Electron Hydrodynamics and Hall Viscosity

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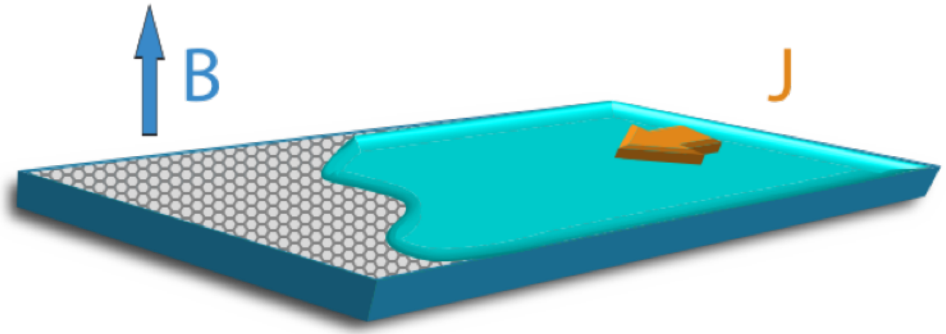
UC Berkeley

Nordita August 2018

[Scaffidi, Nandi, Schmidt, Mackenzie, and Moore, Phys. Rev. Lett. **118**, 226601]



Acknowledgements

- UC Berkeley: Joel Moore
- MPI Dresden: Nabhanila Nandi, Burkhard Schmidt, Philip Moll, Pallavi Kushwaha, Andrew Mackenzie

Could it finally be true?

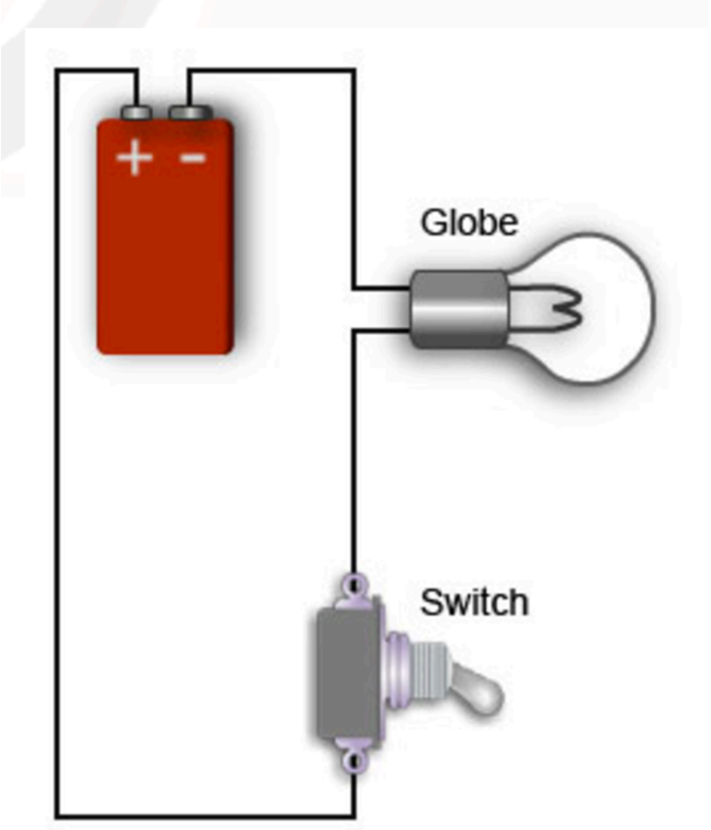


Figure 1: Electrical

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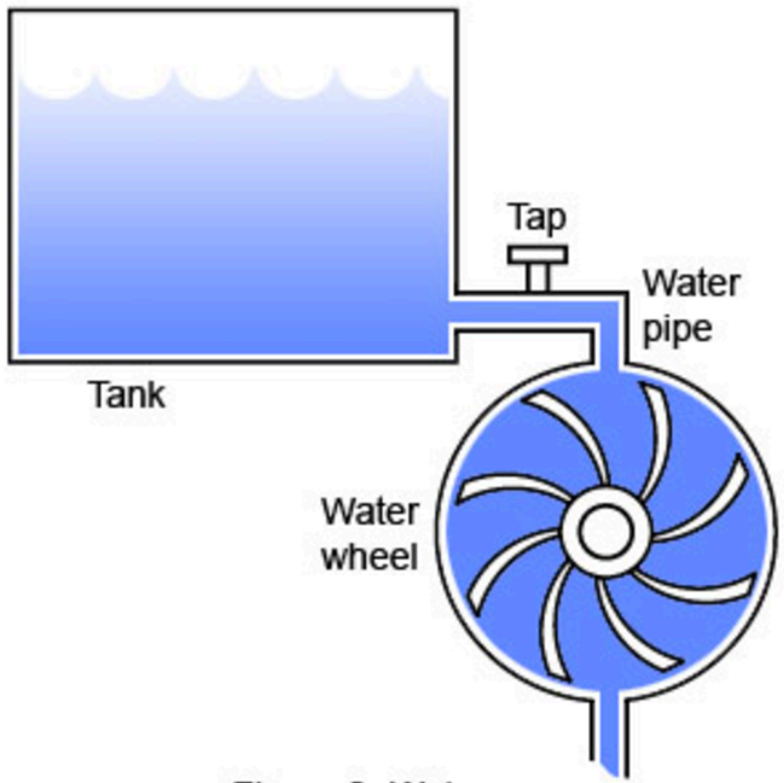
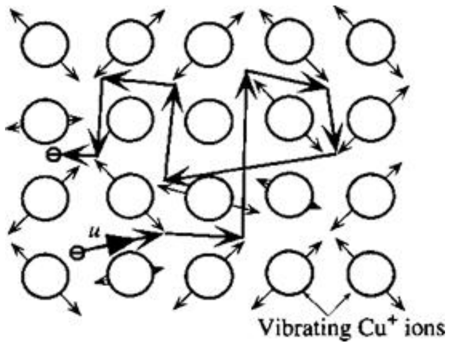


Figure 2: Water

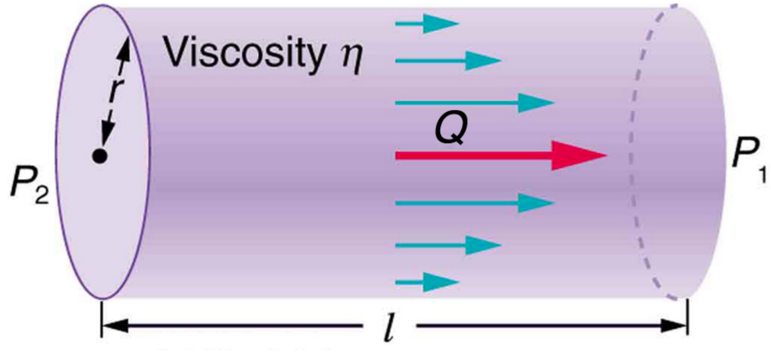
Metal versus water

Electrons in a metal



- Resistance arises through “extrinsic” processes due to the lattice: impurities, phonons,umklapp,...

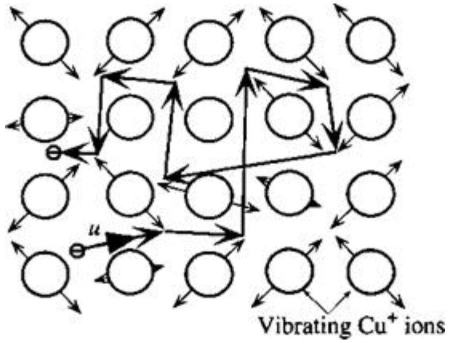
Water in a pipe



- Resistance arises through an “intrinsic” process: viscosity

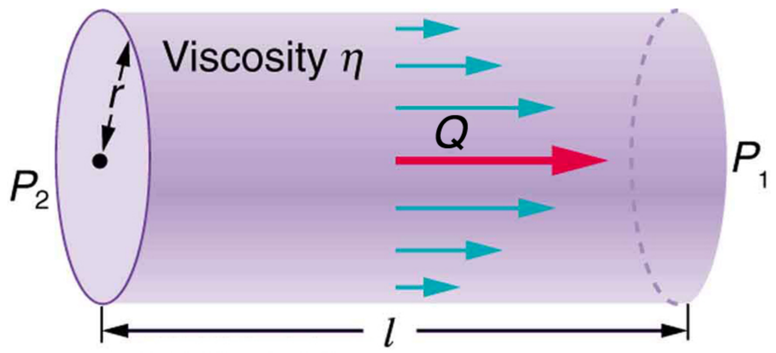
Metal versus water

Electrons in a metal



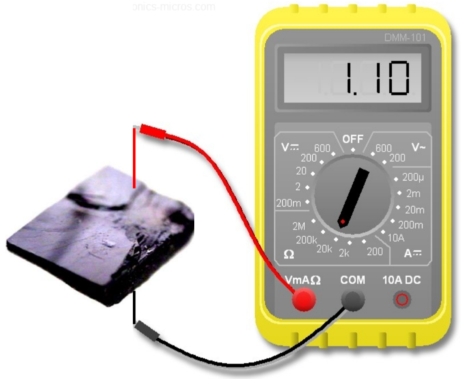
- Resistance arises through “extrinsic” processes due to the lattice: impurities, phonons,umklapp,...

Water in a pipe



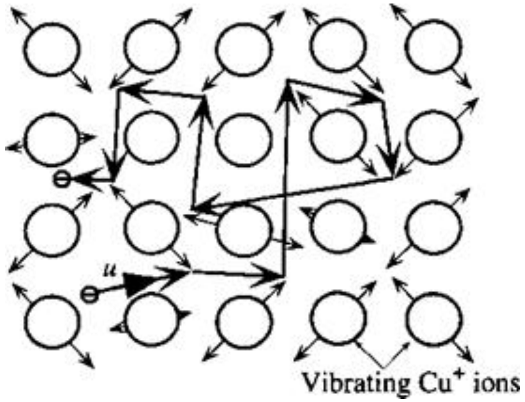
- Resistance arises through an “intrinsic” process: viscosity

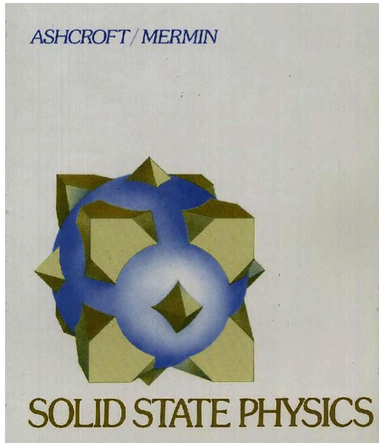
Conventional metallic transport [Drude 1900]



$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

Mean free time of electrons, set by defects and vibrations of the lattice

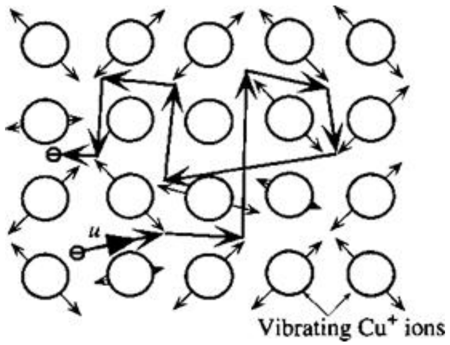




Electron-electron scattering[5] plays a relatively minor role in the theory of conduction in solids, for reasons to be described in Chapter 17.

