# Strange insulators from pinninng of superstructure in holography

**Alexander Krikun**
*(Instituut Lorentz, Leiden)*

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# References


| | | |
|---|---|---|
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# Outline

1. **Spontaneous vs. explicit breaking of translations**
2. **Mott insulator as a commensurate charge density wave**
3. **Holographic Mott insulator**
4. **Toy model: Helix**
5. **Scaling properties of the pinned state**

# Translational invariance

Suppose the Lagrangian is translationally invariant

$$S = \int dx^d \mathcal{L}_0[\psi], \qquad \partial_{x_i} \mathcal{L}_0 = 0$$

Then the momentum is conserved as a corresponding Noeter charge

$$\partial_t P_i = 0$$

# Explicit breaking

If we add a position dependent term in the Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \lambda \mathcal{L}_1(x)$$

Then the Lagrangian is not invariant anymore

$$\partial_x \mathcal{L} \neq 0$$

The momentum is not conserved

$$\partial_t P \neq 0$$

and decays with characteristic time scale

$$\tau_P \sim \lambda$$

# Spontaneous breaking

Given the translationally invariant Lagrangian the ground state may nonetheless exhibit the spontaneous superstructure

$$\partial_x \mathcal{L}_0 = 0, \qquad \psi_0 = \psi_0(x), \qquad ||\psi_0|| \equiv \Sigma$$

The Noeter theorem still holds though and the momentum is conserved $\partial_t P = 0$.
The ground state is continuously degenerate since any $\psi_0(x + \delta x)$ is also a ground state.
There is a gapless sliding mode in the linearized spectrum: **the Goldstone boson.** $\psi_{GB} \equiv \partial_x \psi_0(x)$

$$\mathbf{E}[\psi_0(x + dx)] \equiv \mathbf{E}[\psi_0(x)] + \delta \mathbf{E}[\partial_x \psi_0] dx = \mathbf{E}[\psi_0(x)]$$

$$\delta \mathbf{E}[\partial_x \psi_0] = 0$$

# Explicit and Spontaneous breaking

If one adds an x-dependent deformation in the system with superstructure, this gives mass to the *pseudo*-Goldstone

$$\mathbf{E}[\psi_0(x+dx)] = \mathbf{E}[\psi_0(x)] + \lambda \mathbf{E}_1[\psi_0(x)]$$

$$\delta \mathbf{E}[\partial_x \psi_0] \sim \lambda \Sigma$$

The effective mass is proportional to the product of explicit and spontaneous symmetry breaking scales.
**The momentum mediated transport is gapped.**

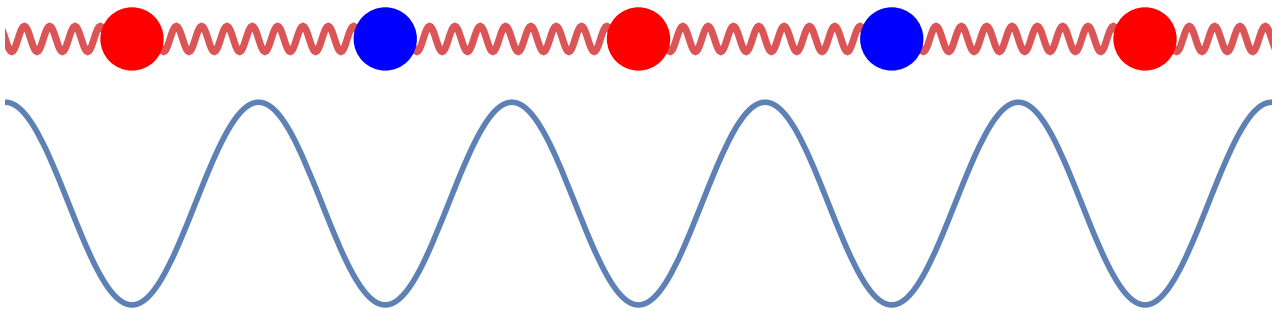
# Mott insulator as commensurate charge density wave

