Holographic Abrikosov lattice

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Introduction

Use the gauge/gravity correspondence as a tool to investigate the dynamics of strongly coupled CFTs at finite temperature, charge density and external magnetic field.

- Attempt to understand universal features of strongly coupled condensed matter systems found in the vicinity of quantum critical points.

- Explore black hole physics: construct and study novel charged black hole solutions that asymptote to AdS.

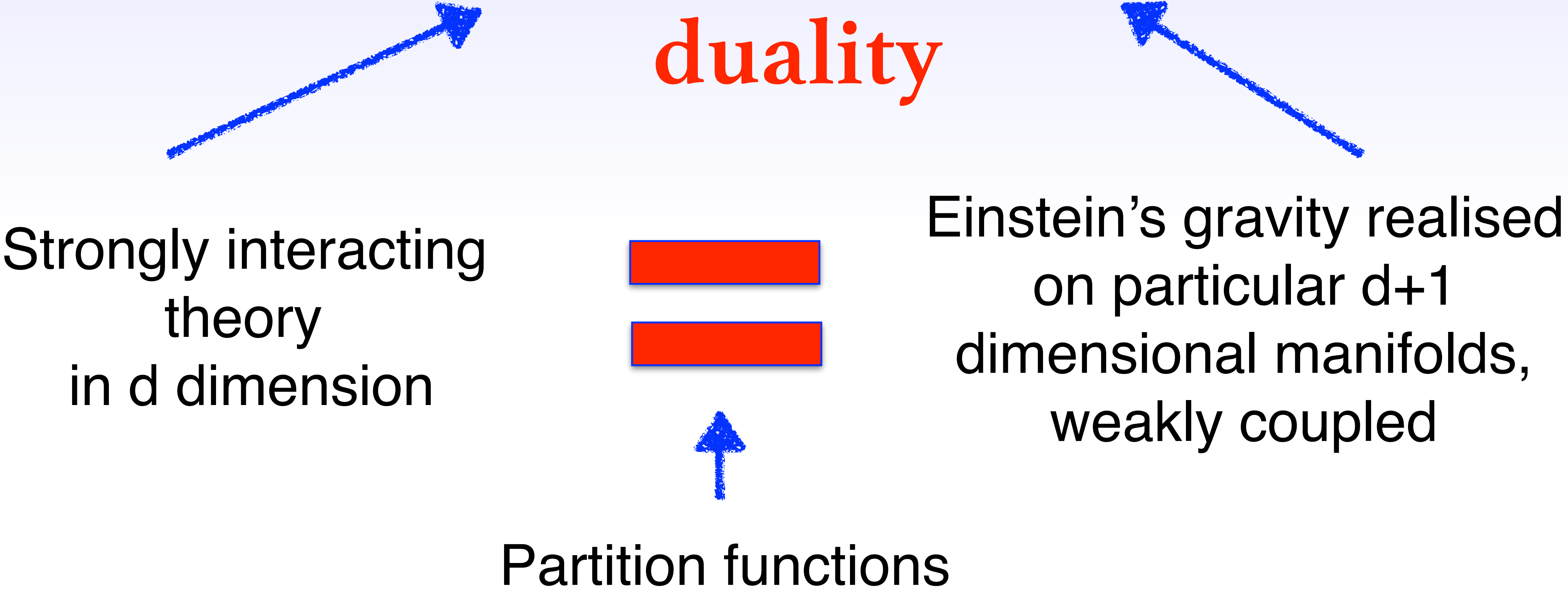
Questions to address?

Specific questions that can be addressed:

- What type of phases are possible and what are the properties of each phase?
- What kind of ground states are possible? cooling down or classification of IR geometries?
- How do these phases compete? What is the dual phase diagram?

But what is Holography?
and how does it work?

Gauge/ Gravity duality



Strongly interacting theory
in d dimension

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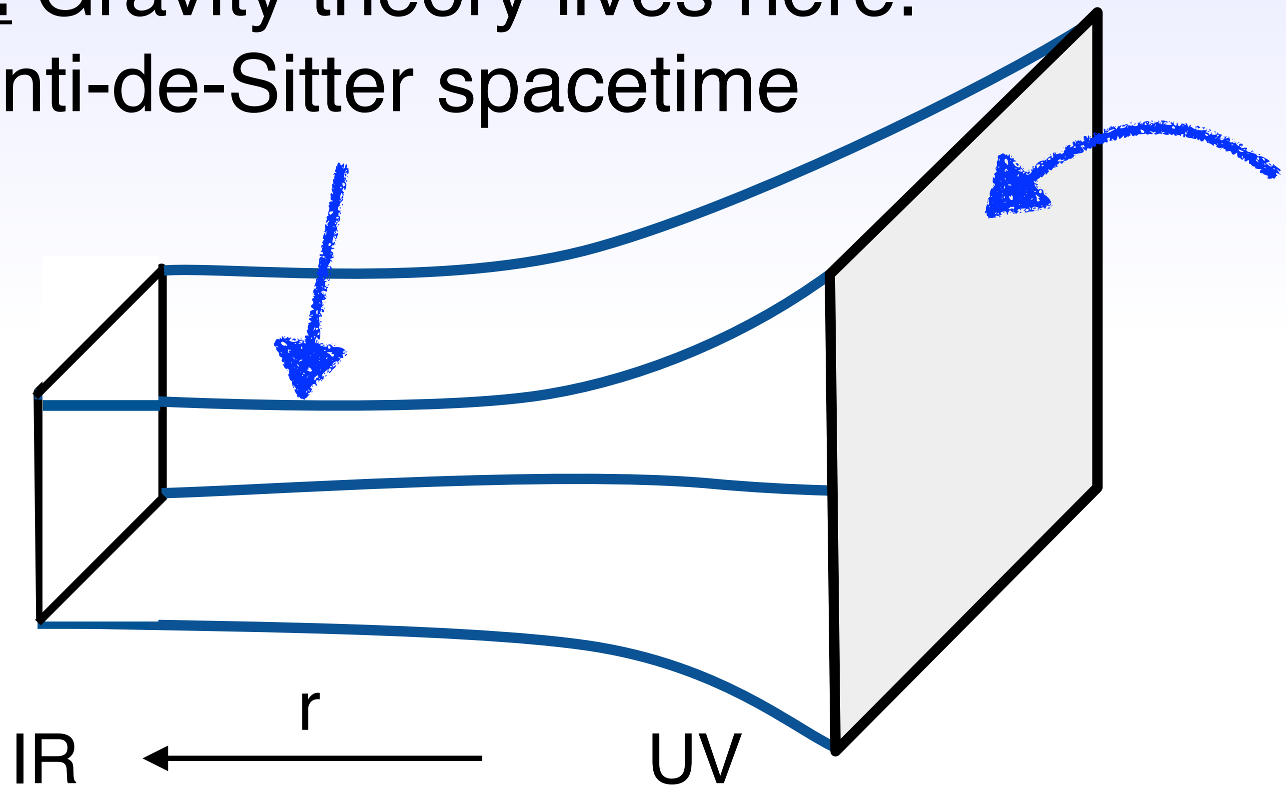
Einstein's gravity realised on particular d+1 dimensional manifolds, weakly coupled

Partition functions

The dictionary

Simplest set-up: T=0,B=0, no charge density.