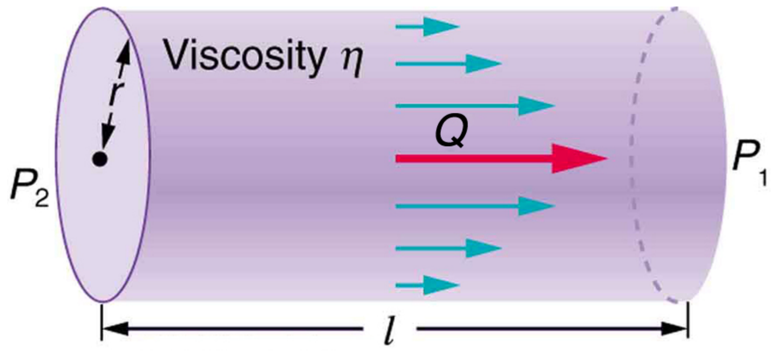
Metal versus water

Electrons in a metal



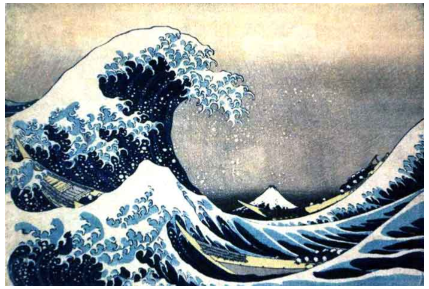
- Resistance arises through “extrinsic” processes due to the lattice: impurities, phonons,umklapp,...

Water in a pipe

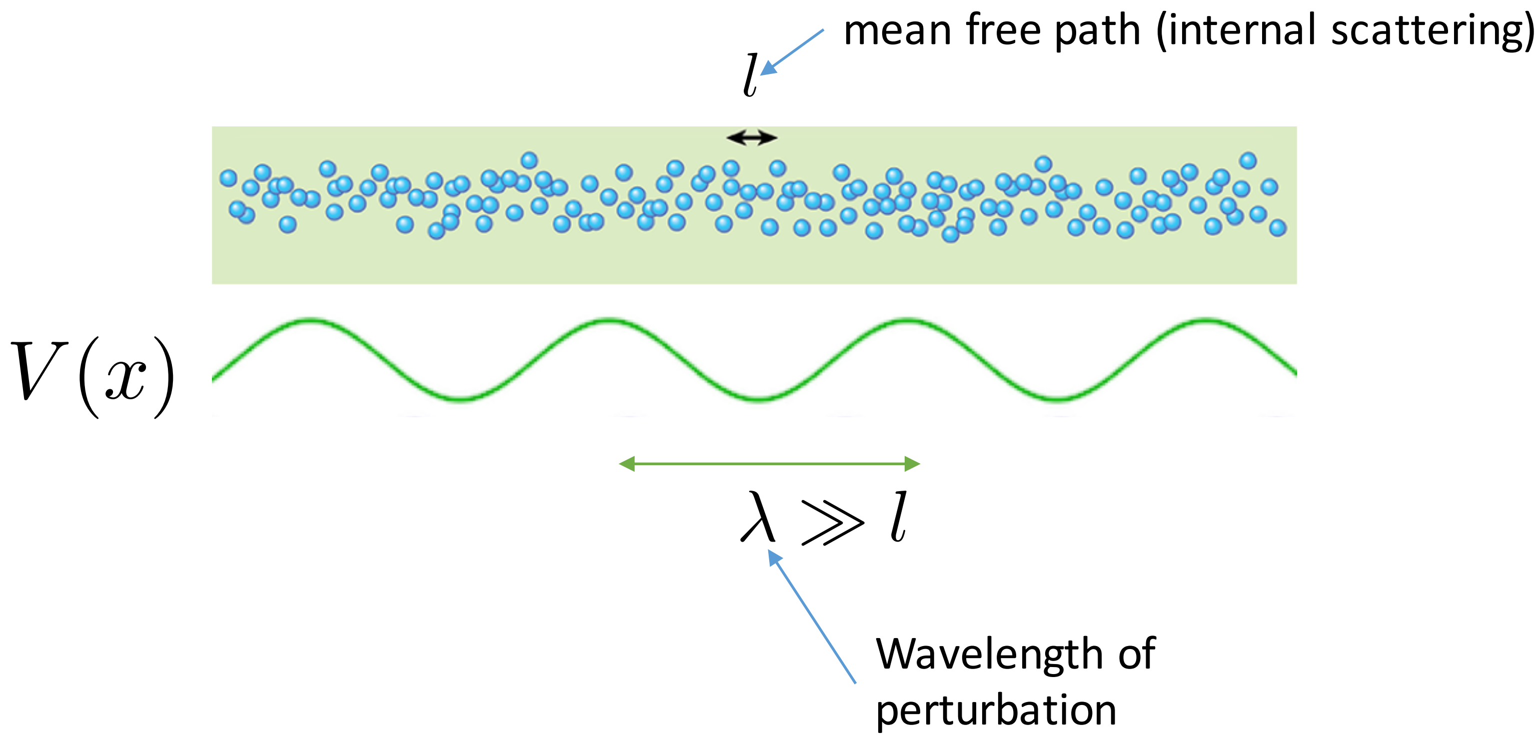


- Resistance arises through an “intrinsic” process: viscosity

Hydrodynamics

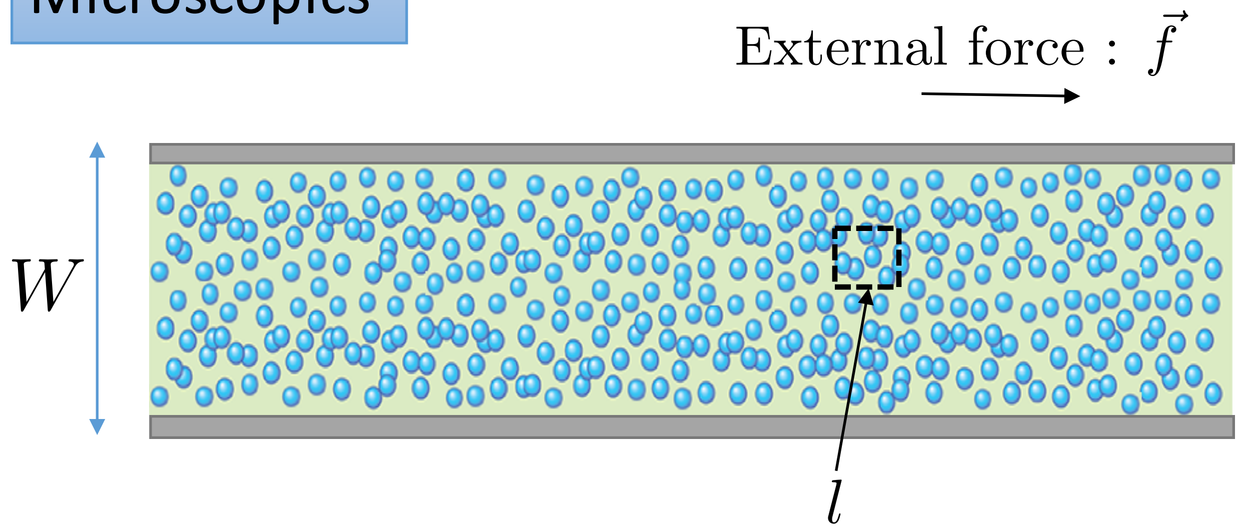


- Universal description of fluids based on conserved quantities: momentum, energy, charge,…
- Applications in nearly all fields of physics
- Works at length/time scales much larger than the microscopic ones
- The more interacting, the better!

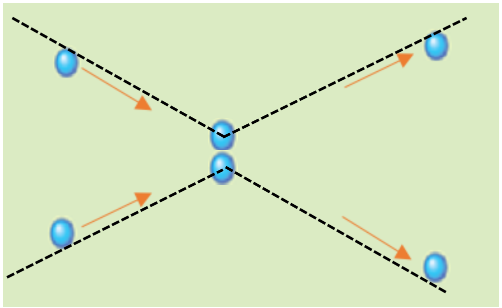


Viscous fluid

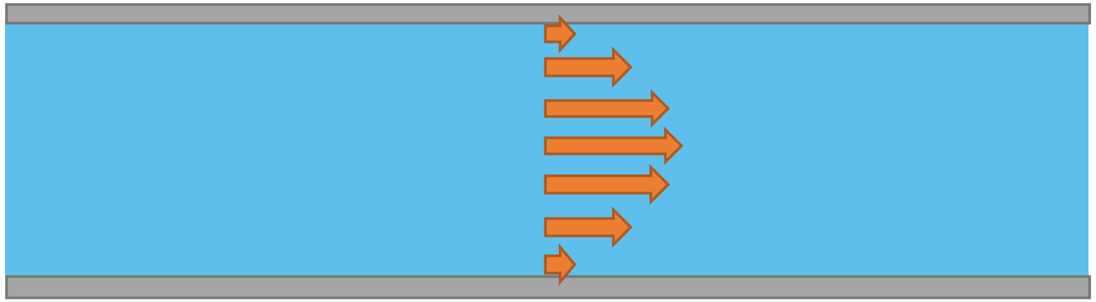
Microscopics



$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$



Hydrodynamics



Navier-Stokes

$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} + \frac{\vec{f}}{m}$$

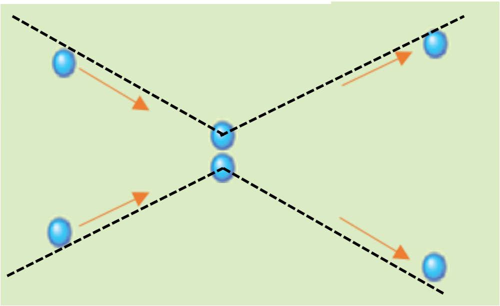
Hydrodynamic flow of electrons possible if…

$$l_{MC} \ll W \ll l_{MR}$$

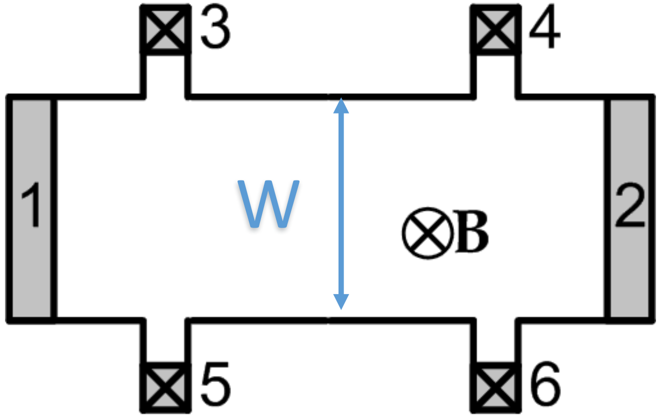
Momentum-conserving mean free path

- Normal e-e

$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$



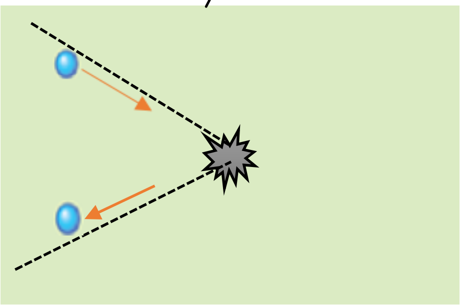
Sample size



Momentum-relaxing mean free path:

- Impurities
- Phonons
- Umklapp e-e

$$\vec{v} \neq \vec{v}'$$

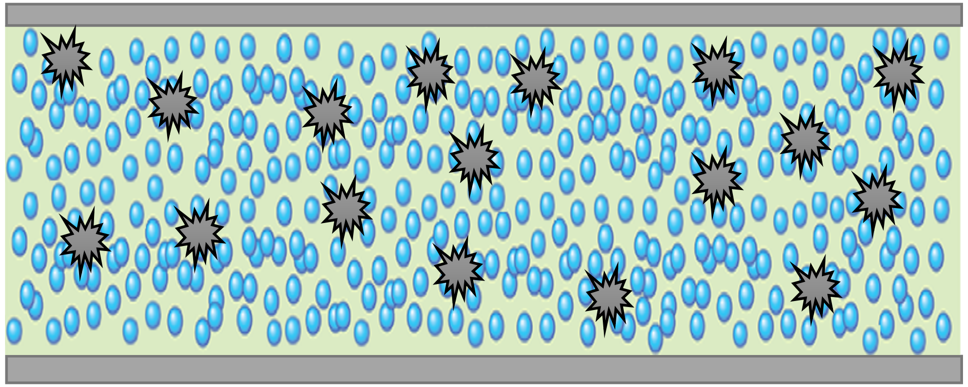


[Gurzhi 1963] [*Spivak, Kivelson 2006]* [Hruska, Spivak 2011] *[Andreev Kivelson Spivak 2011]*
[Torre,Tomadin,Geim,Spivak 2011] [Levitov Falkovich 2016] many more papers…

