“Transport and Black Hole Horizons”

Talk at “Bounding Transport and Chaos in Condensed Matter and Holography”

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Based on work with:

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Outline

1 Motivation/Setup

2 Holographic Lattices

3 Spontaneous Modulated Phases

4 Summary

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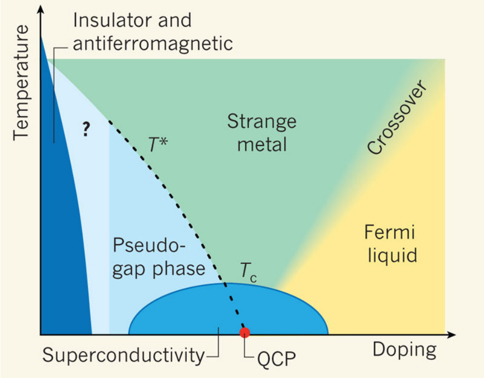
The Cuprates



Appeal to the holographer :

- Non Quasi-Particle physics
- Strong momentum relaxation $\rightarrow$ Incoherent transport
- Competing orders with broken translations

Fourier/Ohm law

- Experiments on transport of conserved charges probe collective degrees of freedom
- Apply electric field $e^{-i\omega t}E_0$ and temp gradient $e^{-i\omega t}\nabla \ln T_0$
- Extract response of electric $J^i(\omega)$ and thermal current $J_Q^i(\omega) = -{T^i}_t(\omega) - \mu\, J^i(\omega)$
- Transport coefficients are packaged in Ohm/Fourier law

$$\begin{pmatrix} J(\omega) \\ J_Q(\omega) \end{pmatrix} = \begin{pmatrix} \sigma(\omega) & T\alpha(\omega) \\ T\bar{\alpha}(\omega) & T\bar{\kappa}(\omega) \end{pmatrix} \begin{pmatrix} E_0 \\ -\nabla \ln T_0 \end{pmatrix}$$

- From linear response e.g. $\sigma = G_{JJ}(\omega, k=0, k'=0)/(i\,\omega)$
- The DC limit $\omega \to 0$ is experimentally interesting

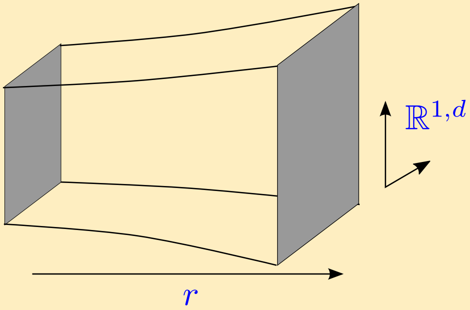
Setup

To model in holography:

- CFT with a global $U(1)$
- Finite chemical potential μ_0
- Finite temperature T_0
- Introduce periodic sources that can relax momentum:
  - Local chemical potential $\mu(\mathbf{x})$
  - Local temperature $T(\mathbf{x})$
  - Local strain $g_{ij}(\mathbf{x})$
- Probe with external electric field $\nabla\delta\mu = E$ and temp gradient $-\nabla\delta T/T = \zeta$ to extract conductivities

$\rightarrow$ Powerful gravitational techniques give access to the RG flow

AdS/CMT



The CFT vacuum is modelled by AdS_{d+2}

$$ds^2 = r^2 \left(-dt^2 + d\mathbf{x}_d^2\right) + \frac{dr^2}{r^2}$$

AdS/CMT

Schematically the bulk action is

$$\mathcal{L} = R_{d+2} + \Lambda - \frac{1}{4}F^2 + \mathrm{matter}$$

- Use the relevant/marginal ones to deform the boundary theory

$$S = S_{CFT} + \int d^{d+1}x\, \phi(x)\, \mathcal{O}(x)$$

- Introduce a black hole (brane) horizon in the bulk to raise temperature T

AdS/CMT

Universal deformations are

- The stress tensor

$$ds^2 = r^2 \left(-dt^2 + d\mathbf{x}_d^2 + \delta g_{\mu\nu}(\mathbf{x}) dx^\mu dx^\nu\right) + \frac{dr^2}{r^2} + \cdots$$

