Memory matrix approach to DC resistivity near an Ising-nematic quantum critical point

Xiaoyu Wang

University of Chicago

Xiaoyu Wang and Erez Berg, in preparation

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Quantum critical metal

- Describes the phenomenon where the critical fluctuations associated with a quantum critical point is coupled to itinerant electrons (i.e., a Fermi surface)
- Examples include antiferromagnetism, nematicity, charge order …
- Commonly observed in high Tc superconductors (cuprates and iron pnictides/chalcogenides)

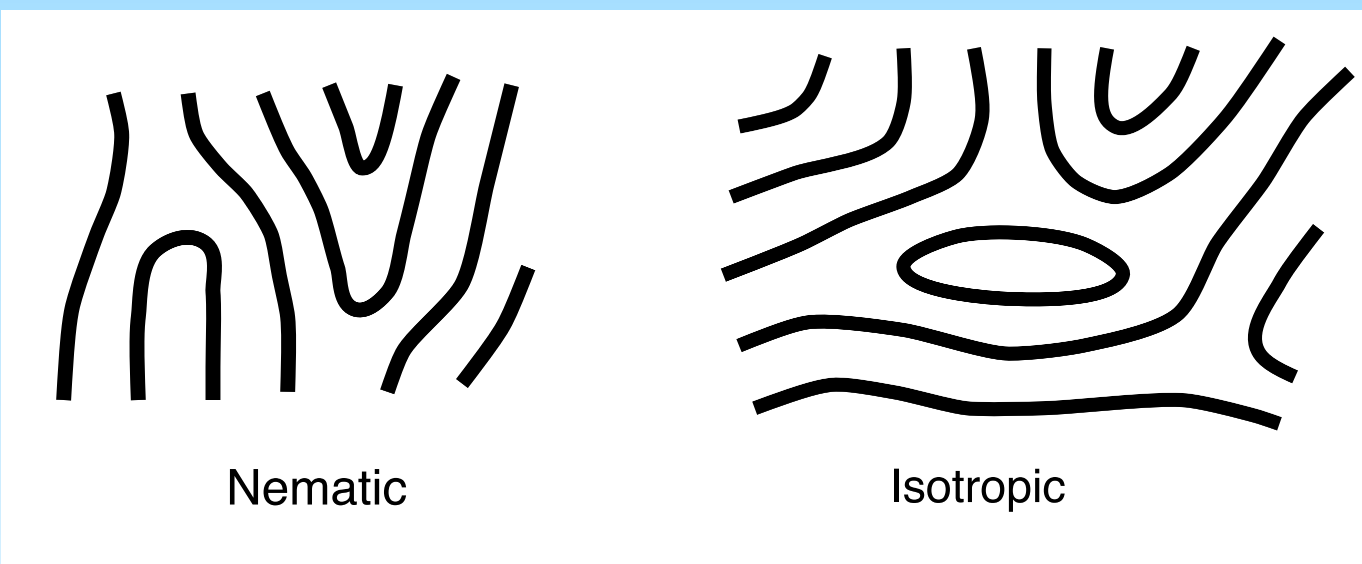
Less understood are transport properties

- quasiparticle (Boltzmann) vs. non q.p. (memory matrix, holography)
- Even in Boltzmann, transport lifetime different from q.p. lifetime
- Mechanism for current relaxation
  - disorder; Umklapp
  - compensated metal

- **Electrical transport properties near an Ising-nematic QCP**
- Extended Boltzmann approach (memory matrix with many slow operators)
- Temperature-dependent DC resistivity
- Summary & Outlook

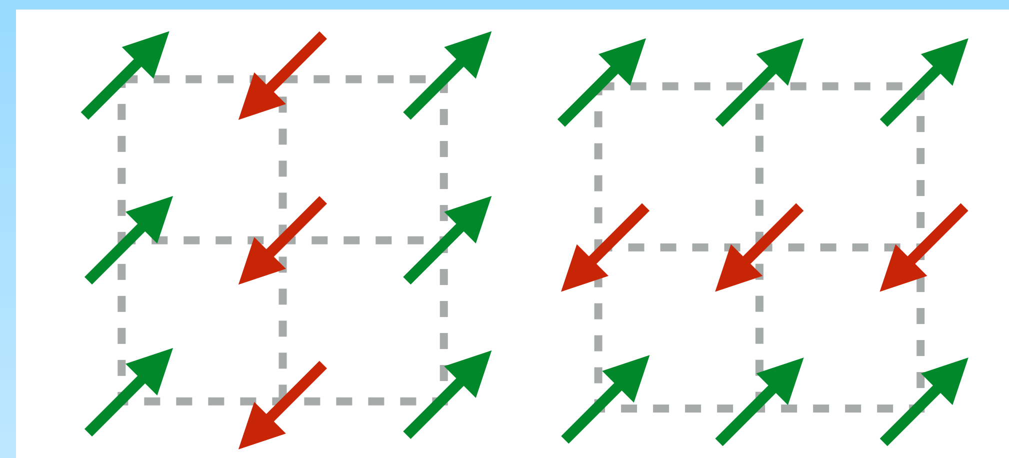
What is an electronic nematic phase?

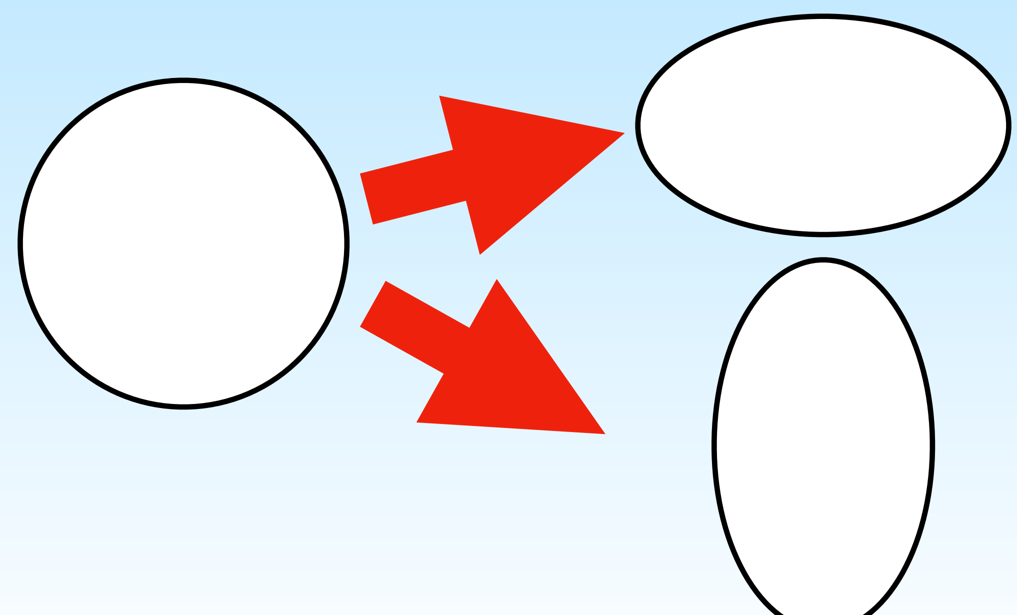
- Electrons spontaneously break rotational symmetry w.r.t. z-axis
  - isotropic -> U(1)
  - tetragonal (square lattice) -> Z_2 (Ising)



Kivelson, Fradkin & Emery, Nature '98

- Arises as vestigial or a principle phase
  - charge/spin density wave
  - Pomeranchuk instability (l=2)



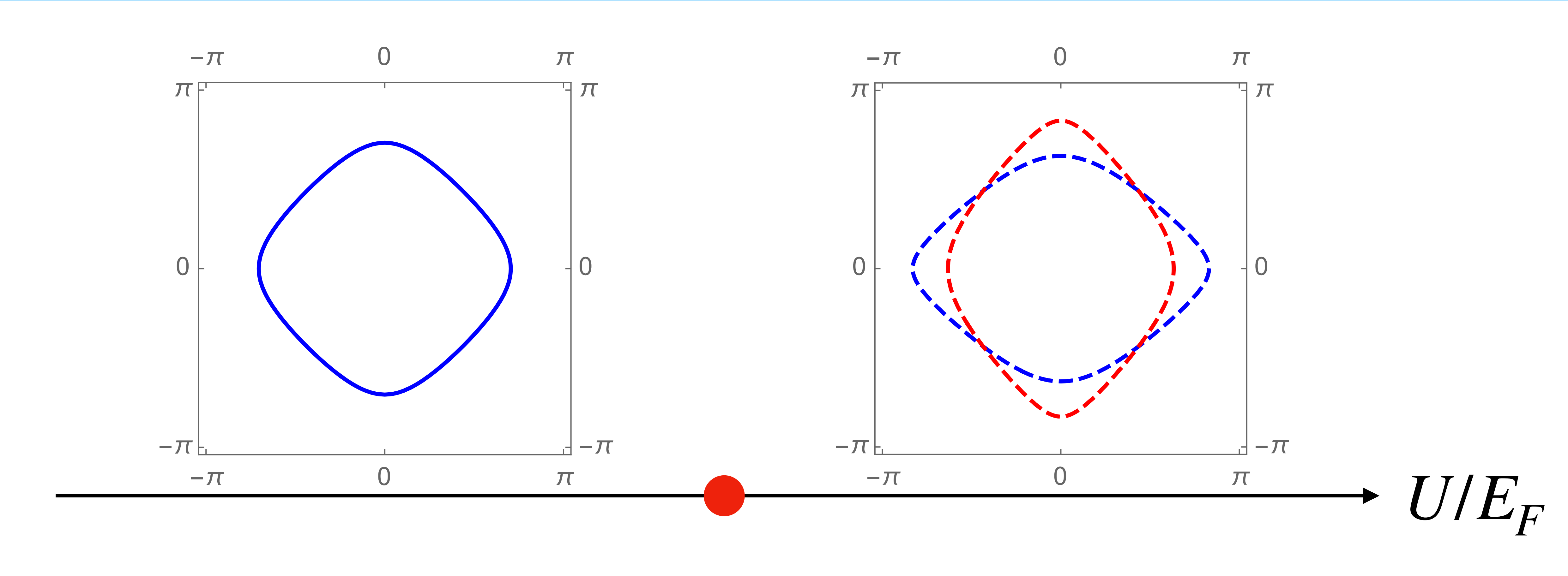


A toy model

- Hamiltonian with attractive interaction in the nematic channel

$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} - \mu\right) c^{\dagger}_{\mathbf{k}} c_{\mathbf{k}} - U \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \left(f_{\mathbf{k}} c^{\dagger}_{\mathbf{k}-\frac{\mathbf{q}}{2}} c_{\mathbf{k}+\frac{\mathbf{q}}{2}} \right) \left(f_{\mathbf{k}'} c^{\dagger}_{\mathbf{k}'+\frac{\mathbf{q}}{2}} c_{\mathbf{k}'-\frac{\mathbf{q}}{2}} \right)$$

