Kavli Institute for Theoretical Sciences
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Topological semimetals from holography

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arXiv.1511.05505, 1604.01346, 1801.09357 and work in progress

Topological states of matter

- Classification of states of matter
- Quantum matter: Landau-Ginzberg paradigm, order parameter, symmetry breaking
- Topological states of matter: beyond the Landau-Ginzburg paradigm; nontrivial topology in the quantum wave function; certain properties stable under small perturbations; quantum topological phase transition
- Examples: quantum hall state, topological insulators, topological semimetals……

Topological states of matter with interactions

- Most known topological states of matter: based on weak coupling
- Topological states of matter with strong interactions:
  - In lab: iridium oxide materials (Shitade et al., 2009), transition metal oxide heterostructures (Xiao et al., 2011) , the Kondo insulator SmB6 (Dzero et al., 2012, 2010; Wolgast et al., 2013)
  - Possible consequences of strong interactions: topological structure destroyed; new topological structures arise;

Motivation:

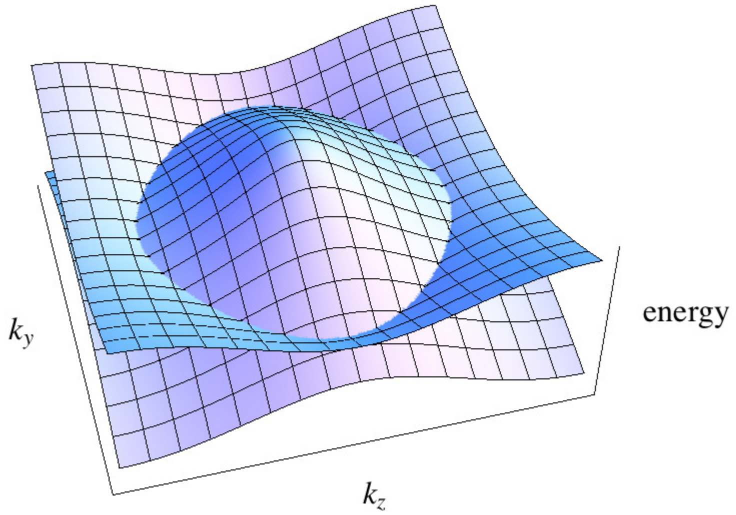
- Topological states of matter with strong interactions: difficulty in direct condensed matter calculations, especially for topological semimetals;
- Holography;
- New entry in the holographic dictionary: topological states of matter;
- New predictions from holography for transport properties;

Outline:

- Weyl/nodal line semimetals;
- Holographic Weyl/nodal line semimetal: model, transport properties;
- The topological structure: where is the bulk topology?
- Topological invariants: from Green functions
- Summary and open questions

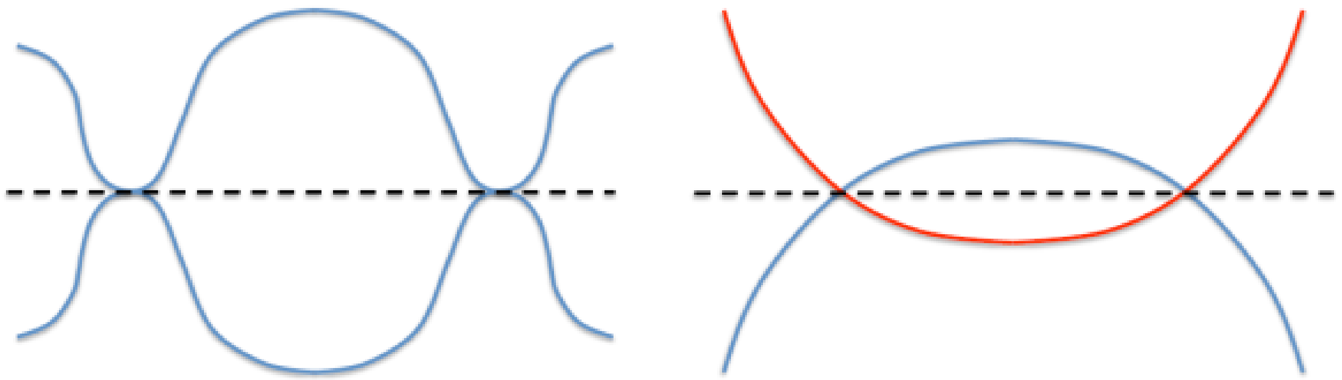
I Topological semimetals in condensed matter

- Many particle systems: emergent semimetals with band crossing
- Dirac semimetal   Weyl semimetal   nodal line semimetal



I Topological semimetals in condensed matter

- Fermi nodal points/lines protected by topology
- Topological charge
- Accidental VS topological (Picture taken from C. Fang, et.al, 2016)

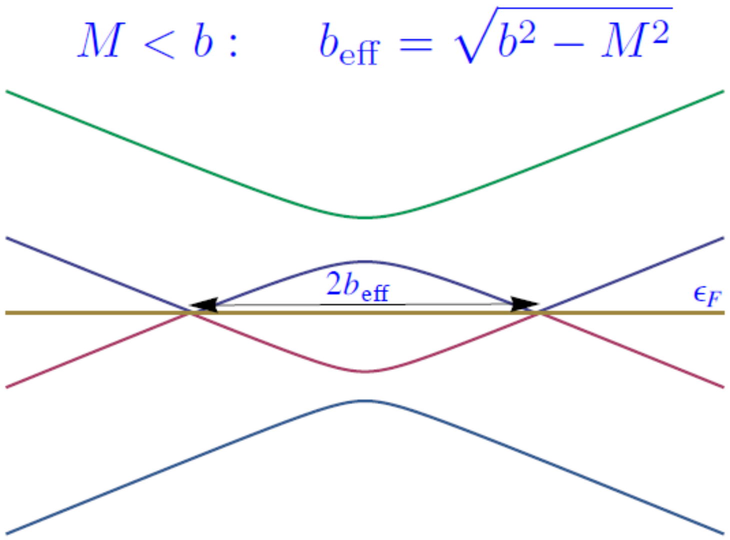


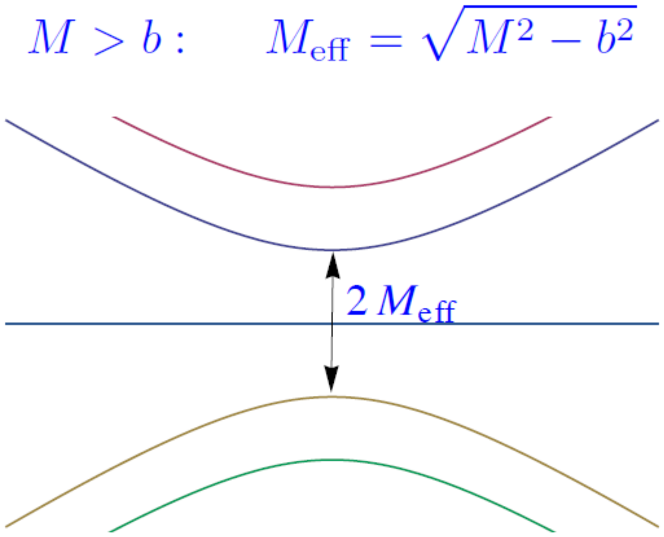
I Topological semimetals in condensed matter

- Weakly coupled field theory model: Weyl semimetal
- A QFT model *(Kostolecky et al. ; Jackiw; Burkov, Balents; Grushin)*

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu + M + \gamma_5 \gamma_z b)\psi$$

- Topological phase transition:





I Topological semimetals in condensed matter

- Anomalous Hall effect:

$$\vec{J} = \frac{1}{2\pi^2}\vec{b}_{\text{eff}} \times \vec{E}$$

- Weyl semimetal phase

$$M < b: \quad b_{\text{eff}} = \sqrt{b^2 - M^2} \qquad \mathcal{L}_{\text{eff}} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma_5\gamma_z b_{\text{eff}})\psi$$

