# Quantum tunneling driven by three-body scattering in bosonic Josephson junctions

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We studied quantum dynamics of ultacold atomic gases in symmetric single-particle potential with separated minima. Two level approximation for Bose-Einstein condensates is used modified for effects due to two- and three- body particles interactions. Classical Hamiltonian in phase space is derived in order to study time-evolution of inter-trap population imbalance and phase diference.

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## INTRODUCTION

B.D. Josephson predicted that there is a finite probability finding Cooper pairs on thin dielectric barrier linking two superconductors[1]. Both non-stationary and stationary quantum tunneling are observed [2] depending if the sandwich system (Josephson junction) is biased to an external voltage or not. Superfluid Helium shows similar oscillation patterns [3]: there is formal analogy with superconductors the voltage being replaced by pressure exerted on the ends of small aperture connecting superfluids.

In Bose Einstein condensate (BEC) of atomic gases leaked in double-well single particle potentials collective quantum states formed in its local minima exchange particles as well by the tunneling effect [4]. Besides linear Josephson current non-linear transport develops in the system of bosons when interaction energy increases, eventually leading to macroscopic quantum self-trapping (MQST) [5, 14]. Bosonic Josephson junctions of exciton-polaritons, quasi-particles arising in strong coupling of semiconductor excitons and confined photon mode[6, 7], are considered [9–12] and recently experimentally reported [13]. Polaritonic Josephson junctions demonstrate in addition to the aforementioned anharmonicity (being consequence of two-particle scattering) and intrinsic Josephson-type dynamics related to polariton pseudospin, i.e polarization degree of freedom [8].

In theoretical treatment of Josephson junctions of cold atoms and polaritons one usually resort to two-mode approximation postulating localized solutions of Schr odinger equation for adjacent wells but neglecting interactions [14]. Many-body-effects are included then by standard time-dependent Gross-Pitaevskii equation (GP) for a given Josephson-type potential [5, 14]which long-range order parameter is taken sum of the two localized modes. The postulated modes are not eigenstates of the system but overlap, allowing constant coupling of particles seating in different traps. This is reasonable approximation as far as the BEC density is low [15]. With the interactions becoming non-negligible in the junction, instead starting from ordinary Schrdinger equation it is necessary [16, 17] to consider steady-state solutions of Schr odinger equation with cubic non-linearity (GP equations) in the first step of the formalism. The cubic term accounts for two-body contact interactions becoming significant in magnitude. In the low interaction limit the last approach called variational tunneling method (VAT)[16] corrects the initial model in making tunneling parameter a function of instantaneous values of phase and population difference between traps.

BEC of high densities was engineered on microfabricated magnetic trap ($2-3\times10^6$ atoms [18]) or in one-dimensional $^{87}Rb$ Bose gas waveguide [19]. Stability of BECs is maintained against matter-wave collapse in these experiments. Notwithstanding the predictions of GP theory, fails of long-range order parameter surpasses $na^3 \approx 10^{-3}$ bound, where $n$ is the condensate density and $a$ is s-wave scattering-length [15]. Increase in the number of atoms per unit volume inevitably leads to three-body contact interaction of atomic species as probabilities of scattering increase [20, 21]. In the former reference authors extended GP model with an effective potential depending on the square of the density (quintic term) to account for three-body scattering. Their repulsiveness cancels attractive interactions of two-body scattering leading to the stability and preservation of coherent properties of BEC. This kind of stability is well known for an other phenomenon: dissipative solitons [22], if optical pumping in the system matches losses due to recombination processes. It is also known that in BEC of $^{87}Rb$ atoms imaginary part representing recombination is three to four orders of magnitude lesser then the real one being three-body scattering energy [21] providing long coherence time during which integrability can be assumed [23].

In this article we have studied Josephson junction of BEC of cold atoms focusing on the effect of three-body scattering on tunneling current. Expanding long-range order parameter of the total system on symmetric and antisymmetric basis we derived classical Hamiltonian in terms of standard variables: population imbalance and phase difference between traps. We apply Lagrangian formalism using appropriate non-linear constrain for the situation we studied motivated by VAT technique used in the reference [16]. Phase diagrams clearly shows appearance of new tunneling regimes associated with three-body scattering. Full second quantization Hamiltonian is derived perturbatively in terms of angular momentum, i.e su(2) algebra. We discussed occurrence of Shapiro resonances when the tunneling energy equals one due to quintic nonlinearities. Realistic system properties are considered based on present experimental facilities.