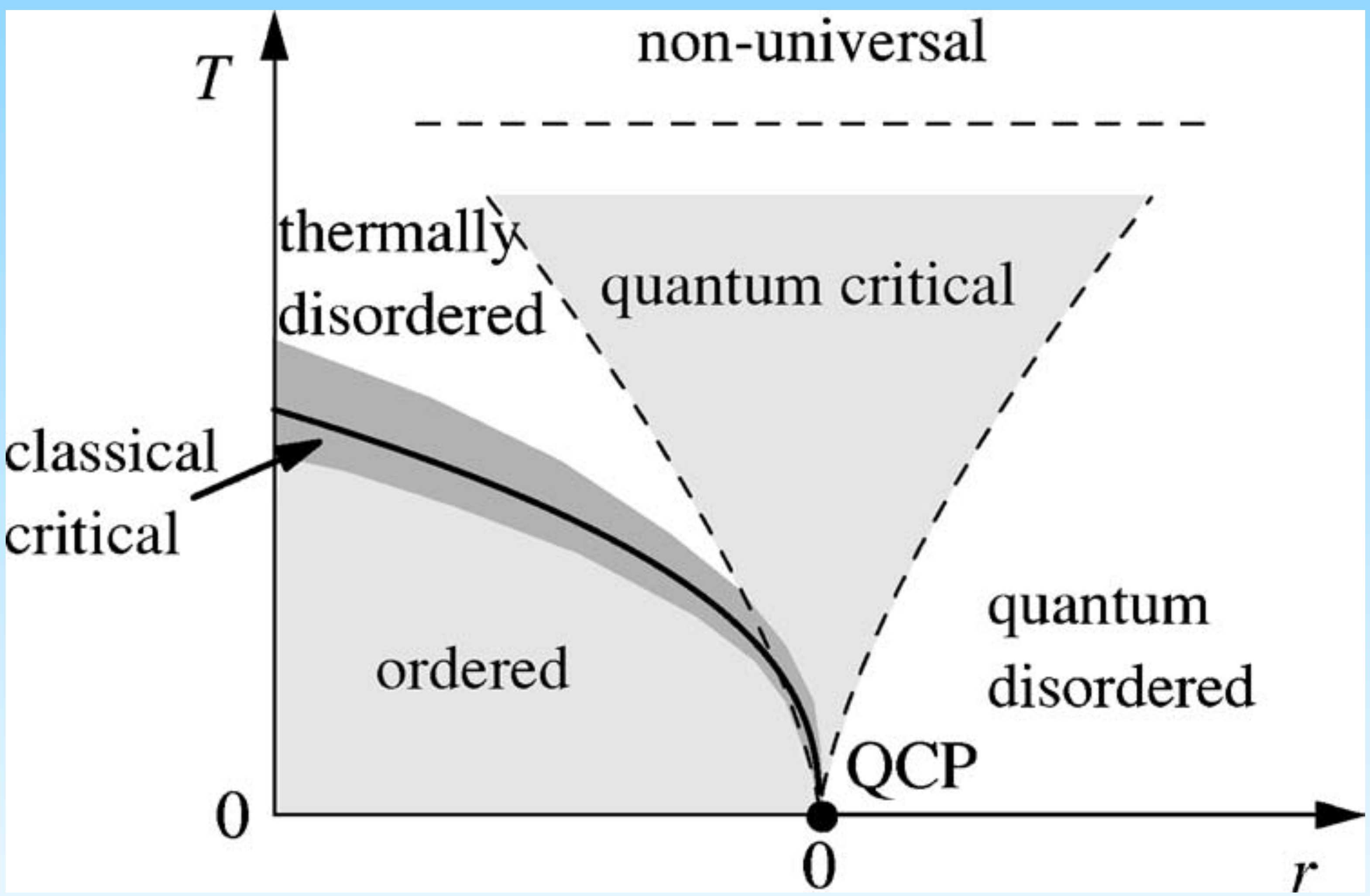
- Ising nematic form factor $f_{\mathbf{k}} = \cos k_x - \cos k_y$
  - odd under 90-degree rotation



- Hertz-Millis theory

$$H_U \Rightarrow -\sum_{\mathbf{q}} \frac{|\phi_{\mathbf{q}}|^2}{2U} + \sum_{\mathbf{kq}} \phi_{\mathbf{q}} \left(f_{\mathbf{k}} c^{\dagger}_{\mathbf{k}-\frac{\mathbf{q}}{2}} c_{\mathbf{k}+\frac{\mathbf{q}}{2}} \right)$$

- Integrate out fermions; study Gaussian fluctuations



Löhneysen et al., RMP ’07

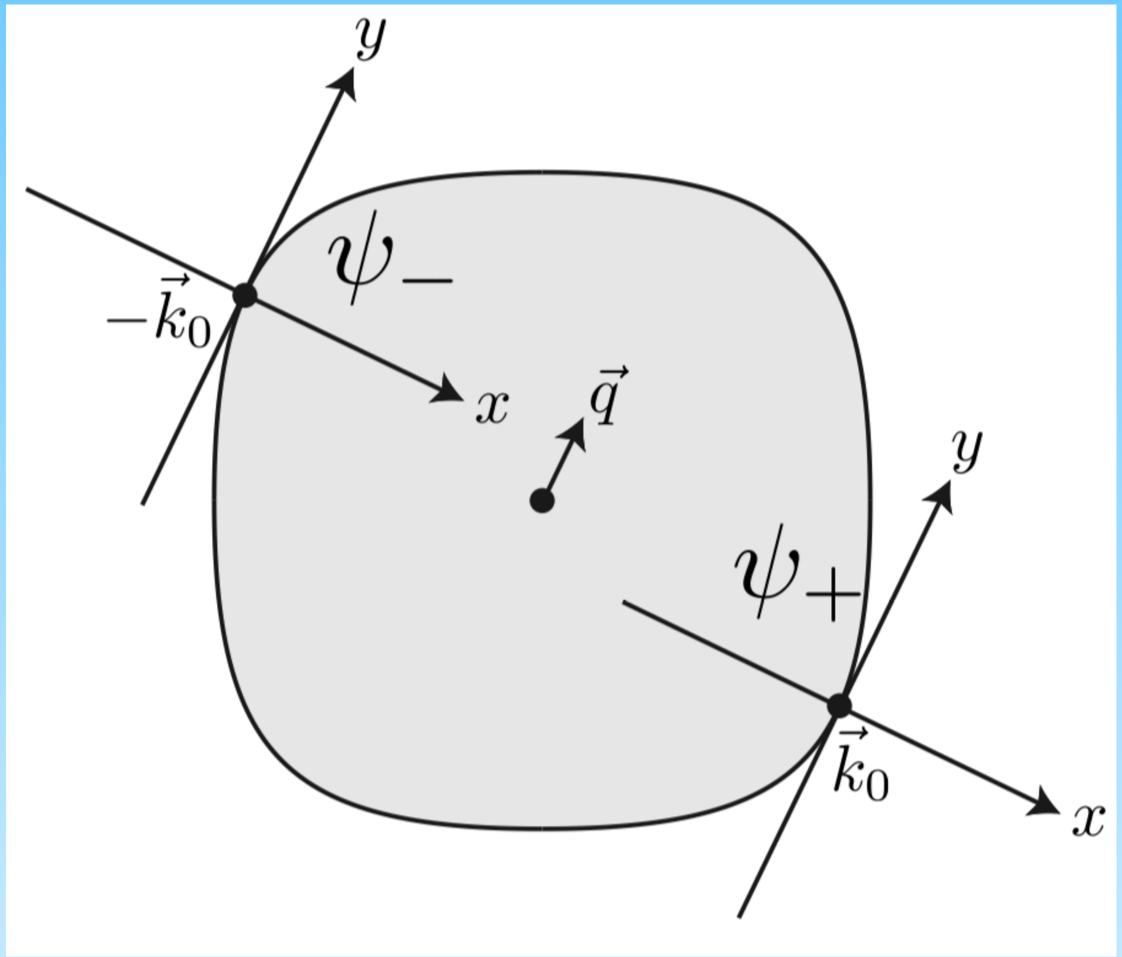
- “Quantum critical fan”: characteristic energy scale smaller than $k_B T$
- Landau damping

$$D_{\phi}^{-1} \sim r + \mathbf{q}^2 - i\frac{\gamma\Omega}{|\mathbf{q}|}$$

- Dynamical critical exponent

$$\xi_\tau \sim \xi^z; \;\; \xi \sim |r|^{-\nu}$$

- Fermions cannot be integrated out !



Metlitski & Sachdev, PRB ‘10

- Feedback of critical bosons on fermions

$$\Sigma(\omega) \sim i\ \mathrm{sign}(\omega)\,|\,\omega\,|^{2/3}$$

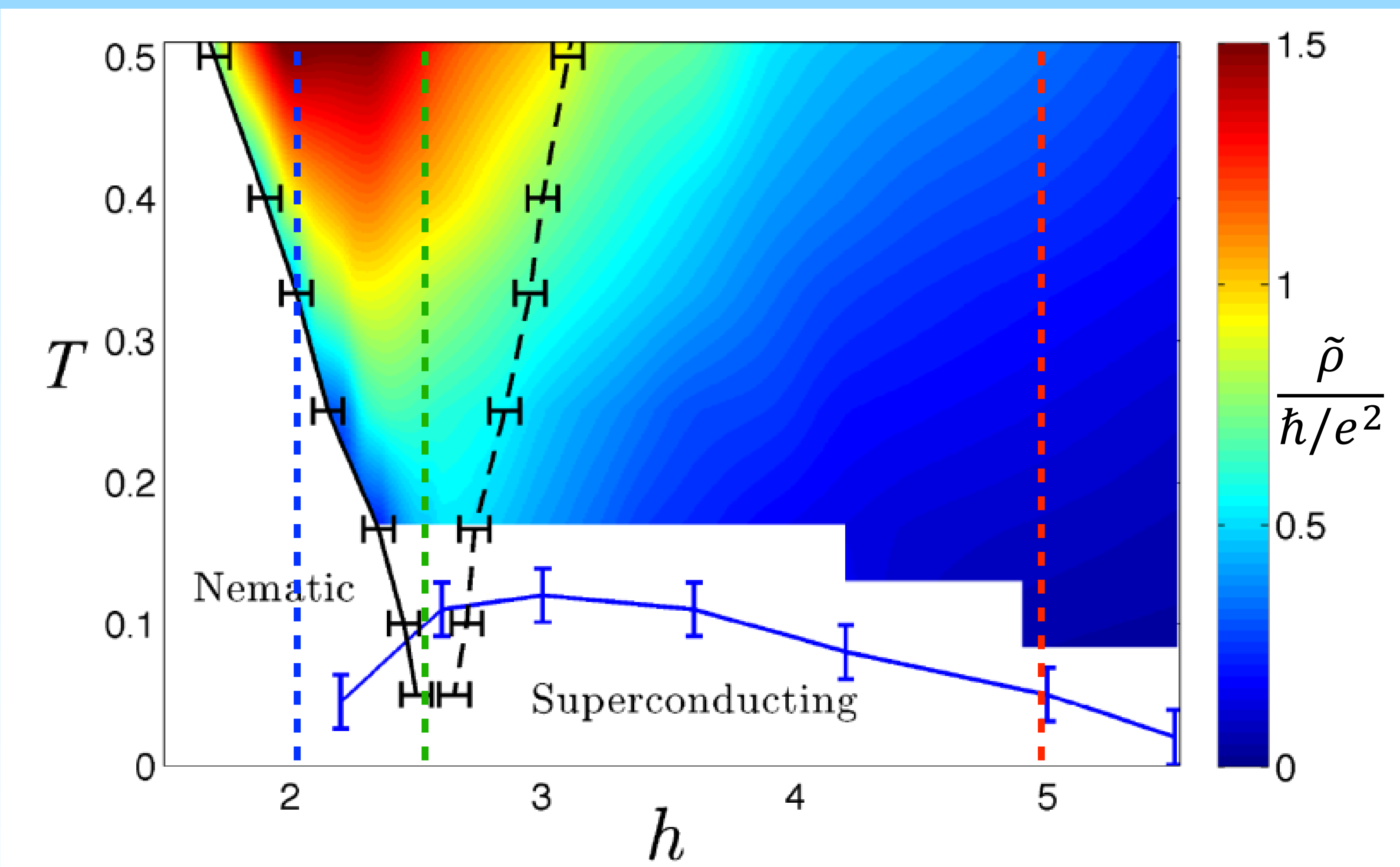
$$G(\mathbf{k},\omega) = \frac{1}{\omega - \varepsilon_{\mathbf{k}} - \Sigma(\omega)}$$

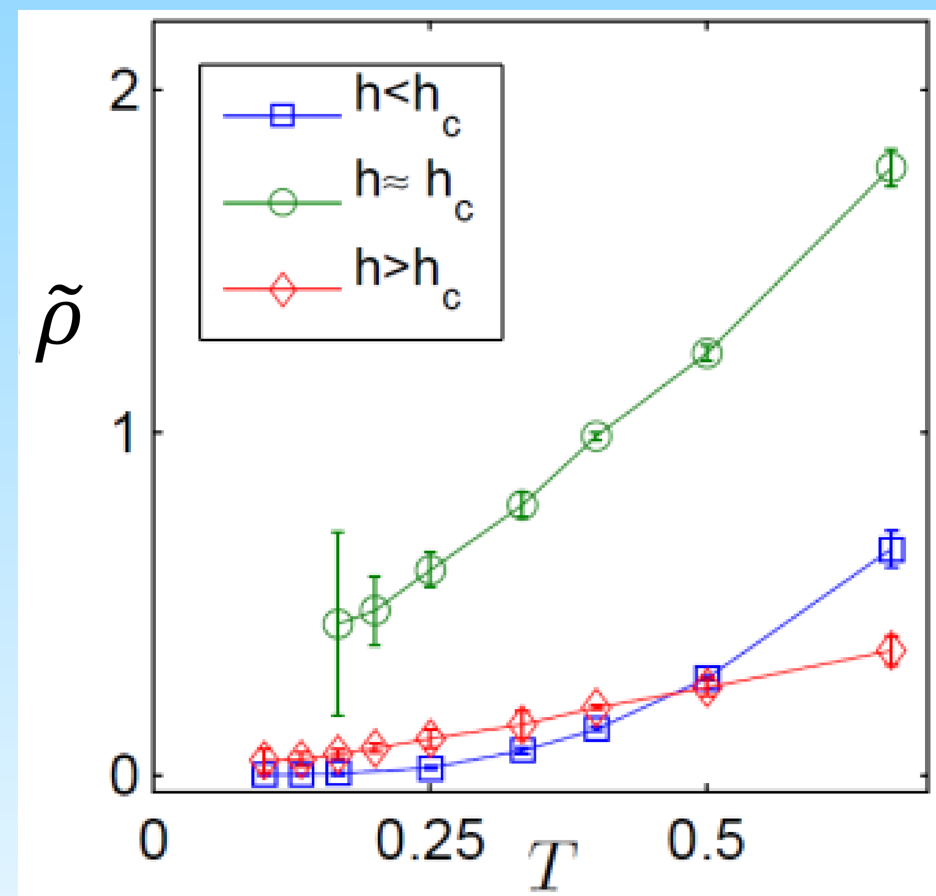
- Low-energy physics described by non-Fermi liquid coupled to Landau damped critical fluctuations

What about transport properties?