# Mott insulator

The crystal of charged particles can freely slide

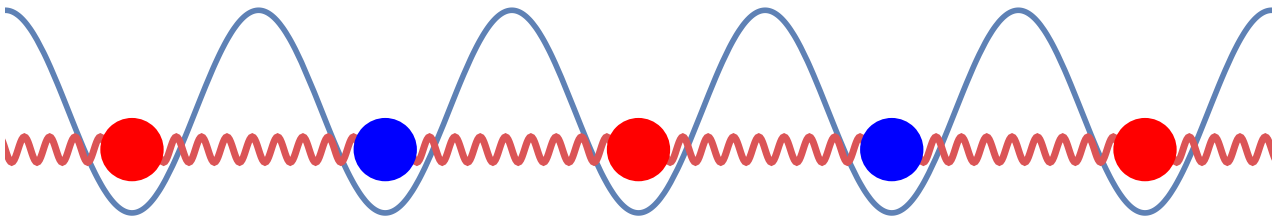
# Mott insulator

The weak pinning by ionic lattice produces resistivity



# Mott insulator

The strongly pinned crystal is a Mott insulator



# Holographic Mott insulator

# Holographic Mott insulator

Consider the model with inhomogeneous spontaneous superstructure

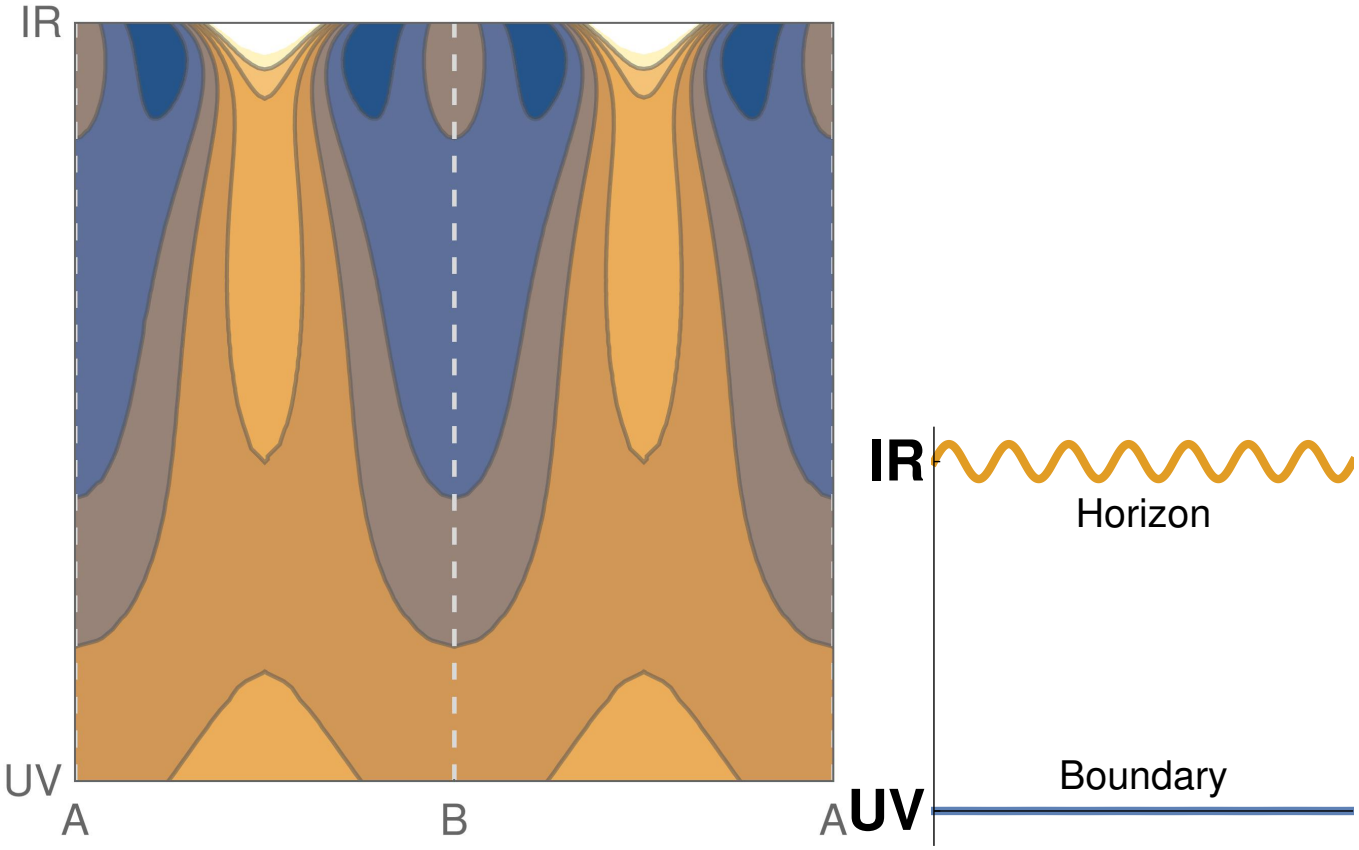
$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\psi)^2 - \frac{\tau(\psi)}{4}F^2 - V(\psi)\right) - \frac{1}{2}\int \vartheta(\psi)F\wedge F$$

and explicit lattice

$$\mu(x) = \mu_0\big(1 + A\cos(qx)\big)$$

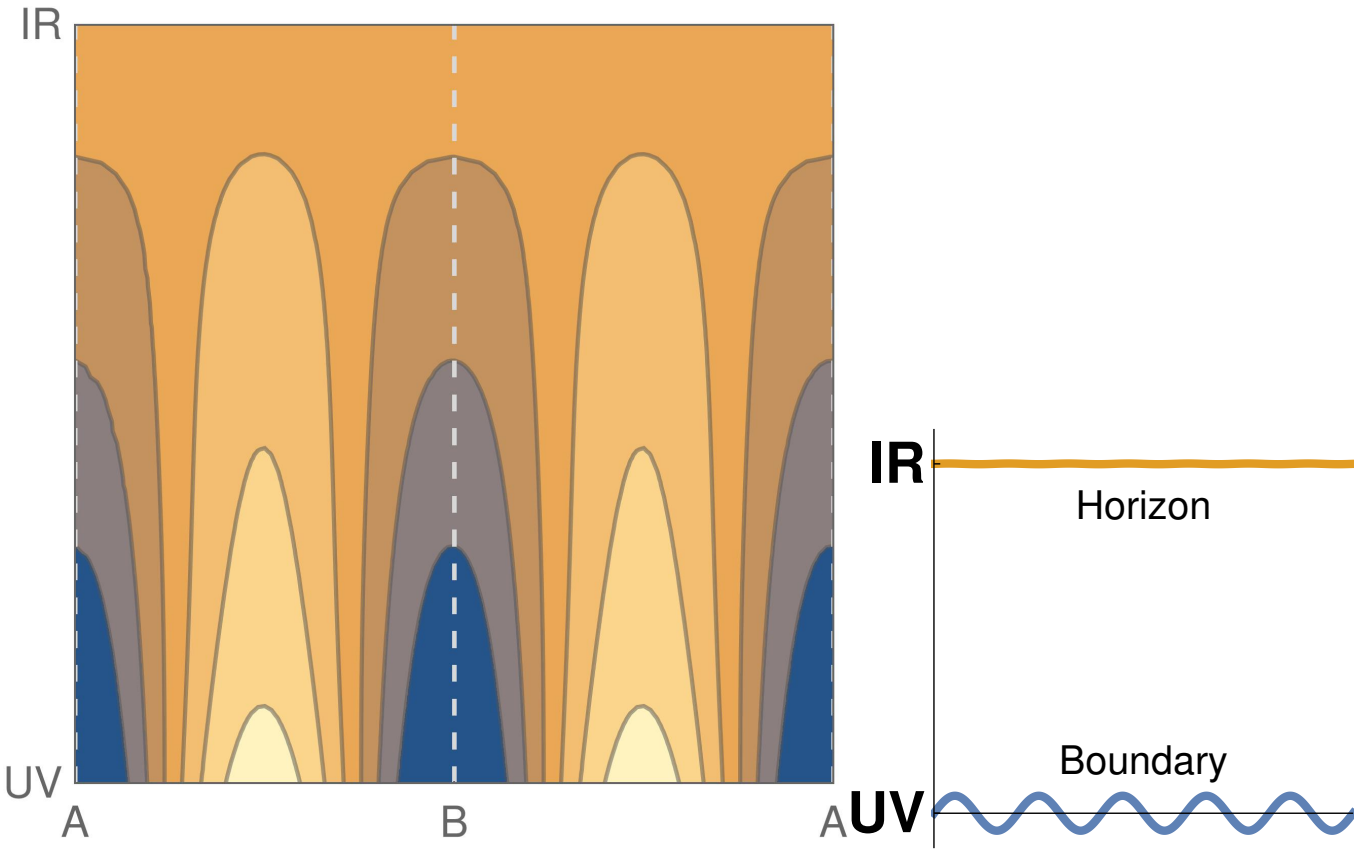
# Spontaneous structure

The spontaneous structure is **relevant in IR**.



