Quantum chaos in an electron-phonon bad metal

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With: Erez Berg, Steve Kivelson.

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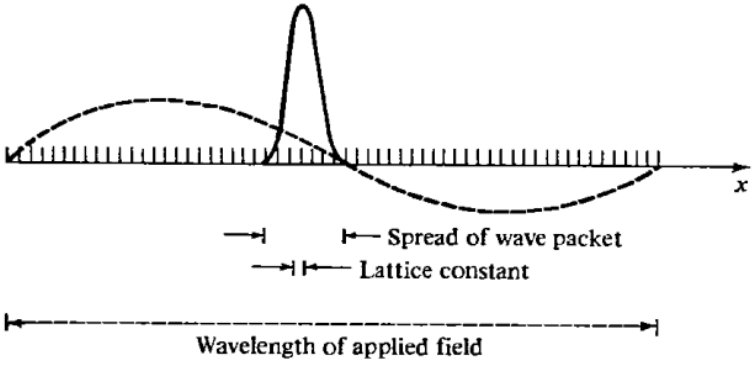
Ioffe-Regel limit

Metallic transport, resistivity saturation, and bad metals.

Quasiparticle description

- **Boltzmann theory:**
  - Electrons as coherent quasiparticles
  - Distribution function $f_p(r)$
  - wavepackets in both r and p.
  - $l_{mfp} \gg \lambda_F$ or $E_F\tau \gg 1$.

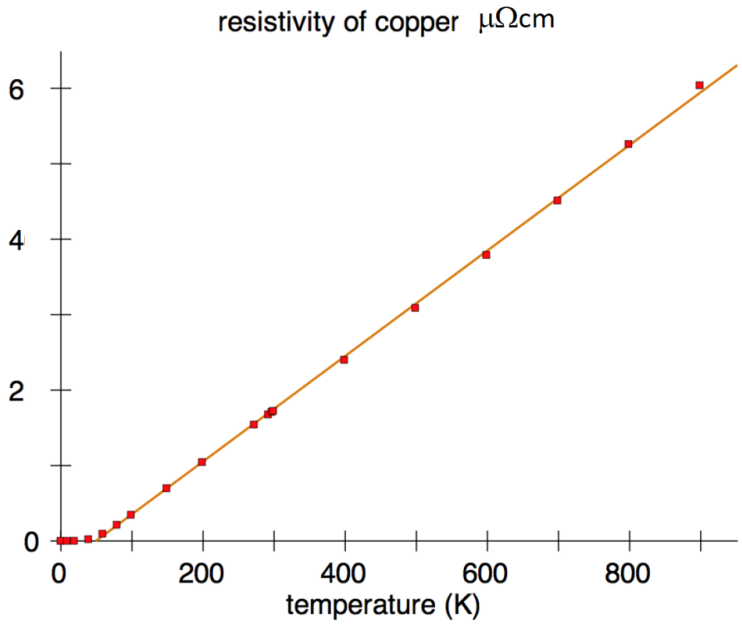


Ashcroft and Mermin

Quasiparticle description

- Drude formula
  - Electrons as coherent quasiparticles.
  - $\sigma_{2D} = \frac{e^2}{h} E_F \tau.$
- Electron – phonon coupling about ω_D
  - $\tau^{-1} \propto T$
  - $\rho \propto T$

Quasiparticle description

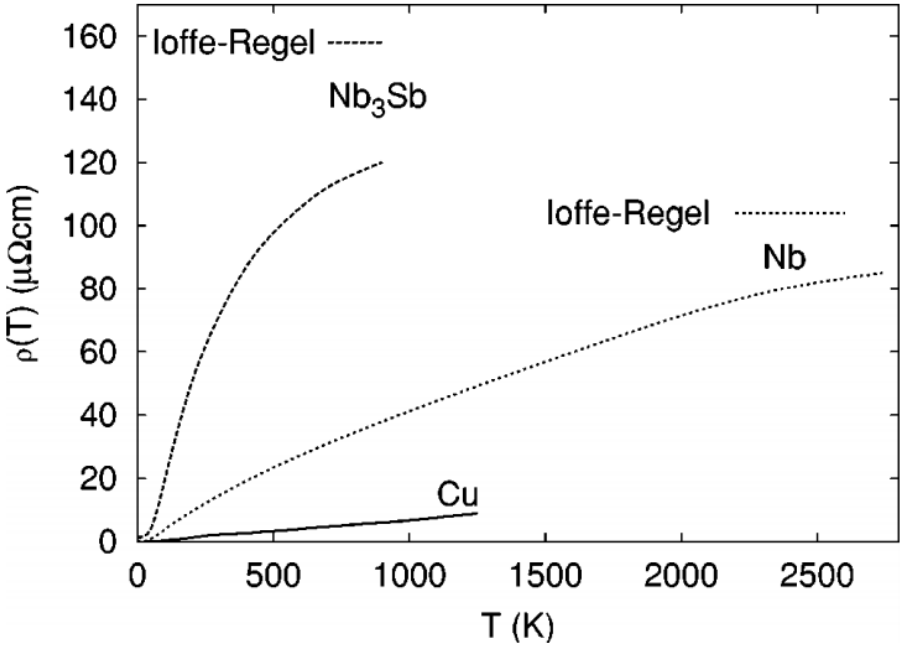


$$\rho \propto T$$

Quasiparticle description

- Key concept: **Ioffe-Regel Limit**
  - Limit on the resistivity in the qp picture:
    - $E_F\tau > 1$ and $\sigma = \frac{e^2}{h} E_F\tau$.
    - $\sigma_{2D} \gtrsim \frac{e^2}{h}$
    - $\rho_{3D} \lesssim 100\mu\Omega cm$

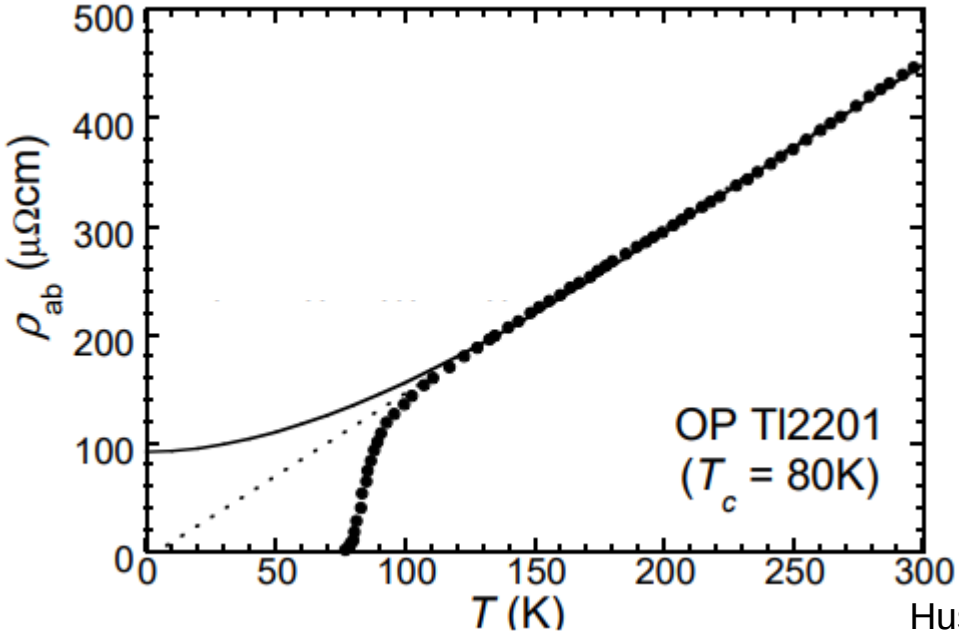
Resistivity saturation



- Resistivity saturation

Gunnarsson et al.,
RMP 75 (2003)

Bad Metals



Hussey, (2001)

- Breakdown of qp picture:
  - Bad metals violate the Ioffe-Regel limit
  - Strong el-el interactions.
  - Linear-T resistivity.

