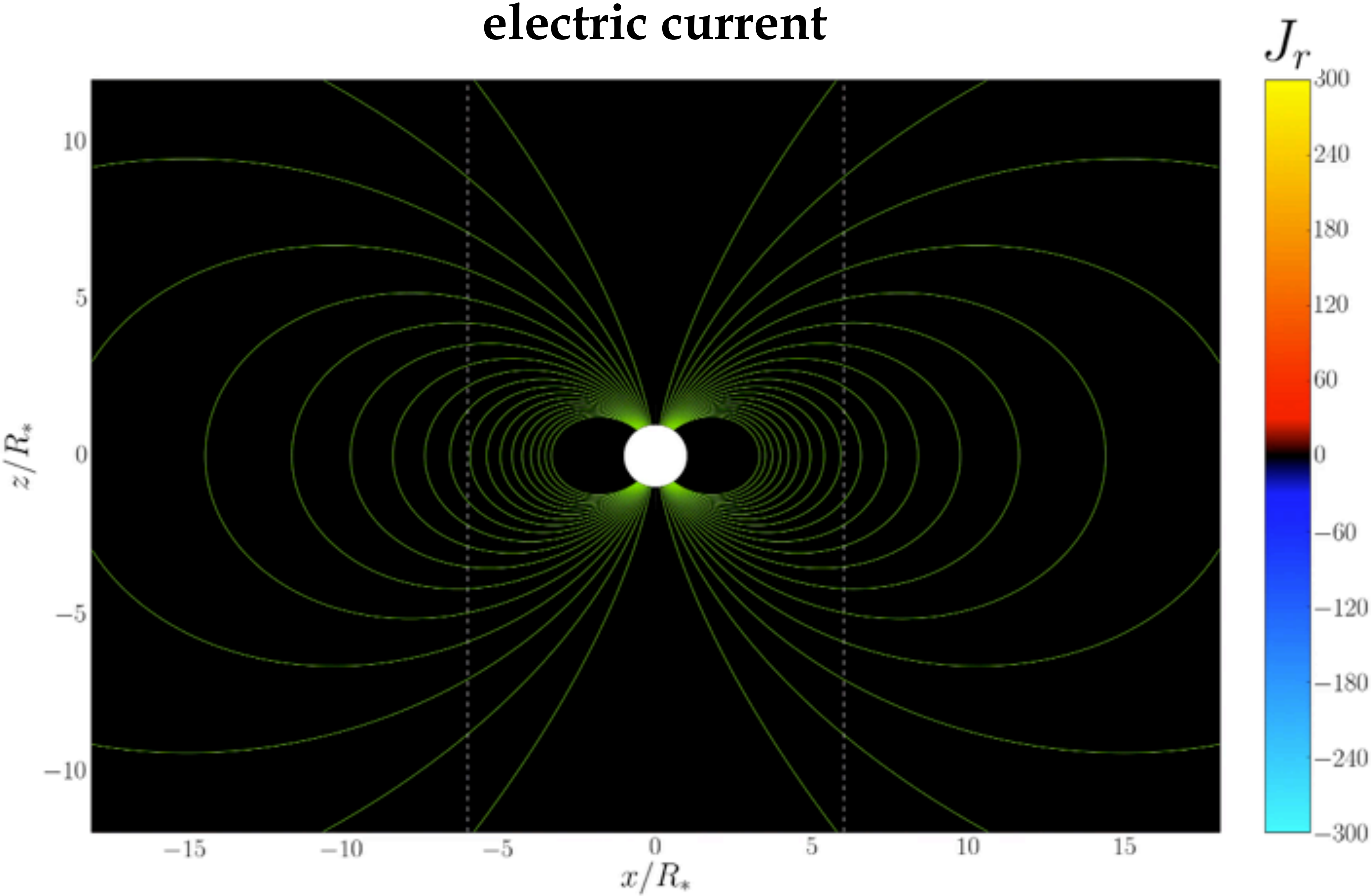
- Start with a non-rotating star and spin it up. E will be induced
- Particles lifted from the star will move in the self-consistent electromagnetic field
- E and B: fixed inside the star, calculated from Maxwell equations outside the star
- Accelerated particles emit photons
- High-energy photons convert to e+-

Method: **Particle in cell (PIC) + pair creation:**

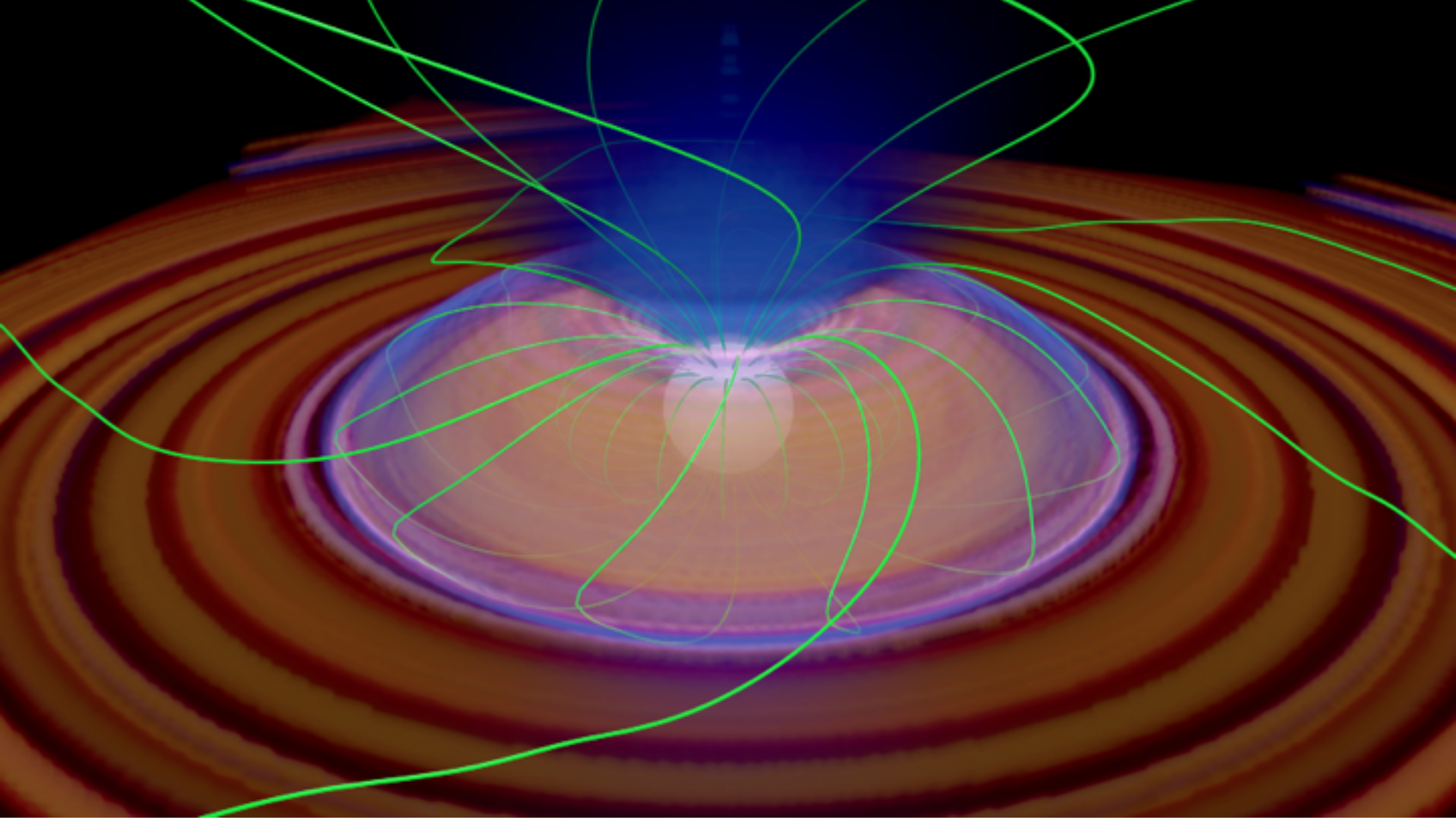
- fields calculated on a (curvilinear) grid
- particles followed individually
- photon emission, tracing, and pair creation: Monte-Carlo