## MODEL

As we consider spatially homogenous quantum transport we reduce to 1+1 dimensional space for the order parameter $\Psi = \Psi(x,t)$ normalized to total number of atom $\int_{\mathbb{R}} \mathrm{d}x\Psi = N$. It obeys following Lagrangian

$$\mathcal{L} = \frac{i\hbar}{2}(\Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t}) - \mathcal{H}, \tag{1}$$

with Hamiltonian of the effective potential consisting of the double-trap external potential $V(x)$ and the cubic-quintic nonlinearities

$$\mathcal{H} = \frac{\hbar^2}{2m}|\nabla\Psi|^2 + (V(x) + \frac{g_2}{2}|\Psi|^2 + \frac{g_3}{3}|\Psi|^4)|\Psi|^2 \tag{2}$$

$g_2$ and $g_3$ are $s$-wave scattering length dependent interaction strengths [21]. Variating eq.(1) with respect to dual of the order parameter $\Psi^*$ one immediately obtain GP equation with quintic term. To study tunneling effect we decompose

$$\Psi(x,t) = \alpha_+(t)\Psi_+(x) + \alpha_-(t)\Psi_-(x) \tag{3}$$

where symmetric (+) and antisymmetric (−) states being linear combinations of localized eigenstates of double-well potential $\Psi_\pm = (1/\sqrt{2})(\Psi_1 \pm \Psi_2)$ participate with $\alpha_\pm(t)$ complex coefficients. They are taken to fulfill

$$E_\pm|\Psi_\pm|^2 = \frac{\hbar^2}{2m}|\nabla\Psi_\pm|^2 + (V(x) + \frac{g_2}{2}|\Psi_\pm|^2 + \frac{g_3}{3}|\Psi_\pm|^4)|\Psi_\pm|^2. \tag{4}$$

with proper energies $E_\pm$. Plugging eq.(3) the Hamiltonian (2) after averaging over spatial coordinate becomes

$$\langle\mathcal{H}\rangle = \frac{1}{2}\mu_{+-}\gamma_1 + |\alpha_\pm|^2(E_\pm - \frac{1}{2}\mu_{\pm\pm} - \frac{1}{3}\nu_{\pm\mp} + \frac{1}{3}\nu_{\pm\mp}\gamma_2) + |\alpha_\pm|^4\frac{1}{2}\mu_{\pm\pm} + |\alpha_\pm|^6\frac{1}{3}\nu_{\pm\pm}, \tag{5}$$

with summing over all repeated subscripts. The overlap integrals of the real valued modes are defined as

$$\mu_{ij} = g_2\int_{\mathbb{R}} dx\Psi_i^2\Psi_j^2, \tag{6}$$

$$\nu_{ij} = g_3\int_{\mathbb{R}} dx\Psi_i^4\Psi_j^2, \tag{7}$$

$(i,j = +,-)$ and terms mixing complex expansion coefficients reads

$$\gamma_l = a_{1l}|\alpha_+\alpha_-|^2 + a_{2l}\alpha_+^2\alpha_-^{*2} + a_{3l}\alpha_+^{*2}\alpha_-^2, \tag{8}$$

vectors $a_l$ are $a_1 = (4,1,1)$ and $a_2 = (9,3,3)$. Relation for expansion coefficients of total wave function to basis of localized functions $\alpha_{1,2}(t) = |\alpha_{1,2}(t)|\exp(i\theta_{1,2}(t)) = (1/\sqrt{2})(\alpha_+(t) \pm \alpha_-(t))$ with help of standard variables in Josephson junctions physics: population imbalance $z(t) = (|\alpha_1(t)|^2 - |\alpha_2(t)|^2)/N$ and phase difference $\phi(t) = \theta_1(t) - \theta_2(t)$ map terms $|\alpha_\pm|^2$ and $\alpha_+\alpha_-^*$ figuring in eq. (5) and eq. (8) to

$$|\alpha_\pm|^2 = \frac{N}{2}\left(1 \pm \sqrt{1-z^2}\cos\phi\right), \tag{9}$$

$$\alpha_+\alpha_-^* = \frac{N}{2}\left(z - i\sqrt{1-z^2}\sin\phi\right). \tag{10}$$

