A quantum hydrodynamical description for chaos

Mike Blake - MIT

Massachusetts Institute of Technology

MAB, Hyunseok Lee and Hong Liu
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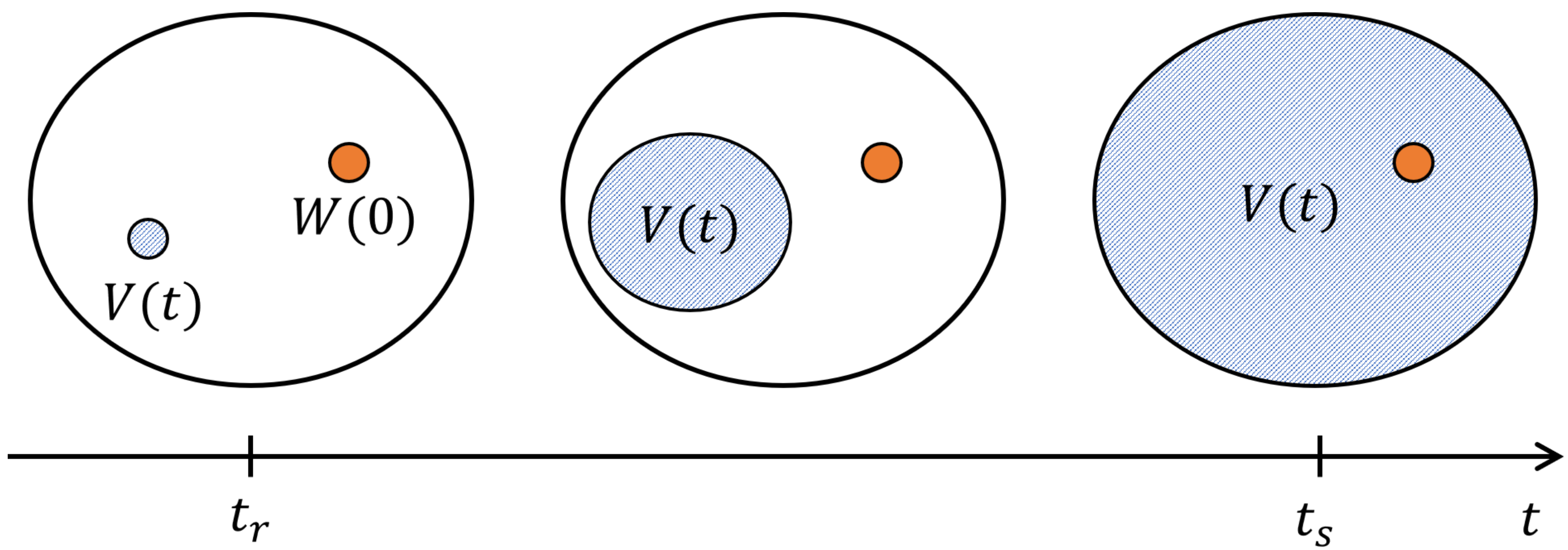
Effective Theories of Chaos

- Much recent progress in characterising scrambling/chaos in quantum many body-systems.
- Most studies focus on specific models (SYK, holography, spin chains).
- Goal of this talk is to suggest an effective field theory description in terms of hydrodynamics.

Chaos and Scrambling

- Scrambling/chaos describes growth of operators with time



- Simple measure provided by expectation value of commutators

$$C(t) = \langle [W(t), V(0)]^2 \rangle_{\beta_0}$$

- In semiclassics this grows exponentially in a chaotic system

$$\{x(t), p(0)\} = \frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda_L t} \quad \Longrightarrow \quad C(t) \sim \hbar^2 e^{2\lambda_L t}$$

- Many-body quantum chaos: For interacting quantum systems with many degrees of freedom

$$C(t) \sim \frac{1}{\mathcal{N}} e^{\lambda t}$$

$$t_r \ll t \ll t_s$$

$$\lambda \leq \frac{2\pi k_B}{\hbar \beta_0}$$

Maldacena, Shenker & Stanford

- For operators separated in space typically get ballistic spreading e.g. in SYK chains/holography

$$\langle [W(t,x), V(0,0)]^2 \rangle_{\beta_0} \sim \frac{1}{\mathcal{N}} e^{\lambda(t - x/v_B)}$$

(Different spatial dependence often seen in non-maximally chaotic systems)

- Exponential behaviour originates from out-of-time ordered correlation functions

Time Ordered $\langle W(t)W(t)V(0)V(0)\rangle$

Out-of-time Ordered $\langle W(t)V(0)W(t)V(0)\rangle$

Hydrodynamics

- Hydrodynamics is a universal description of dynamics of conserved quantities (e.g. energy) on long distances.
- Naively this has nothing to do with chaos and scrambling.
- But various hints at a deep connection:

1. Schwarzian action is a hydrodynamic theory.
2. In holography chaos originates from gravitational shock waves.
3. In many chaotic systems butterfly velocity is related to energy diffusion constant.

