Applications of Generalized Global Symmetries

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1. Generalized global symmetries
2. Phases of electromagnetism
3. Magnetohydrodynamics
4. Future directions

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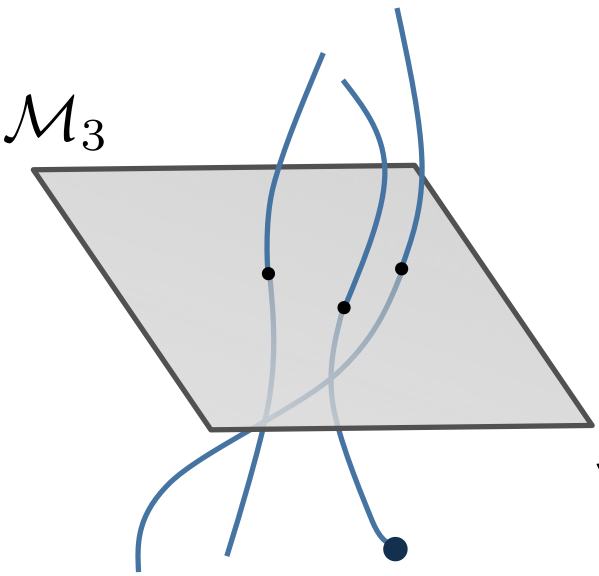
Global symmetries

Let's review ordinary U(1) 1-index currents first:

$$\nabla_\mu j^\mu = 0 \qquad d \star j = 0$$

An ordinary current counts particles; "catch them all" by integrating on a co-dimension 1 subspace (a "time-slice"):



$$Q = \int_{\mathcal{M}_3} \star j$$

Local operators are charged under it:

$$\nabla_\mu j^\mu(x)\mathcal{O}(y) \sim iq\delta(x-y)\mathcal{O}(y)$$

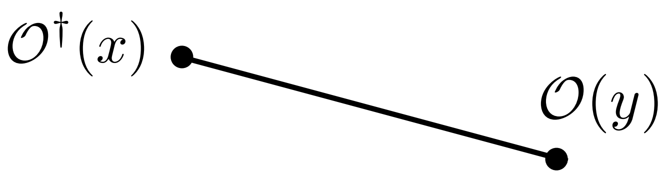
Because local operators are 0-dimensional, we call these 0-form symmetries.

Global symmetries

What are global symmetries good for?

Landau told us: use them to classify phases of matter. Two cases:

1. Unbroken: in this phase, all objects that you count are gapped, and charged correlation functions decay exponentially:



$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y)\rangle \sim \exp(-m|x-y|)$$

2. Spontaneously broken: something has condensed, and charged correlation functions factorize and saturate at large distances:

$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y)\rangle \sim \langle \mathcal{O}^\dagger\rangle\langle \mathcal{O}\rangle$$

CFTs describe phase transitions: they sit in between these two cases.

Goldstone modes

Non-trivial fact: if a charged operator develops a vev, then there is a gapless Goldstone mode in the spectrum.

$$\langle \mathcal{O}^\dagger(x)\mathcal{O}(y)\rangle \sim \langle \mathcal{O}^\dagger\rangle\langle \mathcal{O}\rangle \quad \longrightarrow \quad J = d\theta$$

The dynamics of this Goldstone mode is very simple: at low energies, action is completely fixed by symmetries.

$$S = \int d^4x \ \rho_s (d\theta)^2 + \cdots$$

So: it is useful to study global symmetries.

Generalized global symmetries

Claim (Gaiotto, Kapustin, Seiberg, Willet):

There are other kinds of global symmetries.

$$J^{\mu} \to J^{\mu\nu}$$

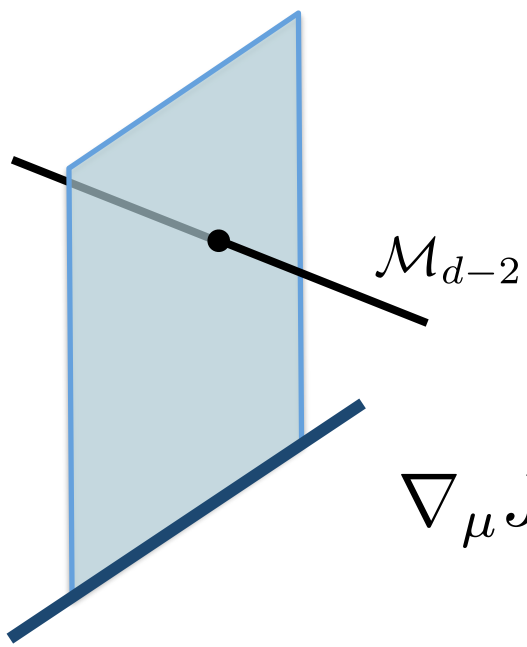
Idea: consider higher-form currents.