- Gapped phase

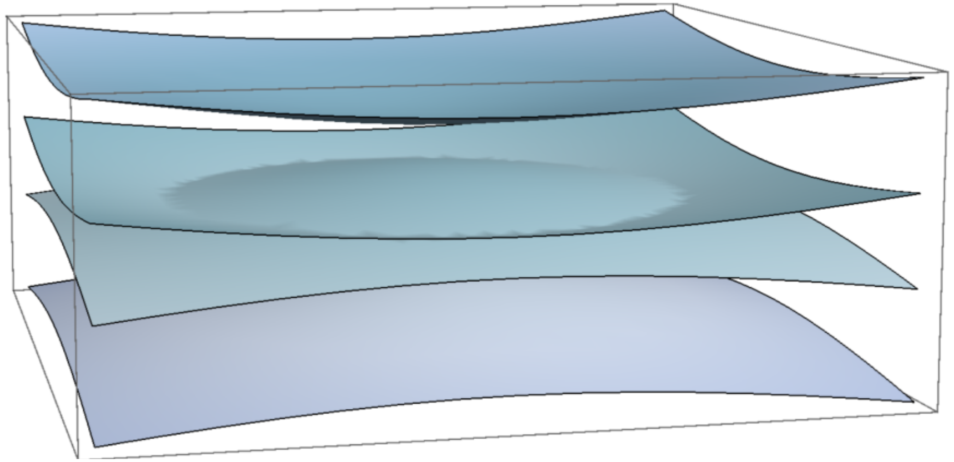
$$M > b: \quad M_{\text{eff}} = \sqrt{M^2 - b^2} \qquad \mathcal{L}_{\text{eff}} = \bar{\psi}(i\gamma^\mu\partial_\mu + M_{\text{eff}})\psi$$

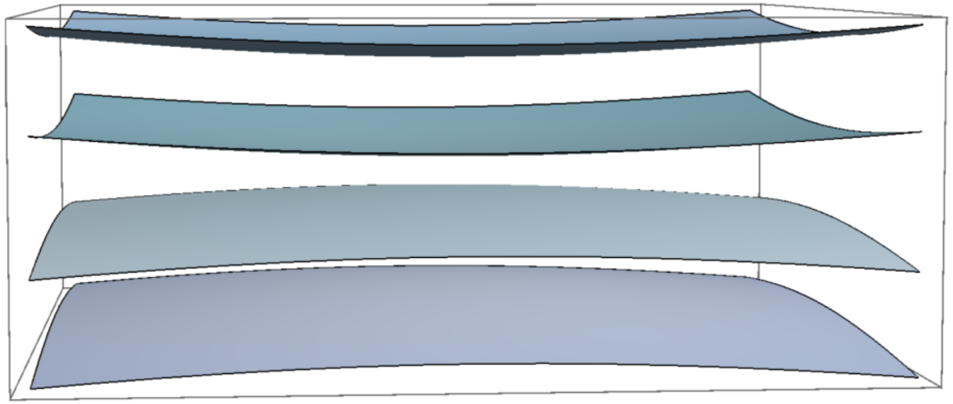
I Topological semimetals in condensed matter

- Weakly coupled field theory model: nodal line semimetal

$$\mathcal{L} = i\bar{\psi}\left(\gamma^\mu \partial_\mu - m - \gamma^{\mu\nu} b_{\mu\nu}\right)\psi \qquad \gamma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$m^2 < 4b_{xy}^2 \qquad\qquad m^2 > 4b_{xy}^2$$





II Holographic semimetals

Not independent operators,
real/imaginary part self duality

$$\bar{\psi}\Gamma^{\mu\nu}\Gamma^5\psi = \frac{i}{2}\epsilon^{\mu\nu}_{\rho\sigma}\bar{\psi}\Gamma^{\rho\sigma}\psi$$

$$\partial_\mu J^\mu = 0\,,$$
$$\partial_\mu J^\mu_5 = im\bar{\psi}\gamma^5\psi + 2ib_{\mu\nu}\bar{\psi}\gamma^{\mu\nu}\gamma^5\psi$$

Operator	Field
$m\bar{\psi}\psi$	U(1)A charged scalar with source
$\bar{\psi}\Gamma_5\Gamma_z\psi$	axial field Az
$\bar{\psi}\gamma^{\mu\nu}\psi$	massive two form field B

Other important ingredients:

gravity+cosmological constant;
$U(1)_A$ and $U(1)_V$ gauge field;
chiral anomaly (and mixed axial gravitational anomaly) represented by a special form of Chern-Simons term;

II Holographic semimetals

- Holographic action

U(1)$_V$

U(1)$_A$

Chern-Simons term dual to chiral anomaly

$$S=\int d^5x\sqrt{-g}\bigg[\frac{1}{2\kappa^2}\Big(R+\frac{12}{L^2}\Big)-\frac{1}{4}\mathcal{F}^2-\frac{1}{4}F^2+\frac{\alpha}{3}\epsilon^{abcde}A_a\Big(3\mathcal{F}_{bc}\mathcal{F}_{de}+F_{bc}F_{de}\Big)$$

$$-(D_a\Phi)^*(D^a\Phi)-V_1(\Phi)-\frac{1}{3\eta}\left(\mathcal{D}_{[a}B_{bc]}\right)^*\left(\mathcal{D}^{[a}B^{bc]}\right)-V_2(B_{ab})-\lambda|\Phi|^2B^*_{ab}B^{ab}\bigg]$$

U(1)$_A$ charged scalar

mass parameters determined by scaling dimensions

with $\lambda_1\phi^4$ potential term

U(1)$_A$ charged two form field, *a better way is to use the first order action that obeys the self dual property automatically (G.E.Arutyunov, et al., 1998;R. Alvares, et al. 2011;):*

$$-2g_B\left(i\frac{1}{6}(B\wedge H^\dagger-B^\dagger\wedge H)+m_B|B|^2\right)$$

$$H=dB-iA^5\wedge B$$

- Bulk configurations and boundary conditions

Weyl

$$\phi|_{r\to\infty}\sim\frac{M}{r}+\cdots$$

$$A_z|_{r\to\infty}\sim b+\cdots$$

Nodal line

$$\phi|_{r\to\infty}\sim\frac{M}{r}+\cdots$$

$$B_{xy}|_{r\to\infty}\sim br+\cdots$$

II Holographic semimetals

- Weyl/nodal line semimetals share the same mathematical structure;
- With backreactions to gravity (well defined probe limit exists, though subtle near the critical point);

- In both cases:
- three types of near horizon solutions at zero temperature: flowing to different UV values;
- a quantum phase transition from a topologically nontrivial phase to a trivial phase;

II.1 Holographic Weyl semimetals

IR

Near horizon T=0 solutions

Topological solution

$$ds^2 = u(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u} + hdz^2$$
$$A = A_z dz\,, \quad \Phi = \phi\,.$$

$$u = r^2,\ \ h = r^2,\ \ A_z = a_1 + \frac{\pi a_1^2 \phi_1^2}{16 r} e^{-\frac{2a_1 q}{r}},$$
$$\phi = \sqrt{\pi}\phi_1 \left(\frac{a_1 q}{2r}\right)^{3/2} e^{-\frac{a_1 q}{r}}$$

$$M/b < 0.744$$

Critical solution

$$ds^2 = u_0 r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u_0 r^2} + h_0 r^{2\beta} dz^2$$
$$A_z = r^\beta, \quad \phi = \phi_0\,.$$

$$(t, x, y, r^{-1}) \ \rightarrow \ s(t, x, y, r^{-1}) \ \text{ and } \ z \ \rightarrow \ s^\beta z$$

$$M/b \simeq 0.744$$

Trivial solution

$$ds^2 = u(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u} + hdz^2$$
$$A = A_z dz\,, \quad \Phi = \phi\,.$$

$$u = (1 + \frac{3}{8\lambda}) r^2,\ \ h = r^2,\ \ A_z = a_1 r^{\beta_1},\ \ \phi = \sqrt{\frac{3}{\lambda}} + \phi_1 r^{\beta_2}$$