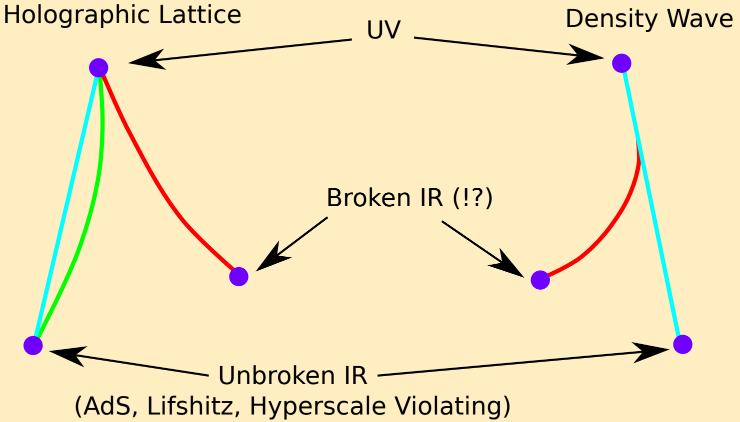
- The chemical potential

$$A = \mu(\mathbf{x})\, dt + \cdots$$

- Subleading terms give the VEVs
- The dual action is

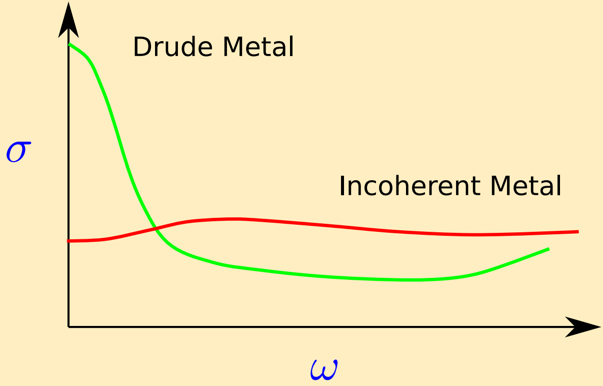
$$S = S_{CFT} + \int \mu\, J^t + \frac{1}{2}\delta g_{\mu\nu} T^{\mu\nu}$$

RG Flows with broken translations



- Strong lattices can lead to new IR fixed points with broken translations $\rightarrow$ Incoherent transport
- Similar to the spontaneously broken ones
- Transport at low frequencies is different

Transport Properties



- Weak lattices gives small momentum relaxation τ^{-1} rates and $\sigma(\omega \to 0) \propto \tau$
- Strong lattices lead to incoherent transport. Anything universal at low frequencies?
- Focus on DC limit

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General Considerations

Define theory on a spatially periodic manifold

$$ds^2 = -dt^2 + \gamma_{ij}(\mathbf{x})\, dx^i dx^j$$
$$A^{ext} = \mu(\mathbf{x})\, dt$$

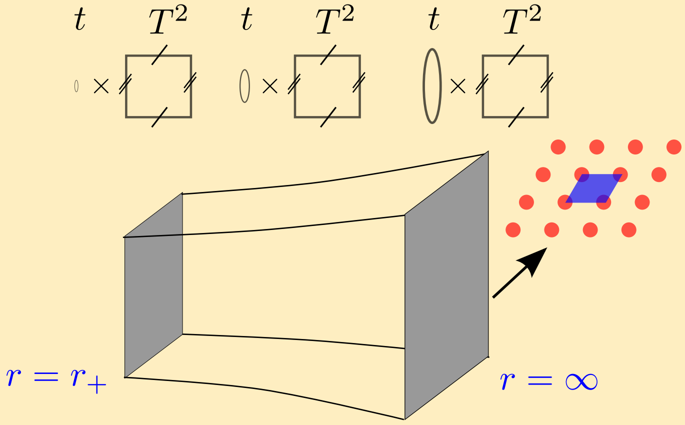
with ∂_t a symmetry. To study transport:

- Perturb by temperature gradient and electric field

$$\delta ds^2 = 2\,\phi_T\, dt^2, \qquad \delta A^{ext} = -\mu\,\phi_T\, dt + \phi_E\, dt$$
$$\partial_i \ln T = -\zeta_i = -\partial_i \phi_T, \qquad E_i = -\partial_i \phi_E$$

- Compute response for conserved electric δj^μ and heat currents $\delta J_Q^\mu = -\delta T^\mu{}_t - \mu\,\delta J^\mu$
- In the DC limit we have $\partial_t \phi_T = \partial_t \phi_E = 0$ and ζ, E are closed one-forms.

Setup for “lattices”



- Introduce periodic lattice (deformation) on the boundary [Hartnoll, Hofman][Horowitz, Tong, Santos]
- Focus on simple black hole topologies [AD, Gauntlett, Griffin, Melgar]

DC conductivities from BH horizons

- Bulk theory is Einstein-Maxwell
- Consider E/M charged, static black branes

$$ds^2 = -UG\,dt^2 + \frac{F}{U}\,dr^2 + ds^2(\Sigma_d)$$
$$A = a_t\,dt$$
$$ds^2(\Sigma_d) = g_{ij}(r,x)dx^i dx^j$$

- Asymptotically, $r \to \infty$

$$U \to r^2, \qquad F \to 1, \qquad a_t(r,x) \to \mu(x)$$
$$G \to \bar{G}(x), \qquad g_{ij}(r,x) \to r^2\gamma_{ij}(x)$$

- Local μ, T and strain

DC conductivities from BH horizons

- Close to the horizon as $r \to 0$

$$ds^2 = -4\pi T\, r\, dt^2 + (4\pi T\, r)^{-1}\, dr^2 + \gamma^{(h)}_{ij}(\mathbf{x})\, dx^i\, dx^j$$
$$A = r\, \rho^{(h)}(\mathbf{x})\, dt$$

- The horizon quantities $\gamma^{(h)}_{ij}$, $\rho^{(h)}$ are different objects from the field theory γ_{ij} and ρ
- In the limit long lattice periods L limit

$$\gamma^{(h)}_{ij}(\mathbf{x}) = c_\gamma\, \gamma_{ij}(\mathbf{x}) + \mathcal{O}(L^{-2})$$
$$\rho^{(h)}(\mathbf{x}) = c_\rho\, \rho(\mathbf{x}) + \mathcal{O}(L^{-2})$$

- Limit to connect to hydro/gravity

DC conductivities from BH horizons

For the perturbation:

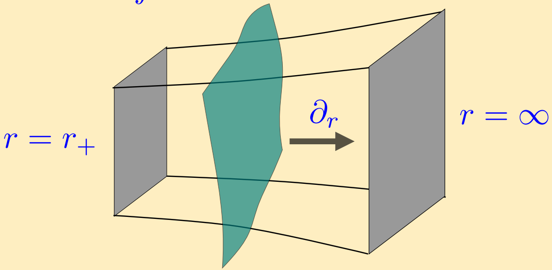
$$\delta(ds^2) = \delta g_{\mu\nu}(r,x)dx^\mu dx^\nu + 2GU\phi_T\,dt^2,$$
$$\delta A = \delta a_\mu(r,x)dx^\mu + (a_t\,\phi_T + \phi_E)\,dt$$

- Split perturbation in terms with and without sources
- Look for a perturbative solution with ∂_t a symmetry
- Complicated system of linear PDE's:
  - $g_{\mu\nu} \to \frac{1}{2}\,(d+2)\,(d+3) - (d+2)$ functions
  - $A_\mu \to (d+2) - 1$ functions