Three regimes

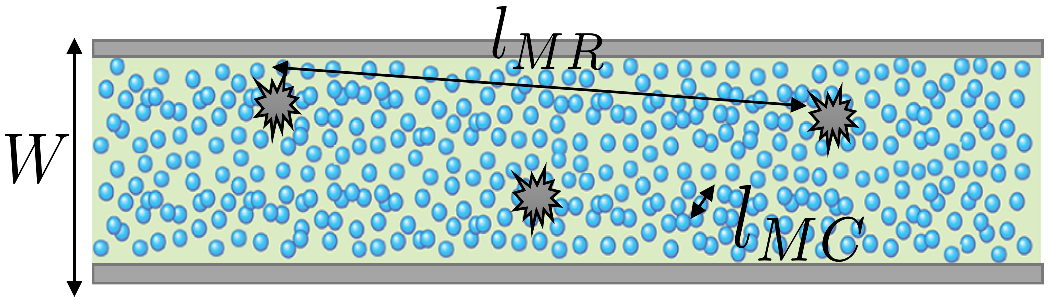
Ohmic

$$l_{MR}, l_{MC} \ll W$$



Viscous

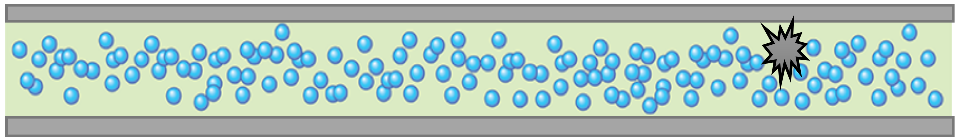
$$l_{MC} \ll W \ll l_{MR}$$



Diffuse-Ballistic

"Knudsen flow"

$$W \ll l_{MR}, l_{MC}$$



Electron hydro - Why is it interesting?

- New regime of transport

$$\cancel{\vec{j} = \sigma \vec{E}} \quad \Longrightarrow \quad \boldsymbol{\eta} \nabla^2 \vec{j} = -\frac{e^2 n}{m} \vec{E}$$

- Realize "exotic" hydro in the lab:

For electrons in a solid: $\cancel{H = \frac{P^2}{2m}} \quad \Longrightarrow \quad H(\mathbf{k})\psi(\mathbf{k}) = E(\mathbf{k})\psi(\mathbf{k})$

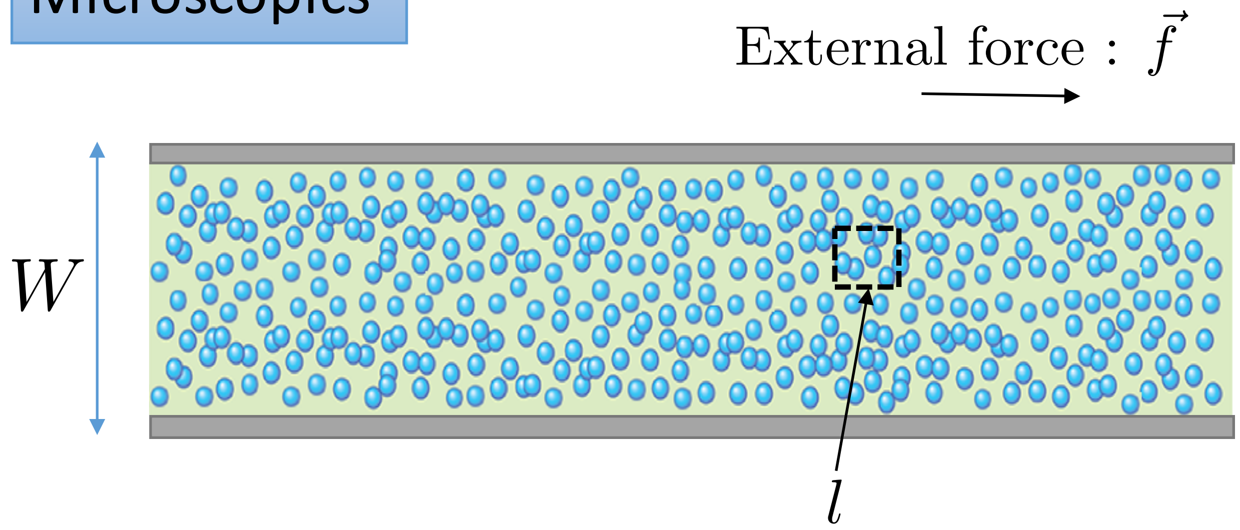
- First step towards transport in unconventional metals?

Outline

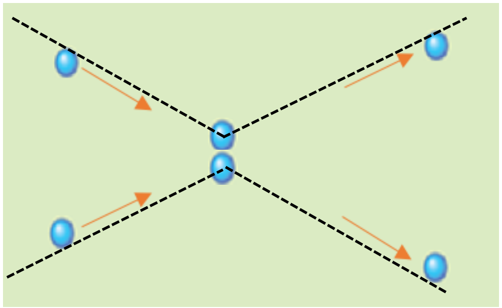
- Electron hydrodynamics – What and why?
- Viscous Fermi liquids
- Using magnetic fields to detect viscous effects
- Conclusion and outlook

Viscous fluid

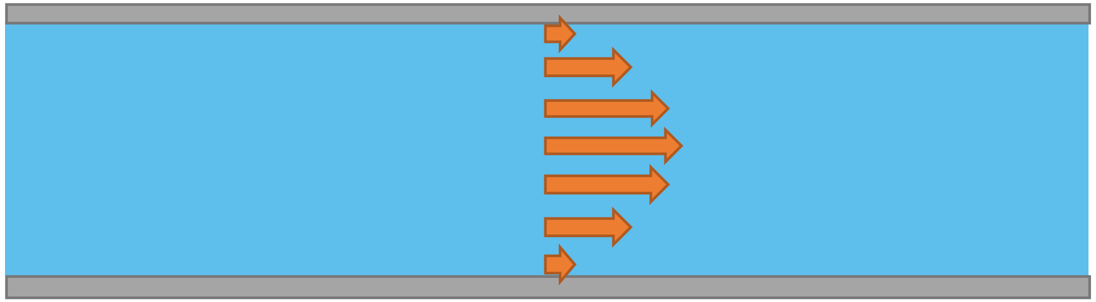
Microscopics



$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$



Hydrodynamics



Navier-Stokes

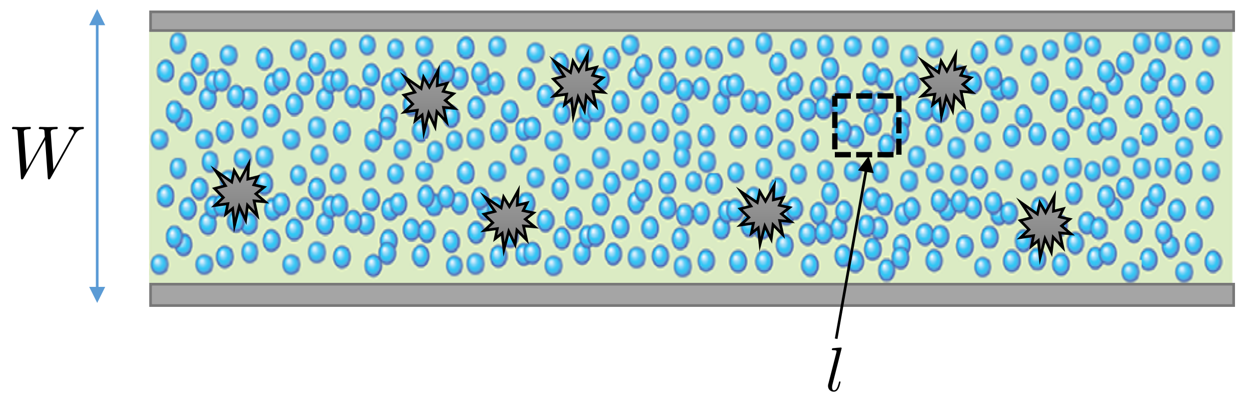
$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} + \frac{\vec{f}}{m}$$

Viscous electronic fluid

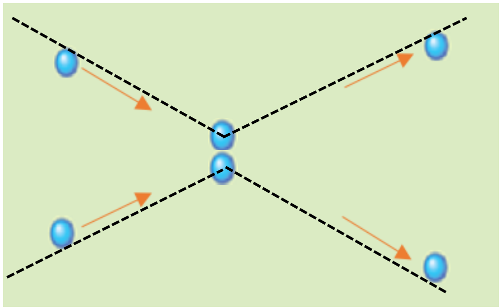
$$\vec{j} = ne\vec{v}$$

Microscopics

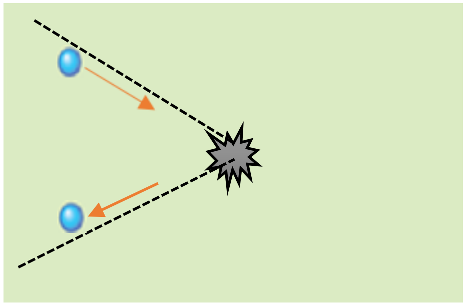
External field: $\vec{E}$



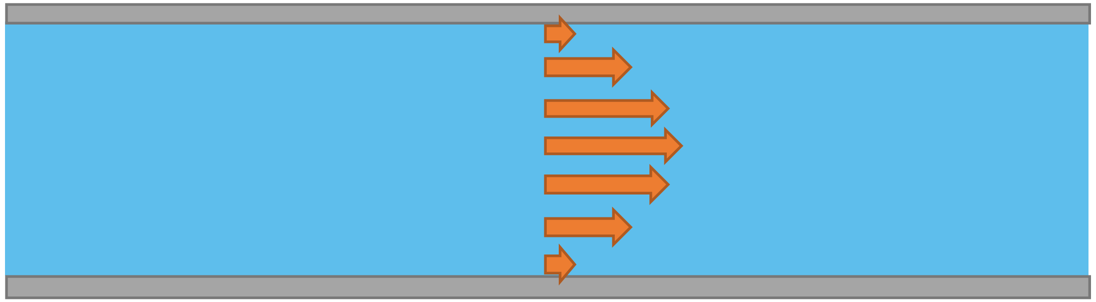
$$\vec{v}_1 + \vec{v}_2 = \vec{v}_1' + \vec{v}_2'$$



$$\vec{v} \neq \vec{v}'$$



Hydrodynamics



$$\partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v} = \boldsymbol{\eta} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e\vec{E}}{m}$$

Two regimes

$$\partial_t \vec{v} = \boldsymbol{\eta}\nabla^2\vec{v} - \frac{1}{\tau_{MR}}\vec{v} + \frac{e\vec{E}}{m}$$

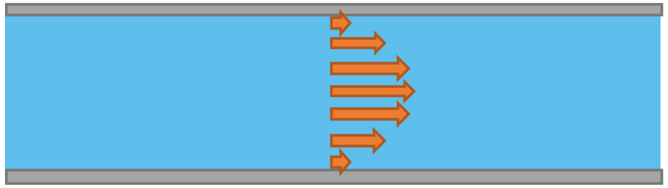
$$\vec{j} = ne\vec{v}$$

If term dominates

If term dominates

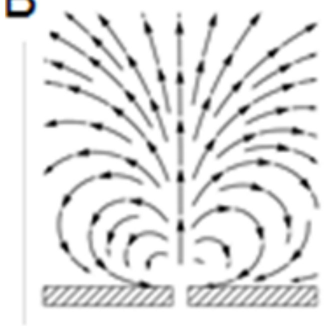
Viscous

Non-local relation between current and electric field!