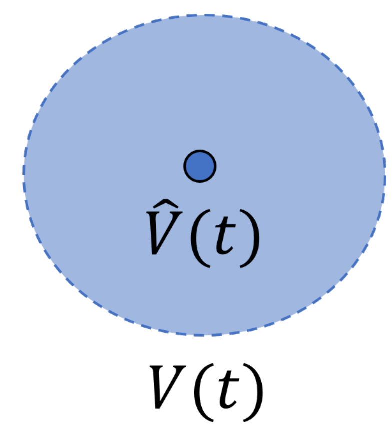
However…

- Chaos involves dynamics on timescales which lie beyond a derivative expansion $\tau \sim 1/\lambda$
- Also need to be able to calculate OTOCs in a hydrodynamic theory.
- Recently there has been a new formulation of hydrodynamics as a quantum field theory.
- Extends validity of hydrodynamics to a quantum non-perturbative level.

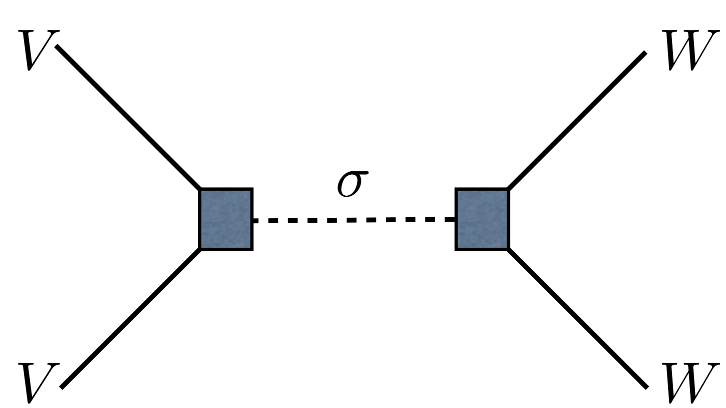
Crossley, Glorioso & Liu

- Proposal: Quantum hydrodynamic theory has an exponentially growing mode $\sigma(t) \sim e^{\lambda t}$ associated to energy conservation*

$$V(t) = f\big(\hat{V}(t), \sigma\big) =$$



- Chaos originates from coupling of bare operator $\hat{V}(t)$ to this mode.



* at least for maximal chaos

Hydrodynamics with energy conservation

- In 0+1 dimensions only conserved quantity is energy. Dynamical variables of hydrodynamic theory

$$\beta(t) = \beta_0/\partial_t\sigma$$

Comoving fluid time $\quad \sigma(t)$

Noise field $\quad \sigma_a(t)$

- Simple generalisation to $\sigma(t,\vec{x}) \quad \sigma_a(t,\vec{x})$ for higher dimensional systems with only energy conservation.

- Hydrodynamics is most general action for these variables subject to certain symmetries.
- Full hydro action non-perturbative in derivatives and can be non-local at scales β_0
- Energy density and energy flux are functions of $\sigma(t,\vec{x})$ $\sigma_a(t,\vec{x})$.
- Equation of motion is just energy conservation.

Hydrodynamic description of chaos

- Propose action has extra symmetry

$$e^{-\lambda\sigma} \to e^{-\lambda\sigma} + c$$

(motivation from gravity)

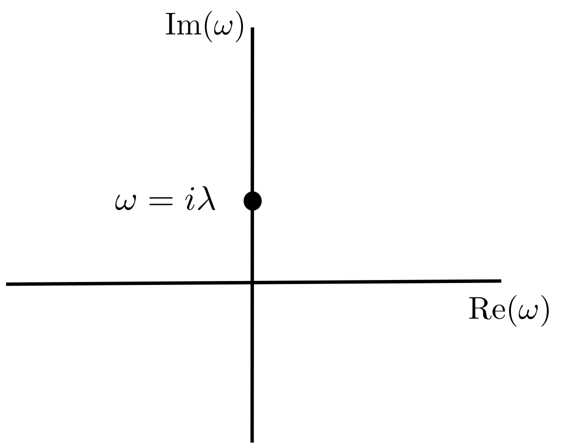
- Guarantees solution of form

$$\sigma = t + ce^{\lambda t}$$

- Energy/energy flux invariant under symmetry and do not see exponential mode (not an instability!)