Saturating vs. bad metals

- Saturating metals
  - Obey the MIR limit.
  - Electron – phonon systems.
  - Good BCS superconductors.
- Bad metals
  - Violate the MIR limit.
  - Electron – electron systems.
  - High Tc superconductors.

Outline

- Large-N electron – phonon model
- Transport Coefficients
  - Resistivity – saturation / bad metal.
  - Thermal conductivity.
- Chaos
  - Chaos is linked with thermal transport.

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Electron-phonon model

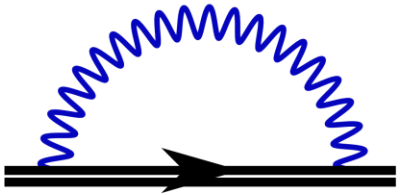
Large N model which displays resistivity saturation or bad metallic behavior.

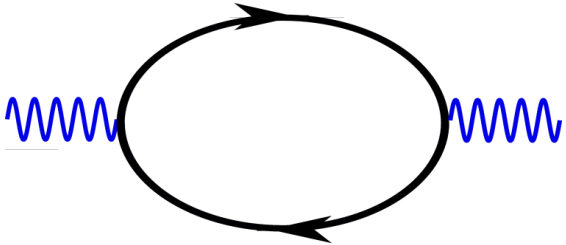
Electron-phonon model

- $N \gg 1$ bands of electrons $c_a(k), a \in [1, N]$.
  - Fermi energy E_F.
  - Bandwidth Λ.
- $N^2 \gg N$ phonon modes $X_{ab}(k), a, b \in [1, N]$.
  - Dispersionless optical phonons.
  - Einstein frequency ω_0.

Single particle properties

- N electron bands, $N^2 \gg N$ phonon modes:
  - Strongly renormalized electrons
    - $\Sigma'' \propto O(1)$.
  - Weakly renormalized phonons
    - $\Pi'' \propto \frac{1}{N}$.





Electron-phonon model

- Motivation for large N:
  - Realistic (?)
    - E.g. A_3C_{60} has $N_{ph} = 189$, $N_{el} = 3$.
  - Strong coupling in controlled manner.
  - Suppresses lattice instabilities.

Electron-phonon model

- Electrons degenerate:
  - $E_F \gg T$.
- Phonons in the classical limit:
  - $\omega_0 \ll T$.
- Strong coupling:
  - $\lambda = \frac{\alpha^2 \nu}{K} \gg 1$ – strong coupling.
  - λT may be larger than E_F, Λ.

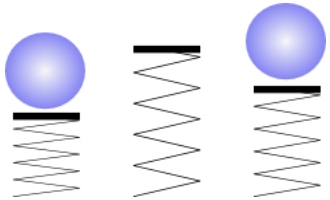
Single particle properties

- Strongly renormalized electrons
  - $\Sigma'' = \sqrt{\lambda T \times E_F} > E_F$ for $T > E_F/\lambda$
- Weakly renormalized phonons
  - $\Pi'' = \frac{1}{N}\frac{\omega_0^2}{T}$ for $T > E_F/\lambda$

Electron-phonon model

On-site coupling ("Holstein")

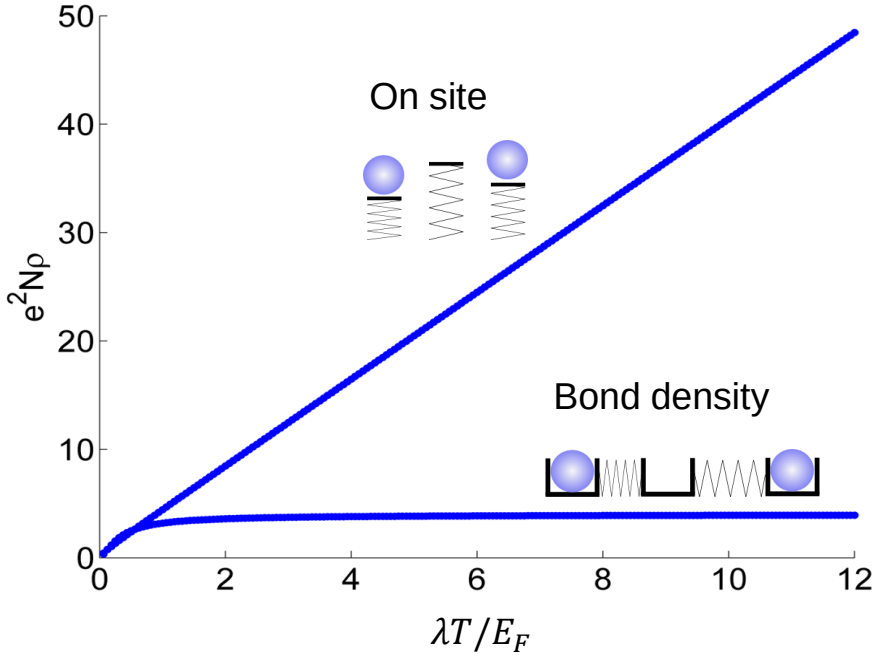
$$H_{int} = \frac{\alpha}{\sqrt{N}} \sum_{i,ab} X_{i,ab} c_{i,a}^{\dagger} c_{i,b}$$



Bond-density coupling ("SSH")

$$H_{int} = \frac{\alpha}{\sqrt{N}} \sum_{i,ab} X_{i,ab} c_{i,a}^{\dagger} c_{i+1,b}$$

Transport: resistivity

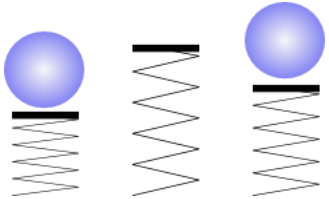


- Resistivity depends on coupling.
- Saturation of resistivity in SSH model due to phonon-assisted channel.