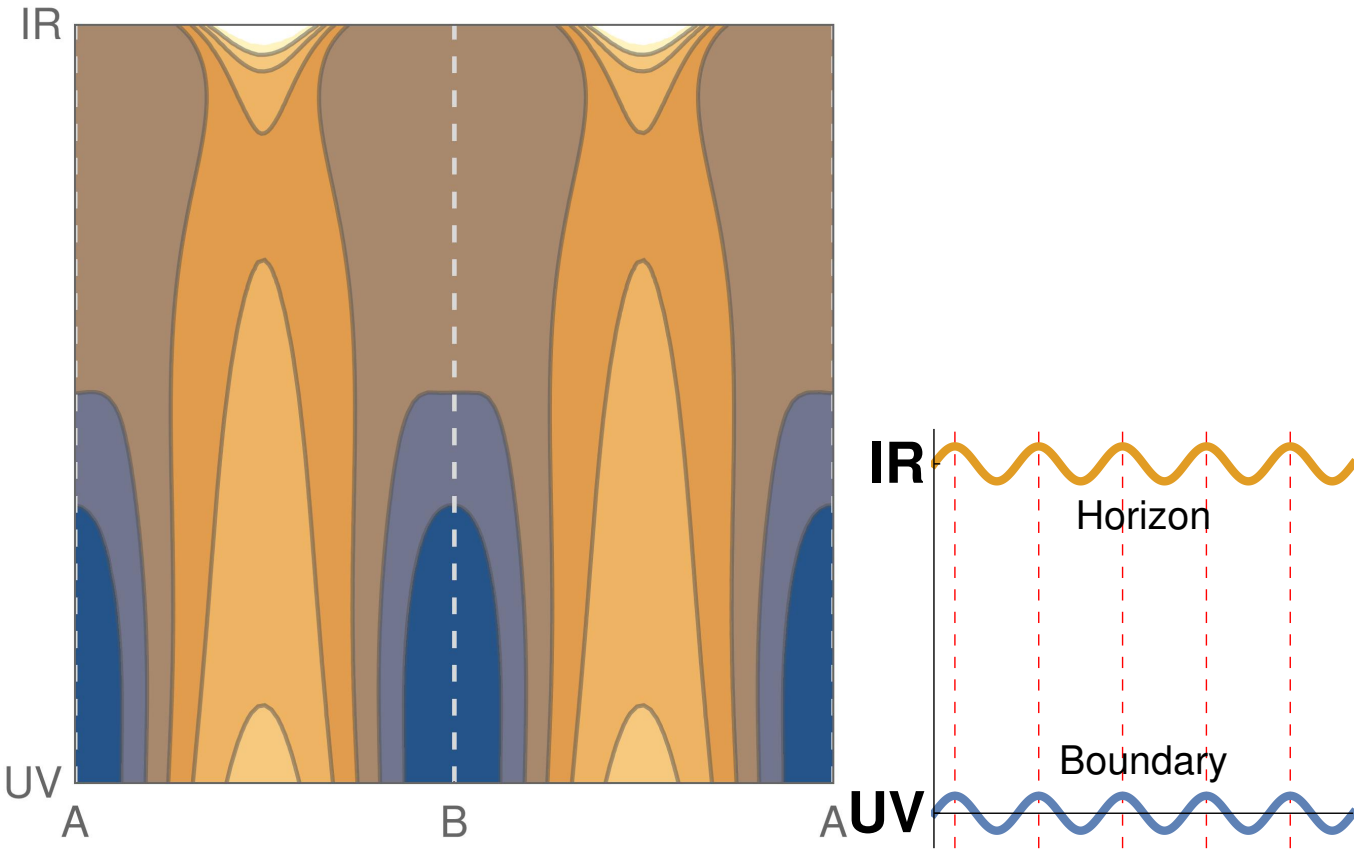
# Explicit lattice

Explicit lattice is **irrelevant in IR**



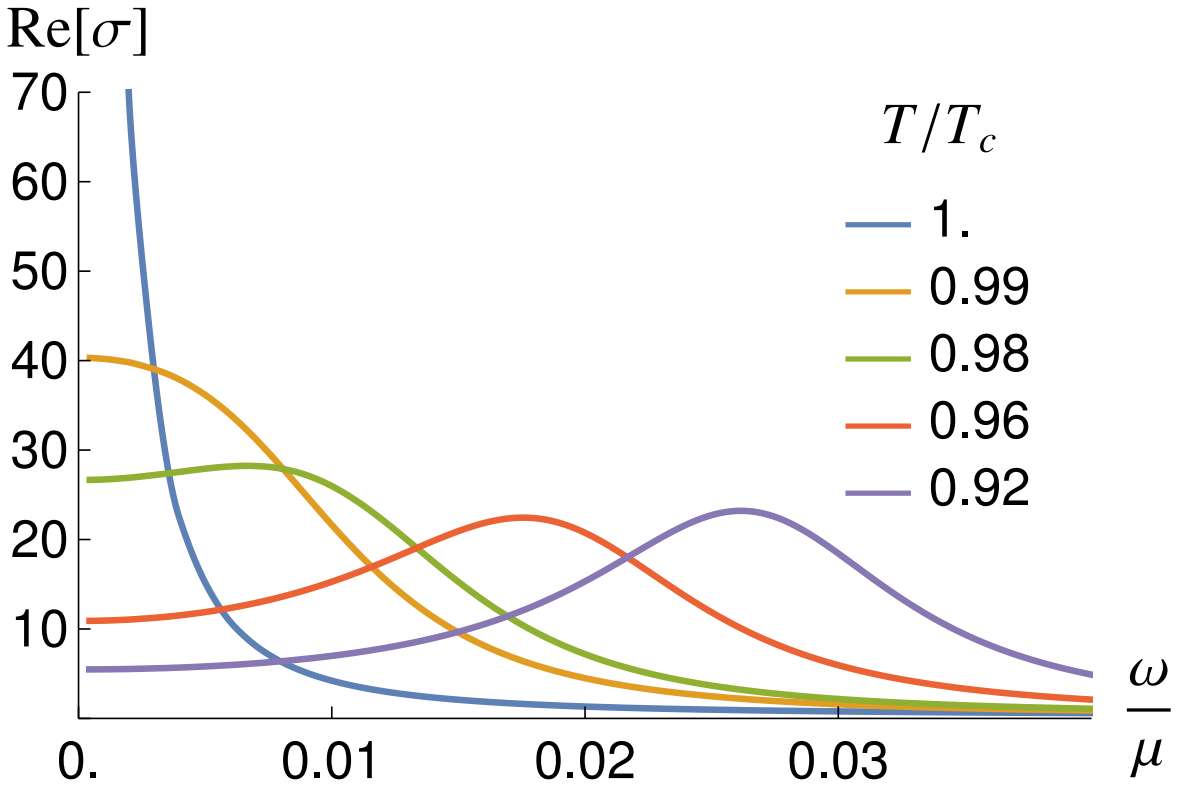
# Lock in

The **lock in** happens in the bulk



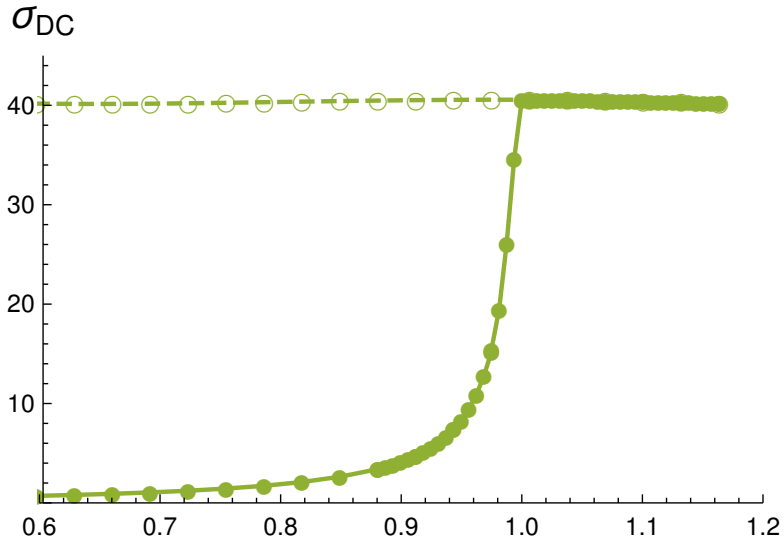
# AC conductivity

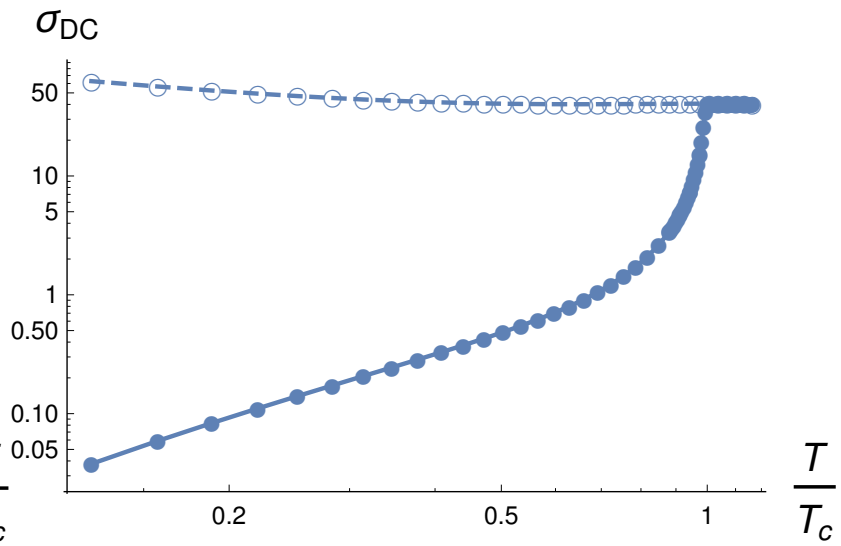
The spectral weight is **shifted** due to the pinning of Goldstone



# Insulating state

The pinned state is gapless insulating




What are the features of this state?

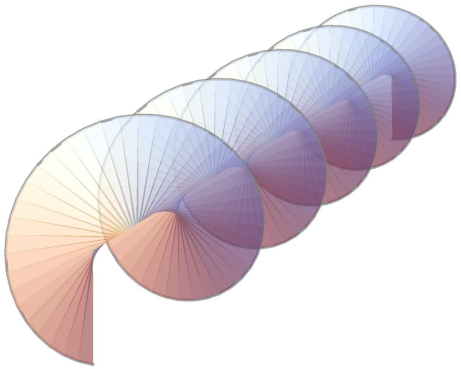
# Weak pinning
## *Helical toy model*

# Helical background

Same physics can be studied in the **homogeneous helical lattice**

