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Holographic fermions in striped superconductors

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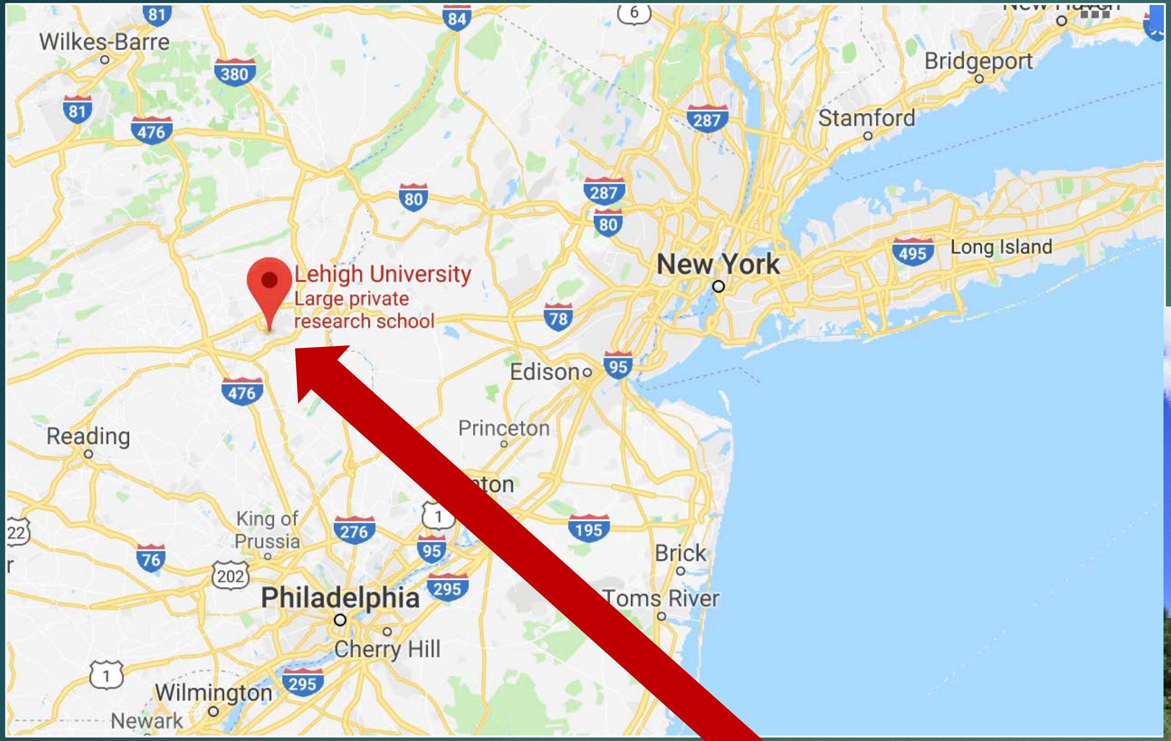
Lehigh University



Fall 2018:
Faculty job opening at Lehigh
in High Energy Theory (see AJO)

Today:

- **Part I**: Holographic Realization of Intertwined Orders (Pair Density Wave)
  S.C., Li Li, Jie Ren Phys.Rev.D95 (2017) no.4, 041901 and JHEP 1708 (2017) 081

- **Part II**: Fermionic spectral functions in striped superconducting phases
  SC, L. Li, J. Ren arXiv:1807.11730 and work in progress

Holography as a Theoretical Laboratory

Study solvable models that may be in the same universality class as strongly correlated QM phases
→ Can we understand **the basic mechanisms** underlying the dynamics and unconventional properties of these systems?

Draw qualitative and quantitative lessons → look for **universal features**

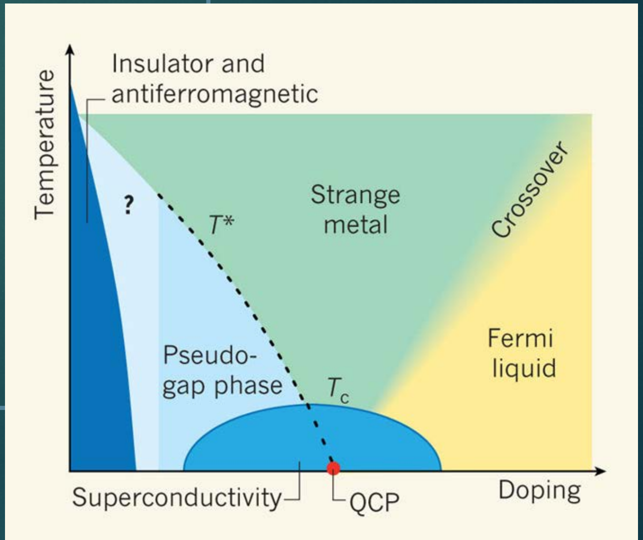
Solvable often implies working with overly simplified bottom-up **toy models**

Also **broader questions in gravity** → to what extent is it **emergent**?

Holography as a Theoretical Laboratory

Challenges for understanding strongly coupled QM phases of matter

- Breakdown of Fermi-liquid theory, no quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
- Rich structure of emergent IR phases
- Different scales in the system
- Long-range entanglement
- …



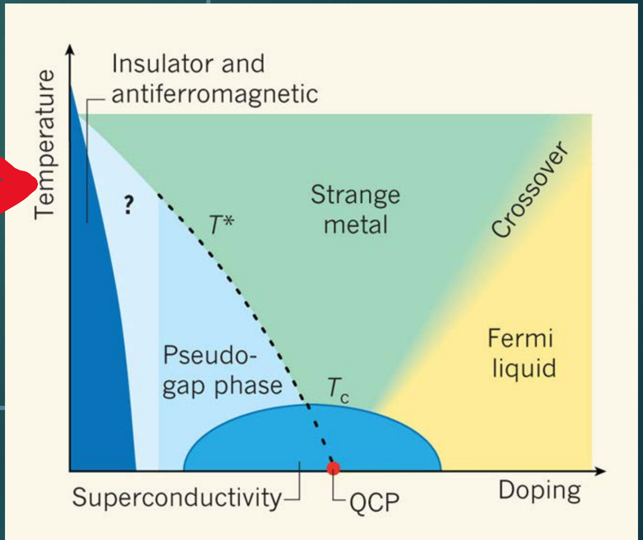
Holography as a Theoretical Laboratory

Challenges for understanding strongly coupled QM phases of matter

- Breakdown of Fermi-liquid theory, no quasiparticles
- An intrinsically complex phase diagram exhibiting a variety of orders
- Rich structure of emergent IR phases
- Different scales in the system

Can we identify generic imprints of these symmetry breaking mechanisms and build intuition for their phenomenology?

Can we understand the structure of Fermi surfaces in strongly correlated electron matter?



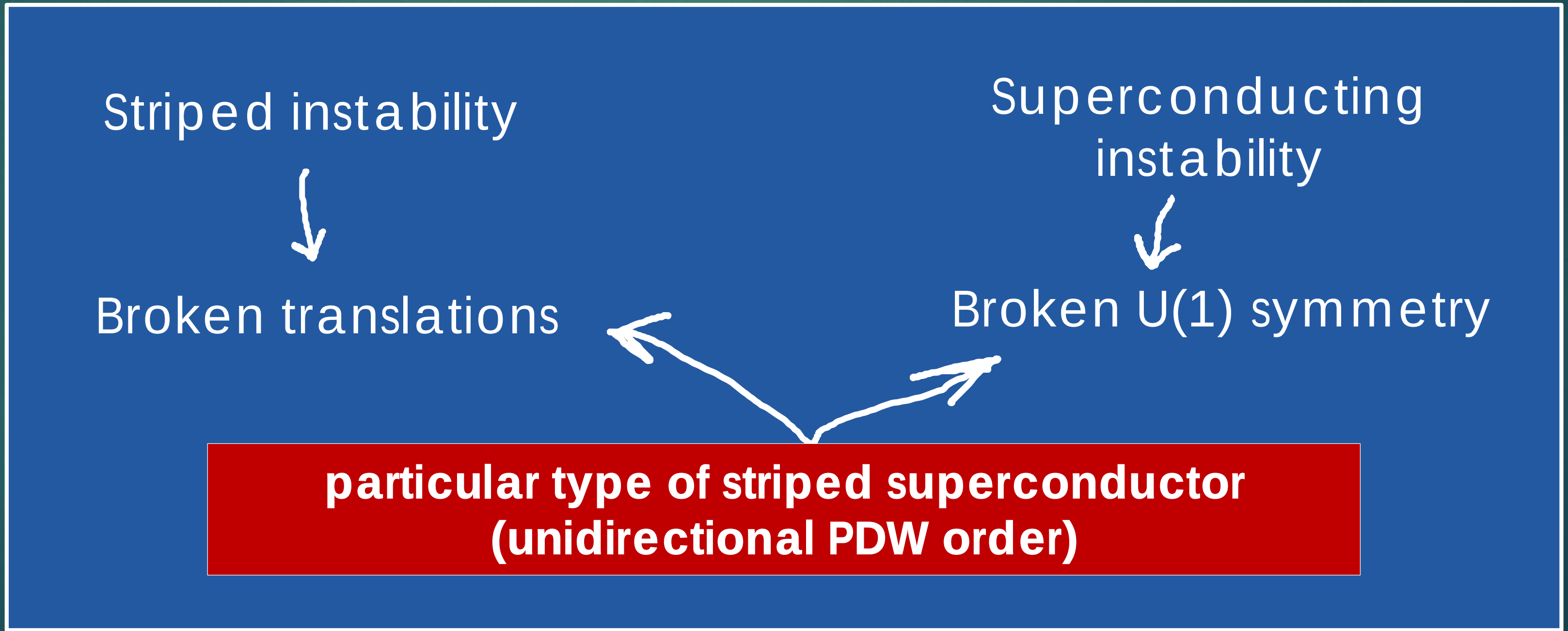
Holographic Realization of Intertwined Orders

Phases don't always compete or simply co-exist. Often they are closely intertwined

Goal:

- **Break translational and U(1) symmetry spontaneously at same time, by same mechanism**

