Need for (sound) speed and holography

Niko Jokela

UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

C.Ecker, C.Hoyos, NJ, D.Rodríguez, A.Vuorinen 1609.03480,1707.00521

Bounding Transport and Chaos
in Condensed Matter and Holography
NORDITA – 14 September 2018

- Is speed of sound bounded at strong coupling?
- Why do we care?
- In this talk 3+1d, but many comments apply equally well elsewhere

Outline

1. Speed of sound, equation of state
2. Hints for a bound in holographic models
3. Breaking bounds

Intro

$$\boxed{v_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_s}$$

- Plays an important role in hydro
- By dim.analysis $p \to T^4$, $T \to \infty$, $\epsilon = T\frac{\partial p}{\partial T} - p \to 3p$ and $v_s^2 \to 1/3$
- Approach $1/3$ always from below? E.g. in QCD from asymptotic freedom:

$$v_s^2 \approx \frac{1}{3} + \frac{5N_c}{36\pi}\beta(\alpha) < \frac{1}{3} \ .$$

- What about strongly interacting systems?

Equation of state and stiffness

$$\boxed{v_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_s}$$

- Equation of state fixes

$$p = p(\epsilon)$$

- In a CFT

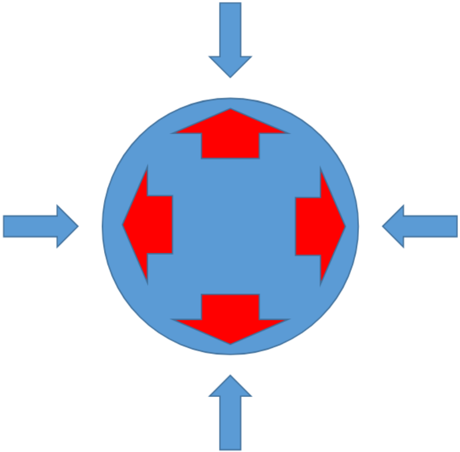
$$\langle T^\mu_\mu \rangle = 0 = -\epsilon + 3p \to v_s^2 = 1/3$$

- Causality restricts the EoS:

$$p \leq \epsilon$$

Equation of state and stiffness

- Compress the fluid and the energy increases
- The pressure also increases which opposes compression
- The larger $\partial p/\partial \epsilon$, the less compressible the fluid is
- The EoS is stiffer (softer) the larger (smaller) v_s



Examples of soft EoS in holography

- First computed in $\mathcal{N} = 4$

[Policastro-Son-Starinets'02]

$$v_s^2 = \frac{1}{3}$$

- "Mass deformed $\mathcal{N} = 4$", $\mathcal{N} = 2^*$:

[Benincasa-Buchel-Starinets'05]

$$v_s = \frac{1}{\sqrt{3}} \left(1 - \frac{\Gamma(3/4)^4}{3\pi^4} (m_f / T)^2 - \frac{1}{18\pi^2} (m_b / T)^4 + \dots \right) .$$

- Klebanov-Strassler $\mathcal{N} = 1$:

[Aharony-Buchel-Yarom'05]

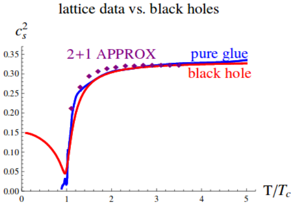
$$v_s^2 = \frac{1}{3} - \frac{2}{9} \frac{1}{\log \frac{T}{\Lambda}} + \dots$$

Class of holographic models and lattice QCD

- For a family of models at high temp
  [Gubser-Nellore'08, Cherman-Cohen-Nellore'09, Hohler-Stephanov'09]

$$S = \frac{1}{2\kappa_5^2}\int d^5x\left(R - \frac{1}{2}(\partial\phi)^2 + V(\phi)\right)$$

$$v_s^2 \simeq \frac{1}{3} - C(\Delta)(LT)^{\Delta-4}$$



- Also for several scalars below conformal value
  [Cherman-Nellore'09]
- Scalars at finite density:
  [Yang-Yuan'17]

$$\left(\frac{\partial p}{\partial \epsilon}\right)_\mu \leq \frac{1}{\sqrt{3}}$$

