Slow relaxation and diffusion in holographic quantum critical systems

Richard Davison
Harvard University

1705.07896 with M. Blake, S. Sachdev
1808.05659 with S. Gentle, B. Gouteraux

Bounding Transport and Chaos Program
Nordita, August 2018

Planckian timescale in transport

- The Planckian timescale is important in quantum many-body systems

$$\tau_P = \frac{\hbar}{k_B T}$$

Sachdev, Zaanen,.....

Argued to provide a lower limit on the timescale for a variety of processes.

- Does it fundamentally limit the transport properties of these systems?

e.g.

$$D = v^2 \tau \gtrsim v^2 \tau_P \ \ ?$$

Hartnoll

- What are the appropriate v and τ for systems without quasiparticles?

Diffusive transport in holographic theories

- Holographic duality gives us tools to test these ideas.

- In the quantum critical region of many holographic theories

$$D_T = \frac{z}{4\pi(z-1)} v_B^2 \tau_P$$

- Suggests

  (1) thermal diffusion is related to the propagation of chaos

  (2) the characteristic timescale of thermal diffusion is τ_P

The special case z=1

- But when z=1, the thermal diffusivity is parametrically larger than τ_P

- These phases have collective excitations with parametrically long lifetimes

$$\tau_{eq} \gg \tau_P$$

- Local equilibration is achieved only at times $t \gtrsim \tau_{eq}$

- This timescale governs the thermal diffusivity

$$D_T = \frac{2}{d_{eff}} v_B^2 \tau_{eq}$$

- Consistent with some previous conjectures

Hartnoll
Lucas

Outline of the talk

1 Holographic quantum critical systems

2 Thermal diffusivity in generic cases

3 Slow equilibration due to irrelevant deformations

Holographic quantum criticality I

- Use holography to study QFTs in the quantum critical regime.

Charmousis, Gouteraux et al

- I will restrict to translationally invariant systems (for simplicity).

- Start with a CFT and deform it by turning on

chemical potential for U(1) charge $\langle \rho \rangle \neq 0$

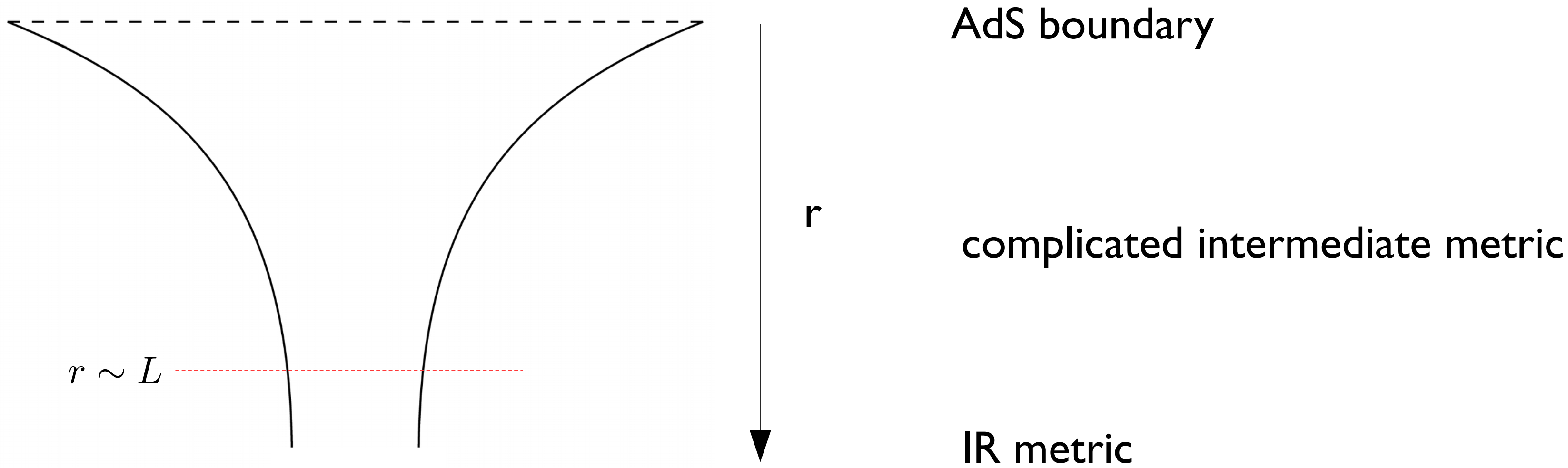
source for scalar operator

$\longrightarrow$ generates an RG flow that ends at a different IR fixed point.