Bulk: Gravity theory lives here.
Anti-de-Sitter spacetime

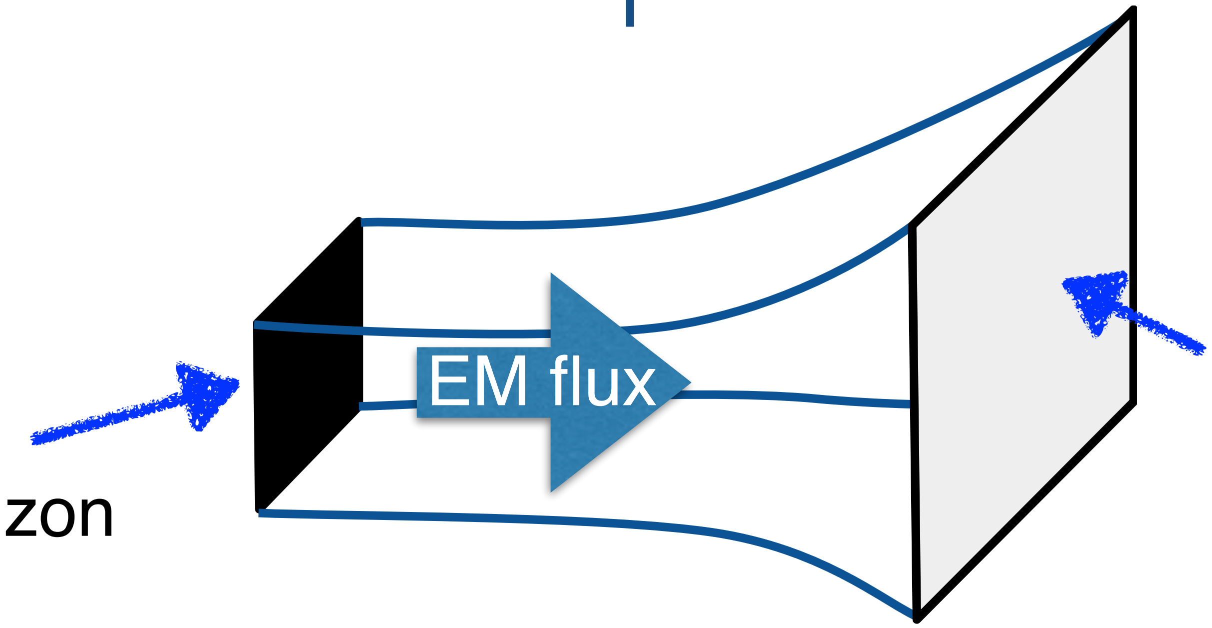
Boundary: Flat.
Strongly coupled
matter lives here:
CFT.

Emergent holographic direction,
captures the resolution scale.
[G,R]=0

The dictionary

Add temperature, charge density, magnetic field, etc

Boundary	Bulk
finite temperature, T	BH with the same T
electrons, μ	electric field
in magnetic field, B	magnetic field



BH horizon

Thermal field theory at finite density and B

Putting everything together

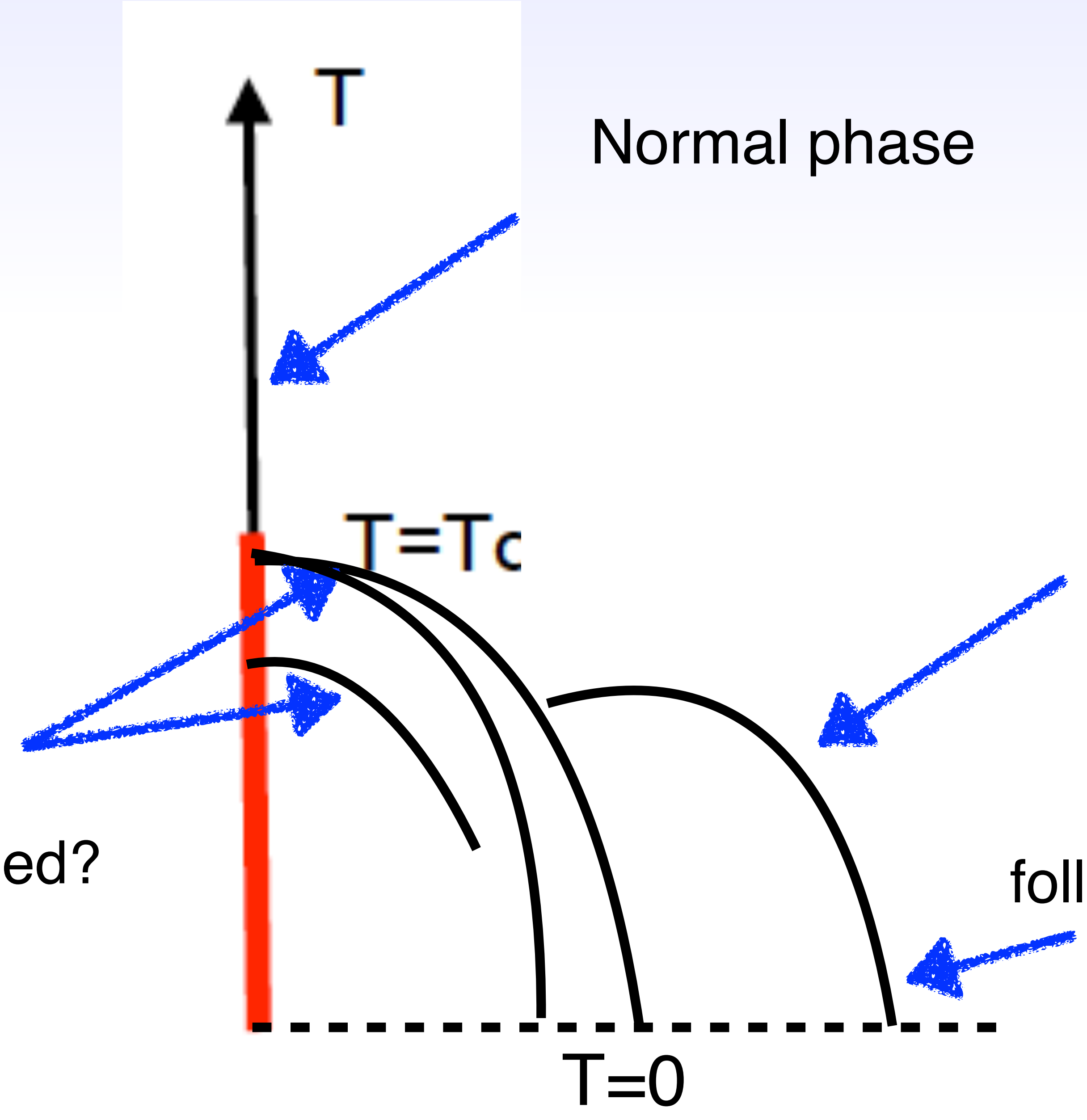
Idea: Distinct phases of matter correspond to different branches of black holes

⇩

Goal: given the desired field theory deformations (e.g. B, μ), construct **all** possible branches of black holes within a theory.

