Interpolating between strong and weak coupling in thermal QFTs with gravity duals

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S.Grozdanov and A.Starinets:
Teor. Mat. Fiz. 182, no. 1, 76 (2014)
1611.07053 [hep-th]
S.Grozdanov, N.Kaplis and A.Starinets
1605.02173 [hep-th]
S.Grozdanov, N.Gushterov, P.Tadic and A.Starinets
18XX.XXXXX [hep-th]
J.Casalderrey Solana, S.Grozdanov, and A.Starinets
1806.10997 [hep-th]

BOUNDING TRANSPORT AND CHAOS IN CONDENSED MATTER AND HOLOGRAPHY
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Motivation and three examples:

weak-strong coupling interpolation for

1) Zero-temperature observables

2) Thermodynamic observables

3) Transport

Motivation: ongoing experimental programs and theoretical advances of the last two decades

Experiments:

Experiments on heavy ion collisions at RHIC (2000-current) and LHC (2010-current)
(relativistic, “many-body”, ***strongly interacting***, non-equilibrium “hot” system)

Romatschke and Romatschke, ``Relativistic Fluid Dynamics Out of Equilibrium : Ten Years of Progress in Theory and Numerical Simulations of Nuclear Collisions,'' arXiv:1712.05815 [nucl-th]. Busza, Rajagopal and van der Schee, ``Heavy Ion Collisions: The Big Picture, and the Big Questions,’’ arXiv:1802.04801 [hep-ph].

Experimental realization (1995-1999) of new classes of quantum “many-body” systems
(e.g. ultra-cold atomic Bose and Fermi gases),
current extensive study of their collective behavior
(non-relativistic, “many-body”, ***strongly interacting***, non-equilibrium “cold” system)

Theory:

Gauge-string duality: A “new” (1997) non-perturbative tool to study ***strongly interacting*** quantum systems
(zero or finite temperature/density, relativistic and non-relativistic, equilibrium and non-equilibrium – but for limited class of theories/parameters)

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

- Field content:

$A_\mu \qquad \Phi_I \qquad \Psi_\alpha^A \qquad$ all in the adjoint of $SU(N)$

$I = 1 \ldots 6 \qquad A = 1 \ldots 4$

- Action:

$$S = \frac{1}{g_{YM}^2} \int d^4x \, \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_I)^2 - \frac{1}{2}[\Phi_I, \Phi_J]^2 + i\bar{\Psi}\Gamma^\mu D_\mu \Psi - \bar{\Psi}\Gamma^I [\Phi_I, \Psi] \right\}$$

- Large N: effective coupling = 't Hooft coupling $\lambda = g_{YM}^2 N$

(super)conformal field theory = coupling doesn't run

Interpolation between weak and strong coupling: *exact results are rare (even at T=0)…*

Example (old & beautiful): expectation value of a circular Wilson loop in

$\mathcal{N} = 4\ SU(N_c)$ SYM in $d = 4$ in the limit $N_c \to \infty, \lambda \equiv g^2_{YM} N_C$

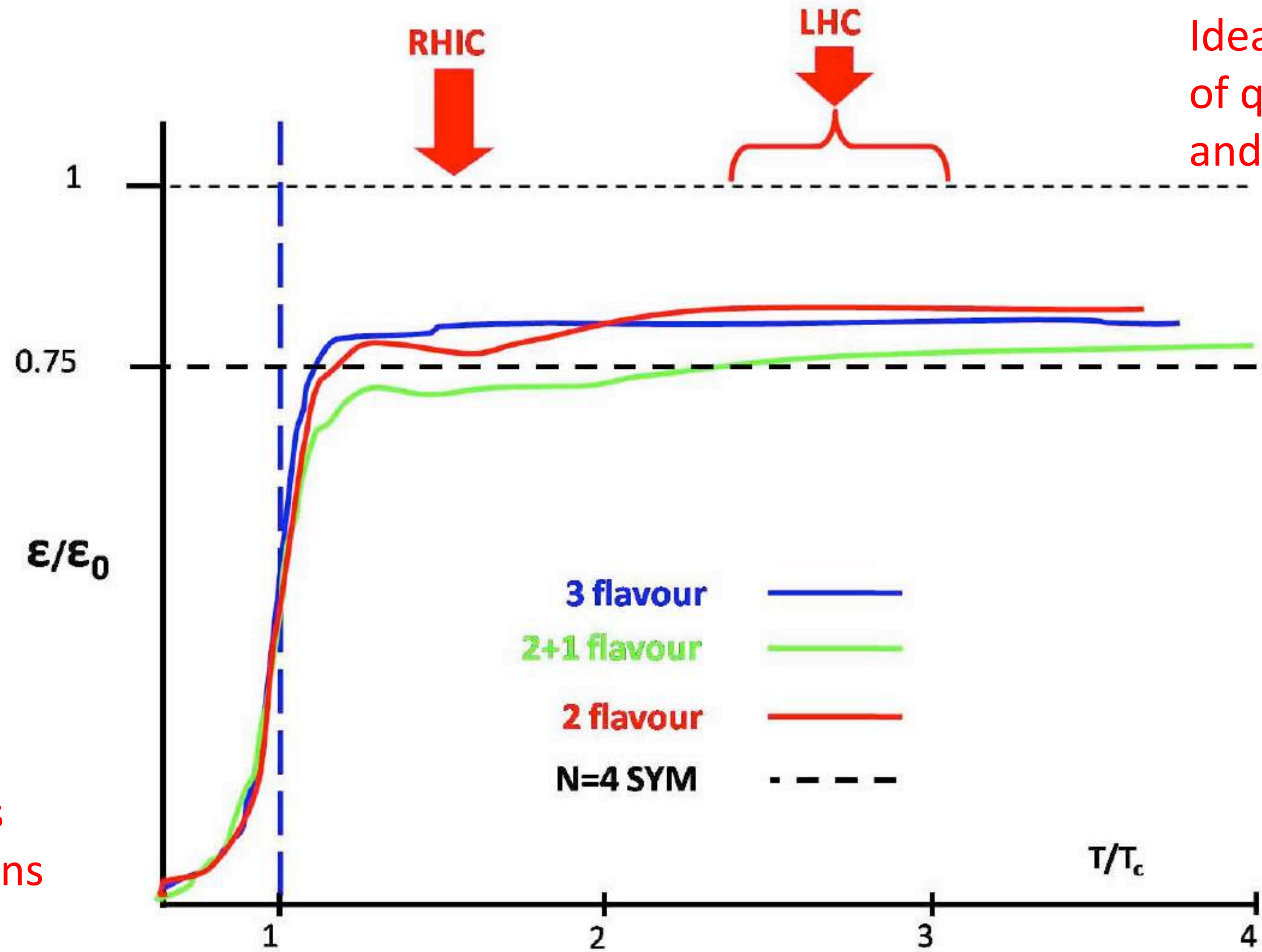
$$\langle W_C \rangle = \frac{2}{\sqrt{\lambda}} I_1\left(2\sqrt{\lambda}\right)$$

$$\langle W_C \rangle = 1 + \frac{\lambda}{4} + \frac{\lambda^2}{48} + \cdots \qquad \lambda \ll 1$$

$$\langle W_C \rangle \sim \sqrt{\frac{2}{\pi}} \frac{e^{\sqrt{2\lambda}}}{(2\lambda)^{3/4}} + \cdots \qquad \lambda \gg 1$$

Erickson, Semenoff and Zarembo (2000)

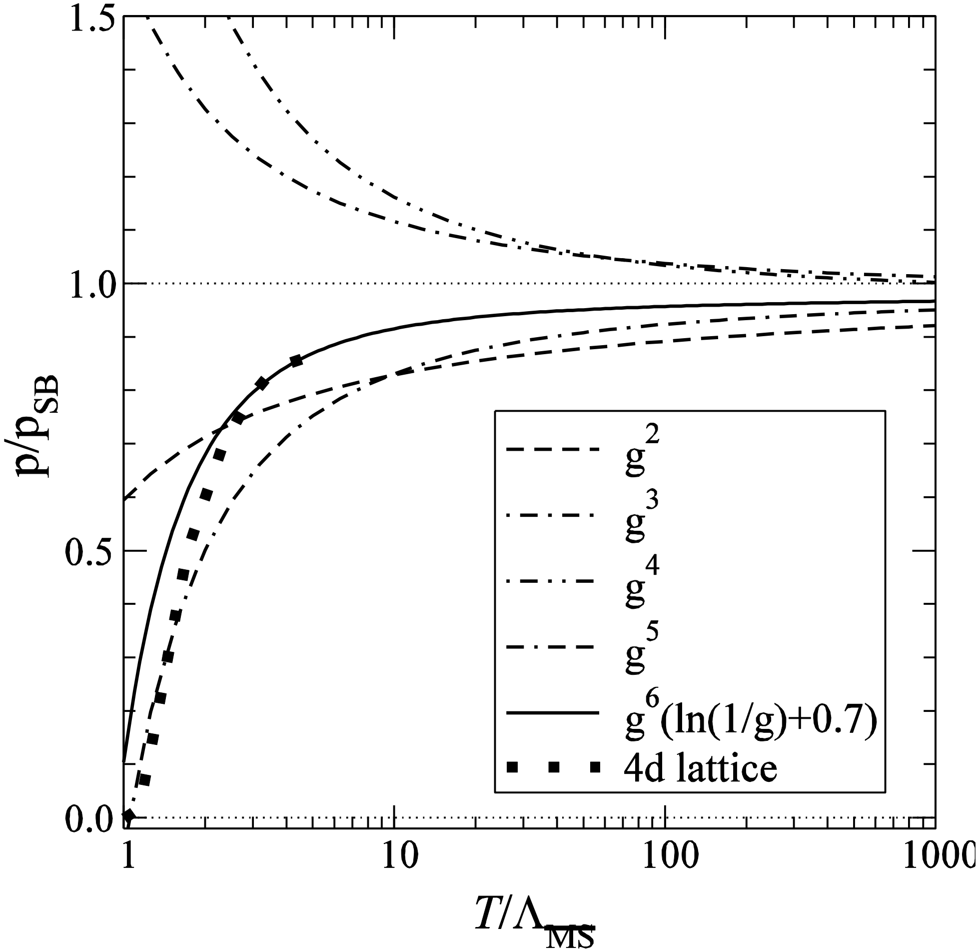
Energy density vs temperature for various gauge theories



Ideal gas
of hadrons

Figure: artist's impression based on LQCD, from Myers and Vazquez, 0804.2423 [hep-th]

Pressure in perturbative QCD



Entropy density of $\mathcal{N}=4$ SYM in the planar limit ($N \to \infty$)