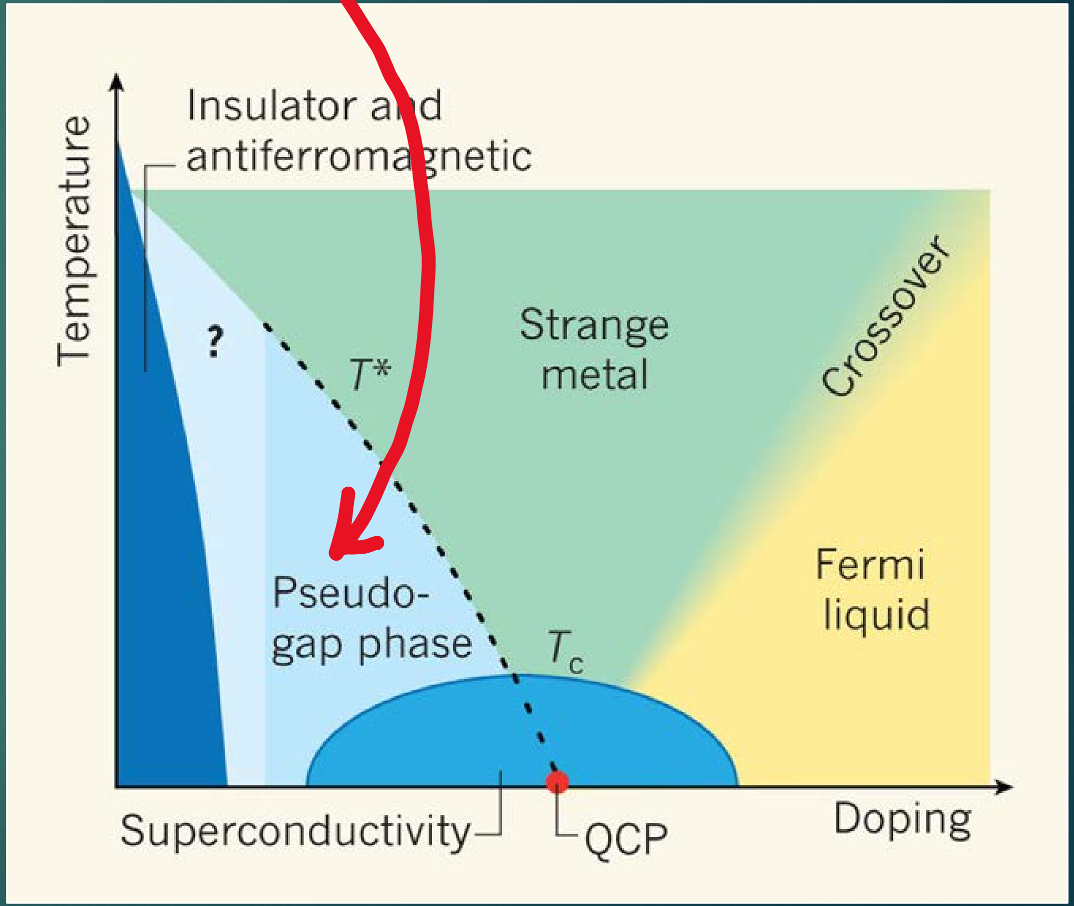
In our model:



Motivation

1. Realize some of the features of **Pair Density Wave (PDW) order** of high temperature superconductors (cuprates)

PDW phase:
- intertwines CDW, SDW and SC orders in a very specific way
- Evidence in pseudo-gap of of cuprate high T_c superconductor $La_{2-x} Ba_x Cu O$ (LBCO)



2. Explore properties of Fermi surface in the pseudo gap and striped strongly correlated phases more generally
- detached segments of the Fermi surface (Fermi arcs)

Holographic Toy Model of PDW Order

Features of PDW order we focus on:

- Scalar condensate (superconducting order) is spatially modulated + its oscillations average out to zero (no homogeneous component)

$$\langle O_\chi \rangle \propto \cos(k\,x)$$

- Charge density is modulated and oscillates at twice the frequency of the condensate

$$\rho(x) = \rho_0 + \rho_1 \cos(2k\,x)$$

Contrast to co-existing CDW + SC orders:

- Scalar condensate has a uniform component, and oscillates at the same frequency as the CDW

E. Berg, E. Fradkin, S.A. Kivelson and J.M. Tranquada, *Striped superconductors: how spin, charge and superconducting orders intertwine in the cuprates*, *New J. Phys.* **11** (2009) 115004 [arXiv:0901.4826].

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CAN REALIZE BOTH BUT FOCUS ON PDW TODAY for concreteness

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The Holographic Model

S.C., L. Li, J. Ren (1612.04385, 1705.05390)

4D Bottom-up Model (2+1 dual QFT)

$$S = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2\Lambda + \mathcal{L}_m]$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Keep in mind:

- Toy Model ("minimal") to get specific phase we are after
- Can be improved at the cost of adding a more complicated matter sector (more realistic order parameter)

- **See Cai, Li, Wang, Zaanen [e-Print: arXiv:1706.01470] for different realization of PDW order**

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$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Two real scalars

Two U(1) vector fields

$$F = dA \qquad \tilde{F} = dB$$

Field content:

- Gravity
- Two real scalars χ and θ
- Two U(1) vector fields A_μ and B_μ with different physical interpretations:
  - A_μ → **charge density** of field theory
  - B_μ → **spectator field** or proxy for "spin" density or **second species of charge carriers**

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$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Generalized "Stuckelberg Superconductor"

Generalizes standard holographic SC
→ allows for more general couplings

Stuckelberg mechanism:
local gauge invariance encoded in

$$\theta \to \theta + \alpha(x^\mu)\,, \qquad A_\mu \to A_\mu + \frac{1}{q_A}\partial_\mu\alpha(x^\mu)$$

The Holographic Model

S.C., L. Li, J. Ren (1612.04385, 1705.05390)

Crucial coupling **for seeding spatially modulated instabilities**
c=0 → leading unstable mode is not striped

4D Bottom-up Model (2+1 dual QFT)

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$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

$$Z_A(\chi) = 1 + \frac{a}{2}\chi^2 \qquad Z_B(\chi) = 1 + \frac{b}{2}\chi^2 \qquad \mathcal{K}(\chi) = \frac{\kappa}{2}\chi^2$$

$$Z_{AB}(\chi) = c\,\chi \qquad V(\chi) = \frac{1}{2}m^2\chi^2$$

Theory chosen so that symmetric phase $\chi = \theta = B_\mu = 0$ is described by standard charged black hole in AdS

The Holographic Model

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4D Bottom-up Model (2+1 dual QFT)

$$S = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2\Lambda + \mathcal{L}_m]$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{Z_A(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{Z_B(\chi)}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{Z_{AB}(\chi)}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - V(\chi)\,,$$

Note:

- At some critical temperature the system becomes **unstable to the condensation of χ and B_μ**
- **Scalar hair will be spatially modulated** (breaking of translational invariance)
- Properties of condensate sensitive to whether $q_A\, q_B = 0$ or not (we will keep q_A non-zero since the charge density will be associated with A).
- Today **focus on $q_B = 0$ (PDW)** as opposed to CDW + SC (both charges non-zero)

Onset of Instabilities – Building Intuition

Analytical T=0 analysis gives some insight **(violation of BF bound)**

Working to leading order in perturbations:

$$\delta\chi = \varepsilon\, w(r) \cos(k\, x)\,, \qquad \delta B_t = \varepsilon\, b_t(r) \cos(k\, x)$$

$$w'' + \#\, w' + c\, b_t' + m_{eff}^2(k)\, w = 0$$

$$b_t'' + \#\, b_t' + c\, w' + \tilde{m}_{eff}^2(k)\, b_t = 0$$

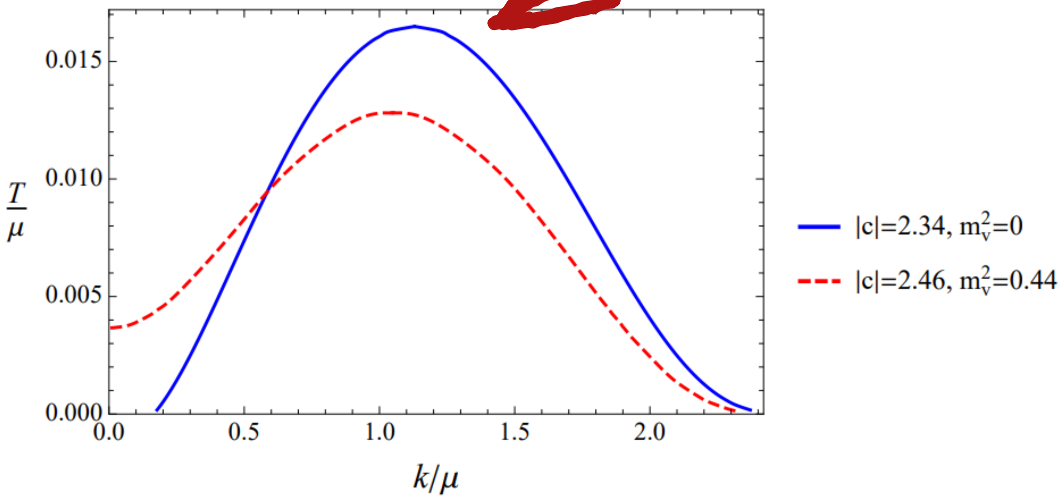
Coupled through $Z_{AB} F \tilde{F}$

There are nonzero k values at which the **scaling dimension becomes imaginary** → instability

Critical Temperature for Instability

For a given k there will be a normalizable zero mode appearing at a particular T → T_c

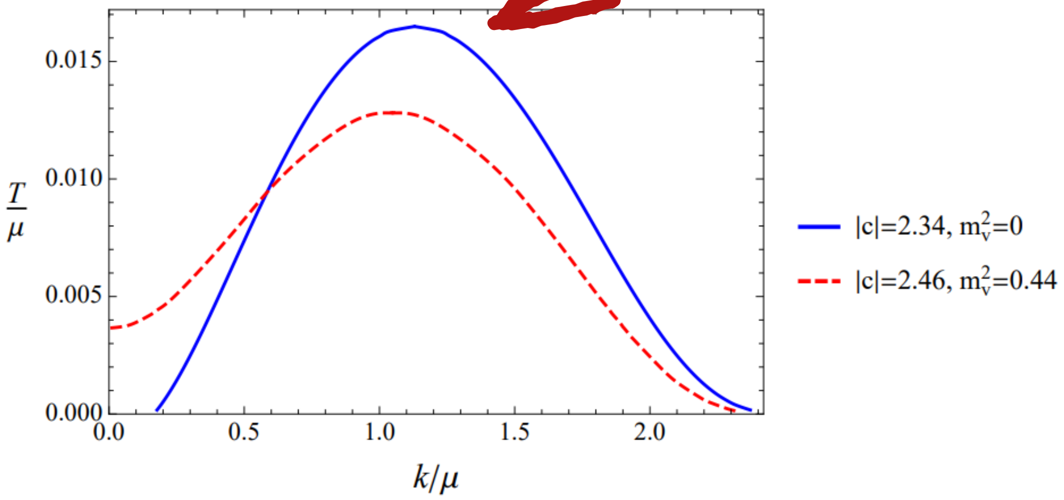
Condensate is driven by a spatially modulated mode



Minimize free energy →
thermodynamicall preferred solution

Critical Temperature for Instability

Simple observation: Condensation at nonzero k will always occur at higher T_c than in homogeneous phase (because of generic bell shape of instability curve)



spatial modulations "enhance" the superconducting critical temperature
→ Facilitate the transition