Competition of phases:

- Superconductors, SM phases



Normal phase

Further instabilities
may occur

Instabilities of
normal phase:
which one is realised?

For ground state,
follow preferred branch
down to T=0

Holographic Superconductors in external magnetic field

Holographic Superconductors

Phases of matter in which a charged operator spontaneously acquires an expectation value.

Holographically:

- minimal ingredients:

- finite temperature		black holes in AdS
- finite charge density	$\Longrightarrow$	U(1) gauge field
- an order parameter		charged scalar (s-wave)

- Manifestation of transition: at low T, a new branch of BHs with non-trivial order parameter emerges: U(1) spontaneously broken.

No hair theorems
only in flat space

Strongly coupled SC in magnetic field?

The model: simplest Holo-SC [Hartnoll,Herzog,Horowitz]

$$S = \frac{1}{16\pi\, G} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} F^2 - |(\nabla_\mu + i\, q\, A_\mu)\, \psi|^2 - V \right)$$

Potential: it controls the superconducting ground state

$$V = -6 + m^2\, |\psi|^2 + \frac{u}{2}\, |\psi|^4$$

W-shape

$m^2 < 0,\ u > 0$

to have AdS

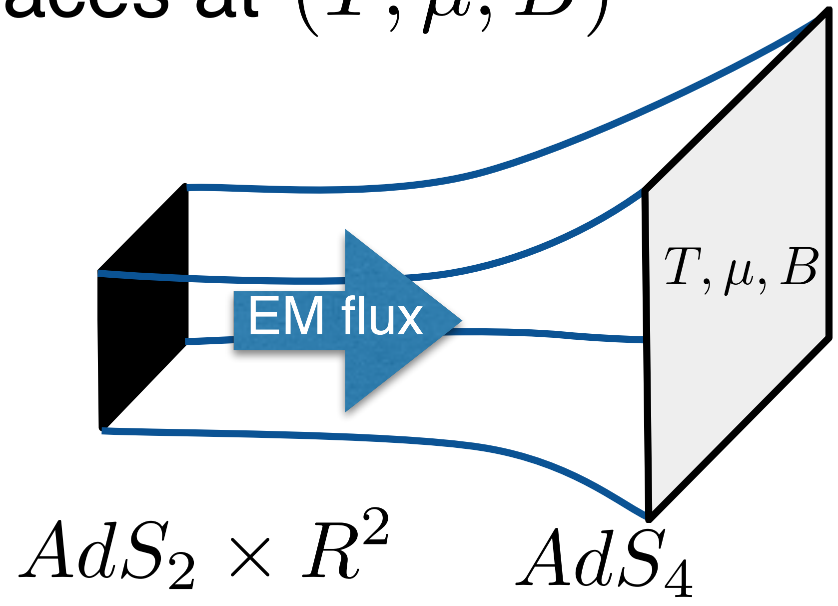
mass term, fixes scaling dimension

$$m^2 = -2 \Rightarrow \Delta = 1, 2$$

$$\psi = \frac{c_s}{r} + \frac{c_v}{r^2} + \cdots \quad r \to \infty$$

Solutions:

- Unit radius AdS_4 vacuum solution: $A = \psi = 0$
  Dual to a d = 3 CFT with a conserved U(1) currents.
- Electric AdS-RN black brane: $\psi = 0,\ A \to \mu dt$
  Dual field theory places at finite temperature and chemical potential, (T, μ)
- Dyonic AdS-RN black brane: $\psi = 0,\ A \to \mu\, dt + Bxdy$
  Dual field theory places at (T, μ, B)



Step 1: Instabilities near the horizon of the BH **EASY**

Study perturbations around the near-horizon limit of the electric AdS-RN:

- Modes tend to be more unstable in this region.
- Use the AdS_2 BF bound criterion to check for instabilities: if the bound is violated, the theory is unstable (converse not always true)

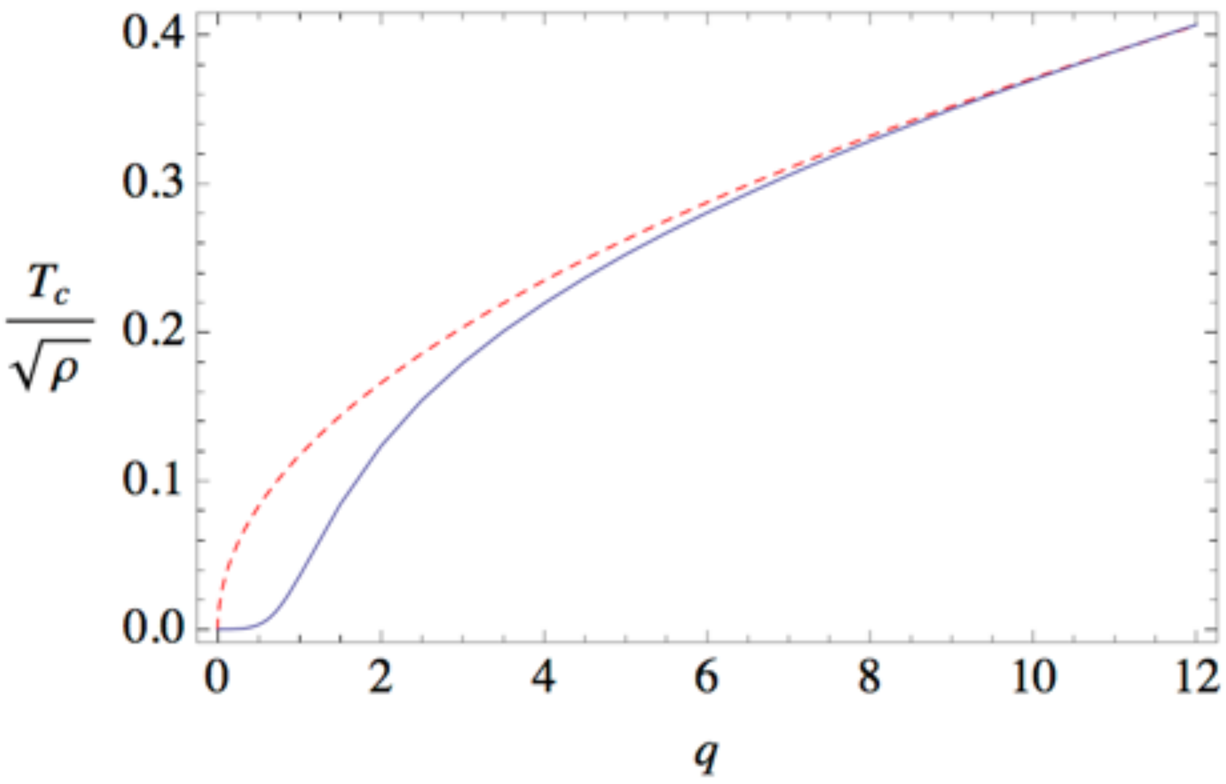
$$\Box_{AdS_2}\psi - L^2_{(2)}M^2\psi = 0, \quad M^2 = m^2 - 2q^2$$