Time = 0.08

electric current

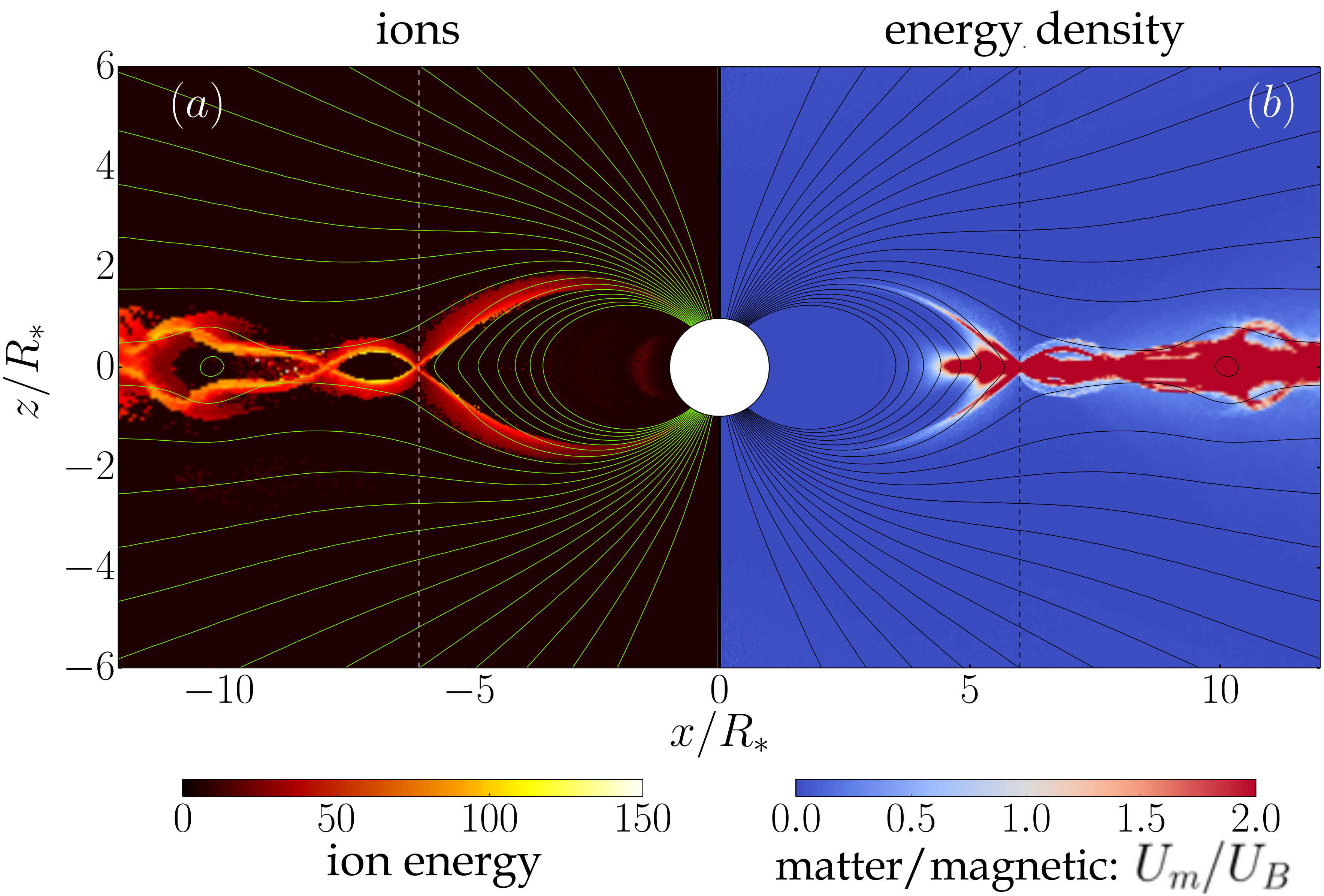


Chen & AB 2014

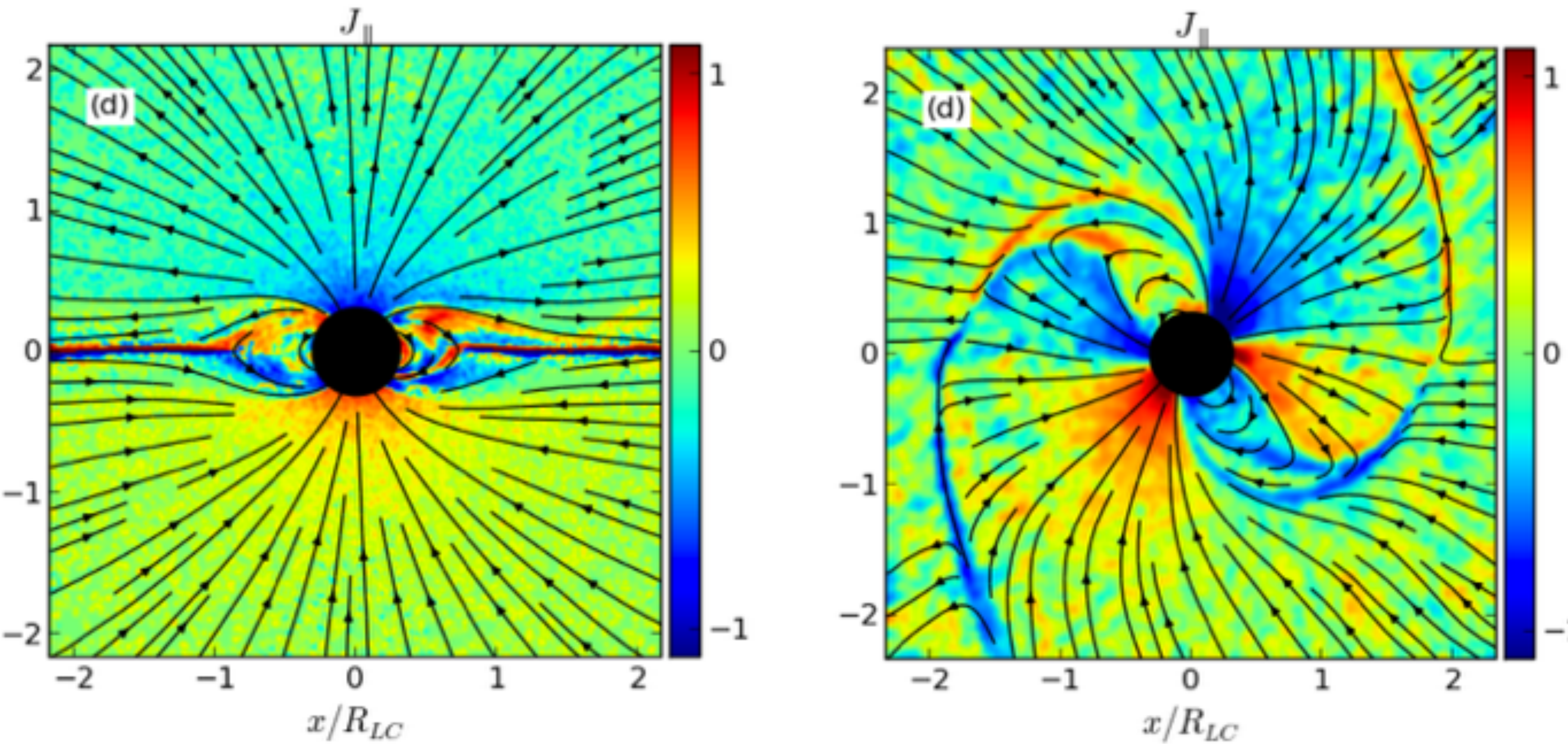


aligned rotator

charge density: - blue + orange



ions
energy density
(a)
(b)
z/R_*
x/R_*
6
4
2
0
−2
−4
−6
−10
−5
0
5
10
0
50
100
150
ion energy
0.0
0.5
1.0
1.5
2.0
matter/magnetic: U_m/U_B



Pilippov et al. 2015

Scales

$$R_\star \sim 10 \text{ km} \qquad R_{\rm LC} = \frac{c}{\Omega} = (10 - 10^4) R_\star$$

$$\lambda_p = \frac{c}{\omega_p} \qquad \omega_p^2 = \frac{4\pi n e^2}{m_e} \qquad e n_{\rm GJ} = -\frac{\mathbf{B} \cdot \mathbf{\Omega}}{2\pi c}$$

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Typical pulsars: $\frac{R_\star}{\lambda_p} \sim 10^6$ – not doable, as λ_p must be resolved => rescaling

$$\omega_p^2 = 2\omega_B \Omega \qquad \Omega \ll \omega_p \ll \omega_B$$

$$\omega_B = \frac{eB}{m_e c} \approx 1.8 \times 10^{19} B_{12} \text{ rad/s} \qquad \Omega \sim 10^2 \text{ rad/s}$$

Energy scales

Rotation-induced voltage: $\Phi_0 = \frac{\mu\Omega^2}{c^2}$ $\gamma_0 = \frac{e\Phi_0}{m_e c^2}$

$$\gamma_0 = \frac{1}{4}\left(\frac{R_\star}{R_{\rm LC}}\right)\left(\frac{R_\star}{\lambda_p}\right)^2$$

No way to reach true energies needed for pair creation =>

1) rescaling the threshold for discharge (rescaling photon emission rates and their conversion probabilities)
2) rescaling the energies of secondary pairs