$$\boldsymbol{\eta}\nabla^2\vec{j} = -\frac{e^2 n}{m}\vec{E}$$

B

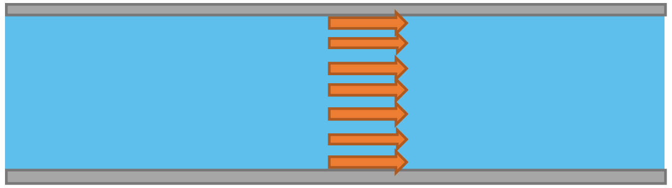


$$\nabla \times \vec{j} \neq 0$$

$$\bar{\rho} = \frac{m}{e^2 n}\eta\frac{12}{W^2}$$

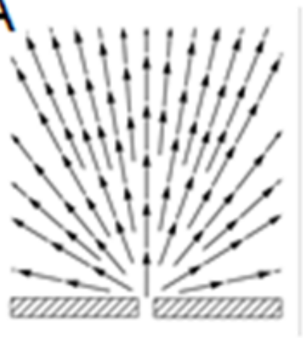
Ohmic

$$\sigma = \frac{ne^2}{m}\tau_{MR}$$



$$\vec{j} = \sigma\vec{E}$$

A



$$\nabla \times \vec{j} = 0$$

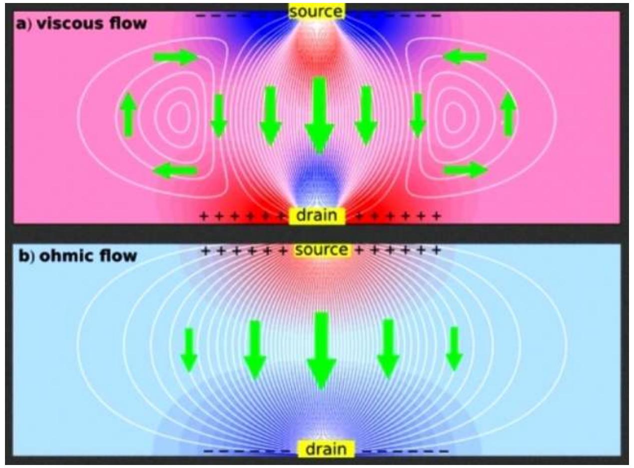
$$\bar{\rho} = \frac{m}{e^2 n}\frac{1}{\tau_{MR}}$$

"Viscous electronics"

$$\cancel{\vec{j} = \sigma \vec{E}} \Longrightarrow \eta \nabla^2 \vec{j} = -\frac{e^2 n}{m} \vec{E}$$

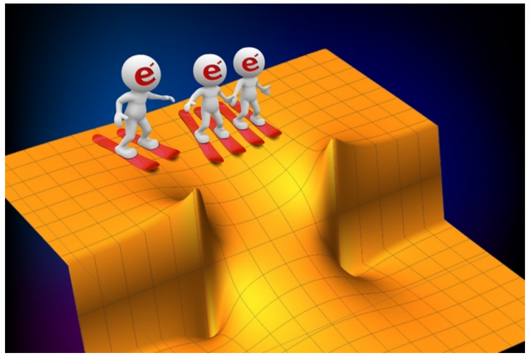
- Non-local relation between current and electric field can lead to many interesting effects:

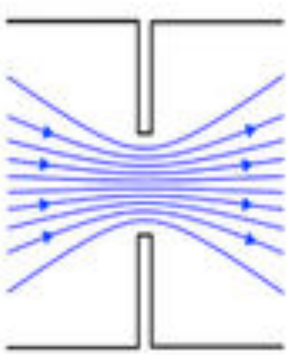
Current vortices



[Levitov, Falkovich, Nature Physics '15]

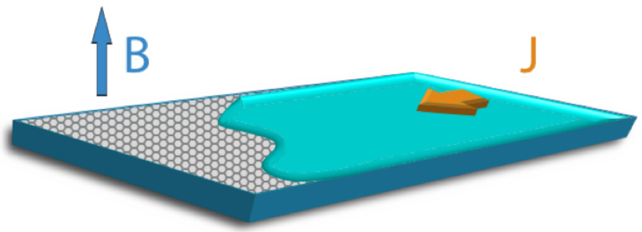
Higher-than-ballistic conduction





[Guo et al, PNAS '17]
[Krishna Kumar et al, Nature Physics '17]

Viscous Hall effect



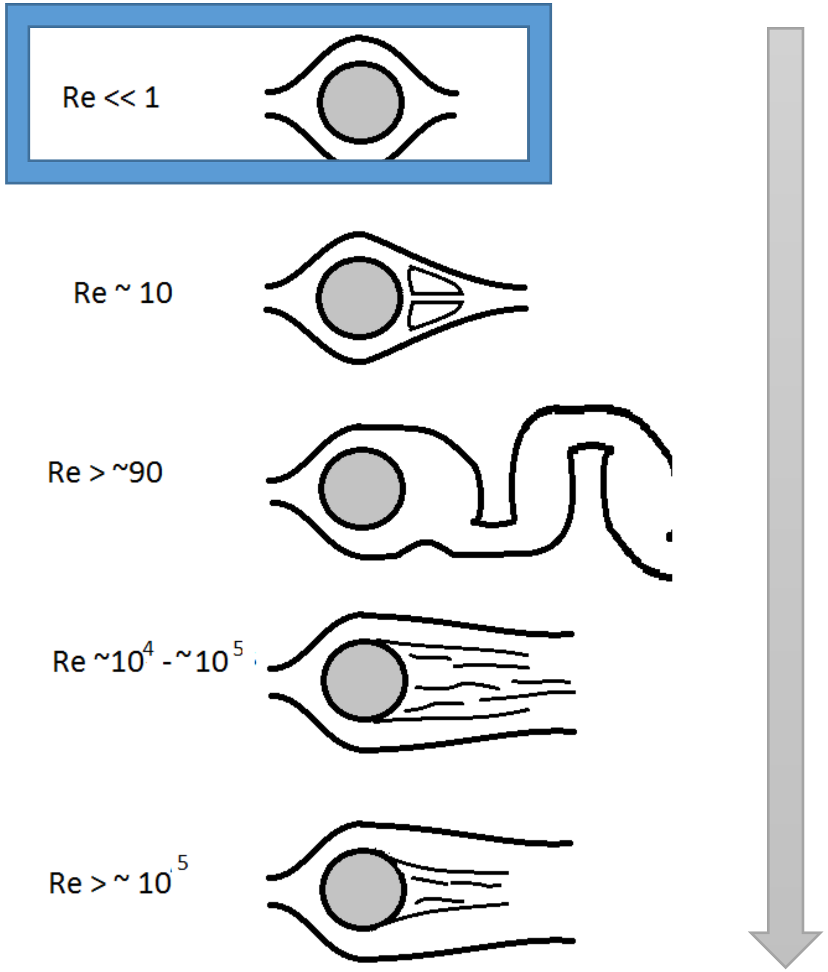
[TS et al, PRL '17]

Where did the convection term go?

$$\partial_t \vec{v} + \vec{v}\cdot\nabla\vec{v} = \boldsymbol{\eta}\nabla^2\vec{v} - \frac{1}{\tau_{MR}}\vec{v} + \frac{e\vec{E}}{m}$$

$$Re = \frac{\vec{v}\cdot\nabla\vec{v}}{\eta\nabla^2\vec{v}}$$

$$Re = \frac{v_{\text{drift}}W}{\eta} < \frac{v_{\text{drift}}}{v_F} \times \frac{l_{MR}}{l_{MC}} \ll 1$$



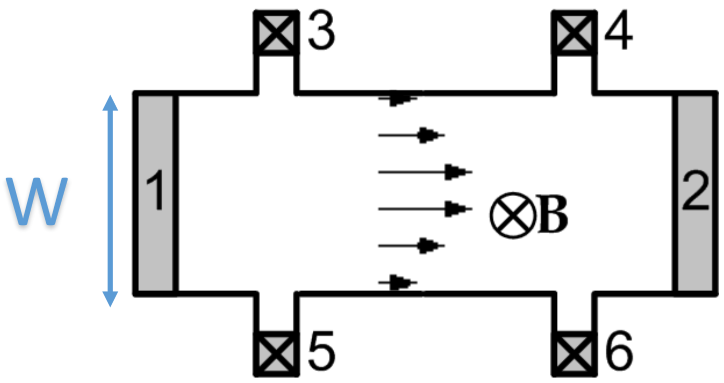
Outline

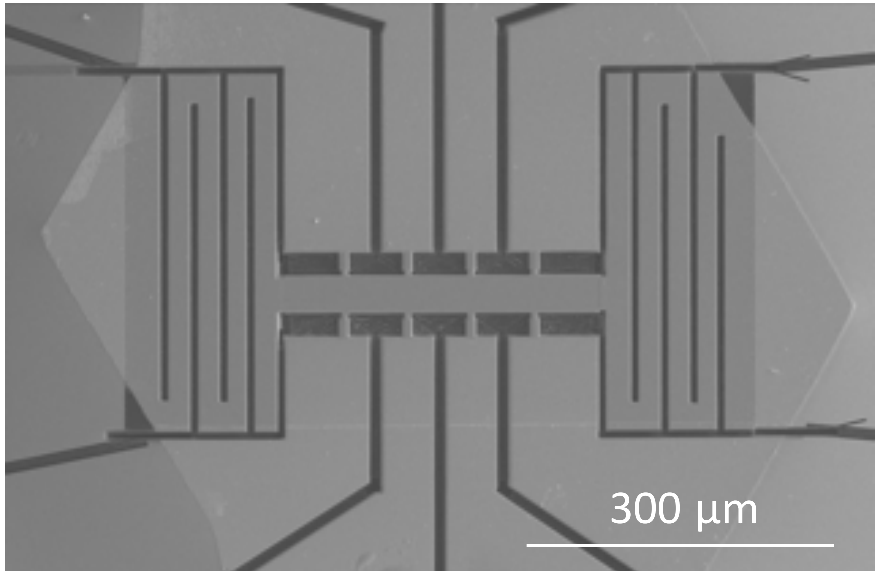
- Electron hydrodynamics – What and why?
- Electron hydrodynamics in good metals
- **Using magnetic fields to detect hydro effects**
- Conclusion and outlook

[TS et al, PRL ‘17]

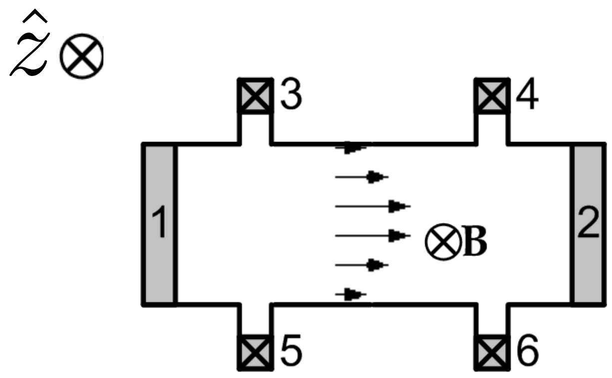
How to identify hydro effects?