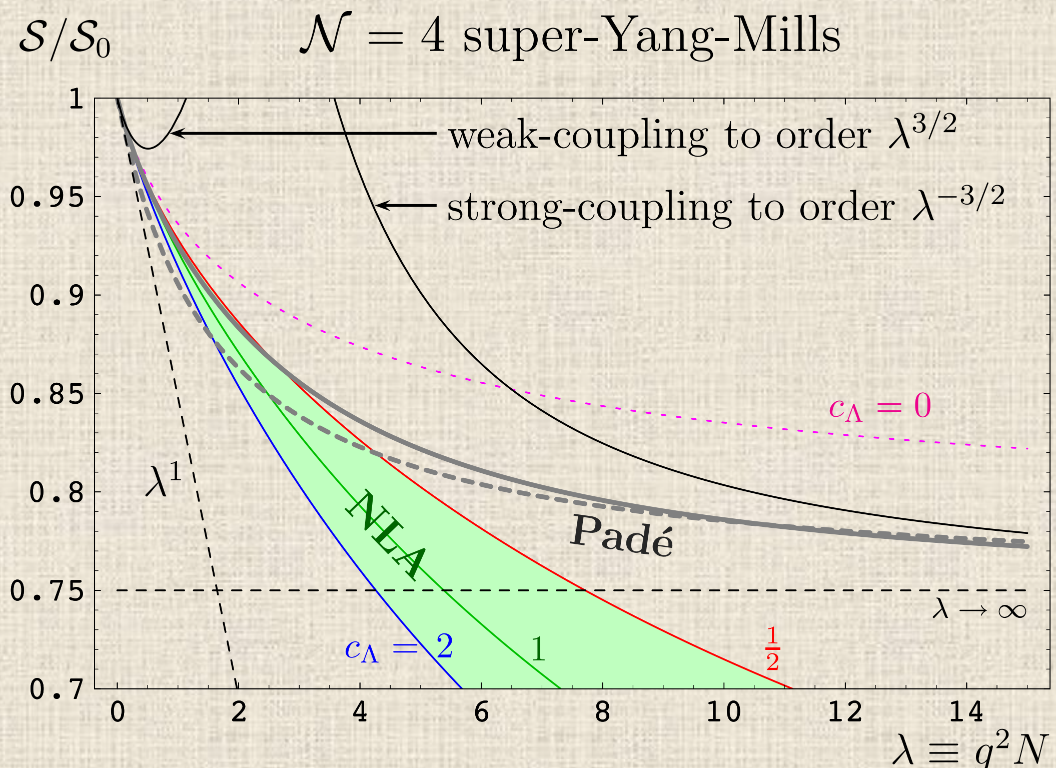


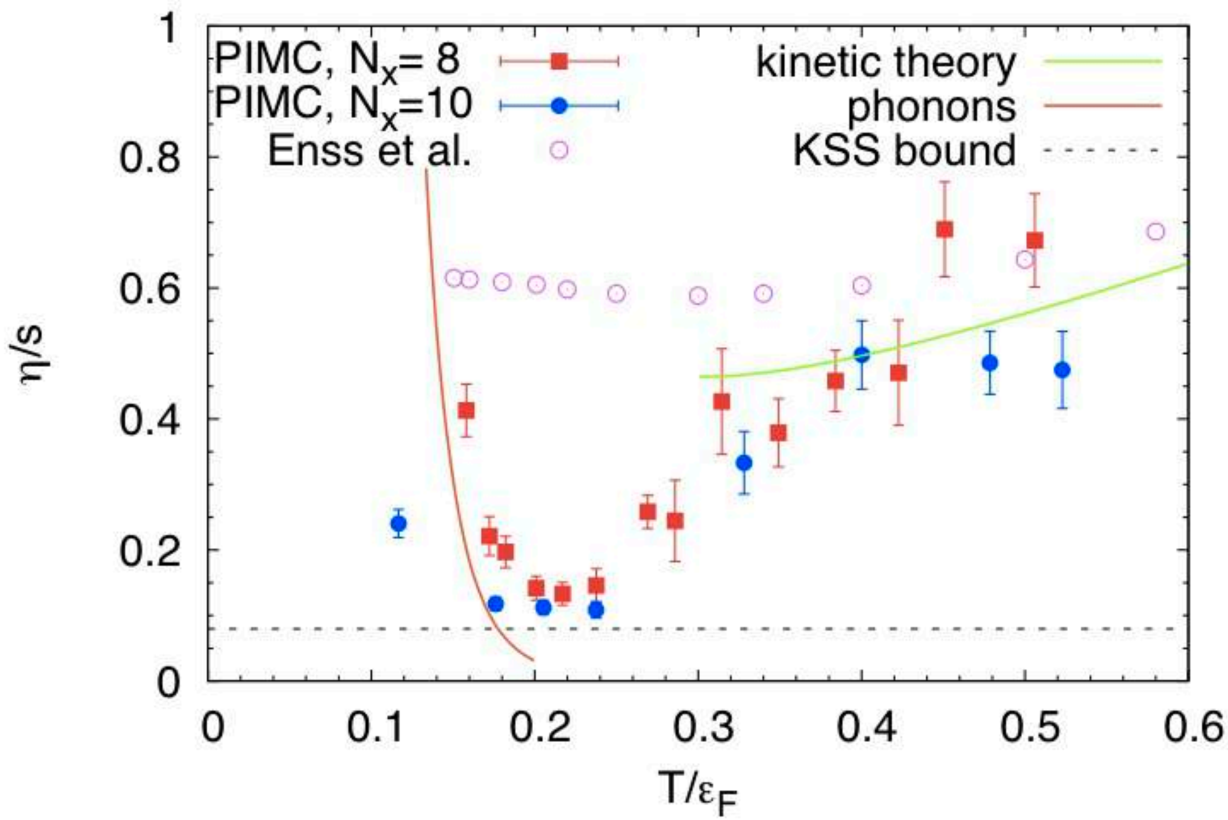
Fig from Blaizot, Iancu, Kraemmer and Rebhan, hep-ph/0611393

$\lambda \ll 1$ $\qquad s/s_0 = 1 - \frac{3}{2\pi^2}\lambda + \frac{\sqrt{2}+3}{\pi^3}\lambda^{3/2} + \cdots$ Fotopoulos and Taylor, hep-th/9811224

$\lambda \gg 1$ $\qquad s/s_0 = \frac{3}{4} + \frac{45}{32}\zeta(3)\,\lambda^{-3/2} + \cdots$ Gubser, Klebanov and Tseytlin, hep-th/9805156

$s_0 = \frac{2\pi^2}{3} N_c^2 T^3$ - Stefan-Boltzmann (free gas)

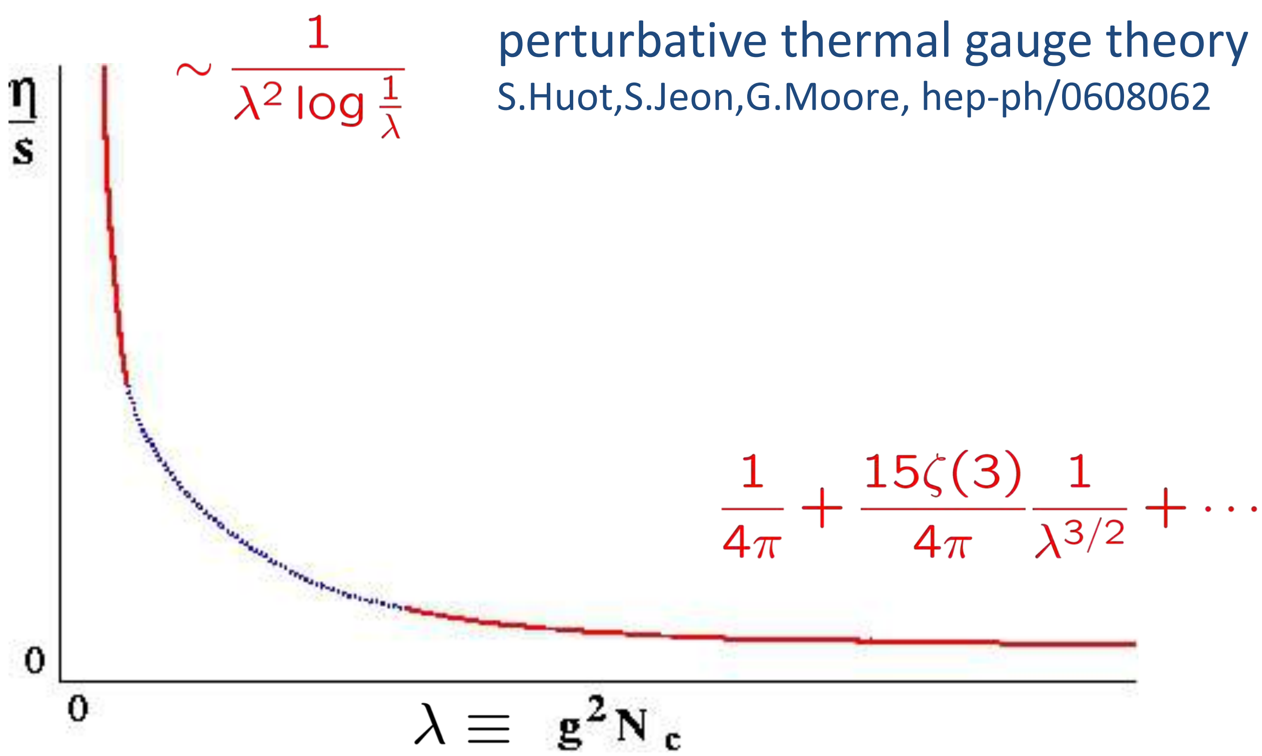
Viscosity-entropy ratio in Unitary Fermi gas



G.Vlazlowski, P.Magierski, J.E.Drut, arXiv:1204.0270 [cond-mat.quant-gas]

$$(\eta/s)_{\min} \sim 1.38 \text{ in units of } \frac{\hbar}{4\pi k_B}$$

Shear viscosity in $\mathcal{N} = 4$ SYM



Correction to $1/4\pi$: Buchel, Liu, A.O.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Plan

Transport properties and analytic structure of correlation functions in weakly interacting many-body quantum system (particles or quasiparticles)

Transport properties and analytic structure of correlation functions in strongly interacting many-body quantum systems (from holography - dual gravity)

Real systems are at intermediate coupling (e.g. QGP)

The problem of interpolation between weak and strong coupling is non-trivial

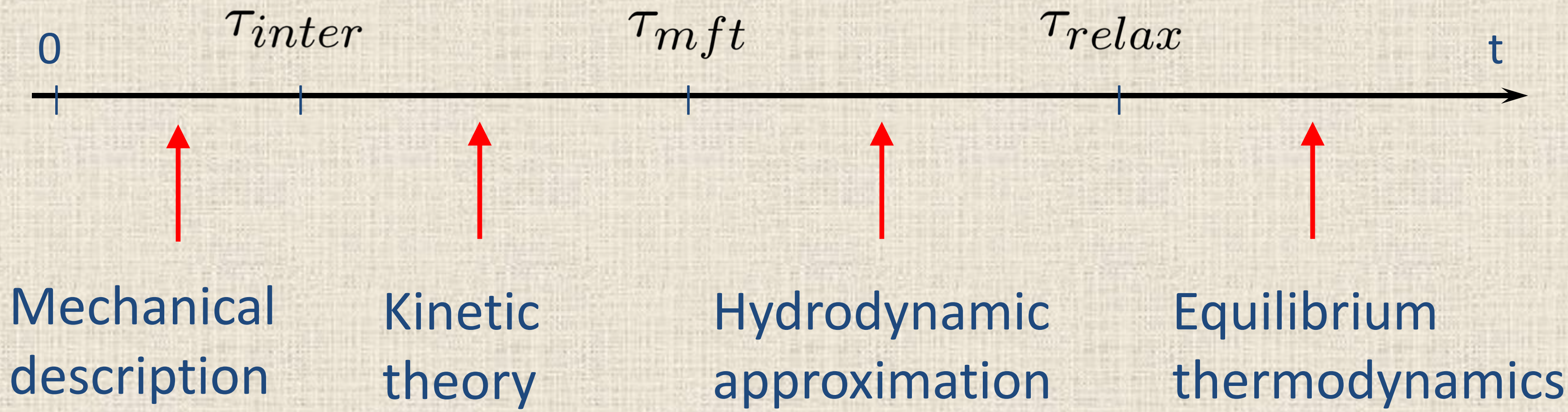
We compute (inverse) coupling corrections using two dual higher-derivative actions - R^2 (Gauss-Bonnet) and R^4 (dual to N=4 SYM) and argue that results are consistent with expectations from (interpolated) weak coupling calculations

Weak coupling

Hydrodynamic regime in kinetic theory

Hierarchy of times (e.g. in Bogolyubov's kinetic theory)

$$\tau_{mft} \ll t \ll T_{existence}$$



Hierarchy of scales

$$l_{mfp} \ll l \ll L$$

(L is a macroscopic size of a system)

The hydrodynamic regime (continued)

Degrees of freedom

0 | τ_{inter} | τ_{mft} | τ_{relax} | t

Mechanical description

Coordinates, momenta of individual particles

Kinetic theory

Coordinate- and time-dependent distribution functions

Hydrodynamic approximation

Local densities of conserved charges

$\epsilon(x,t)$

Equilibrium thermodynamics

Globally conserved charges

E

Hydro regime: $\boxed{\tau_{\text{micro}} \ll \tau \ll t_{\text{global}}}$ $\boxed{l_{\text{micro}} \ll l \ll L_{\text{global}}}$

Relaxation time in kinetic theory

Kinetic equation
$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = C[F]$$

Linearized by
$$F(t,\mathbf{r},\mathbf{p}) = F_0(\mathbf{r},\mathbf{p})\left[1+\varphi(t,\mathbf{r},\mathbf{p})\right]$$

Leads to
$$\frac{\partial \varphi}{\partial t} = -\frac{p_i}{m}\frac{\partial \varphi}{\partial r^i} + \frac{\partial U(r)}{\partial r^i}\frac{\partial \varphi}{\partial p_i} + L_0[\varphi]$$

For spatially homogeneous distributions:
$$\varphi(t,\mathbf{p}) = e^{-\nu t}h(\mathbf{p})$$