- D3-probe-D7

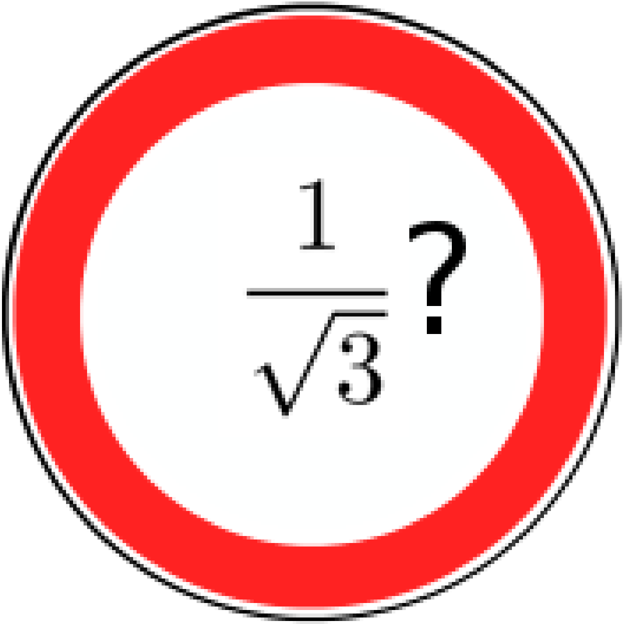
[Mateos-Myers-Thomson'07]

$$v_s^2 = \frac{1}{3} - \frac{\lambda_{\rm YM}}{24\pi^2}\frac{N_f}{N_c}\left(mc + \frac{1}{3}mT\frac{\partial c}{\partial T}\right)$$

- Also at $m < \mu \neq 0 = T$:

[Karch-Son-Starinets'08,Kulaxizi-Parnachev'08]

$$v_s^2 = \frac{1-(m/\mu)^2}{3-(m/\mu)^2} = \frac{1}{3} - \frac{2}{9}(m/\mu)^2 + \dots$$


$\frac{1}{\sqrt{3}}$?

Field theory counterexample

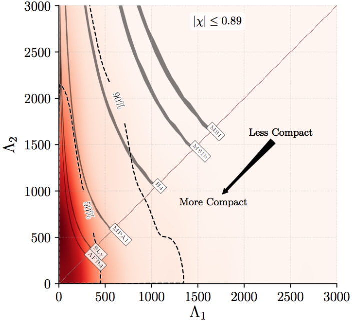
- Consider QCD at finite isopin density
- Can do lattice, no sign problem
- For $\mu_I > m_\pi$ the pion condenses
- Get larger speeds:

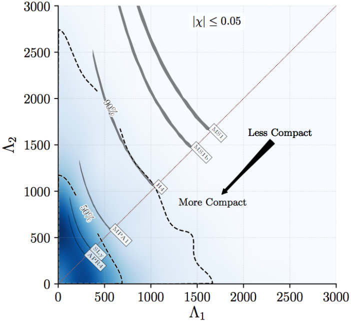
[Son-Stephanov'00]

$$v_s^2 = \frac{1-(m_\pi/\mu_I)^2}{1+3(m_\pi/\mu_I)^2} \ \rightarrow \ 1 \ , \ \ m_\pi/\mu_I \rightarrow 0$$

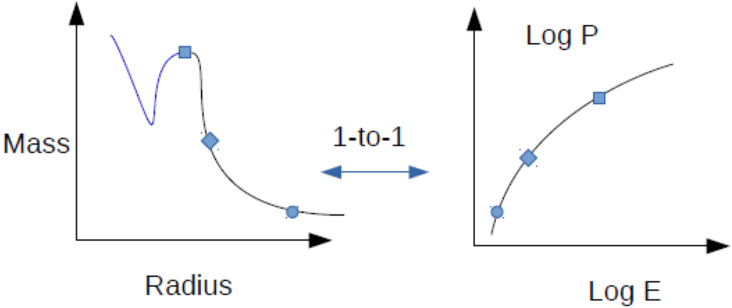
Neutron star mass measurements

- Two-solar-mass stars
  [Demorest et al.'10]
  [Antoniadis et al.'13]
- Low squishiness
  [LIGO/Virgo'17]