(effectively changing Planck constant)

but preserving hierarchy $1 \ll \gamma_s \ll \gamma_{\rm thr} < \gamma_0$

Worries

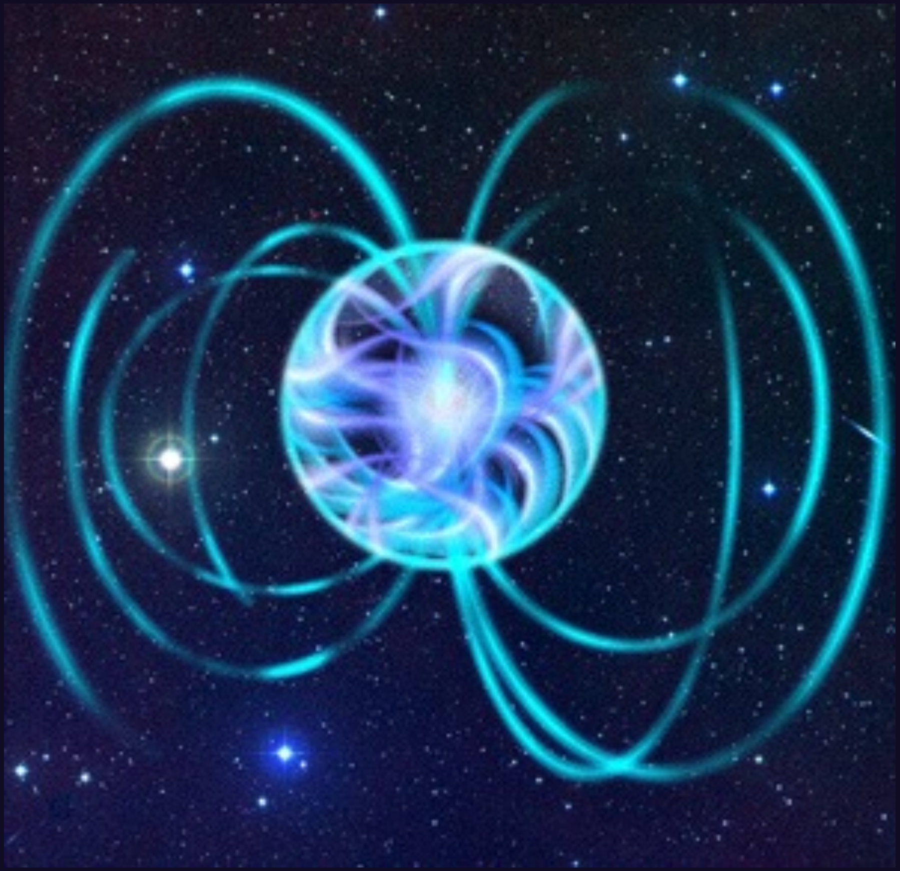
– unresolved gyration near the star
– small light cylinder radius
– low multiplicity of pair creation
– low γ_s
– no synchrotron emission from secondary pairs
– small m_i/m_e

* what is lost after re-scaling?
* how should the simulation be scaled to real pulsars?
* is it possible to implement full radiative transfer?

What questions should be asked?

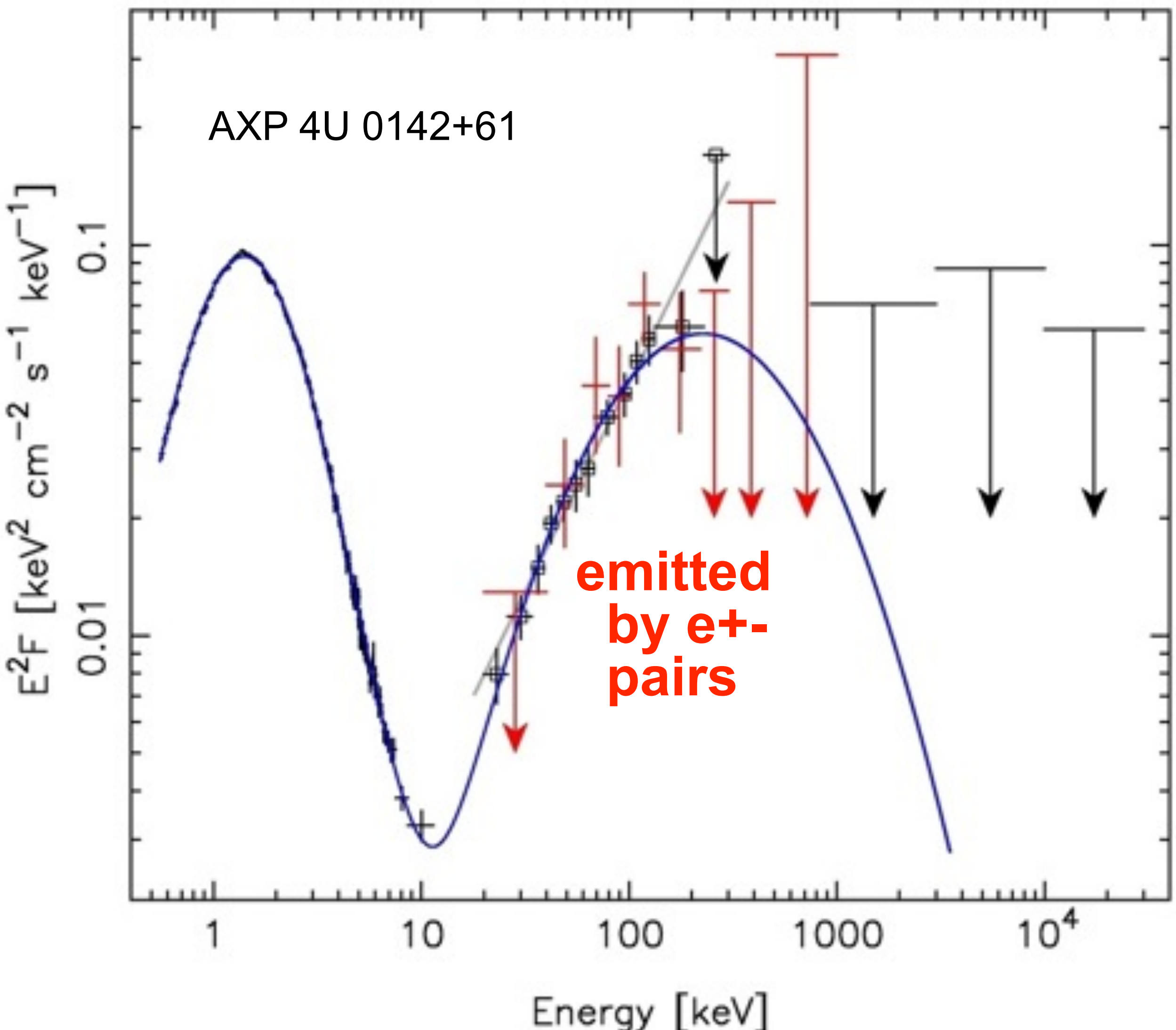
- Conditions for activating a pulsar
- Where is the "gap"? Where is energy dissipated? (inside and outside LC)
- Where are pairs created?
- What is the mechanism of current closure?
- Where are coherent radio waves produced and by what mechanism?
- What governs the diversity of pulsars? (rotation rate? misalignment? multipoles?)

Magnetars

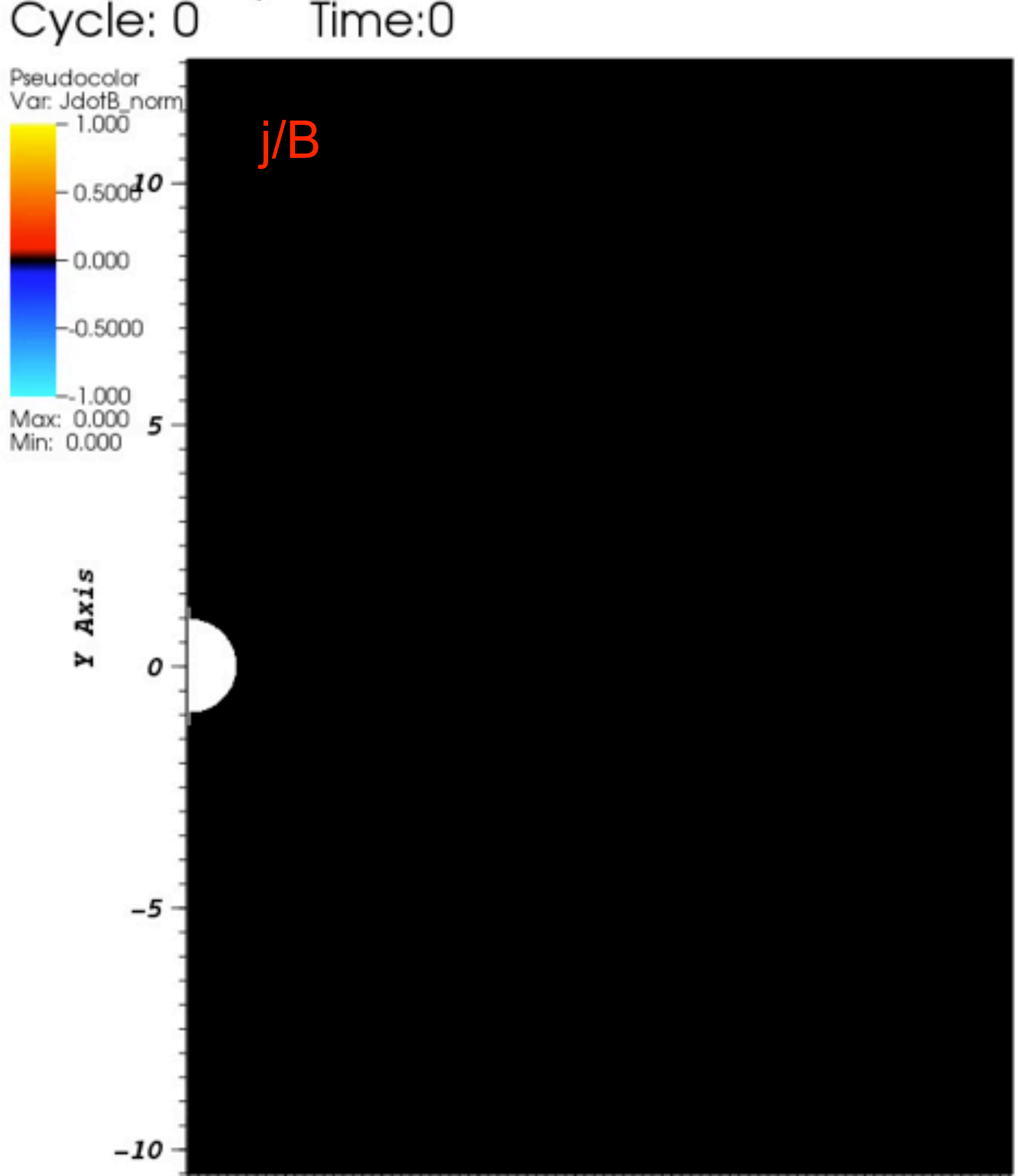


X-ray spectra: two peaks

(luminosity 10^{35}-10^{36} erg/s)