- Examples include actions for SYK/SYK chains.

Correlation functions



- Chaos behavior shows up in hydro Green's function

$$G_R = \langle \sigma(t, x_i)\sigma(0)\rangle_{\beta_0} \;\; \sim e^{\lambda t}$$

- Also have a series of poles in lower half plane described by solutions to

$$\omega + ik^2 D(\omega, k^2) = 0 \qquad D = D_E + \mathcal{O}(\omega, k^2)$$

- These are physical hydrodynamic poles that also appear in $G^R_{T_{00}T_{00}}$. They determine v_B !

- Combination of two gives ballistic behavior

$$G_R \sim \int d\omega dk \frac{e^{-i\omega t + ikx}}{(\omega - i\lambda)(\omega + ik^2 D(\omega, k^2))}$$

$$\sim \theta(t) e^{\lambda(t - x/v_B)}$$

$$\lambda + k^2 D(i\lambda, k^2) = 0 \qquad k = ik_0 \qquad v_B = \frac{\lambda}{k_0}$$

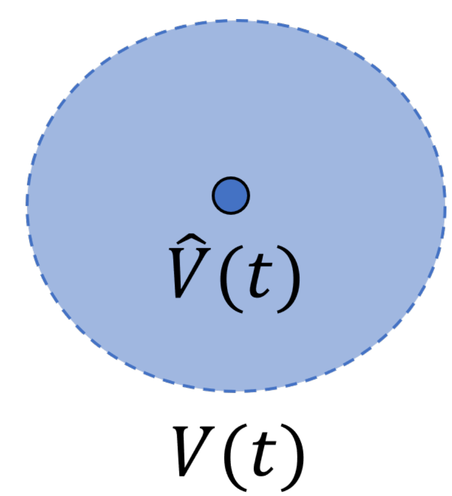
- Example of SYK chain corresponds to choice $D(\omega, k^2) = D_E$ which gives result $v_B^2 = D_E \lambda$

c.f. Gu, Qi & Stanford

Four-point functions

- Coupling of operators to hydrodynamic mode

$$V(t) = f\left(\hat{V}(t), \sigma\right) =$$

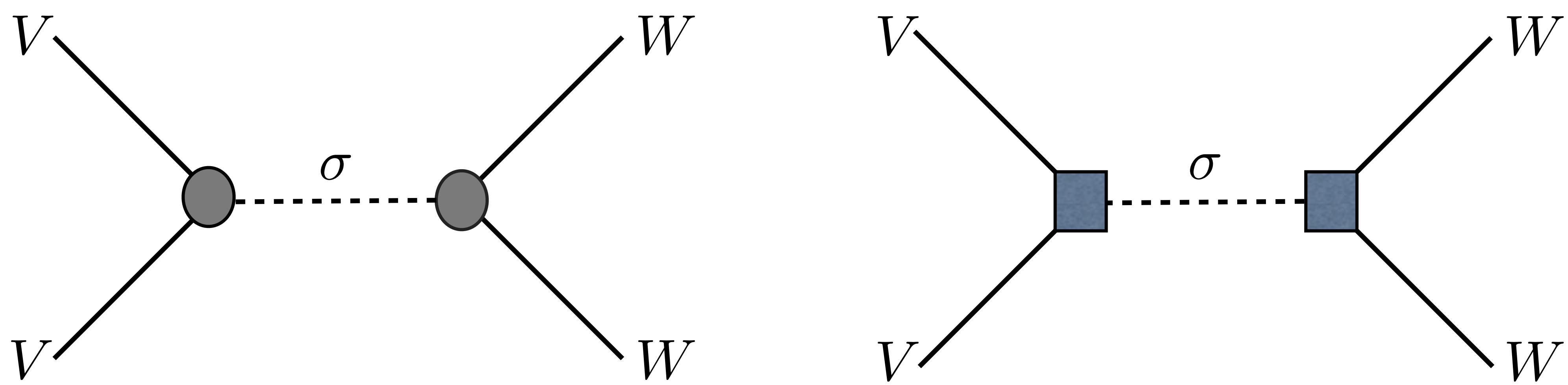


- Both TO and OTO correlation functions can be related to Green's functions of σ (for large $\mathcal{N}$)