- The model can be studied using QMC without fermion sign problem!

"Resistivity proxy": $\tilde{\rho} \equiv \frac{\partial_\tau^2 \Lambda(\beta/2)}{2\pi\Lambda^2(\beta/2)} \approx \frac{\int_0^T d\omega\, \omega^2 \sigma(\omega)}{T\left[\int_0^T d\omega \sigma(\omega)\right]^2}$

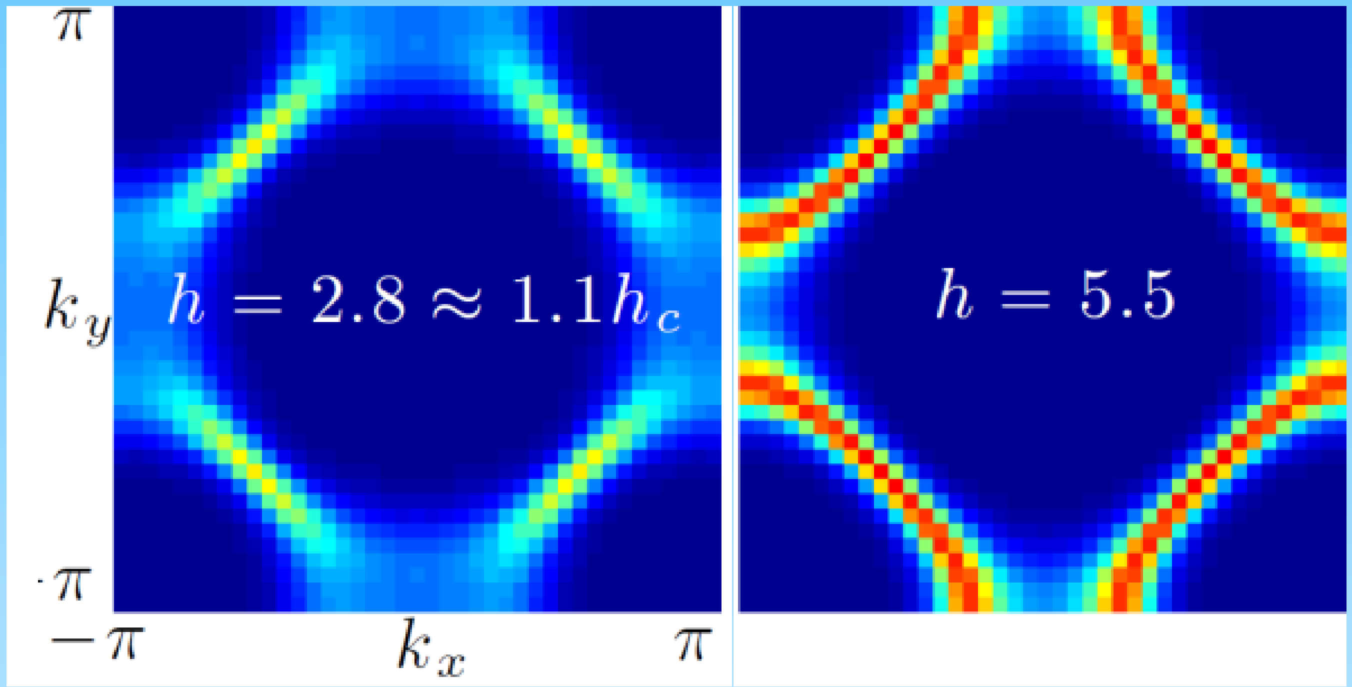




Lederer, Schattner, Berg & Kivelson, PNAS '17

What regime are we in?

$A(\mathbf{k}, \omega)$

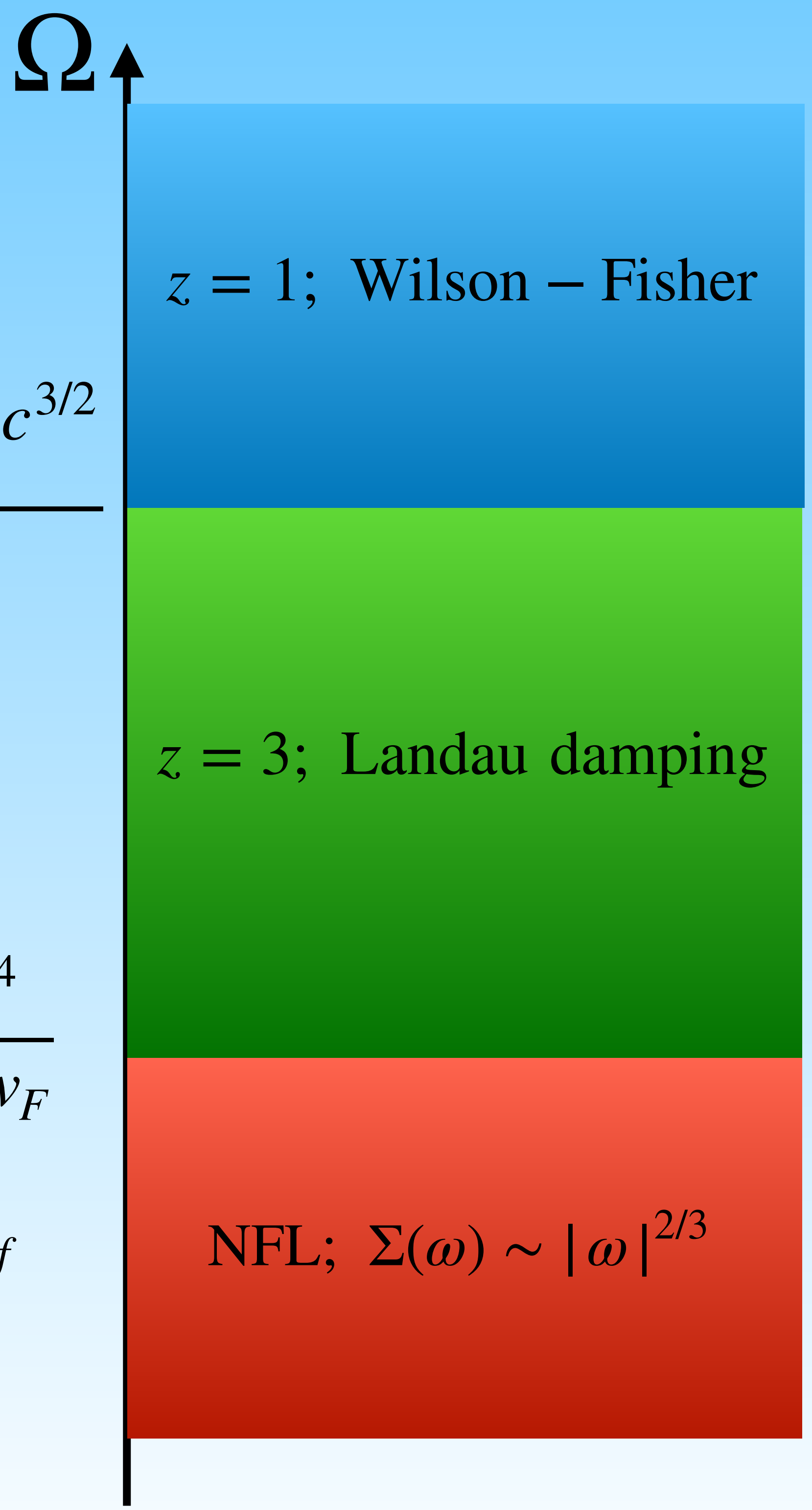


Lederer, Schattner, Berg & Kivelson, PNAS '17

- Despite strong coupling, FS smearing $\sim T^{\alpha} \ll E_F$
- Some form of extended Boltzmann equation should work

* argument similar to why kinetic theory works for lattice SYK model, cf. Avishkar Patel & Subir Sachdev

What regime are we in?



$\Omega_b \sim \dfrac{\sqrt{N_f}\lambda c^{3/2}}{v_F}$

$\Omega_f \sim \dfrac{\lambda^4}{N_f v_F}$

$T_c \sim \Omega_f$

$$D_\phi^{-1} \sim r + \mathbf{q}^2 - i\frac{\gamma\Omega}{|\mathbf{q}|} - \frac{\Omega^2}{c^2}$$

$$G^{-1}(\mathbf{k},\omega) = \omega - \varepsilon_{\mathbf{k}} - i\ \mathrm{sign}(\omega)\frac{v_F}{N_f}|\gamma\omega|^{2/3}$$

- either $\lambda^2/v_F \ll 1$ or $N_f \gg 1$
- Parametrically large temperature regime governed by extended Boltzmann transport

$$\Omega_f < T < \Omega_b \ll E_F$$

- where Landau damping entails complicated multi-particle scattering process, but electrons are coherent

Transport in NFL regime: work in progress!

- Electrical transport properties near an Ising-nematic QCP
- Extended **Boltzmann approach (memory matrix with many slow operators)**
- Temperature-dependent DC resistivity
- Summary & Outlook

Memory matrix approach

- Memory matrix approach is about identifying “slow operators”

Given a set of slow operators : $\{A, B, C, \dots\}$

In linear response : $$\Phi_{AB}(\omega) \approx \sum_{CD} \chi_{AC} \left(\frac{1}{M(\omega) - i\omega\chi} \right)_{CD} \chi_{DB}$$

where $$M_{CD}(\omega) = \frac{\mathrm{Im}\mathcal{G}^R_{\dot{C}\dot{D}}(\omega)}{\omega}$$

Hartnoll, Lucas & Sachdev, *Holographic Quantum Matter*

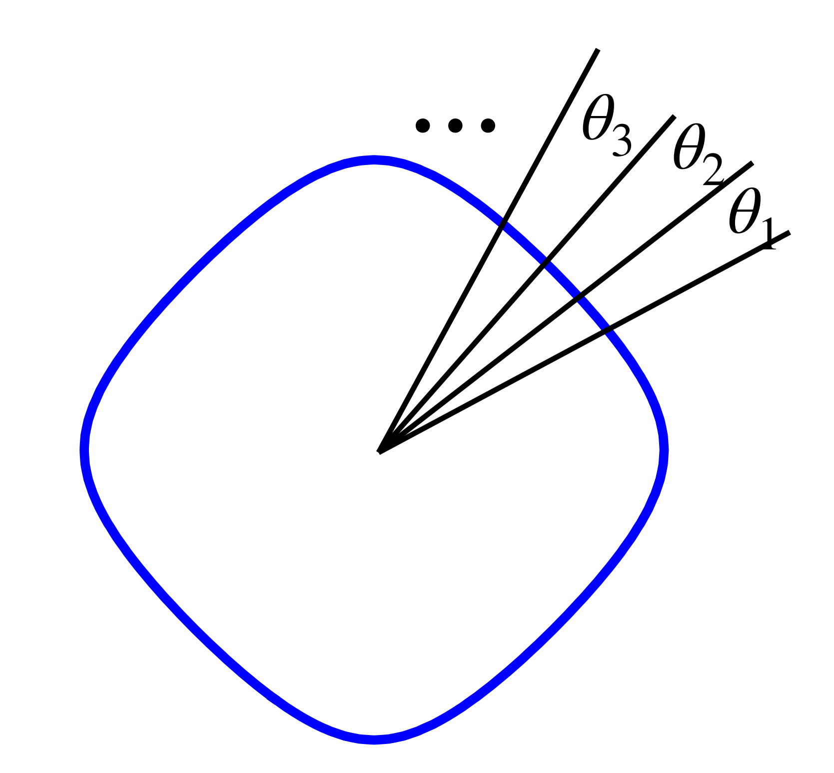
- Example: weak impurity scattering

$\{J, P\}$ $$\dot{P} = \int_{\mathbf{r}} h(\mathbf{r}) \nabla O(\mathbf{r})$$

$$\sigma_{JJ}(\omega) \approx \frac{\chi^2_{JP}}{M_{PP} - i\omega\chi_{PP}}$$

$$M_{PP}(\omega) \sim h_0^2 \int_{\mathbf{q}} q_x^2 \frac{\mathrm{Im}\mathcal{G}^R_{OO}(\mathbf{q}, \omega)}{\omega}$$