Den Hartog et al. (2008)



twisted closed magnetosphere: j-bundle formation and slow untwisting

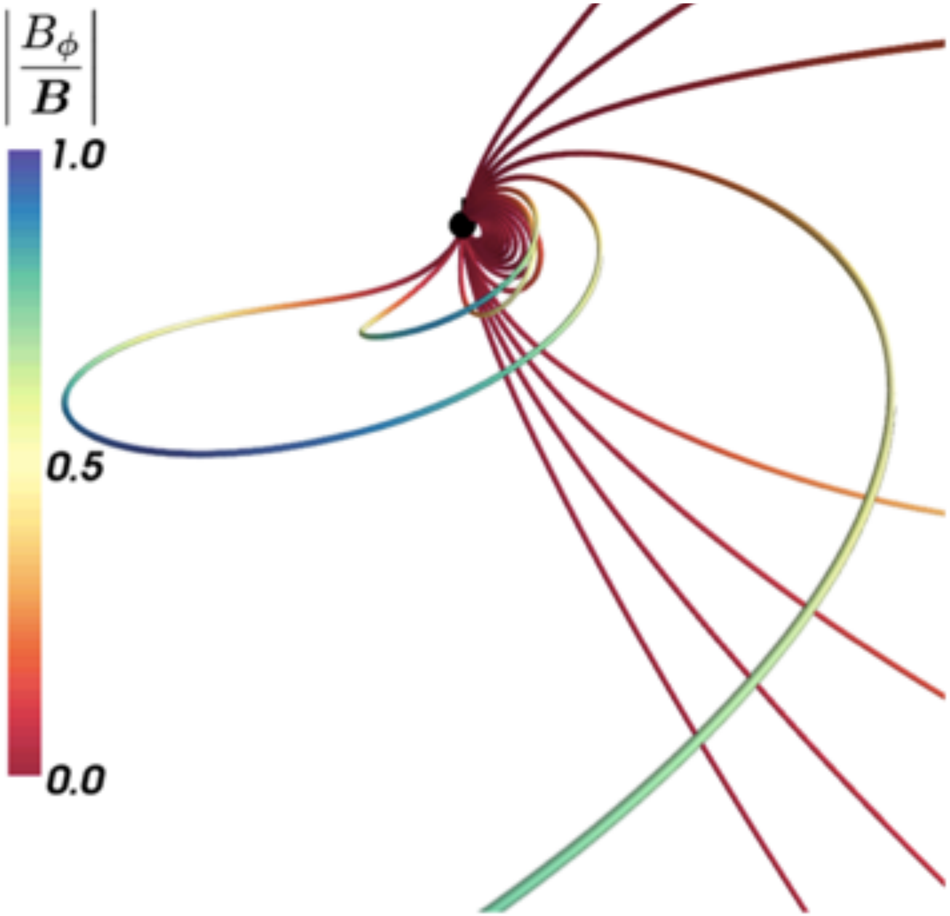
Chen & AB 2017:

