Holographic Tensor Models

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September 13, 2018

Context: a survey

- 2015: SYK: disordered condensed matter system of N Majorana fermions

$$H = -\sum_{1\leq i<j<k<l\leq N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$$

$$\{\chi_i, \chi_j\} = \delta_{ij} \quad \left\langle J_{ijkl}^2 \right\rangle = \frac{3!}{N^3} J^2$$

  - solvable at large N,
  - presents black hole features (approximate conformal symmetry and maximal chaos, at strong coupling)

$\to$ simplest model of holography ($nAdS_2/nCFT_1$)

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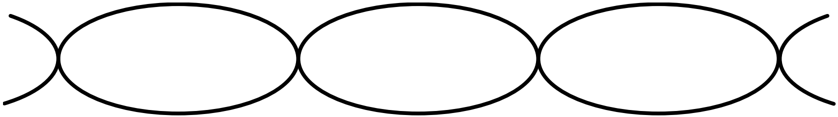
- 2016: ['16 Witten] A non-disordered quantum model obeys the same leading order Schwinger-Dyson equations as SYK, allegedly with a clearer holographic dual. It was inspired by tensor models.

Predecessors

Vector models:

$$Z(g) = \int \mathcal{D}\phi \; e^{-\frac{1}{2}\phi^2 - \frac{g}{4!N}\phi^4}$$



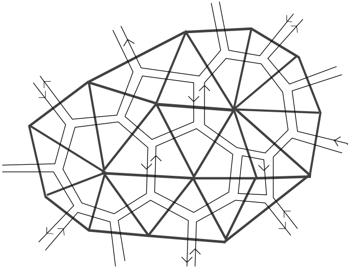
(['03 Moshe et Zinn-Justin] for a review)

Geometry of branched polymers (fractal trees).
[In $d = 3$ ($N = 1$) dual to 3d Ising at T_c and $d \geq 2$ to higher-spin theory in AdS_{d+1}.]

Predecessors

Matrix models:

$$Z(g) = \int \mathcal{D}M\mathcal{D}M^\dagger \; e^{-\frac{1}{2}\,\mathrm{Tr}\,|M|^2 - \frac{g}{3!\sqrt{N}}\,\mathrm{Tr}\,|M|^3} = \sum_h N^{2-2h} Z_h(g)$$



['81 Polyakov, '86 Kazakov, '88 David, '16 Miller, Sheffield, ...]
(['93 DiFrancesco et al] for a review)

Geometry of planar graphs.
[Discretization of 2d Liouville theory and its continuous limit (double scaling) is the Brownian map.]

Tensor models: Aim and Tools

Tensor models: most obvious attempt to quantize gravity in higher dimensions. But first attempts ['91 Ambjorn et al., '92 Boulatov, etc.] were plagued by singular manifolds and didn't have a large N expansion.
What brought the success?...

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The field: $T_{a_1 \ldots a_D}$ rank D (unsymmetrized) tensor, transforms under $G^{\otimes D}$ (G of rank N):

$$T'_{b^1 \ldots b^D} = \sum_a U^{(1)}_{b^1 a^1} \ldots U^{(D)}_{b^D a^D} T_{a^1 \ldots a^D} \;, \quad U^{(i)} \in G \;.$$

Index stands for (discretized) space or is an abstract dof.

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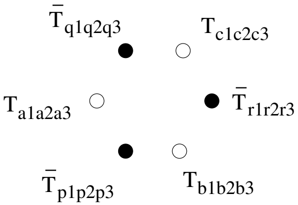
Action and Observables: $G^{\otimes D}$-invariants ("bubbles").

Tensor invariants as Colored Graphs

Example $(D = 3)$, $G = U(N)$

$$\sum \delta_{a^1 p^1}\delta_{a^2 q^2}\delta_{a^3 r^3} \quad \delta_{b^1 r^1}\delta_{b^2 p^2}\delta_{b^3 q^3} \quad \delta_{c^1 q^1}\delta_{c^2 r^2}\delta_{c^3 p^3}$$
$$T_{a^1 a^2 a^3}\, T_{b^1 b^2 b^3}\, T_{c^1 c^2 c^3}\, \bar{T}_{p^1 p^2 p^3}\, \bar{T}_{q^1 q^2 q^3}\, \bar{T}_{r^1 r^2 r^3}$$

White (black) vertices for T $(\bar{T})$.