- Idea: Look at finite size corrections to transport coefficients
- Problem: how to distinguish from ballistic effects?
- Solution: use magnetic field





Navier-Stokes under magnetic field



$$\partial_t \vec{v} = \eta_{xx}\nabla^2\vec{v} - \frac{1}{\tau_{MR}}\vec{v} + \frac{e}{m}\vec{E}$$

Add B field

$$\partial_t \vec{v} = \eta_{xx}\nabla^2\vec{v} - \frac{1}{\tau_{MR}}\vec{v} + \frac{e}{m}(\vec{E} + \vec{v}\times\vec{B}) + \eta_{xy}\nabla^2\vec{v}\times\hat{z}$$

Lorentz force

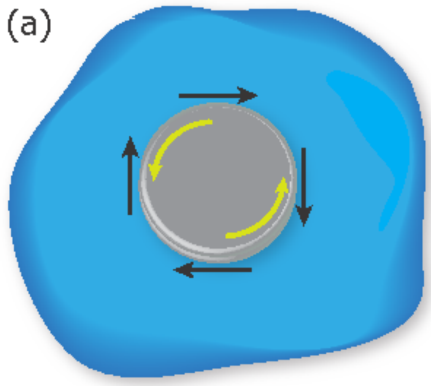
Hall viscosity

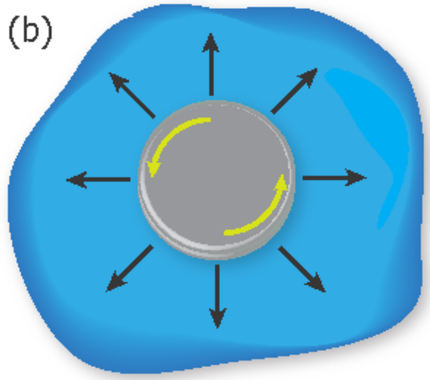
A tale of two viscosities

$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity

Hall viscosity





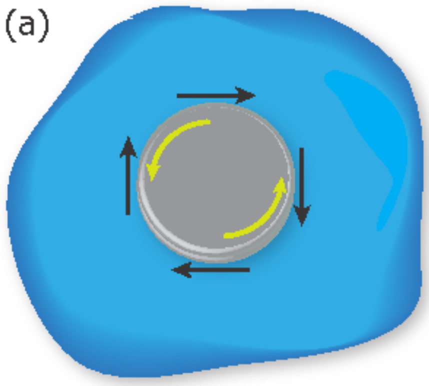
Probed by ρ_{xx}

Probed by ρ_{xy}

A tale of two viscosities

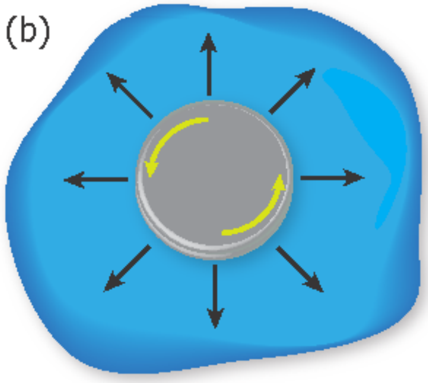
$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m} (\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity



Probed by ρ_{xx}

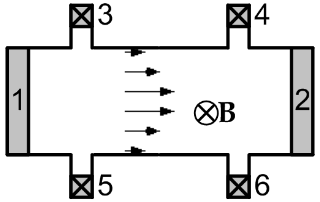
Hall viscosity



Probed by ρ_{xy}

Hydrodynamic solution

[TS et al, PRL '17]



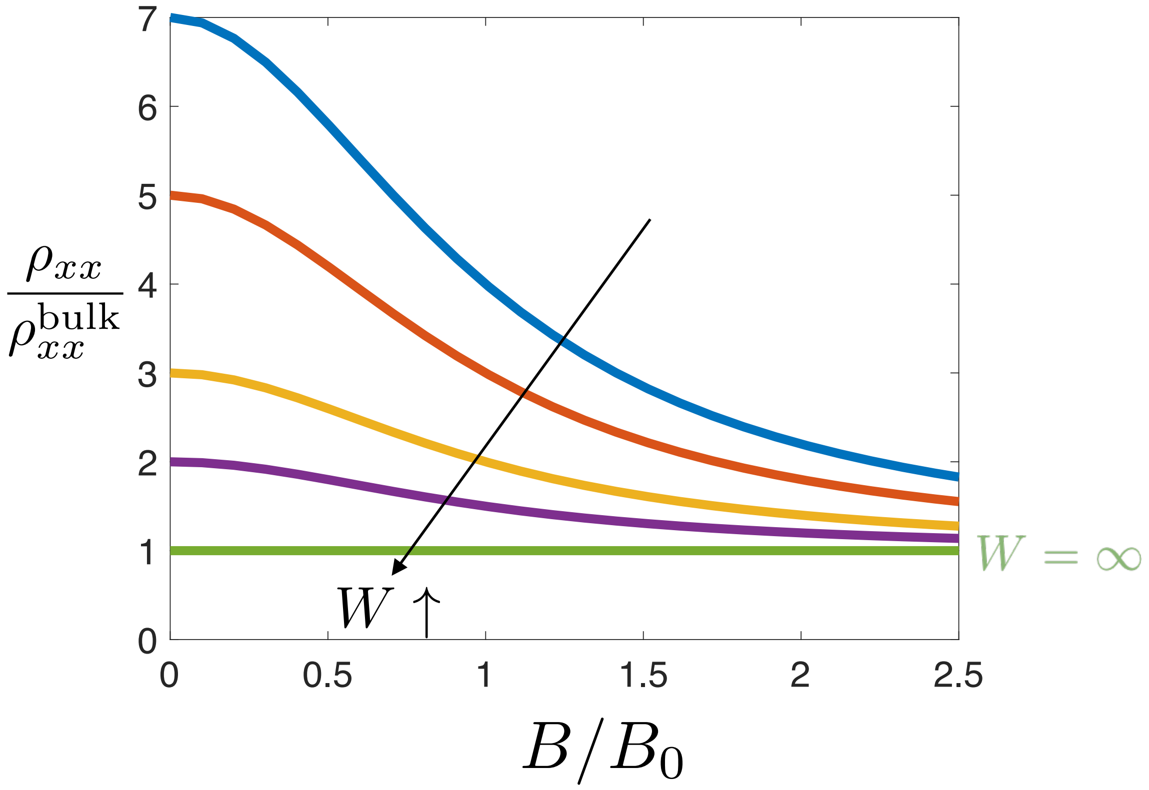
$$\partial_t \vec{v} = \eta_{xx}\nabla^2\vec{v} + \eta_{xy}\nabla^2\vec{v}\times\vec{z} + \frac{e}{m}(\vec{E}+\vec{v}\times\vec{B}) - \frac{1}{\tau_{MR}}\vec{v}$$

$$\rho_{xx} \simeq \rho_{xx}^{bulk} + \frac{m}{e^2 n}\eta_{xx}\frac{12}{W^2}$$

$$\rho_{xx}^{bulk} = \frac{m}{e^2 n}\frac{1}{\tau_{MR}}$$

Result from Boltzmann theory for a charged Fermi liquid:

$$\eta_{xx}(B) \sim \eta_{xx}(B=0)\frac{1}{1+(B/B_0)^2}$$



Navier-Stokes is not enough

In theory:

$$l_{MC} \ll W \ll l_{MR} \quad \Longrightarrow \quad \text{Navier-Stokes}$$

In practice:

$$l_{MC} \lesssim W \lesssim l_{MR} \quad \Longrightarrow \quad \text{Kinetic theory}$$

$f\left(\vec{r}, \vec{k}\right)$: quasi-particle density

$$\boxed{\partial_t f + \vec{v}\cdot\nabla_{\vec{r}} f + \frac{e}{m}(\vec{E} + \vec{v}\times\vec{B})\cdot\nabla_{\vec{v}} f = -\frac{1}{\tau_{MR}} f - \frac{1}{\tau_{MC}} I[f]}$$