$$S = \int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{4}W^2\right)$$
$$- \frac{\gamma}{6}\int d^5x A\wedge F\wedge F - \frac{\kappa}{2}\int d^5x B\wedge F\wedge W$$

$$\omega_1 = dx,$$
$$\omega_2 = \cos(px)dy - \sin(px)dz,$$
$$\omega_3 = \sin(px)dy + \cos(px)dz.$$

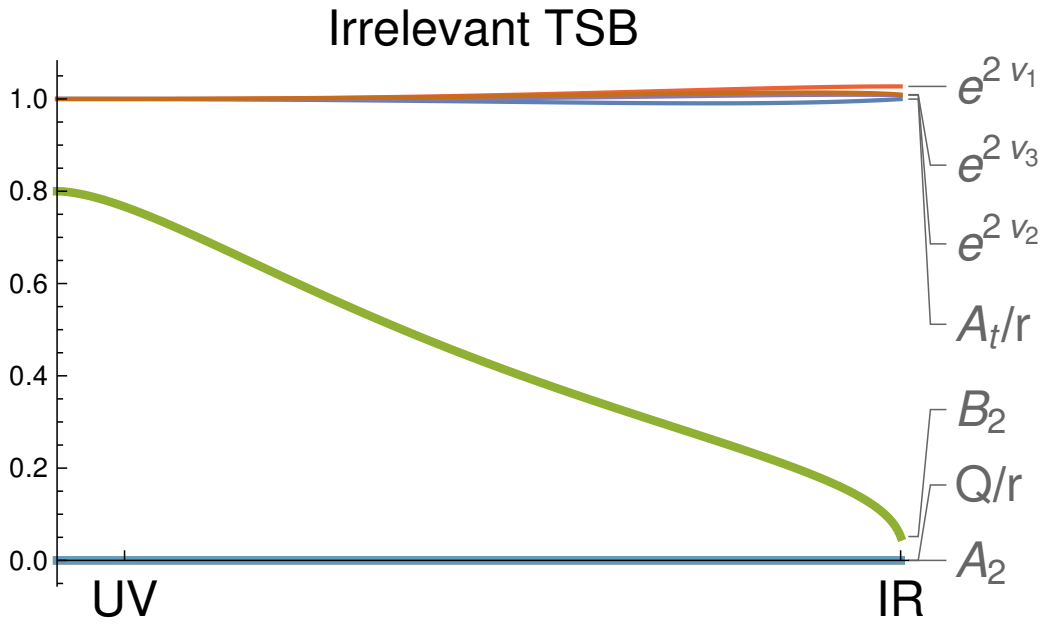


$$A = A_t dt + A_2\omega_2, \qquad A_t\Big|_{r\to\infty} = \mu$$
$$B = B_t dt + B_2\omega_2, \qquad B_2\Big|_{r\to\infty} = \lambda$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1}{\omega_1}^2 + e^{2v_2}(\omega_2 + Qdt)^2 + e^{2v_3}{\omega_3}^2,$$