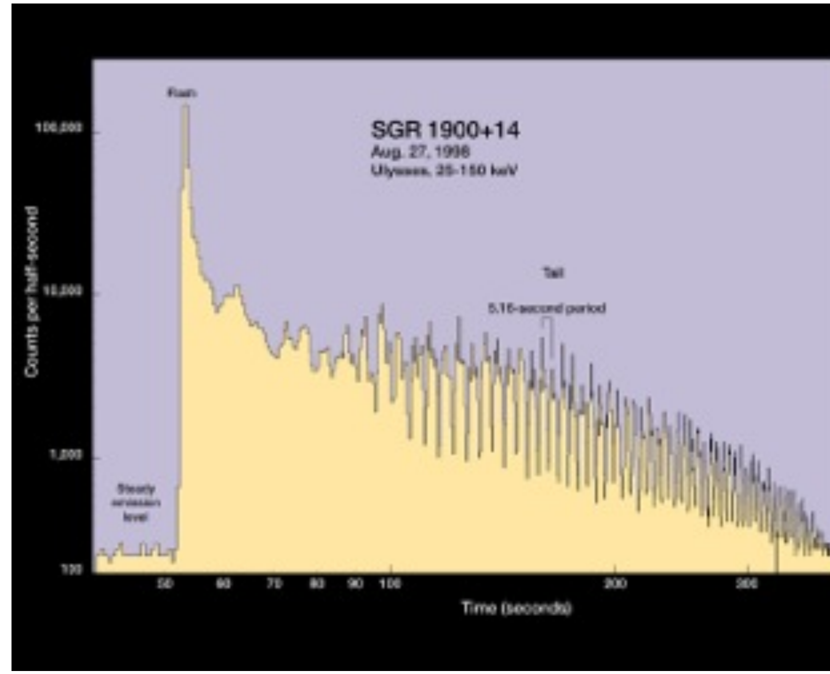
PIC simulations

e+- discharge controls magnetosphere evolution

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

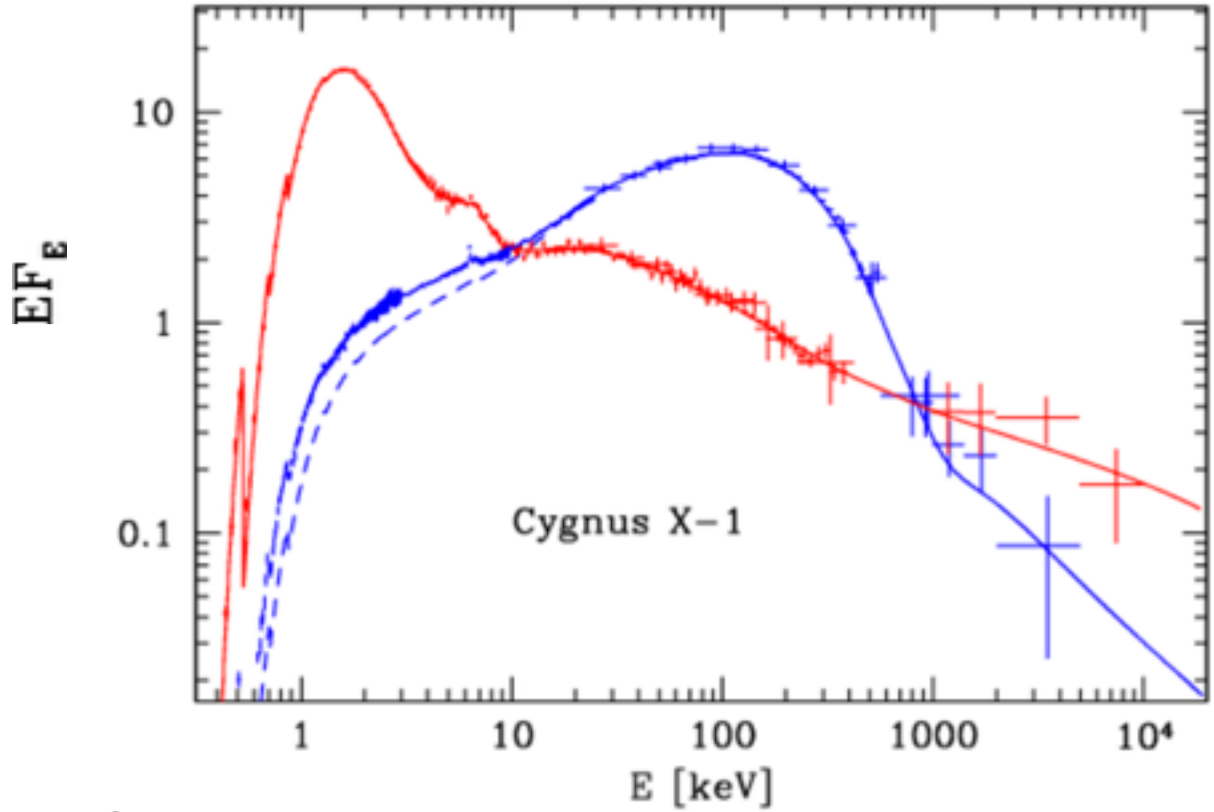
Over-twisted magnetospheres: flares



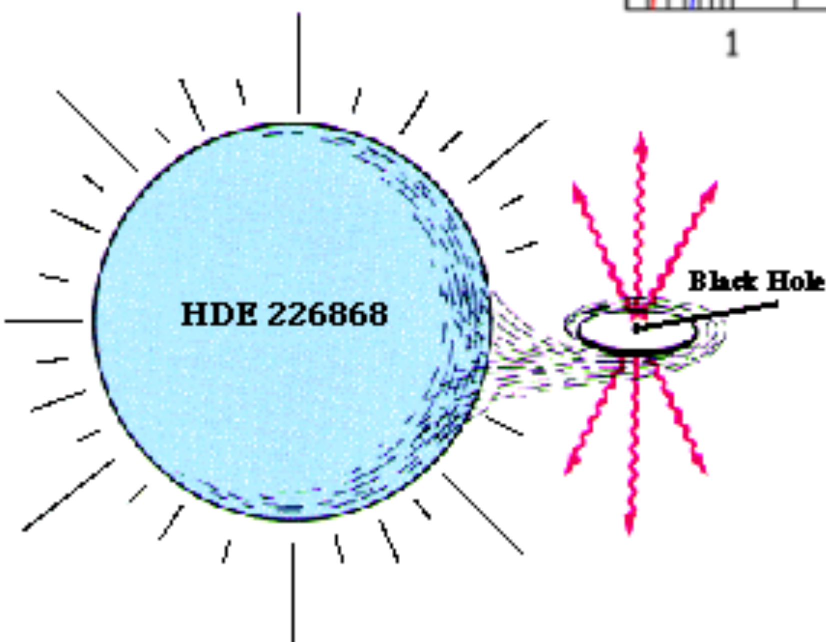


Parfrey et al. 2013

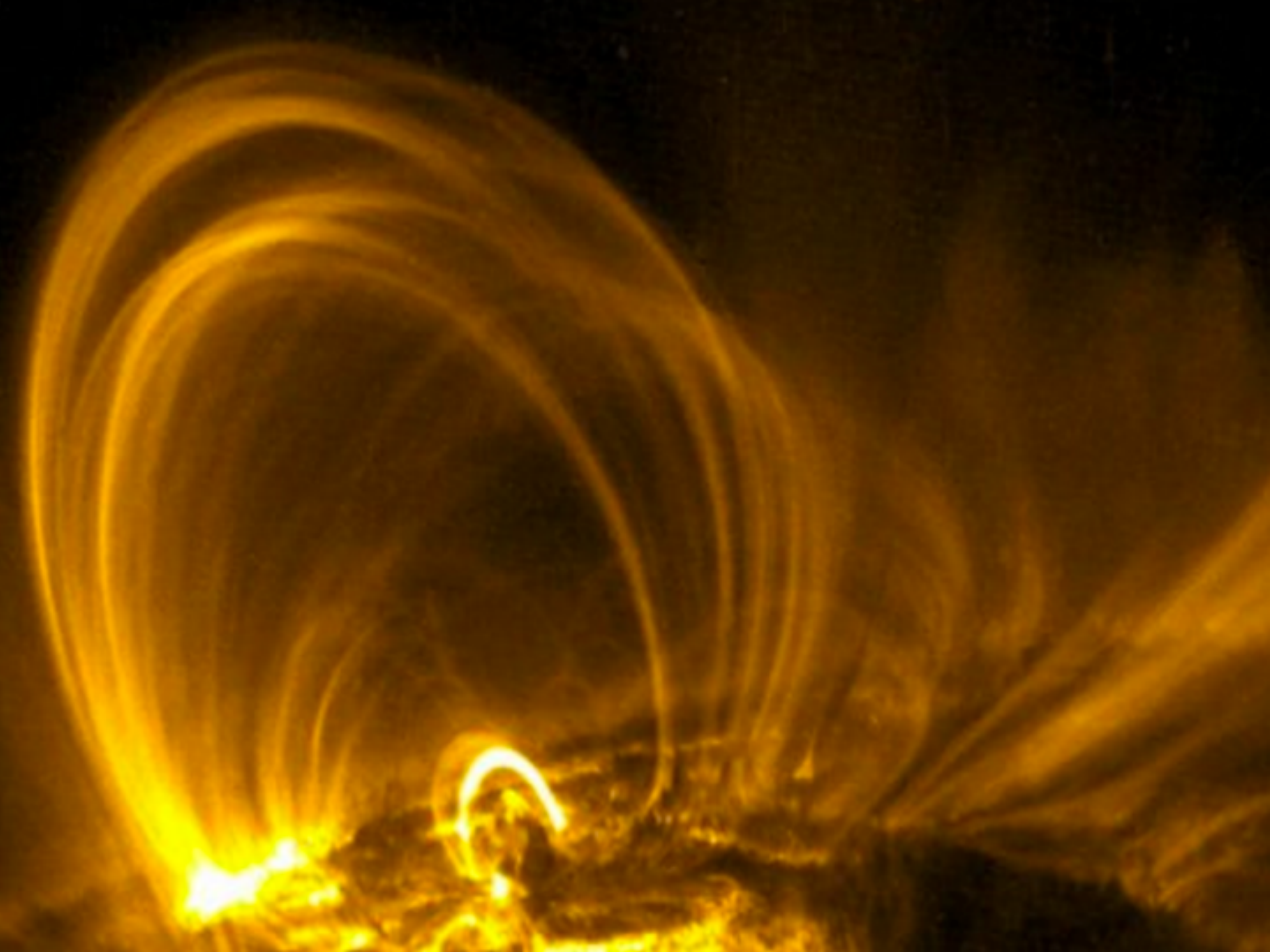
II. Magnetic flares near accreting black holes

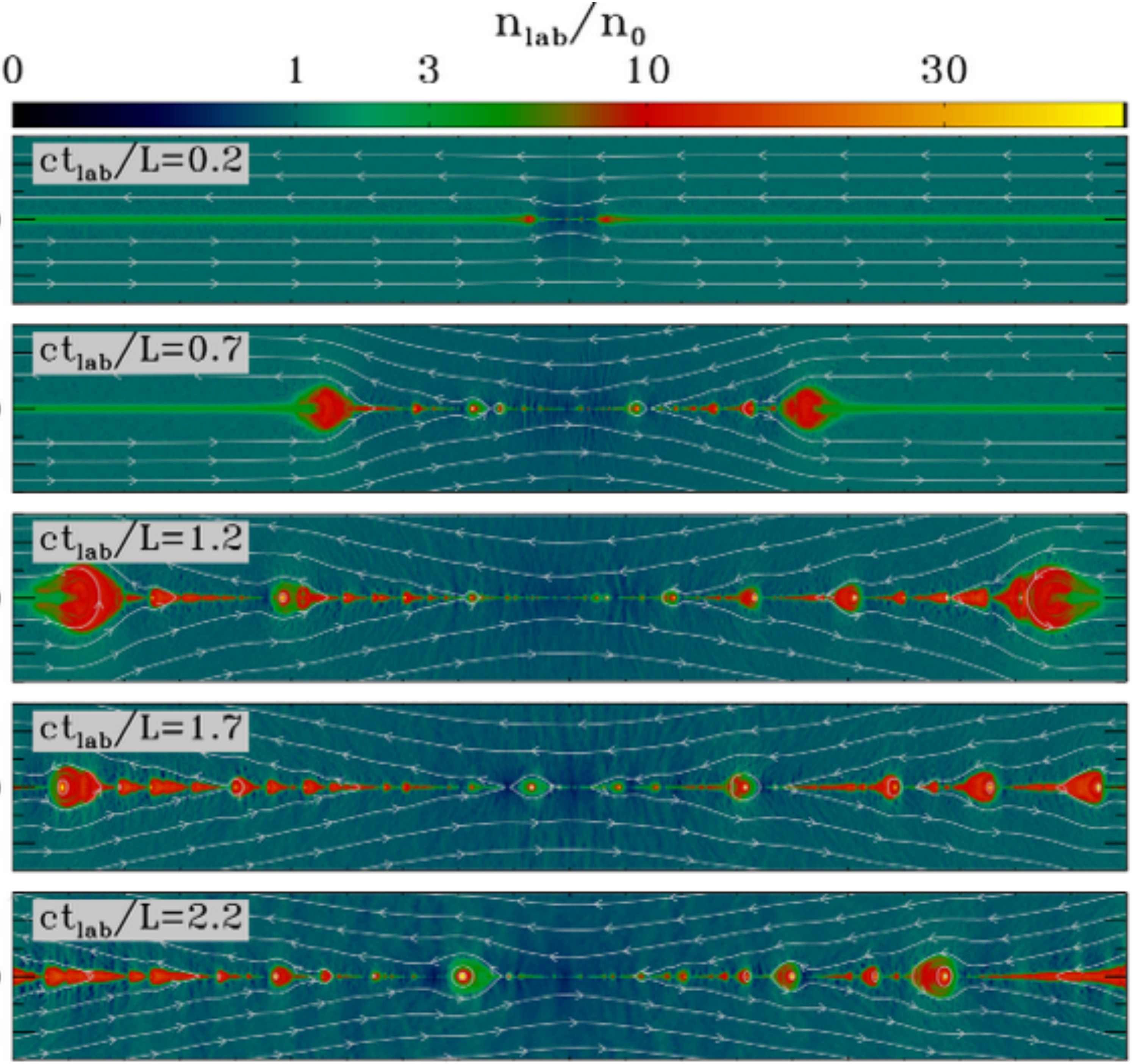


McConnell et al. 2002



"hybrid Comptonization"





Self-similar chain of plasmoids: $r_{\mathrm{L}} < w < 0.1s$

Uzdensky et al. 10

Melzani et al. 14

Sironi, Spitkovsky 14

Guo et al.16

Sironi et al.16

Werner et al.16

Radiative magnetic reconnection:

1. Magnetization: $\boxed{\sigma = \frac{B^2}{4\pi\rho c^2} = \frac{2U_B}{\rho c^2}} = 1 - 10^3$

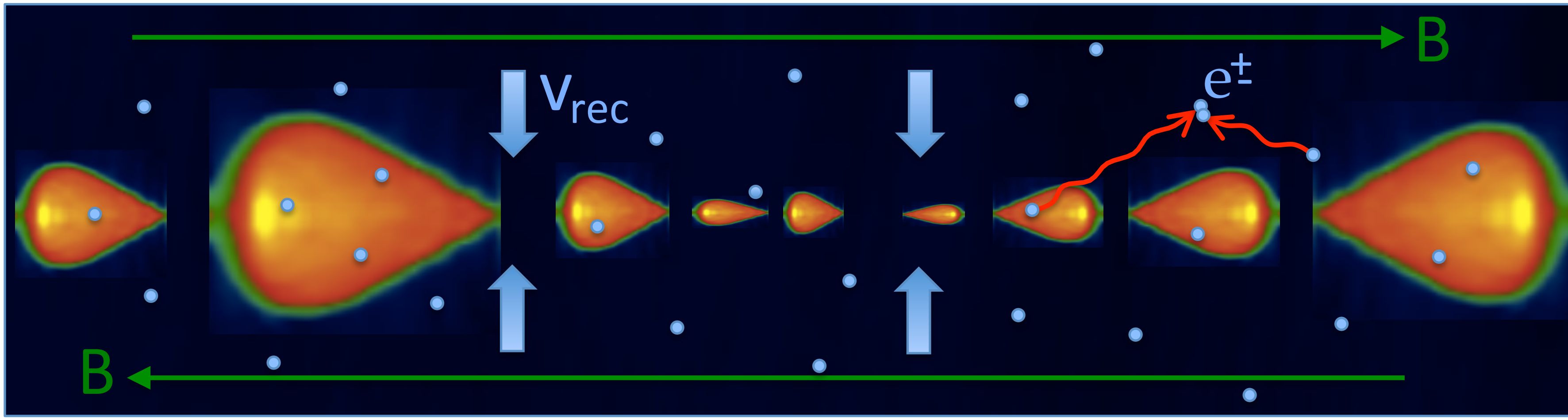
2. Compactness: cooling time vs. light crossing time

$$\frac{t_{\rm IC}}{s/c} = \frac{3}{4\gamma_e \ell_{\rm rad}} \qquad \ell_{\rm rad} = \frac{U_{\rm rad}\sigma_{\rm T} s}{m_e c^2}$$

$$U_{\rm rad} c \sim U_B v_{\rm rec} \qquad \ell_{\rm rad} \sim \frac{v_{\rm rec}}{c} \ell_B$$

$$\boxed{\ell_B = \frac{U_B \sigma_{\rm T} s}{m_e c^2}} \sim 10^3$$

Reconnection in the radiative regime (high compactness parameter)



$\ell \gg 1 \quad \Rightarrow$

1. Plasmoids are cooled
2. Energetic photons (>1 MeV) convert to e+- pairs

AB 2017

Mechanism of pair creation: photon-photon collisions $\gamma + \gamma \rightarrow e^+ + e^-$

threshold	$E_1 E_2 > (m_e c^2)^2$
cross section near threshold	$\sigma_{\gamma\gamma} \sim 0.1\sigma_{\rm T}$
optical depth	$\tau_{\gamma\gamma} = \sigma_{\gamma\gamma} n_\gamma s > 1$

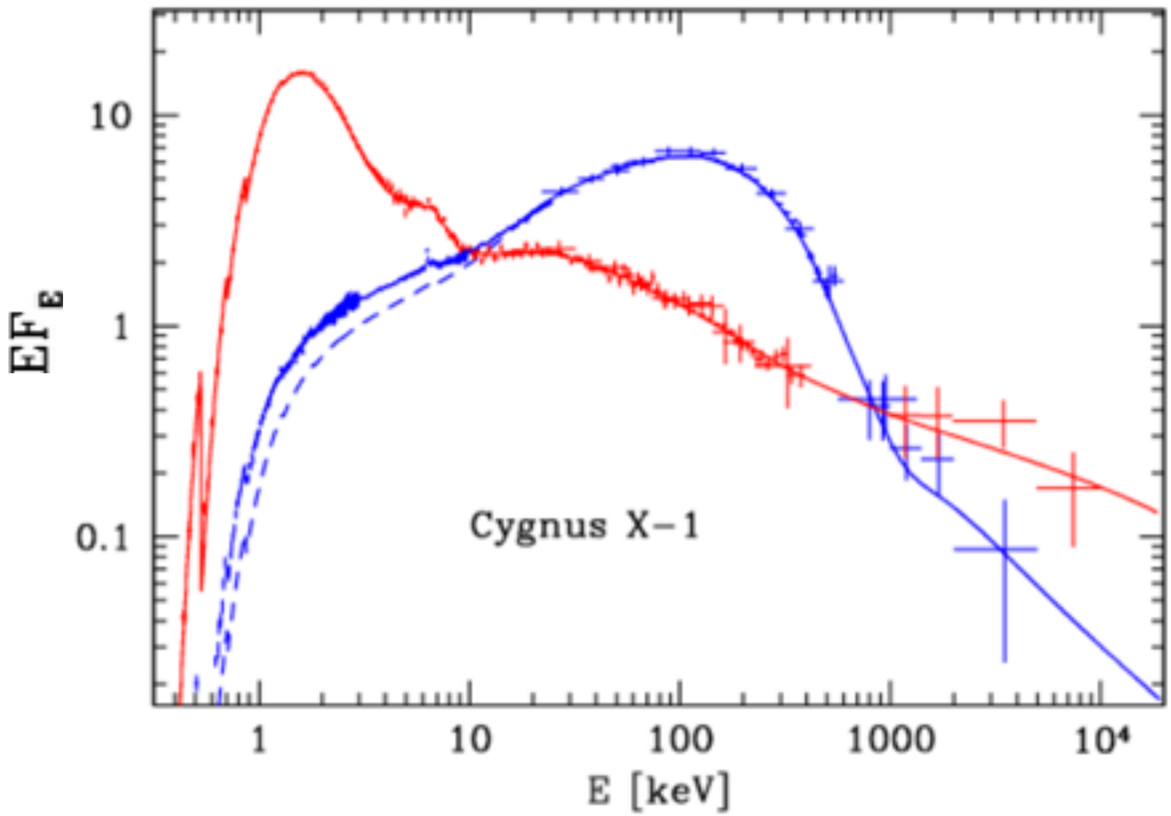
Bulk motion of pair-loaded chain plasmoids

Magnetic stresses push plasmoids: $$f_{\rm push} = \xi\,\frac{U_B}{w}$$

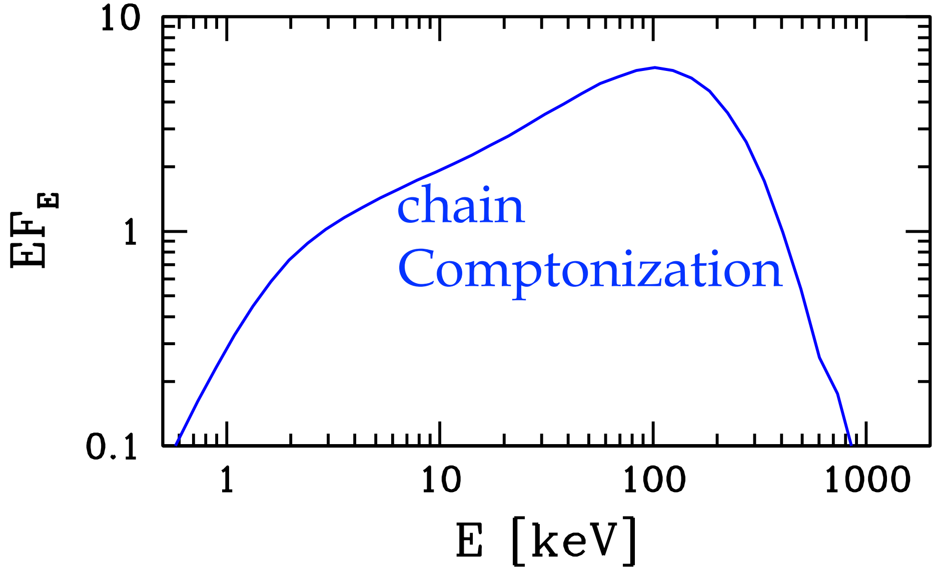
Radiation exerts drag: $$f_{\rm drag} \approx \beta\gamma^2 U_{\rm rad}\sigma_{\rm T} n_\pm$$

Drag-limited motion: $$\gamma \approx (\tau_\star/\tau_{\rm pl})^{1/2} \qquad (\gamma \le \sigma^{1/2})$$

$$\tau_\star \equiv \xi\,\frac{U_B}{U_{\rm rad}} \approx \frac{\xi}{\beta_{\rm rec}}$$



McConnell et al. 2002



AB 2017

Kinetic simulations of black hole flares

Step 1: "switch on" Compton cooling in a PIC simulation of reconnection (Sironi & AB, in preparation)

Next implement:

- synchrotron cooling
- Comptonization (radiative transfer) $\tau_{\rm T} \sim 1$
- pair creation (nonthermal cascade)
- thermalization by Coulomb collisions
- synchrotron self-absorption

2D should be sufficient; Vlasov's method may be preferred

Challenges: rescaling!