Current operator

On-site coupling ("Holstein")

$$J = \int d^d k \, v_{\boldsymbol{k}} c_{\boldsymbol{k}}^{\dagger} c_{\boldsymbol{k}} \equiv J_0$$



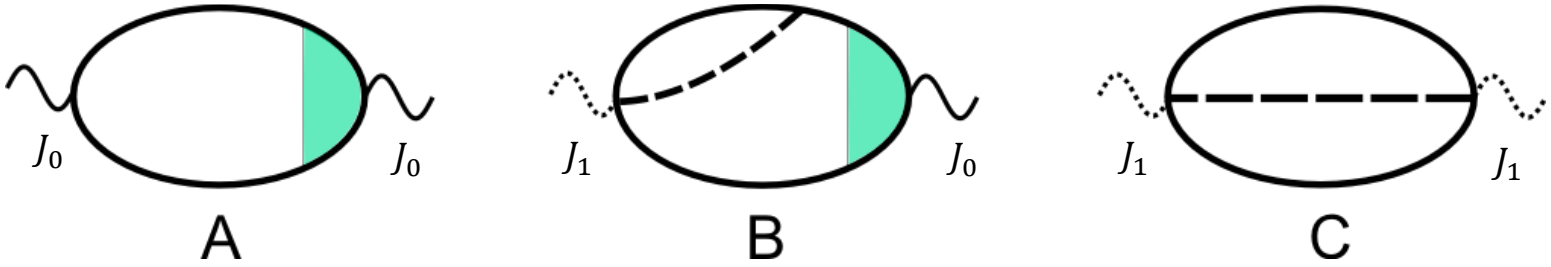
Bond-density coupling ("SSH")

$$J = J_0 + i \frac{\alpha}{\sqrt{N}} \sum_i X_i c_{i+1}{}^{\dagger} c_i$$

$$\equiv J_0 + J_1$$

Conductivity

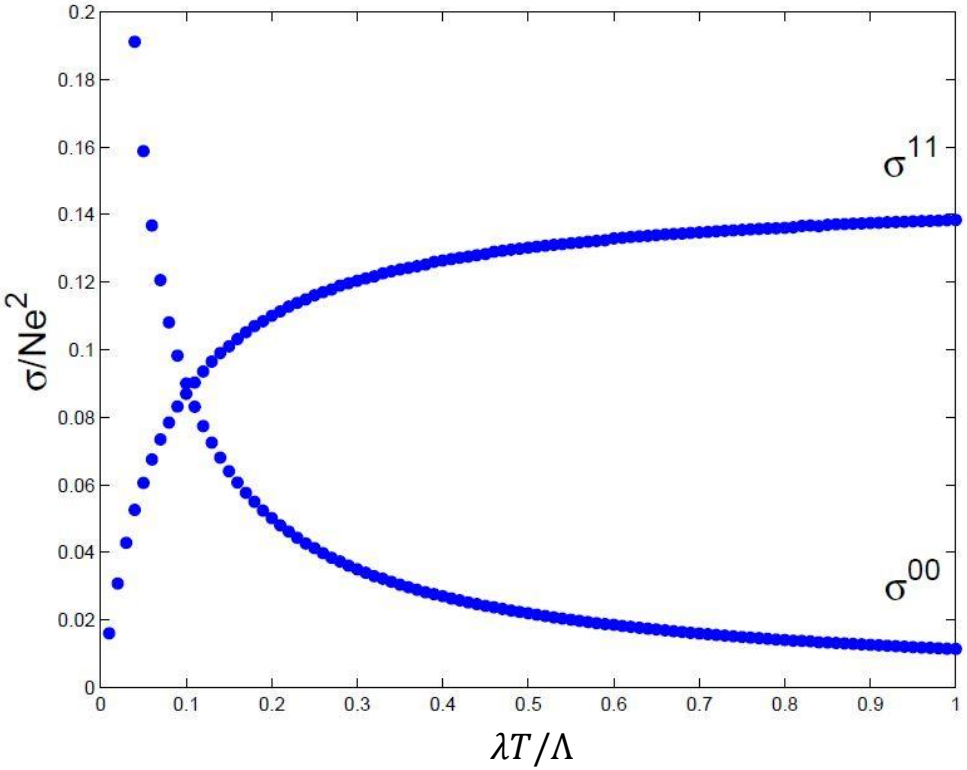
- Kubo formula: $\sigma = \lim_{\omega \to 0} \frac{\Im \Pi^{JJ}(\omega)}{\omega}$



$$J_0 = i \sum_i \left[{c_{i+1}}^\dagger \, c_i - {c_i}^\dagger \, c_{i+1} \right]$$

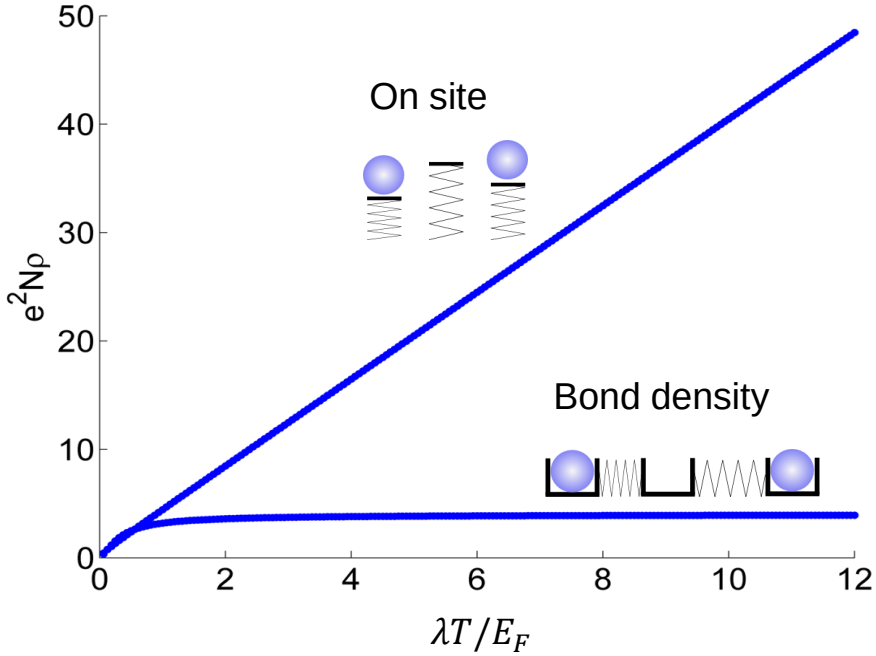
$$J_1 = i \frac{\lambda}{\sqrt{N}} \sum_i X_i \left[{c_{i+1}}^\dagger \, c_i - {c_i}^\dagger \, c_{i+1} \right]$$