# Irrelevant breaking

At small $\lambda$ the lattice is **irrelevant in IR**



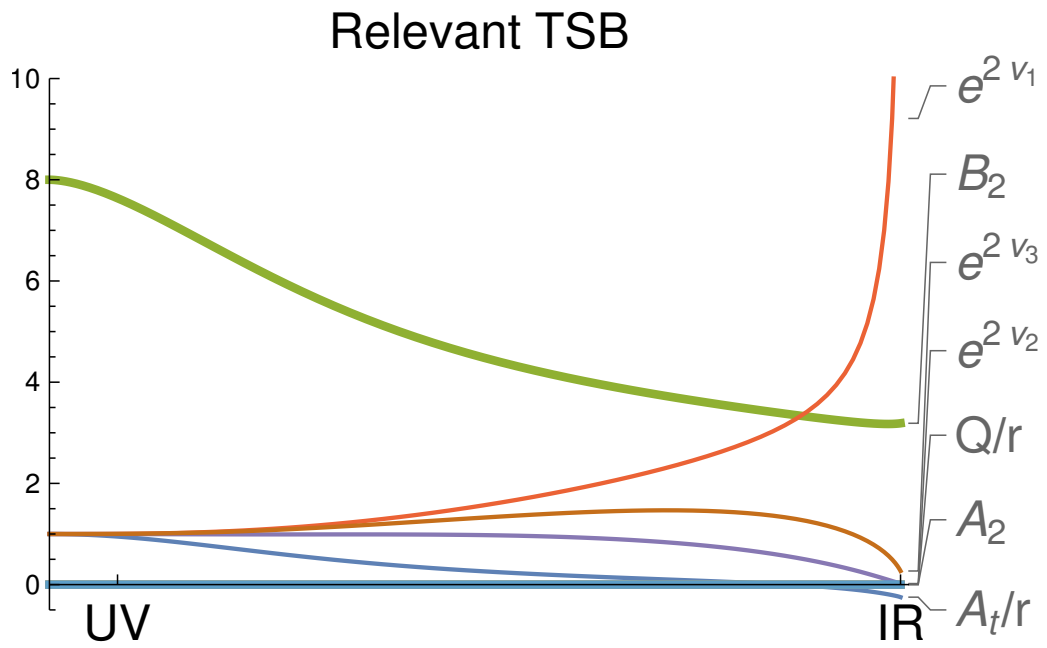

Zero $T$ geometry scales near horizon ($r = 0$) as

$$U \sim r^2, \qquad e^{v_1} \sim 1, \qquad e^{v_2} \sim 1, \qquad e^{v_3} \sim 1$$

$$A_t \sim r, \qquad B_2 \sim 0$$

# Relevant breaking

At large $\lambda$ the lattice is **relevant in IR**



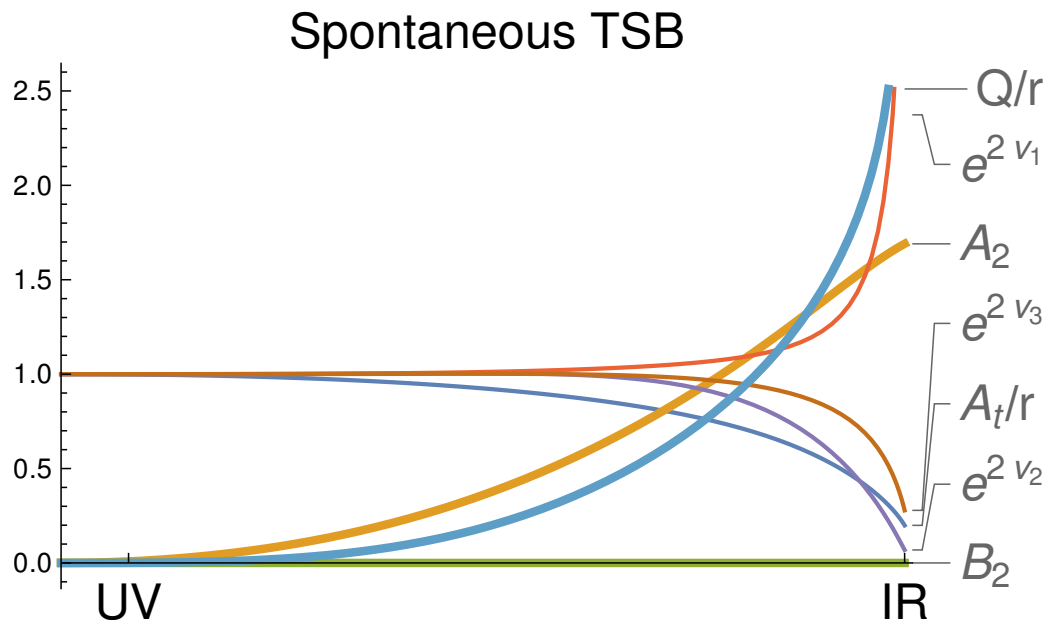

Zero $T$ geometry scales near horizon ($r = 0$) as

$$U \sim r^2, \qquad e^{v_1} \sim r^{-1/3}, \qquad e^{v_2} \sim r^{2/3}, \qquad e^{v_3} \sim r^{1/3}$$