Radial Hamiltonian

- Imagine radial foliation by hypersurfaces e.g. normal to ∂_r
- Radial evolution Hamiltonian is sum of constraints

$$H_{\partial_r} = \int N\,\mathcal{H} + N_\mu\,\mathcal{H}^\mu + D\,\mathcal{G} + \text{b.t.}$$



- At infinity they yield Ward identities

$$\nabla_\mu \langle T^{\mu\nu}\rangle = F^{\mu\nu}\,\langle J_\nu\rangle\,, \qquad \nabla_\mu \langle J^\mu\rangle = 0, \qquad \langle T^\mu{}_\mu\rangle = \text{anom}$$

- Meaningful but not closed system without e.g. hydro

DC conductivities from BH horizons

Examine constraints close to the horizon

- Impose infalling conditions
- Define

$$v_i \equiv -\delta g_{it}^{(0)}, \qquad w \equiv \delta a_t^{(0)}\,,$$
$$p \equiv -4\pi T\frac{\delta g_{rt}^{(0)}}{G^{(0)}} - \delta g_{it}^{(0)}\,g_{(0)}^{ij}\nabla_j\,\ln G^{(0)}$$

DC conductivities from BH horizons

Constraints on the horizon give

$$\mathcal{H}^t \Rightarrow \quad \nabla_i v^i = 0$$

$$\mathcal{G} \Rightarrow \quad \nabla^2 w + v^i \nabla_i a_t^{(0)} = -\nabla_i E^i$$

$$\mathcal{H}^j \Rightarrow \quad 2\,\nabla^i \nabla_{(i} v_{j)} + a_t^{(0)} \nabla_j w - \nabla_j\, p = -4\pi T\, \zeta_j - a_t^{(0)} E_j$$

- Solve for a Stokes flow on the curved black hole horizon
- Closed system of equations in d dimensions
- Not a derivative expansion
- Related work
  [Damour][Thorne, Price][Eling, Oz][Bredberg, Keeler, Lysov, Strominger]
- What does this have to do with the boundary?
  [Komar][Policastro, Son, Starinets][Iqbal, Liu][Blake, Tong]

DC conductivities from BH horizons

- The perturbative steady state geometry is stationary

$$ds^2 = -H^2\,(dt+\alpha)^2 + ds^2\,(M_{D-1})$$
$$A = A_t\,(dt+\alpha) + \beta$$

with ∂_t Killing

- Diffeo invariance

$$t \to t + \Lambda_E(y^m), \qquad \alpha \to \alpha + d\Lambda_E(y^m)$$

- Gauge invariance

$$A(t,y^m) \to A(t,y^m) + d\Lambda_M(y^m), \qquad \beta \to \beta + d\lambda_M$$

- Assume absence of anomalies

DC conductivities from BH horizons

- Diffeos + Gauge invariance constrain the lower dim bulk action

$$\mathcal{L} = \mathcal{L}(d\alpha, d\beta; \mathcal{D}d\alpha, \mathcal{D}d\beta, \ldots)$$

- The EOMs of α and β give two divergence free antisymmetric tensors

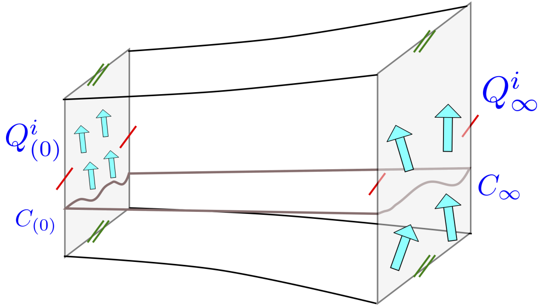
$$\nabla_m V^{mn} = 0, \qquad \nabla_m W^{mn} = 0$$