Non-uniform current density

Lorentz force

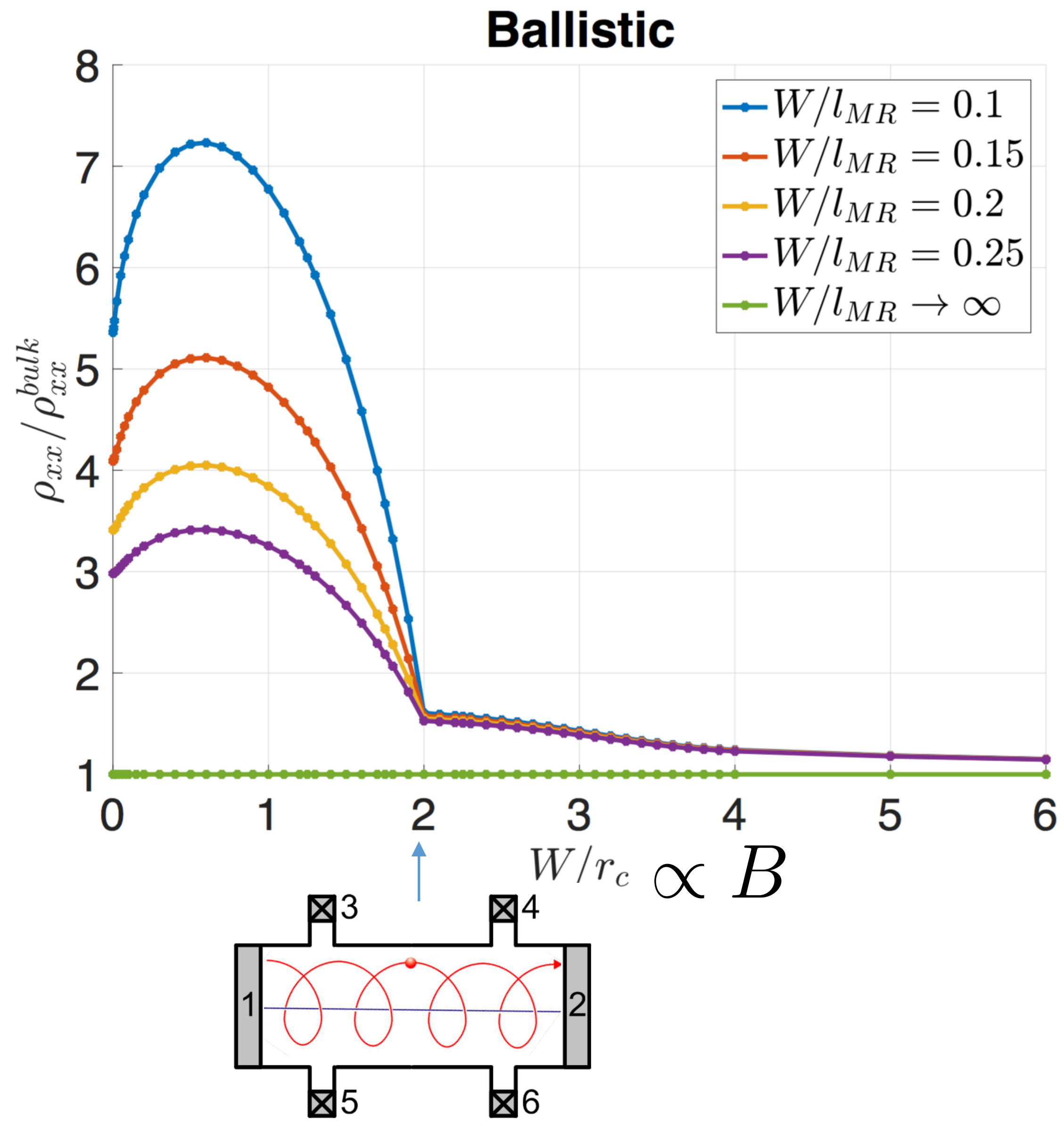
Momentum-relaxing scattering

Momentum-conserving scattering

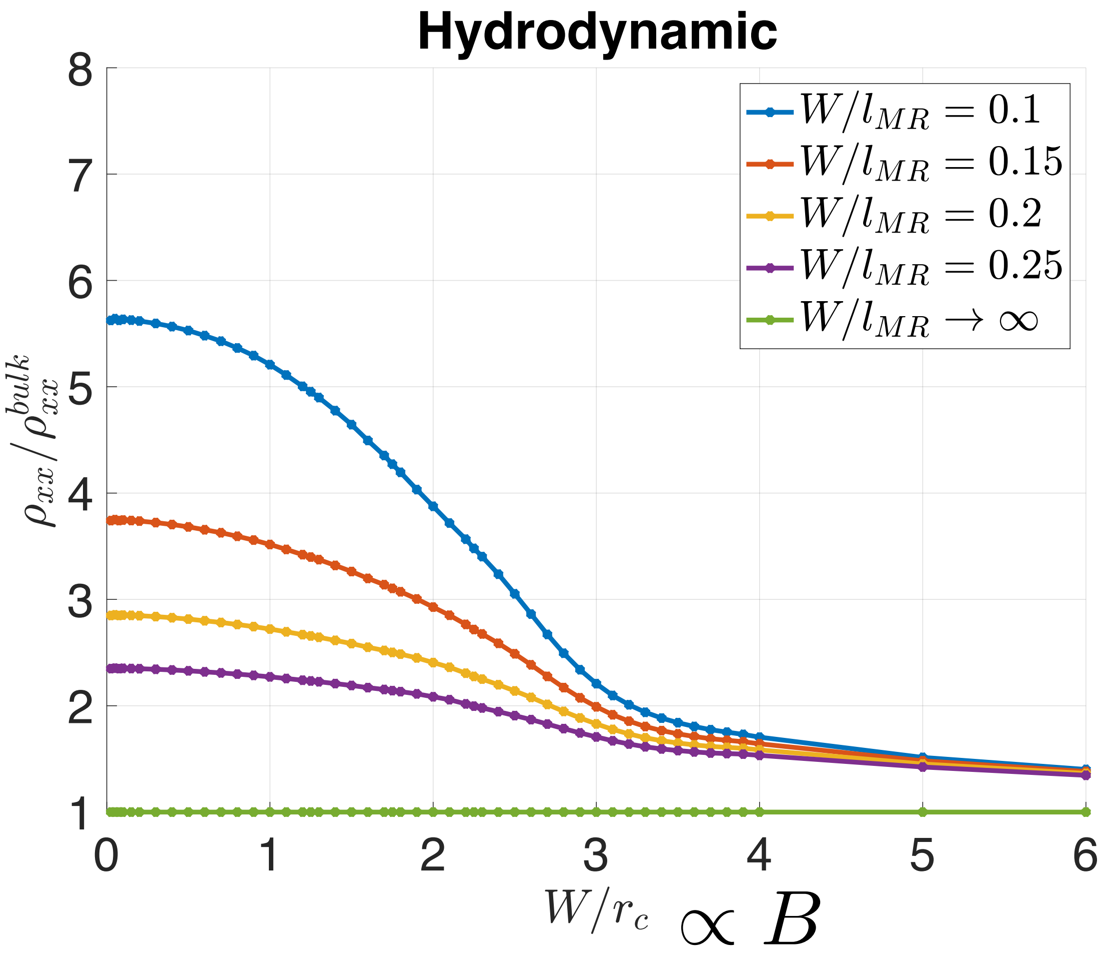
+ boundary condition for diffuse scattering at the boundaries

Results of Boltzmann-hydro: magnetoresistance

$$W \ll l_{MR}, l_{MC}$$



$$l_{MC} \ll W \ll l_{MR}$$

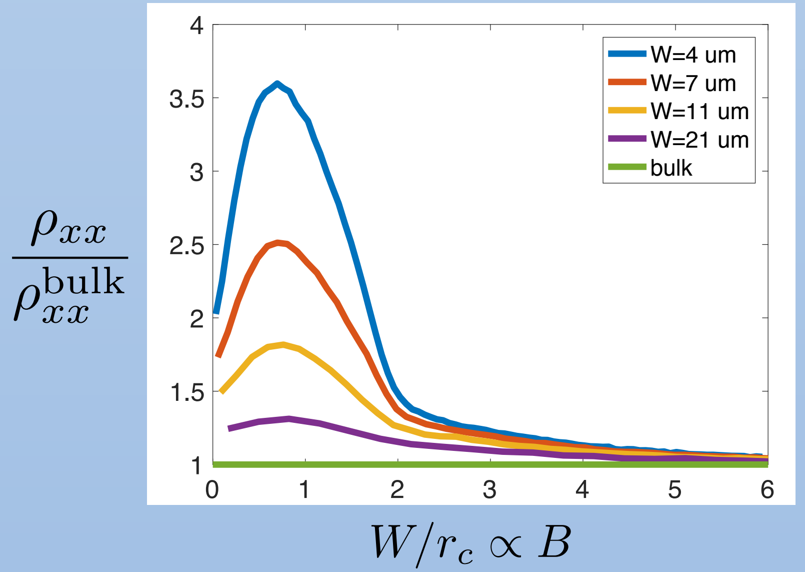


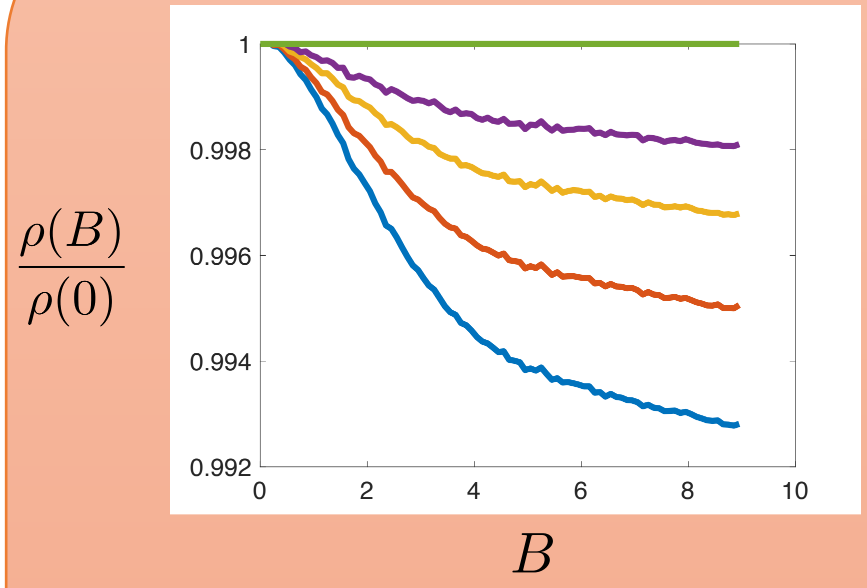
Preliminary results for PdCoO2

T=2 K

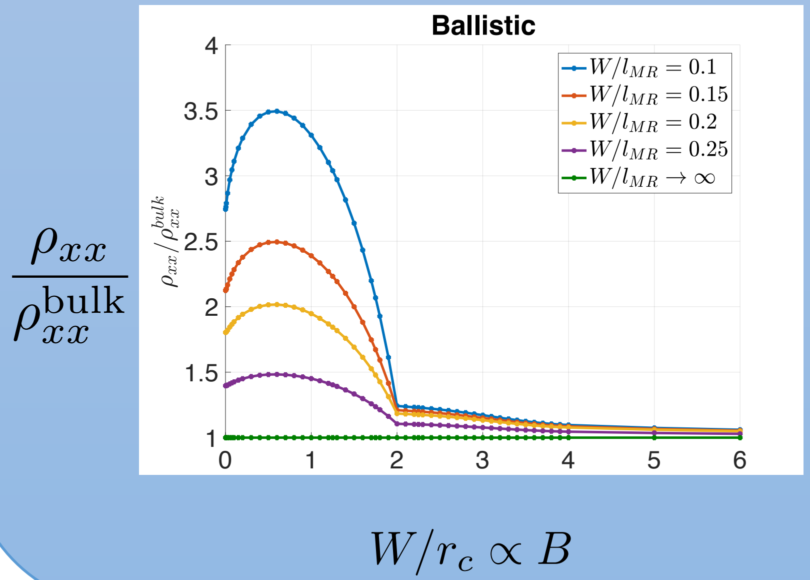
T=100 K

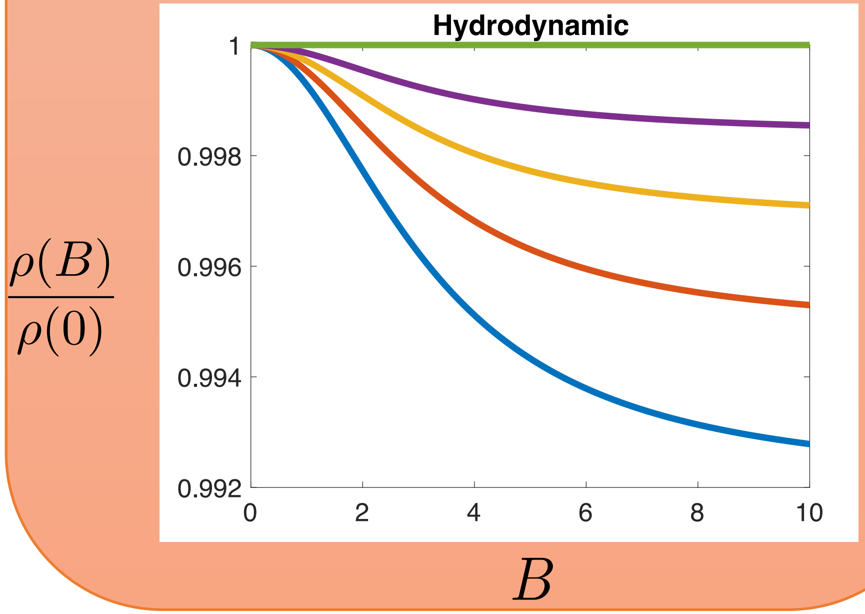
Experiment:





Theory:



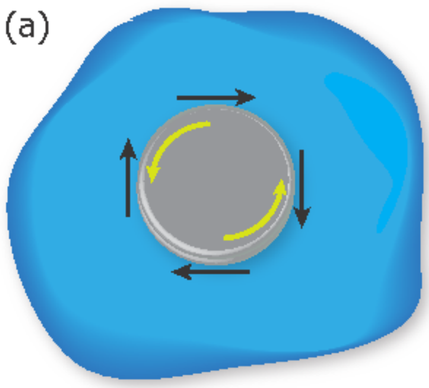


[Nandi, TS, Moore, Mackenzie, to appear (2018)]

A tale of two viscosities

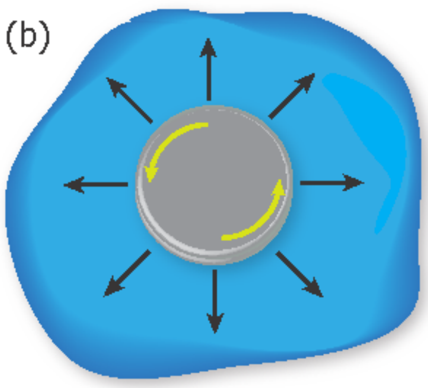
$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} - \frac{1}{\tau_{MR}} \vec{v} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) + \eta_{xy} \nabla^2 \vec{v} \times \hat{z}$$

Shear viscosity



Probed by ρ_{xx}

Hall viscosity

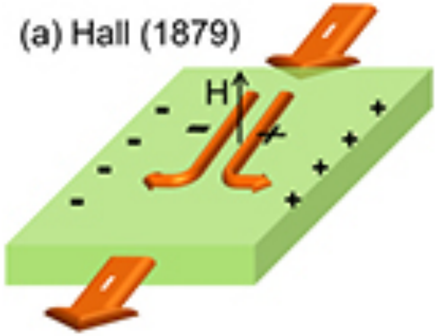


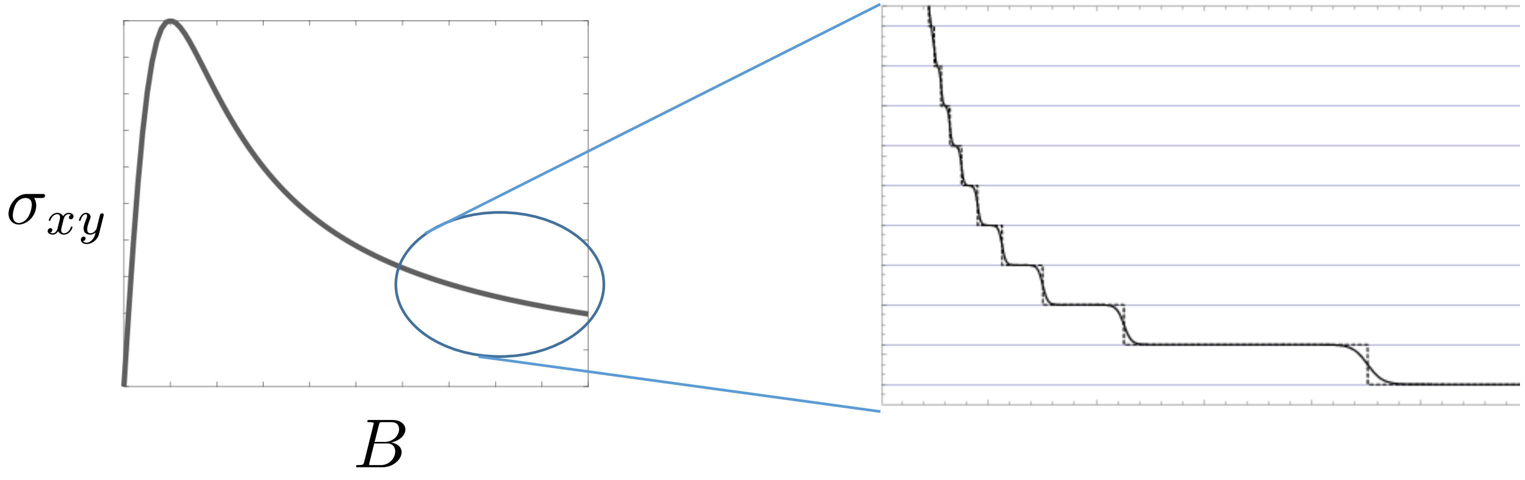
Probed by ρ_{xy}

Historical perspective on Hall effects

Classical Hall Effect [Hall 1879]