Conductivity



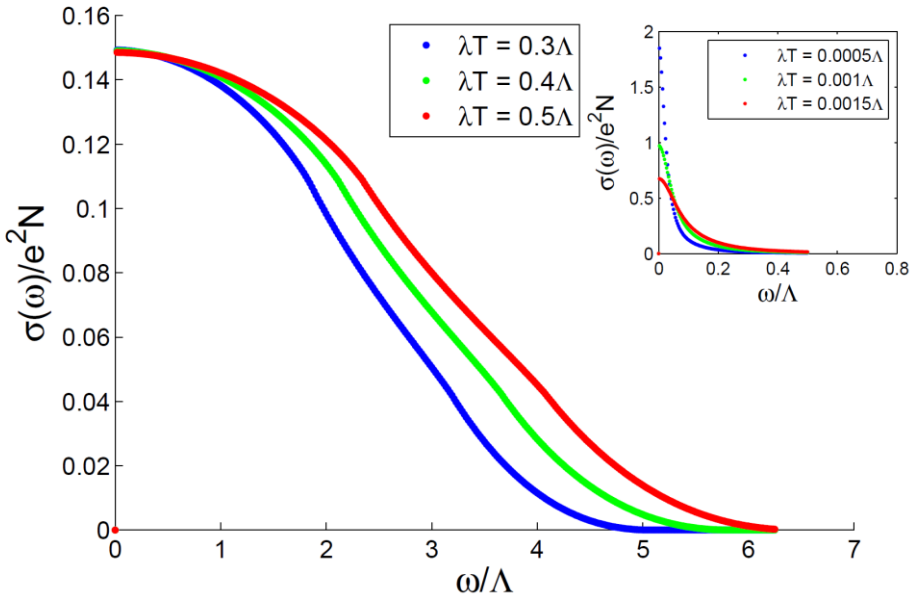
- 00 channel decays as $\frac{1}{T}$.
- 11 channel saturates to $\sigma_0 \sim \sigma_{MIR}$.
- 01 channel is negligible.

Transport: resistivity



- Resistivity depends on coupling.
- Saturation of resistivity in SSH model due to phonon-assisted channel.

Transport: optical conductivity



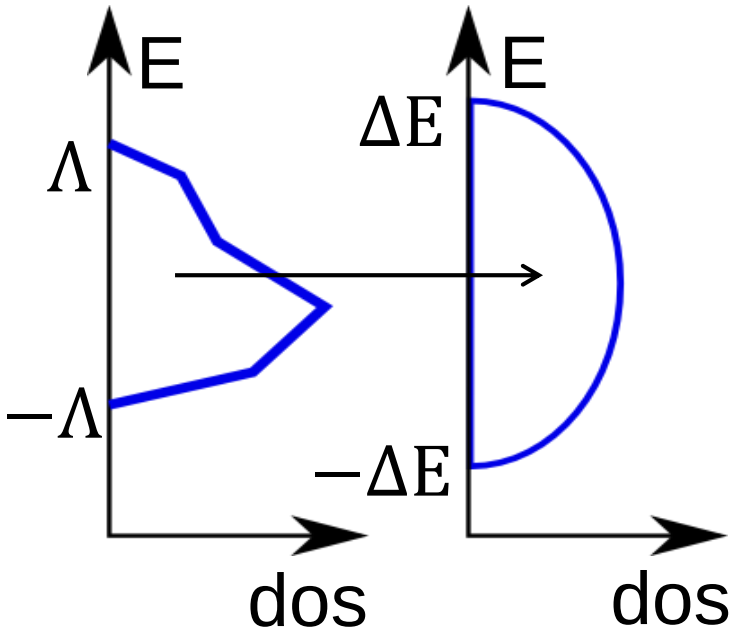
- Optical conductivity for bond density coupling.
- At high T, the relevant energy scale is $\Delta E = \sqrt{\lambda T \times E_F}$.

Electron-phonon model

- Interaction term dominant at high T:

$$\mathrm{H} = \frac{\alpha}{\sqrt{N}} \sum_{ab=1}^{N} X_{ab} c_a^{\dagger} c_b$$

- $\left\langle X_{\alpha\beta}^2 \right\rangle \approx T/K.$
- Phonons act as a random matrix in ab space.



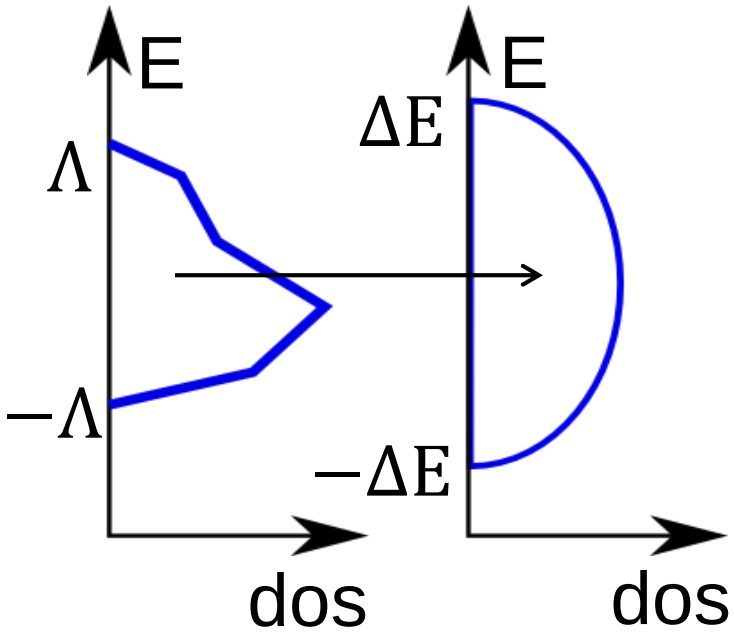
Electron-phonon model

- Interaction term dominant at high T:

$$\mathrm{H} = \frac{\alpha}{\sqrt{N}} \sum_{\alpha\beta=1}^{N} X_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta}$$

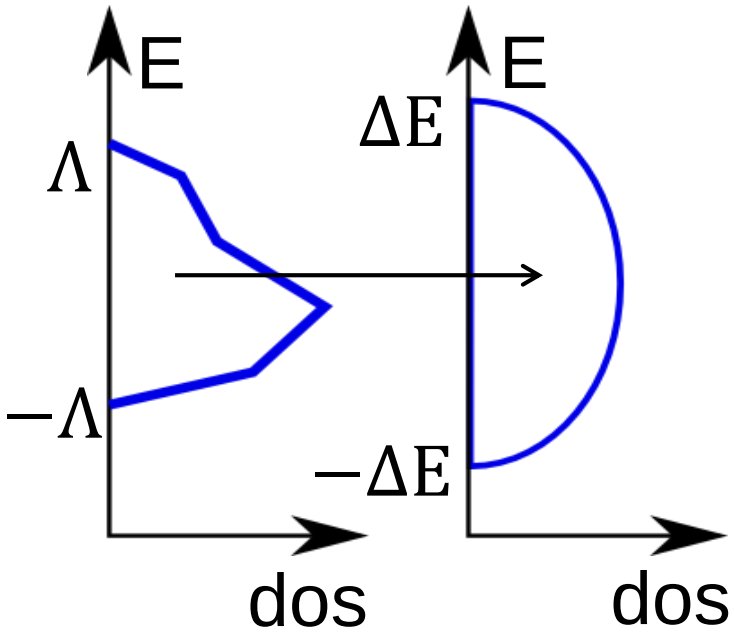
- Random matrix theory:

$$\Delta E_{eff} = \sqrt{\lambda T \times E_F}$$



Electron-phonon model

- Effective bandwith
  - $W \sim \sqrt{\lambda T/\nu}$
- Electron decay rate
  - $\Sigma'' \sim \sqrt{\lambda T/\nu}$
- Compressibility:
  - $\chi \sim 1/\sqrt{\lambda T/\nu}$