- We want to probe the physics of these IR fixed points.

Holographic quantum criticality II

- We know the form of the holographic dual of such a QFT.



- The metric in the interior reflects the scaling symmetries of the IR fixed point:

$$ds^2 = \left(\frac{r}{L}\right)^{\frac{2\theta}{d}} \left[-\left(\frac{L}{r}\right)^{2z} L_t^2 dt^2 + \left(\frac{L}{r}\right)^2 L_x^2 d\vec{x}^2 + \frac{\tilde{L}^2 dr^2}{r^2} \right] + \ldots$$

z : dynamical critical exponent $\qquad$ $d - \theta$: effective dimensionality

Two classes of IR fixed points

- Spacetimes like this are classical solutions of the action

$$S = \int d^{d+2}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{Z(\phi)}{4}F^2 - V(\phi)\right)$$

with

$$\phi = \sqrt{\frac{2}{d}(d-\theta)(dz-d-\theta)}\log\left(\frac{r}{L}\right) + \ldots \qquad A_t = L_t A_0\left(\frac{r}{L}\right)^{\theta-d-z-2\Delta} + \ldots$$

- The IR fixed points come in two different categories

1). A_0 is a marginal coupling: $z \neq 1$ and $\Delta = 0$

2). A_0 is an irrelevant coupling: $z = 1$ and $\Delta < 0$

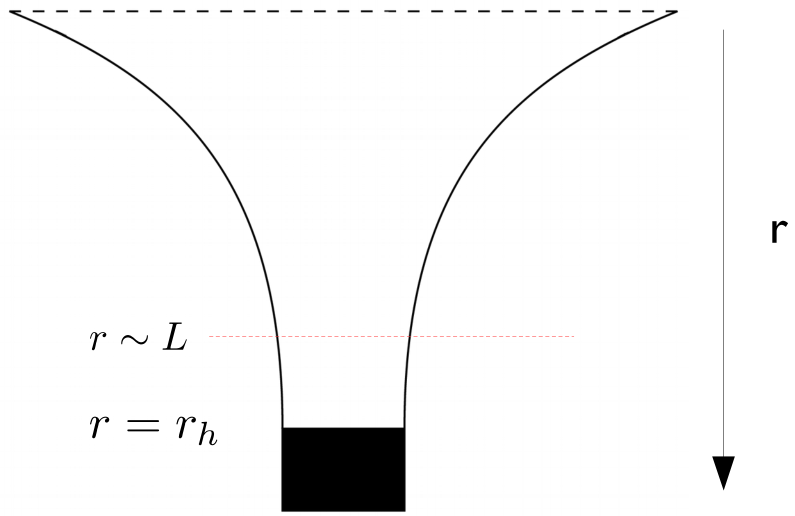
- I will always assume the coupling is non-zero: $A_0 \propto \langle\rho\rangle \neq 0$

Non-zero temperature and IR observables

- Want to probe the physics controlled by the IR fixed point of the QFT.

  i.e. the properties of the QFT controlled by the IR part of the spacetime.

- At non-zero temperatures, there is an event horizon in the spacetime.



- At small T, the scaling of the fixed point still determines IR properties.

  (as in the quantum critical region near a quantum phase transition)

Transport properties of holographic theories

- The QFTs have a conserved energy and a conserved U(1) charge.

- Their conductivities are infinite because of momentum conservation

  e.g.

$$\sigma(\omega) = \frac{\rho^2}{\epsilon + P}\left(\frac{i}{\omega} + \delta(\omega)\right) + \ldots$$

- Isolate the transport that is independent of momentum conservation:

$$\delta\rho_{inc} = s^2 T \delta\left(\rho/s\right)$$

RD, Gouteraux, Hartnoll

  This obeys

$$\partial_t \delta\rho_{inc} + \nabla \cdot \delta j_{inc} = 0 \qquad\qquad \delta j_{inc} = sT\delta j - \rho\delta q$$

Hydrodynamics of incoherent charge density

- Relativistic hydro should take over after local equilibration occurs

$$\sigma_{\rm inc}(\omega) = \sigma_{\rm inc}^{dc} + \ldots$$

- The 'incoherent' charge diffuses

$$\sigma_{\rm inc}(\omega, k) = \frac{i\omega}{i\omega - Dk^2}\sigma_{\rm inc}^{dc} + \ldots \qquad D = \frac{\sigma_{\rm inc}^{dc}}{\chi_{\rm inc}}$$

- At low temperatures $D \to D_T = \dfrac{\kappa}{T\left(\partial s/\partial T\right)_\rho}$

- What governs this thermal diffusivity?