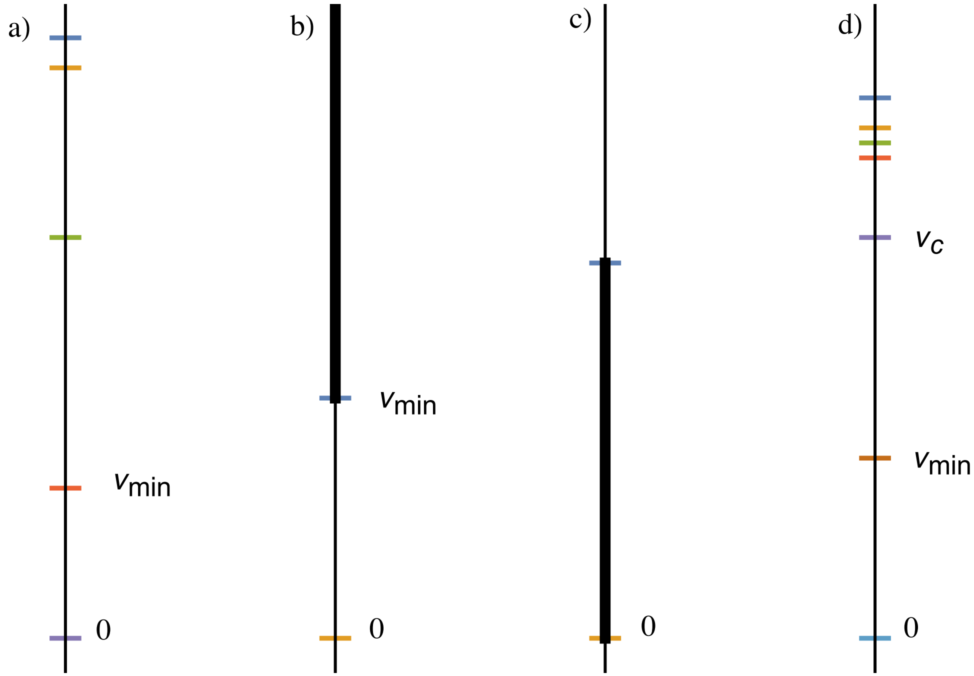
Eigenvalue problem:
$$-\nu h = L_0[h]$$

Solution:
$$\varphi(t,\mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$$

Spectrum of linearized kinetic operator

(at zero spatial momentum, i.e. all hydro modes are at 0)

Wang Chang & Uhlenbeck (1952), Grad (1963)



a) Discrete spectrum, $U = \alpha/r^4$

b) Continuous spectrum with a gap, $U = \alpha/r^n, \ \ n > 4$

c) Continuous gapless spectrum, $U = \alpha/r^n, \ \ n < 4$

d) Hod spectrum

Relaxation time in kinetic theory (continued)

$$\varphi(t,\mathbf{p}) = \sum_n C_n e^{-\nu_n t} h_n(\mathbf{p})$$

$$-\nu h = L_0[h]$$

The hierarchy of relaxation times is determined by the spectrum of the linearized kinetic operator

$$\tau_R = 1/\nu_{min}$$

For weakly inhomogeneous systems:

$$\frac{\partial F}{\partial t} + \frac{p_i}{m}\frac{\partial F}{\partial r^i} - \frac{\partial U(r)}{\partial r^i}\frac{\partial F}{\partial p_i} = -\frac{F - F_0}{\tau_R}$$

Krook-Gross-Bhatnagar (KGB) equation (1959) a.k.a. “RTA”

Transport is then essentially determined by the relaxation time, e.g. shear viscosity is

$$\eta = \tau_R\, s\, T$$

Of course, the situation is significantly more complicated for generic weakly interacting quantum systems (relativistic or not) at finite temperature and/or density

Resummations typically lead to effective kinetic theory (AGD, Popov, AMY++). Transport is determined by the spectrum of kinetic operator. Partial results exist, yet e.g. the analytic structure of correlators of gauge-invariant operators is generically unknown (but see recent work by Guy D. Moore, 1803.0073).

G.D.Moore, ``Stress-stress correlator in phi^4 theory: Poles or a Cut?,'' arXiv:1803.00736 [hep-ph].

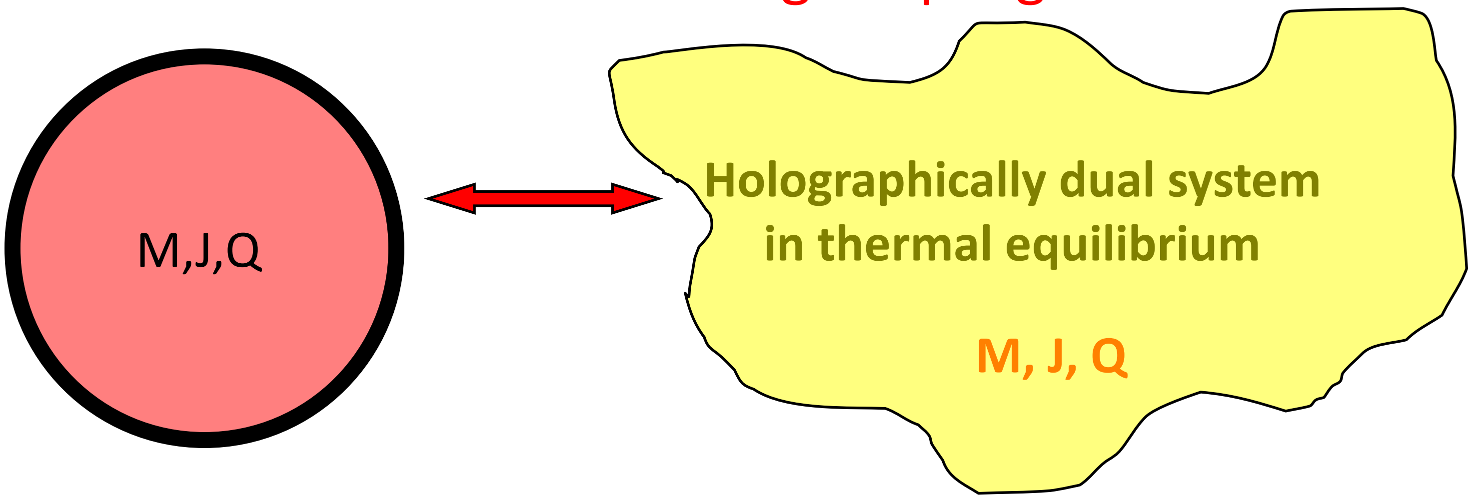
A.Kurkela and U.A.Wiedemann, ``Analytic structure of nonhydrodynamic modes in kinetic theory,'' arXiv:1712.04376 [hep-ph].

P.Romatschke, ``Retarded correlators in kinetic theory: branch cuts, poles and hydrodynamic onset transitions,'' [arXiv:1512.02641 [hep-th]].

Strong coupling

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations (and fluctuations of other fields) $\Longleftrightarrow$ Deviations from equilibrium

$$g_{\mu\nu}^{(0)} + h_{\mu\nu} \quad \Longleftrightarrow \quad ????$$

$$A_{\mu}^{(0)} + a_{\mu}$$

$$j_i = -D\partial_i j^0 + \cdots$$

$$"\Box"\ h_{\mu\nu} = 0 \quad "\Box" a_{\mu} = 0 \quad \Longleftrightarrow \quad \partial_t j^0 + \partial_i j^i = 0$$

and B.C.

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

Birmingham, Sachs, Solodukhin, 2001; Son and AOS, 2002

In quantum field theory, the dispersion relations such as

$$\omega = \pm v_s\, q - \frac{i}{2(\epsilon + P)} \left(\frac{4}{3}\eta + \zeta\right) q^2$$

appear as poles of the retarded correlation functions, e.g.

$$\langle T_{00}(k)\, T_{00}(-k)\rangle \sim \frac{q^2\, T^4}{\omega^2 - q^2/3 + i\omega q^2/3\pi T}$$

- in the hydro approximation - $\omega/T \ll 1\,, \quad q/T \ll 1$

Fluid dynamics is an effective theory valid in the long-wavelength, long-time limit

Fundamental degrees of freedom = densities of conserved charges

Equations of motion = conservation laws + constitutive relations^*

Example I

$$\left.\begin{aligned} &\partial_a J^a = 0 \\ &J^i = -D\,\nabla^i\,J^0 + \cdots \end{aligned}\right\} \quad \partial_t J^0 = D\,\nabla^2 J^0 + \cdots$$

Example II

$$\left.\begin{aligned} &\partial_a T^{ab} = 0 \\ &T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots \end{aligned}\right\}$$

Navier-Stokes eqs
Burnett eqs
…..

* Modulo assumptions e.g. analyticity

** E.o.m. universal, transport coefficients depend on underlying microscopic theory

Consider relativistic neutral conformal fluid in a d-dimensional (curved) space-time

$$T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots$$

Including only terms with first and second derivatives of fluid velocity:

$$\begin{aligned}\Pi^{ab} = & -\eta\sigma^{ab} \\ & + \eta\tau_{\Pi}\left[{}^{\langle}D\sigma^{ab\rangle} + \frac{1}{d-1}\sigma^{ab}\left(\nabla\cdot u\right)\right] \\ & + \kappa\left[R^{\langle ab\rangle} - (d-2)u_c R^{c\langle ab\rangle d}u_d\right] \\ & + \lambda_1\sigma^{\langle a}{}_c\sigma^{b\rangle c} + \lambda_2\sigma^{\langle a}{}_c\Omega^{b\rangle c} + \lambda_3\Omega^{\langle a}{}_c\Omega^{b\rangle c}\end{aligned}$$

Transport coefficients (in conformal case): $\eta\,, \tau_{\Pi}\,, \kappa\,, \lambda_1\,, \lambda_2\,, \lambda_3$

Non-conformal case: 2 first order coefficients, 15 (10) second order coefficients (see S.Bhattacharyya, 1201.4654 [hep-th])

Beyond second order hydrodynamics

Tensors structures appearing in the derivative expansion have been analyzed using computer algebra in 1507.02461 [hep-th] by Grozdanov & Kaplis.

At third order, there are 20 relevant structures in the conformal case and 68 in the non-conformal one.

This still needs an entropy current analysis similar to the one in S.Bhattacharyya, 1201.4654 [hep-th]

Example: dispersion relations in conformal case

$$\omega = -i\frac{\eta}{\varepsilon+P}k^2 - i\left[\frac{\eta^2\tau_\Pi}{(\varepsilon+P)^2} - \frac{\theta_1}{2(\varepsilon+P)}\right]k^4 + \cdots$$

$$\omega = \pm c_s k - i\Gamma k^2 \mp \frac{\Gamma}{2c_s}\left(\Gamma - 2c_s^2\tau_\Pi\right)k^3 - i\left[\frac{8\eta^2\tau_\Pi}{9(\varepsilon+P)^2} - \frac{\theta_1+\theta_2}{3(\varepsilon+P)}\right]k^4 + \cdots$$

Here $c_s = 1/\sqrt{3}$ $\quad \Gamma = \eta/(\varepsilon + P)$

Notations used in the derivative expansion

$$D \equiv u^a \nabla_a$$

$$\Delta^{ab} \equiv g^{ab} + u^a u^b$$

$$A^{\langle ab \rangle} \equiv \frac{1}{2} \Delta^{ac} \Delta^{bd} \left(A_{cd} + A_{dc} \right) - \frac{1}{d-1} \Delta^{ab} \Delta^{cd} A_{cd} \equiv {}^{\langle} A^{ab \rangle}$$

$$\sigma^{ab} = 2^{\langle} \nabla^a u^{b \rangle}$$

$$\Omega^{ab} = \frac{1}{2} \Delta^{ac} \Delta^{bd} \left(\nabla_c u_d - \nabla_d u_c \right)$$

*Hydro definitions differ in the literature – see footnote 91 on page 128
of M.Haehl, R.Loganayagam, M.Rangamani, 1502.00636 [hep-th]
See Appendix B in S.Grozdanov, AOS, 1611.07053 [hep-th]

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

In the regime described by a gravity dual the correlator can be computed using gauge theory/gravity duality

Kubo formulas for second order transport coefficients

First order transport coefficients can be computed from two-point functions of the corresponding operators using Kubo formulas

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

Similarly, second transport coefficients can be computed from three-point functions

$$\lambda_2 = 2\eta\tau_\Pi - 4 \lim_{p,q\to 0} \frac{\partial^2}{\partial p^0 \partial q^z} G_{RAA}^{xy,ty,xz}(p,q)$$

Moore, Sohrabi, Saremi, 2010, 2011; Arnold, Vaman, Wu, Xiao, 2011

Schwinger-Keldysh generating functional

$$G_{RA\ldots}^{ab,cd,\ldots}(0,x,\ldots) = \left.\frac{(-i)^{n-1}(-2i)^n \delta^n W}{\delta h_{A\;ab}(0)\delta h_{R\;cd}(x)\ldots}\right|_{h=0} = (-i)^{n-1} \langle T_R^{ab}(0) T_A^{cd}(x)\ldots\rangle$$

$$W\left[h^+,h^-\right] = \ln \int \mathcal{D}\phi^+\mathcal{D}\phi^-\mathcal{D}\varphi \exp\left\{ i\int d^4x^+ \sqrt{-g^+}\mathcal{L}\left[\phi^+(x^+),h^+\right] \quad - \int_0^\beta d^4y \mathcal{L}_E\left[\varphi(y)\right] - i\int d^4x^- \sqrt{-g^-}\mathcal{L}\right.$$

How to compute second order transport coefficients?