$$L^2_{(D)}M^2 \geq -\frac{(D-1)}{4} \quad \Rightarrow \quad \boxed{m^2 - 2q^2 \geq -\frac{3}{2}}$$

Step 2: Determine T_c **relatively EASY**

- Consider linearised perturbations around the full AdS-RN black hole
- Solve 1 linear 2nd order ODE for the scalar subject to boundary conditions: $\psi = \frac{c_s}{r} + \frac{c_v}{r^2} + \cdots$

[Hartnoll,Herzog,Horowitz]

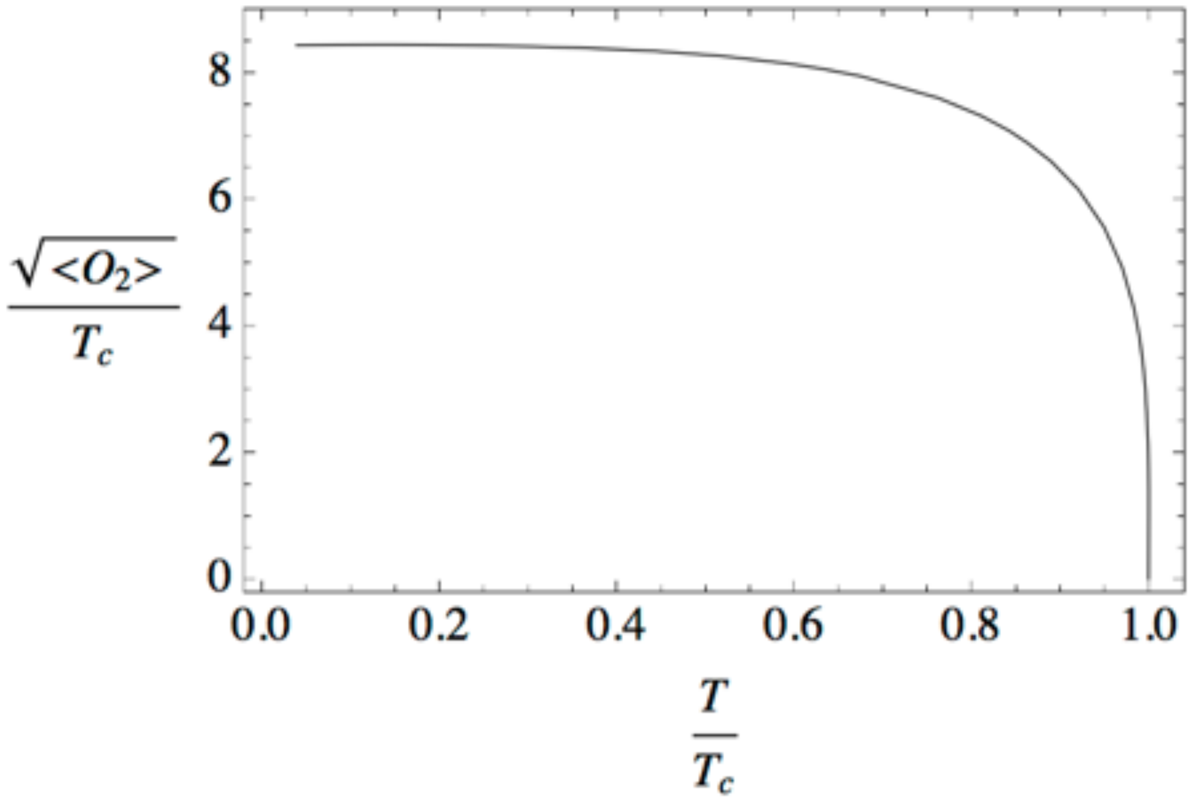


Step 3: Backreacted solutions DIFFICULT

(non-linear coupled ODEs/2D PDEs/3D PDEs)

- Necessary: is the new branch of black holes preferred? What is the ground state?
- Solve 4 non-linear ODEs subject to boundary conditions.

[Hartnoll,Herzog,Horowitz]



$$\frac{\sqrt{\langle O_2 \rangle}}{T_c} \sim \left(1 - \frac{T}{T_c}\right)^{1/2}$$

Value of u in the potential is important at this step

Ground states [Gubser, Nellore]

- For q large, AdS-AdS domain wall: $q\, L_{IR}\psi_{IR} > 1$

$$\begin{aligned} \psi_{IR} &= \sqrt{\frac{-m^2}{u}} \qquad \longleftrightarrow \qquad \psi = 0 \\ L_{IR} &= \sqrt{\frac{-6}{V_{IR}}} \qquad \longleftrightarrow \qquad L = 1 \end{aligned}$$

- For q small, AdS to Lif in IR.

Conductivity: [Hartnoll,Herzog,Horowitz]

- delta function as zero frequency with additional weight
- indications of deviation from quasiparticle picture

Turn on a magnetic field?

