Planckian bound on the decay of simple operators

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Stanford Physics

Many-Body Quantum Chaos, Bad Metals, and Holography; NORDITA

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- the Planckian time scale

$$\boxed{\tau \gtrsim \frac{\hbar}{k_{\mathrm{B}}T}}$$

has been conjectured to bound "quantum dynamics" in many-body systems with **few-body interactions**: e.g.

$$H = t_{ij}c_i^\dagger c_j + J_{ab}\sigma_a\sigma_b + K_{aij}\sigma_a c_i^\dagger c_j + U_{ijkl}c_i^\dagger c_j^\dagger c_k c_l$$

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- Example 1: bounds on decay rates hold in many interacting QFTs [Sachdev; `cond-mat/9810399`]

$$\langle c^\dagger(t)c(0)\rangle \sim \mathrm{e}^{-\gamma t}, \quad \gamma \lesssim \frac{k_\mathrm{B}T}{\hbar}.$$

- the heuristic derivation of this bound: starting with

$$\Delta E \Delta t \gtrsim \hbar$$

and estimating that for a local operator

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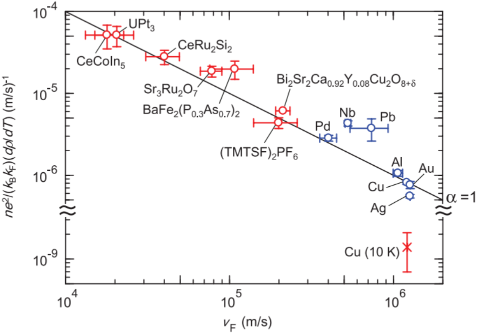
- but rigorously, Δt is (as far as I know) always related to dephasing of the *many-body wave function*...

- Example 2: bound on the transport time in the resistivity in strange metals? [Zaanen (2004)]

$$\rho = \frac{m}{ne^2\tau_{\mathrm{tr}}} \lesssim \frac{m}{ne^2}\frac{k_{\mathrm{B}}T}{\hbar}$$



[Bruin, Sakai, Perry, Mackenzie (2013)]

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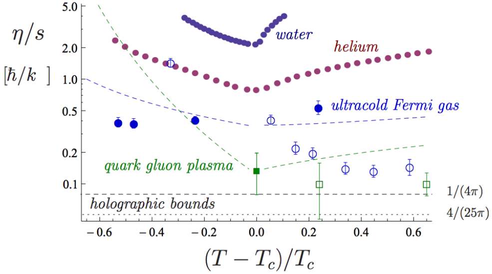

Viscosity and Diffusion Bounds

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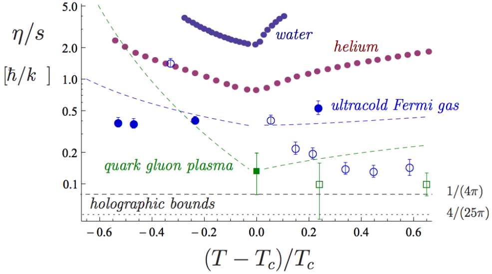

- Example 4: diffusion [Hartnoll; 1405.3651]

$$D \gtrsim v^2 \frac{\hbar}{k_{\mathrm{B}} T}.$$

- Example 5: the "chaos bound" on out-of-time-ordered correlation functions (OTOCs): if

$$\langle A(t)B(0)A(t)B(0)\rangle \propto 1 - \frac{1}{N}\mathrm{e}^{\lambda_{\mathrm{L}}t}$$

where the Lyapunov rate [Maldacena, Shenker, Stanford; 1503.01409]

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- for now: OTOCs are inspired by

$$\left|\frac{\partial x_i(t)}{\partial x_j(0)}\right| \sim \mathrm{e}^{\lambda_{\mathrm{L}}t}$$

in a classical chaotic system

Counterexamples

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  - 5 (OTOC growth): 1+1 dimensional free fermion

$$\{c^\dagger(x,t), c(0,0)\} \sim \delta(x-t)$$

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- *all* prior counterexamples now appear consistent