Patching the Fermi surface



- Typical nemato-electronic scattering has

$$D_\phi^{-1} \sim \mathbf{q}^2 - i\frac{\gamma\Omega}{|\mathbf{q}|} \qquad \Omega \sim T;\ \ q \sim T^{1/z}$$

- "Patching" the Fermi surface
  - slow inter-patch relaxation
  - small Lorenz number

Metzner, Fradkin, Haldane, Hartnoll …

- Set of slow operators $\{n_{\hat{k}} \,|\, n_{\hat{k}} \equiv \int \mathrm{d}k_\perp n_{\mathbf{k}}\}$
- Memory matrix describes collision integral

$$M_{\hat{k}\hat{k}'}(\omega) = \frac{\mathrm{Im}\mathcal{G}^R_{\dot{n}_{\hat{k}}\dot{n}_{\hat{k}'}}(\omega)}{\omega} \quad \text{where } \dot{n}_{\hat{k}} = i[H, n_{\hat{k}}]$$

- Conductivity $J \approx \oint_{\hat{k}} \mathbf{v}_{\hat{k}} n_{\hat{k}}$

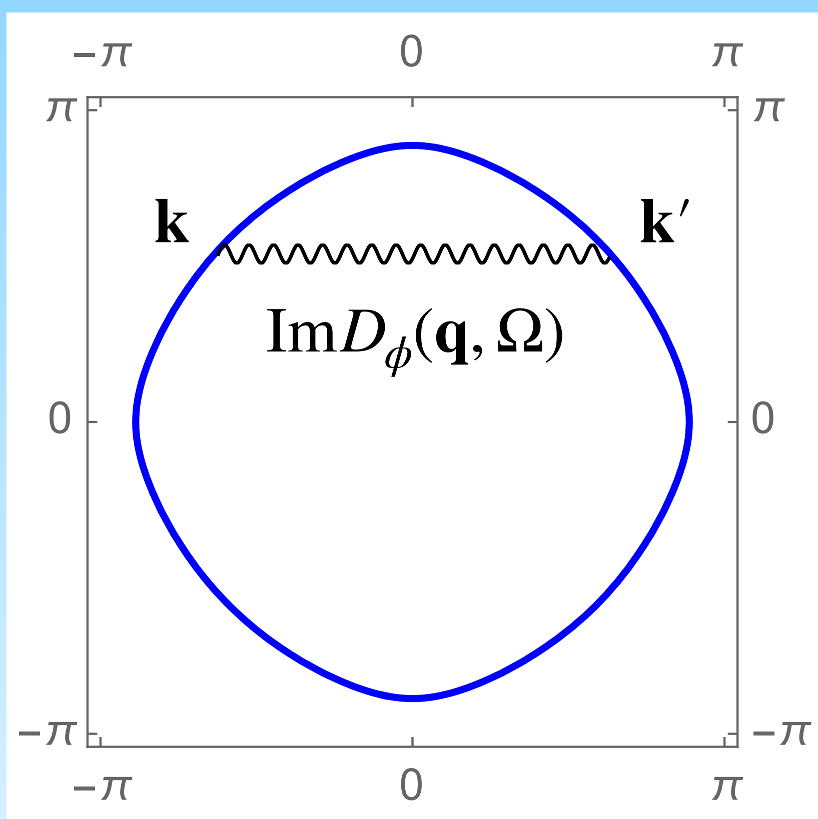
Why not isolate out current and momentum?

$$\sigma_{JJ}(\omega) \approx \oint_{\hat{k}\hat{k}'} (\mathbf{v}_{\hat{k}} \cdot \mathbf{v}_{\hat{k}'})\Phi_{\hat{k}\hat{k}'}(\omega);\ \text{ where } \Phi_{\hat{k}\hat{k}'}(\omega) \approx \oint_{\hat{k}_1\hat{k}_2} \chi_{\hat{k}\hat{k}_1}\left(\frac{1}{M - i\omega\chi}\right)_{\hat{k}_1\hat{k}_2} \chi_{\hat{k}_2\hat{k}'}$$

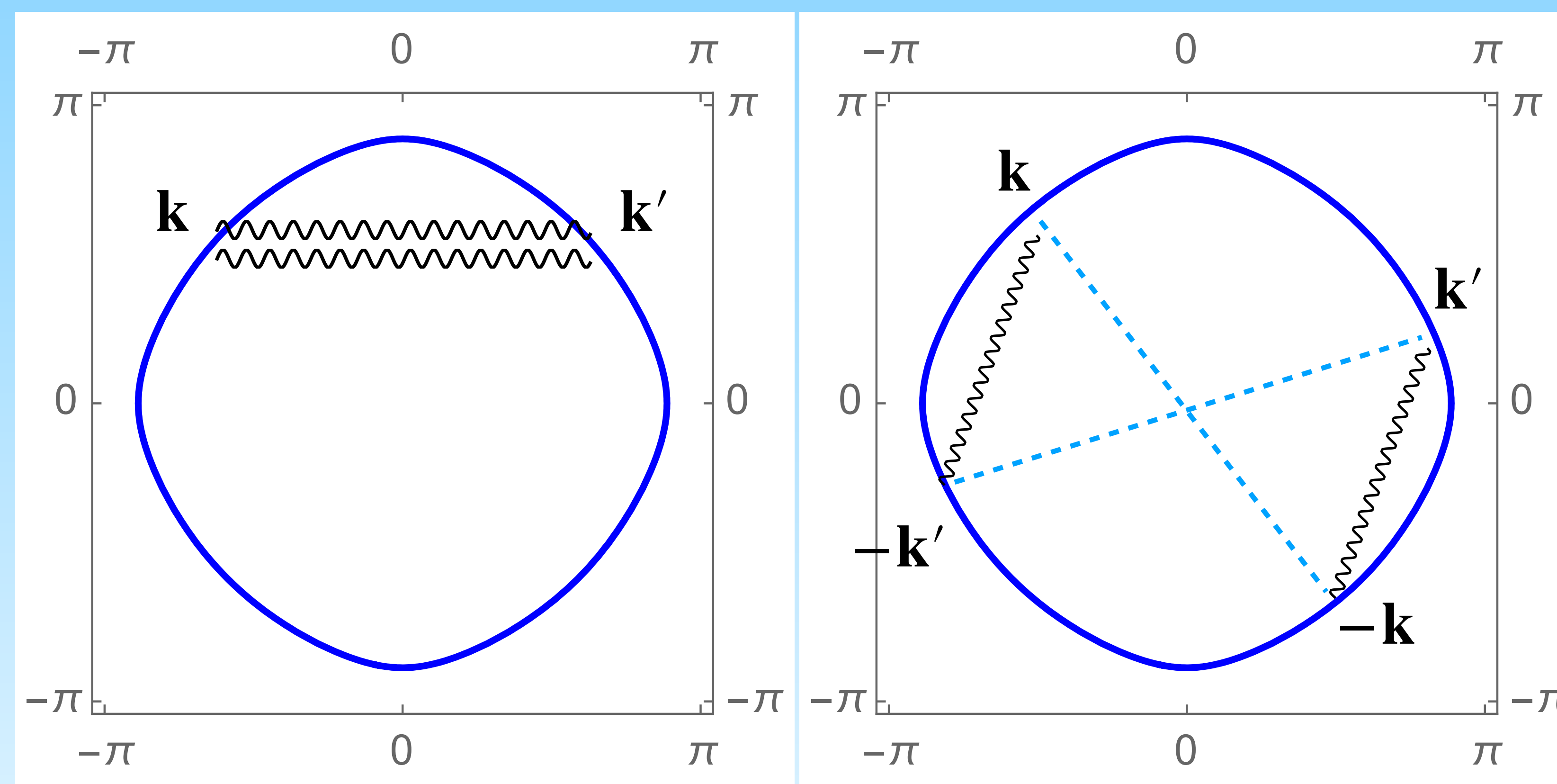
Memory matrix

- Two low-energy degrees of freedom
  - critical nematic fluctuations + electrons near FS