Scaling of thermal diffusivity

- D_T is governed by the IR fixed point (near-horizon spacetime) at small T

- Dimensional analysis

$$\longrightarrow \qquad [D_T] = 2\,[x] - [t] = -2 + z$$

- Temperature is the only scale $\quad [T] = z$

$$\longrightarrow \qquad D_T \sim T^{1-2/z}$$

- An explicit calculation yields

$$D_T = F(L_t, L_x, \rho \ldots) \times T^{1-2/z} = F(\mathrm{UV\ sources}) \times T^{1-2/z}$$

The butterfly velocity

- D_T looks very complicated in these units. Are there more natural units?

- The butterfly velocity v_B is controlled by the IR fixed point

Blake

- v_B is a measure of the speed at which chaotic effects propagate

$$C(x,t) = -\langle [W(x,t), V(0,0)]^2 \rangle_T \sim e^{2\pi T(t - x/v_B)}$$

Shenker & Stanford
Roberts & Stanford

- Explicit calculation in holographic theories

$$v_B^2 = G(L_t, L_x, \rho \ldots) \times T^{2-2/z} = G(\mathrm{UV\ sources}) \times T^{2-2/z}$$

Blake
Roberts & Swingle

consistent with dimensional analysis near the IR fixed point.

Thermal diffusivity in holographic theories

- In units of the butterfly velocity

$$D_T = \frac{F(L_t, L_x, \rho, \ldots)}{G(L_t, L_x, \rho, \ldots)} \times v_B^2 \tau_P$$

IR quantum critical scaling ensures the coefficient is T-independent.

- Explicit calculation of the coefficient gives

$$D_T = \frac{z}{4\pi(z-1)} v_B^2 \tau_P$$

Blake, RD, Sachdev

It depends **only** on the dynamical critical exponent z

- v_B and τ_P control thermal diffusion in all these cases.

Geometric explanation of the result

- Origin: D_T and v_B depend only on the metric near the horizon.

e.g. $$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + h(r)d\vec{x}^2 \qquad f(r) \sim L_t^{-2} r^{\#} \qquad h(r) \sim L_x^{-2} r^{\#}$$

$$\kappa = 4\pi \frac{f'(r_h)}{f''(r_h)} \qquad T\left(\frac{\partial s}{\partial T}\right)_\rho = \frac{2-\theta}{z} 4\pi h(r_h) \qquad v_B^2 \tau_P = \frac{2\pi}{h'(r_h)}$$

$$\longrightarrow \quad D_T = \frac{z}{2\pi\,(2-\theta)} \frac{f'(r_h)h'(r_h)}{f''(r_h)h(r_h)} \times v_B^2 \tau_P$$

- Holds for more complicated actions and matter field profiles.

- Analogous results found in some other non-holographic cases

Gu, Qi, Stanford
Patel & Sachdev

- Suggests chaos and thermal diffusion originate from same underlying dynamics

Blake, Lee, Liu

What about z=1?

- Why does this break down for critical phases with z=1 ?

$$D_T \to \infty \qquad\qquad v_B^2 \sim T^0$$

- The problem is with D_T calculated from the IR scaling geometry

- D_T is sensitive to the irrelevant coupling A_0

$$D_T \sim \frac{1}{T} \times \frac{T^{2\Delta}}{A_0^2} \qquad\qquad \Delta < 0$$

- There is an apparent contradiction with a proposed upper bound

$$D \lesssim v^2 \tau_{eq}$$

Hartnoll
Lucas

Relaxation time

- τ_{eq} is set by the longest-lived pole of the retarded Green's functions

- Calculate the conductivity of the U(1) charge $\sigma(\omega) = -\frac{i}{\omega}\lim_{r\to 0}\left(r^{2-d}\frac{a_x'(r)}{a_x(r)}\right)$

- Use the variable $\tilde{a}_x = \frac{a_x}{sT + \rho A_t(r)}$

$$\frac{d}{dr}\left[\sqrt{\frac{-g_{tt}}{g_{rr}}}Z(\phi)g_{xx}^{d/2-1}\left(sT+\rho A_t\right)^2\tilde{a}_x'\right] + \omega^2\sqrt{\frac{g_{rr}}{-g_{tt}}}Z(\phi)g_{xx}^{d/2-1}\left(sT+\rho A_t\right)^2\tilde{a}_x = 0$$