$$A_t \sim r^{5/3}, \qquad B_2 \sim B_2^0 + B_2^1 r^{4/3}$$

# Spontaneous breaking

At $T < T_c$ the **spontaneous breaking is relevant in IR.**



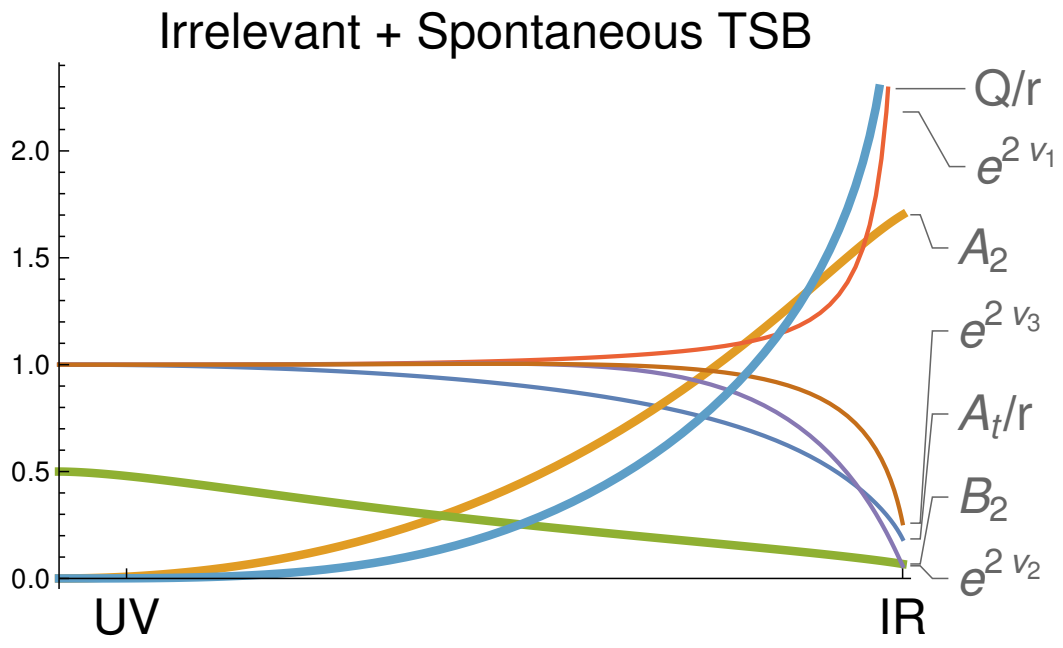

Zero $T$ geometry scales near horizon ($r = 0$) as

$$U \sim r^2, \qquad e^{v_1} \sim r^{-1/3}, \qquad e^{v_2} \sim r^{2/3}, \qquad e^{v_3} \sim r^{1/3}$$

$$A_t \sim r^{5/3}, \qquad A_2 \sim A_2^0 + A_2^1 r^{4/3}, \qquad Q \sim r^{2/3}$$

# Spontaneous and explicit breaking

Small explicit lattice does not affect near horizon geometry



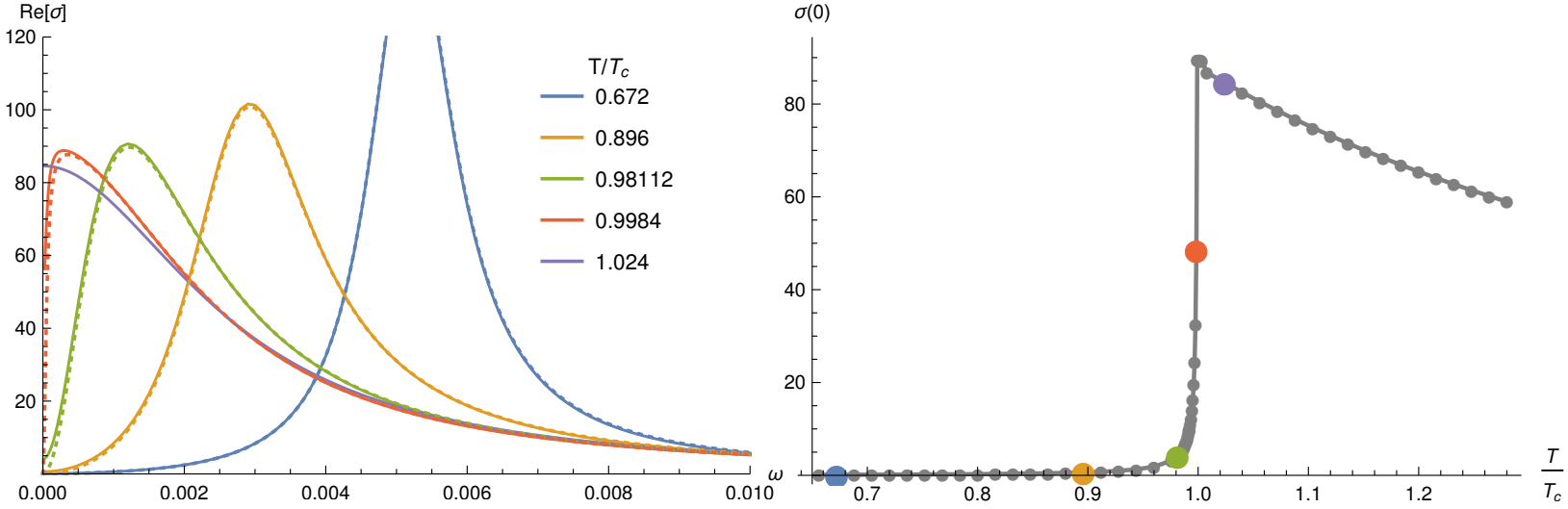

Zero $T$ geometry scales near horizon ($r = 0$) as

$$U \sim r^2, \qquad e^{v_1} \sim r^{-1/3}, \qquad e^{v_2} \sim r^{2/3}, \qquad e^{v_3} \sim r^{1/3}$$

$$A_t \sim r^{5/3}, \qquad A_2 \sim A_2^0, \qquad Q \sim r^{2/3}, \qquad B_2 \sim 0$$