A First Definition of Operator Size

- let's begin by considering a system of N spin-$\frac{1}{2}$s. the space of Hermitian operators is

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- pick some O(1) R. operator $\mathcal{O}$ is simple $\iff$ $\mathcal{S}[\mathcal{O}] \le R$.
- **problem:** under time evolution:

$$\mathcal{O}(t) = \mathrm{e}^{\mathrm{i}Ht}\mathcal{O}\mathrm{e}^{-\mathrm{i}Ht}$$

does not depend on temperature T!

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- sum of OTOCs gives us average size: $(A(t)|\mathcal{S}|A(t))$

[Roberts, Stanford, Streicher; 1802.02633]

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- invoke memory matrix formalism: if ω_* is a pole of

$$\hat{\sigma}_{AB}(\omega) = [\chi(M + N - \mathrm{i}\omega\chi)^{-1}\chi]_{AB}$$

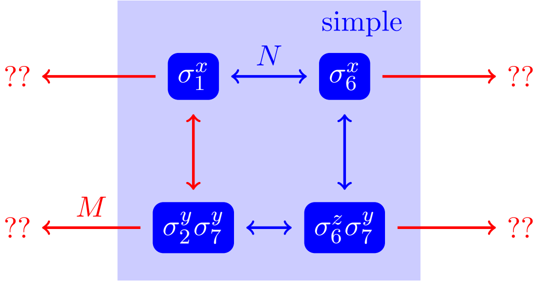
$$M_{AB} = \mathrm{i}(A|\mathfrak{p}\mathcal{L}\mathfrak{q}(\mathfrak{q}\mathcal{L}\mathfrak{q} - \mathrm{i}\omega)^{-1}\mathfrak{q}\mathcal{L}\mathfrak{p}|B),$$

$$N_{AB} = (A|\mathfrak{p}\mathcal{L}\mathfrak{p}|B), \quad \chi_{AB} = (A|\mathfrak{p}|B)$$

then $|\mathrm{Im}(\omega_*)|^{-1}$ is a lifetime of a simple operator

$$\hat{\sigma}_{AB}(\omega) = [\chi(M + N - \mathrm{i}\omega\chi)^{-1}\chi]_{AB}$$

- M_{AB} is positive semidefinite: contains **"dissipation"** – the decay of simple operators into complicated operators
- N_{AB} is antisymmetric: **"dissipationless"** – rotation of simple operators into each other
- χ_{AB} is generally a thermodynamic scale factor (but right now is the identity)



- assume Hamiltonian H is k-local:

$$H = J_i^{\alpha}\sigma_i^{\alpha} + J_{ij}^{\alpha\beta}\sigma_i^{\alpha}\sigma_j^{\beta} + \cdots + J_{i_1\cdots i_k}^{\alpha_1\cdots\alpha_k}\sigma_{i_1}^{\alpha_1}\cdots\sigma_{i_k}^{\alpha_k}$$

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- constrain M and N:

$$|(B_{R'}|\mathcal{L}|A_R)| \leq \frac{\min(R, R')}{\tau_*} \times \begin{cases} 1 & |R' - R| < k \\ 0 & \text{otherwise} \end{cases}$$

where τ_* is a "Lieb-Robinson" time ($\propto J$'s)

$$M = \begin{pmatrix} & \vdots & 0 \\ \cdots & 0 & 0 \\ 0 & 0 & M^{(R)} \end{pmatrix}, \quad N \sim \begin{pmatrix} & \vdots & 0 \\ \cdots & \frac{R-k}{\tau_*} & \frac{R-k}{\tau_*} \\ 0 & -\frac{R-k}{\tau_*} & \frac{R}{\tau_*} \end{pmatrix}$$

sizes:
$R - 2k + 1$ to $R - k$
$R - k + 1$ to R

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- integrate out more operators: $R \to R-k$. we estimate

$$M^{(R-k)} \lesssim \frac{(R-k)^2}{\tau_*^2 M^{(R)}}, \quad \Longrightarrow \quad M^{(R)} \lesssim \frac{R}{\tau_*}.$$

