Higher-form symmetry in theory of elasticity

and holographic models with transverse sound

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Based on work with Sašo Grozdanov 1801.03199
and work in progress with Sašo Grozdanov and Andy Lucas

Holography is an effective theory contructed from **global symmetry**

$$\boxed{\begin{aligned} Z_{QFT}[g_{\mu\nu}, a_\mu] &= \left\langle e^{i \int d^{d+1}\sqrt{-g}(T^{\mu\nu}g_{\mu\nu} + j^\mu A_\mu)} \right\rangle, \\ &= \exp\left(iS_{\text{gravity}}[G_{ab}, A_a]\right) \end{aligned}}$$

where

$$\boxed{G_{\mu\nu}(u, x^\mu)\Big|_{u\to 0} \sim g_{\mu\nu}(x^\mu), \qquad A_\mu(u, x^\mu)\Big|_{u\to 0} \sim A_\mu}$$

Implying that

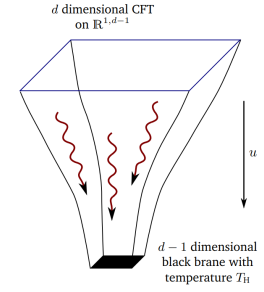
$$\partial_\mu \langle T^{\mu\nu}\rangle = 0, \qquad \partial_\mu \langle j^\mu \rangle = 0$$

* Toy models for strongly coupled QFT
* **Hydrodynamic** behaviour of QFTs at strong coupling

  Bandurin et. al.; Crossno et. al. ; Moll et. al. ’16

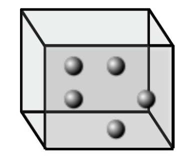
  Policastro, Son & Starinets; Kovtun, Son & Starinets ’01-’05
* Universal lower bound for η/s

  Kovtun, Son & Starinets ’05



Ordinary vs higher-form conserved currents

Ordinary or 0-form global symmetry



- Count number of **"point" particles** via **1-form current**

$$Q = \int dV_\mu j^\mu$$

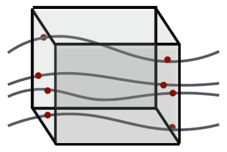
- (For $U(1)$ global symmetry) transform **local operator** at point x

$$\mathcal{O}(x) \quad \to \quad e^{i\alpha Q}\,\mathcal{O}(x)\,e^{-i\alpha Q} = e^{i\alpha}\mathcal{O}(x)$$

Higher-form global semmetry

- Count number of **"lines"** via **2-form current**

$$Q = \int dS_{\mu\nu} J^{\mu\nu}$$



- (For $U(1)$ global symmetry) transform **line operator** define on a curve γ

$$\mathcal{W}(\gamma) \quad \to \quad e^{i\alpha Q}\,\mathcal{W}(\gamma)\,e^{-i\alpha Q} = e^{i\alpha}\mathcal{W}(\gamma)$$

Gaiotto, Kapustin, Seiberg & Willet '14

Lake '18

Why do we care about these symmetries ?

Because they are abundance in nature

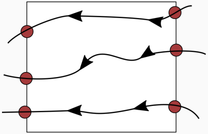
- Topological phases of matter

Nussinov & Ortiz '08; Kapustin & Thorngren '13; Kapustin & Seiberg '14;...

- Ordinary system such as $E\&M$ or theory of elasticity

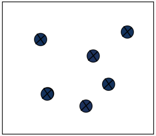
QED in $2+1d$: $S = \int d^3x F_{\mu\nu}F^{\mu\nu}$

$$\underbrace{d\star F}_{\text{Electric flux}} = 0, \qquad \underbrace{d\star j}_{\text{Magnetic flux}} = dF = 0$$



Dual to superfluid in 2+1d: $S = \int d^3x (\partial\phi)^2$

$$d\star(d\phi) = 0, \qquad d\star(\star d\phi) = d^2\phi = 0$$



- Other systems with or without supersymmetry

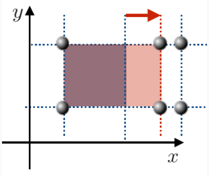
Gaiotto,Kapustin, Seiberg & Willet '14;
Cordova, Dumitrescu &Intrilligator '18

Typically, it is described by dynamics of **displacement field** ϕ^I

$$\boxed{S = \sum_{I,J=1,2} \int d^3x \left(\rho(\partial_t \phi^I)^2 + C^{ij}_{IJ} \partial_i \phi^I \partial_j \phi^J \right) = \sum_{IJ} \int d^3x \, C^{\mu\nu}_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J}$$