Need for speed



- Maximum mass depends on the EoS $\rightarrow$ need stiff
- Bound on v_s strongly disfavored

[Bedaque-Steiner'14]

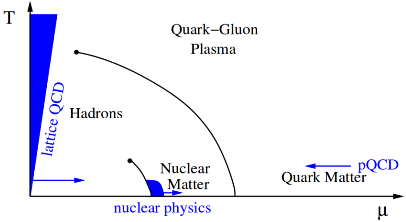
- Relativistic kinetic theory, causality implies (Taub's inequality):

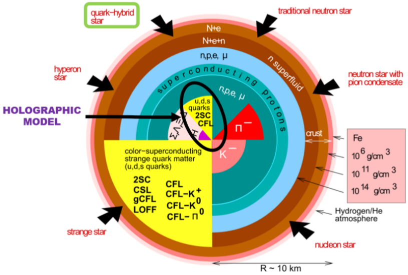
$$\tau = \frac{\epsilon(\epsilon - 3p)}{\rho_m^2} > 1$$

- D3-probe-D7:

$$\tau_{D7} = \frac{3}{4}\left(1 + \frac{m^2}{3\mu_q^2}\right) \;\rightarrow\; \frac{3}{4} \leq \tau_{D7} \leq 1$$

- Taub's inequality is violated $\leftrightarrow$ strongly coupled and no quasiparticle description






NEED FOR
SPEED
NO LIMITS

Breaking bounds in holography

Let's list several cases where $1/\sqrt{3}$ is exceeded:

- Dp/Dq:($n|p \perp q$), n=common spatial directions ($q \geq p$) [lots,review:NJ-Ramallo'14]

$$v_s^2 = \frac{1}{n + \frac{p-3}{4}(p+q-2n-8)}$$

Dp/Dq	n	$SUSY$	$v_s^2 > 1/n$
$Dp/D(p+4)$	p	✓	$p>3$
$Dp/D(p+2)$	$p-1$	✓	$p>3$
Dp/Dp	$p-2$	✓	$p>3$
$D4/D8/\overline{D8}$	3	✗	✓
$D3/D7'$	2	✗	✗
$D2/D8'$	2	✗	✓

- Non-relativistic Lifshitz scaling, get stiff EoS:

$$(t, x^{i=1,\dots,n}) \to (\lambda^z t, \lambda x^{i=1,\dots,n}) \Rightarrow p = \frac{z}{n}\epsilon$$

- Dp/Dq-like with (Lifshitz optional) hyperscaling violation [Järvelä-NJ-Ramallo'16]

Breaking it non-trivially

- Common to all above trivial cases: non-CFT in the UV
- With UV AdS, can get

$$v_s^2 > \frac{1}{3}$$

  - Scalars at finite density [Hoyos-NJ-Rodriguez-Vuorinen'16,Ecker-Hoyos-NJ-Rodriguez-Vuorinen'17]
    (rest of the talk)
  - Double trace deformations [Anabalon-Andrade-Astefanesei-Mann'17]
  - Dynamical magnetic field [Grozdanov-Poovuttikul'17]
  - Backreacted flavors, flowing down to anisotropic IR [NJ et al.'18]

Holographic models that break it non-trivially

- A family of Einstein-Maxwell and charged scalar models

$$S = \frac{1}{16\pi G_5}\int d^5x \left(R - L^2 K(\phi) F^2 - |D_\mu\phi|^2 - V(\phi)\right)$$
$$D_\mu\phi = (\partial_\mu - iqA_\mu)\phi\ ,\ \ m^2L^2 = \Delta(\Delta-4)$$

- Bottom-up models:

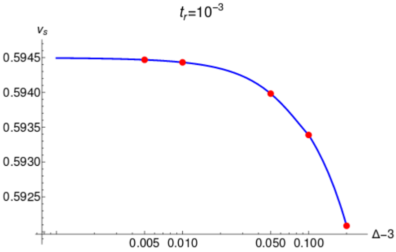
$$K(\phi) = 1\ ,\ \ V(\phi) \simeq -\frac{12}{L^2} + m^2|\phi|^2 + \frac{V_4}{2L^2}|\phi|^4 + \dots$$