Higher degree terms of left hand side of eq. (9) and complex conjugate and its powers of eq. (10) can be easily deduced from the last expressions thus the Hamiltonian (5) transforms to

$$\langle\mathcal{H}\rangle = \frac{A+B_2}{2}z^2 + \sum_{m=0}^{3} B_m\left(1-z^2\right)^{m/2}\cos^m\phi \tag{11}$$

with coefficients per number of particles

$$\frac{2A}{N} = \frac{N}{4}\left(\frac{23}{2}\mu_{+-} - \mu\right) + \frac{N^2}{4}\left(\frac{39}{2}\nu_S - \nu\right), \tag{12}$$

$$\frac{2B_0}{N} = (E_+ + E_-) + \frac{1}{2}\left(\frac{N}{2} - 1\right)\mu$$
$$\frac{1}{3}\left(\frac{N^2}{4} - 1\right)\nu + \frac{N}{8}\left(N\nu_S + \mu_{+-}\right),, \tag{13}$$

in difference with cubic nonlinearity case $B_l$ becomes function of population imbalance

$$B_1(z) = B_1^{'} + B_1^{''}z^2, \tag{14}$$

$$\frac{2B_1^{'}}{N} = \Delta E - \frac{1}{2}\Delta\mu - \frac{1}{3}\Delta\nu + \frac{N}{2}\left(\Delta\mu + \frac{N}{2}\Delta\nu + \frac{N}{4}\Delta\nu_A\right) \tag{15}$$

$$\frac{2B_1^{''}}{N} = \frac{19N^2}{8}\Delta\nu_A, \tag{16}$$

$$\frac{2B_2}{N} = \frac{N}{4}\left(\mu - \frac{1}{2}\mu_{+-} + N\left(\nu - \frac{1}{2}\nu_S\right)\right), \tag{17}$$

$$\frac{2B_3}{N} = \frac{N^2}{12}(\Delta\nu - \frac{3}{2}\Delta\nu_A). \tag{18}$$

$\Delta E = E_+ - E_-$, $\Delta\mu = \mu_{++} - \mu_{--}$, $\Delta\nu = \nu_{++} - \nu_{--}$,$\mu = \mu_{++} + \mu_{--}$,$\nu = \nu_{++} + \nu_{--}$, $\nu_S = \frac{1}{2}(\nu_{+-} + \nu_{-+})$, $\Delta\nu_A = \frac{1}{2}(\nu_{+-} - \nu_{-+})$ ,

$$\langle\mathcal{H}\rangle = \frac{A}{2}z^2 + B_1(z)\left(1-z^2\right)^{1/2}\cos\phi + \frac{3}{4}B_3\left(1-z^2\right)^{3/2}\cos\phi \quad (19)$$

$$\frac{1}{2}B_2\left(1-z^2\right)\cos 2\phi + \frac{1}{4}B_3\left(1-z^2\right)^{3/2}\cos 3\phi + B_0 + \frac{1}{2}B_2,$$

$$\dot{z} = -B_1(z)\left(1-z^2\right)^{1/2}\sin\phi - \frac{3}{4}B_3\left(1-z^2\right)^{3/2}\sin\phi \quad (20)$$

$$-B_2\left(1-z^2\right)\sin 2\phi - \frac{3}{4}B_3\left(1-z^2\right)^{3/2}\sin 3\phi,$$

$$\begin{aligned}\dot{\phi} &= -Az + B_1^{'}z\left(1-z^2\right)^{-1/2}\cos\phi + \quad (21)\\ &- B_1^{''}\left(2-3z^2\right)z\left(1-z^2\right)^{-1/2}\cos\phi\\ &+ \frac{3}{2}B_3 z\left(1-z^2\right)^{1/2}\cos\phi + B_2 z\cos 2\phi\\ &+ \frac{3}{4}B_3 z\left(1-z^2\right)^{1/2}\cos 3\phi,\end{aligned}$$


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