Fluid-gravity correspondence [Bhattacharyya et al, 2007]

Quasinormal spectrum [Baier et al, 2007]

Kubo formulas & three-point functions
[Moore, Sohrabi, Saremi, 2010, 2011; Arnold, Vaman, Wu, Xiao, 2011]

First and second order transport coefficients of *conformal* holographic fluids *to leading order* in supergravity approximation

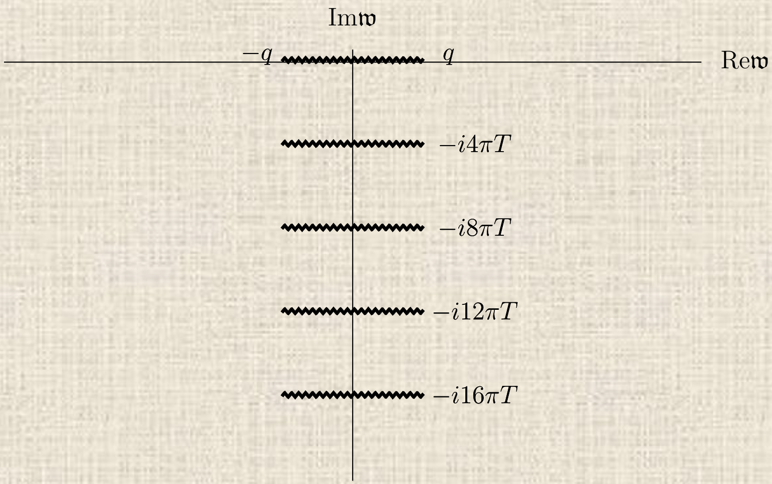
$$
\begin{aligned}
\eta &= s/4\pi\,, \\
\tau_\Pi &= \frac{d}{4\pi T}\left(1+\frac{1}{d}\left[\gamma_E+\psi\left(\frac{2}{d}\right)\right]\right)\,, \\
\kappa &= \frac{d}{d-2}\,\frac{\eta}{2\pi T}\,, \\
\lambda_1 &= \frac{d\eta}{8\pi T}\,, \\
\lambda_2 &= \left[\gamma_E+\psi\left(\frac{2}{d}\right)\right]\frac{\eta}{2\pi T}\,, \\
\lambda_3 &= 0
\end{aligned}
$$

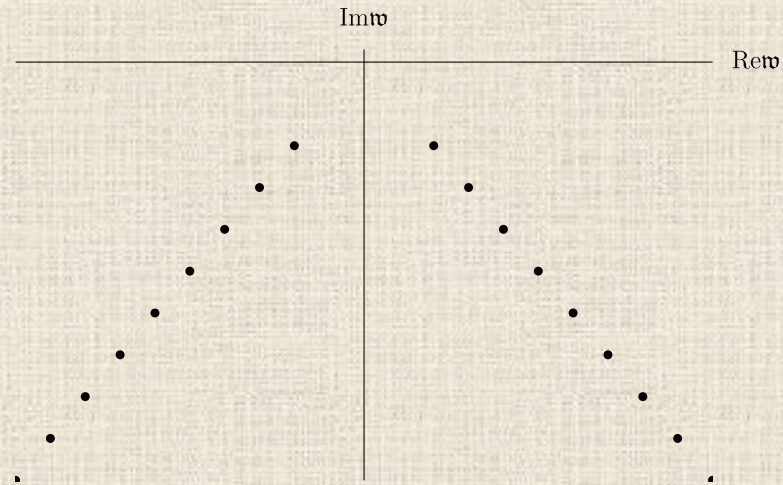
Bhattacharyya et al, 2008

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Cuts versus poles: a mystery

Singularities of a Green's function in the complex frequency plane





Weak (vanishing) coupling

Hartnoll, Kumar (2005)

Strong (infinite) coupling

AOS (2002)

We should be able to interpolate between the two limits…

Coupling constant corrections to N=4 SYM transport coefficients

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \gamma\mathcal{W}\right) \qquad \gamma = \lambda^{-3/2}\zeta(3)/8$$

$$\eta = \frac{\pi}{8}N_c^2T^3\left(1 + 135\gamma + \dots\right)$$

$$\tau_\Pi = \frac{(2-\ln 2)}{2\pi T} + \frac{375\gamma}{4\pi T} + \dots$$

$$\kappa = \frac{N_c^2T^2}{8}\left(1 - 10\gamma + \dots\right)$$

$$\lambda_1 = \frac{N_c^2T^2}{16}\left(1 + 350\gamma + \dots\right)$$

$$\lambda_2 = -\frac{N_c^2T^2}{16}\left(2\ln 2 + 5\,(97 + 54\ln 2)\,\gamma + \dots\right)$$

$$\lambda_3 = \frac{25N_c^2T^2}{2}\,\gamma + \dots$$

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Curvature squared corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{R^2} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R - 2\Lambda + L^2\left(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{r_+^3}{2\kappa_5^2}\left(1 - 8\left(5\alpha_1 + \alpha_2\right)\right) + \ldots$$

$$\eta\tau_\Pi = \frac{r_+^2\left(2-\ln 2\right)}{4\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2\left(23 + 5\ln 2\right)}{12\kappa_5^2}\alpha_3 + \ldots$$

$$\kappa = \frac{r_+^2}{2\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{25 r_+^2}{6\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_1 = \frac{r_+^2}{4\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2}{12\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_2 = -\frac{r_+^2\ln 2}{2\kappa_5^2}\left(1 - \frac{26}{3}\left(5\alpha_1 + \alpha_2\right)\right) - \frac{r_+^2\left(21 + 5\ln 2\right)}{6\kappa_5^2}\alpha_3 + \ldots$$

$$\lambda_3 = -\frac{28 r_+^2}{\kappa_5^2}\alpha_3 + \ldots$$

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Gauss-Bonnet corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{r_+^2}{2\kappa_5^2}\left(1 - 4\lambda_{GB}\right)$$

$$\eta\tau_\Pi = \frac{r_+^2\left(2 - \ln 2\right)}{4\kappa_5^2} - \frac{r_+^2\left(25 - 7\ln 2\right)}{8\kappa_5^2}\lambda_{GB} + \ldots$$

$$\kappa = \frac{r_+^2}{2\kappa_5^2} - \frac{17r_+^2}{4\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_1 = \frac{r_+^2}{4\kappa_5^2} - \frac{9r_+^2}{8\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_2 = -\frac{r_+^2\ln 2}{2\kappa_5^2} - \frac{7r_+^2\left(1 - \ln 2\right)}{4\kappa_5^2}\lambda_{GB} + \ldots$$

$$\lambda_3 = -\frac{14r_+^2}{\kappa_5^2}\lambda_{GB} + \ldots$$

Brigante, Myers, H.Liu, Myers, Shenker, Yaida, 2008

Shaverin, Yarom, 2012

Note: $2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = 0$

Non-perturbative Gauss-Bonnet corrections to transport coefficients of a (hypothetical) strongly coupled liquid

$$S_{GB} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$\eta = \frac{s}{4\pi}\gamma^2 = \frac{s}{4\pi}(1 - 4\lambda_{GB})$$

Brigante et al, 2008

$$\tau_\Pi = \frac{1}{2\pi T}\left(\frac{1}{4}(1+\gamma)\left(5+\gamma-\frac{2}{\gamma}\right) - \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

Banerjee and Dutta, 2011

$$\lambda_1 = \frac{\eta}{2\pi T}\left(\frac{(1+\gamma)(3-4\gamma+2\gamma^3)}{2\gamma^2}\right)$$

Grozdanov and AOS, 2014

$$\lambda_2 = -\frac{\eta}{\pi T}\left(-\frac{1}{4}(1+\gamma)\left(1+\gamma-\frac{2}{\gamma}\right) + \frac{1}{2}\log\left[\frac{2(1+\gamma)}{\gamma}\right]\right)$$

Grozdanov and AOS, 2014

$$\lambda_3 = -\frac{\eta}{\pi T}\left(\frac{(1+\gamma)(3+\gamma-4\gamma^2)}{\gamma^2}\right)$$

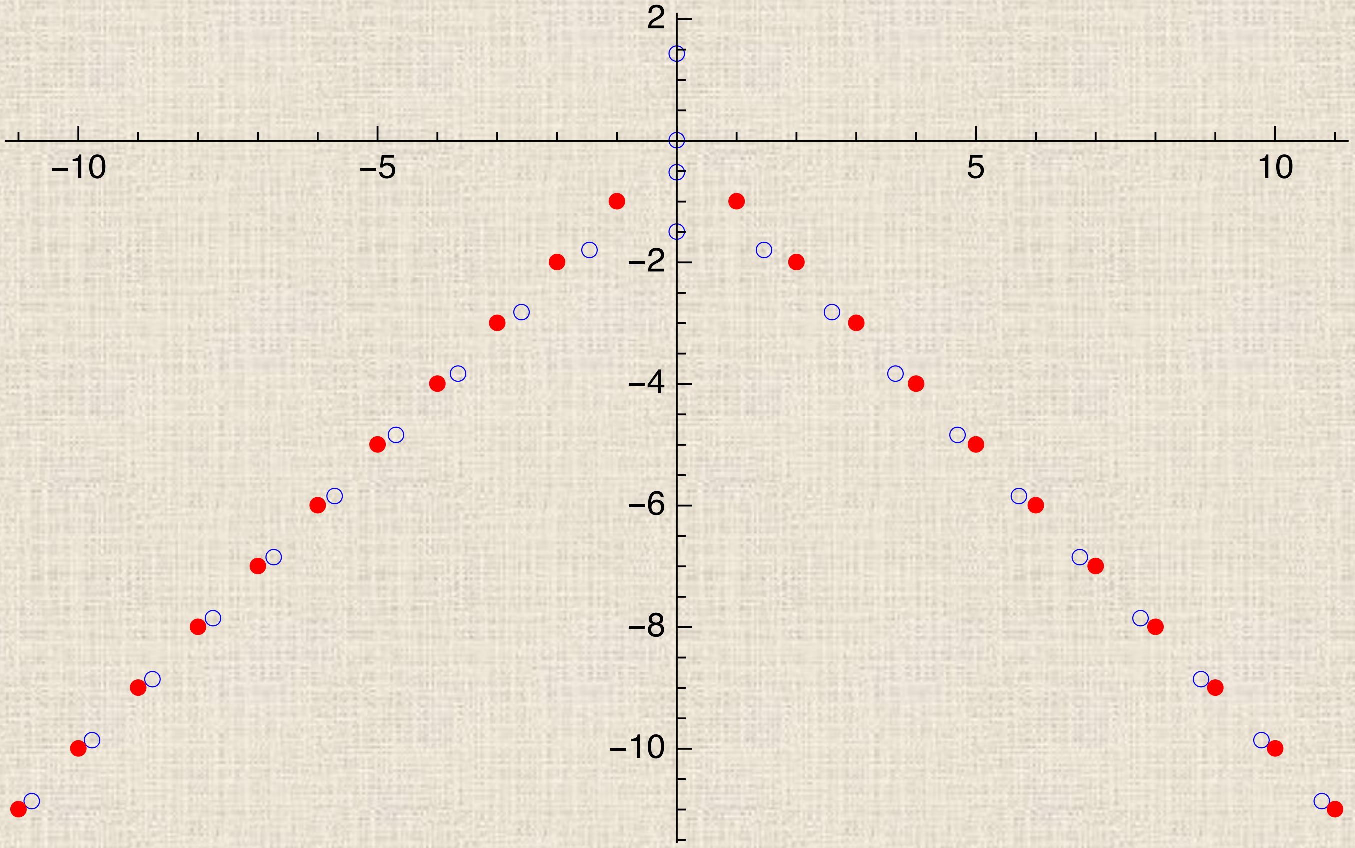
Grozdanov and AOS, 2014

$$\kappa = \frac{\eta}{\pi T}\left(\frac{(1+\gamma)(2\gamma^2-1)}{2\gamma^2}\right)$$