- Two class of processes under RPA



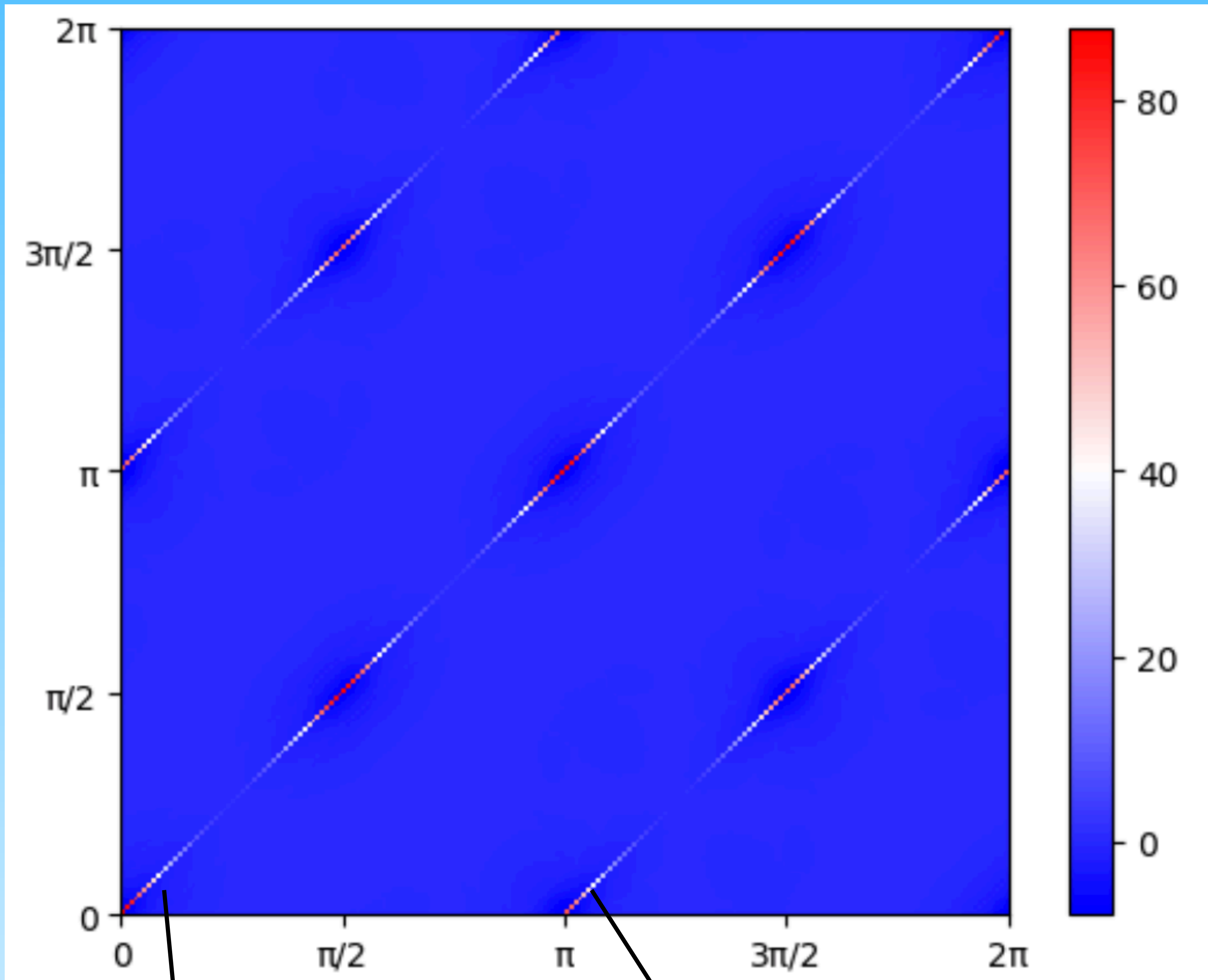
Class I

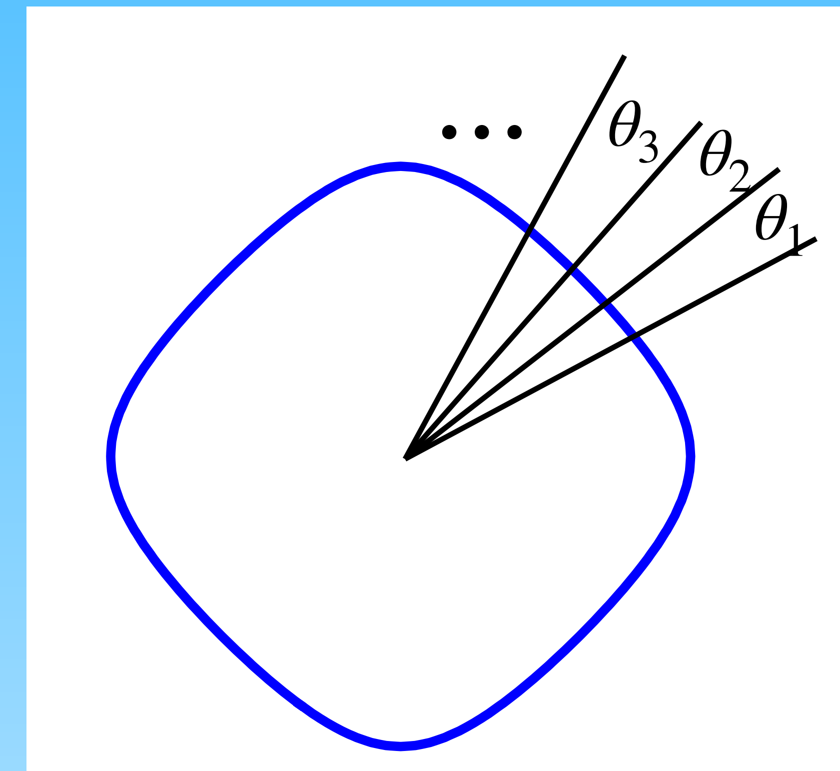


$\mathbf{q} = \mathbf{k} - \mathbf{k}'$

$\mathbf{q} = \mathbf{k} + \mathbf{k}'$

Class II

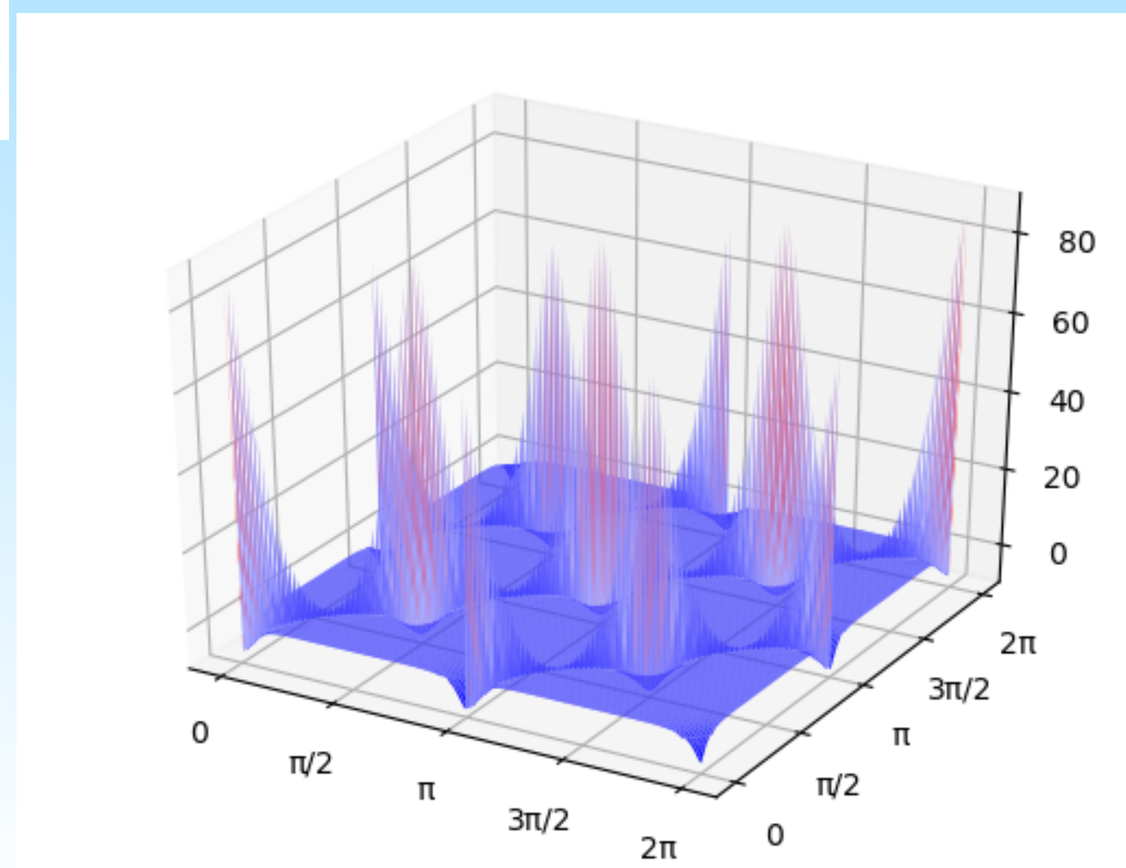




$M_{\hat{k}\hat{k}'}(\omega \to 0)$

momentum exchange

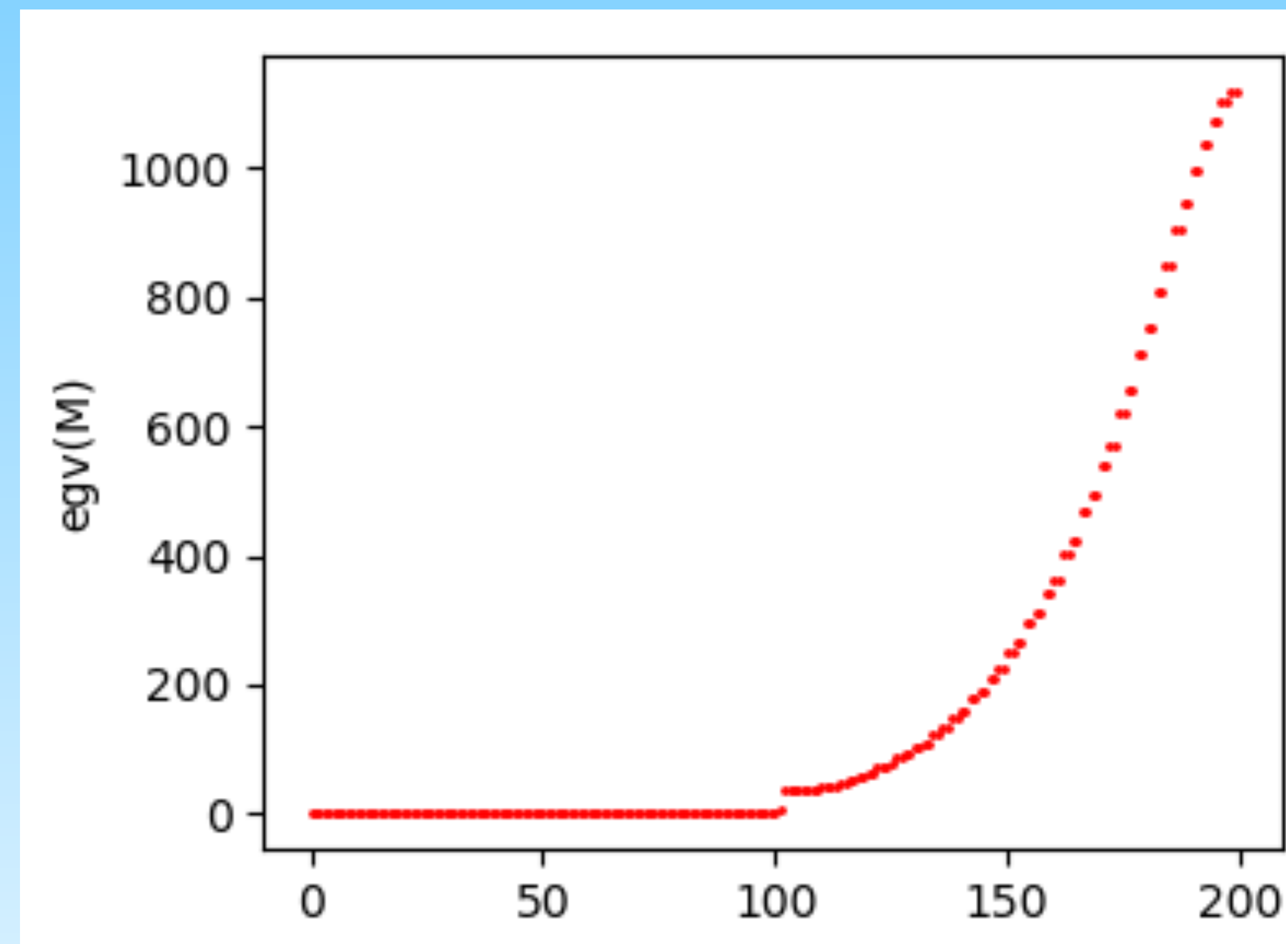
head-on



Memory matrix structure

- 2D electron system without impurity or Umklapp, all odd angular harmonics are conserved

Ledwith, Guo, Shytov & Levitov, arXiv:1708.02376

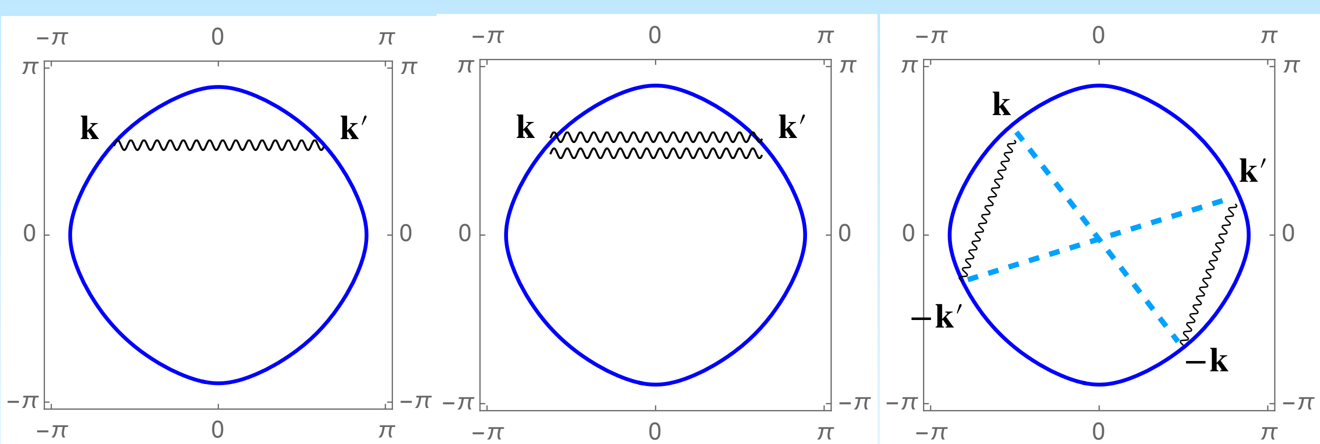


- In our language:

$$M^{(1)}_{\hat{k}\hat{k}'} = -2M^{(2,+)}_{\hat{k}\hat{k}'} = 2M^{(2,-)}_{\hat{k},-\hat{k}'}$$

- s.t.