Generalized global symmetries

In the rest of this talk, we study 2-index currents, i.e.

$$\nabla_\mu J^{\mu\nu} = 0 \qquad d \star J = 0$$

A 2-index current counts strings; as they don't end in space or time, "catch them all" by integrating on a co-dimension-2 subspace:



$$Q = \int_{\mathcal{M}_{d-2}} \star J$$

Line operators are charged under it:

$$\nabla_\mu J^{\mu\nu}(x) W_C \sim iq \int_C dx^\nu \delta(x - X_C) W_C$$

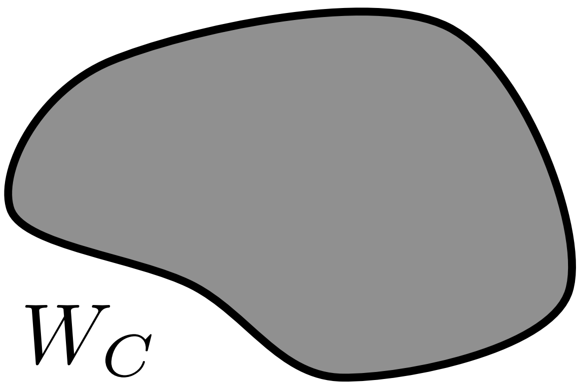
Because lines are 1-dimensional, these are called 1-form symmetries.

Generalized global symmetries

What are generalized global symmetries good for?

Follow Landau: use them to classify phases of matter. Two cases:

1. Unbroken: in this phase, objects that you count (strings) are gapped (have tension). Charged line operators have an area law.

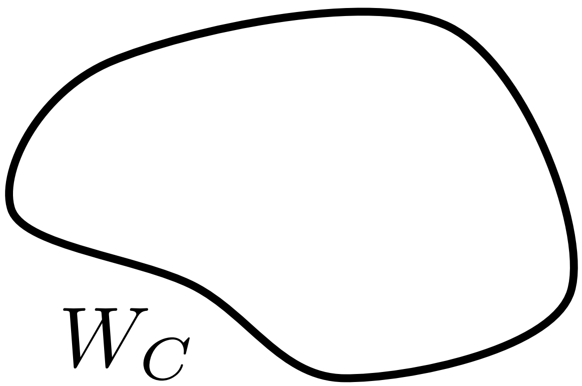


$$\langle W_C \rangle \sim \exp\left(-T_{p+1} \mathrm{Area}[C]\right)$$

This is the line-like analogue of an exponentially decaying correlation function of local operators.

Generalized global symmetries

2. Spontaneously broken: in this phase, strings have "condensed", i.e. have no tension. More precisely: charged line operators develop perimeter laws at large distances:



$$\langle W_C \rangle \sim \exp(-T_p \mathrm{Perimeter}[C])$$

Depends only locally on the line operator: thus this is the line-like analogue of a factorized correlation function of local operators.

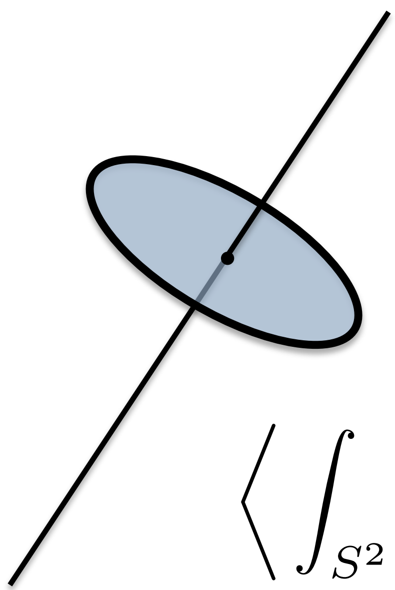
Goldstone?