$$M/b > 0.744$$

UV

II.1 Holographic Weyl semimetals

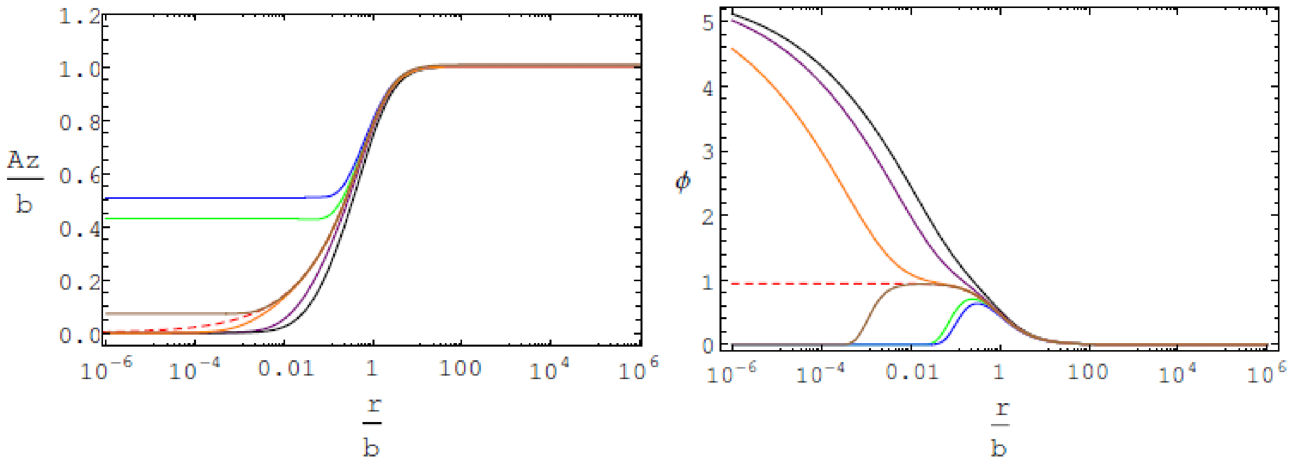
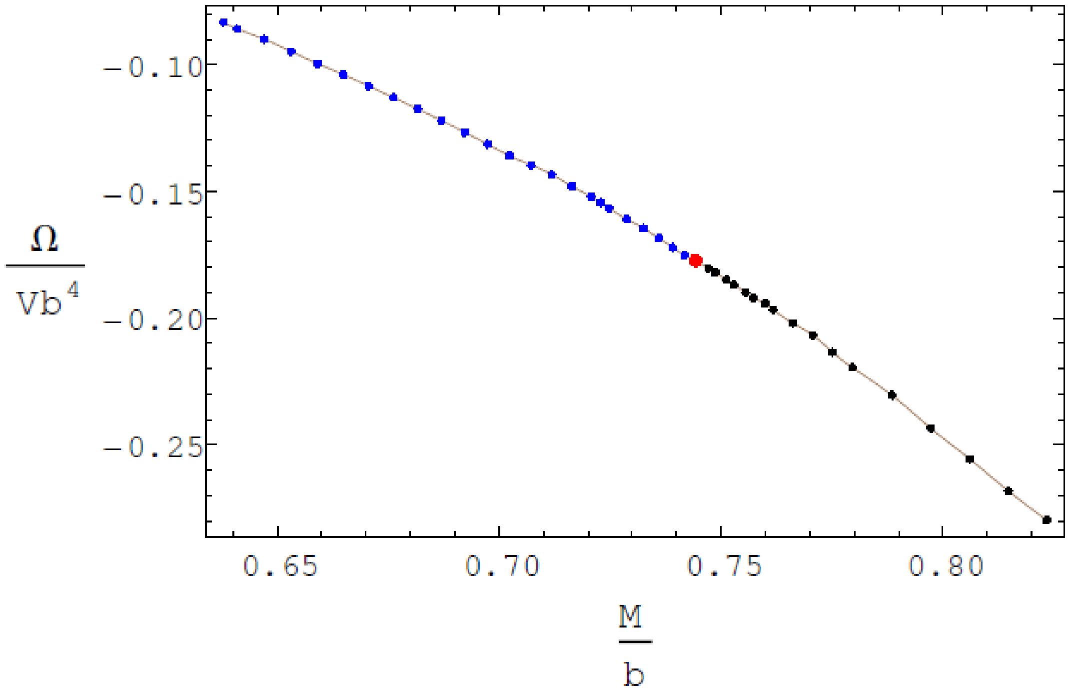


Figure 2. The bulk profile of background A_z and ϕ for $M/b =$ 0.695 (blue), 0.719 (green), 0.743 (brown), 0.744 (red-dashed), 0.745 (orange), 0.778 (purple), 0.856 (black).

As M/b approaches near the critical point, the two solutions approach the critical solution and develop a critical region near the IR

II.1 Holographic Weyl semimetals

Free energy density:



very continuous phase transition

II.1 Holographic Weyl semimetals

Order parameter: anomalous Hall conductivity

Kubo formula $$\sigma_{mn} = \lim_{\omega\to 0} \frac{1}{i\omega}\langle J_m J_n\rangle(\omega, \vec{k}=0)$$

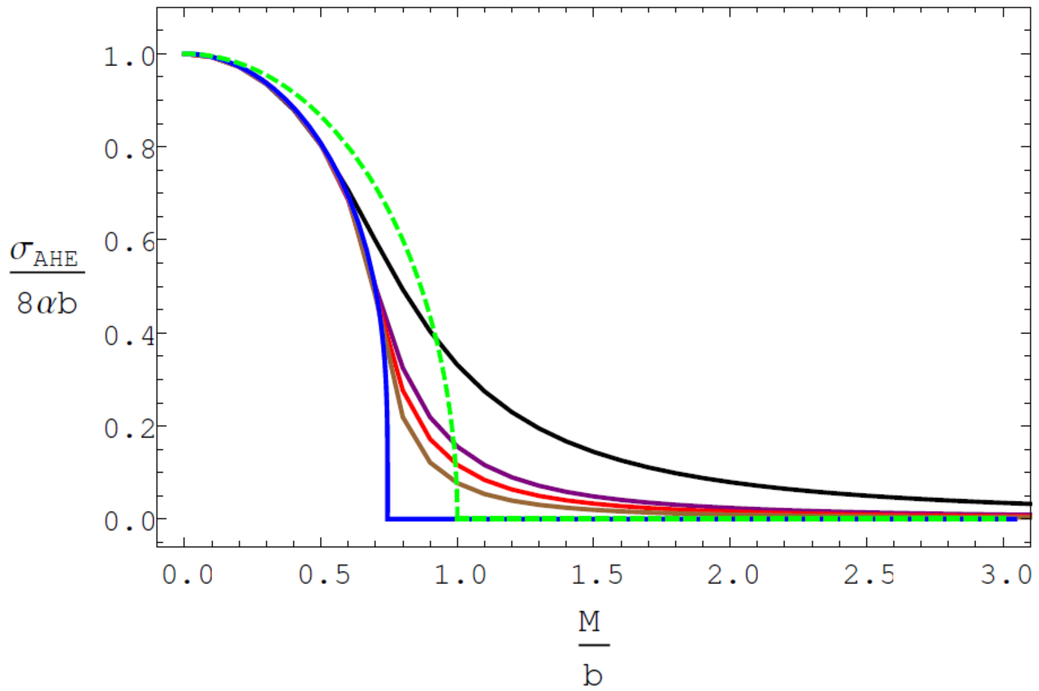
determined by the IR value of the axial gauge field

$$\sigma_{xy} = -8\alpha A_z(r_0)$$

Only nonvanishing for the topological nontrivial solution

Topological trivial solution: restoration of time reversal symmetry in the IR

II.1 Holographic Weyl semimetals



blue: T=0 holographic model; green dashed: weakly coupled T=0;
black: T/b=0.1; purple: T/b=0.05; red: T/b=0.04; brown: T/b=0.03;

$$(\sigma_{\text{AHE}}/b) \propto ((M/b)_c - M/b)^{\alpha} \qquad \alpha \approx 0.211$$

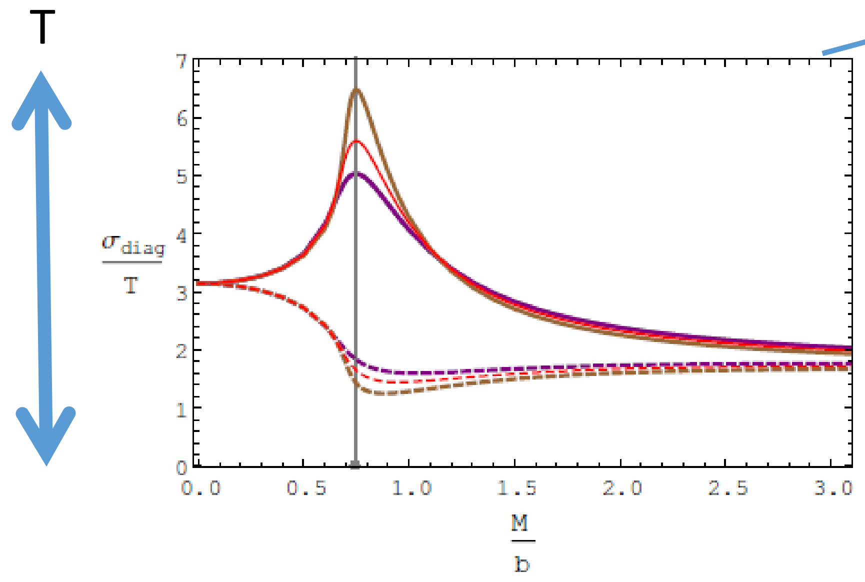
in contrast to the field theory result: 0.5