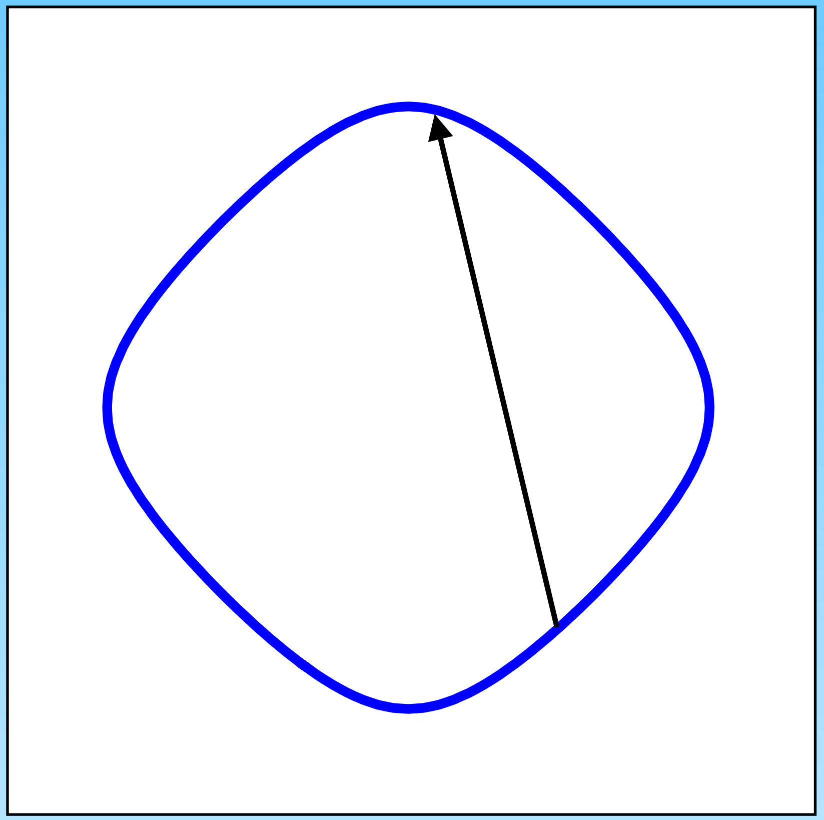
$$M^{\text{tot}}_{\hat{k}\hat{k}'} = \frac{1}{2}\left(M^{(1)}_{\hat{k}\hat{k}'} + M^{(1)}_{\hat{k},-\hat{k}'}\right)$$

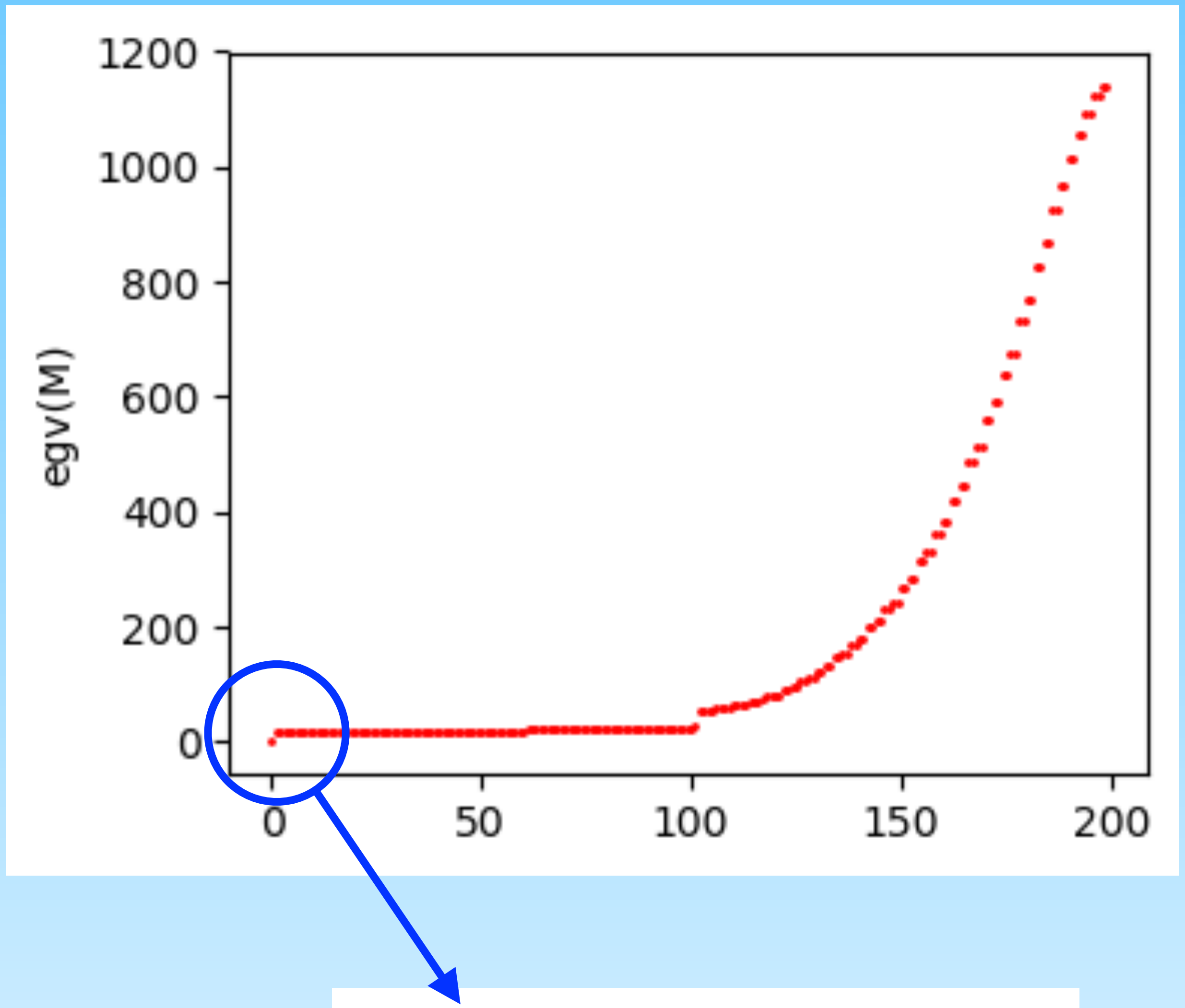


Typically only class I diagram considered, (wrongly) leading to momentum relaxation

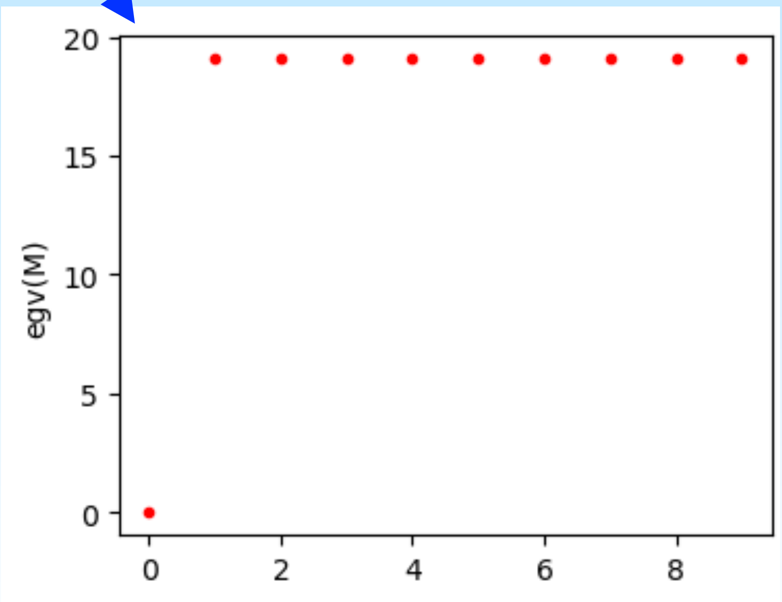
Dell'Anna & Metzner, PRL '07

Weak random disorder

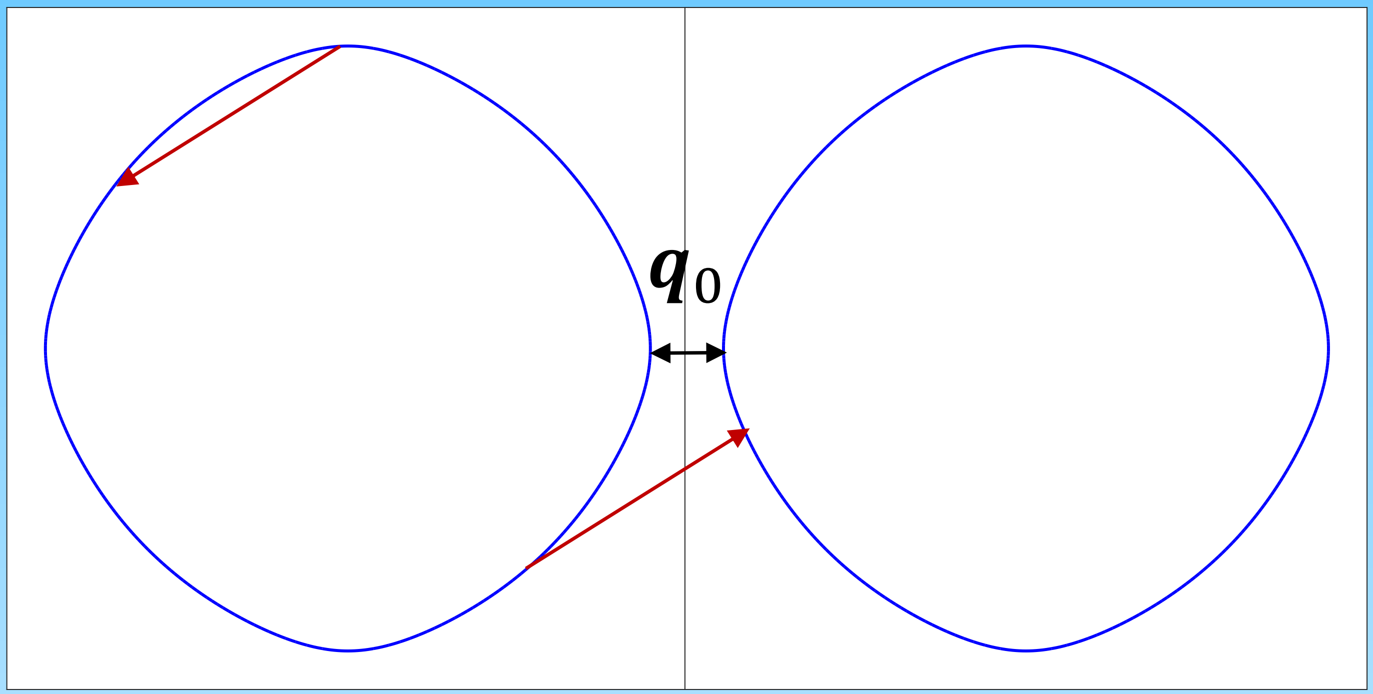


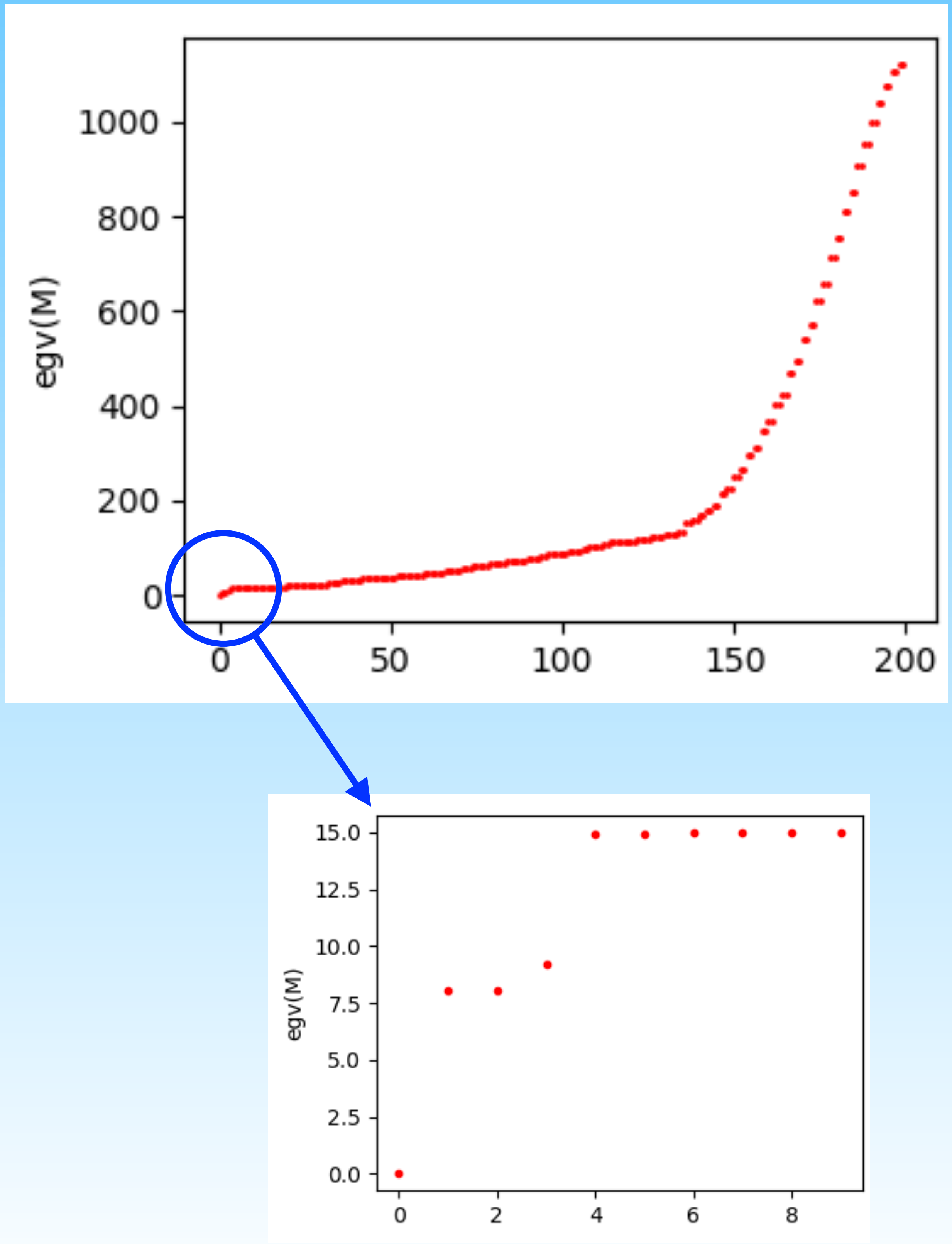


- All odd harmonics develop non-zero relaxation
- In particular, momentum is no longer conserved!