We are studying a thin 2D thin superfluid layer:
- for non-zero B, flux lines will penetrate uniformly
- still need to establish is there exist a critical B

SC destroyed as
soon as B non-zero

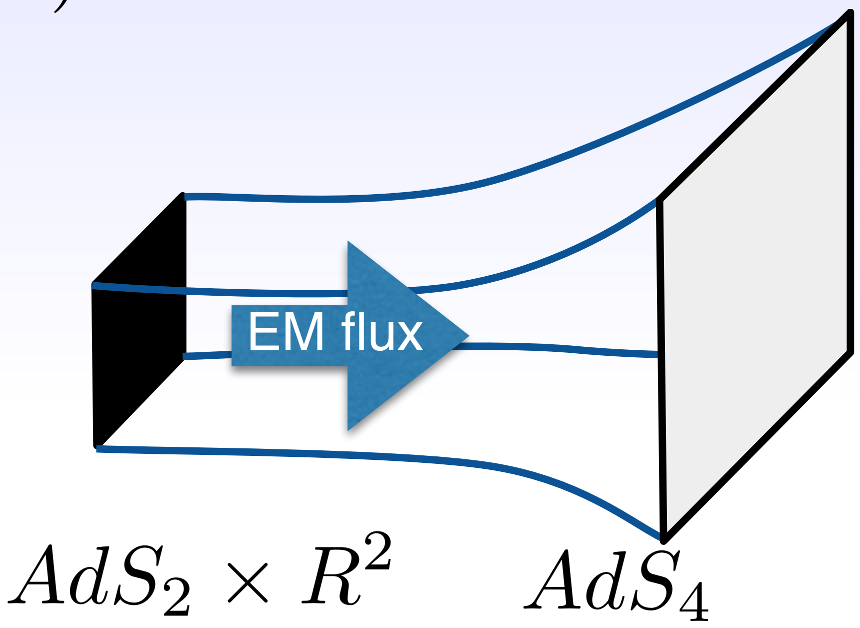
2D Vortex lattice:
SC destroyed only
at the vortex core

Not a droplet lattice
No assumptions, no limits

[Hartnoll,Herzog,Horowitz]
[Albash,Johnson]

Starting Point:

Dyonic AdS-RN black brane: (T, μ, B)
$\psi = 0\,, A = \mu dt + Bxdy$



Step 1:

B contributes in the effective IR mass, but with opposite sign: as B increases, less and less unstable.

$$M^2 = m^2 - 2q^2 + 24q^2B^2 + \mathcal{O}(B^3)$$

Step 2: Determine T$_c$

1 linear PDE for the scalar. Separate variables:

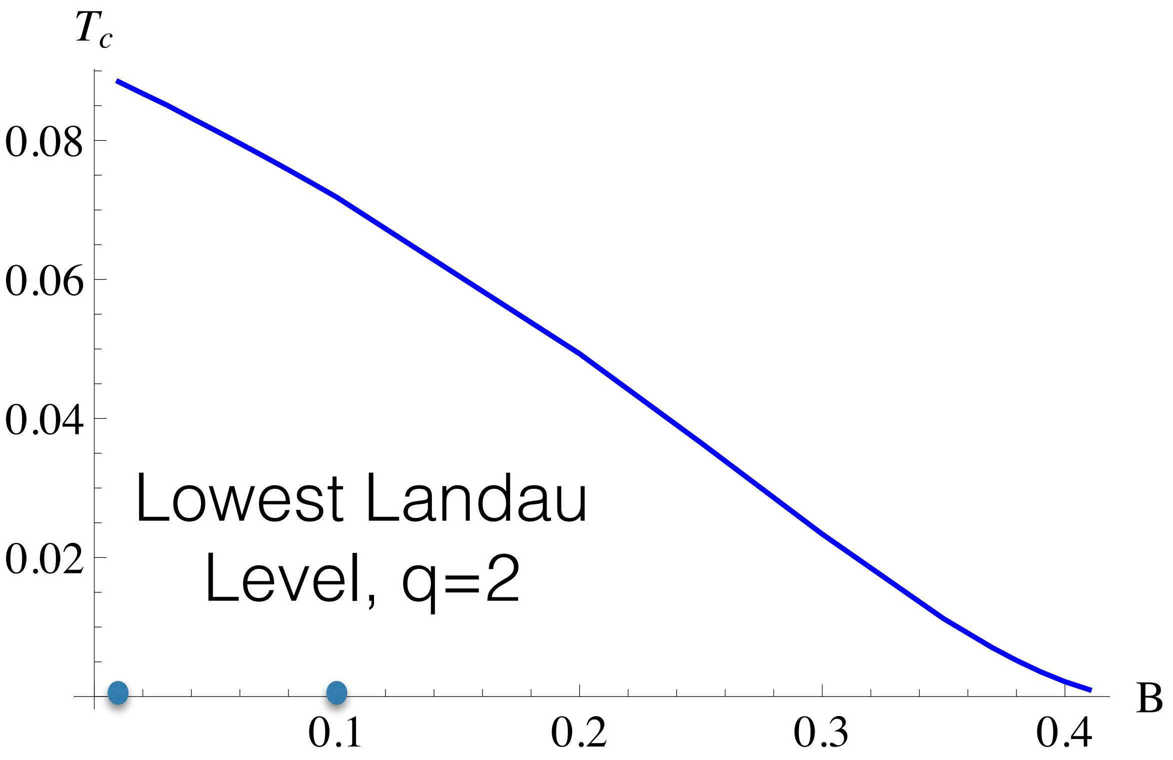
$$\psi(r,x,y) = h(r)\Phi(x,y)$$

- Eigenvalue eqn can be solve fully, subject to boundary conditions:

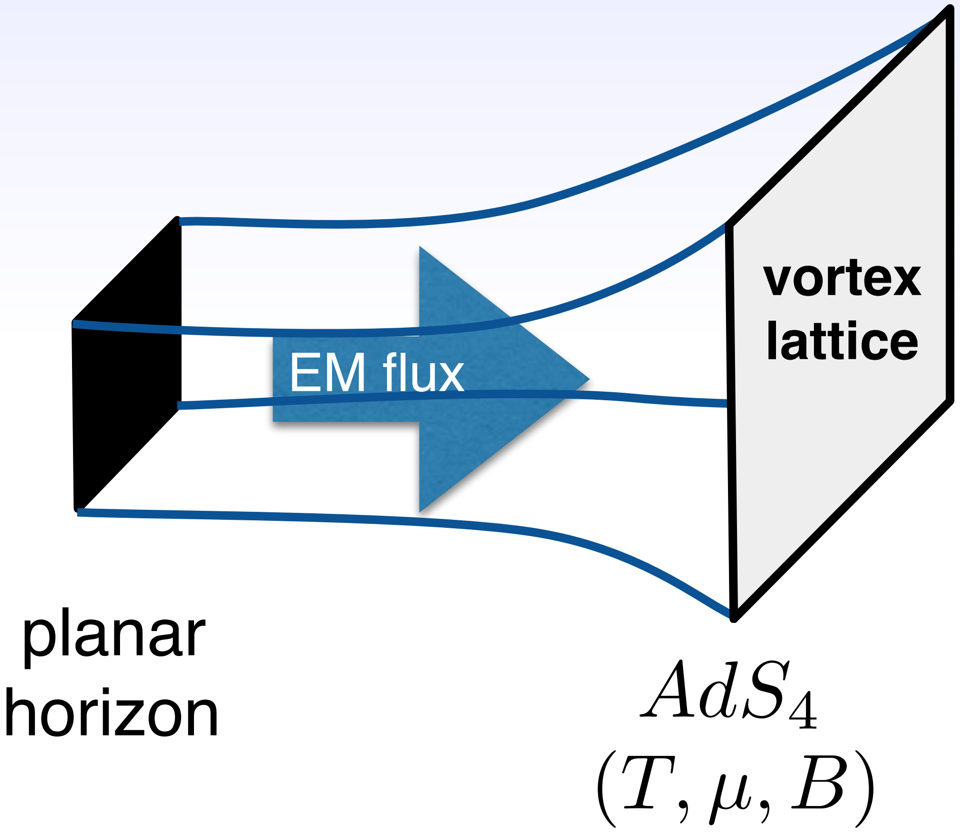
$$D\Phi = \lambda\Phi$$

$$\lambda_j = -qB(2j + 1)$$ Landau Level

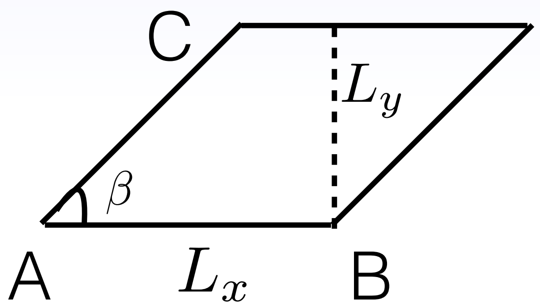
- Radial equation, to solved numerically (depends on λ): T$_c$(j,q,B)



Step 3: DIFFICULT (non-linear coupled ODEs/2D PDEs/3D PDEs)



Each vortex cell: L_x, L_y, v



$$(x, y) \sim (x + L_x, y)$$

$$(x, y) \sim (x + v\, L_y, y + L_y)$$

$$\cos\beta = \frac{v}{\sqrt{v^2 + 1}}$$

Flux quantisation

Actually the problem on the (x, y) plane is only quasi periodic!

$$A = a - Bx dy$$

problematic!

$$D_\mu \psi\left(r, x + v\, L_y, y + L_y\right) = D_\mu \psi\left(r, x, y\right) + \boxed{i\, \psi\, \left(-\delta^x_\mu\, qBL_y\right)}$$

Resolution:

$$\boxed{\begin{aligned} \psi\left(r, x + L_x, y\right) &= \psi\left(r, x, y\right) \\ \psi\left(r, x + v\, L_y, y + L_y\right) &= e^{i\, qBL_y\, x}\, \psi\left(r, x, y\right) \end{aligned}}$$

Compatibility implies quantisation condition: $\boxed{qBL_xL_y = 2\pi n}$

Dimension of moduli space reduced: 2d

flux quanta carried by the vortex

Setting up the problem

- Translation invariance broken in 2 spatial directions: all components are on; **16 PDEs in 3 variables**, (r, x, y)!

- **Gauge fixing:** [Headrick, Kitchen,Wiseman].
  Use the DeTurck trick to dynamically fix the gauge:
  metric, gauge field

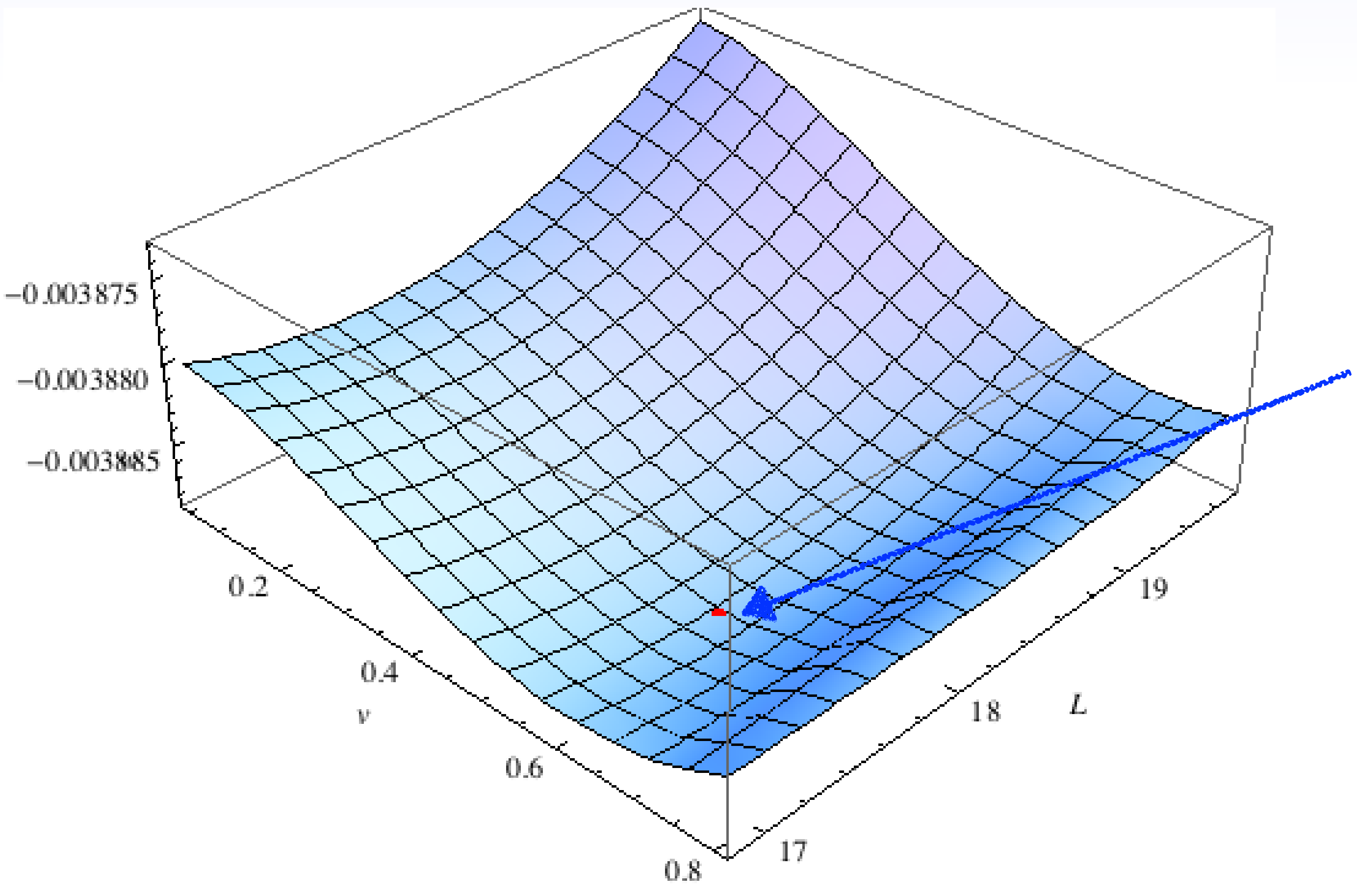
- **Boundary conditions:**

- AdS asymptotics, no sources (spontaneous)
- Regularity at the horizon
  x- direction periodic
  y- quasi-periodic

- Solve the equations numerically using spectral methods on the Hamilton cluster. Check convergence.