$\boxed{\sigma \sim 1 - 1000}$ OK

$$U_B \sim \frac{\dot{M}(GMr)^{1/2}}{4\pi r^2 H}$$

size $s \sim r_g = 2GM/c^2$

$$B = (8\pi U_B)^{1/2} \sim 10^8 \left(\frac{M}{10M_\odot}\right)^{-1/2} \text{ G}$$

$$\frac{c}{\omega_B r_g} \sim 10^{-11} \left(\frac{M}{10M_\odot}\right)^{-1/2}$$

High-energy particles from X-points

X-point acceleration: $$t_X \sim \frac{\bar{\gamma}_e m_e c}{eB} \sim 10^{-9} \left(\frac{M}{10 M_\odot}\right)^{-1/2} \frac{r_g}{c}$$

Synchrotron cooling: $$t_{\rm syn}^{\rm min} \sim \frac{m_e c}{\sigma_{\rm T} U_B \bar{\gamma}_e} \sim 10^{-5} \frac{r_g}{c}$$

$$(\gamma_e \sim \sigma)$$

III. Shock waves in GRBs

$n_\gamma / n_b \sim 10^5$ **radiation mediated shocks**

Zeldovich, Raizer 1966
Weaver 1976
Blandford, Payne 1981
Budnik et al. 2010
Levinson 2012
AB 2017

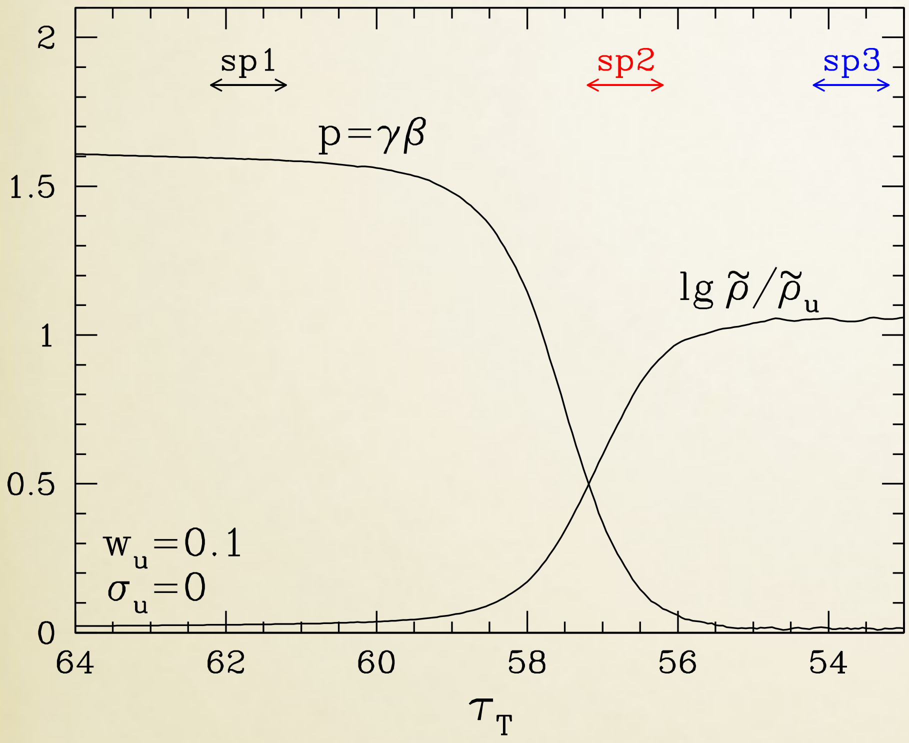
Do GRB shocks generate energetic particles?
Do GRB shocks create e+- pairs?

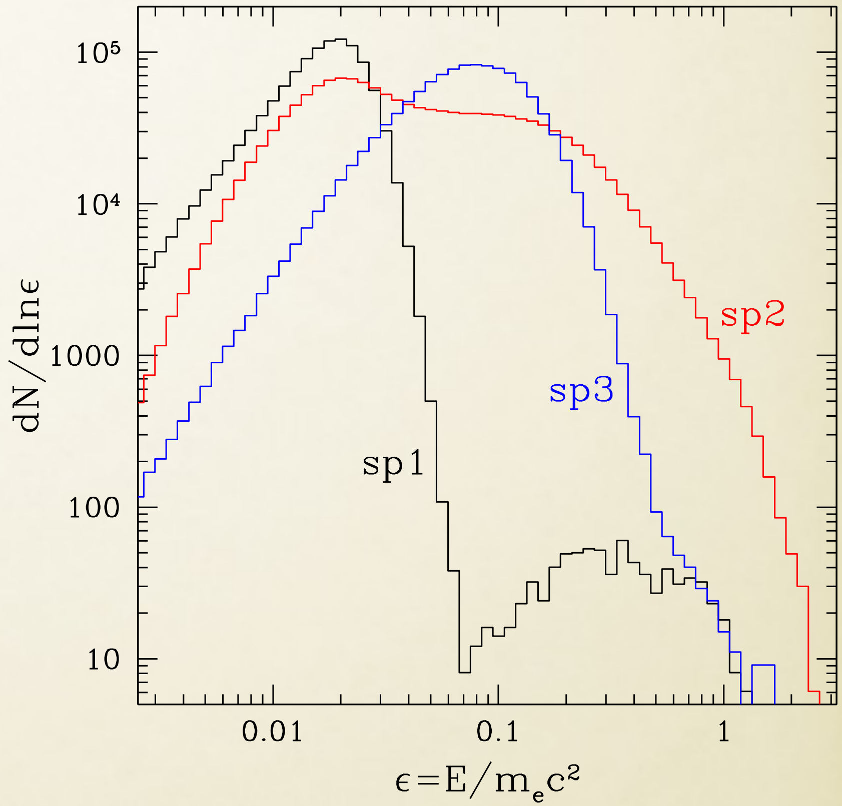
Radiation MHD from first principles: “Photon In Cell”

Fluid motion: Lagrangian grid

Radiation: individual photons
Monte-Carlo scattering

Radiation mediated shock (B=0)



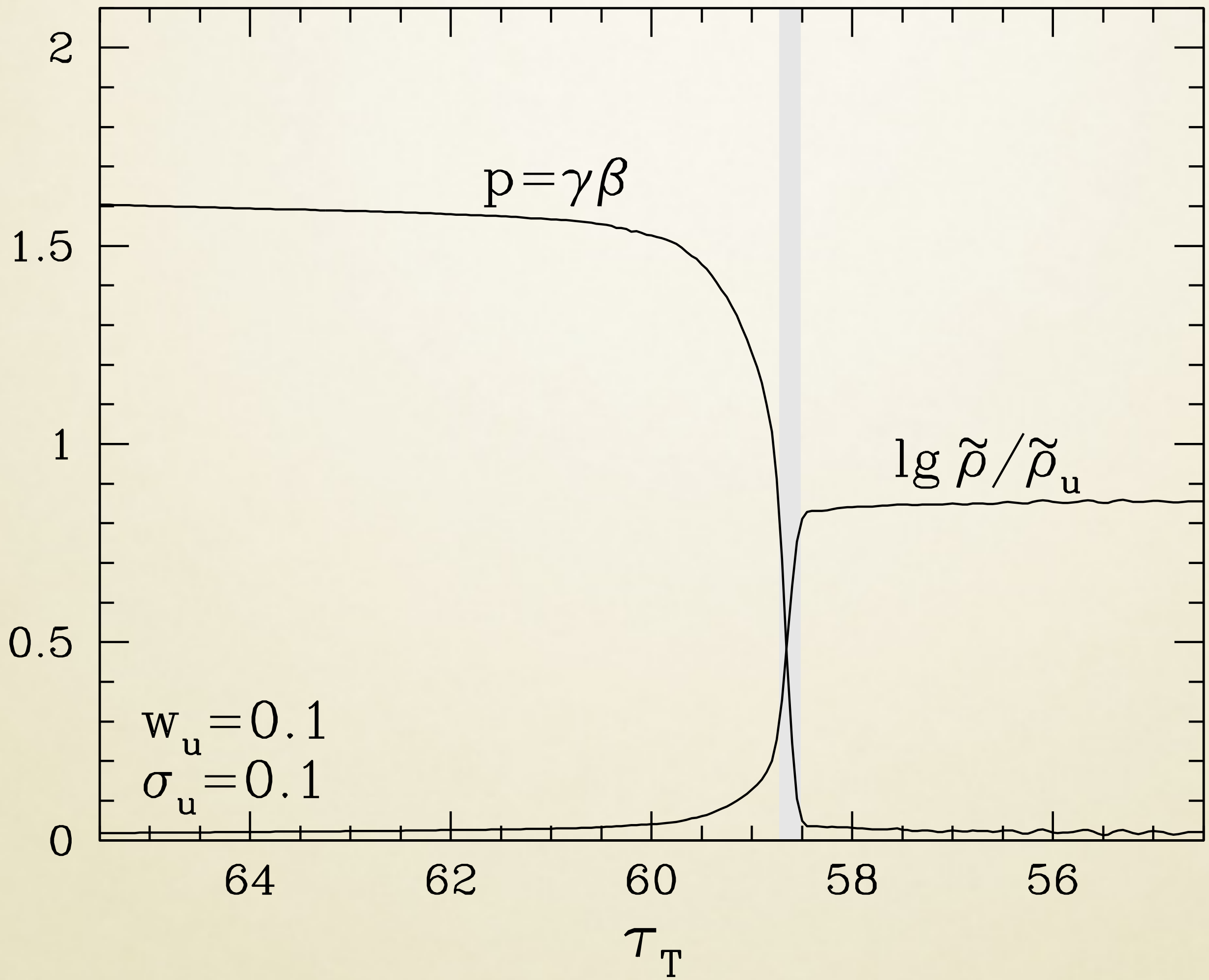


Bulk Comptonization, Klein-Nishina, and pair creation

$$n_\pm \sim 10^{-3} n_\gamma \qquad Z_\pm \sim 10^2 \left(\frac{n_\gamma/n_b}{10^5}\right)$$