Umklapp





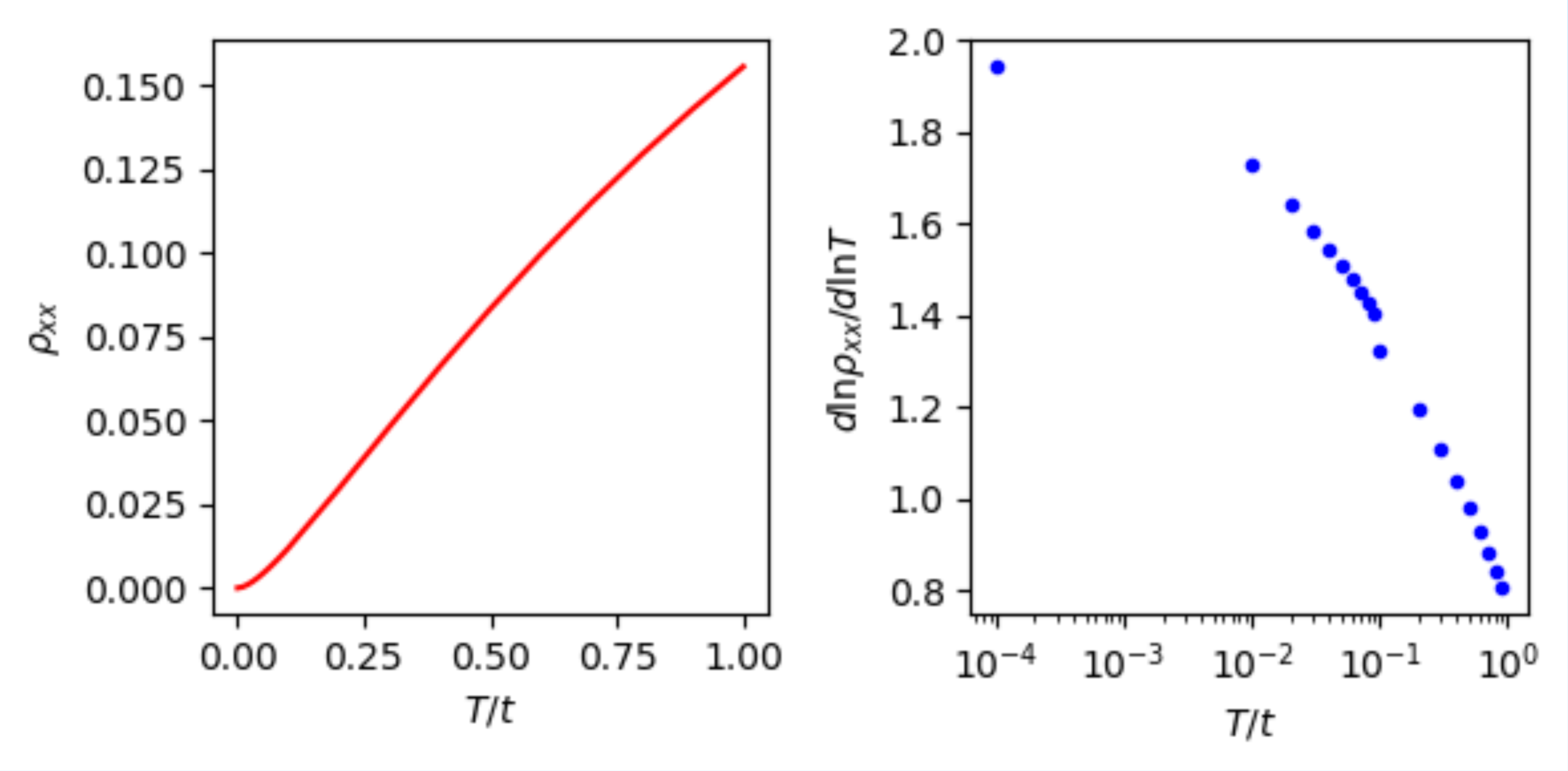
- All odd harmonics develop non-zero relaxation
- In particular, momentum is no longer conserved!

* Another interesting case is compensated metal, where current and momentum have zero overlap

- Electrical transport properties near an Ising-nematic QCP
- Extended Boltzmann approach (memory matrix with many slow operators)
- **Temperature-dependent DC resistivity**
- Summary & Outlook

DC resistivity

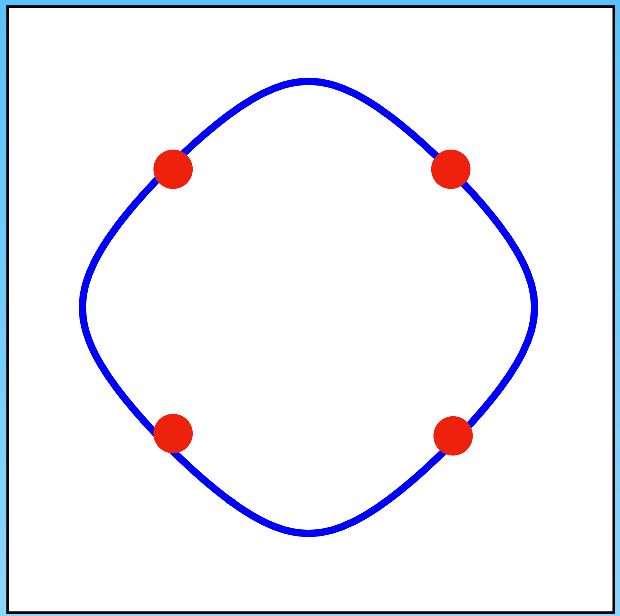
- Weak random disorder adds a constant resistivity
  - sanity check!
- Umklapp scattering gives rise to strong T-dependence, smooth crossover from T^2 at low-T to sub-linear at high-T

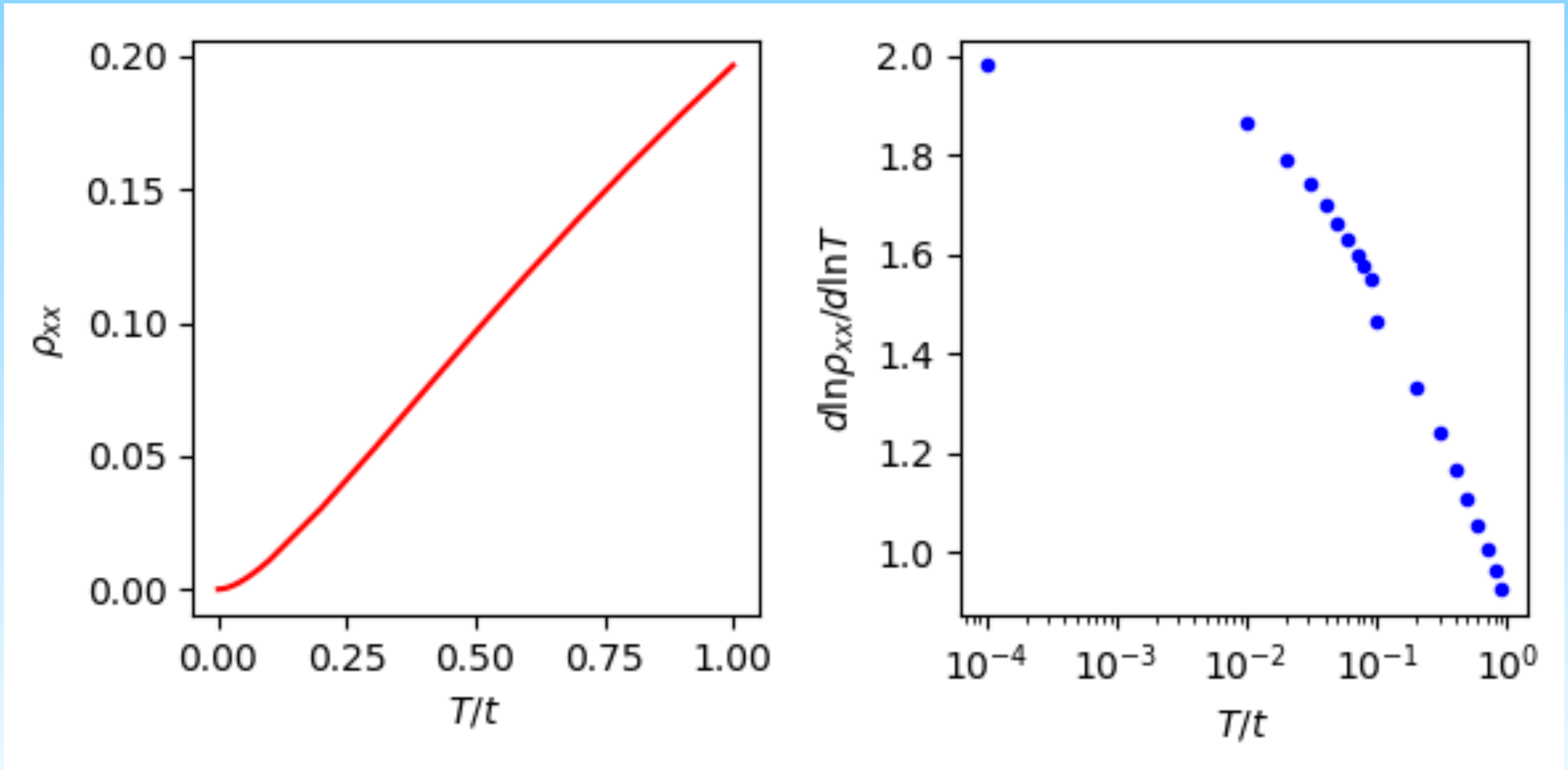


- At sufficiently low T, $\rho \sim T^2$ from noncritical fluctuations (even at QCP)