Goldstone's theorem for broken symmetries

Analog of Goldstone's theorem (NI, Hofman; Lake). First, strip off the perimeter:

$$\overline{W}_C \equiv W_C \exp(+T_p \mathrm{Perimeter[C]})$$

Take C to be a straight line, consider:



$$\nabla_\mu J^{\mu\nu}(x)\overline{W}_C \sim iq \int_C dx^\nu \delta(x - X_C)\overline{W}_C$$

This is a divergence! Integrate it over the inside of a S^2 of radius R, take vev.

$$\left\langle \int_{S^2} J(R)\overline{W}_C \right\rangle \sim iq\langle \overline{W}_C \rangle$$

$$\boxed{\langle J(R)\overline{W}_C \rangle \sim \frac{1}{R^2}}$$

Power-law correlation: so a perimeter law implies a gapless Goldstone mode! What are these new gapless modes?

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Maxwell EM

Consider an example: Maxwell electrodynamics

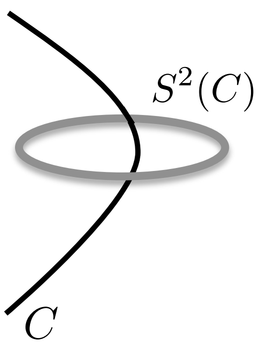
$$\boxed{J^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}}$$

Conserved, by Bianchi:

$$\nabla_\mu J^{\mu\nu} = 0$$

This means that magnetic field lines don't end. $J^{\mu\nu}$ counts magnetic flux density.

Charged line operator is the t'Hooft line. In path integral, demand that



$$\int_{S^2(C)} F = 2\pi$$ on a curve *C*

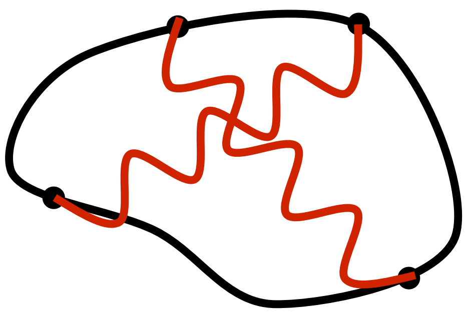
Physically: corresponds to the worldline of a magnetic monopole.

Phases of EM

First, consider EM with no matter:

$$\boxed{S = -\frac{1}{4g^2}\int d^4x\, F^2}$$

This is a free theory. How is the 1-form symmetry realized?
Calculate t'Hooft line, find perimeter law:



$$\langle W_C \rangle \sim \exp(-T_p \mathrm{Perimeter}[C])$$

Symmetry is spontaneously broken:
Goldstone mode is the photon!

$$J = d\tilde{A} \qquad d \star J = 0$$

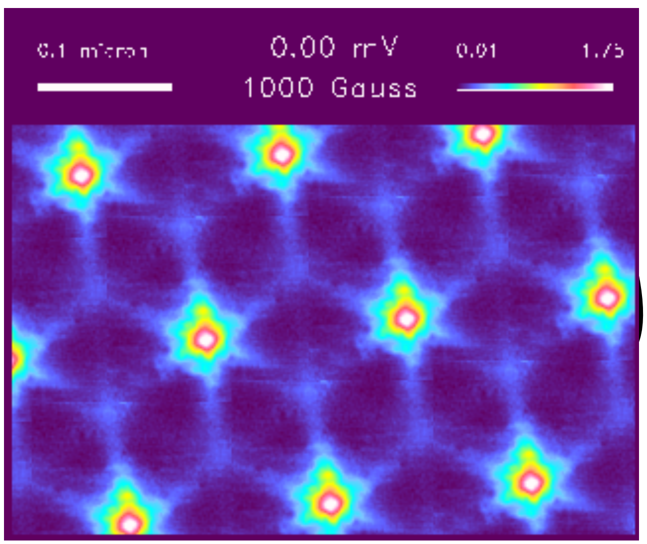
Explains "why" the 4d photon is massless.