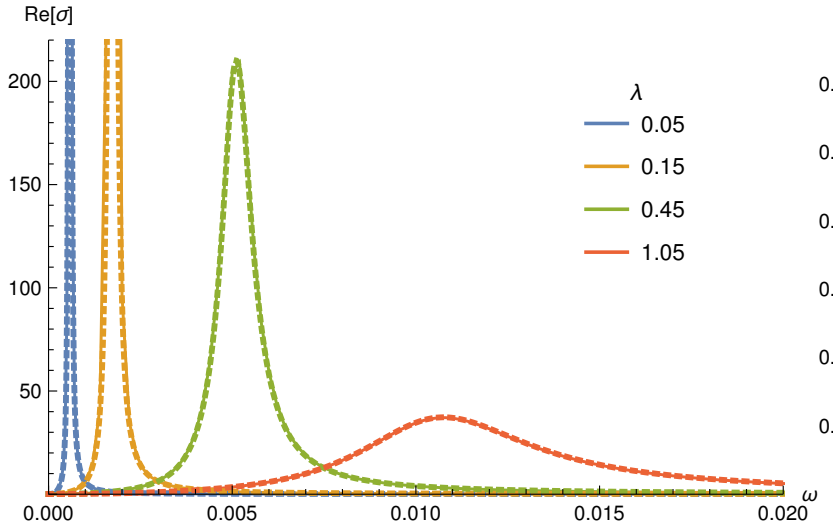
# Pinning of Goldstone

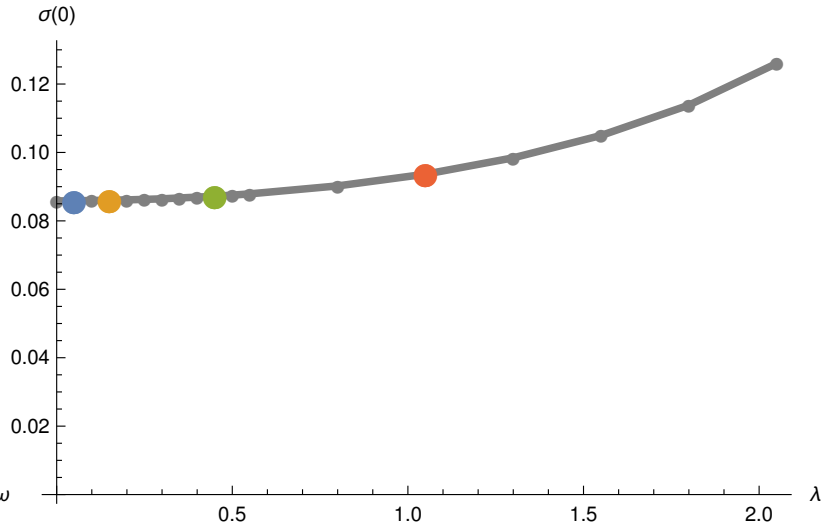
Again, at $T < T_c$ (fixed $\lambda$) the Goldstone is gapped.



# Pinning of Goldstone

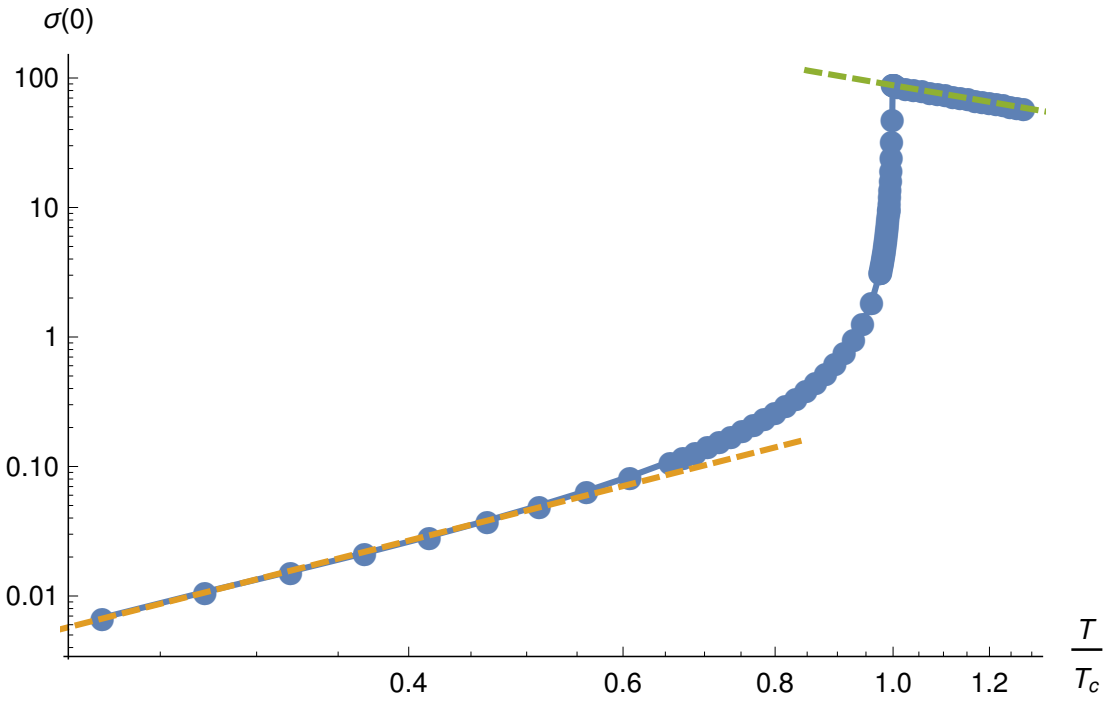
At fixed $T$ the DC conductivity is insensitive to $\lambda$!



# Strange insulator

The new insulating state exhibits the new scaling




Can we understand that?

# DC from horizon

One can compute DC conductivity in terms of **horizon data.**

In the case of the helix the leading terms behave as

$$\sigma_{DC} = e^{v_2+v_3-v_1} \frac{B_2^2}{B_2^2 + A_2^2} T^{4/3} + O(T^{8/3})$$

| | | |
|---|---|---|
| Relevant explicit: | $A_2 = 0$ | $\sigma_{DC} \sim T^{4/3}$ |
| Pure spontaneous: | $B_2 = 0$ | $\sigma_{DC} \sim T^{8/3}$ |