Maslov, Yudson & Chubukov '11

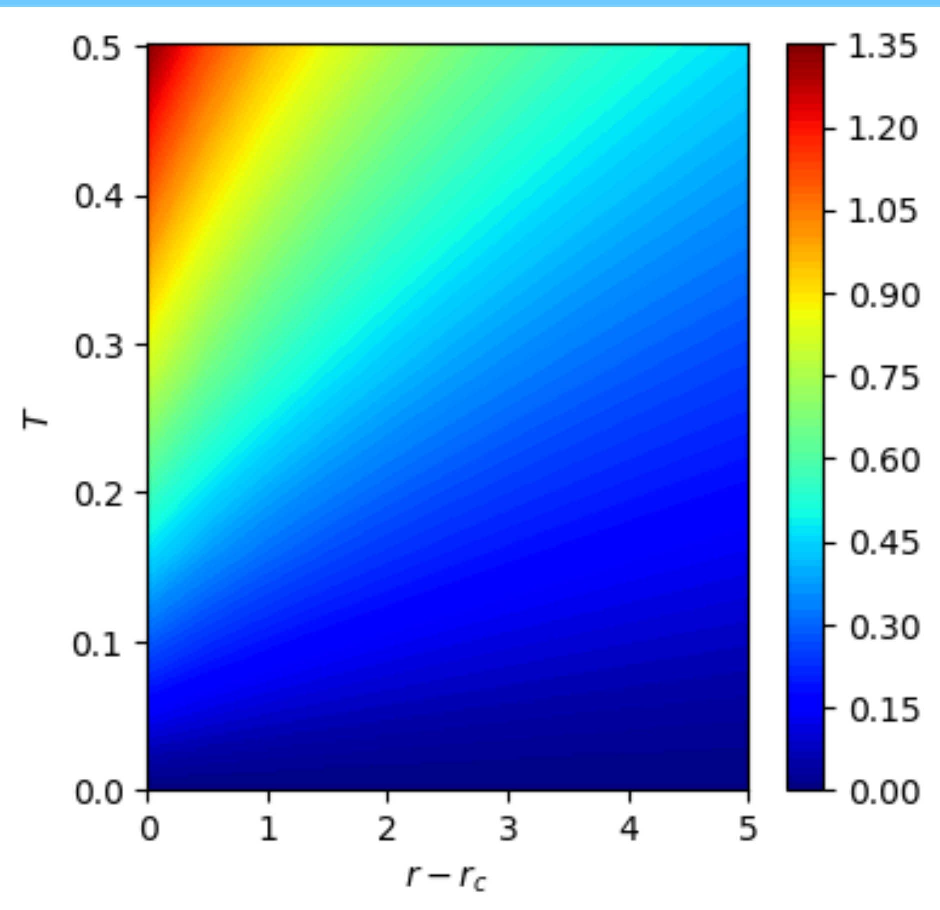
- Cold spots do not particularly matter

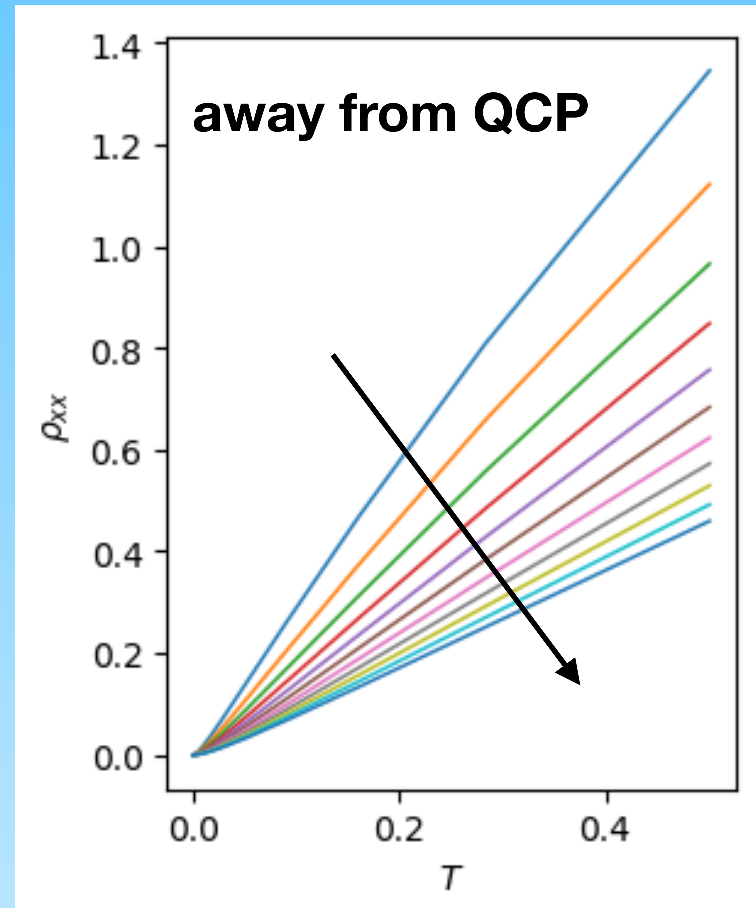




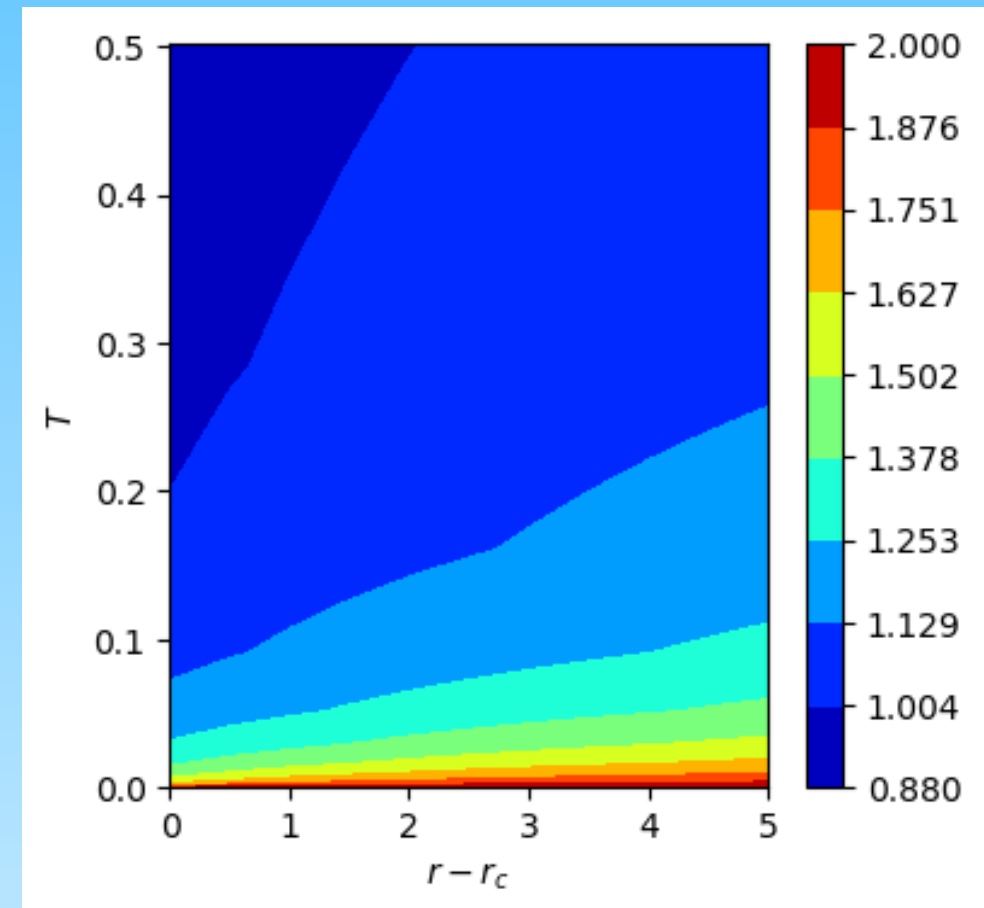
DC resistivity

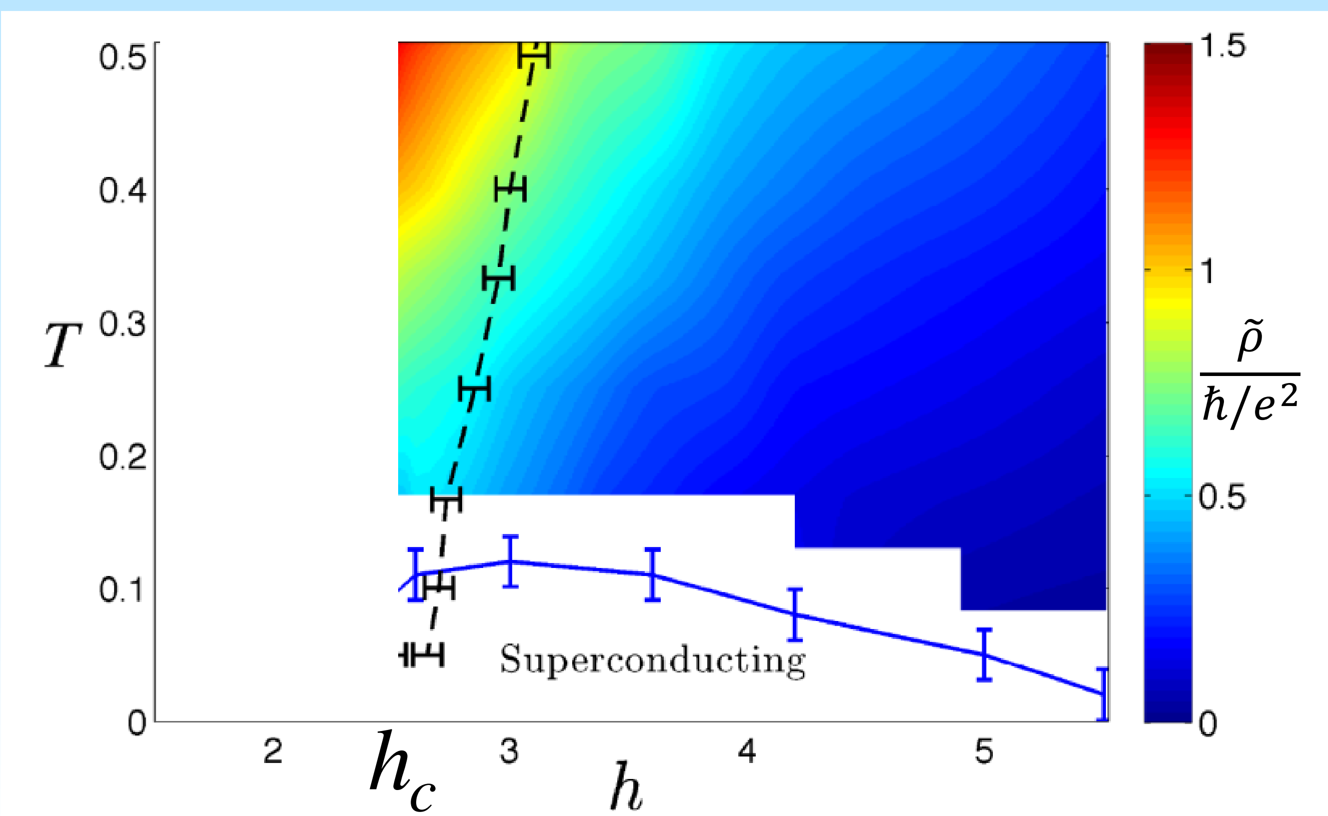
ρ_{xx}

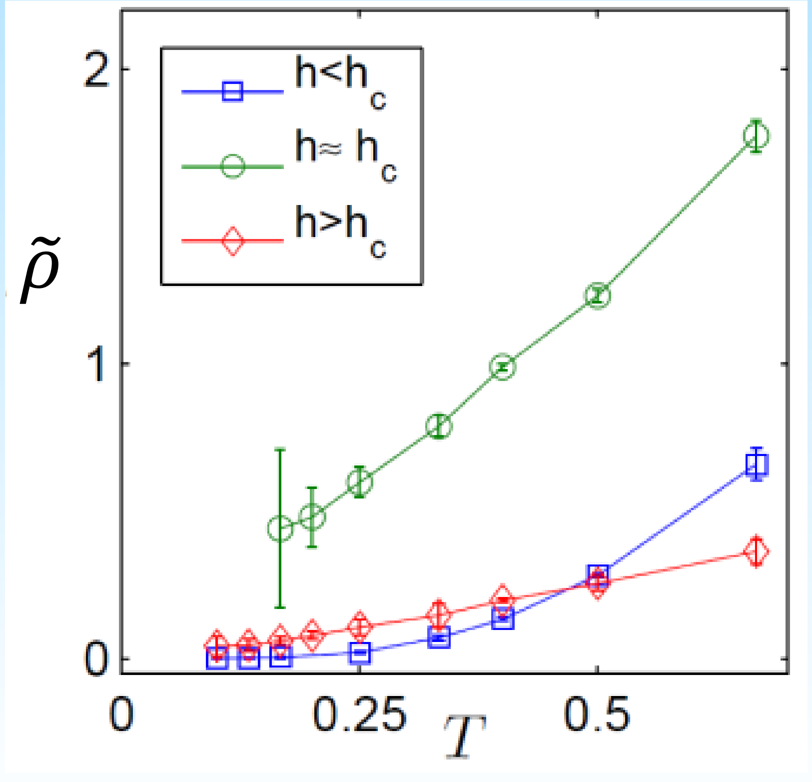




$d \ln \rho_{xx} / d \ln T$







Conclusion

- In quantum critical metals, there is a finite-temperature region governed by extended Boltzmann transport
- Memory matrix is a good approximation to calculating the collision integral
- Temperature dependence of the DC resistivity does not show one particular power law scaling; smooth crossover from T^2 at low-T to sub-linear at high-T

- Compensated metal ($\chi_{JP} \to 0$)?
- Density wave QCPs that break translation symmetry?
- Generalization to bad metal regime?
- Connection to experiments?

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