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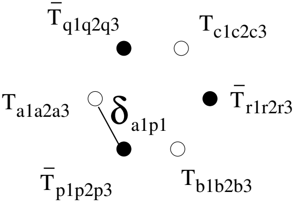
Edges for $\delta_{a^c q^c}$



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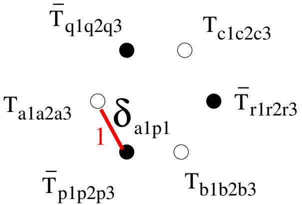

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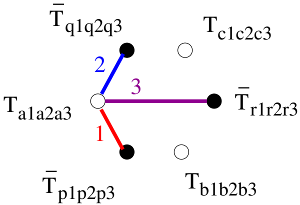

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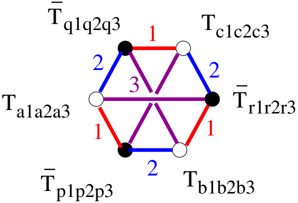

Tensor invariants as Colored Graphs

Example $(D = 3)$, $G = U(N)$

$$\mathrm{Tr}_{\mathcal{B}}(T, \bar{T}) = \sum \prod_{v} T_{a_v^1 \dots a_v^D} \prod_{\bar{v}} \bar{T}_{q_{\bar{v}}^1 \dots q_{\bar{v}}^D} \prod_{c=1}^{D} \prod_{e^c=(w,\bar{w})} \delta_{a_w^c q_{\bar{w}}^c}$$

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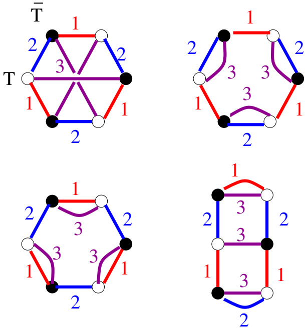

Single trace models

The action: "single trace" invariant

$$S(T,\bar T) = \underbrace{\sum T_{b^1\dots b^D}\,\bar T_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{b^c q^c}}_{\text{unique quadratic invariant}} + \underbrace{\sum_{\substack{\text{connected graphs } \mathcal{B}\\ \text{with D colors}}} t_{\mathcal{B}}\,\mathrm{Tr}_{\mathcal{B}}(T,\bar T)}_{\text{interaction}}$$

The partition function:

$$Z(t_{\mathcal{B}}) = \int [d\bar T dT]\; e^{-N^{D-1}S(T,\bar T)}$$

The gauge invariant observables:

$$\mathrm{Tr}_{\mathcal{B}}(T,\bar T)$$

Objective: compute $\log Z$, $\langle \mathrm{Tr}_{\mathcal{B}_1}(T,\bar T)\dots \mathrm{Tr}_{\mathcal{B}_2}(T,\bar T)\rangle$

Feynman expansion

$$S(T,\bar{T}) = \sum T_{b^1\dots b^D}\,\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{b^c q^c} + \sum_{\substack{\text{connected graphs } \mathcal{B} \\ \text{with D colors}}} t_{\mathcal{B}}\,\mathrm{Tr}_{\mathcal{B}}(T,\bar{T})\ ,$$

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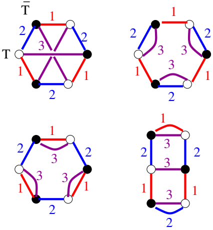
Feynman expansion:

- Taylor expand in $t_{\mathcal{B}} \to$ graphs with D colors



$$Z(t_{\mathcal{B}}) = \sum \int_{T,\bar{T}} e^{-N^{D-1}\left(\sum T_{b^1\dots b^D}\bar{T}_{q^1\dots q^D}\prod_{c=1}^{D}\delta_{b^c q^c}\right)} \mathrm{Tr}_{\mathcal{B}_1}(T,\bar{T})\,\mathrm{Tr}_{\mathcal{B}_2}(T,\bar{T})\dots$$