# Dengerously irrelevant mode

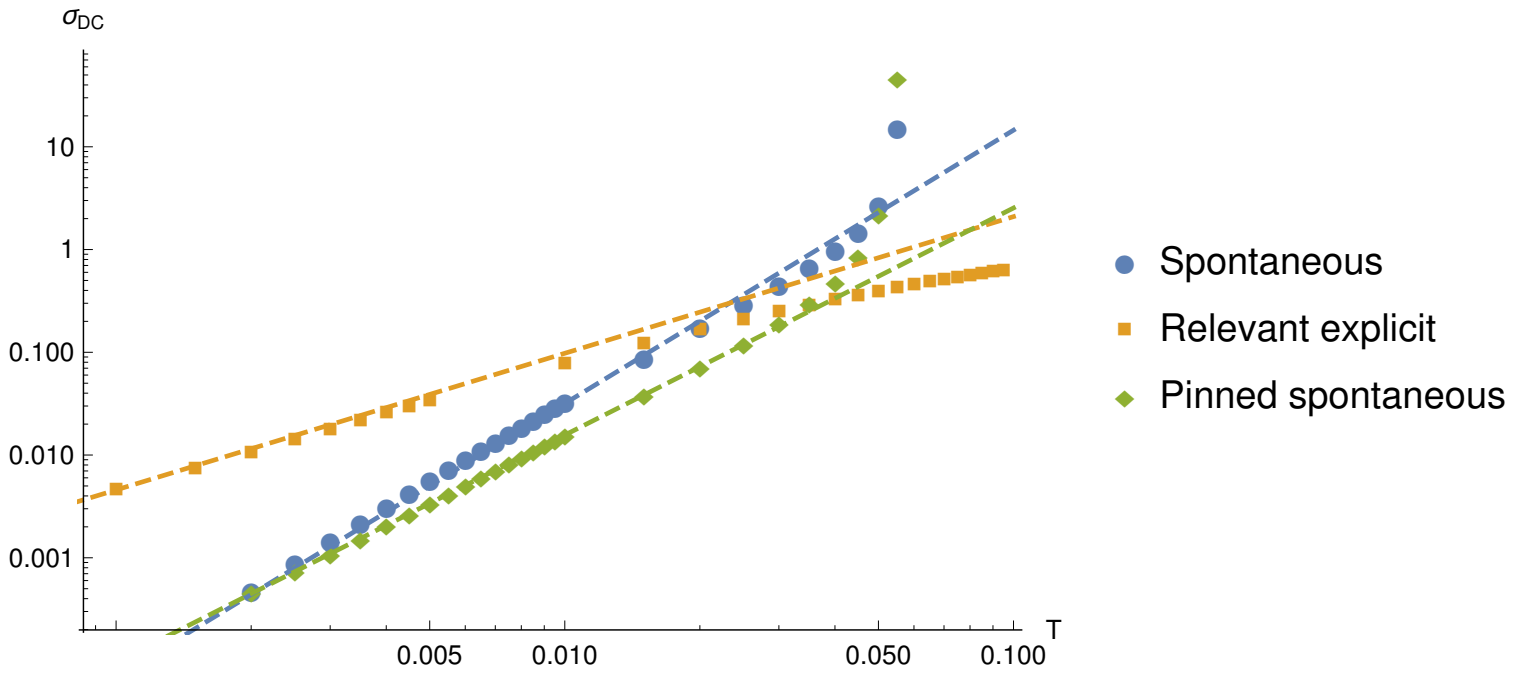
In case of pinning there is an **irrelevant** $B_2$ mode which can contribute to the scaling of conductivity

$$B_2 \sim \lambda r^{\delta}, \qquad \delta = \frac{1}{6}(-5 + \sqrt{145}) \approx 1.17 \tag{1}$$

$$\sigma_{DC} \sim e^{v_2+v_3-v_1} \frac{\lambda^2}{A_2^2} T^{4/3+2\delta} + O(T^{8/3})$$

# IR Scaling

These predictions match with numerics!

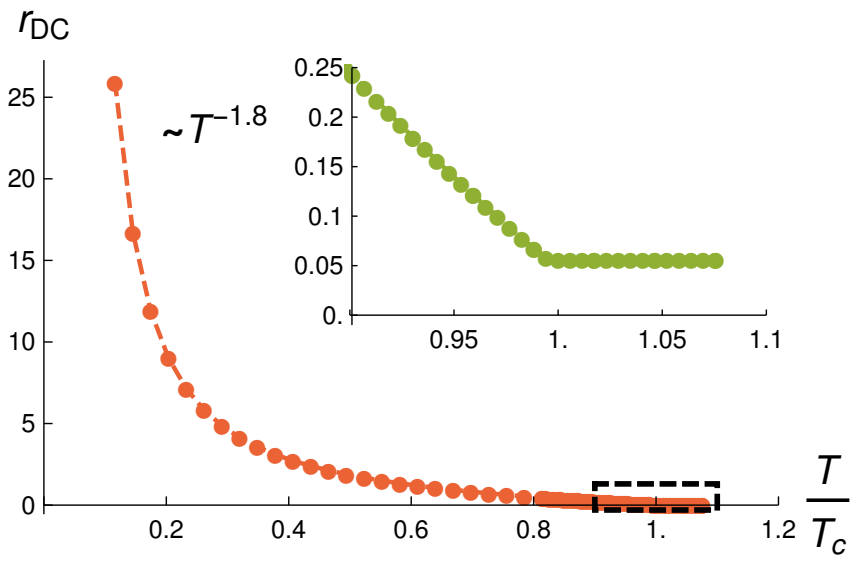


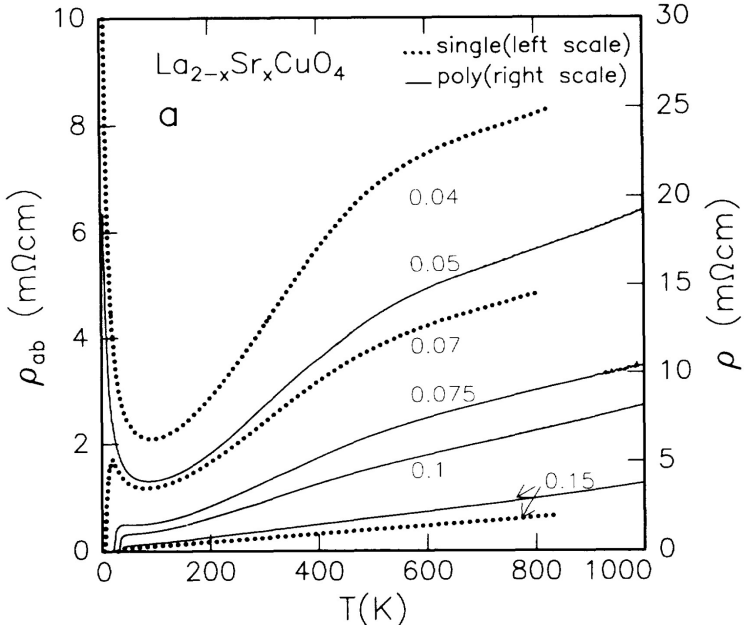
# Conclusion

- When the spontaneous structure is pinned, the insulating state of the new class arizes
- The conductivity is not controlled by momentum relaxation rate
- Instead the incoherent conductivity of the purely spontaneous state plays the key role

# Experimentally relevant

Strange gapless insulating states arise in the CDW pseudogap region of High-Tc superconductors




H.Takagi *et al.* PRL 69, 2975