“Abrikosov” vortex lattice

Analysing the moduli space of oblique n=1 lattices: $\{L_x, v\}$
free energy for $T/T_c = 0.7$, B=0.01



Expected from Landau-Ginzburg, but only for T~T$_c$
[Abrikosov, ‘57]

↓

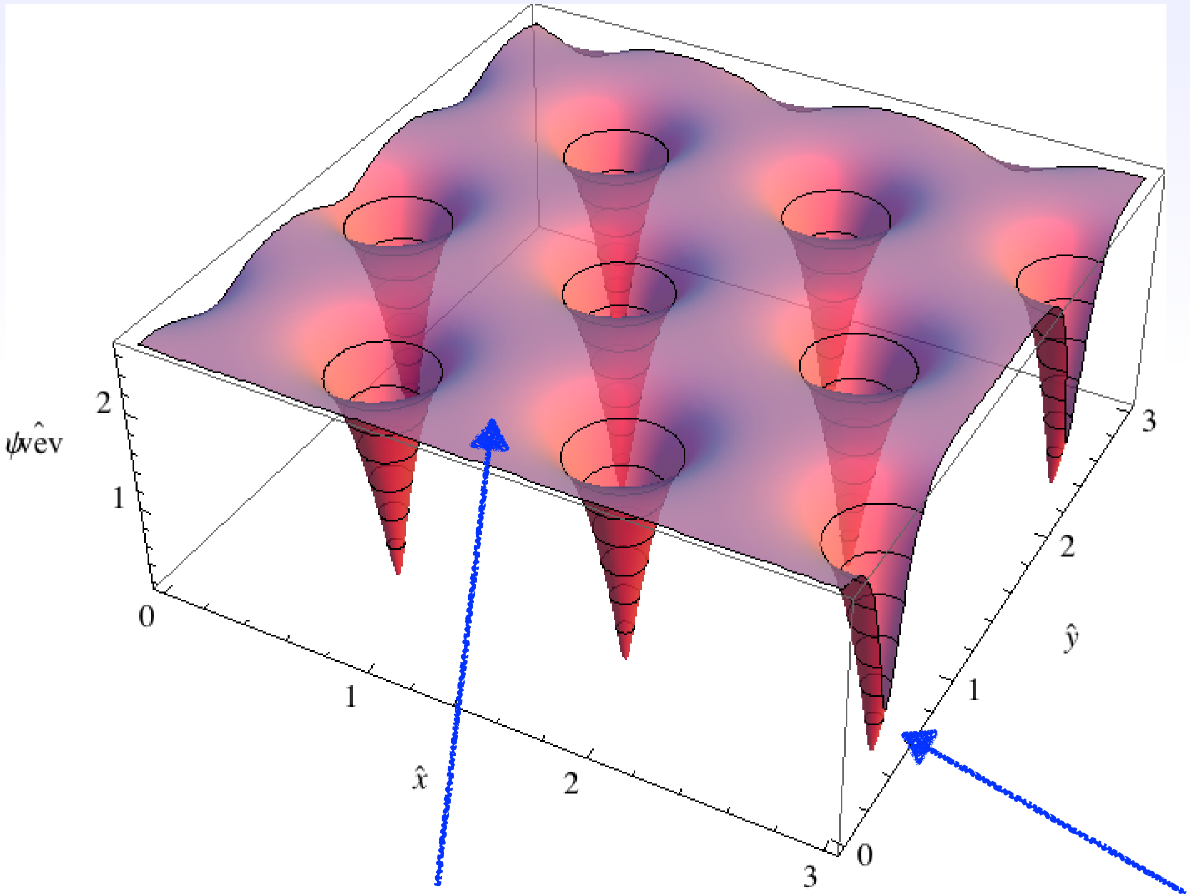
Preferred configuration corresponds to a **triangular** lattice with $L_x = L_y$.
Persists at lower T, higher B.