Phases of EM

Can instead consider EM in a Higgs/superconducting phase:

$$S = \int d^4x \left[-\frac{1}{4g^2} F^2 + \boxed{\rho^2 (A - d\theta)^2} \right]$$

Low energy spectrum is gapped: no massless particles. Magnetic flux is confined: if you try to make it, it ends up trapped in a heavy Abrikosov vortex.



The tension of the vortex worldsheet implies an area law for the t'Hooft line.

$$\langle W_C \rangle \sim \exp\left(-T_{p+1} \mathrm{Area}[C]\right)$$

Thus the 1-form symmetry in this phase is unbroken.

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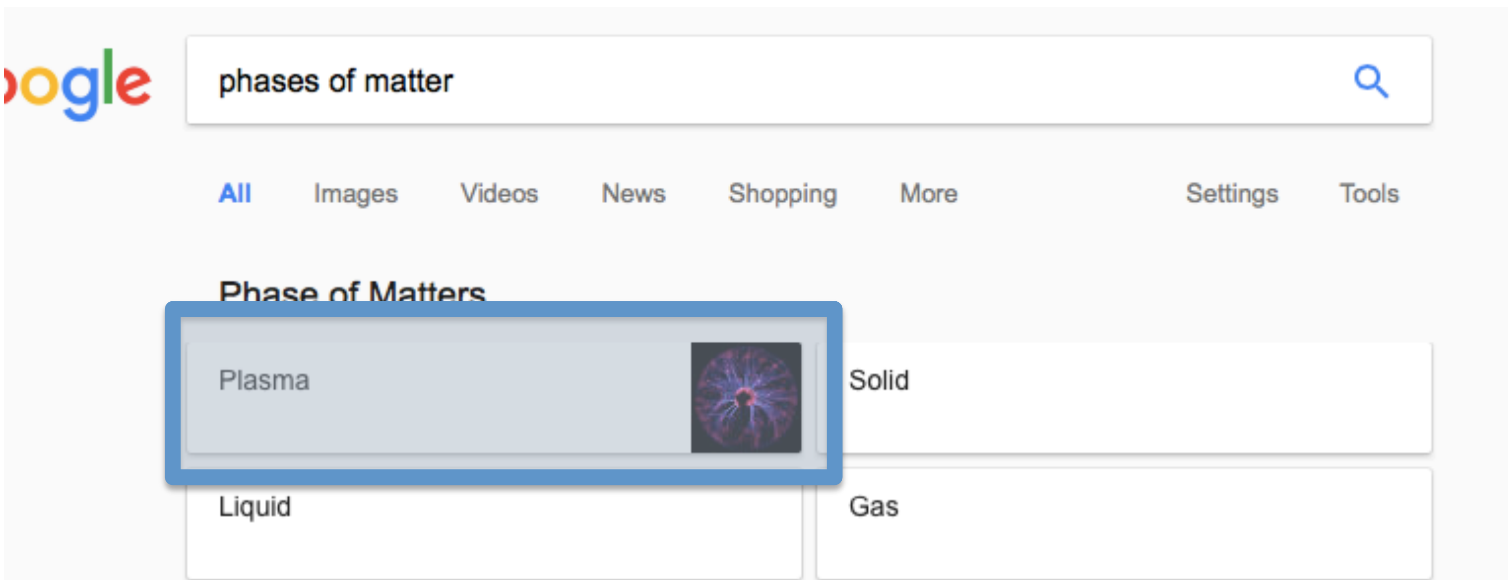
What is magnetohydrodynamics?

Something else you might want to do: place EM at finite temperature. We expect the structure of conserved currents to be important here.

To understand this, first: what are the phases of matter?

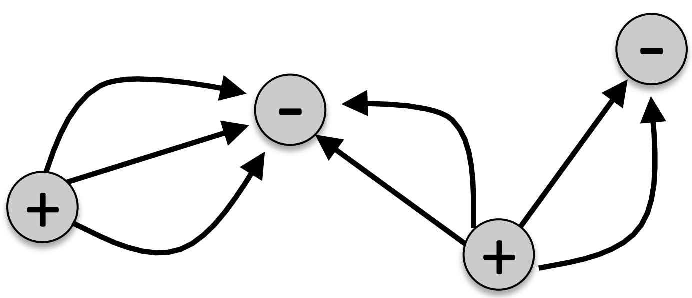


What is plasma? Why is it different from liquid?