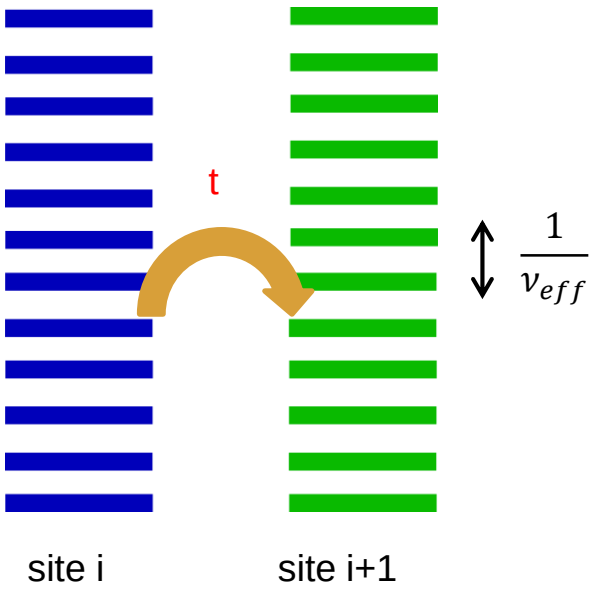


Electron-phonon model

- Incoherent transport
- Diffusion constant given by

  Fermi's Golden rule:

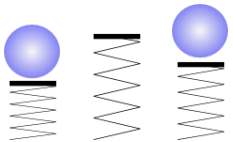
$$D \sim t_{hop}^2 \nu_{eff} \sim t_{hop}^2 / \sqrt{\lambda T \times E_F}$$



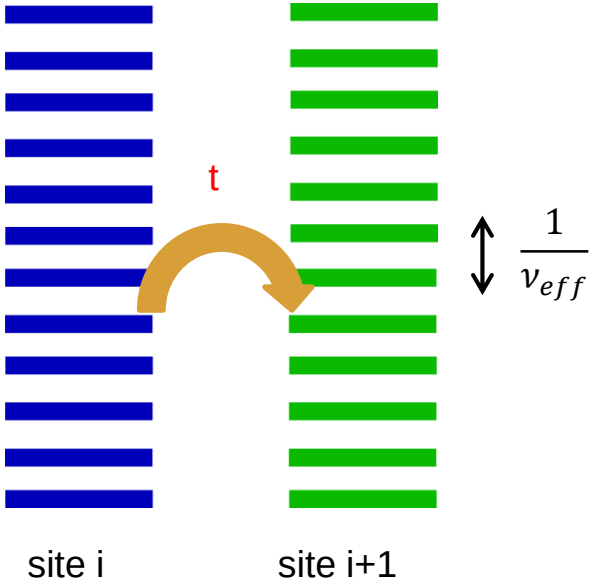
Electron-phonon model

$$D \sim t_{hop}^2 \nu_{eff} \sim t_{hop}^2 / \sqrt{\lambda T \times E_F}$$

- On site model: $t_{hop} = t \rightarrow D \propto 1/\sqrt{T}$



- Bond density model: $t_{hop} \propto \sqrt{T} \rightarrow D \propto \sqrt{T}$



Electron-phonon model

- Einstein relation:
  - $\sigma = \chi D$
- Resistivity:
  - Site density: $\rho \sim T$
  - Bond density: $\rho \sim const.$
- Temperature dependence originates from both χ and D.

Thermal conductivity

- Thermal conductivity is dominated by the phonons:
  - There are N^2 phonons and only N electrons.
  - The phonons are long lived, $\Pi'' \sim \frac{1}{N}$.

Thermal conductivity

- If the phonons are dispersive, Boltzmann theory gives

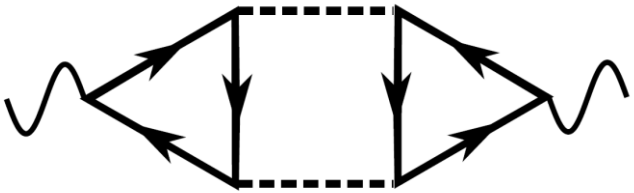
$$\kappa = N^2 \frac{\mathrm{v}_{\mathrm{ph}}^2}{\Pi''} = \mathrm{N}^3 \mathrm{v}_{\mathrm{ph}}^2 \frac{\mathrm{T}}{\omega_0^2}$$

Thermal conductivity

- Non-dispersive phonons:
  - The el-ph interaction gives a correction to phonon velocity:

$$\tilde{v}_{ph} = \frac{1}{N}\frac{\omega_0}{T}\frac{E_F}{\Sigma''}v_{el}$$

- we use the Kubo formula to find:



$$\kappa = N^2\frac{\tilde{v}_{\mathrm{ph}}^2}{\Pi''}$$

Electron-phonon model - summary

- N electrons, N^2 phonons.
- Decay rate:
  - $\Sigma'' = \sqrt{\lambda T \times E_F}$
  - $\Pi'' = \frac{1}{N}\frac{\omega_0^2}{T}$
- Resistivity:
  - Site density: $\rho \sim T$ above the MIR limit
  - Bond density: $\rho \sim const.$
- Phonons responsible for κ.

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Chaos

Scrambling rate and butterfly velocity in the el-ph model.

Quantum chaos

$$\langle |[V(x,t), W(0)]|^2 \rangle \propto e^{\lambda_L \left(t - \frac{|x|}{v_B}\right)}$$

- V, W – generic local observables.
- λ_L – rate at which local information is scrambled.
- v_B – velocity of the effects of a local perturbation.