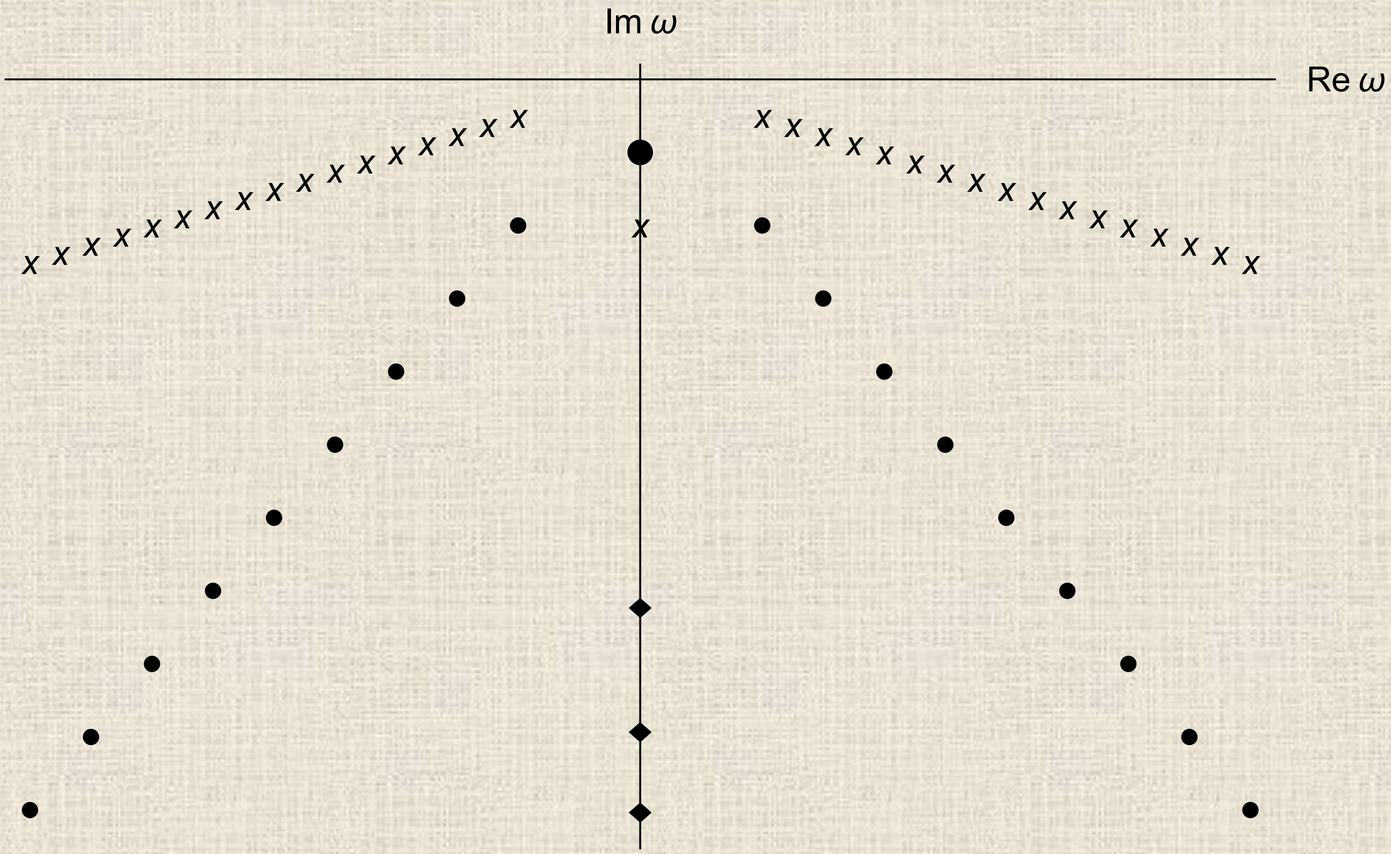
Banerjee and Dutta, 2011

$$H(\lambda_{GB}) = 2\eta\tau_\Pi - 4\lambda_1 - \lambda_2 = -\frac{\eta}{\pi T}\frac{(1-\gamma_{GB})(1-\gamma_{GB}^2)(3+2\gamma_{GB})}{\gamma_{GB}^2} = -\frac{40\lambda_{GB}^2\eta}{\pi T} + \ldots$$

Poles (blue) and zeros (red) of a typical retarded correlator at infinite coupling (dual gravity results)

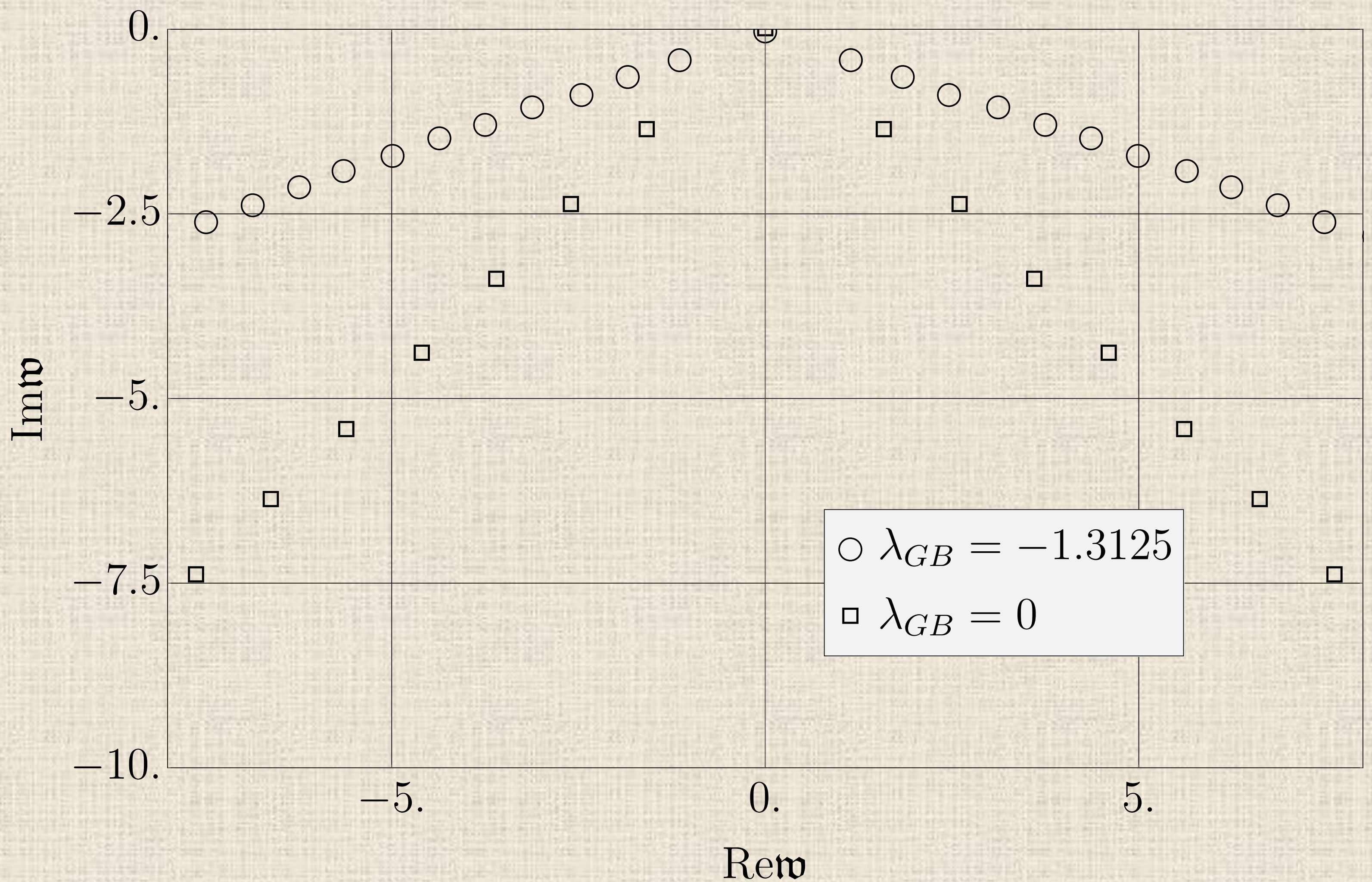


Singularities of stress-energy tensor Green's function at infinite (black dots) and finite (black crosses and diamonds) coupling

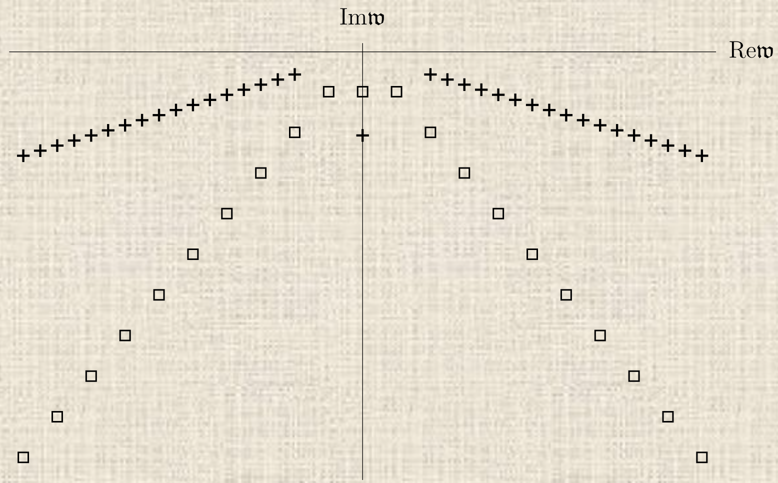


Earlier work: Stricker, 1307.2736 [hep-th]; Waeber, Schäfer, Vuorinen and Yaffe, 1509.02983 [hep-th].

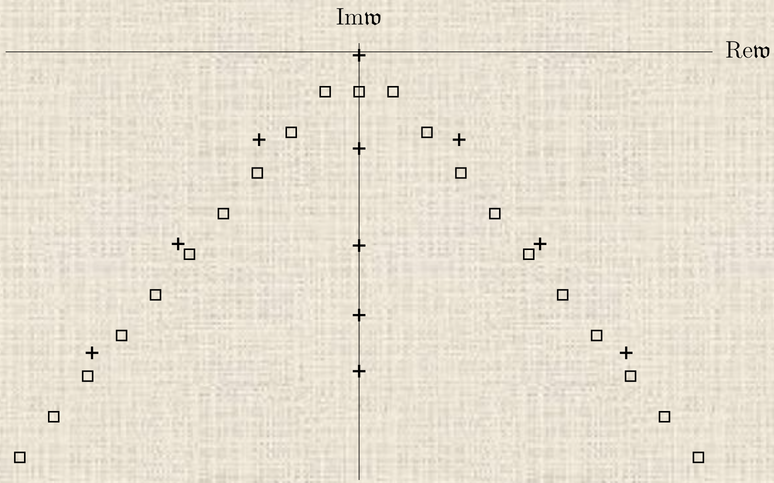
Quasinormal spectrum of Gauss-Bonnet black brane
vs
AdS-Schwarzschild black brane (numerical data)



Singularities of stress-energy tensor Green's function in different regimes of viscosity-entropy ratio (shear channel)