Plasma

Textbooks describe Maxwell EM coupled to light electrically charged matter at finite temperature, e.g.

$$S = \int d^4x \left[-\frac{1}{g^2} F^2 + \bar{\psi} \left(\partial\!\!\!/ + A\!\!\!/ \right) \psi \right]$$



Consists of a gas of charges interacting via electric fields.

Symmetries?

$$d \star J = 0 \quad dJ = g^2 \bar{\psi} \gamma^\mu \psi$$

There is no physical symmetry associated with j_{el} ; though this is conserved, it is a gauge current and is not useful in the hydro theory (Debye screening sets it to zero at long distances).

Generalized global symmetries at T > 0

This symmetry structure defines a hydrodynamic theory.

Study hydro with these conserved currents:

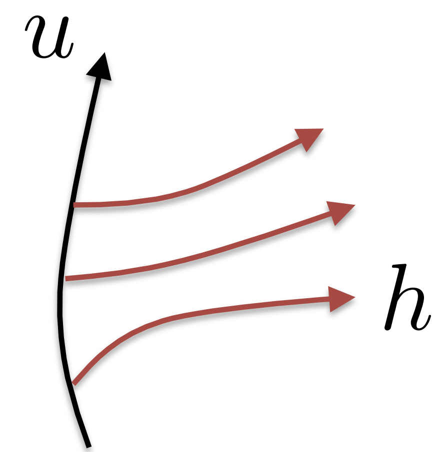
$$T^{(\mu\nu)} \qquad\qquad J^{[\mu\nu]}$$

This "plasma" is the universality class of the theory called magnetohydrodynamics in textbooks.

In the rest of the talk, we will systematically develop this theory from this point of view and compare it with what we find in textbooks. Its structure is completely fixed. I used Maxwell EM to motivate it, but it does not enter into the construction.

Ideal MHD

What are the degrees of freedom?



Two vectors:

$u^\mu(x)$ Fluid velocity.

$h^\mu(x)$ Background magnetic field.

$$u^2 = -1, h^2 = 1, u \cdot h = 0$$

Two scalars:

$T(x)$ Temperature.

$\mu(x)$ Chemical potential (of 2-form charge; not "ordinary" chemical potential).

Currents in ideal MHD

In hydrodynamics, we now systematically build the microscopic currents out of the fluid degrees of freedom in a derivative expansion.

At zeroth order, find:

$$J^{\mu\nu} = 2\rho u^{[\mu} h^{\nu]}$$
$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} - (\mu\rho) h^\mu h^\nu$$

$\rho(\mu, T)$ is the density of magnetic flux.

$\mu\rho$ is the tension in the field lines.

Must specify equation of state $p(\mu, T)$

Equations of motion of ideal MHD

Equations of motion are the conservation equations:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^{\mu\nu} = 0$$

Massage them into

$$\boxed{\nabla_\mu (s u^\mu) = 0}$$

Entropy current is conserved in ideal hydrodynamics.

$$(\epsilon + p) u^\mu \nabla_\mu u^\nu + \nabla^\nu p - \boxed{\nabla_\mu (\mu \rho h^\mu h^\nu)} = 0$$

Compare to Lorentz force law on charged particle:

$$m \ddot{x}^i + \partial_i p = \boxed{(q \vec{v} \times \vec{B})^i}$$

Capturing the same physics, but we never put in Maxwell's equations!

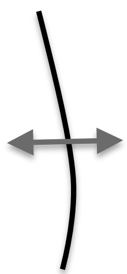
Interlude: traditional MHD

Is this the same as textbook treatments? (Hernandez, Kovtun).

Only if we pick the equation of state to be:

$$p(T,\mu) = p_0(T) + \boxed{\frac{1}{2}g^2\mu^2}$$

g is electromagnetic coupling: this form assumes that the magnetic field and fluid pieces decouple in the stress tensor.



$$\partial_T \partial_\mu p = 0$$

No entropy in magnetic fields.

No symmetry based reason for this to happen -- our formalism does not know about *g*. Can do MHD at strong electromagnetic coupling.

Dissipative magnetohydrodynamics

We now go beyond ideal hydro.