- if interaction (factor) graph of H is regular:

[Bentsen, Gu, Lucas; 1805.08215]

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- τ_* bounds both growth and decay of operators
- in regular theories with “classical” operator dynamics:

$$\tau_\mathrm{L} \ge \frac{1}{k-1}\frac{1}{M^{(1)}}$$

  - SYK: [Roberts, Stanford, Streicher; 1802.02633]
  - random unitary circuit: [Nahum, Vijay, Haah; 1705.08975]

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- σ_1^x, $\sigma_1^y\sigma_4^z$, etc., are largely made of simple operators
- if $|s)$ is the eigenbasis of $\mathcal{S}$, we write

$$|A(t)) = \sum_s a_s(t)|s).$$

our conjecture is that if $|A(0)) = |s_0)$ $(s_0 < R)$:

$$\sum_{s\le R} |a_s(t)|^2 \gtrsim \mathrm{e}^{-\gamma t}, \quad \gamma \lesssim \frac{k_\mathrm{B}T}{\hbar}$$

- probe chaos using a sum of OTOCs:

$$(A(t)|\mathcal{S}|A(t)) = \frac{1}{8}\sum_{\alpha=1}^{3}\sum_{i=1}^{N}([\sigma_i^\alpha, A(t)]|[\sigma_i^\alpha, A(t)]) = \sum_s s|a_s(t)|^2 \le C\mathrm{e}^{\lambda_\mathrm{L} t}, \quad \lambda_\mathrm{L} \le \frac{2\pi k_\mathrm{B} T}{\hbar}?$$

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- for local operators, $\sum$ error $\sim N^0$. postulating that error $\sim N^0$ for operator $|A)$ of size $\le R/6$:

$$(A(t)|\mathfrak{p}|A(t)) > \frac{1}{2}, \quad \text{for all } t \lesssim \frac{1}{T}$$

so simple operator lifetime $\gtrsim 1/T$

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Free Theories

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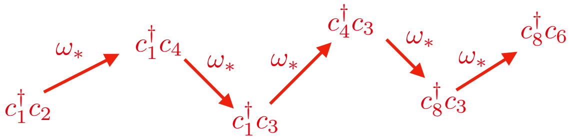

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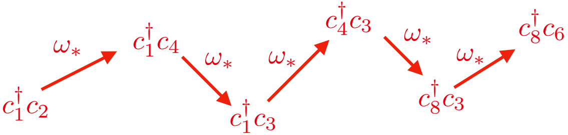

- this resolves all free fermion objections to Planckian bounds

- a subtler counterexample to the chaos (OTOC) bound may arise in heterogeneously connected systems: e.g.

$$H = \frac{1}{R^2} \sum_{A,B=1}^{R} \sum_{i=1}^{N} J_{ABi}^{\alpha\beta\gamma} \sigma_A^\alpha \sigma_B^\beta \sigma_i^\gamma$$

with $\frac{R}{\log N} \to \infty$, $\frac{R}{N} \to 0$ as $N \to \infty$.

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- random unitary circuit on this (hyper)graph:

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- does this happen with fixed H at finite T?

- now we return to transport bounds:

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  - e.g., Drude theory of magnetotransport

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- if we write $D = v^2\tau_{\mathrm{diff}}$, the same comments also apply to τ_{diff} in many disordered systems

- in a conventional disordered metal, τ_{tr} is the decay rate of a specific fermion bilinear $J_{ij}c_i^\dagger c_j$, and is finite at $T=0$:

$$c_1^\dagger c_2 \xrightarrow{\omega_*} c_1^\dagger c_4 \xrightarrow{\omega_*} c_1^\dagger c_3 \xrightarrow{\omega_*} c_4^\dagger c_3 \xrightarrow{\omega_*} c_8^\dagger c_3 \xrightarrow{\omega_*} c_8^\dagger c_6$$