$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$



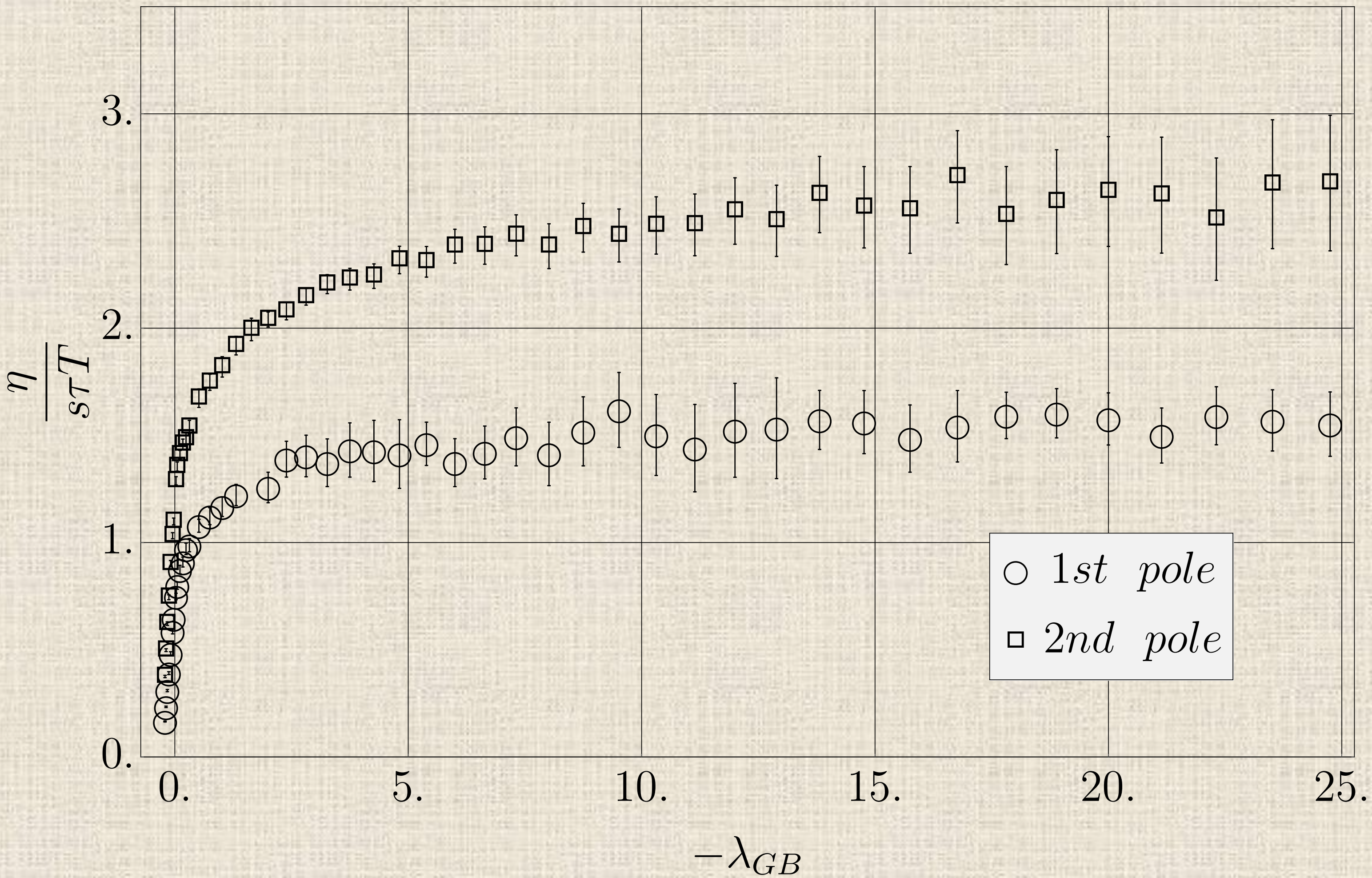
$$\frac{\eta}{s} < \frac{1}{4\pi}$$

White squares: poles at infinite coupling
Crosses: poles at finite coupling

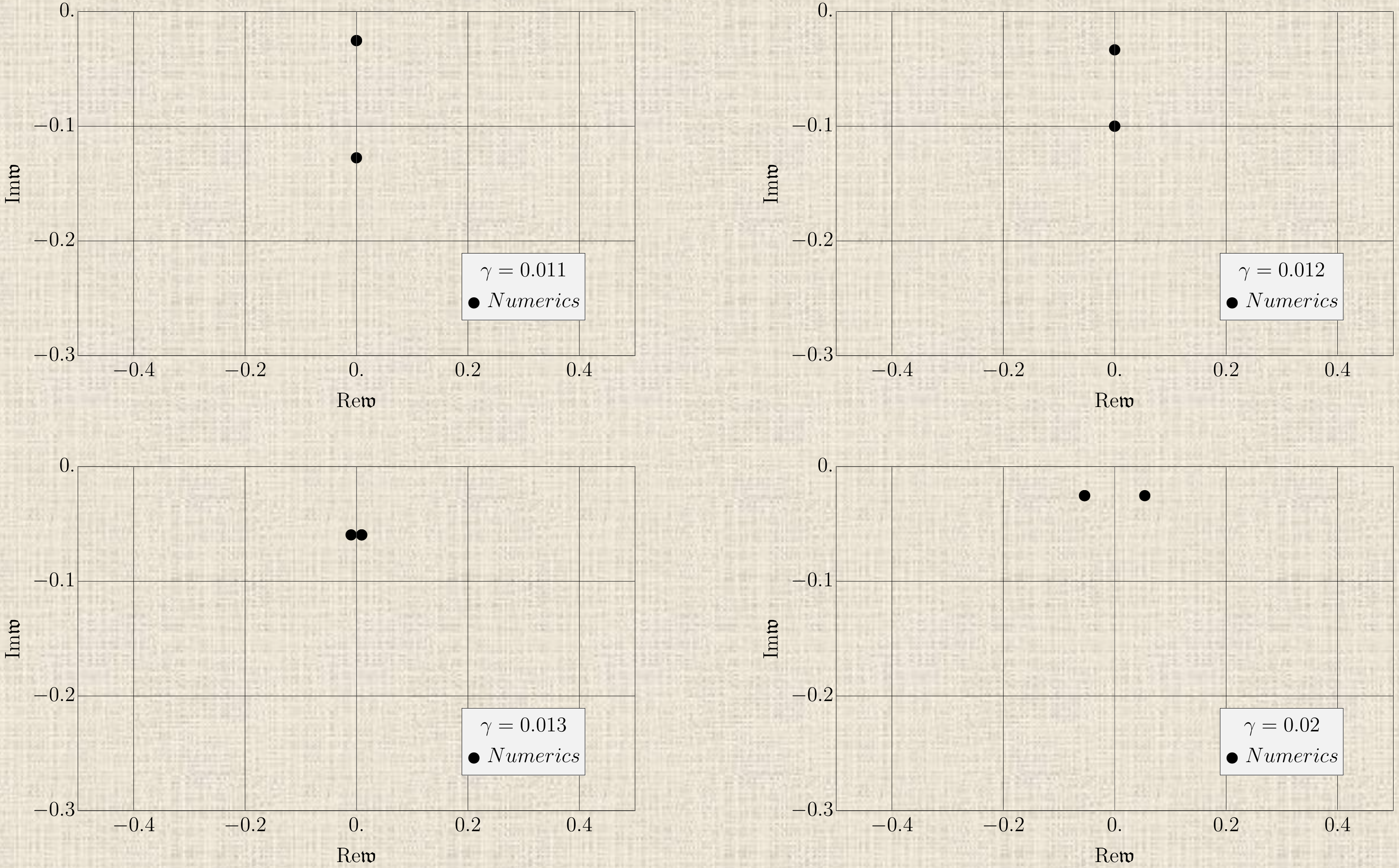
On the “unreasonable effectiveness” of kinetic theory at strong coupling

Recall that in kinetic theory $\eta = const\; s\, \tau_R\, T$

What happens at large but finite coupling, with $\tau_R = 1/|\mathrm{Im}\,\omega_F|$?