- Close to the boundary they give the heat and electric currents

$$J^i{}_{\infty} \equiv 2(\gamma^{\infty}_{D-1})^{1/2} V^{ri}_{\infty}\,, \qquad J^i_{Q\,\infty} \equiv -2(\gamma^{\infty}_{D-1})^{1/2} W^{ri}_{\infty}$$

  are the QFT electric and heat current densities
- Can also understand this through Wald's formalism [Liu, Lu, Pope]

DC conductivities from BH horizons



This relates that fluxes of the currents we are after to fluxes of horizon currents

$$\int_{C_\infty} *J_{Q\,\infty} = \int_{C_{(0)}} *J_{Q\,(0)}, \qquad \int_{C_\infty} *J_\infty = \int_{C_{(0)}} *J_{(0)}$$

DC conductivities from BH horizons

- Solutions for v^i, w and p are uniquely fixed by sources E and ζ
- Then

$$
\begin{aligned}
J^i_{(0)} &= \frac{s}{4\pi}\left(\partial^i w + E^i\right) + \rho\, v^i \\
J_Q{}^i_{(0)} &= T s\, v^i
\end{aligned}
$$

$$s = 4\pi\,\sqrt{g_{(0)}}, \quad \rho = \sqrt{g_{(0)}}\, a_t^{(0)}$$

- To find field theory currents $\bar{\mathcal{J}}^i_\infty$ and $\bar{\mathcal{Q}}^i_\infty$ in e.g. $d=2$

$$\bar{\mathcal{J}}^1 = \int dx^2\, J^1_\infty, \quad \bar{\mathcal{J}}^2 = \int dx^1\, J^2_\infty$$

- Can define horizon DC conductivities σ_H, α_H, $\bar{\alpha}_H$ and $\bar{\kappa}_H$
- Argument shows they are equal to the field theory DC conductivities

Applications

- Anomalous scaling for the Hall angle from holography
  [AD, Blake]
- Bounds on conductivity from holography
  [Grozdanov, Lucas, Sachdev, Schalm]
- Diffusion and Chaos (?!?)
  [Blake][Lucas, Steinberg][AD, Blake][Blake, Davison, Sachdev]

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Setup for modulated phases

- No explicit breaking of translations

$$ds^2 = -UG\,dt^2 + \frac{F}{U}\,dr^2 + ds^2(\Sigma_d)$$
$$A = a_t\,dt$$
$$ds^2(\Sigma_d) = g_{ij}(r,x)dx^i dx^j$$

- Demand AdS asymptotics, at $r \to \infty$

$$U \to r^2, \qquad F \to 1, \qquad a_t(r,x) \to \mu$$
$$G \to 1, \qquad g_{ij}(r,x) \to r^2\delta_{ij}$$

- Local μ, T, surface forces

Setup for modulated phases

- Close to the horizon as $r \to 0$

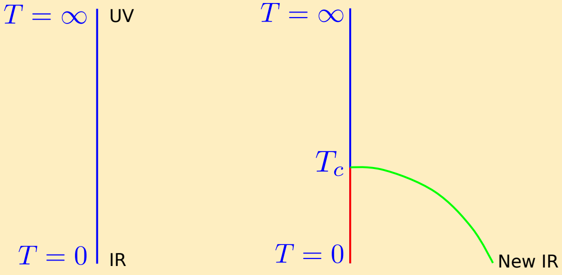
$$ds^2 = -4\pi T\, r\, dt^2 + (4\pi T\, r)^{-1}\, dr^2 + \gamma^{(h)}_{ij}(\mathbf{x})\, dx^i\, dx^j$$
$$A = r\, \rho^{(h)}(\mathbf{x})\, dt$$

- The horizon quantities $\gamma^{(h)}_{ij}$, $\rho^{(h)}$ break translations but the field theory metric and couplings don't