A Cartoon Picture

Gravity Side

Standard AdS-RN black hole supported by U(1) gauge field A_μ

$\chi = B_\mu = 0$

T

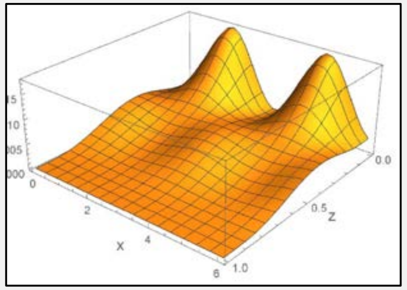
T_c

Dual Field Theory

Global U(1) symmetry

Spatially modulated instabilities
System unstable to condensation of χ and B_μ

New type of striped IR geometry



χ, B_μ nontrivial

U(1) and translational symmetry broken spontaneously at the same time

$\langle O_\chi \rangle \sim \cos(k_c\, x)$

$\rho_B = \langle J^t_B \rangle \sim \cos(k_c\, x)$

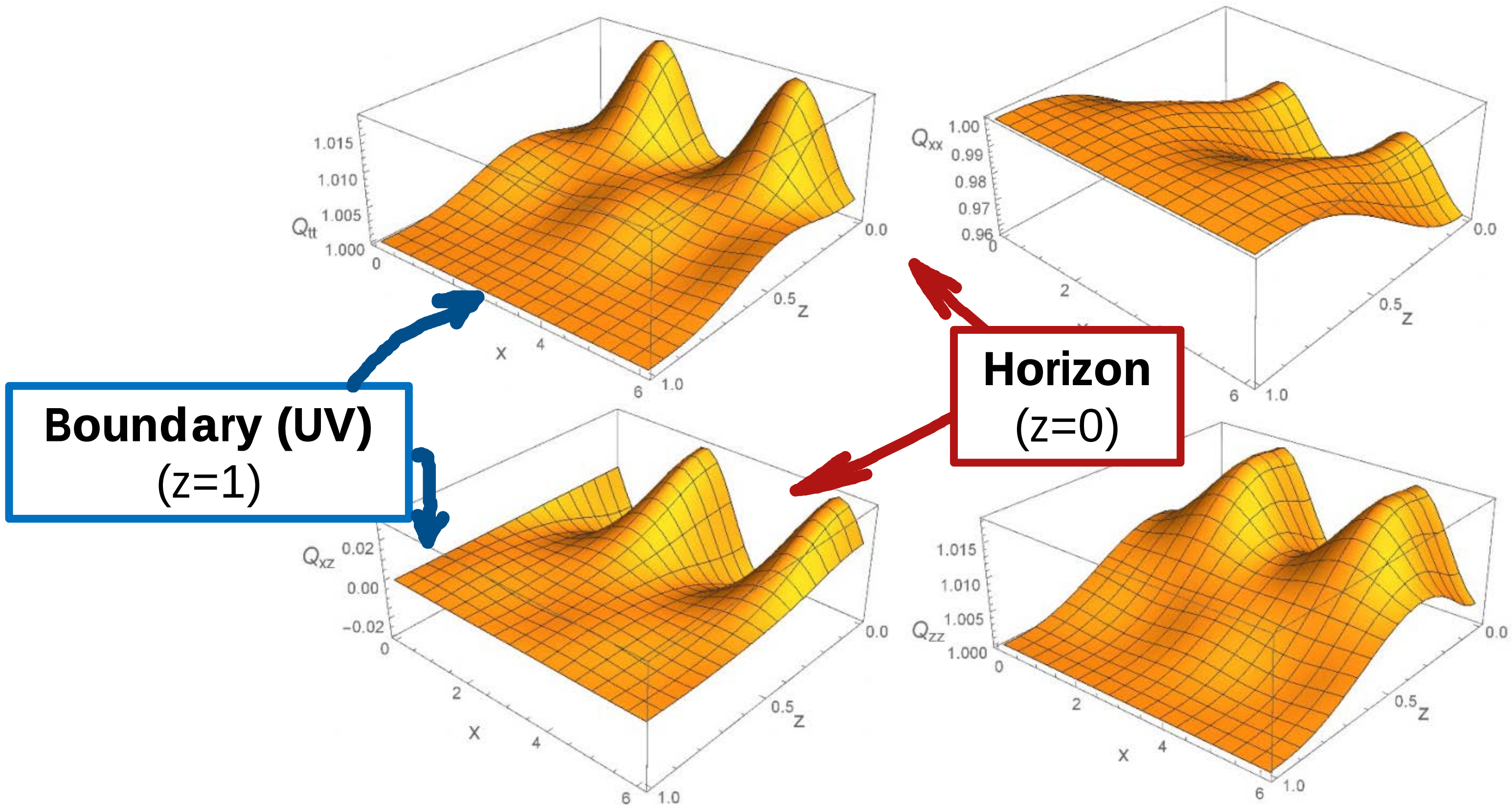
$\rho_A = \langle J^t_A \rangle \sim \cos(2k_c\, x)$

Modulated condensate and charge density

The Striped Geometry (PDW)

Black Hole Geometry

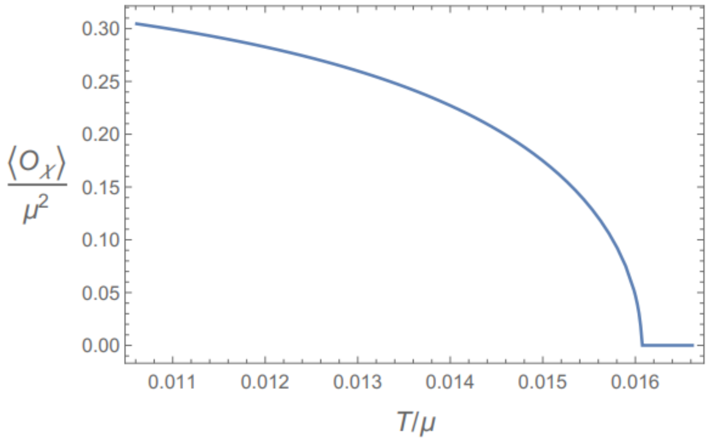
$$ds^2 = \frac{r_h^2}{L^2(1-z^2)^2}\left[-F(z)Q_{tt}\,dt^2 + \frac{4z^2L^4Q_{zz}}{r_h^2F(z)}\,dz^2 + Q_{xx}(dx - 2z(1-z^2)^2Q_{xz}dz)^2 + Q_{yy}\,dy^2\right]$$



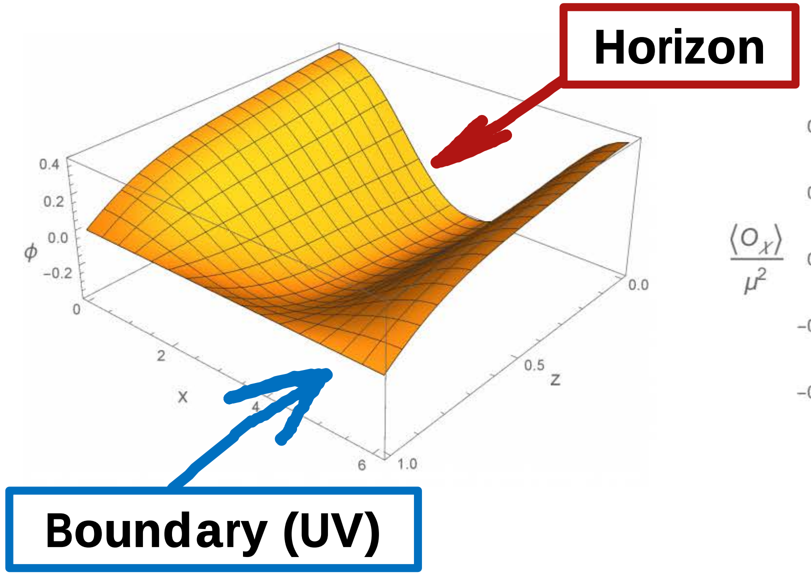
NOTE: spatial modulations are imprinted on the horizon (IR)

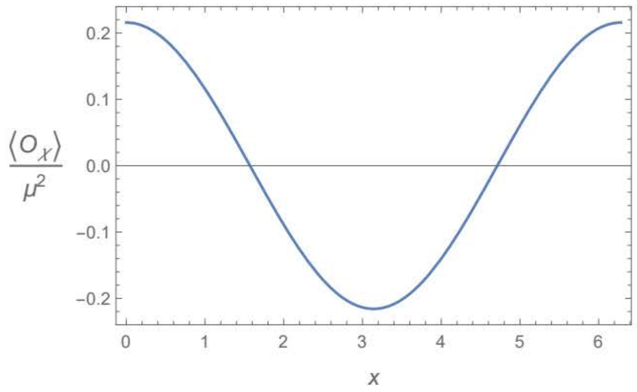
→ Stripes are relevant deformation of the UV CFT

The Scalar Field Condensate (PDW)



VEV of scalar condensate as a function of T at x=0



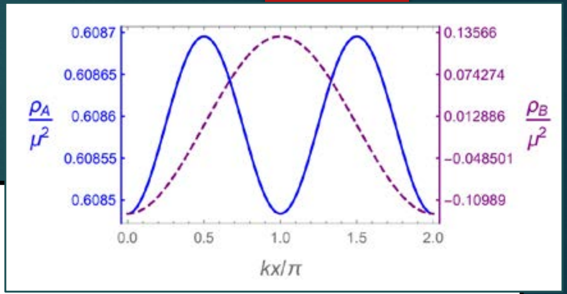


$$\langle O_\chi \rangle \sim \cos(k_c\, x)$$

Scalar condensate oscillations average out to zero

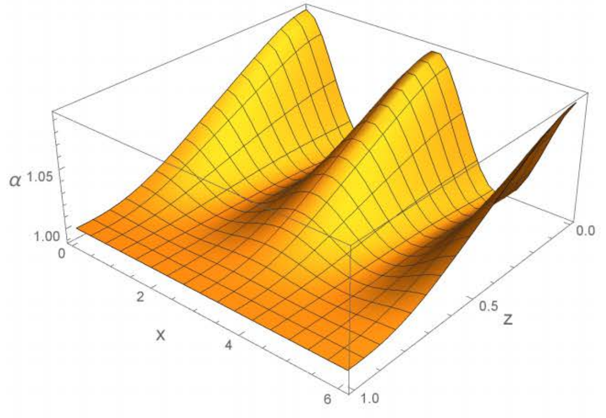
(PDW feature)

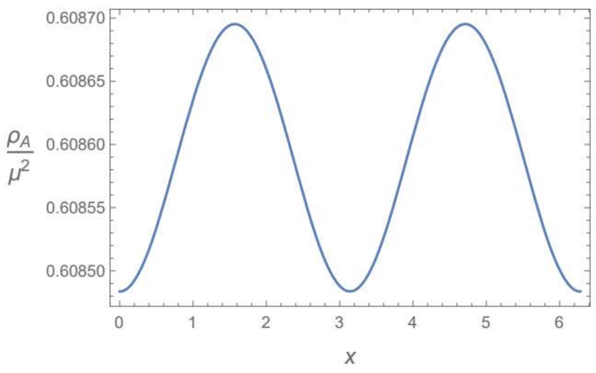
Vector Field Profiles (PDW)