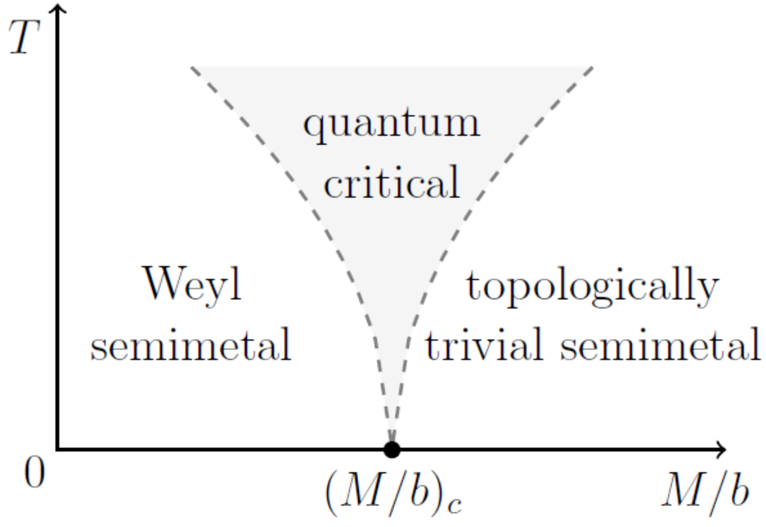
II.1 Holographic Weyl semimetals

transverse (solid) and longitudinal (dashed) electric conductivities for different temperatures.

between a topological semimetal and a topologically trivial phase

a topological trivial semimetal (partially gapped)





the Lifshitz critical point governs the physics in the quantum critical region

II.1 Holographic Weyl semimetals: prediction of odd viscosity

Advantage of holography in the study of transport properties of many body systems:

- *Zero or finite temperature: black hole;*
- *Real time, direct calculation in Minkowski signature; no need of Wick rotation;*
- *Solve for perturbations in classical gravity;*
- *On-shell action of perturbations give the transport coefficients;*

Prediction of this holographic model: odd viscosity

- Broken time reversal symmetry: **odd viscosity**
- Axisymmetric system with a time reversal breaking parameter: **two independent odd(Hall) viscosities**

viscosity tensor $$\eta_{ij,kl} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}[G^R_{ij,kl}(\omega, 0)]$$

$$\eta_{H_\parallel} = -\eta_{xz,yz} = \eta_{yz,xz} \qquad \eta_{H_\perp} = \eta_{xy,T} = -\eta_{T,xy}$$

$$xx - yy$$

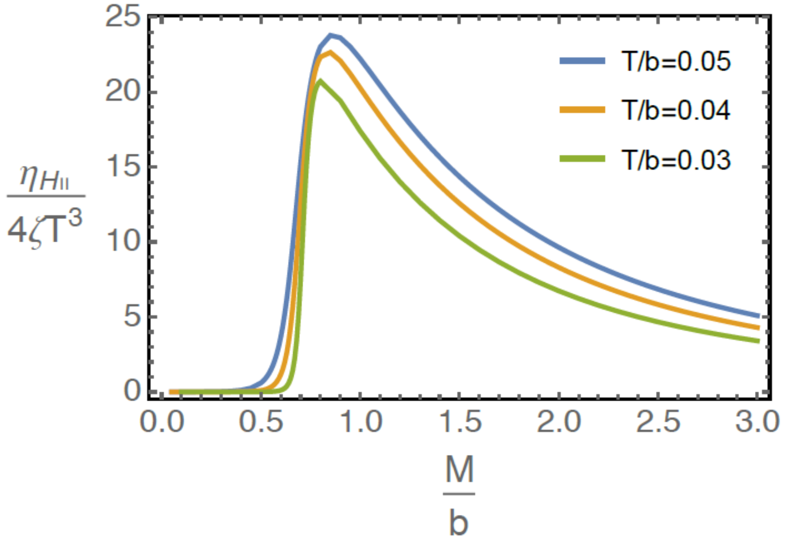
- $\eta_{H_\perp}$ *Hall viscosity in the plane orthogonal to b*
- $\eta_{H_\parallel}$ *specific to axisymmetric three dimensional systems*

Odd viscosity in holographic Weyl semimetal

viscosity determined by IR properties: effective b goes to zero, no substantial odd viscosity expected

gravitational anomaly

$$\eta_{H_{\parallel}} = \eta_{yz,xz} = -\eta_{xz,yz} = 4\zeta \frac{q^2 A_z \phi^2 f^2}{h}\bigg|_{r=r_0}$$



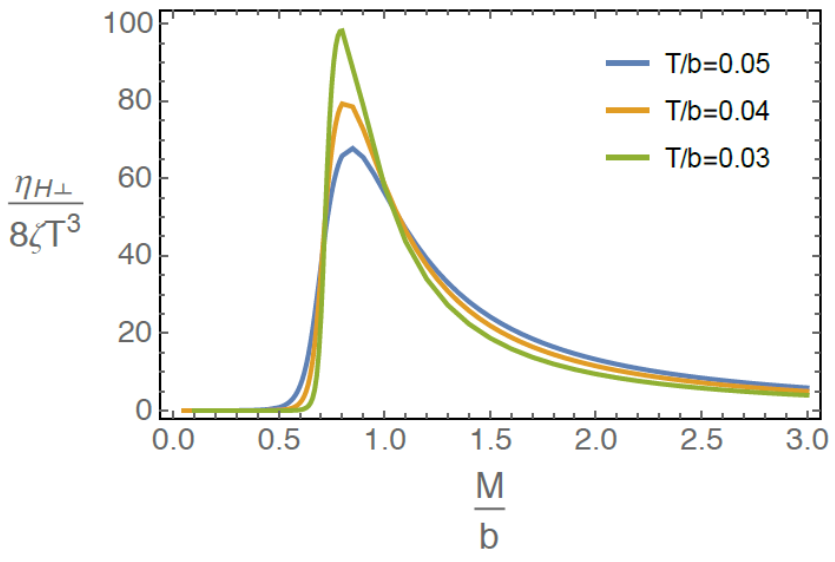
- *highly suppressed in the Weyl semimetal phase;*
- *rises steeply entering the quantum critical region;*
- *peaks at the critical point and drops slowly as M/b increases, finally reaching zero*

Odd viscosity in holographic Weyl semimetal

gravitational anomaly

$$\eta_{H_\perp} = 8\zeta q^2\phi^2 f A_z\Big|_{r=r_0}$$

IR properties



qualitatively the same as the other one

- *highly suppressed in the Weyl semimetal phase;*
- *rises steeply entering the quantum critical region;*
- *peaks at the critical point and drops slowly as M/b increases, finally reaching zero*

II.2 Holographic semimetals: nodal line

$$ds^2 = u(-dt^2 + dz^2) + \frac{dr^2}{u} + f(dx^2 + dy^2)$$

$$\Phi = \phi(r)\,,$$

$$B_{xy} = B(r)\,.$$

near horizon solutions at T=0

IR

nodal line semimetal solution

$$u = \frac{1}{8}(11 + 3\sqrt{13})r^2\Big(1 + \delta u r^{\alpha_1}\Big)\,,$$

$$f = \sqrt{\frac{2\sqrt{13}}{3} - 2\, b_0 r^{\alpha}}\Big(1 + \delta f r^{\alpha_1}\Big)\,,$$

$$\phi = \phi_0 r^{\beta}\,,$$

$$B = b_0 r^{\alpha}\Big(1 + \delta b r^{\alpha_1}\Big)\,,$$

UV M/b<critical value

critical solution

$$u = u_0 r^2 (1 + \delta u r^{\beta})\,,$$

$$f = f_0 r^{\alpha} (1 + \delta f r^{\beta})\,,$$

$$\phi = \phi_0 (1 + \delta \phi r^{\beta})\,,$$

$$B = b_0 r^{\alpha} (1 + \delta b r^{\beta})\,,$$

UV critical M/b

trivial solution

$$u = (1 + \frac{3}{8\lambda_1})r^2\,,$$

$$f = r^2\,,$$

$$\phi = \sqrt{\frac{3}{\lambda_1}} + \phi_1 r^{\frac{2\sqrt{160\lambda_1^2 + 84\lambda_1 + 9}}{3 + 8\lambda_1} - 2}\,,$$