Quantum chaos

$$\left\langle |[V(x,t), W(0)]|^2 \right\rangle \propto e^{\lambda_L\left(t - \frac{|x|}{v_B}\right)}$$

- Bound on λ_L: (Maldacena et al., JHEP, 2016)

$$\lambda_L \leq 2\pi T/\hbar$$

Quantum chaos

- Non quasiparticle systems are expected to saturate the bound.
  - SYK model.
- Systems with quasiparticles have parametrically smaller λ_L:
  - Fermi liquids: $\lambda_L \propto T^2$
  - Weakly coupled large-N systems: $\lambda_L \propto 1/N$. (Chowdhury and Swingle, 2017; Julia's talk yesterday)

Quantum chaos and transport

- The quantities of chaos define a diffusion constant

$$D_L = \frac{v_B^2}{\lambda_L}$$

- Question: in a generic system, can we bound

$$D \geq D_L > \frac{v_B^2}{T}?$$

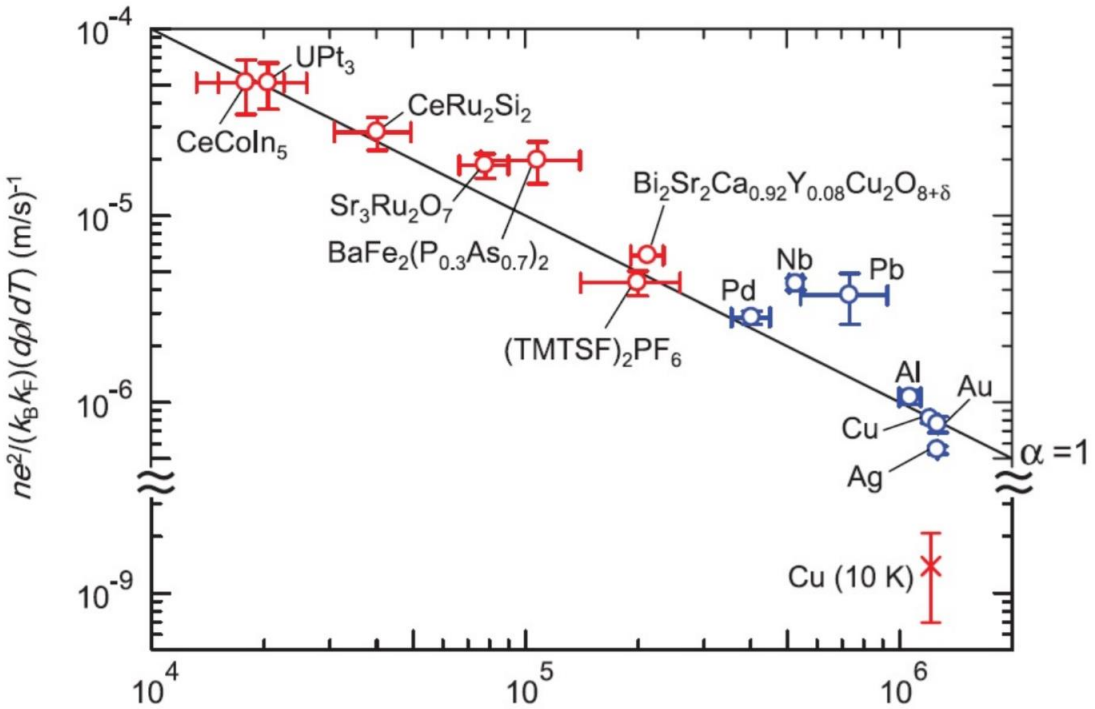
(Hartnoll, Nature 2015; Blake, PRL, 2016)

Quantum chaos and bad metals

This conjecture is pertinent to bad metals:

- Non-quasiparticle transport
- Universality in bad metals:
  - Many display $\rho(T) \propto T$ over large range of temperatures.
  - Many display $\tau^{-1} = T$ – dissipative timescale. (Bruin et al., Science)

Quantum chaos and bad metals



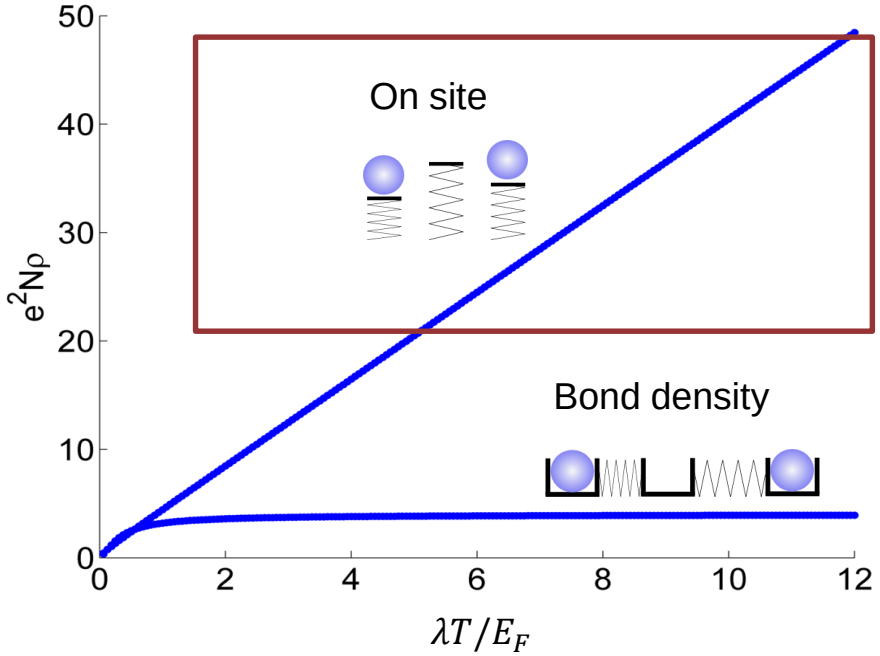
Bruin et al., Science

Quantum chaos and bad metals

We ask:

- Is quantum chaos related to the transport coefficients in bad metals?
- does the bound on the scrambling rate lead to a universal bound on transport in these materials?
- Is a strongly incoherent metal necessarily also strongly chaotic?

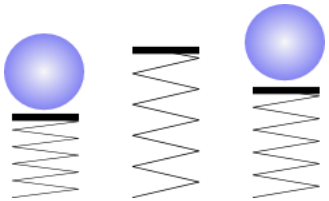
Quantum chaos and bad metals



- Solvable model for a high-T bad metal.

Chaos in el-ph model

- We calculate
  - The scrambling rate λ_L
  - The butterfly velocity v_B
- For
  - Dispersive phonons
  - Non-dispersive phonons.



- In the bad metallic regime of the on-site model.