Vector field A_t profile

$$\rho_A = \langle J_A^t \rangle \sim \cos(2k_c\, x)$$

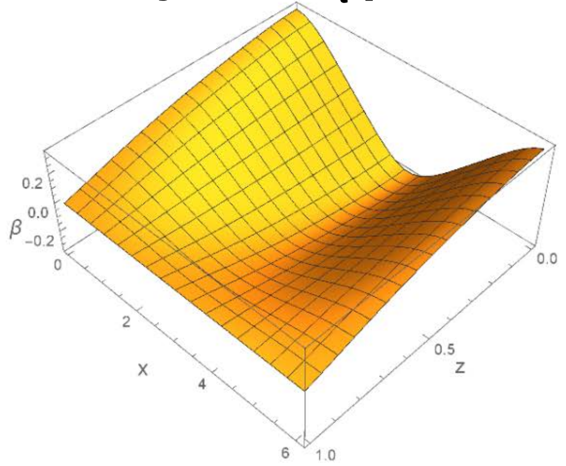


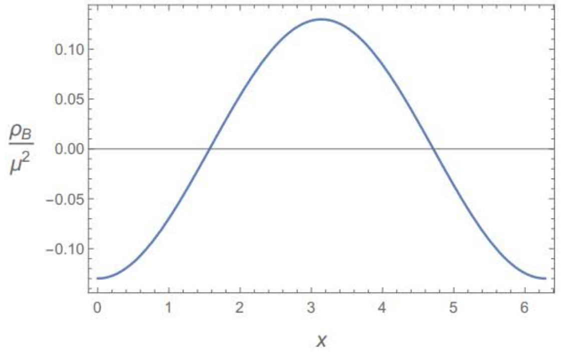


charge density oscillates twice as fast as scalar condensate (PDW feature) and CDW order induced

Auxiliary field B_t profile

$$\rho_B = \langle J_B^t \rangle \sim \cos(k_c\, x)$$





Second "charge" density (**proxy for SDW**?) oscillates at same frequency as scalar condensate

Next:

Examine fermionic spectral functions in this spontaneously generated striped superconducting phase, including the effects of explicit breaking of translations

Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1807.11730

- A lot of work on fermionic response in holography

  Single fermion spectral function computations

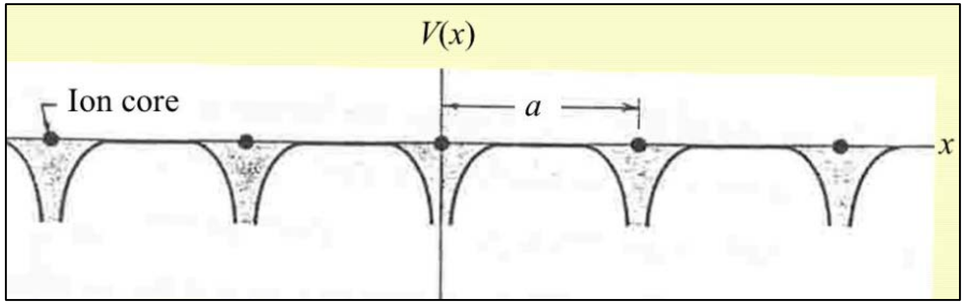
  - **Cubrovic, Zaanen, Schalm** Science 325 (2009) 439
  - **Faulkner, Liu, McGreevy, Vegh** PRD 83 (2011) 125002, Science 329 (2010) 1043

  See e.g. Iqbal, Liu and Mezei, arXiv:1110.3814 for a review

- Most studies focused on **cases with translational invariance or homogeneous lattices**

To make contact with real materials important to include effects of **periodic lattices**

(also, many rich **striped phases** in strongly correlated electron systems)



Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1807.11730

- Very few holographic studies on **fermions in inhomogeneous systems**
- Our work is motivated by and builds on:
  - **Y. Liu, K. Schalm, Y.W. Sun and J. Zaanen [arXiv:1205.5227]**

    Perturbatively small periodic modulation of chemical potential, neglecting backreaction
  - **Y. Ling, C. Niu, J.P. Wu, Z.Y.Xian and H.B. Zhang [arXiv:1304.2128]**

    Included backreaction
- Among features identified in these works: **anisotropic FS and appearance of a gap**
- BUT: in these studies of inhomogeneous systems the **lattice is irrelevant in the IR** (explicit breaking of translations only)

Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1807.11730

- Our PDW phase offers a framework with **a periodic structure that is IR relevant** (crystalline structure **generated spontaneously**)
  - We will add a **source in the UV to break translations explicitly (ionic lattice)** along the same direction as the spontaneous breaking

$$\mu(x) = \mu\left[1 + a_0 \cos(px)\right]$$

  - Recall our PDW phase had modulated charge density

$$\rho_A = \langle J_A^t \rangle \sim \cos(2k_c\, x),$$

- We will work with $p = 2k_c$ → **commensurate case** (**one scale in the problem**)

(Leave the incommensurate case for future work)

Holographic Fermions in Striped Superconductors

SC, L. Li, J. Ren, arXiv:1807.11730

Setup:

- Place a probe fermion in the resulting geometry, study Dirac equation numerically

$$S_{probe} = \int d^4x \sqrt{-g}\, i\bar{\Psi}(\not{D} - m)\Psi$$

$$D_\mu = \partial_\mu + \frac{1}{4}\omega_{ab\mu}\Gamma^{ab} - iqA_\mu$$

- Solutions will reflect the periodicity of the background (Bloch expansion)

- **Our main interest:**
  - **Formation of a Fermi surface at strong coupling**
  - **Gap → what determines its size?**
  - **Destruction of Fermi surface?**

Role of spontaneous vs. explicit translational symmetry breaking

Breaking Translations – Spontaneous vs. Explicit

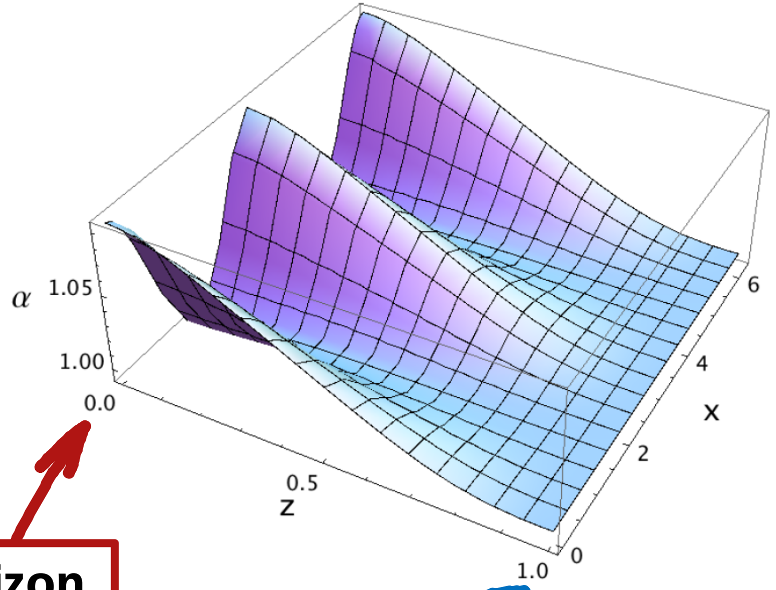
Gauge field profile

PDW order
(spontaneous breaking)

$$\rho_A = \langle J^t_A \rangle \sim \cos(2k_c\, x).$$

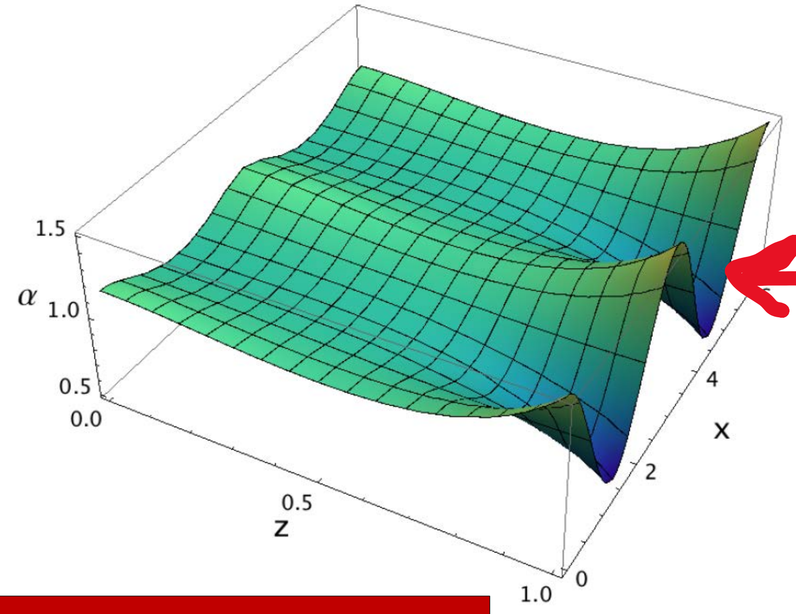
PDW + Ionic Lattice
(explicit breaking in UV, spontaneous in IR)

$$\mu(x) = A_t(1, x) = \mu[1 + a_0 \cos(p\, x)]$$



Horizon
(z=0)

Boundary (UV)