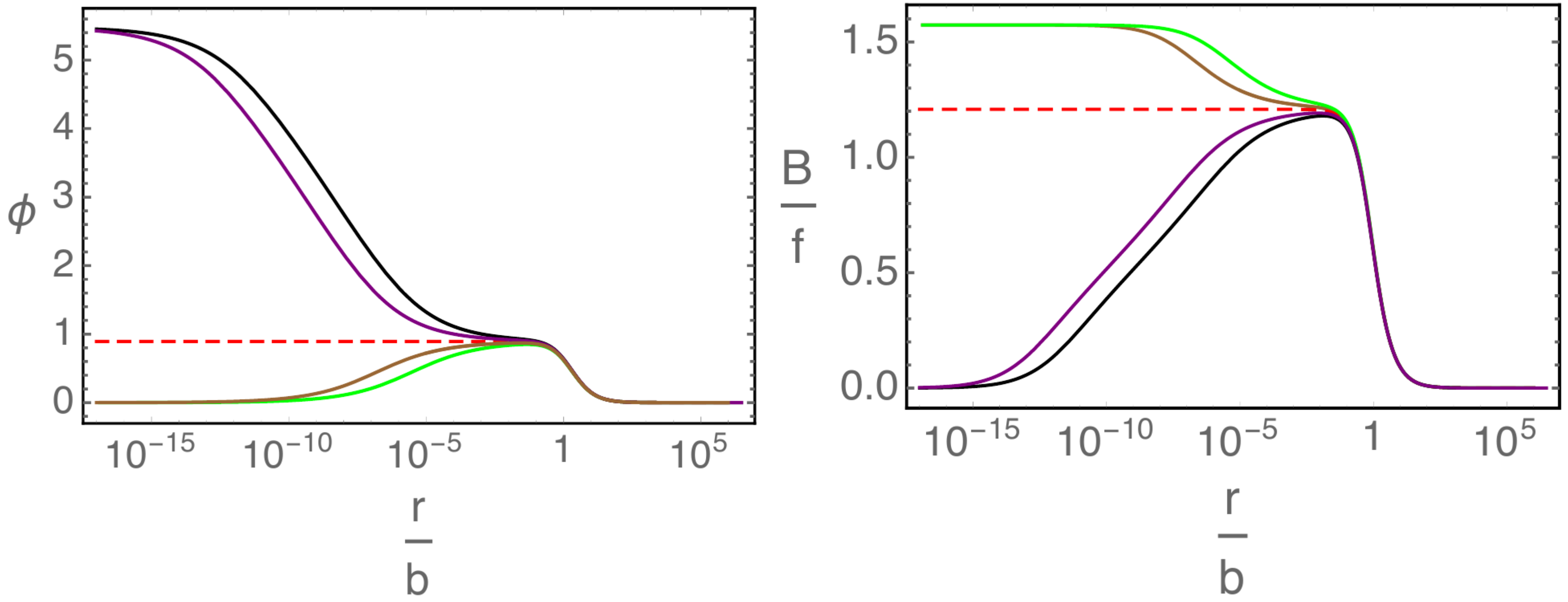
$$B = b_1 r^{2\sqrt{2}\sqrt{\frac{3\lambda + \lambda_1}{3 + 8\lambda_1}}}\,,$$

UV M/b>critical value

UV

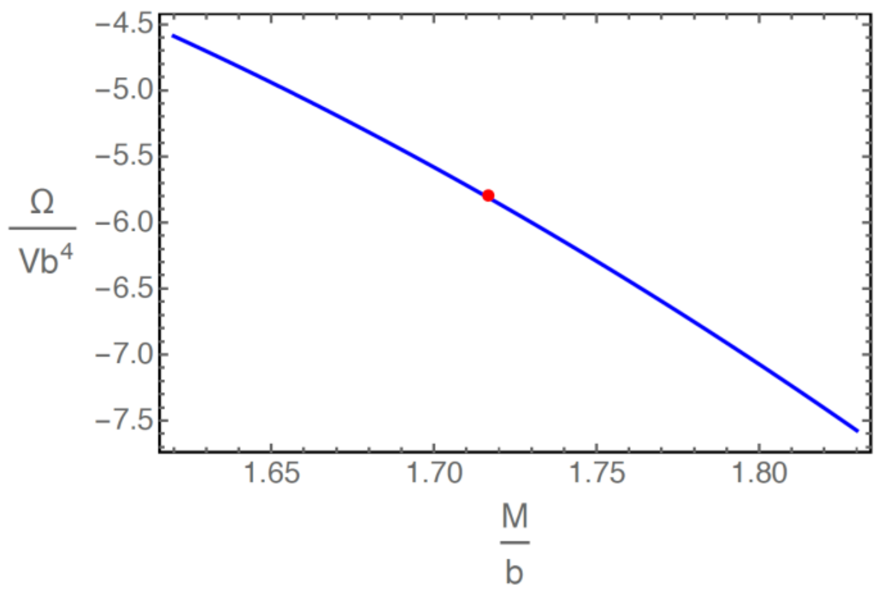
II.2 Holographic semimetals: nodal line

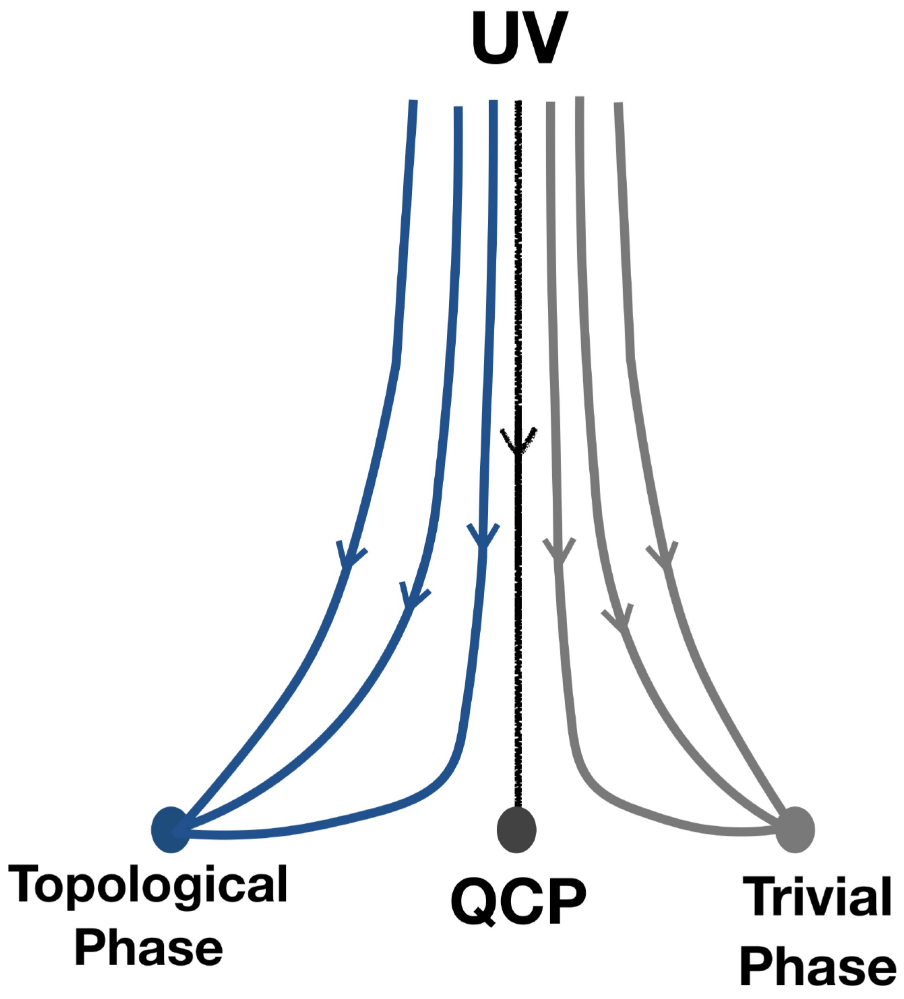
- Bulk configurations



II.2 Holographic semimetals: nodal line

Free energy and phase transition





III The topological structure

- Where does the topology lie?
- topological invariant from Green function;
- a bulk topological structure;
- Equations for B and phi

$$\phi'' + \left(\frac{3u'}{2u} + \frac{f'}{f}\right)\phi' - \left(m_1^2 + \lambda_1\phi^2 + \frac{2\lambda B^2}{f^2}\right)\frac{\phi}{u} = 0\,,$$

$$\frac{B''}{\eta} + \frac{B'}{\eta}\left(\frac{3u'}{2u} - \frac{f'}{f}\right) - \frac{B}{u}(m_2^2 + \lambda\phi^2) = 0\,,$$

- Ignoring the interaction term, behavior determined by IR conformal dimension

$$B \sim c_B r^{-\delta_-^B} \qquad \phi \sim c_\phi r^{-\delta_-^\phi}$$

III the topological structure

- With interaction in the IR

$$B \sim c_B r^{-\delta_-^B} \qquad \phi \sim c_\phi r^{-\delta_-^\phi}$$

c_B and c_ϕ cannot both be nonzero

- Three types of near solutions at leading order:

$$c_\phi = 0 \text{ while } c_B \neq 0$$

$$c_B = 0 \text{ while } c_\phi \neq 0$$

$$c_\phi = c_B = 0$$

The interaction term modifies the IR scaling dimension of at least one of the fields.

III the topological structure

- For a nodal line/Weyl semimetal solution, with this near horizon behavior, we cannot find a small perturbation in this background with $\phi \sim c_\phi r^{-\delta_-^\phi}$ or $\phi \sim c_\phi$ in the case with $\lambda_1 \phi^4$ term, which could gap or partially gap the semimetal.

- The quantum phase transition mechanism here for topological semimetals is different from the BF bound mechanism for many holographic quantum phase transitions, e.g. holographic superconductors, metal-insulator phase transitions, etc..