- Periods will be fixed by the theory, no large period limit

Finite chemical potential

Phase transitions

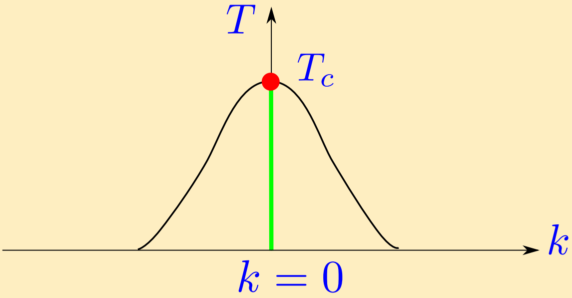


Theory can develop symmetry breaking instabilities:

- At $T >> \mu$ the normal phase bh's are stable
- At $T < T_c$ there exist tachyonic modes
- Zero mode at $T = T_c$ gives rise to broken phase black hole branch
- Dual operator ϕ takes a VEV $< \phi >$
- First examples were superfluids [Gubser][Hartnoll, Herzog, Horowitz]

Finite chemical potential

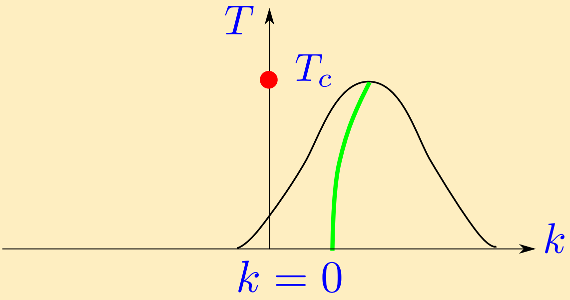
- In most cases the order parameter does not break translations
- Translation breaking static modes exist at at $T < T_c$



- The one at $k = 0$ wins in the phase diagram

Finite chemical potential

- At finite chemical potential/magnetic field static modes can start appearing at finite wavelengths



- Thermodynamically preferred black branes will break horizon $+$ field theory translations

Finite chemical potential

Helical order in $D = 5$

Bulk Einstein-Maxwell + CS term

[Nakamura, Ooguri, Park] [AD, Gauntlett]

$$\mathcal{L} = R + \Lambda - \frac{1}{4}F^2 + \lambda\, \varepsilon^{\alpha\beta\gamma\delta\epsilon} A_\alpha F_{\beta\gamma} F_{\delta\epsilon}$$

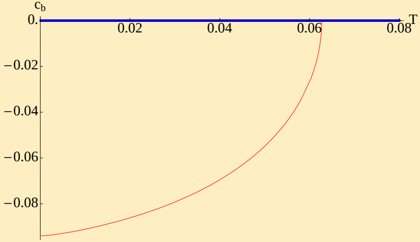
- Broken phase develops helical current density/magnetisation

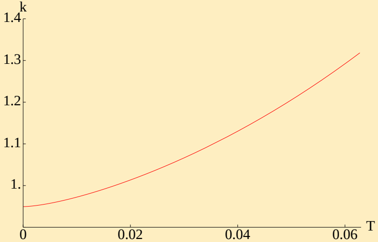
$$< J_y >= c_b \sin(kx), \quad < J_z >= c_b \cos(kx)$$

- Spontaneous breaking of translations
- Ground states with broken translations

Modulated phases

Helical order





- Helical black holes come with the period k as a parameter
- Fix k by minimising the free energy density
- Preferred k changes with temperature

Modulated phases

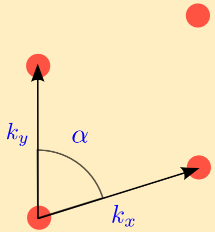
Inhomogeneous phases in $D = 4$

$$\mathcal{L} = R - \frac{1}{2}\partial\phi^2 - V(\phi) - \frac{1}{4}Z(\phi)\,F^2 + \frac{1}{4}\vartheta(\phi)\,\epsilon_{abcd}F^{ab}F^{cd}$$