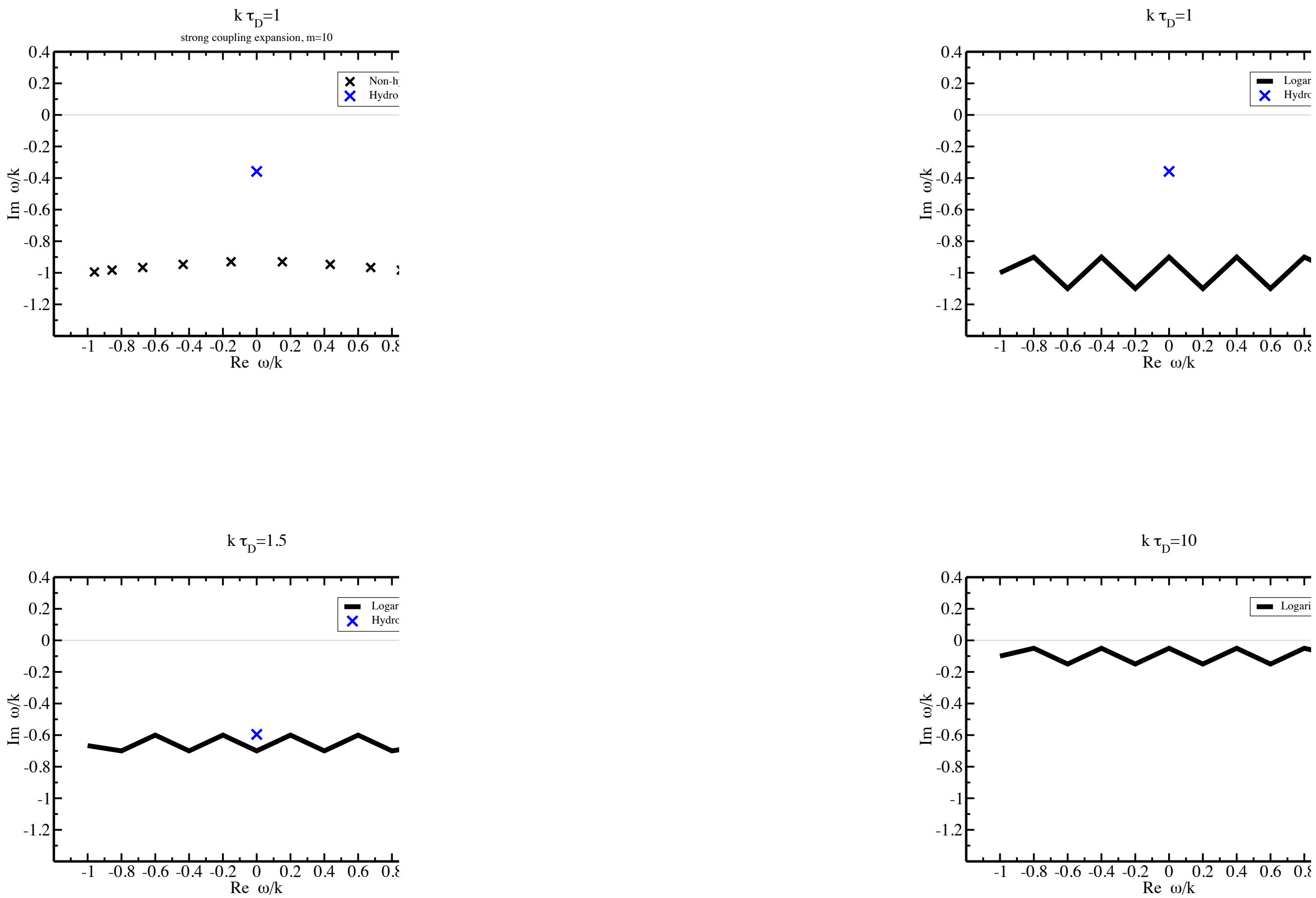


Breakdown of hydrodynamics at (large) finite coupling



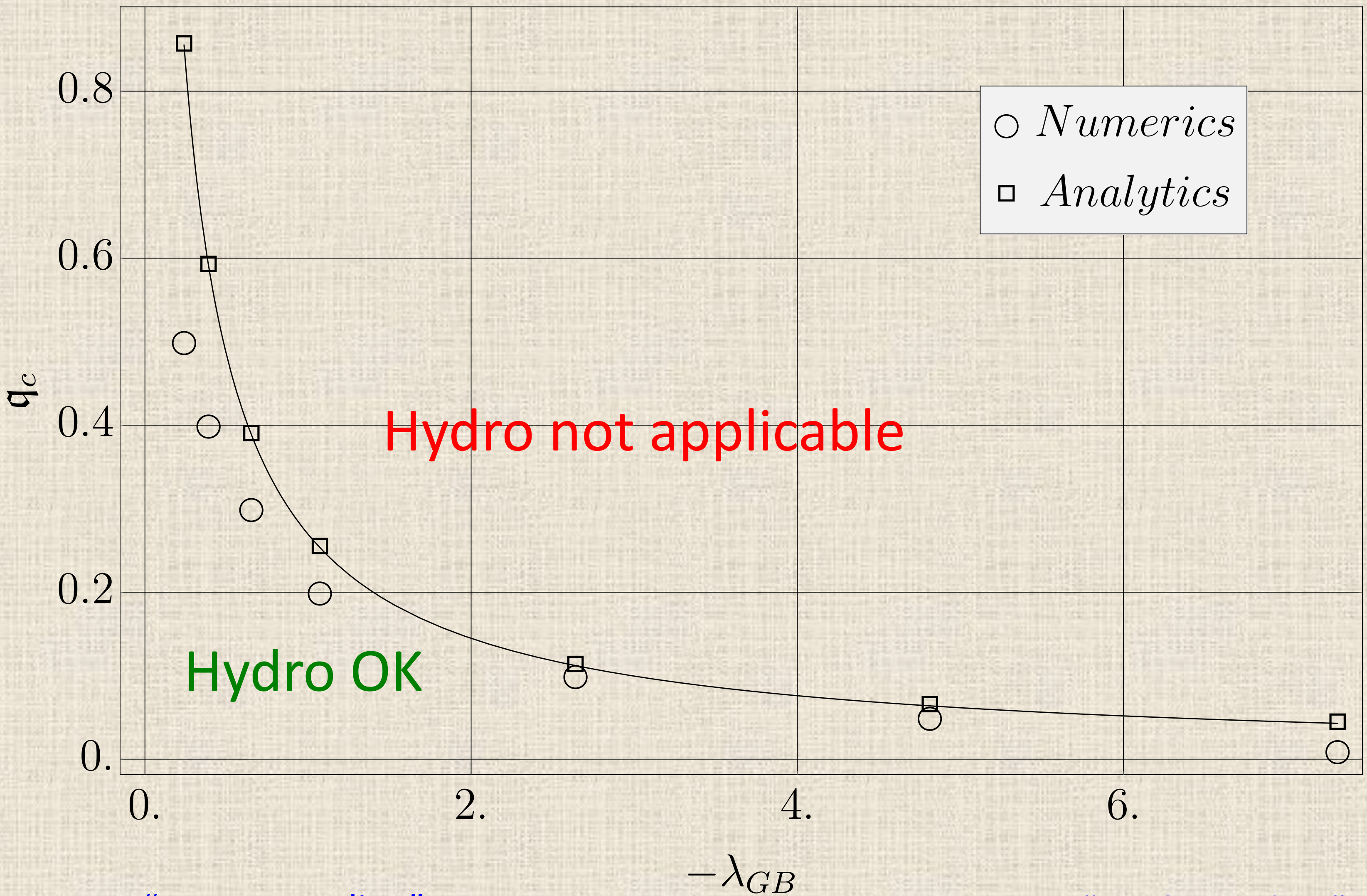
$$q_c = q_c(\lambda)$$

Kinetic theory (relaxation time approximation)



Figs from Paul Romatschke, 1512.02641 [hep-th]
See also A.Kurkela and U.Wiedemann, 1712.04376 [hep-ph]

“Applicability of hydrodynamics” as a function of coupling



“strong coupling” “weak coupling”

Transport peak of spectral functions at large finite coupling

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

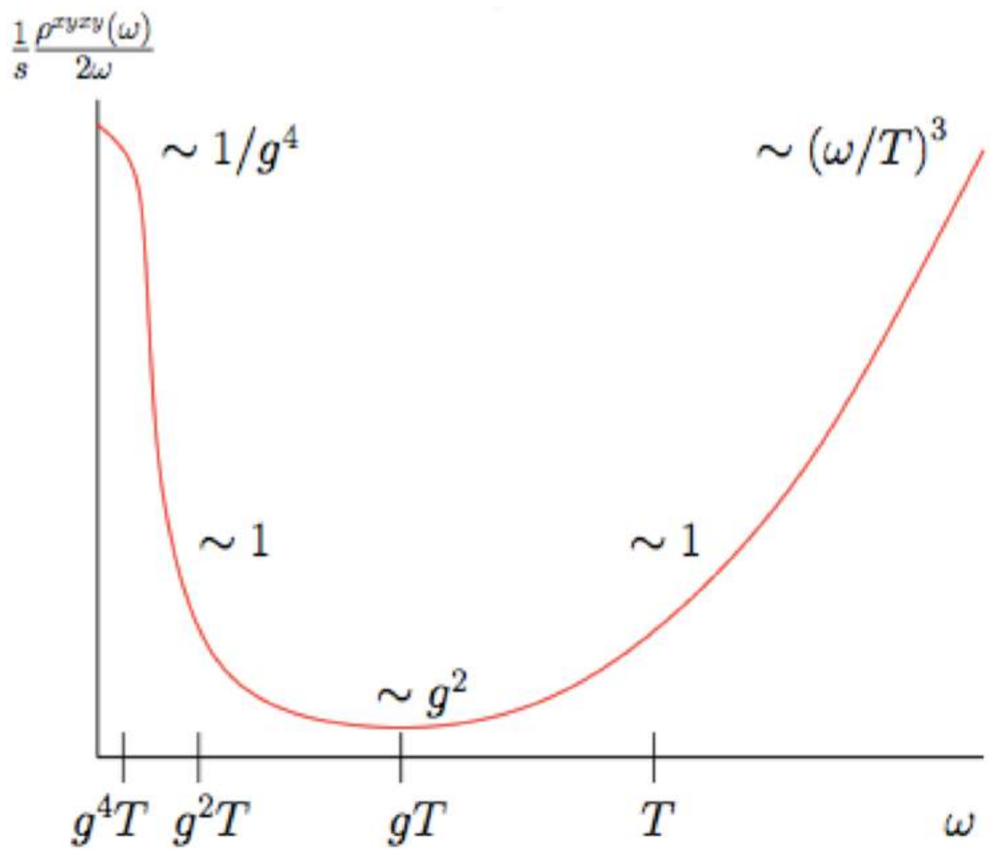
$$\chi_{xy,xy}(k) = \int d^4x e^{-ikx} \langle [T_{xy}(x) T_{xy}(0)] \rangle = -2\, \mathrm{Im}\, G^R_{xy,xy}$$

Viscosity is determined by the height of the peak of the spectral function at w=0.

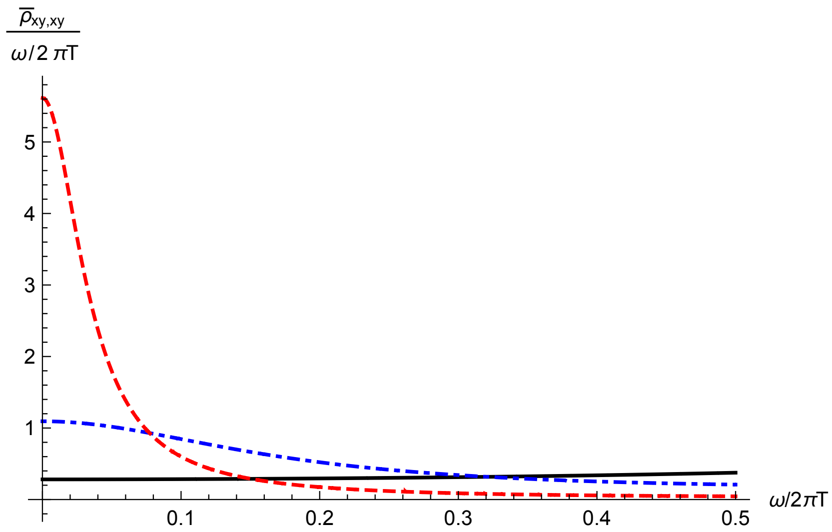
The peak is affected by the singularities of the correlator in the complex w plane.

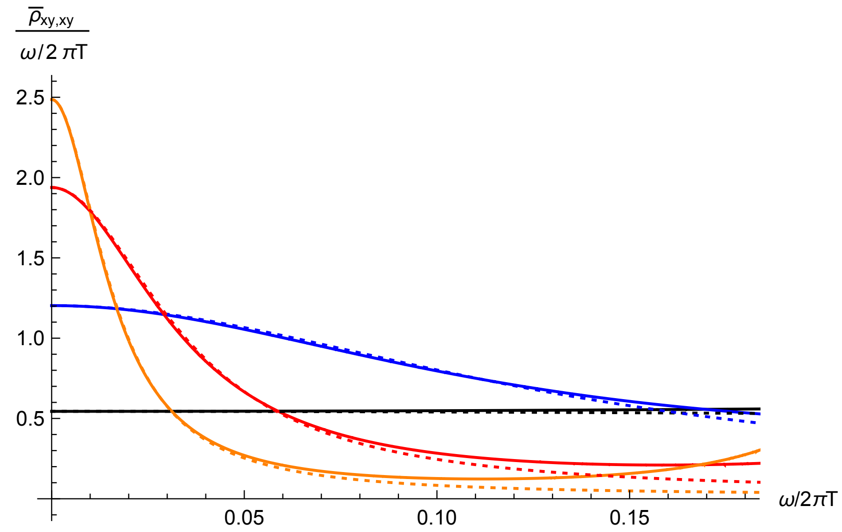
What kind of singularities? Are they the same at weak and strong coupling?

Transport peak in QCD at finite temperature (sketch)



Transport peak of spectral functions at large finite coupling





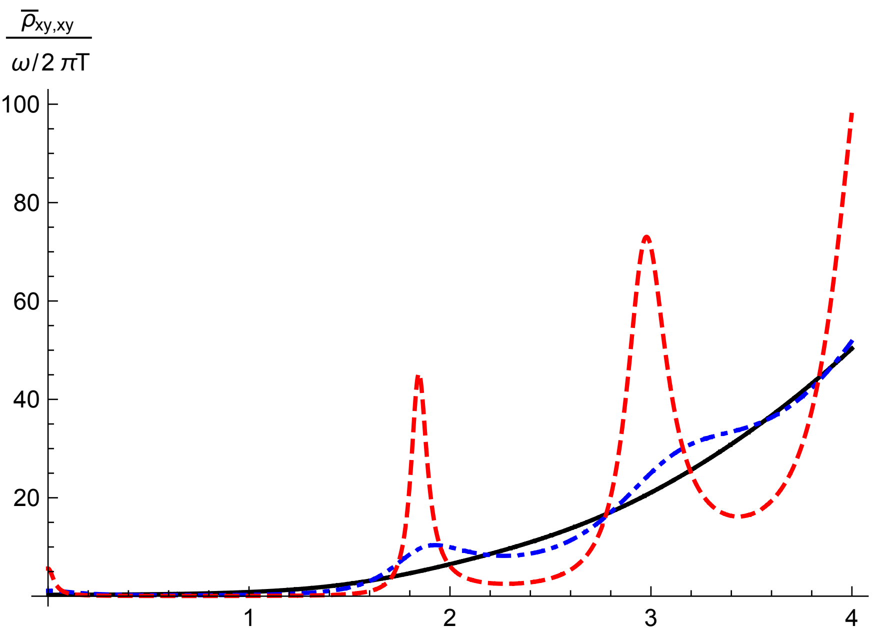
$\mathcal{N} = 4$ SYM

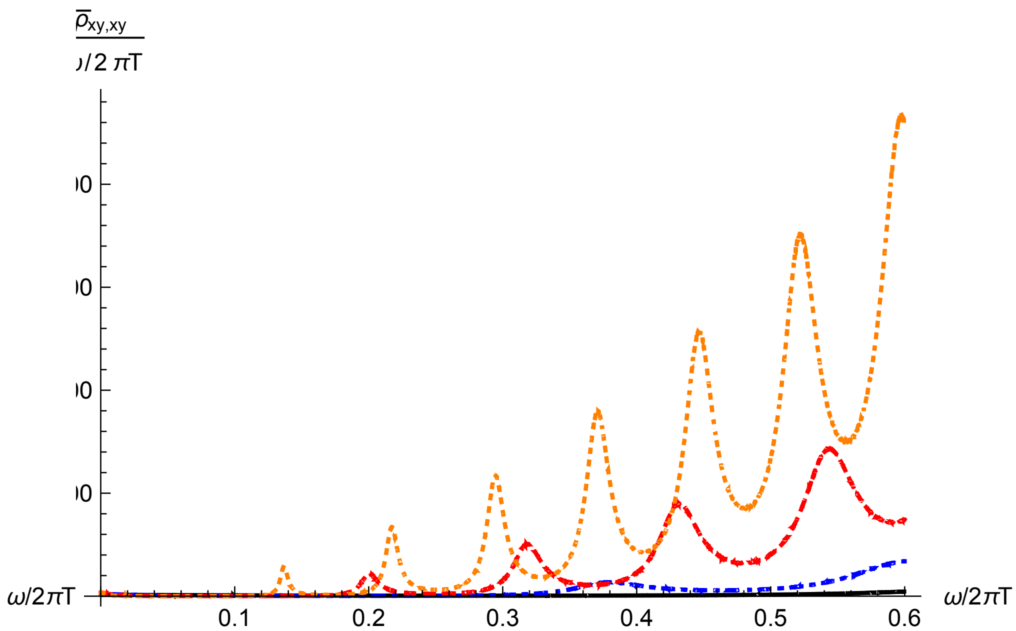
Gauss-Bonnet

Note: Black solid line is the spectral function at infinite coupling

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

Quasiparticle peaks at high frequency





$\mathcal{N} = 4$ SYM

Gauss-Bonnet

Linear instability of black brane backgrounds in higher-derivative gravity

Coupling constant corrections to the entropy, viscosity, correlators etc are coming from