↑

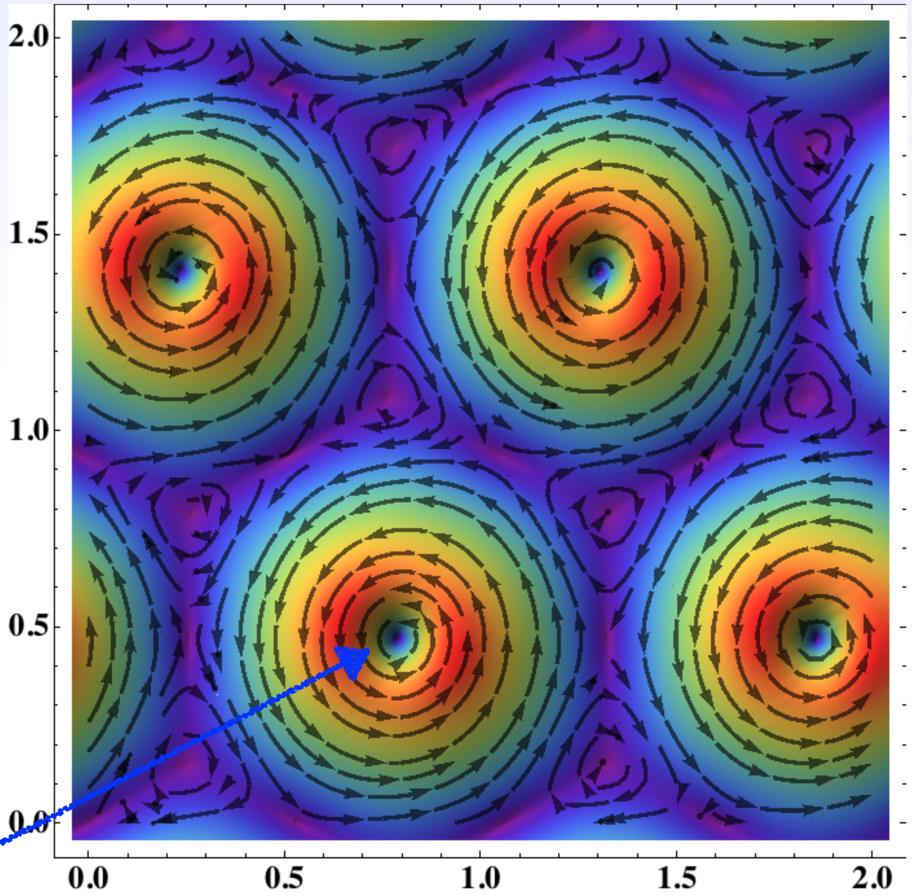
Hints towards novel ground states

Preferred solution

scalar field vev



current



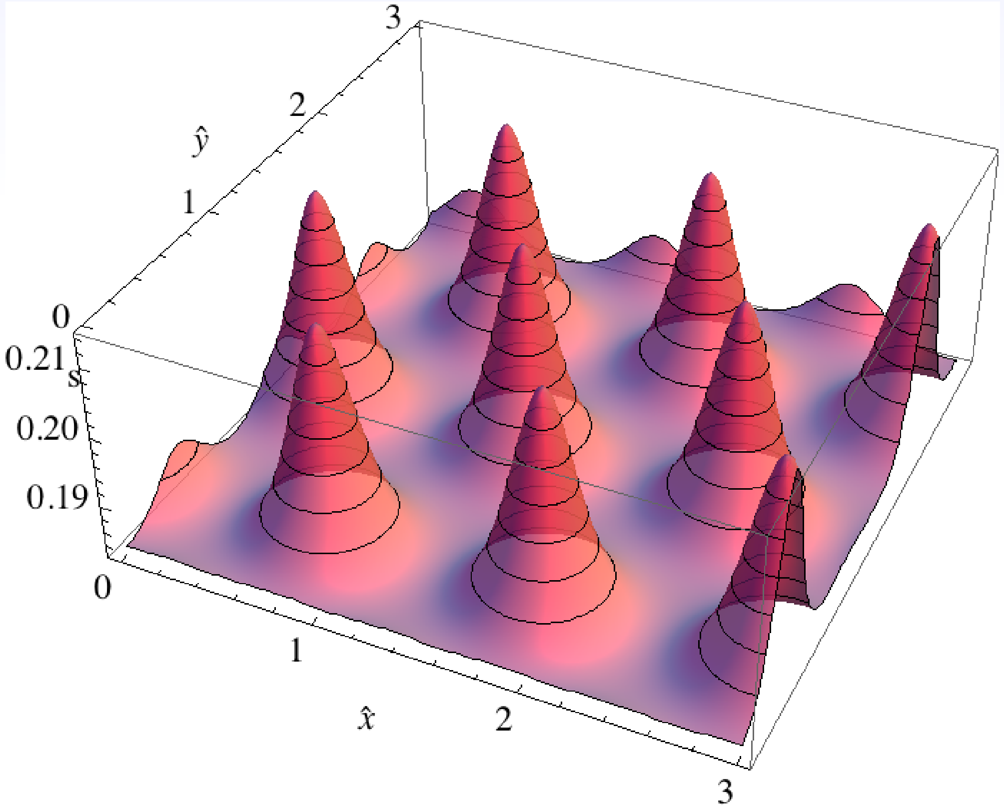
superconducting
state

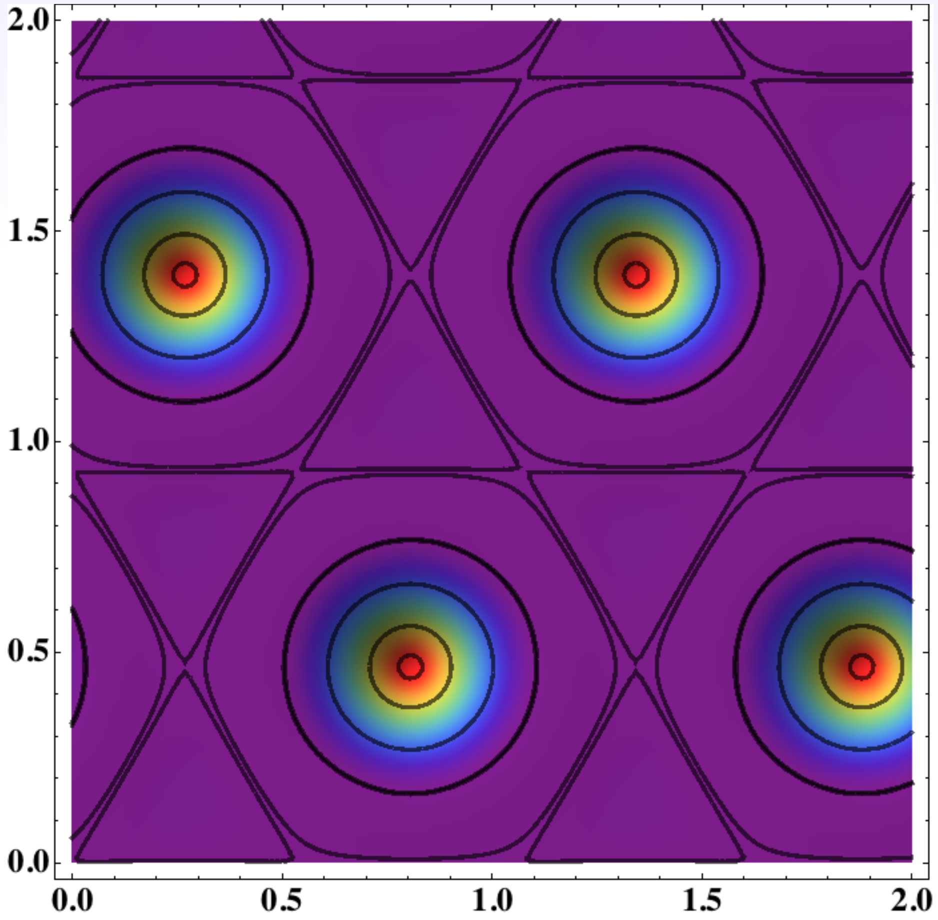
No condensate: normal phase
Flux lines trapped by currents

Thermodynamic quantities

Entropy

Energy-Momentum tensor - T_{tt}





Higher entropy
where no condensate

"Abrikosov" vortex lattice

From the numerics we also see:
Along the preferred branch, the stress tensor is that of a perfect fluid. [Donos,Gauntlett],[Donos,Gauntlet, CP]

average pressure

$$\bar{T}^x_x = \bar{T}^y_y = -\frac{1}{2}\bar{T}^t_t = p$$

bar: average over period

$$\bar{T}^x_y = 0$$

This can easily be understood by the 1st law of thermodynamics when $(T, \mu, B) = fixed$ and $(L_x, v) = vary$:

$$\frac{\delta w}{\delta L_x} = \frac{\delta w}{\delta v} = 0$$

Future directions

- Exploration of parameter space and potentials
- Higher LLs and higher-flux vortices
- Construct the ground state directly and try to match with low-T numerics? connect to BPS configurations?

- Check stability of the configuration: QNM, compute force between vortices?
- Conductivities?
- Spectral functions?

Thanks for listening!