- These terms lead to charge/magnetisation density wave phases at finite chemical potential/magnetic fields
- Standard terms of $N = 2$ SUGRA in $D = 4$ $\rightarrow$ appear in top-down models

Modulated phases

Space of solutions depends on a reciprocal lattice



$$(x', y') \equiv \left(x' + \frac{L_x}{\sin\alpha}\, n_1 - \cot\alpha\, L_y\, n_2, y' + L_y\, n_2 \right)$$

$$ds_3^2 = -dt^2 + dx'^2 + dy'^2$$
$$A = \mu\, dt + B\, x\, dy$$

Need to minimise the free energy density $w(T, \mu, k_x, k_y, \alpha)$

Modulated phases

For fixed T and μ vary the free energy density

$$w = -\bar{s}T - \mu\,\bar{J}^t + \bar{T}^{tt}$$

with respect to $L_x = 2\pi/k_x$, $L_y = 2\pi/k_y$ and α

$$-L_x \frac{\delta w}{\delta L_x} = w + \bar{T}^x{}_x, \quad -L_y \frac{\delta w}{\delta L_y} = w + \bar{T}^y{}_y$$

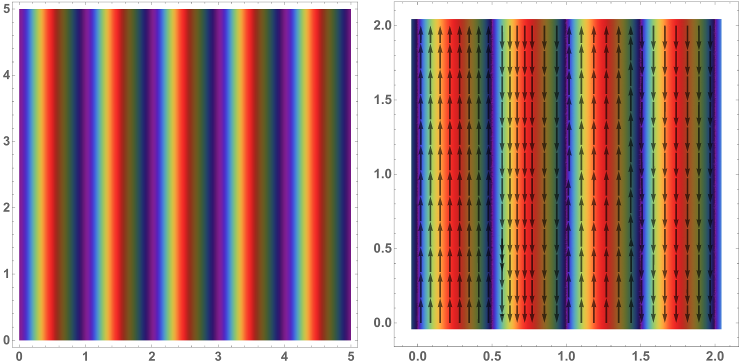
$$\frac{\delta w}{\delta \alpha} = \cot\alpha (w + \bar{T}^x{}_x + \bar{T}^y{}_y) - \frac{L_x L_y}{\sin\alpha} \bar{T}^{xy}$$

Like a perfect fluid on average at the extrema of the free energy density [AD, Gauntlett]

$$w = -p$$

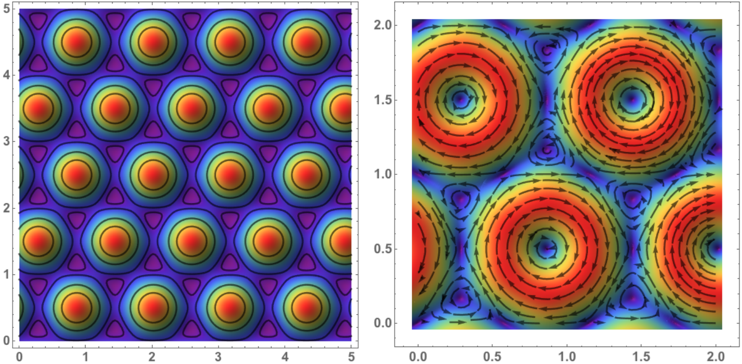
$$\bar{T}^{ij} = p\,\gamma^{ij}$$

Modulated phases



- For $B = 0$ striped structures seem preferred
- Charge density gets modulated
- Spontaneous electric/heat magnetisation current densities appear