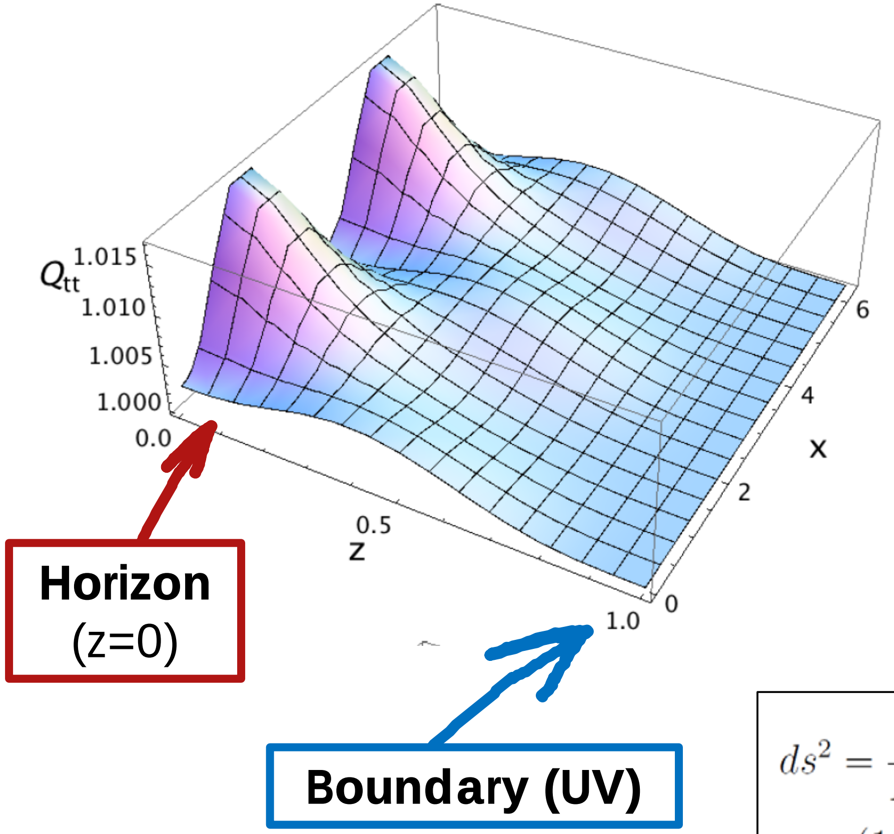


Note: p=2k$_c$ → lattice is commensurate with CDW

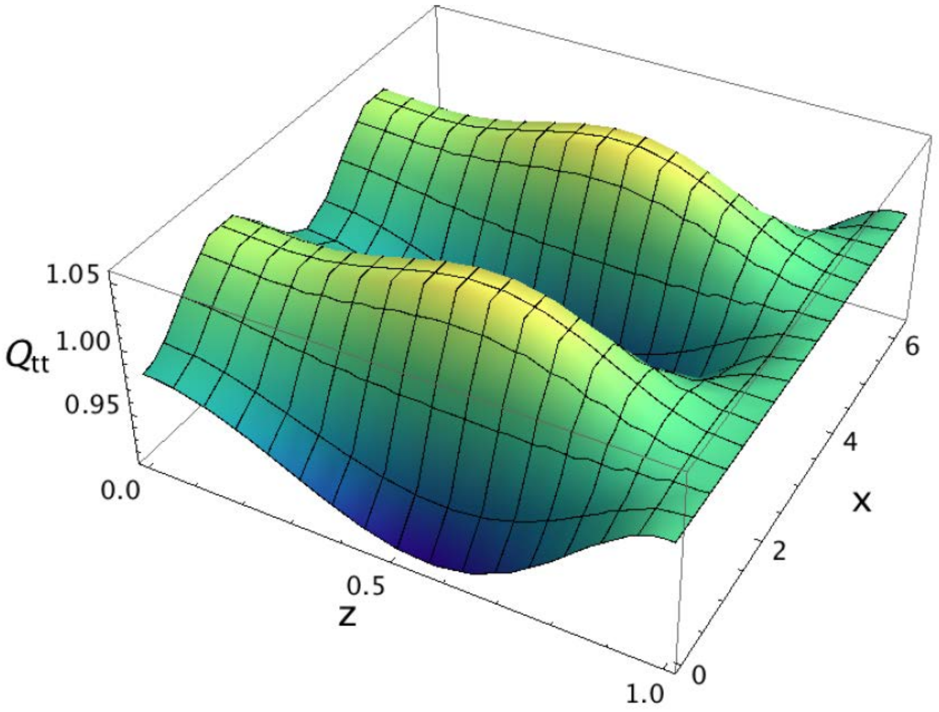
Breaking Translations – Spontaneous vs. Explicit

Geometry - typical metric profile

Pure PDW (spontaneous)



PDW + Ionic lattice



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$$\chi = (1-z^2)\phi\,, \qquad A_t = \mu\, z^2\alpha\,, \qquad B_t = z^2\beta\,, \qquad ($$

Probe fermion and criteria for Fermi surface

- Periodicity of spatially modulated background sets size of Umklapp vector K
- Solutions will reflect periodicity of background (**Bloch expansion**, periodic in x with period 2 π /K)

$$\Psi_\alpha = \int \frac{d\omega dk_x dk_y}{2\pi} \sum_{n=0,\pm1,\pm2,\cdots} \mathcal{F}_\alpha^{(n)}(z,\omega,k_x,k_y) e^{-i\omega t + i(k_x + nK)x + ik_y y}$$

$$k_x \in [-\frac{K}{2}, \frac{K}{2}]$$

momentum level

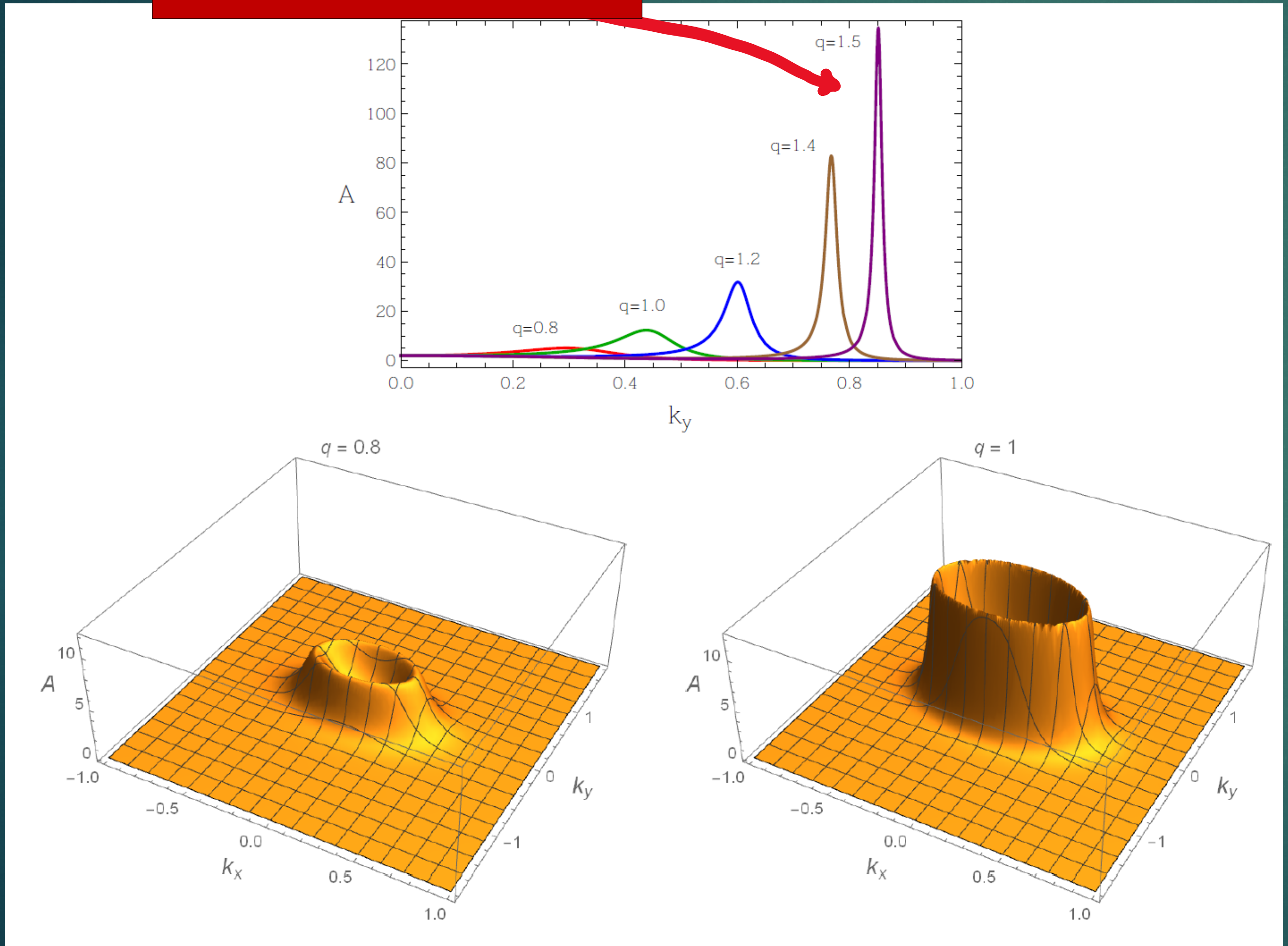
n: Brillouin zone
K: Umklapp vector

- Fermi surface: pole in spectral density at zero temperature as ω→ 0
- **Finite T criteria** to identify Fermi surface (width, frequency and magnitude criteria) introduced in Cosnier-Horeau & Gubser, arXiv:1411.5384
- Spectral function (diagonal momentum basis – **expect dominant response to be in diagonal momentum channel**)

$$A(\omega, k_x, k_y) = \sum_{n=0,\pm1,\pm2,\cdots} \mathrm{Tr}\ \mathrm{Im}[G^R_{\alpha,n;\alpha',n}(\omega,k_x,k_y)]$$

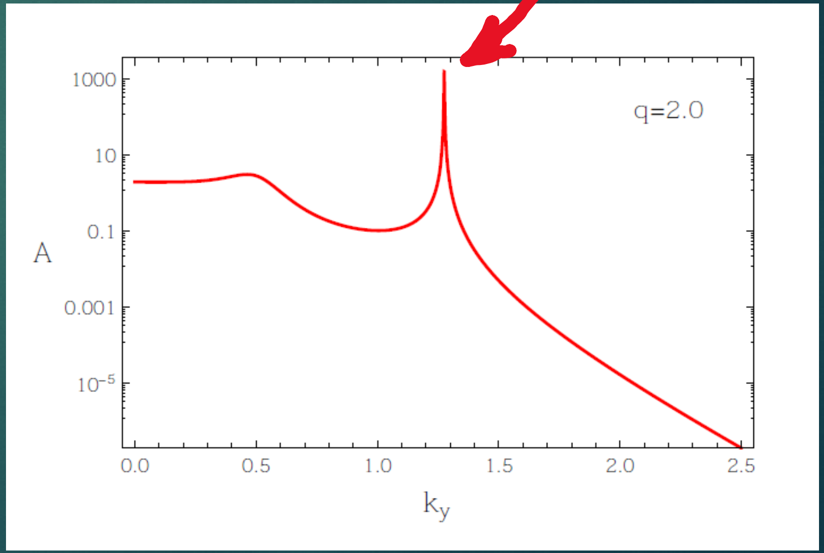
Fermi surface present when fermionic charge is large enough ✓

Not a Fermi Surface



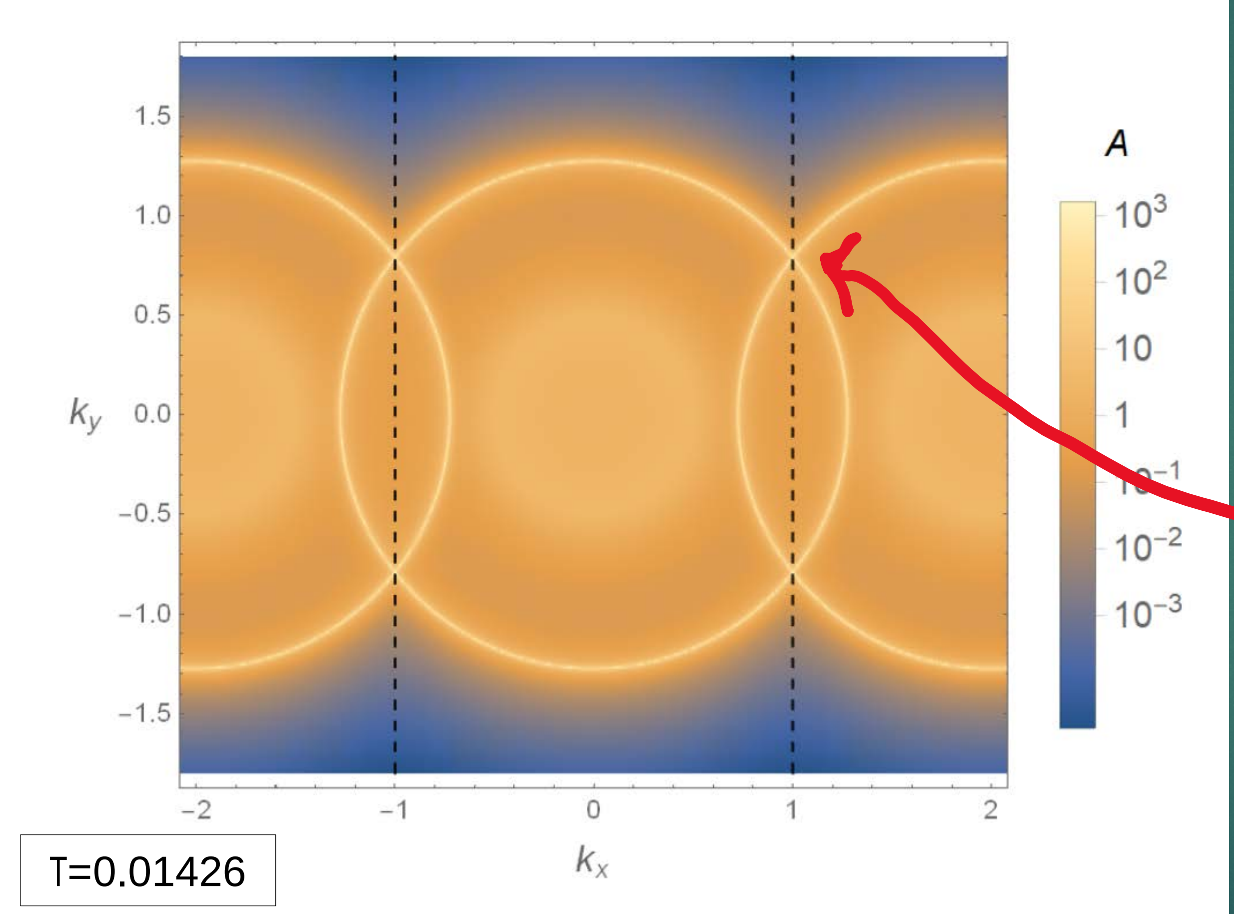
Peaks in spectral function become sharper and higher as q increases