Quantum Hall Effect [von Klitzing 1980]

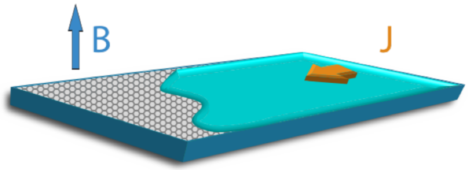


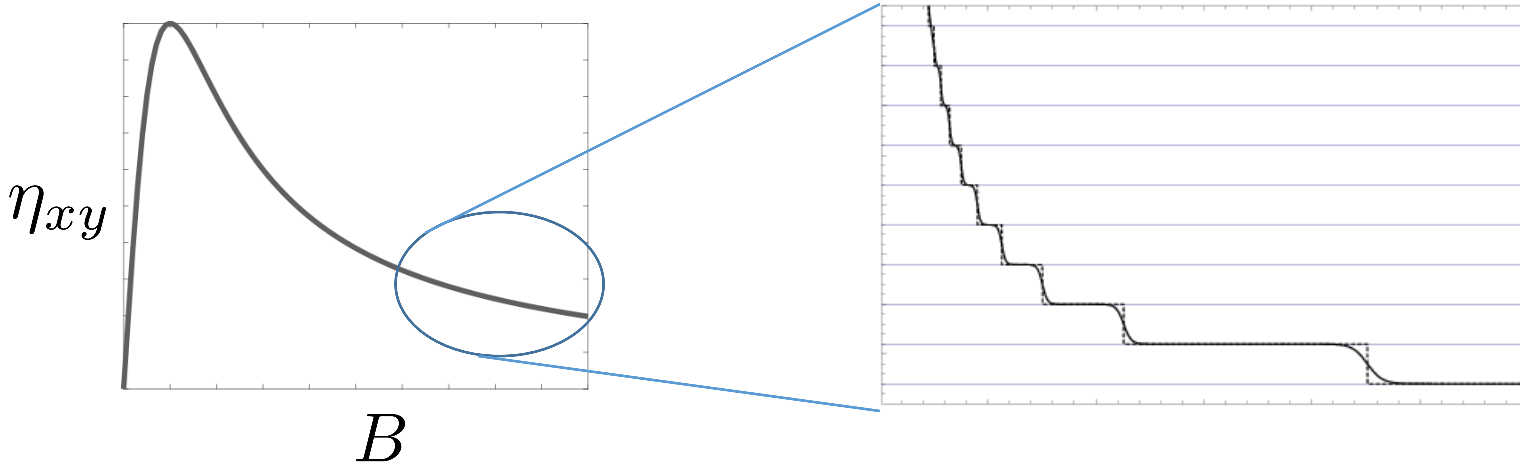


$$\sigma_{xy} = \nu \frac{e^2}{h}$$

Classical Viscous Hall Effect

Quantum Viscous Hall Effect

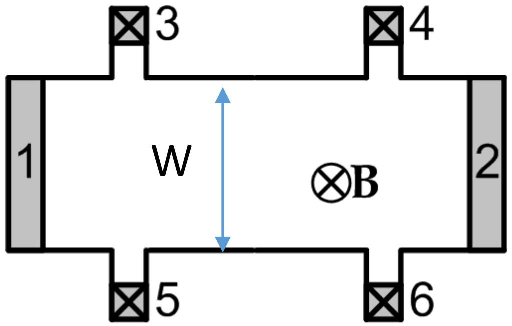




$$\eta_{xy} = \nu n \frac{\hbar}{4}$$

[TS et al, PRL ‘17]

Hall resistivity => Hall viscosity



$$\partial_t \vec{v} = \eta_{xx} \nabla^2 \vec{v} + \eta_{xy} \nabla^2 \vec{v} \times \vec{z} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B})$$

$$\rho_{xy} = \rho_{xy}^{bulk} \left(1 - \eta_{xy} \frac{12}{W^2} \frac{1}{\omega_c}\right)$$

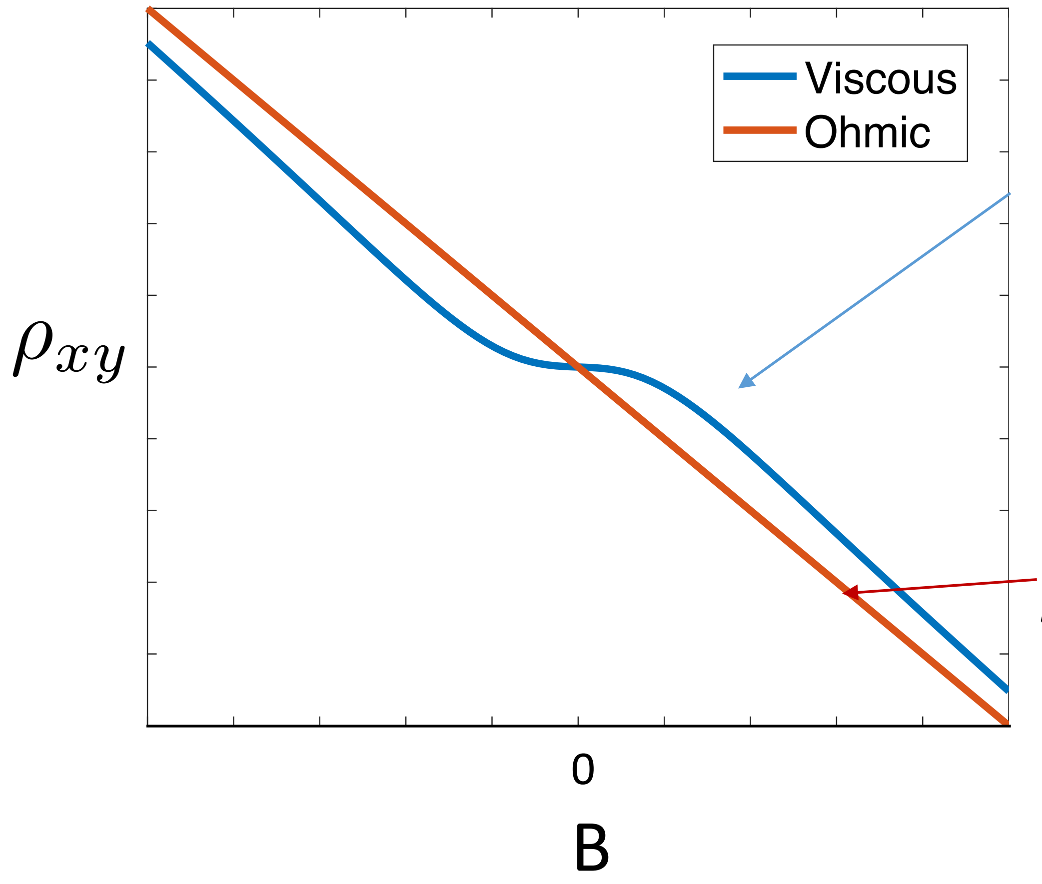
$$\rho_{xy}^{\mathrm{bulk}} = \frac{B}{ne}$$

Hall viscosity could be measured by looking at finite-size effects in Hall resistivity

Nicer probe than ρ_{xx}, because bulk value is “universal”

Hall voltage

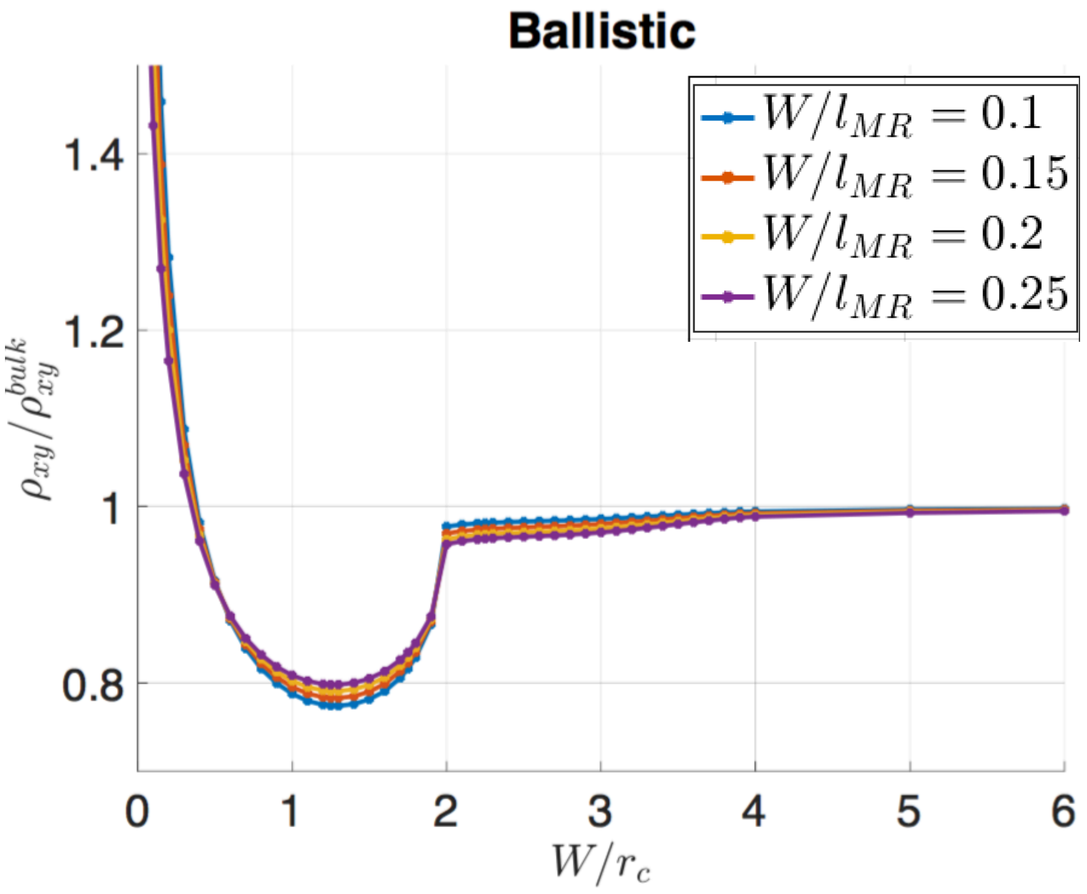
[TS et al, PRL '17]