In real life, there is dissipation. Enters at the next order in the derivative expansion:

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \cdots$$

$$u^\mu u^\nu \cdots \qquad \eta \nabla_\mu u_\nu \cdots$$

Similarly for *J*. Coefficient of each tensor structure is a transport coefficient (viscosity, etc.)

Work it out. Find seven transport coefficients:

$$T^{\mu\nu}_{(1)} \sim \left(\zeta_\perp, \zeta_\parallel, \zeta_\times, \eta_\perp, \eta_\parallel\right) \times \nabla u$$

$$J^{\mu\nu}_{(1)} \sim (r_\perp, r_\parallel) \times (\nabla h - db)$$

On resistivity

Note that in "normal" transport, we compute the conductivity microscopically and then invert "by hand" to get resistivity:

$$r = \frac{1}{\sigma}$$

Our formulation is different: resistivity is itself an intrinsic transport coefficient with a universal Kubo formula:

$$\boxed{r = \lim_{\omega \to 0} \frac{1}{i\omega} \langle E_i(\omega) E_i(-\omega) \rangle_R}$$

Not obviously equivalent! (Resum ladder diagrams to show equivalence at weak coupling)
Interesting to understand when these are different.

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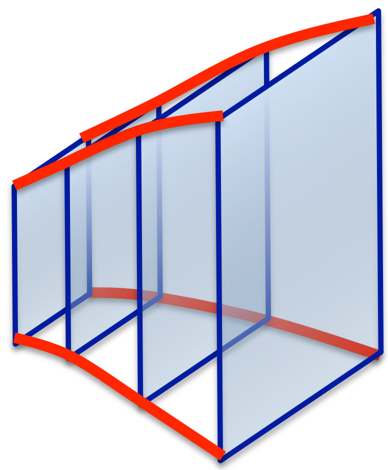
Future directions for MHD

Holographic study of MHD has been started (Grozdanov, Poovuttikul; NI, Hofman). Compute transport coefficients explicitly, check hydro theory. Applications to solids (Grozdanov, Poovuttikul; earlier talk).



Other real-life applications? Consider plasma parameter:

$$\Gamma \sim \frac{\text{Coulomb energy}}{\text{Kinetic energy}}$$

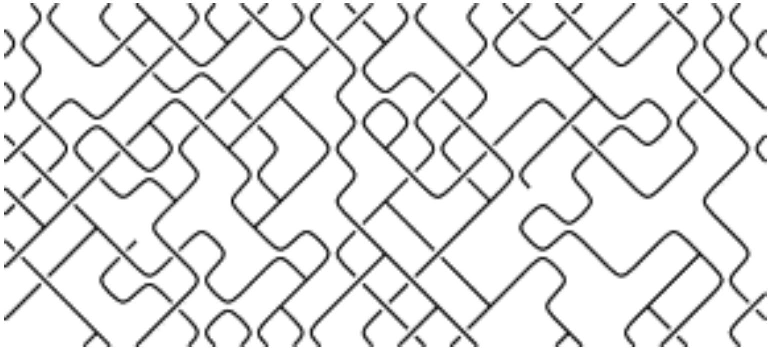
Measures effective strength of electromagnetic interactions. Can be O(100) in “real” life! Our theory may be useful there. Possible applications to astrophysics? (work in progress with S. Gralla).

Future directions for generalized symmetries

In principle, everything that is done with normal global symmetries should now be repeated for generalized ones.

E.g. what is the Goldstone action? (Emergent gauge field, I think.) What happens to Goldstones when we break Lorentz invariance? (See recent work by Armas, Jain for finite T). Anomalies? (Gaiotto, Kapustin, Komargodski, Seiberg…)

More speculatively: is there a connection between generalized symmetry and long-range entanglement (Kitaev, Preskill, Wen) ?



Most speculatively of all: there is one other massless boson in nature. Can the graviton be understood as a Goldstone mode?

Summary

- Generalized global symmetries enforce the conservation of higher-dimensional objects.
- Do everything that normal symmetries do: identify order parameters, spontaneously break, etc.
- If spontaneously broken, they result in gapless Goldstone modes: the photon in our universe can be understood in this way.
- This permits a reformulation of magnetohydrodynamics from a symmetry-based point of view.

The End