There are two kind of conservation law:

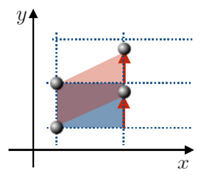
$$\partial_\mu P^\mu_I = 0, \qquad \underbrace{P^\mu_I = C^{\mu\nu}_{IJ} \partial_\nu \phi^J}_{\text{momentum density}}$$

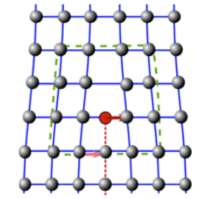


But we also have

$$\partial_\mu J^{\mu\nu}_I = 0, \qquad J^{\mu\nu}_I = \varepsilon^{\mu\nu\lambda} \partial_\lambda \phi_I$$

This system is dual to EM-like theory $a_\mu \to a^I_\mu$





$$S = \sum_{IJ} \int d^3x \, (da^I) \cdot \tilde{C}_{IJ} \cdot (da^J)$$

Kleinert '89; Beekman, Nissinen et. al. '16

We will build EFT from global symmetry

`and ask how far will we get`

`e.g. do we get transverse sound ?`

$*$ Hydrodynamics of $\quad \partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J_I^{\mu\nu} = 0$

- (at ideal level) same eoms as "hydro" for CDW and Wigner crystal

Martin, Parodi & Pershan '72
Delacretaz, Gouteraux, Hartnoll & Karlsson '17

$*$ Holography

- Simplest holographic model with $\partial_\mu J_I^{\mu\nu} = 0$
- Analytic results ?
- Agreement with "hydrodynamic" prediction?

Starting from

$$Z[g_{\mu\nu}, b_{\mu\nu}] = \left\langle \exp\left[\int d+1x\sqrt{-g}\left(T^{\mu\nu}g_{\mu\nu} + J^{\mu\nu}b_{\mu\nu}\right)\right]\right\rangle,$$
$$= \exp\left[-\int d^{d+2}X\sqrt{-G}\,\mathcal{L}[G_{ab}, B_{ab}] + S_{\text{bnd}}\right]$$

with $\mathcal{L} = \frac{1}{3}(dB_I)_{abc}(dB_I)^{abc} + \ldots$ and $B|_{\text{bnd}} \sim b$
Simplest action gives a very simple analytic solution

$$\boxed{\begin{aligned} ds^2 &= \frac{dr^2}{r^2 f} + r^2\left(-f dt^2 + dx^2 + dy^2\right), \\ f(r) &= 1 - \frac{m^2}{2r^2} - \left(1 - \frac{m^2}{2r_h^2}\right), \quad (dB_1)_{txr} = (dB_2)_{tyr} = -m \end{aligned}}$$

- ∗ We'll soon see that $m \sim$ line density
- ∗ This is the same solution as 'linear axion' model (hodge dual in the bulk)

Bardoux,Caldarelli & Charmousis '12; Andrade & Withers '13

- Unusual boundary expansion

$$B_{\mu\nu}(r) = \hat{J}_{\mu\nu} r + \hat{B}_{\mu\nu} + \mathcal{O}(1/r)$$

and **eoms $\nabla_a (dB)^{ab\mu} = 0$ only implies that $\partial_\mu \hat{J}^{\mu\nu} = 0$**

- Not very usual counter term action:

$$S_{c.t.} = +\frac{1}{\kappa(\Lambda)} \int_{r=\Lambda} d^{d+1} \sqrt{-\gamma} \Big(n^a (dB_I)_{a\mu\nu} \Big)^2$$

Typically, we will write

$$\frac{1}{\kappa(\Lambda)} = 1 - \frac{\mathcal{M}}{\Lambda}, \qquad \mathcal{M}/\Lambda \sim \text{finite counter term}$$

Here the finite term is VERY IMPORTANT

Witten '00,...,Cvectic & Papadimitriou '16
for higher dim see Grozdanov & Poovuttikul '17; Hofman & Iqbal '17

- Source can be defined via

$$\boxed{b_{\mu\nu} = \frac{\delta S}{\delta J^{\mu\nu}} = B_{\mu\nu}(\Lambda) - \frac{\Lambda}{\kappa(\Lambda)} J_{\mu\nu}}$$