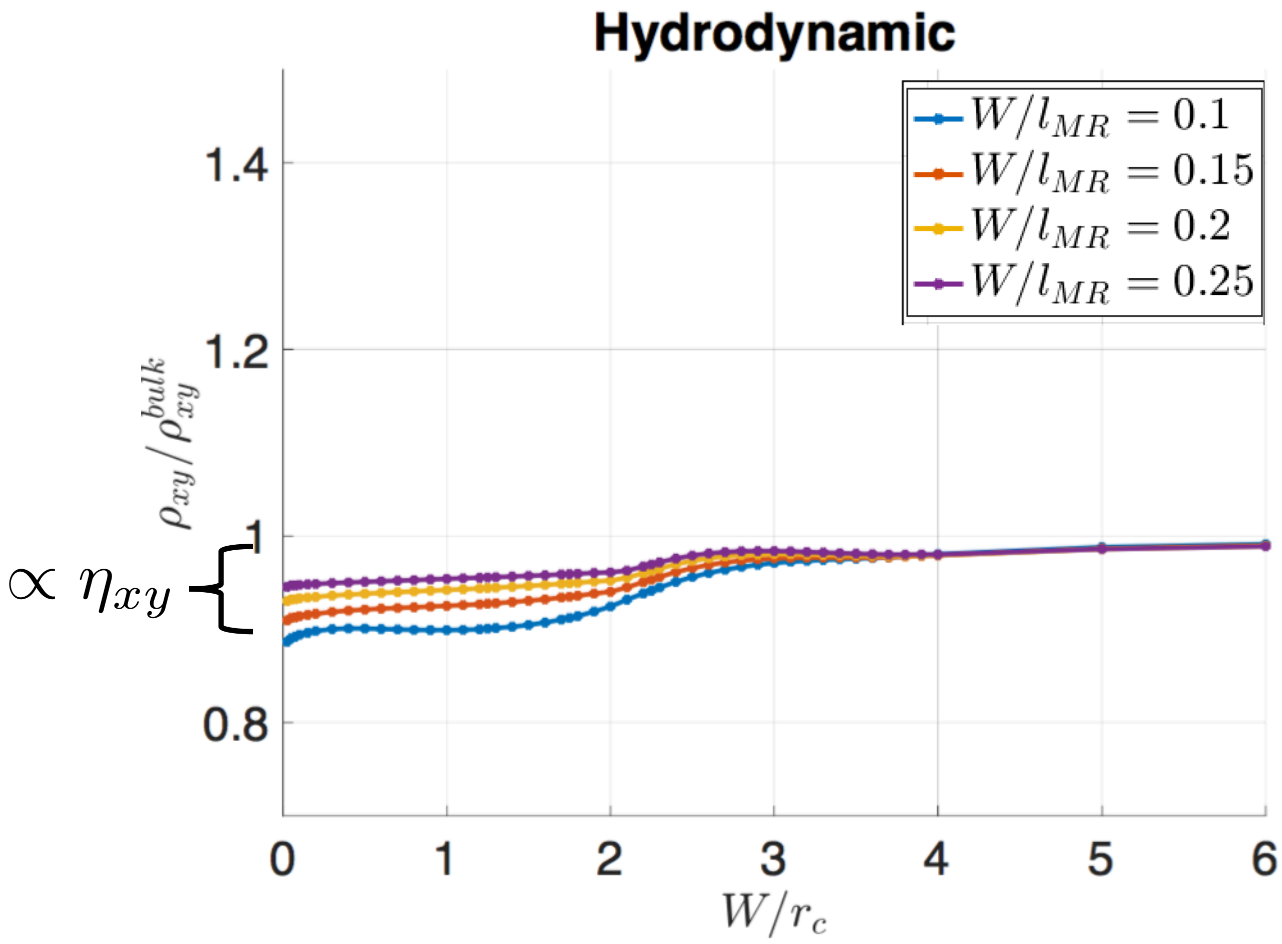


$$\rho_{xy} = \rho_{xy}^{bulk}\left(1 - \eta_{xy}\frac{12}{W^2}\frac{1}{\omega_c}\right)$$

$$\rho_{xy}^{\mathrm{bulk}} = \frac{B}{ne}$$

Results of kinetic theory: Hall resistivity (normalized by bulk value)



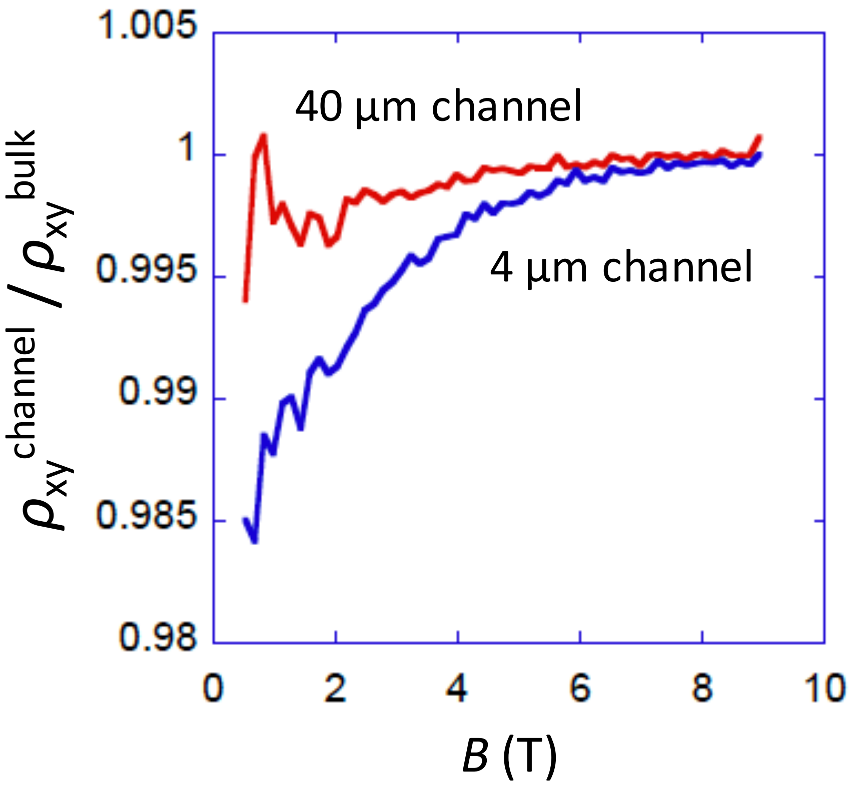


$$\rho_{xy} = \rho_{xy}^{bulk}\left(1 - \eta_{xy}\frac{12}{W^2}\frac{1}{\omega_c}\right)$$

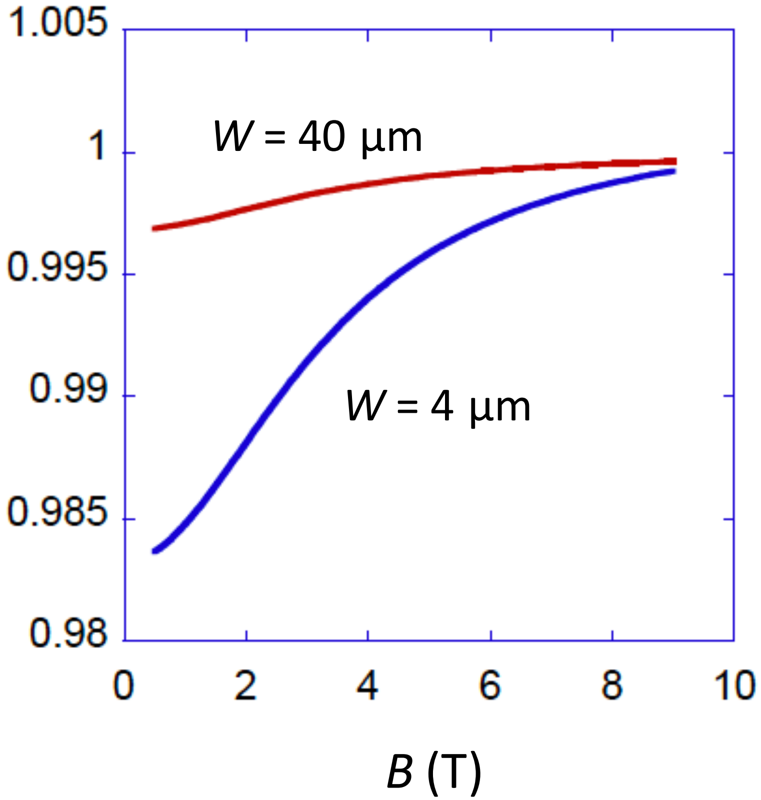
Experimental results

[Nabhanila Nandi, Andrew Mackenzie]

Experiment: $PdCoO_2$ 75 K



Theory



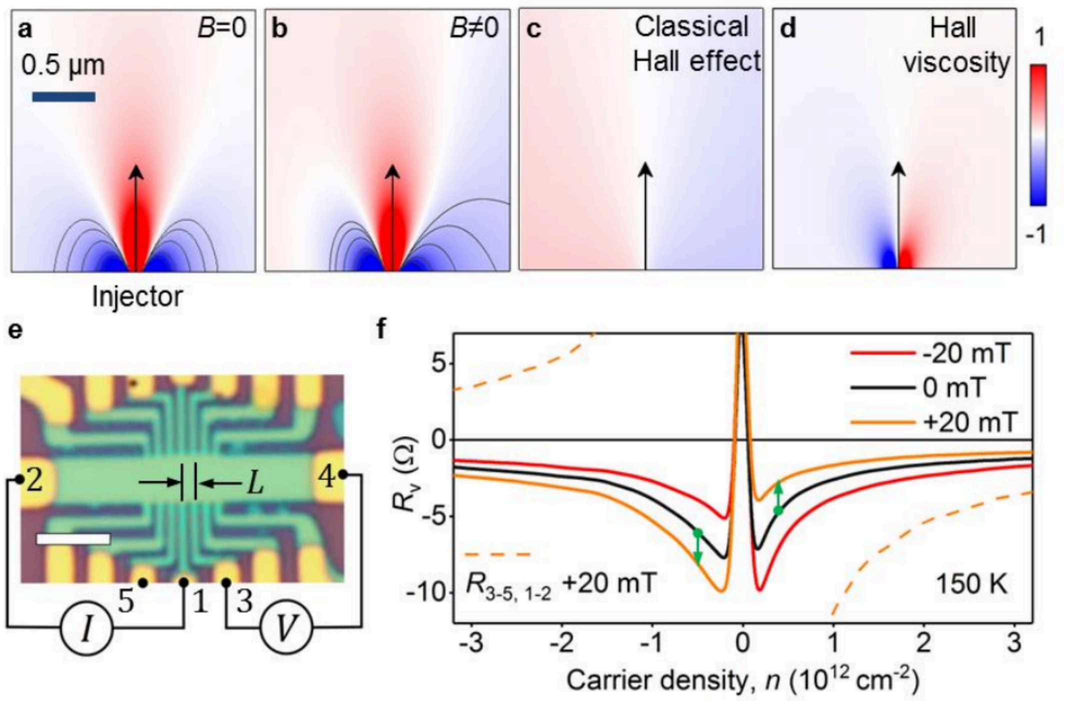
Experimental results: see also

arXiv:1806.01606

Measuring Hall Viscosity of Graphene's Electron Fluid

A. I. Berdyugin, S. G. Xu, F. M. D. Pellegrino, R. Krishna Kumar, A. Principi, I. Torre, M. Ben Shalom, T. Taniguchi, K. Watanabe, I. V. Grigorieva, M. Polini, A. K. Geim, D. A. Bandurin

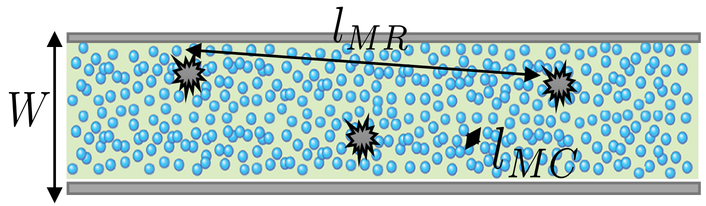
(Submitted on 5 Jun 2018)



Conclusion

- Unlike for many other physical systems where you get it for free, reaching a hydro regime for electrons in a solid is difficult. One is always between a rock and a hard place:

$$l_{MC} \ll W \ll l_{MR}$$



- Size-dependent magnetotransport provides good signatures of hydro

ρ_{xx} $\longrightarrow$ Shear viscosity

ρ_{xy} $\longrightarrow$ Hall viscosity

Acknowledgements

- UC Berkeley: Joel Moore
- MPI Dresden: Nabhanila Nandi, Burkhard Schmidt, Philip Moll, Pallavi Kushwaha, Andrew Mackenzie

Thanks for your attention!