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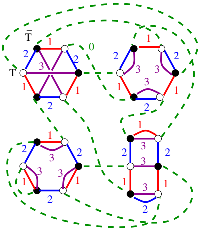
- compute the Gaussian integrals (Wick theorem) $\to$ graphs with $D+1$ colors



$$Z(t_{\mathcal{B}}) = \sum_{\text{graphs } \mathcal{G} \text{ with } D+1 \text{ colors}} A(\mathcal{G})$$

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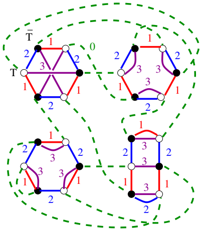
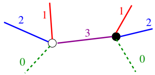


$$Z(t_{\mathcal{B}}) = \sum_{\text{graphs } \mathcal{G} \text{ with } D+1 \text{ colors}} A(\mathcal{G})$$

Each graph $\mathcal{G}$ is embedded in a D dimensional space (Poincaré dual to a triangulation)

Colored graphs and vertex colored triangulations

White and black $D+1$ valent vertices connected by edges with colors $0, 1 \dots D$.



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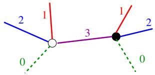
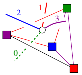
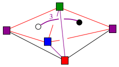


Vertex $\leftrightarrow$ D simplex with colored vertices .



Edges $\leftrightarrow$ gluings along $D-1$ simplices respecting all the colorings



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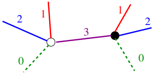

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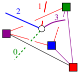

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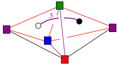
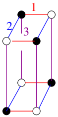
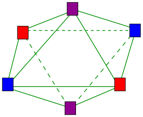


Invariants $\mathrm{Tr}_{\mathcal{B}}$: graphs with D colors $\leftrightarrow$ $D-1$ dimensional boundary triangulations.

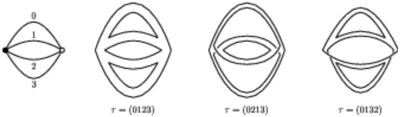




The $1/N$ expansion

Gurau's degree: Choose an order of the $D+1$ colors, this gives a ribbon graph ("jackets"), compute the associated genus ($2 - g_{\mathcal{J}} = V - E + F$), then

$$\omega = \sum_{\mathcal{J}} g_{\mathcal{J}} \quad \text{and} \quad \log Z = \sum_{\mathcal{G}} C_{\mathcal{G}} t^{b(\mathcal{B})} N^{D-\omega(\mathcal{G})}$$



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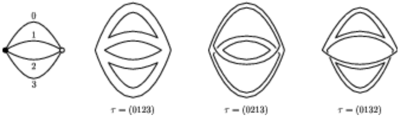
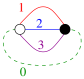


Leading order ($\omega = 0$): melons

Fundamental melon



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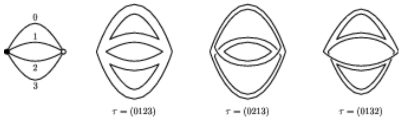
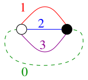
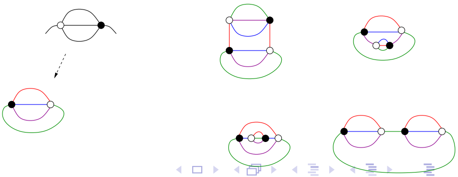


Leading order ($\omega = 0$): melons

Iterative insertions of 2 vertices connected by D edges

Fundamental melon





Important results

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- 2017: Melon dominance in irreps of $O(N)^3$ tensor models [Gurau, Benedetti et al., Carrozza]

The main models:

- Gurau-Witten: ψ_i ($0 \leq i \leq D$) real fermions of rank D

$$\mathbf{S}_{GW} = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - \frac{i^{(D+1)/2} j}{N^{D(D-1)/4}} \psi_0 \psi_1 \cdots \psi_D \right)$$

Symmetry group: $O(N)^{D(D+1)/2}$

The main models:

- Gurau-Witten: ψ_i $(0 \leq i \leq D)$ real fermions of rank D

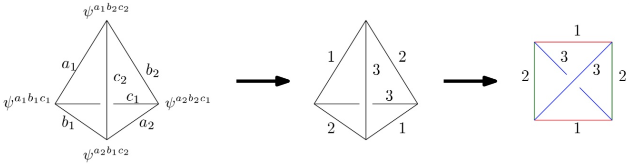
$$\mathbf{S}_{GW} = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - \frac{i^{(D+1)/2} j}{N^{D(D-1)/4}} \psi_0 \psi_1 \cdots \psi_D \right)$$

Symmetry group: $O(N)^{D(D+1)/2}$

- Carrozza-Tanasa-Klebanov-Tarnopolsky: ψ real fermion of rank D

$$\mathbf{S}_{\mathrm{CTKT}}[\psi] = \int \mathrm{d}t \left(\frac{1}{2} \psi_{abc} \partial_t \psi_{abc} + \frac{\lambda}{4N^{3/2}} \psi_{a_1 a_2 a_3} \psi_{a_1 b_2 b_3} \psi_{b_1 a_2 b_3} \psi_{b_1 b_2 a_3} \right) .$$

Symmetry group: $O(N)^D$



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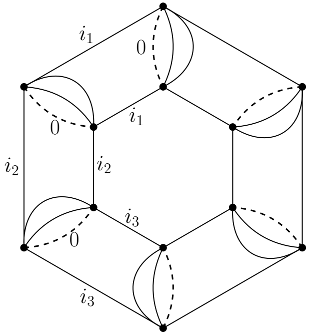
Example:



Those are not distinguished by cSYK, but well by GW.

Their main features:

- <u>Bilocal effective action</u>
  In order to study fluctuations around the conformal solution and path the way towards a holographic interpretation (e.g. "collective field theory"), ['18 Benedetti et al.] introduced 2PI effective action (below for a scalar field):

$$W[J,K] = \log \int \mathcal{D}\varphi \exp\left(-S[\varphi] + J_a\phi_a + \frac{1}{2}\phi_a K_{ab}\phi_b\right),$$

$$\Gamma[\phi, G] = -W[J,K] + \frac{\delta W}{\delta J_a}J_a + \frac{\delta W}{\delta K_{ab}}K_{ab}$$

$$= -W[J,K] + J_a\phi_a + \frac{1}{2}\phi_a K_{ab}\phi_b + \frac{1}{2}G_{ab}K_{ab},$$

$$\Phi_a[J,K] = \frac{\delta W}{\delta J_a}[J,K] \quad G_{ab}[J,K] = \frac{\delta^2 W}{\delta J_a \delta J_b}[J,K],$$

For $G_0^{-1} = \frac{\delta^2 S}{\delta \phi^2}$, expanding the functional integral defining Γ around the classical saddle $\varphi = \phi + f$ (obeying $\Gamma[\phi, G] = S[\phi]$), we get:

Their main features:

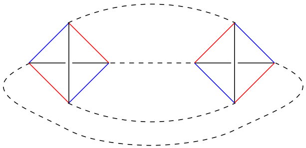
$$\Gamma[\phi, G] = S[\phi] + \underbrace{\frac{1}{2}\,\mathrm{Tr}\left[\log G^{-1}\right]}_{\text{quadradic terms in } f} + \frac{1}{2}\,\mathrm{Tr}\left[G_0^{-1} G\right] + \underbrace{\Gamma_2[\phi, G]}_{\text{generating function of 2PI graphs}} .$$

Then, study equations of motion of ϕ and G through a $1/N$ expansion keeping the relevant graphs.