Chaos in el-ph model

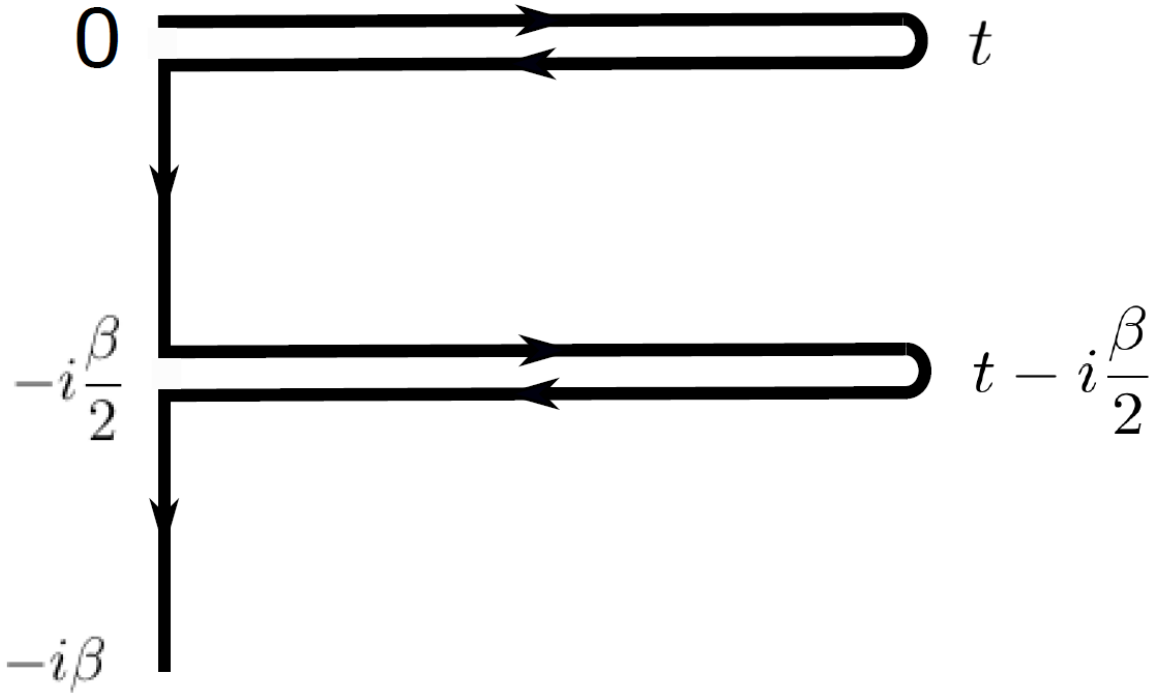
- We find that
  - The scrambling rate is the same for phonons and electrons.
  - Chaos is carried by the phonons.
  - Chaos is weak ($\lambda_L \ll T$).
  - Chaos is related to thermal conductivity.

Chaos in el-ph model

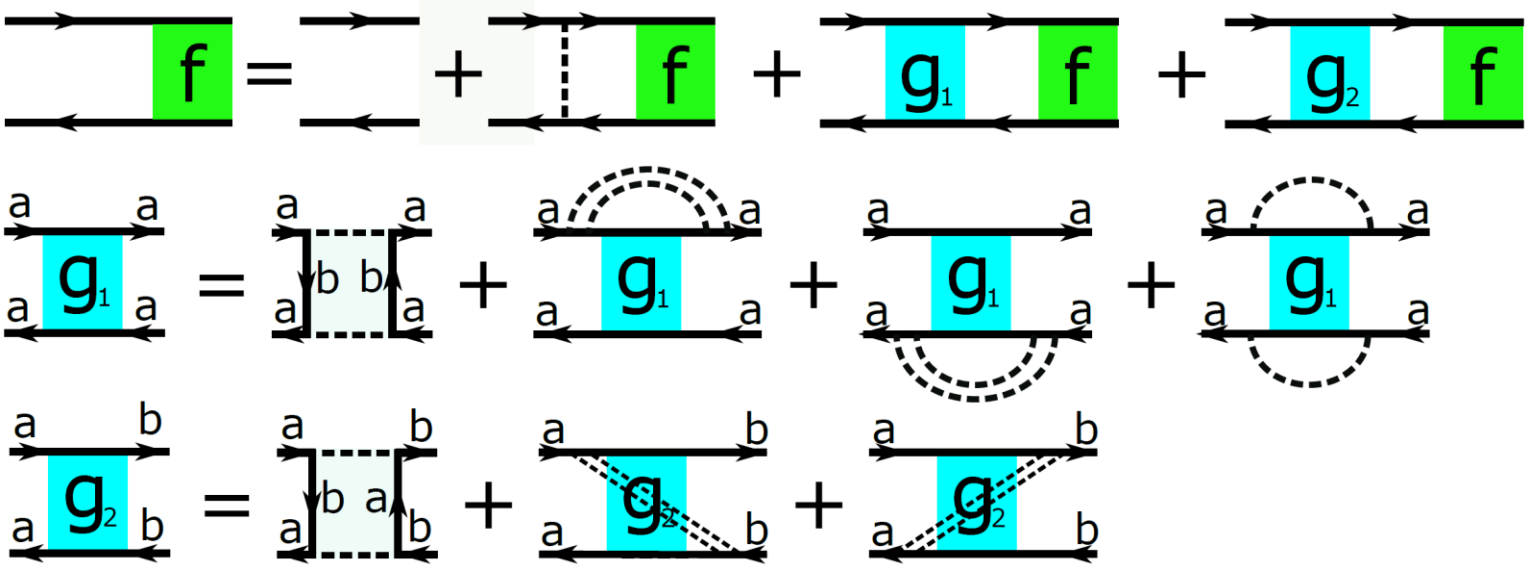
- We calculate the Fourier transform of

$$\tilde{f}(x,t) = \frac{\theta(t)}{N^2} \times \sum_{ab} \mathrm{Tr}\left[\sqrt{\rho}\{c_a(x,t), c_b^\dagger(0,0)\}\sqrt{\rho}\{c_a(x,t), c_b^\dagger(0,0)\}^\dagger\right].$$

Quantum chaos



Quantum chaos



Chaos in el-ph model

We find that in the bad metallic regime

- $\lambda_L = \Pi'' = \frac{1}{N}\frac{\omega_0^2}{T}$ - the phonon decay rate
- $v_B =$
  - v_{ph} for dispersive phonons
  - $\tilde{v}_{ph}$ for non-dispersive phonons

Chaos in el-ph model

We find that in the bad metallic regime

- $\lambda_L = \Pi'' = \frac{1}{N}\frac{\omega_0^2}{T}$ - the phonon decay rate
- Phonons are the longest lived excitations
  - Bottleneck for scrambling.