IV Topological invariants: from Green functions

- Topological invariants for weakly coupled topological systems: distinguish different topology of the quantum wave function (or Hamiltonian) in the momentum space, preserved under homeomorphisms; intrinsic property of the band structure; changes under topological phase transitions
- A simple example for a topological invariant of weakly coupled topological systems: Berry phase

$$c_1 = \frac{1}{2\pi}\int dk_x dk_y \mathcal{F}_{xy}$$

$$\mathcal{F}_{xy} = \partial_{k_x} A_{k_y} - \partial_{k_y} A_{k_x} \text{ and } A_{k_\mu} = i\Sigma_j \langle n_k | \partial_{k_\mu} | n_k \rangle$$

Summed over all occupied states

Ⅳ Topological invariants: from Green functions

- Equivalent expression using Green functions (K. Ishikawa and T. Matsuyama,1986)

$$N = \frac{1}{24\pi^2}\int dk_0 d^2k \mathrm{Tr}[\epsilon^{\mu\nu\rho} G\partial_\mu G^{-1} G\partial_\nu G^{-1} G\partial_\rho G^{-1}]$$

- Could be defined for interacting systems, however, requires integral in the $k_0 = i\omega$ direction
- Topological invariants for interacting systems: the topological Hamiltonian method (Z. Wang and S.C. Zhang, 2012,2013):

The zero frequency Green function contains all topological information.

Topological Hamiltonian: $\mathcal{H}_t(\boldsymbol{k}) = -G(0,\boldsymbol{k})^{-1}$

Topological Green function: $G_t(i\omega,\boldsymbol{k})^{-1} = i\omega - \mathcal{H}_t(\boldsymbol{k})$

Connecting two Green functions: $g_\lambda(i\omega,\boldsymbol{k}) = (1-\lambda)G(i\omega,\boldsymbol{k}) + \lambda G_t(i\omega,\boldsymbol{k}),\ 0 \le \lambda \le 1$

Ⅳ Topological invariants: from Green functions

- Topological Hamiltonian method:
- Topological information contained in $g_{\lambda=1} = G_t(i\omega, \boldsymbol{k})$
- The topological Hamiltonian $\mathcal{H}_t(\boldsymbol{k}) = -G(0, \boldsymbol{k})^{-1}$: an effective Hamiltonian, real eigenvalues: + unoccupied, - occupied;
- Generalized topological invariant:

  replace the Bloch states in the weakly coupled formula by occupied

  eigenstates of $\mathcal{H}_t(k)|n_k\rangle = -E_t|n_k\rangle$ and $E_t > 0$

IV Topological invariants: from Green functions

- Topological invariant for holographic Weyl semimetals:
- A closed surface surrounding the Weyl node

$$C_{\mathrm{Weyl}} = \frac{1}{2\pi} \oint \underbrace{\mathbf{\Omega_k}}_{\text{Berry curvature}} \cdot d\mathbf{S}$$

- Trivial: C=0; Nontrivial: C=integer
- Fermion Green function: probe fermions on the Weyl semimetal background

$$\begin{aligned} S &= S_1 + S_2 + S_{int}, \\ S_1 &= \int d^5x\sqrt{-g}i\bar{\Psi}_1\big(\Gamma^a D_a - m_f - iA_z\Gamma^z\big)\Psi_1, \\ S_2 &= \int d^5x\sqrt{-g}i\bar{\Psi}_2\big(\Gamma^a D_a + m_f + iA_z\Gamma^z\big)\Psi_2, \\ S_{int} &= -\int d^5x\sqrt{-g}(i\phi\bar{\Psi}_1\Psi_2 + i\phi^*\bar{\Psi}_2\Psi_1), \end{aligned}$$

IV Topological invariants: from Green functions

- Topological invariant for pure AdS:

$$G(0,k) \sim \mathcal{N} \begin{pmatrix} \frac{k_\mu \sigma^\mu}{k^{1-2m_f}} & , & 0 \\ 0 & , & -\frac{k_\mu \sigma^\mu}{k^{1-2m_f}} \end{pmatrix}$$

- Poles at k=0; Fermi point;
- Two occupied eigenstates: two chiralities, the same as the free theory;
- For each chirality, there is a topological number 1, -1, total 0;
- Two Weyl points annihilate at the Dirac point;

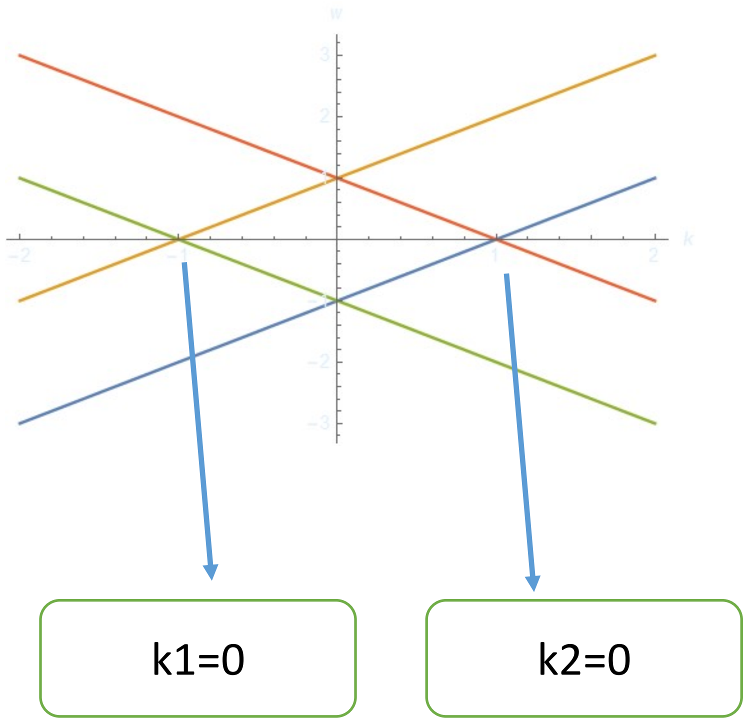
Ⅳ Topological invariants: from Green functions

- Topological invariant for holographic Weyl semimetals:
- The $M/b \to 0$ limit:

$$G \sim \mathcal{N} \begin{pmatrix} \frac{k_{1\mu}\sigma^\mu}{k_1^{1-2m_f}} & , & 0 \\ 0 & , & -\frac{k_{2\mu}\sigma^\mu}{k_2^{1-2m_f}} \end{pmatrix}$$

$$k_1 = \sqrt{k_x^2 + k_y^2 + (k_z - b_0)^2}$$
$$k_2 = \sqrt{k_x^2 + k_y^2 + (k_z + b_0)^2}$$

$$k_z = \pm b_0 \quad C = \pm 1$$



- Semi-analytically, using near far matching method, we could proved that for very small M/b, the topological invariants are the same.

IV Topological invariants: from Green functions

- For holographic nodal line semimetals
- Probe fermions

$$S = S_1 + S_2 + S_{\text{int}} \,,$$

$$S_1 = \int d^5x\sqrt{-g}\, i\bar{\Psi}_1\Big(\Gamma^a D_a - m_f\Big)\Psi_1 \,,$$

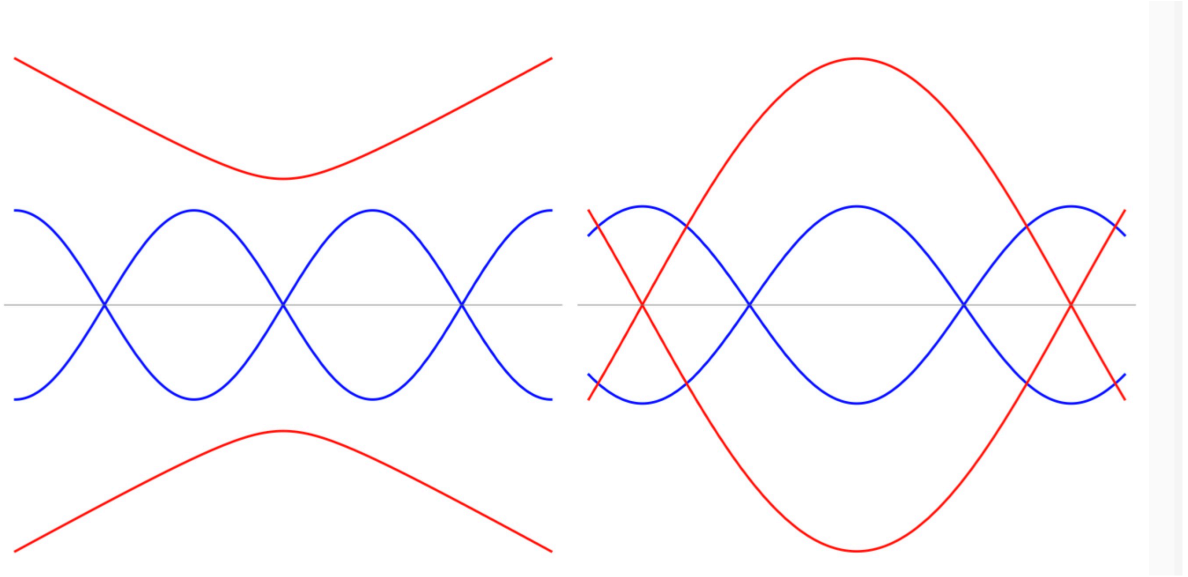
$$S_2 = \int d^5x\sqrt{-g}\, i\bar{\Psi}_2\Big(\Gamma^a D_a + m_f\Big)\Psi_2 \,,$$