- CTKT: (with ansatz $\psi = 0$)

$$\Gamma[0, G] = -\frac{1}{2}\,\mathrm{Tr}\left[\log G^{-1}_{a_1 a_2 a_3 b_1 b_2 b_3}\right] - \frac{1}{2}\,\mathrm{Tr}\left[\partial_t G_{a_1 a_2 a_3 b_1 b_2 b_3}(t, t')\right] + \Gamma_2$$

$\Gamma_2 =$



Their main features:

Using the ∂_t as a $O(N)^3$-breaking term, they reparametrize around the saddle $\bar{G}$

$$G_{\mathbf{ab}} = e^{H_{\mathbf{ac}}} \bar{G}_{\mathbf{cd}} e^{H_{\mathbf{db}}}$$

$$H_{\mathbf{ab}} = H^{(1)}_{a_1 b_1} \delta_{a_2 b_2} \delta_{a_3 b_3} + (\text{perm}) \quad e^{H^{(i)}} \in O(N)$$

and find an effective σ-model

$$\Gamma[0, H] = -\frac{\alpha}{2} \int \mathrm{d}t \; \partial_t H_{ac}(t) \partial_t H_{ac}(t)$$

$\rightarrow \frac{3}{2} N^2$ additional light modes to SYK.

- GW: (same ansatz $\psi = 0$) leads to similar analysis

$\rightarrow \frac{D(D+1)}{4} N^2$ additional light modes to SYK.

[And up to NNNLO, 2PI graphs resum as Tr log-terms, hence interpreted as one-loop gaussian integrals.]

Their main features:

- Group invariants
  They enter the action and constitute the observables. (The building blocks are single-trace.)
  - Vector models: $(\phi_i \phi_i)$
  - Matrix models: $\mathrm{Tr}(M^n)$
  - Tensor models: asymptotically (2k)-vertices $\sim k!$ (for bosons and fermions)
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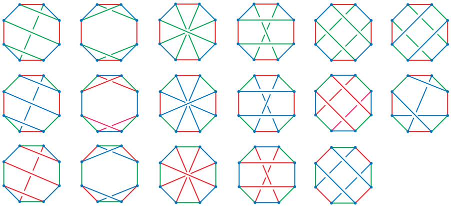
  Example: 8-vertices (gauge invariant) operators



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  - Numerical studies of the spectrum seem to present "dip-ramp-plateau" structure ['17,'18 Krishnan et al.] (But $N = 2$...)
  - Analytic bounds on the spectral range suggest a denser spectral density than in SYK ['18 Klebanov et al.]

Extensions:

- Higher dimensions: "Tensor field theories"
  For bosonic and fermionic models, ranges of dimensions are found without complex conformal dimensions of the bilinears (eg. "prismatic" theories) (bosons: ['17, '18 Klebanov et al.], fermions: ['17 Benedetti et al., Prakash et al.])

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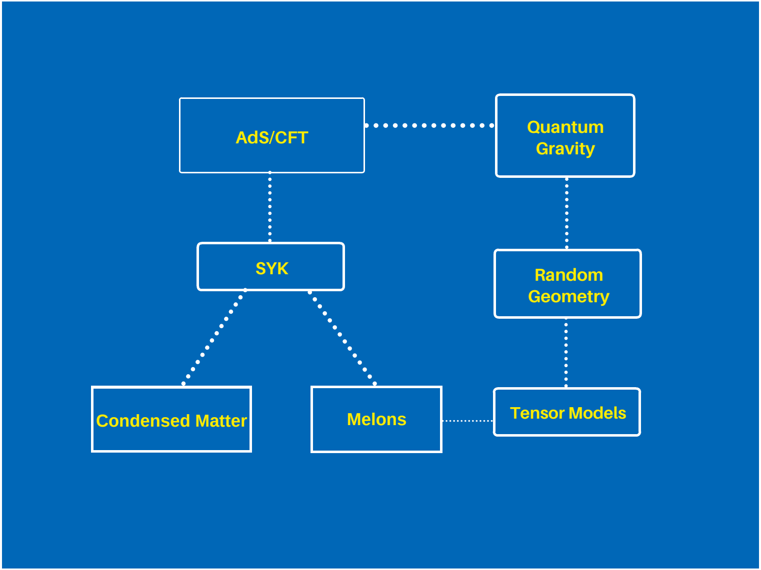
- Dual to richer higher spin theories? ['18 Vasiliev]

Conclusion: connections



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