AB 2017

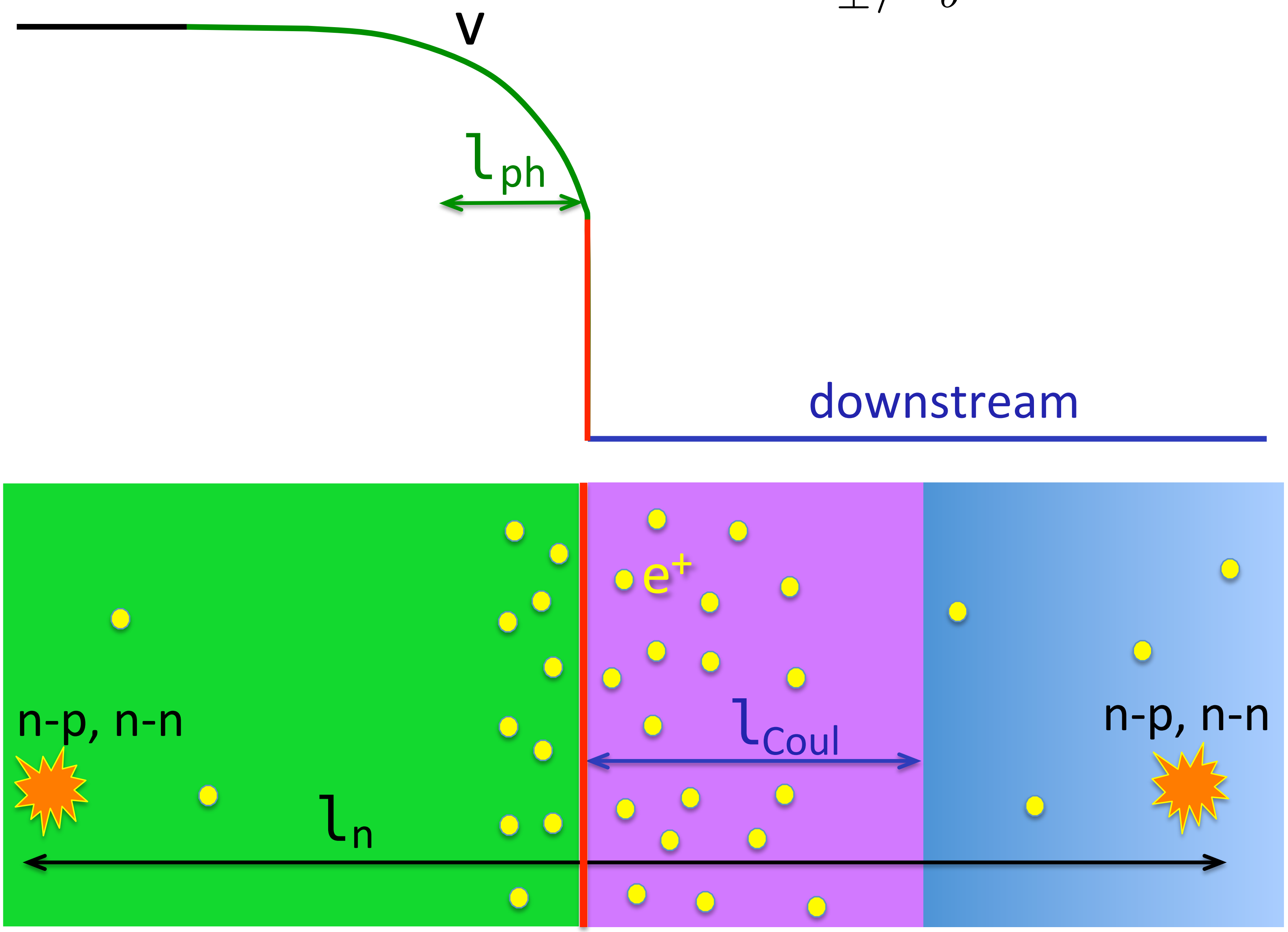
Shock in a magnetized flow



Shock structure:

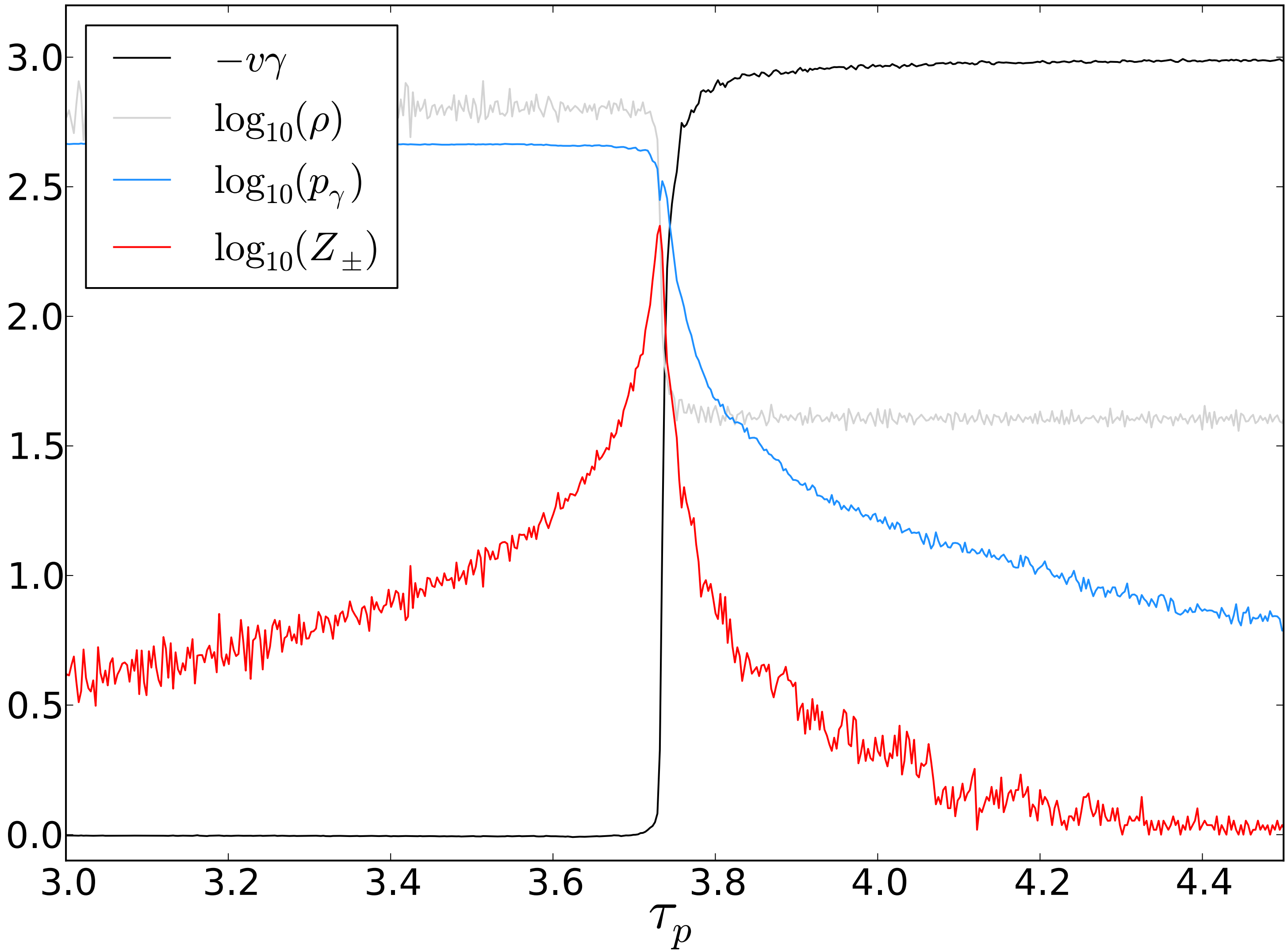
e+- dressed

$n_{\pm}/n_b \sim 10^2$



AB 2017

Shock structure with pair creation



cold plasma; pair creation without high-energy particles!

Lundman, AB, Vurm 2017

Consequences of pair creation:

– pairs increase optical depth and give “grip” to radiation
  => upstream decelerates
– energy per electron is reduced

Velocity profile between upstream and downstream is shaped by radiation pressure + collisionless jump

Pairs in the shock are producers of inverse Compton and synchrotron radiation.

=> injection of e+- stream ahead of the shock

Weakly magnetized relativistic shocks in transparent plasma (particle accelerators):

What happens at high compactness (bright compact objects), where particle acceleration leads to e+- creation?

– generation of long-lived magnetic fields? Derishev et al. 2016
– bootstrap/new nonlinear shock structure?
– characteristic self-regulated radiation spectra?
– limit cycle?

Challenge: huge difference in scale between plasma and radiative processes

Summary

New era of kinetic plasma simulations

+ detailed radiative simulations

Exa-rescaling (depending on the problem)

Magnetic reconnection < shocks < pulsars