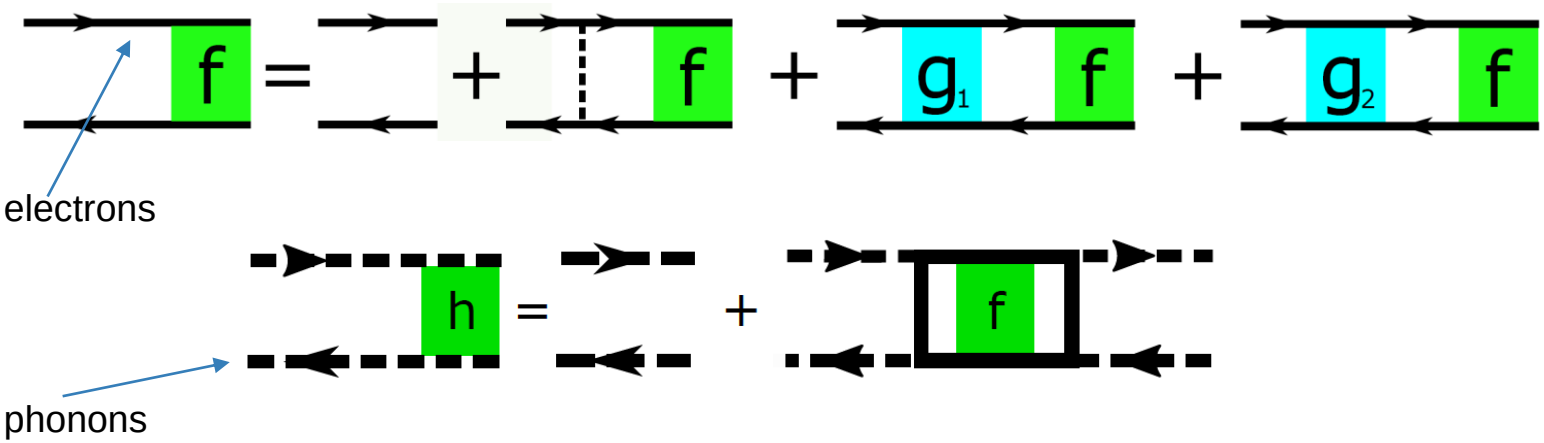
Chaos in el-ph model

We find that in the bad metallic regime

- $D_L = v_B^2/\lambda_L = D_E$ Thermal diffusion constant
- D_L Charge diffusion constant
  - $> D_C$ for dispersive phonons
  - $< D_C$ for non-dispersive phonons

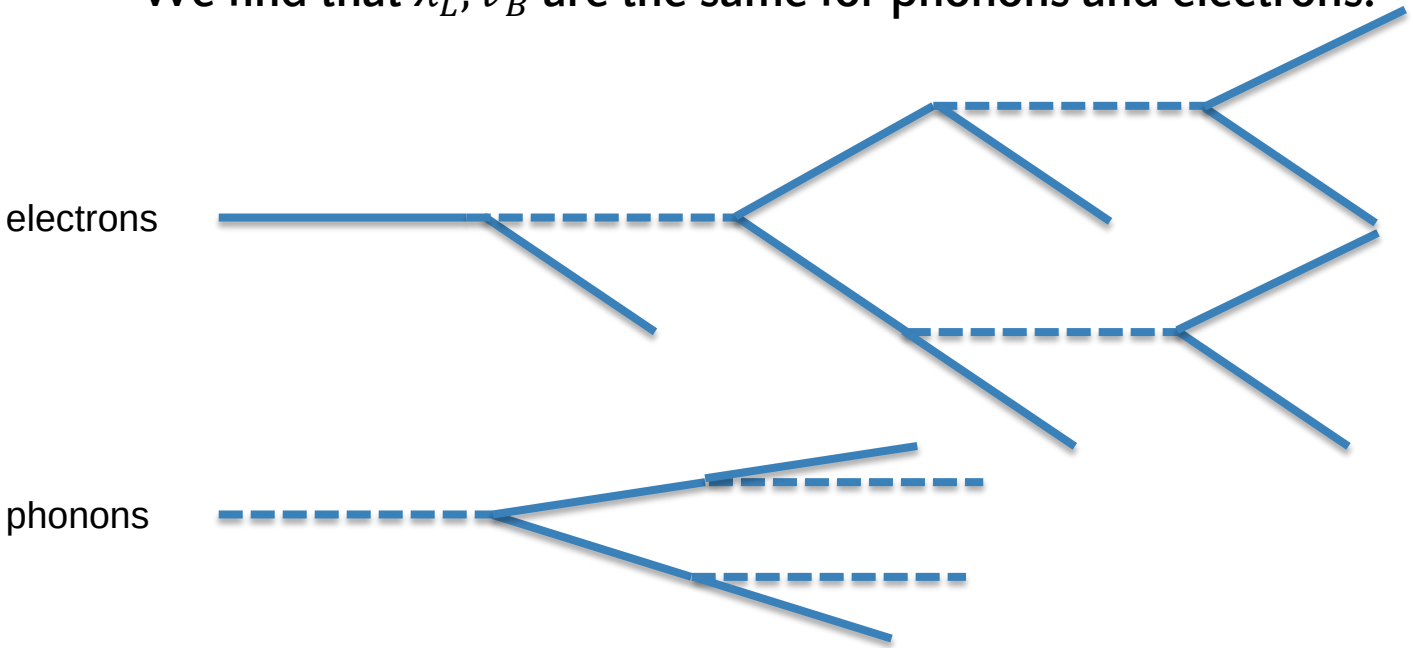
Chaos in el-ph model

We find that λ_L, v_B are the same for phonons and electrons.

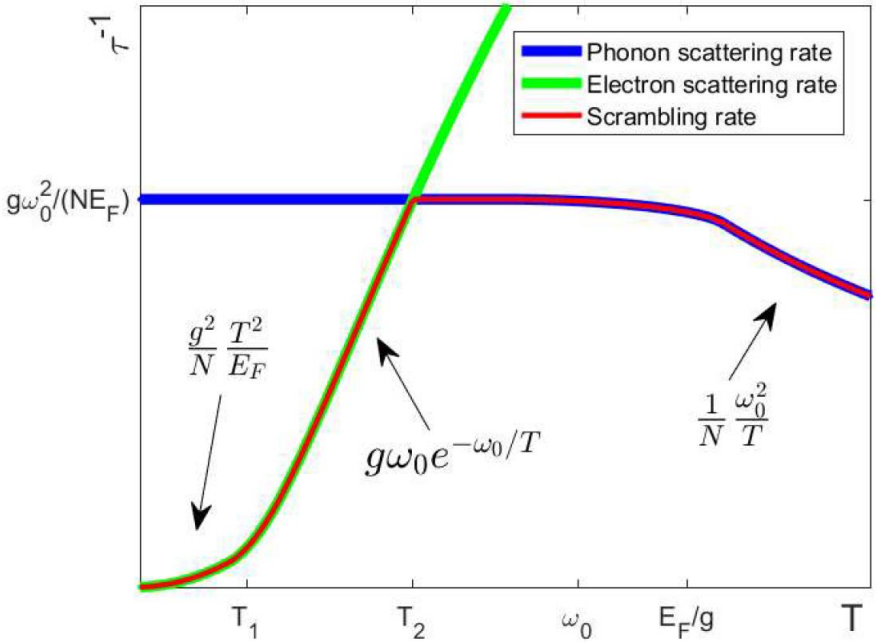


Chaos in el-ph model

We find that λ_L, v_B are the same for phonons and electrons.



Chaos in el-ph model



$$\lambda_L = \min(\Sigma'', \Pi'')$$

Conclusions

- Charge transport and chaos are not connected.
  - In the limit $\omega_0 \to 0$, phonons act as disorder.
  - $\lambda_L \propto \omega_0^2 \to 0$.
  - ρ is finite as $\omega_0 \to 0$.
- There is no bound on charge transport.
- Bad metals are not necessarily strongly chaotic.

Conclusions

- Thermal transport and chaos are connected.
- Both are carried by the phonons.

Conclusions

- Chaos is not related to the decay of any physical quantity
  - Single particle Green's function.
  - Current operator.
- Chaos is related to thermalization
  - Slowest decay rate in the system.

Thanks!

Any questions ?