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{4\cdot 5!}F_5^2 + \gamma e^{-\frac{3}{2}\phi}\mathcal{W} + \dots\right)$$

$$\mathcal{W} = C^{\alpha\beta\gamma\delta}C_{\mu\beta\gamma\nu}C_{\alpha}^{\ \rho\sigma\mu}C^{\nu}_{\ \rho\sigma\delta} + \frac{1}{2}C^{\alpha\delta\beta\gamma}C_{\mu\nu\beta\gamma}C_{\alpha}^{\ \rho\sigma\mu}C^{\nu}_{\ \rho\sigma\delta} \qquad \gamma = \lambda^{-3/2}\zeta(3)L^6/8$$

The corrected 5d metric is

$$ds^2 = \frac{(\pi T L)^2}{u}\left(-e^{a(u)}f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + e^{b(u)}\frac{L^2 du^2}{4u^2 f}$$

$$a(u) = -15\gamma\left(5u^2 + 5u^4 - 3u^6\right), \qquad b(u) = 15\gamma\left(5u^2 + 5u^4 - 19u^6\right)$$

Gubser, Klebanov and Tseytlin, hep-th/9805156; Pawelczyk and Theisen, hep-th/9808126

Linear metric fluctuations satisfy e.o.m. of the type

$$\partial_u^2 Z_1 - \frac{1+u^2}{u\left(1-u^2\right)}\partial_u Z_1 + \frac{w^2 - q^2\left(1-u^2\right)}{u\left(1-u^2\right)^2}Z_1 = \gamma\, G_1\left[Z_1\right]$$

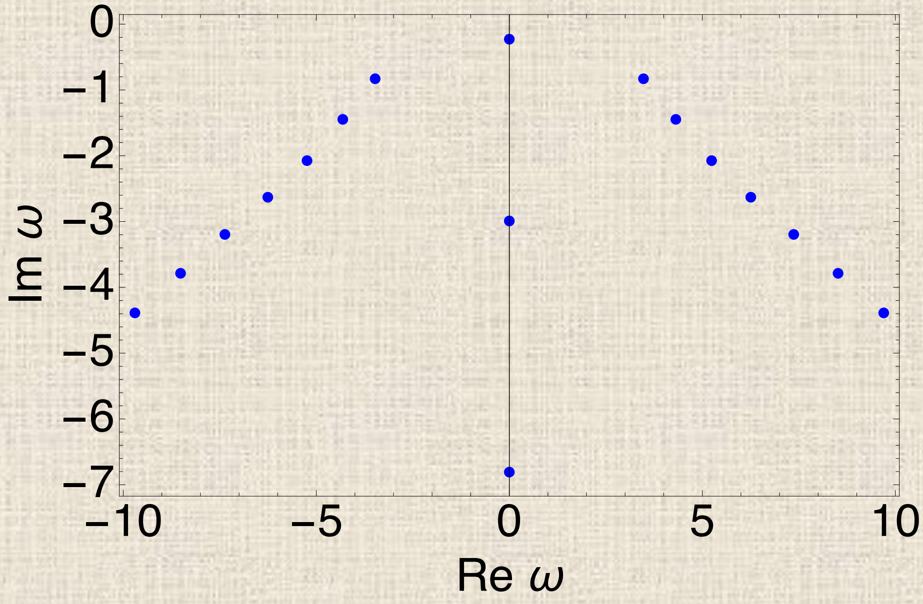
This can be re-written in Eddington-Finkelstein coordinates as

$$-\sqrt{b}\frac{d}{du}\left(\frac{1}{\sqrt{b}}\psi'\right) - 2iw\sqrt{b}\psi' + V_{eff}\psi = 0$$

The sign of the imaginary part of an eigenfrequency is determined by

(Horowitz and Hubeny, 1999)

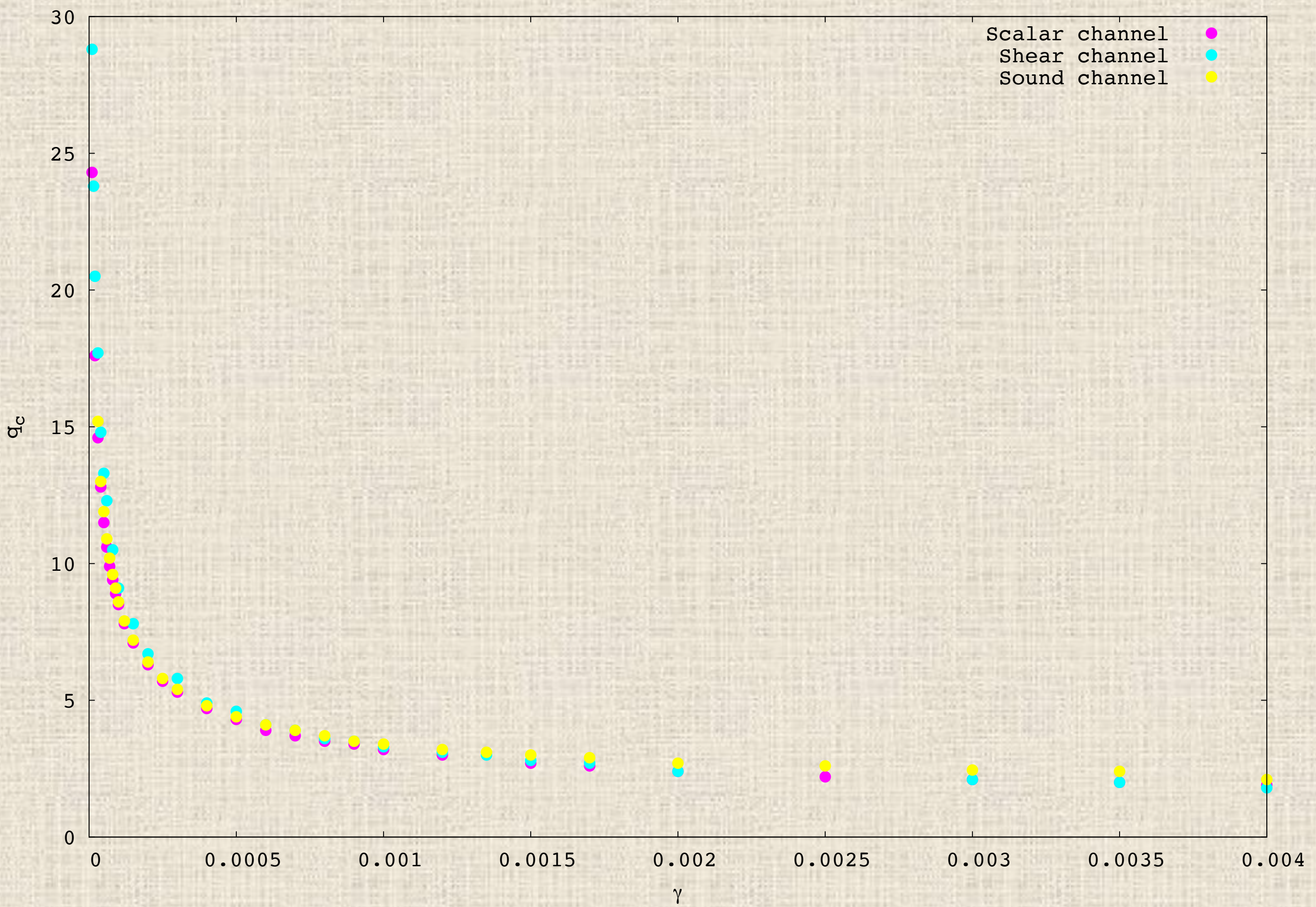
$$\frac{|w|^2|\psi(1)|^2}{\operatorname{Im} w} = -\int_0^1 \frac{du}{\sqrt{b}}\left(|\psi'|^2 + V_{eff}|\psi|^2\right)$$



The instability seems to be generic.

Konoplya and Zhidenko, 2017; Grozdanov, Gushterov, AOS, 2018

Critical momentum vs (inverse) coupling in N=4 SYM



Conclusions & open questions

Finite coupling corrections seem to show qualitatively similar behavior irrespective of the precise structure of higher derivative terms in dual gravity (we did R^2 and R^4)

How robust are the results (structure of higher derivative expansion)?

We observe breakdown of hydrodynamics at coupling-dependent value of a wave-vector. The dependence on coupling suggests that hydrodynamics has a wider applicability range at stronger coupling

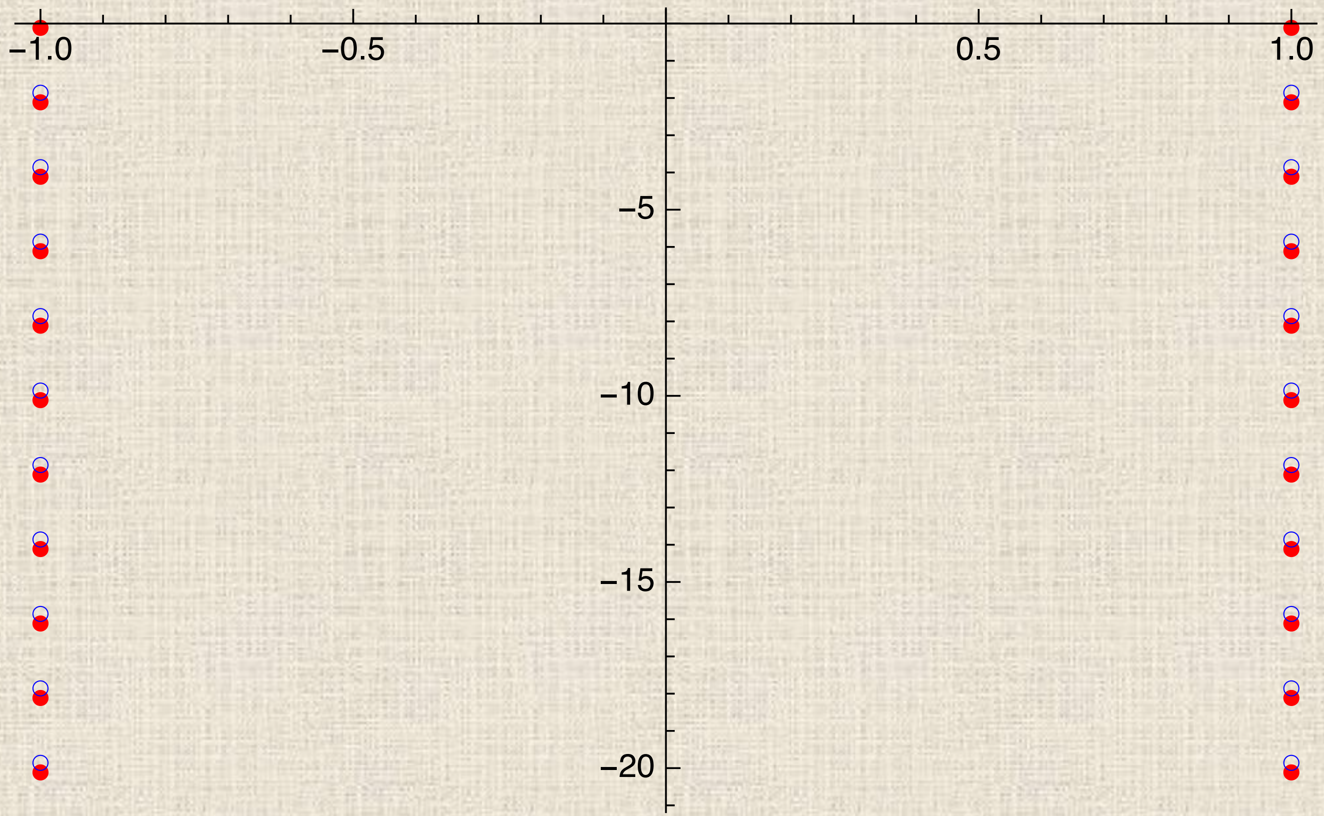
Our results suggest that kinetic theory results may be formally still applicable in the intermediate and strong coupling regime where the use of kinetic theory itself cannot be justified. In particular, transport peak is visible at large finite coupling due to inflow of poles. Compare to pQFT?

We observe qualitatively different analytic structure of correlators depending on whether

$$\eta/s > 1/4\pi \text{ or } \eta/s < 1/4\pi$$

We observe linear instability of the dual metric at finite coupling. Need to explain this.

THANK YOU!



−1.0
−0.5
0.5
1.0
−5
−10
−15
−20