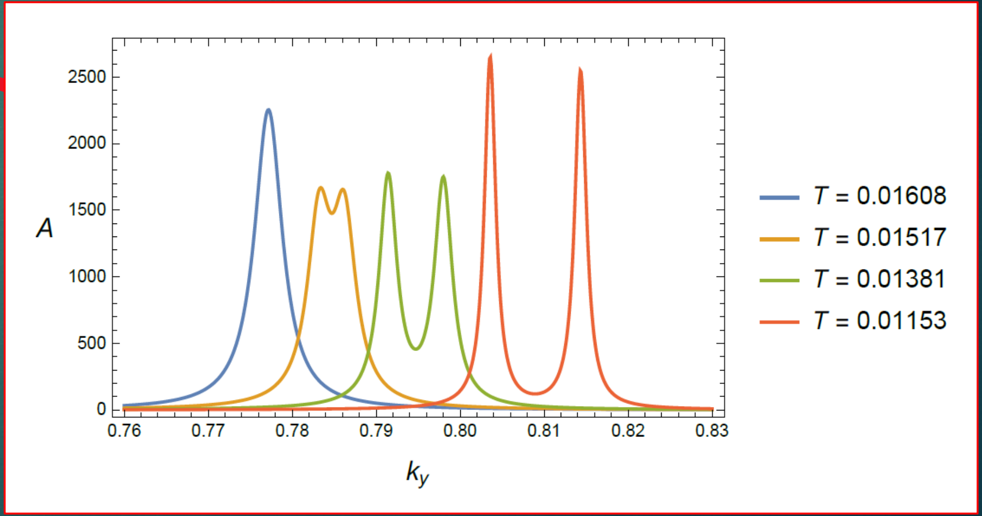
Fermi Surface



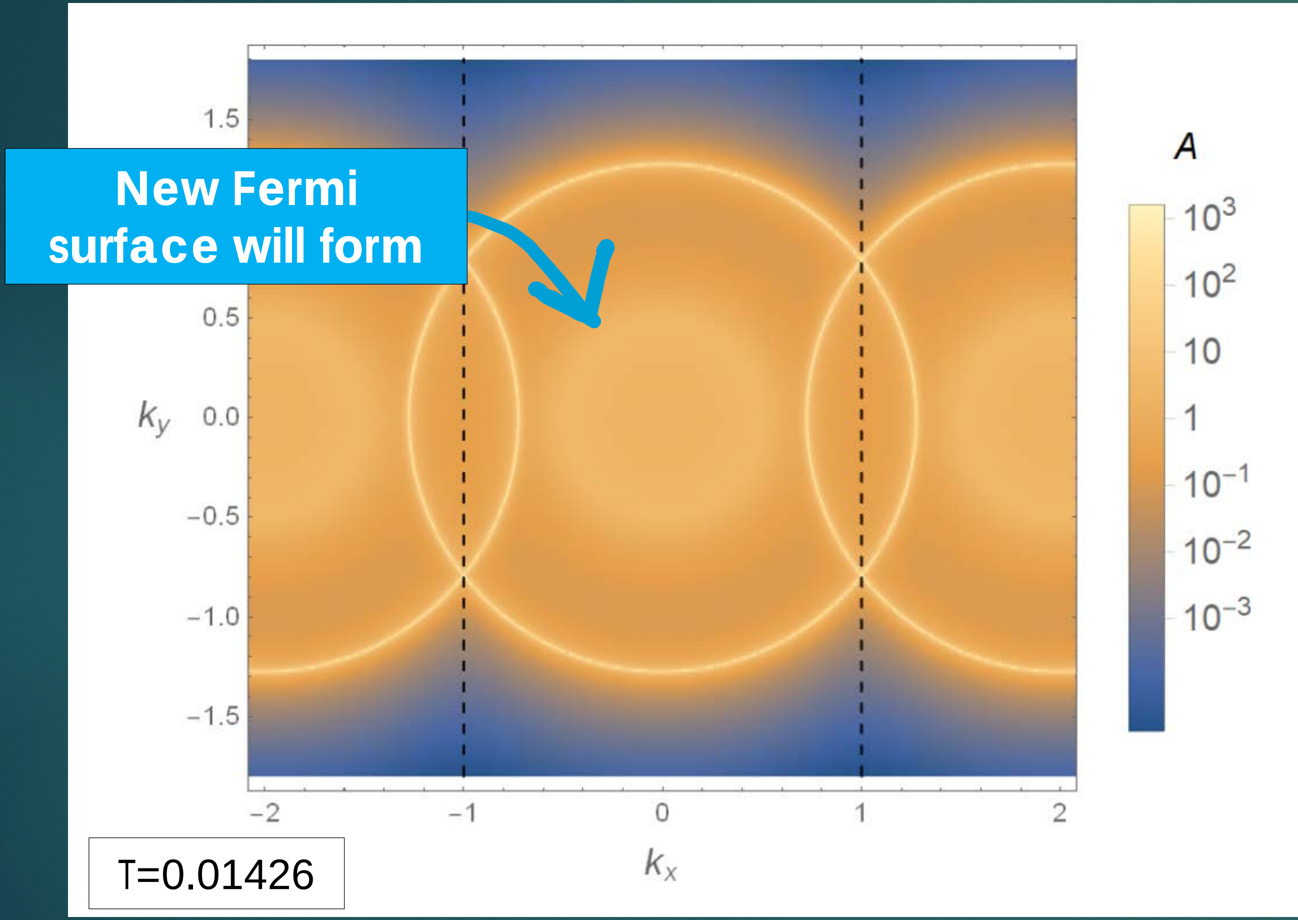
Spontaneous Case (pure PDW):

- Shape becomes more anisotropic as strength of PDW increases (lower T)
- Gap opens up at T_c and increases as temperature is lowered

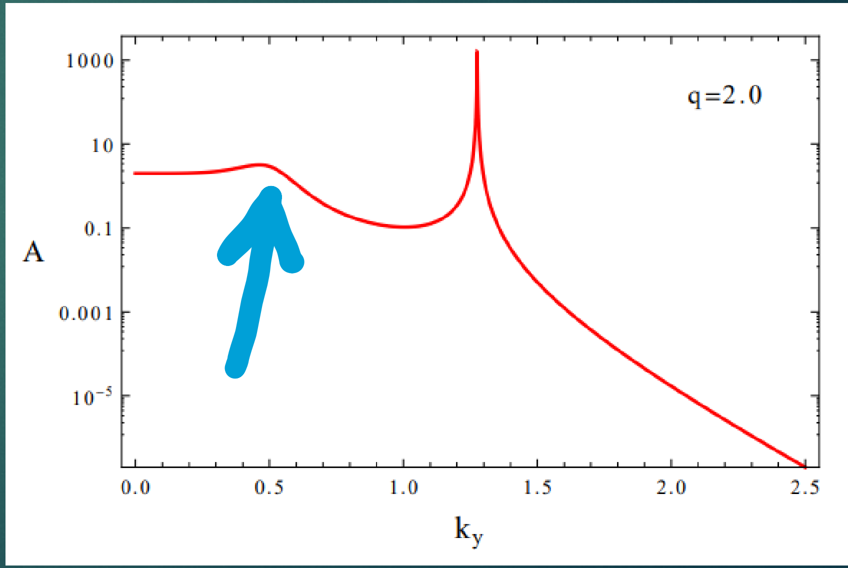




Multiple Fermi surfaces form when fermionic charge is large enough

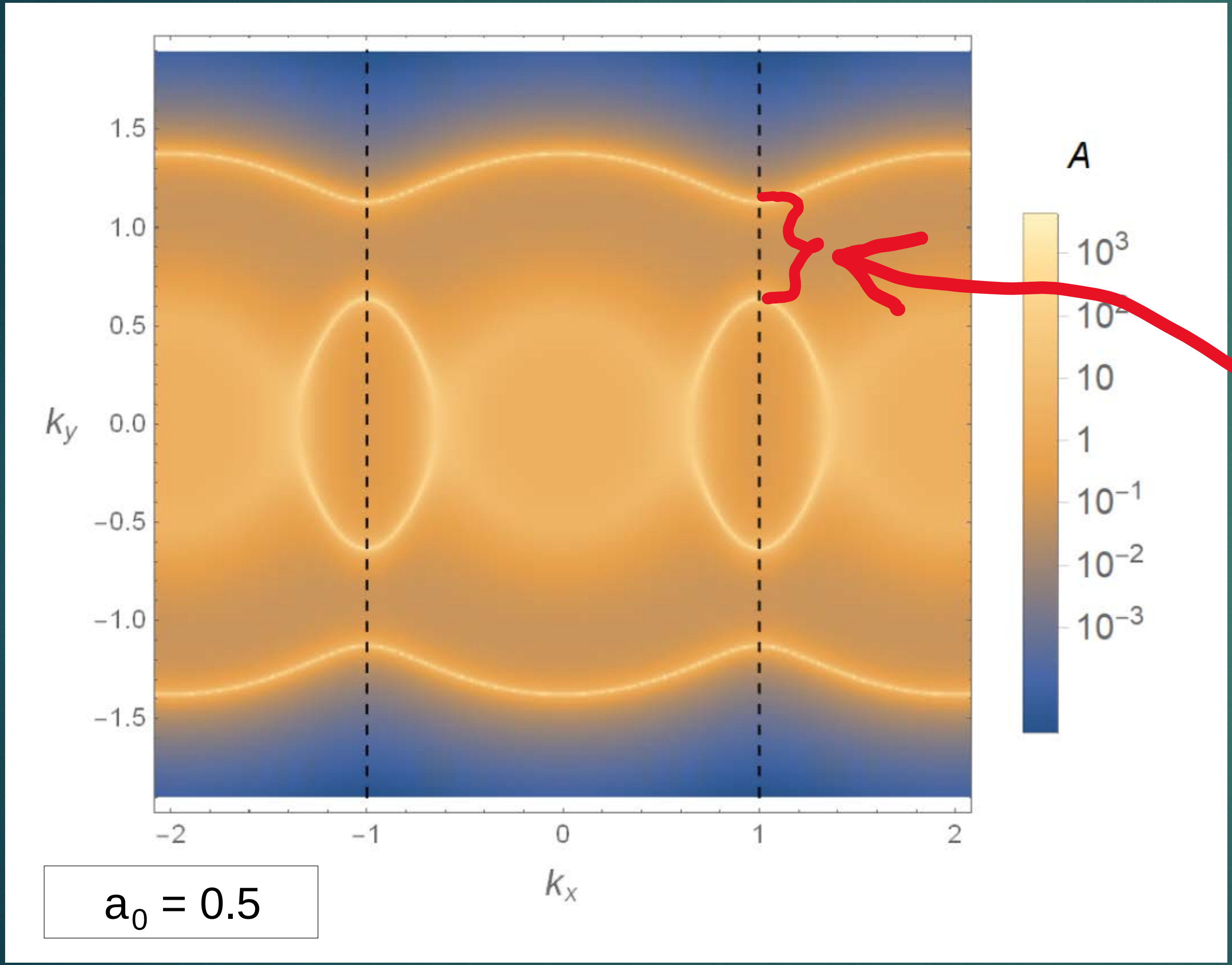


See also S. Gubser and J. Ren arXiv:1204.6315 (analytic fermionic Green's function, several Fermi momenta)