- Look for a perturbative solution $\tilde{a}_x = \left(\frac{r_h - r}{r_h}\right)^{-i\frac{\omega}{4\pi T}}\left(\mathcal{A}_0(r) + \left(\frac{\omega}{4\pi T}\right)\mathcal{A}_1(r) + O(\omega^2)\right)$

$$\longrightarrow \quad \sigma(\omega) = \frac{i}{\omega}\frac{\rho^2}{(sT+\rho\mu)} + \frac{\sigma_0}{(1 - i\omega\tau_{eq})}$$

- We can trust this answer if $\tau_{eq} \gg T^{-1}$

Relaxation time for z=1 critical points

- The lifetime is given by an integral over the entire spacetime

$$\tau_{eq} = \frac{1}{4\pi T}\int_0^{r_h} d\tilde{r}\left(-\frac{s^3T^3Z(\phi(r_h))}{\rho^2(s/4\pi)^{2/d}}\frac{g_{xx}(\tilde{r})}{g_{tt}(\tilde{r})}\frac{d}{d\tilde{r}}\left(\frac{1}{sT+\rho A_t(\tilde{r})}\right)-\frac{1}{r_h-\tilde{r}}\right)$$

Examine the contribution from the IR part of the spacetime.

- The contribution from the IR spacetime diverges at small T when $z=1$

$$\longrightarrow \quad \tau_{eq}(T\to 0) = \frac{L_x^2}{L_t^2}\frac{s^2TZ(\phi(r_h))}{4\pi\rho^2(s/4\pi)^{2/d}} \sim \frac{1}{T}\times\frac{T^{2\Delta}}{A_0^2}$$

RD, Gentle, Gouteraux

There is a collective excitation (QNM) with a parametrically long lifetime.

The dangerously irrelevant coupling

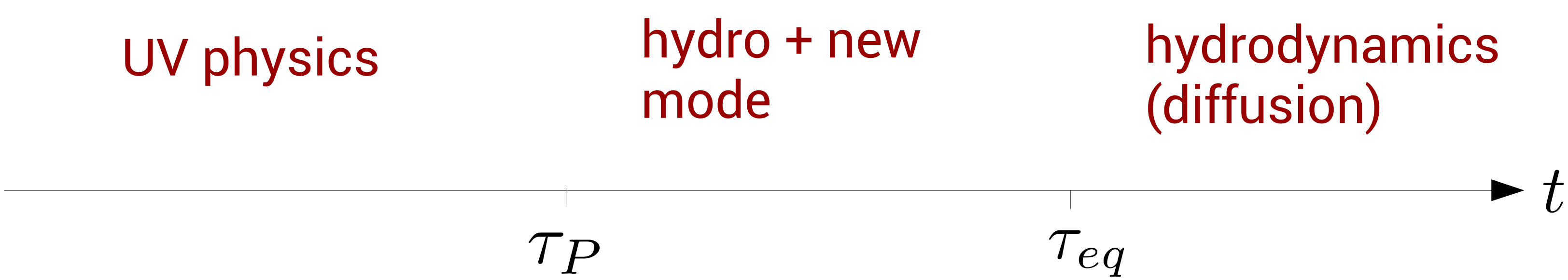
- Dynamics survives over much longer timescales than expected $\tau_{eq} \gg \tau_P$
- This timescale also controls diffusive transport

$$D_T = \frac{2}{d+1-\theta} v_B^2 \tau_{eq}$$

- The late time dynamics are dangerously sensitive to the irrelevant coupling A_0
- Similar to when an irrelevant coupling breaks translational symmetry.
- Also explains a number of other strange properties of these states.

Breakdown of hydrodynamics

- Physical consequence: hydrodynamics breaks down at times $t \lesssim \tau_{eq}$



- What is the new mode and why is its lifetime sensitive to the irrelevant coupling?

- Simplest explanation: uniform perturbations of the incoherent current

$$\partial_t j_{\rm inc} = -\frac{j_{\rm inc}}{\tau_{eq}}$$

Conclusions

- Holography is useful for understanding quantum many-body systems.

$\longrightarrow$ allows explicit calculation of properties of a wide variety of quantum critical systems.

- The thermal diffusivity is governed by the butterfly velocity v_B and the Planckian time τ_P

$\longrightarrow$ do thermal diffusion and the spread of quantum chaos share a common origin?

- The exceptional examples have a collective mode with a parametrically long lifetime $\tau_{eq} \gg \tau_P$

$\longrightarrow$ what is the nature of this new long-lived mode?

why does the irrelevant coupling A_0 determine its lifetime?

Extra slides......