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Disorder-Driven Metal-Insulator Transitions

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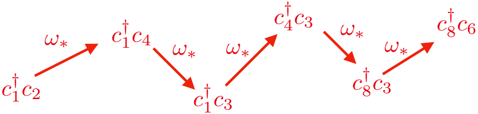
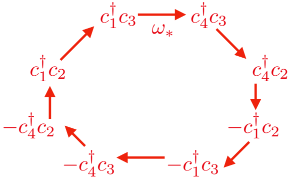


$$\tau_{\mathrm{tr}} \sim \frac{1}{\omega_*}$$

- Anderson localization: $\tau_{\mathrm{tr}} = 0$ without dissipation:



$$\tau_{\mathrm{tr}} = 0$$

(this argument is correct up to exponentially small corrections – but $\tau_{\mathrm{tr}} = 0$ is exact)

- eigenstates of free/interacting localized insulators are robust to local perturbations: [Serbyn, Papić, Abanin; 1305.5554]

$$H = H_0 + \lambda V, \qquad \|\partial_\lambda|\alpha\rangle_\lambda\| < \infty$$

More on Localization

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- since J is local and $\langle\alpha|J|\alpha\rangle = 0$, we conclude $|J)$ contains no null vector of $\mathcal{L}$. hence

$$(J|(\mathcal{L} - \mathrm{i}\omega)^{-1}|J) = 0.$$

as expected, there is no transport in this localized phase

Memory Matrix at the Insulating Transition

- in the memory matrix formalism:

$$M + N = \begin{pmatrix} \delta_1 & -\mathcal{L}_{JX} & -\hat{\mathcal{L}}_J \\ \mathcal{L}_{JX} & \delta_2 & 0 \\ \hat{\mathcal{L}}_J^{\mathsf{T}} & 0 & M_0 + N_0 \end{pmatrix}, \qquad \begin{array}{c} |J) \\ |X) \\ \text{other simple} \end{array}$$

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- in both cases, $\tau_{\mathrm{tr}} \to 0$ due to **dissipationless modes** – no problem with Planckian bounds

When Might Planckian Transport Bounds Work?

- with an (effective) Hamiltonian of the form

$$H = t^{\text{eff}}_{ij} c_i^\dagger c_j + U^{\text{eff}}_{ijkl} c_i^\dagger c_j^\dagger c_k c_l + \cdots\ ,$$

a cartoon for the time evolution of $c_1^\dagger c_1$:

$$\begin{tikzcd}
 & t c_1^\dagger c_4 \arrow[r, dashed] & t^2 c_7^\dagger c_4 \arrow[r, dashed] & t^3 c_1^\dagger c_4 \arrow[r, dashed] & t^4 c_1^\dagger c_9 \arrow[dr, dashed] & \\
c_1^\dagger c_2 \arrow[ur, dashed] & t^7 U c_6^\dagger c_2^\dagger c_3 c_5 \arrow[dr] & t^6 U c_6^\dagger c_2^\dagger c_3 c_1 \arrow[l, dashed] & t^5 U c_6^\dagger c_5^\dagger c_3 c_1 \arrow[l, dashed] & & t^5 c_6^\dagger c_9 \arrow[ll] \\
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- our conjecture: *only* solid arrows occur at rate $\lesssim k_{\mathrm{B}} T/\hbar$
- a bound $\tau_{\mathrm{tr}} \gtrsim \hbar / k_{\mathrm{B}} T$ suggests $U^{\text{eff}} \gg t^{\text{eff}}$:

$$c_1^\dagger c_2 \rightarrow U c_1^\dagger c_5^\dagger c_3 c_4 \rightarrow U^2 c_9^\dagger c_7^\dagger c_5^\dagger c_3 c_4 c_1 \rightarrow U^3 c_8^\dagger c_6^\dagger c_7^\dagger c_5^\dagger c_3 c_4 c_1 c_2$$