- Only difference is in structure of effective vertex

Time Ordered

Out-of-time Ordered



- If coupling respects shift symmetry

No exponential growth

$$\sim 1$$

Exponential growth

$$\sim e^{\lambda(t-x/v_B)}$$

Pole-skipping

- In our theory chaos and energy are described by dynamics of same field σ

- Key predictions: Signatures of chaos in energy density two point function $G^R_{T_{00}T_{00}}(\omega, k)$

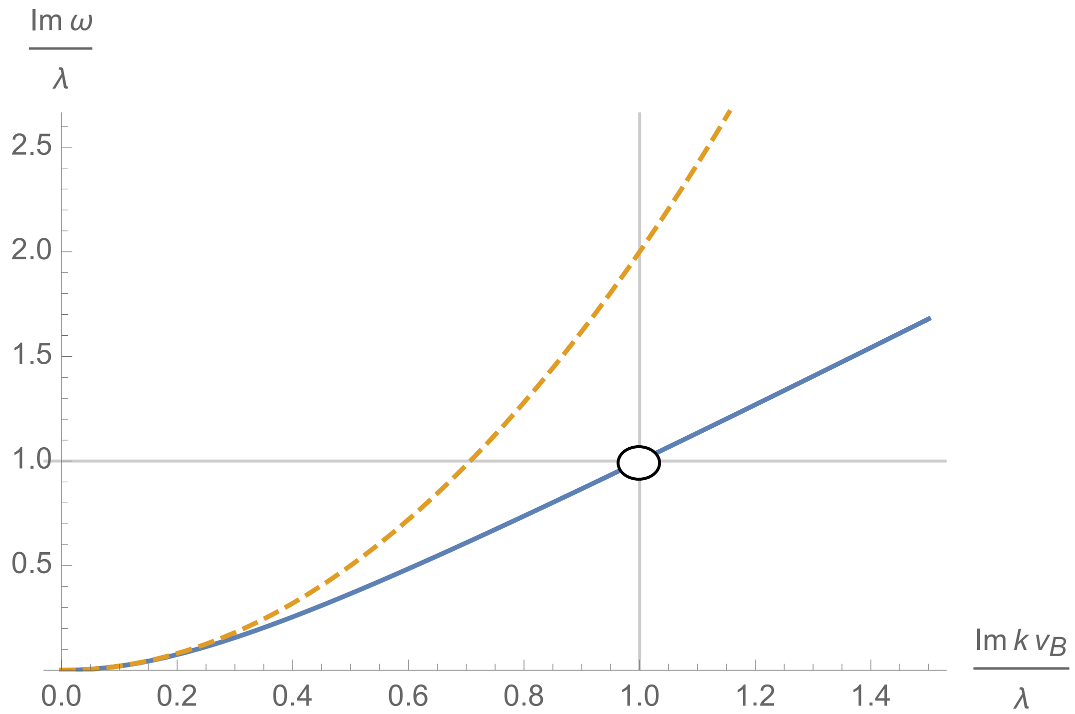
- At low frequencies this will have diffusion pole

$$\omega(k) = -iD_E k^2 + \ldots$$

- Prediction 1: This line of poles passes through the special point

$$\omega = i\lambda \quad k = \frac{i\lambda}{v_B}$$

- Prediction 2: There is also a line of zeroes passing through this point (`Pole-skipping')



- Simplest example is SYK chain

Gu et al

$$G^R_{T_{00}T_{00}}(\omega, k) \sim \frac{i\omega(\omega^2 + \lambda_{max}^2)}{-i\omega + D_E k^2}$$

- First observed holographically in example of AdS5 black holes.

Grozdanov, Schalm & Scopelliti

- We recently found that pole-skipping generically holds in holographic theories dual to Einstein gravity plus matter

Blake, Davison, Grozdanov & Liu

- Pole-skipping and EFT also recently derived for 1+1d CFTs at large c

Haehl and Rozali

Conclusions

- Proposed an effective theory in which exponential growth and ballistic propagation of chaos arise from quantum hydrodynamic field σ
- Theory predicts `pole-skipping' in energy density two point function - a generic feature of many maximally chaotic systems.
- Most of motivation/evidence comes from maximally chaotic systems. Similar effective theory for non-maximal chaos?

Thank you!

Holographic Axion Model

- Translational symmetry breaking parameter m/T
- Dispersion relation changes but in all cases the wave-vector at which the pole passes through $\omega = i\lambda$ is precisely given by $k = i\lambda/v_B$