- Source must not depends on location of the UV cutoff Λ

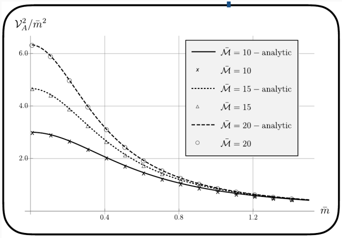
$$\boxed{\frac{\partial b_{\mu\nu}}{\partial \Lambda} = 0, \qquad \Rightarrow \qquad \frac{1}{\kappa(\Lambda)} = 1 - \frac{\mathcal{M}}{\Lambda}}$$

- Find two coupled eoms for

$$Z_+ := \omega\delta B_{1,xy} + k\delta B_{1,ty}, \qquad Z_- = \omega\delta G^y_x + k_x\delta G^y_t$$

and to find spectrum by set source = 0

$$\boxed{\begin{aligned} 0 &= \lim_{u\to 0}\left[Z_+(u) + u\left(1 - u\mathcal{M}/r_h\right)\frac{\partial Z_+}{\partial u}\right], \\ &= \lim_{u\to 0} Z_-(u) \end{aligned}}$$



- Analytic result at small $m/r_h, \omega/r_h, k/r_h$ and large $\mathcal{M}/r_h$

$$\langle T^{ty}T^{ty}\rangle = \frac{\mu\rho k_x^2}{\omega^2 - \mathcal{V}_A^2 k_x^2},$$

$$\mathcal{V}_A^2 = \frac{1}{3}\Big(\mathcal{M}/r_h - 1\Big) + \mathcal{O}(m^2) = \frac{\mu\rho}{\varepsilon + \mathbf{p} - \mu\rho}$$

In agreement with hydro predection

- **High temperature instability** for $r_h/\mathcal{M} > 1$

Comments on different approach

There are mainly two ways people get signature of elasticity

- Models with dynamical field ϕ_I that behave like displacement field

  Esposito, Garcis-Saez, Nicolis & Penco '17; Alberte, Ammon, Baggioli, Jimenez & Pujolas'17
  Amoretti, Arean, Gouteraux & Musso '17

  - They basically have the same global symmetry

- Break the translational symmetry dynamically
  - ⇛ Models with finite k instability at low T
  - ⇛ Track the instability, mostly get inhomogeneous background

  Ooguri & Park '10; Donos & Gauntlett; Rozali, Smyth, Sorkin & Stang; Withers '13;
  Jokela, Jarvinen & Lippert; Cremonini, Li & Ren; Andrade, Krikun, Schalm & Zaanen '17

  At long distance, these models will be described by fluid + elasticity

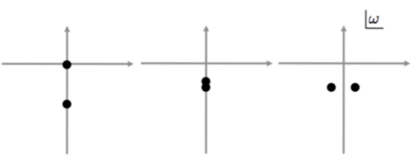
- As far as low-energy excitation (controlled by GS) are concerned,
  they should gives qualitatively the same result

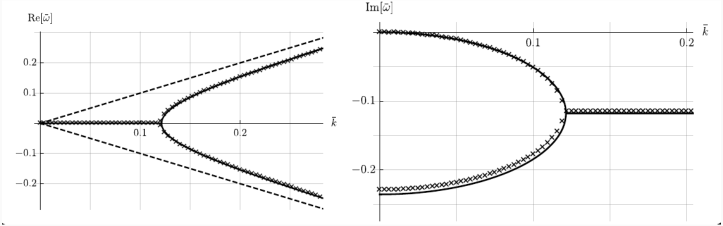
Emergent transverse sound

- Pole collision phenomena : Transverse sound in fluid

  - Almost conserved higher-form currents

    $\tau_{\text{relax}} \sim 1/\omega_{\text{non-hydro}}$





Grozdanov, Kaplis & Starinets '16; Casalderrey-Solana, Grozdanov & Starinets '18 ; Baggioli & Trachenko '18; Yang, Dove, Brazhkin & Trachenko '17

Outlook

- Global symmetry aspects of gauge theory and theory of sSB
  - **Lots things to explore : sSB of higher-form symmetry, BKT transition, anomaly**
  - **Convenient for hydrodynamics, memory matrix approach**
- Classification of phases & transport coefficients
  - **Convenient for hydrodynamics, memory matrix approach**
  - **Natural to add topological defects**
- Interesting new dynamics/instability on gravity dual

It is actually very fun

THANK YOU VERY MUCH FOR LISTENING!