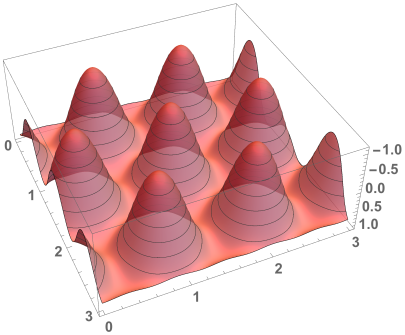
Modulated phases

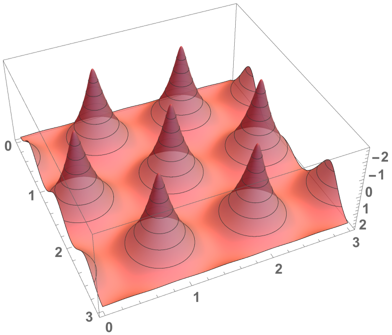


Density waves

- Switching on magnetic field makes other structures preferred [AD, Gauntlett]
- Triangular lattice formations seem to win

Finite chemical potential





Density waves

- At lower temperatures translations breaking effects become stronger
- IR Theory develops point-like defect structure
- New ground states to be found

Conductivity

- The system lives in Minkowski space and is held at constant μ and T
- Operator VEVs in the thermal state break translations
- The momentum charges $P_{(j)} = \bar{T}^t{}_j$ are conserved and overlap with $\bar{J}^j$ and $\bar{J}_Q^j$
- Assuming the preferred thermal phase, relativistic symmetry yields the constraints

$$\mu\,\sigma^{ij}(\omega) + T\alpha^{ij}(\omega) = \frac{i\rho}{\omega}\,\delta^{ij}$$

$$\mu T\bar{\alpha}^{ij}(\omega) + T\bar{\kappa}^{ij}(\omega) = \frac{iTs}{\omega}\,\delta^{ij}$$

[Hartnoll, Herzog] [AD, Gauntlett, Griffin, Ziogas]

- The operator $J^i_{\mathrm{inc}} = Ts\,J^i - \rho\,J^i_Q$ has no overlap with $P_{(j)}$ [Davison, Gouteraux, Hartnoll]

DC Conductivity

- Physical expectation for a normal fluid is $\sigma_{inc}(\omega) = G(\omega)_{J_{inc}J_{inc}}/(i\omega)$ is finite as $\omega \to 0$ with $\sigma_0 = \sigma_{inc}(0)$

- Then at low frequencies

$$\sigma^{ij}(\omega) \to \left(\pi\delta(\omega) + \frac{i}{\omega}\right)\frac{\rho^2}{\varepsilon + p}\delta^{ij} + \sigma_0^{ij}\ ,$$

$$T\bar{\alpha}^{ij} = T\alpha^{ij}(\omega) \to \left(\pi\delta(\omega) + \frac{i}{\omega}\right)\frac{\rho T s}{\varepsilon + p}\delta^{ij} - \mu\sigma_0^{ij}\ ,$$

$$T\bar{\kappa}^{ij}(\omega) \to \left(\pi\delta(\omega) + \frac{i}{\omega}\right)\frac{(Ts)^2}{\varepsilon + p}\delta^{ij} + \mu^2\sigma_0^{ij},$$

- As a $\omega \to 0$ limit

$$\sigma_0^{ij} = \lim_{\omega\to 0}\left[(Ts)^2\sigma^{ij}(\omega) - 2(Ts)\rho T\alpha^{ij}(\omega) + \rho^2 T\bar{\kappa}^{ij}(\omega)\right]$$

DC Conductivity

- Think about DC transport from the bulk
- Adding a boosted background yields a correct perturbation
- DC calculation in the bulk is ill defined
- Form the boost invariant combination of the currents to find [AD, Gauntlett, Griffin, Ziogas]

$$\sigma_0^{ij} = (Ts)^2 \sigma_H^{ij} - 2\,T^2 s\rho\,\alpha_H^{ij} + \rho^2 T \bar{\kappa}_H^{ij}$$

- Generalisable to non-thermodynamically preferred states
- Reproduces earlier results in specific homogeneous models [Amoretti, Arean, Gouteraux, Musso]

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Summary

- Argued that DC transport is fixed by horizon "hydrodynamics"
- Ground states with broken translations? Properties?
- Universal statements?
- Finite frequency?