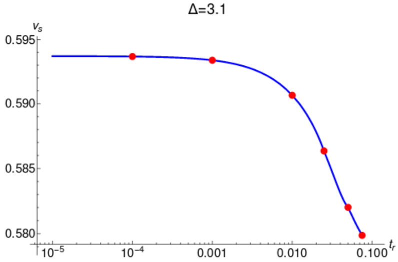
- Top-down: $\mathcal{N}=4$ SU(N_c) SYM with $U(1)$ R-charge (three equal), $\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + m_0 \mathrm{tr}\lambda\lambda$; $q=2$, $\Delta=3$, $V_4 = -\frac{3}{2}$: [Gunaydin-Romans-Warner'86,Bobev-Kundu-Pilch-Warner'10]

$$K(\phi) = \frac{8}{(1-|\phi|^2)^2}\ ,\ \ V(\phi) = -\frac{12}{L^2}\frac{1+|\phi|^4}{(1-|\phi|^2)^2}$$

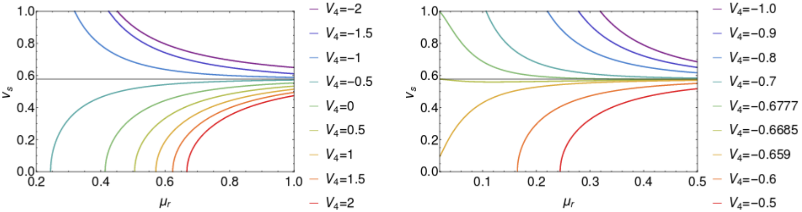
- Finite density $\mu \neq 0$ and break conformal symmetry explicitly by sourcing ϕ

Simplest case



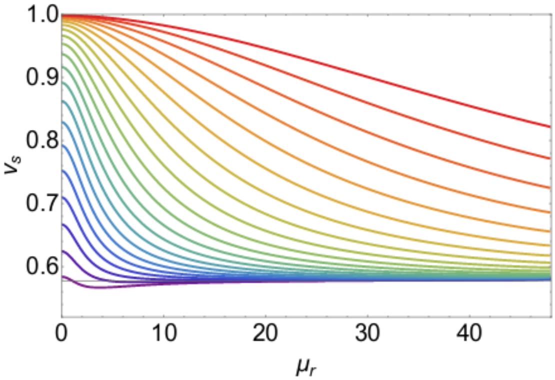


- Start w/ $V(\phi) = -\frac{12}{L^2} + m^2|\phi|^2$
- Technical simplification, we compute $v_s^2 \simeq \left(\frac{\partial p}{\partial \epsilon}\right)_T, \mu \gg T$
- All quantities divided by the source: $t_r = \frac{T}{m_0}$
- Get at most 3% above conformal value
- Need for more speed! Better to go toward $\Delta \to 3$



- Quartic $V(\phi) = -\frac{12}{L^2} + m^2|\phi|^2 + \frac{V_4}{2L^2}|\phi|^4$
- Consider $\Delta = 3, q = 0, t_r = 0.1$
- Making V_4 more negative (positive) increases (decreases) v_s
- In the physical window ($1 \geq v_s \geq 0$):
    - Thermodynamically stable ($\chi = \partial^2 p/\partial\mu^2 > 0$)
    - Dynamically stable (QNMs in LHP at $k = 0$)

Top-down model



- $q = 2,\ \Delta = 3,$ "$V_4 = -\frac{3}{2}$"
- Varying dynamically generated scale $\Lambda = m_0 e^{-\kappa_1}$
- Large hierarchy $\Lambda \gg m_0$ possible $\rightarrow \epsilon \sim p$ when $\mu \sim m_0$
- No instabilities

Conclusions

- There is no bound
- Not trivial to get past $1/\sqrt{3}$ though, need:
  - fine-tuning or
  - dynamically generated scale or
  - breaking some spacetime symmetry
- Physics understanding for getting stiff EoS
  [work in progress w/ Hoyos]
- For neutron star EoS, need to engineer top-down QCD-like model ($q = 0$ and baryon charge)
  [work in progress w/ Hoyos & Henriksson]

Thanks to organizers for a great program!