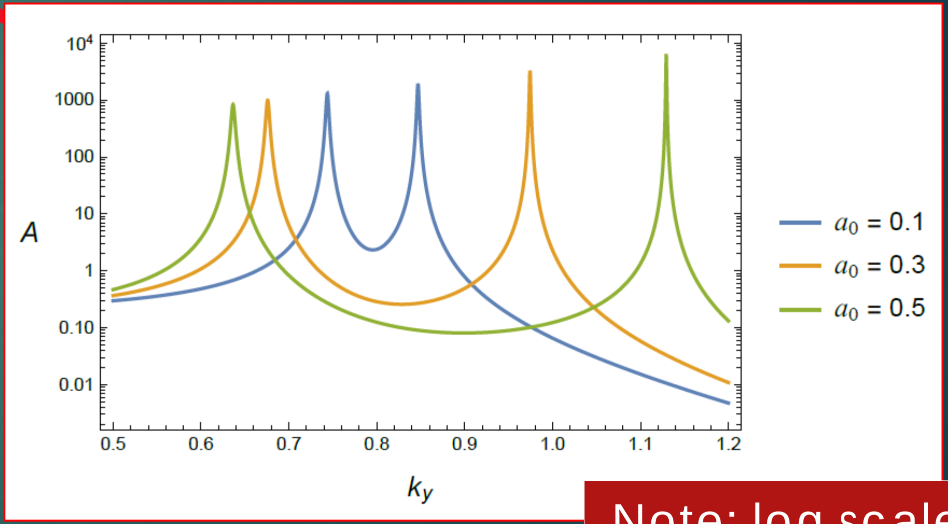


PDW + Ionic Lattice:

More pronounced anisotropy and larger gap ✓



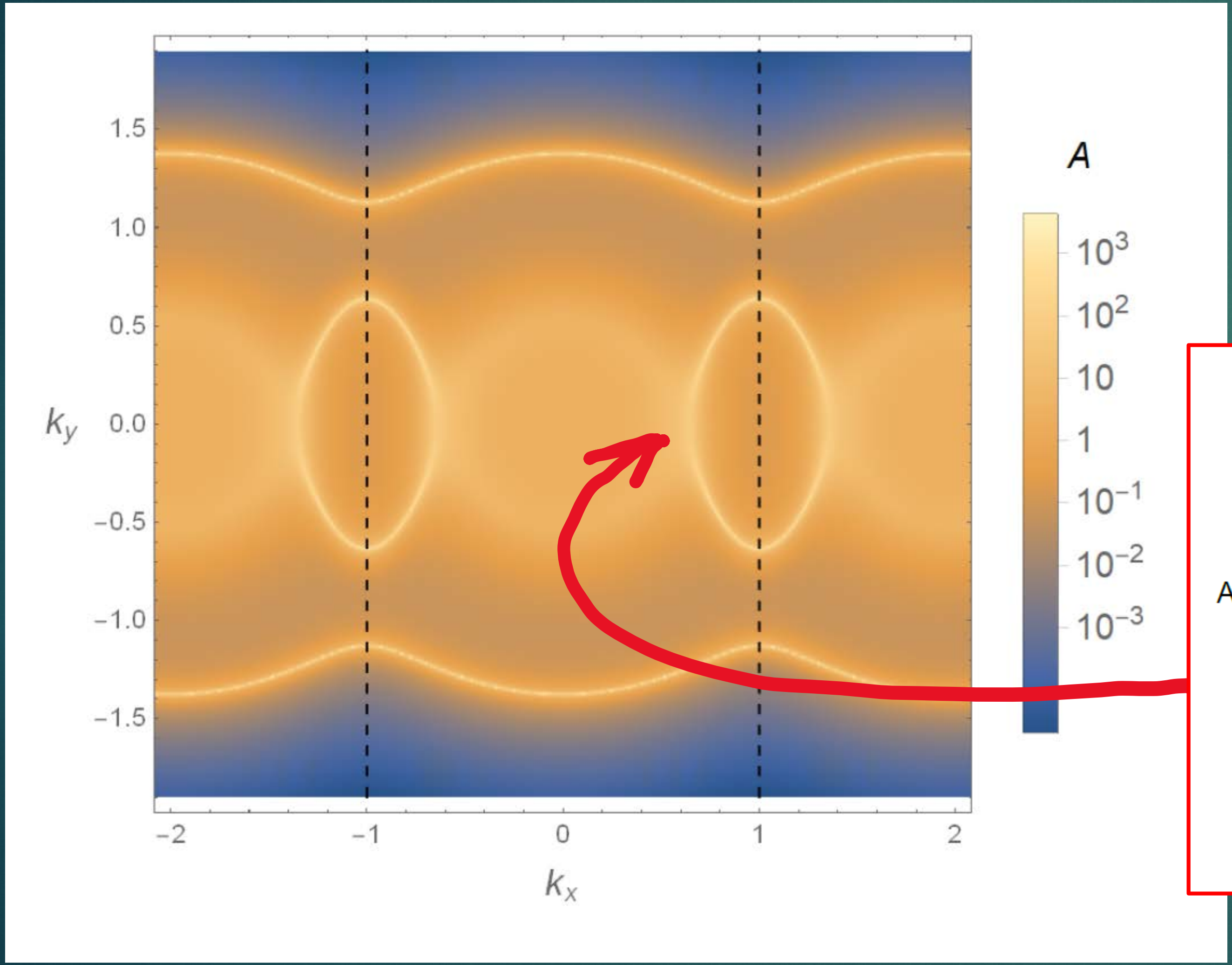
Larger gap at Brillouin zone boundary (increases with lattice amplitude)



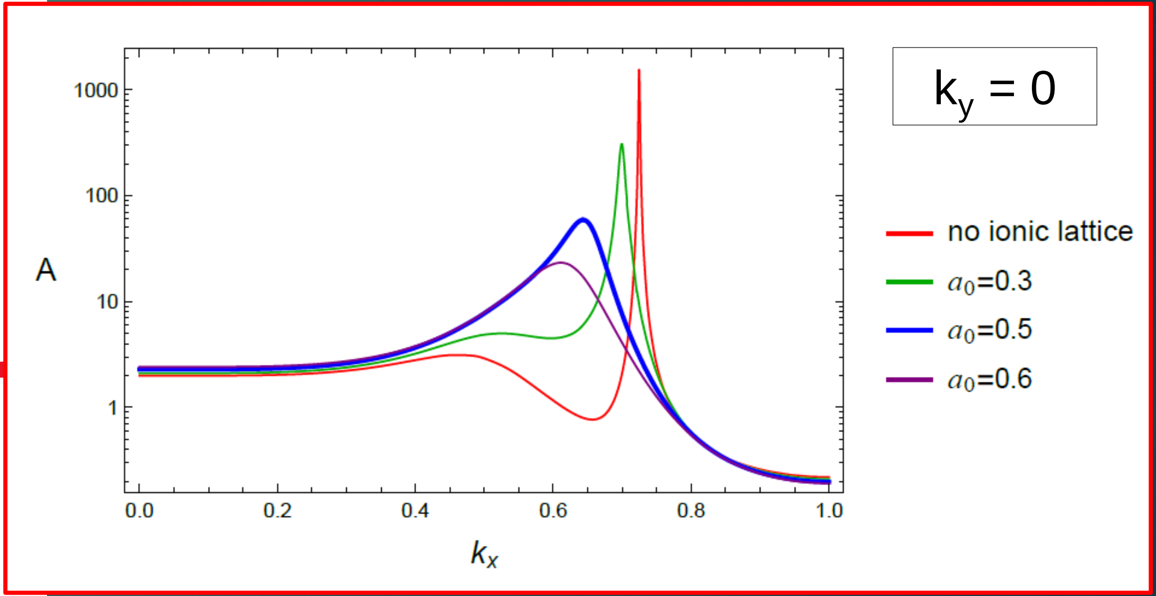
Note: log scale "distorts" peaks

More Interesting Feature:

Parts of Fermi surface gradually dissolve with strong lattice effects

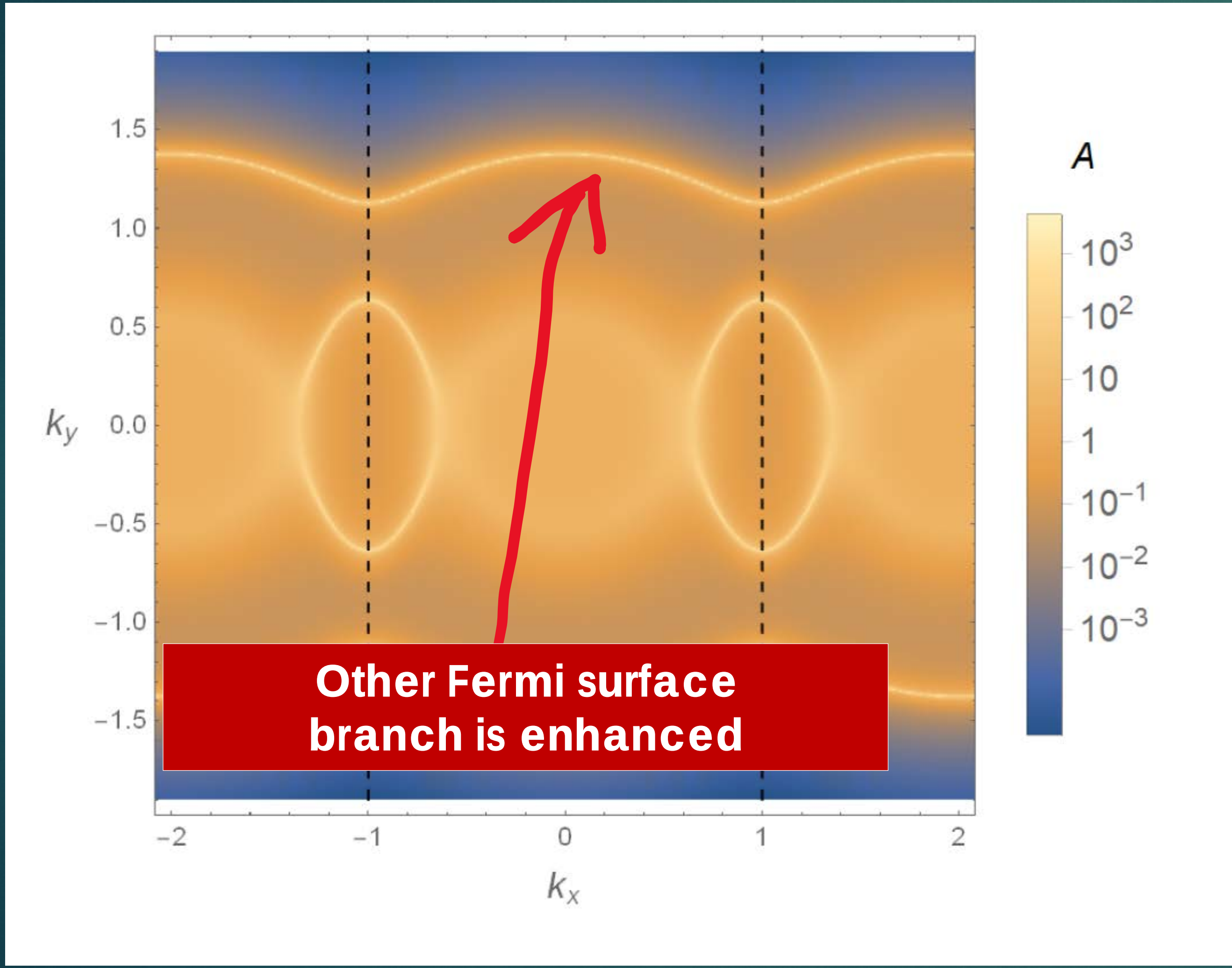


spectral weight peaks suppressed with large lattice strength → Fermi surface gradually dissolves leaving behind detached segments



Interesting Feature:

Fermi surface gradually dissolves with strong lattice effects



spectral weight peaks suppressed with large lattice strength → Fermi surface gradually dissolves leaving behind detached segments

Preliminary analysis (no PDW):

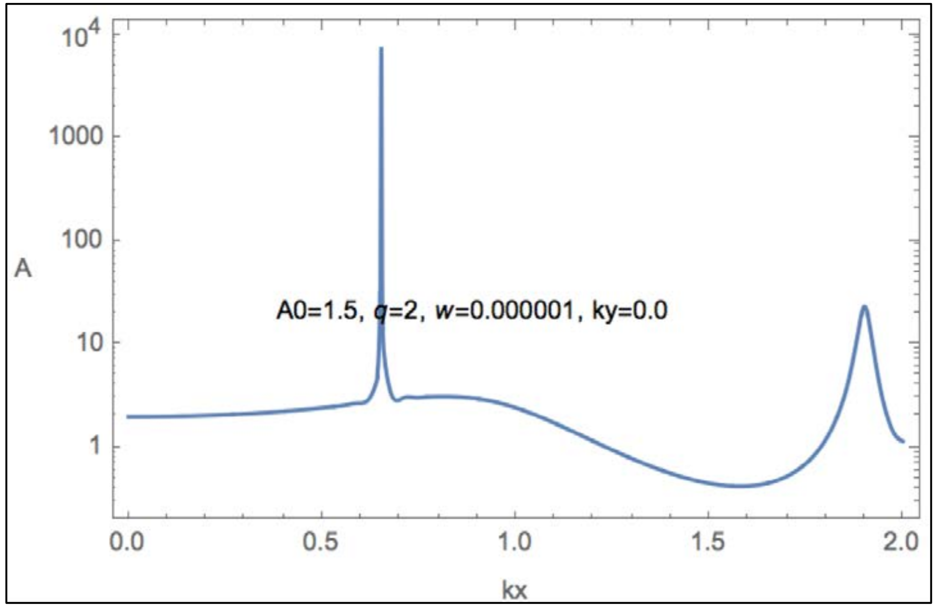
Simple EMD theory with an explicit lattice provided by the source of the neutral scalar [Setup of Horowitz, Santos, Tong, arXiv:1204.0519]

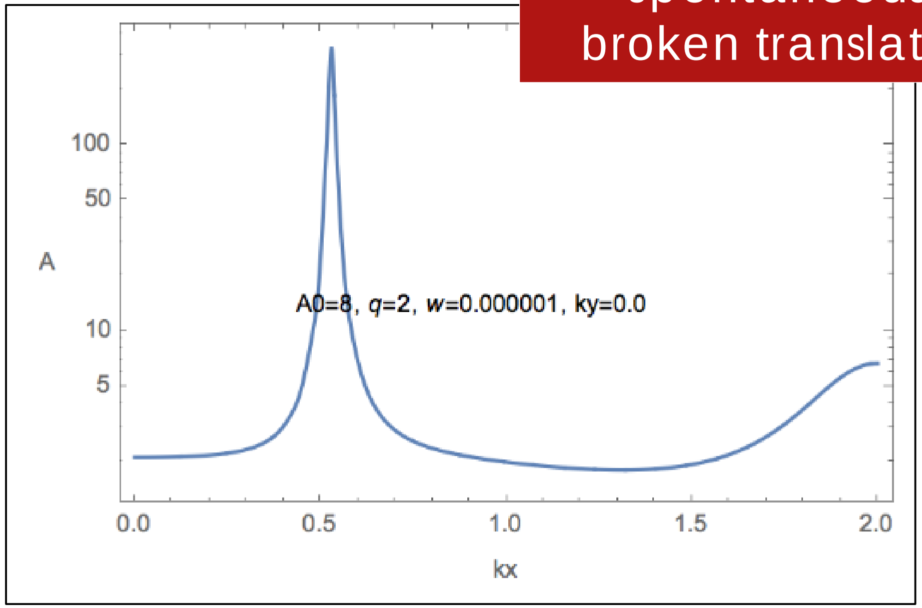
$$S = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2}F_{ab}F^{ab} - 2\nabla_a\Phi\nabla^a\Phi - 4V(\Phi)\right]$$

$$\phi_1(x) = A_0\cos(k_0 x)$$

No superconducting order here and no spontaneously broken translations

Same spectral function suppression observed with strong lattice:





Laundry list of questions to address…

Disappearance of Fermi surface:

- suppression of spectral weight in our analysis occurs when the explicit ionic lattice is strong
  → UV effect (also in simpler EMD models w/out spontaneously generated stripe and SC orders)

- We **expect that the PDW alone would cause a disappearance of the Fermi surface**
  → To see it must reach very low temperatures

- Periodicity of the PDW and lattice oscillations in our analysis are the same (commensurate)
  → The ionic lattice potential amplifies the effects of PDW.

- Incommensurate case (different scales) → Expect smaller reduction of FS (lattice amplifies PDW)

- **Interesting question is to isolate the contribution from PDW vs. from ionic lattice**
  (many ways to do so, including placing PDW order and ionic lattice along different spatial directions)

Origin of Fermi arcs?

In experiments temperature can obscure the gap (thermal broadening)
Hard to distinguish between:

- **real Fermi arcs**
- **point nodes**
- **a small gap at a node point**

Temperature smearing would produce an apparent arc in all these cases

Photoemission perspective on pseudogap, superconducting fluctuations, and charge order: a review of recent progress

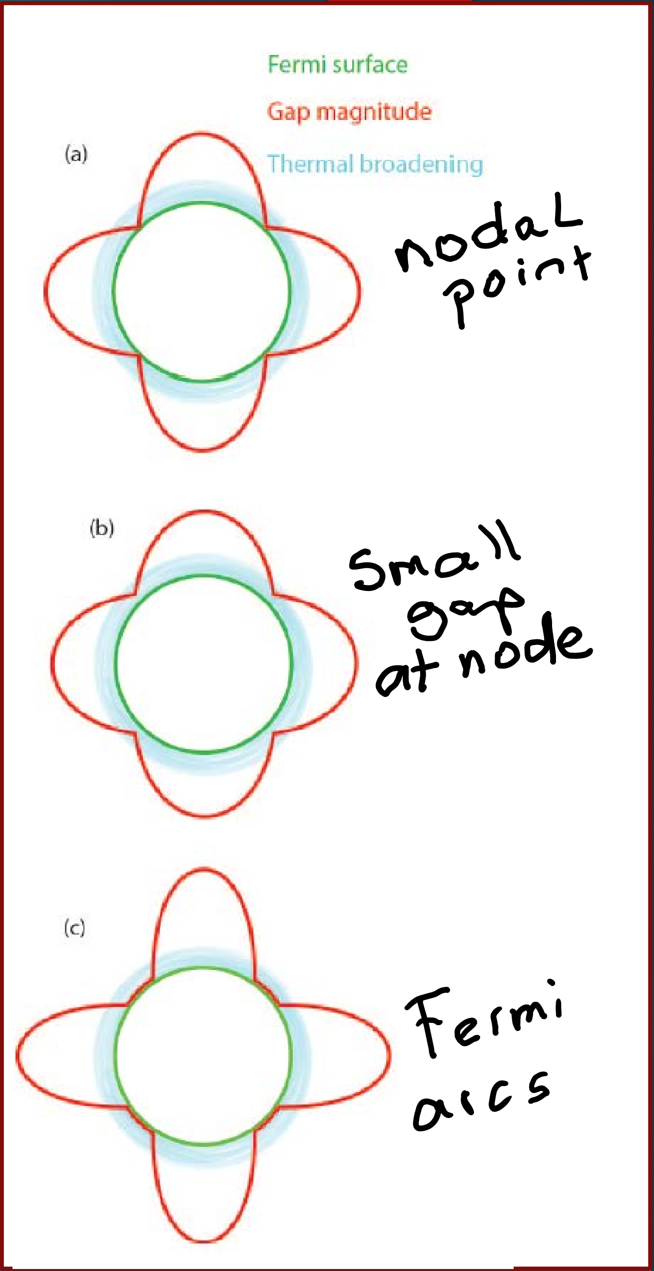
I. M. Vishik[1]

[1] *University of California, Davis*

(Dated: April 2, 2018)

In the course of seeking the microscopic mechanism of superconductivity in cuprate high temperature superconductors, the pseudogap phase—the very abnormal 'normal' state on the hole-doped side—has proven to be as big of a quandary as superconductivity itself. Angle-resolved photoemission spectroscopy (ARPES) is a powerful tool for assessing the momentum-dependent phenomenology of the pseudogap, and recent technological developments have permitted a more detailed understanding. This report reviews recent progress in understanding the relationship between superconductivity and the pseudogap, the Fermi arc phenomena, and the relationship between charge order and pseudogap from the perspective of ARPES measurements.

arXiv:1803.11228



Laundry list of questions to address…

Segmented pieces are left over:

- Can we distinguish between different mechanisms for appearance of "Fermi arcs"?
- Can we rule out idea that they may be pieces of Fermi pockets?
- Temperature dependence of segmented pieces (compare to experiments)?

Need input from CM to ask right questions!

Note: disorder also smears the Fermi surface

- Disorder will pin density wave order but it leads to random pinning

Fermionic response in **T=0 ground state**?

Novel features from **coupling fermion to other orders**? (free fermion so far for simplicity)
[Recall e.g. Benini, Herzog, Yarom 1006.0731, Fermi arcs in holographic d-wave SC]

Thank you