$$S_{\text{int}} = -\int d^5x\sqrt{-g}\Big(i\Phi\bar{\Psi}_1\Psi_2 + i\Phi^*\bar{\Psi}_2\Psi_1 + \mathcal{L}_B\Big) \,,$$

$$\mathcal{L}_B = -i(\eta_2 B_{ab}\bar{\Psi}_1\Gamma^{ab}\gamma^5\Psi_2 - \eta_2^* B_{ab}^*\bar{\Psi}_2\Gamma^{ab}\gamma^5\Psi_1) \,.$$

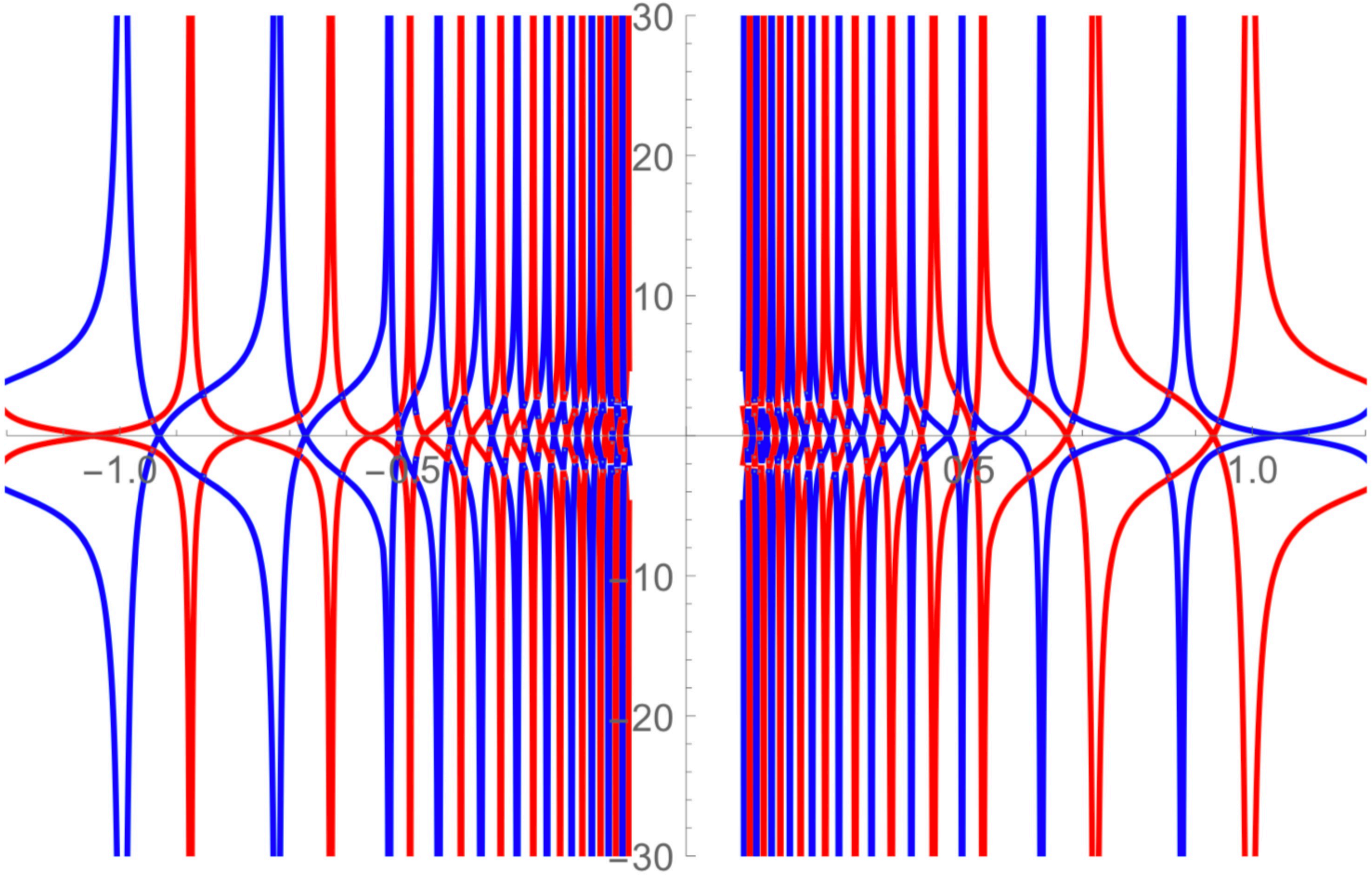
Ⅳ Topological invariants: from Green functions

- Topological invariant for holographic nodal line semimetals:
- Multiple Fermi nodal lines at kz=0



Multiple Fermi nodal lines from the same two bands or different two bands?

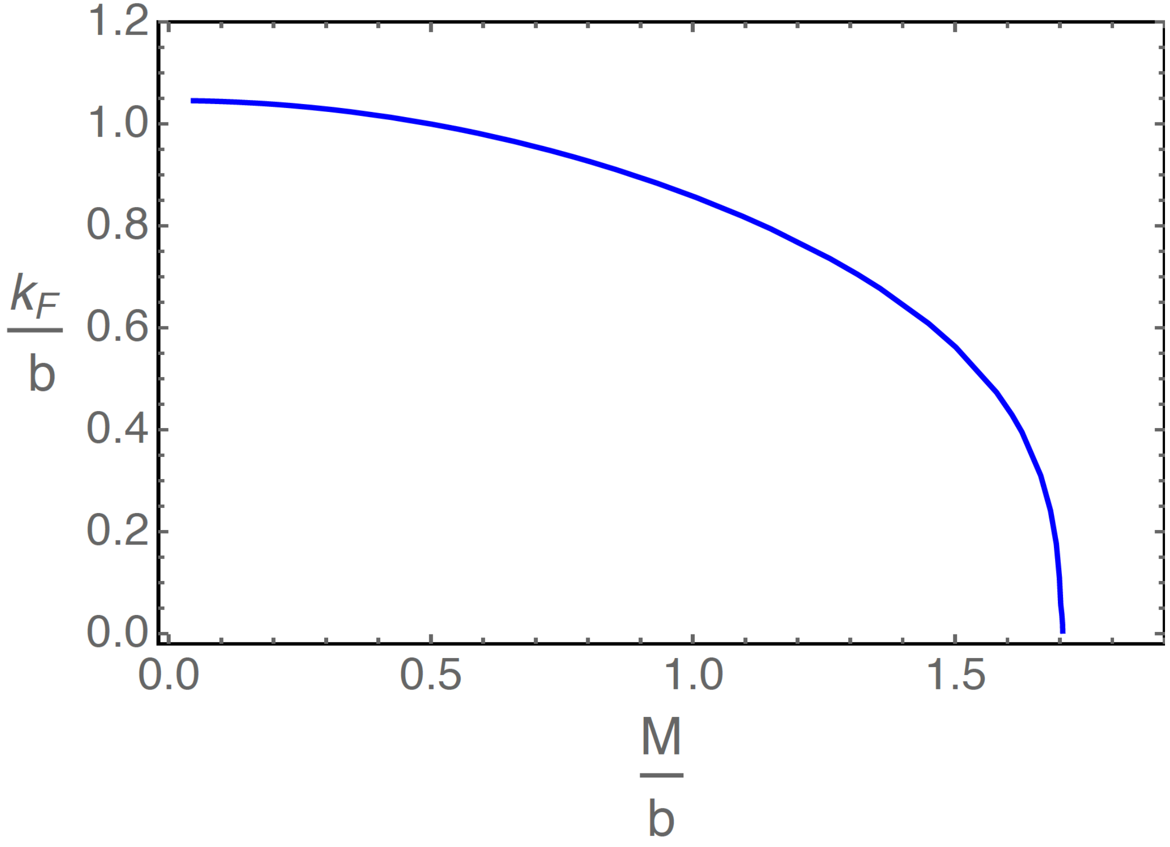
Ⅳ Topological invariants: from Green functions

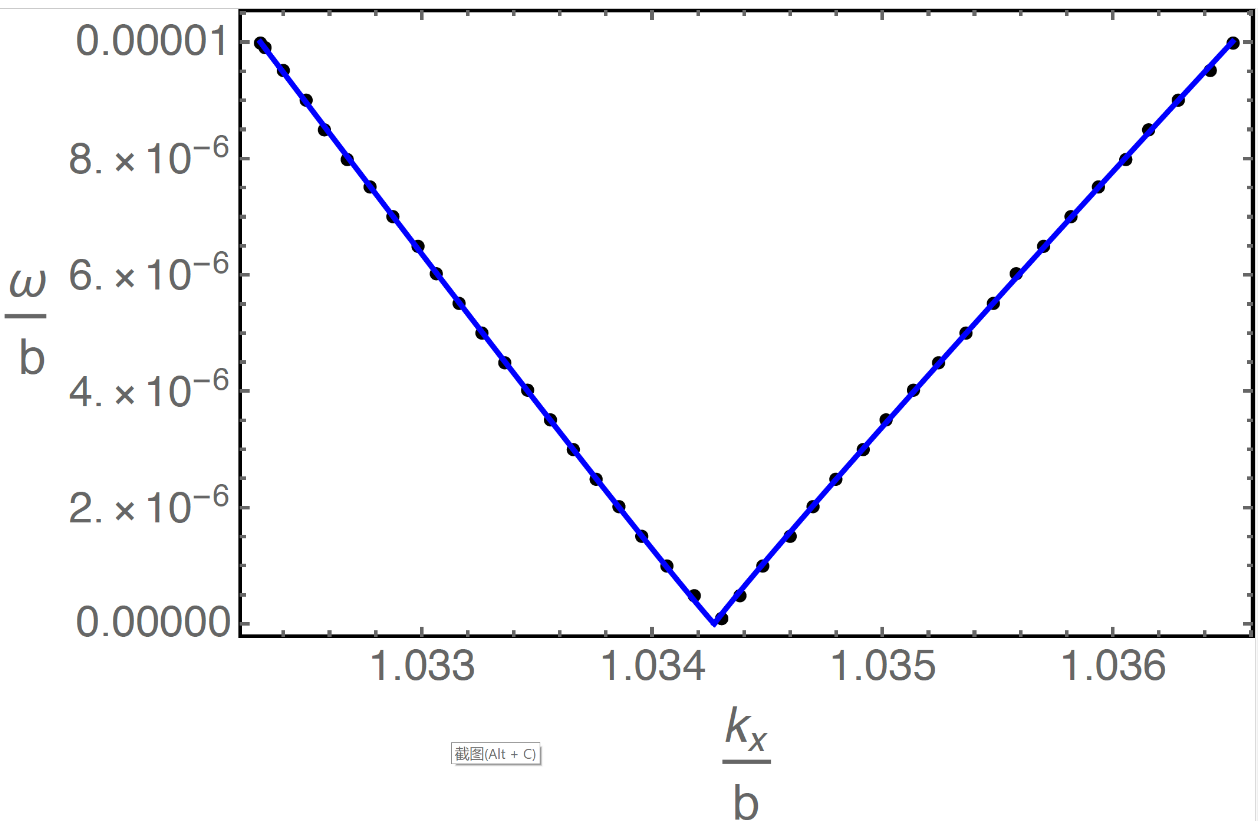


Spectrum constructed from the zero frequency Green functions, which reflects the topological structure of the system.

Zeroes, poles from two different sets of bands

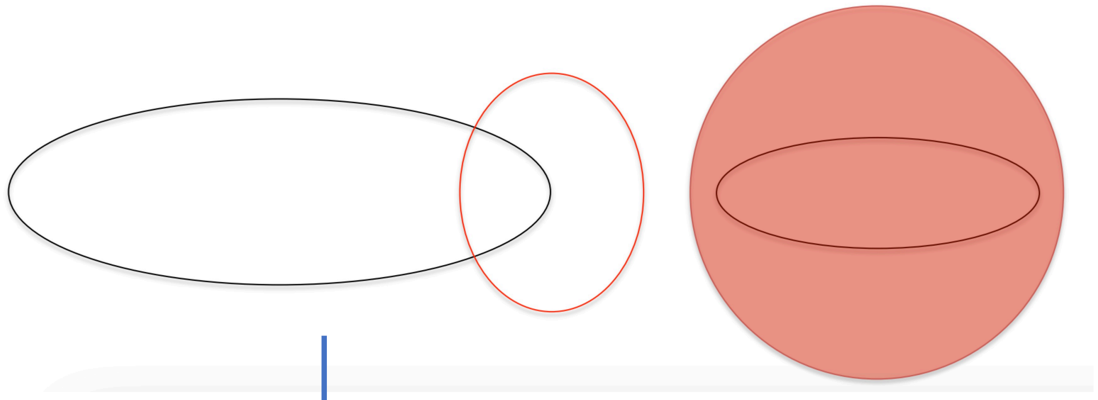
Ⅳ Topological invariants: from Green functions





IV Topological invariants: from Green functions

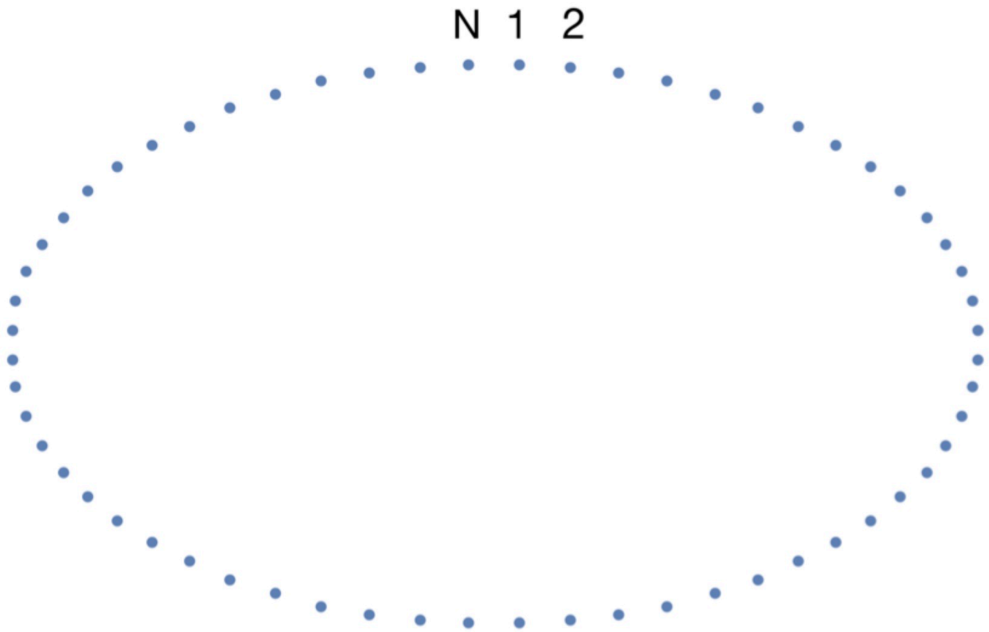
- Topological invariant for holographic nodal line semimetals:
- Two types of topological invariants



Berry phase along a circle

IV Topological invariants: from Green functions

- Topological invariant for holographic nodal line semimetals
- Discrete Berry phase



$$e^{-i\phi_{12}} = \frac{\langle n_1 | n_2 \rangle}{|\langle n_1 | n_2 \rangle|}$$

Berry phase=Sum of all adjacent phase differences

IV Topological invariants: from Green functions

- Topological invariant for holographic nodal line semimetals:

- Results:
- Berry phase=0 for zeros of the zero frequency Green function;
- Berry phase =Pi for all poles from one set of the bands;
- Berry phase undetermined for all poles from the other set of bands;

IV Summary

- Existence of strongly coupled topological semimetal states;
- Transport properties confirm Weyl/nodal line semimetal phase;
- An intrinsic topological structure in bulk configuration;
- Nontrivial topological invariants from dual Green functions
- Transport predictions from holography:

➢ Substantial odd viscosity in the quantum critical region; new prediction for the physics of the quantum critical region of a Weyl semimetal from holography!

➢ New observational effect from gravitational anomaly!

Open questions

- A better understanding of the deeper organizational principles of strongly coupled topological states of matter and predict more types of strongly coupled topological states of matter
- Holographic framework as a possible method for classification of strongly interacting gapped as well as gapless topological states of matter
- Any new strongly coupled topological states of matter which even do not exist at weak coupling?
- Edge states (M. Ammon, et,al.,Phys. Rev. Lett. 118, 201601 (2017))
- Gapped, Topological superconductors, accidental semimetals?
- Prediction of new transport properties of strongly coupled topological states of matter?

Thank you!